Constraining neutron star equation of state from astrophysics, high and low energy nuclear physics

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I Abstract

We apply the novel equation of state, which includes the surface tension contribution induced by the interparticle interaction and the asymmetry between neutrons and protons, to the study of neutron star properties. This high-quality equation of state is obtained from the virial expansion for the multicomponent particle mixtures that takes into account the hard-core repulsion between them. The considered model is in full concordance with all the known properties of normal nuclear matter, provides a high quality description of the proton flow constraints, hadron multiplicities created during the nuclear-nuclear collision experiments and equally is consistent with astrophysical data coming from neutron star observations. The analysis suggests that the best model parametrisation gives the incompressibility factor K_0, symmetry energy J and symmetry energy slope L at normal nuclear density equal to 200 MeV, 30 MeV and $113.28 - 114.91$ MeV, respectively. The found mass-radius relation for neutron stars computed with this equation of state is consistent with astrophysical observations.

II Model

We consider a multicomponent version of the quantum generalization of the equation of state (EoS) within the surface tension induced by particle interaction (IST) [9, 3]. Recently, the IST EoS was used to describe the experimental data of hadron multiplicities measured at AGS, SPS, RHIC and LHC energies of nuclear collisions [10], as well as their nuclear matter properties [9]. In this work the focus is on the application of IST EoS to the study of neutron star (NS) properties.

Attraction between particles - the mean filed approach.
Degrees of freedom - neutrons, n, protons, p, and electrons, e, that characterize by masses m_i and chemical potentials μ_i, where $i = n, p, e$.
We neglect the Coulomb interaction of electrically charged particles and treat electrons as point like ($R_e = 0$) and noninteracting ones whereas nucleons have the same hard-core radius R.
The phenomenological EoS with the mean filed interaction between particles has the form of the system of two coupled equations for pressure p and IST coefficient σ

$$p = \sum_{i=n,p} p_i^{id}(\mu_i - pv_i - \Sigma s_i + U_{at} \pm U_{sym}) + p^{id}(\mu_e) - p_{at} + p_{sym}\,, \quad (1)$$

$$\Sigma = \sum_{i=n,p} p_i^{id}(\mu_i - pv_i - \alpha\Sigma s_i) R_i\,. \quad (2)$$

p^{id}, n^{id} - the pressure and density of non interacting Fermi particles with spin $\frac{1}{2}$ and quantum degeneracy $g = 2$. If $\mu \geqslant m$, then

$$p^{id}(m,\mu) = \frac{\mu k(2\mu^2 - 5m^2) + 3m^4 \ln\frac{\mu+k}{m}}{24\pi^2}\theta(\mu - m)\,, \quad (3)$$

$$n^{id}(m,\mu) = \frac{\partial p^{id}(m,\mu)}{\partial \mu} = \frac{k^3}{3\pi^2}\theta(m-\mu)\,, \quad (4)$$

where $k = \sqrt{\mu^2 - m^2}$ is the Fermi momentum. Otherwise, $p^{id}(m,\,\mu) = 0$ and $n^{id}(m,\,\mu) = 0$.
Due to the mean filed interaction of neutrons and protons the physical chemical potentials of these particles μ_i are changed to the effective ones ν_i^1 and ν_i^2 due to mean filed interaction through the density dependent interaction potential U [8] and hard-core repulsion accounted by the nucleon eigen volume $v = \frac{4}{3}\pi R^3$ and surface $s = 4\pi R^2$.

$$\nu_i^1 = \mu_i - pv - \sigma s + U(n_B^{id}) \mp U^{sym}(n_B^{id}, I^{id})\,, \quad (5)$$

$$\nu_i^2 = \mu_i - pv - \alpha\sigma s + U_0\,, \quad (6)$$

where dimensionless parameter $\alpha > 1$ [9].
$\alpha = 1.245$ reproduces 4 virial coefficients and provides to the widest causality range [10].
The NS core was modelled within the IST EoS, while its crust was described via the polytropic EoS with $\gamma = \frac{4}{3}$.

Attraction and asymmetry terms

Condition of thermodynamic consistency of model with the mean-filed interaction requires the special relation between the interaction pressure and potential [2, 3, 8]

$$p_{int}(n_B^{id}) = \int_0^{n_B^{id}} dn\, n\, \frac{\partial U(n)}{\partial n}\,, \quad (7)$$

which allows us to define p_{int} if U is known. In this work we use parameterization of the interaction potential suggested in Ref. [2]

$$U(n) = -C_d^2 n^{\varkappa}\,, \quad p_{int}(n) = -\frac{\varkappa}{\varkappa+1} C_d^2 n^{\varkappa+1}\,. \quad (8)$$

This parameterization provides causal behavior of the model EoS at high densities.
Asymmetry between nucleons and protons depends on the baryon charge density n_B^{id} and asymmetry parameter $I^{id} = (n^{id}(m,\nu_n^1) - n^{id}(m,\nu_p^1))/n_B^{id}$ of ideal gas. The requirement of thermodynamic consistency:

$$p^{sym}(n_B^{id}, I^{id}) = I^{id} \int_0^{n_B^{id}} dn\, n\, \frac{\partial U^{sym}(n, I^{id})}{\partial n}\,, \quad (9)$$

$$p^{sym}(n_B^{id}, I^{id}) = n_B^{id} \int_0^{I^{id}} dI\, I\, \frac{\partial U^{sym}(n_B^{id}, I)}{\partial I}\,. \quad (10)$$

Conditions (9) and (10) are satisfied simultaneously only if $p^{sym} = p^{sym}(n_B^{id} I^{id})$ and $U^{sym} = U^{sym}(n_B^{id} I^{id})$, thus

$$p^{sym}(n_B^{id} I^{id}) = n_B^{id} I^{id} \int_0^{n_B^{id} I^{id}} dn\, n\, \frac{\partial U^{sym}(n)}{\partial n}\,. \quad (11)$$

In this work we parametrize the symmetry energy pressure as $p^{sym}(n) = \frac{A^{sym} n^2}{1+(B^{sym} n)^2}$, where A^{sym} and B^{sym} are constants fitted to the values of J and L.
Electro neutrality or equivalently zero net density of the electric charge $n_Q = n_p - n_e = 0$. Beta equilibrium: processes $n \leftrightarrow p + e$ is accounted as $\mu_n = \mu_p + \mu_e$. Note that neutrino contribution is neglected. The energy density of electrically neutral equilibrated mixture

$$\varepsilon = \sum_{i=n,p,e} \mu_i n_i - p = \mu_n n_B - p\,, \quad (12)$$

This expression implicitly defines dependence of the pressure on energy density which is required to solve the Tolman-Oppenheimer-Volkoff (TOV) equation in the closed form.

III Constrains on EoS

Nuclear matter properties:

- Dynamical equilibrium of nuclei and vacuum;
- zero pressure p = 0, normal nuclear density $n_0 \simeq 0.16$ fm^{-3};
- binding energy per nucleon $\simeq 16$ MeV;
- incompressibility factor = 200 - 260 MeV;
- symmetry energy at saturation density $E_{sym}(n = n_B) \equiv J = 30 \pm 4MeV$ and slope $L = 3n\frac{\partial E_{sym}}{\partial n}$ = 20 - 115 MeV.

HEP properties:

- Flow constraint;
- hadron multiplicities.

Astro/Gravity:

- PSR J0348-0432 pulsar: $\mathbf{M_{max} = 2.01(4) M_\odot}$;
- constraints from NS - ... binary systems observation;
- tidal deformability of NS - ... binary components. Love numbers from GW170817 observation.

General requirements:

- causality;
- thermodynamic consistency;
- physical interaction between constituents;
- multicomponent character (n, p, e, ...);
- electric neutrality and β equilibrium.

IV IST origin and effect on EoS

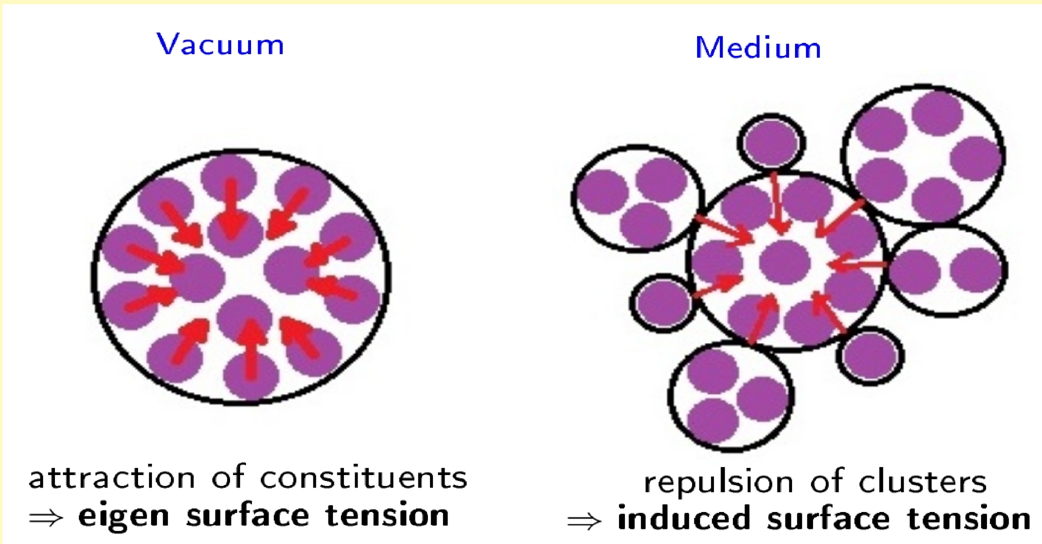

Fig.1. IST appears due to repulsion of particles in different clusters.

- Hard core repulsion only in part is accounted by eigen volume
- The rest corresponds to surface tension and curvature tension Curvature tension can be accounted explicitly or implicitly
- Physical clusters tend to have spherical (in average) shape

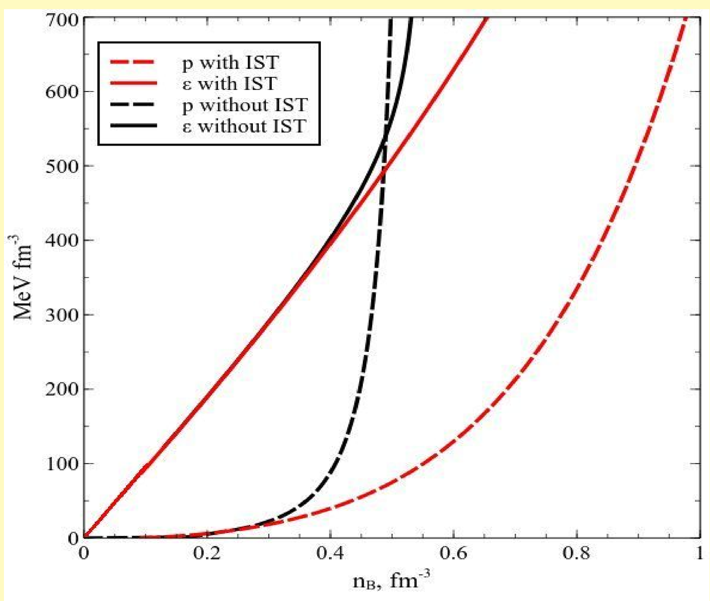

Fig.2. Pressure and energy density for the EoS within the IST (solid curves) and without the IST (dashed curves).

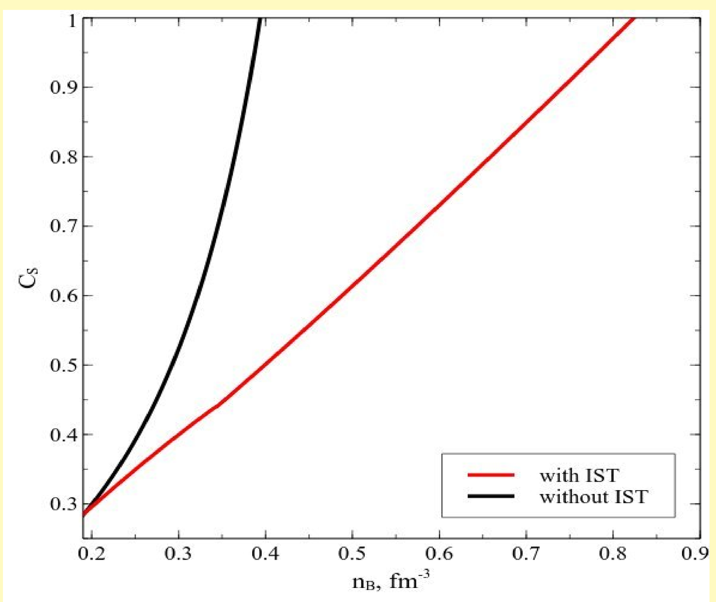

Fig.3. Speed of sound for neutral (protons, neutrons and electrons) and symmetric (protons and neutrons) matter with/without IST.

V Results and discussion

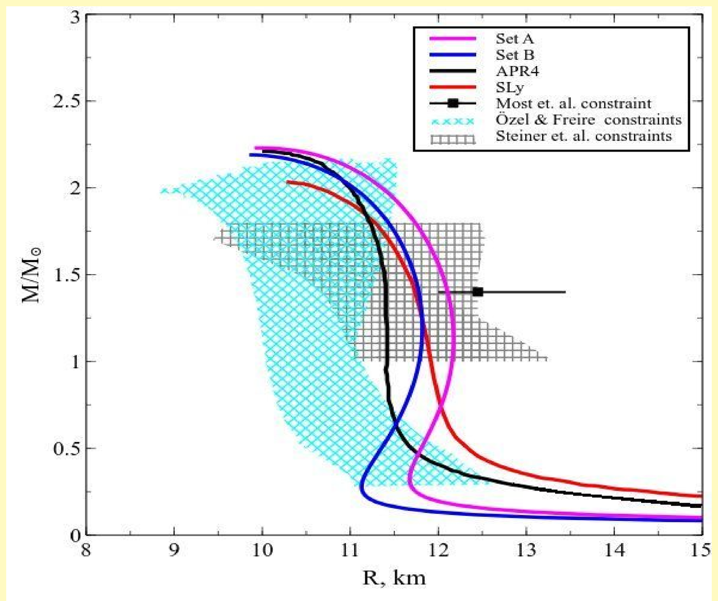

Fig.5. Gravitational mass-radius relation for IST EoS with two sets of model parameters. Constraints are taken from [6, 13].

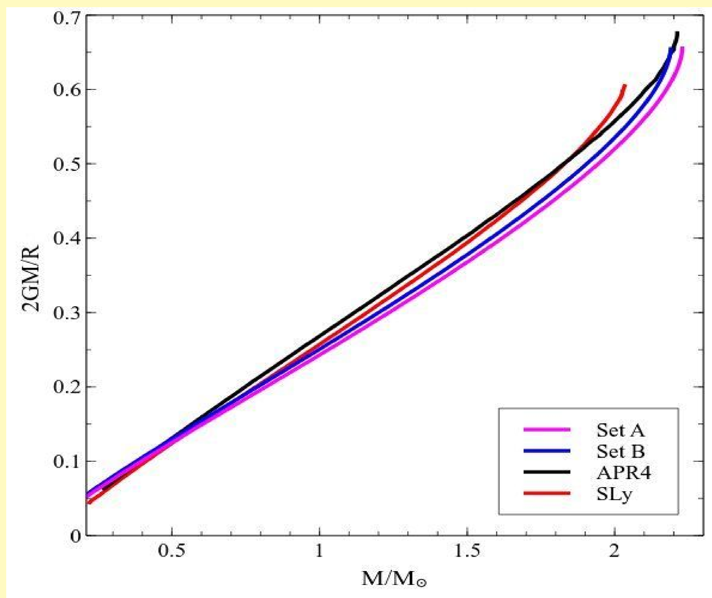

Fig.6. The NS compactness $\frac{2GM}{R}$ as a fucntion of its mass M.

Analysis of the NS mass-radius relation and the star compactness performed for the present EoS, APR4 and SLy EoSs allows us to conclude that the IST EoS is able to describe the NS-NS binary merger data [1].

The flow constraint taken from high-energy nuclear collisions for symmetric baryon matter [4] corresponds to the grey shaded area on Figure 6.

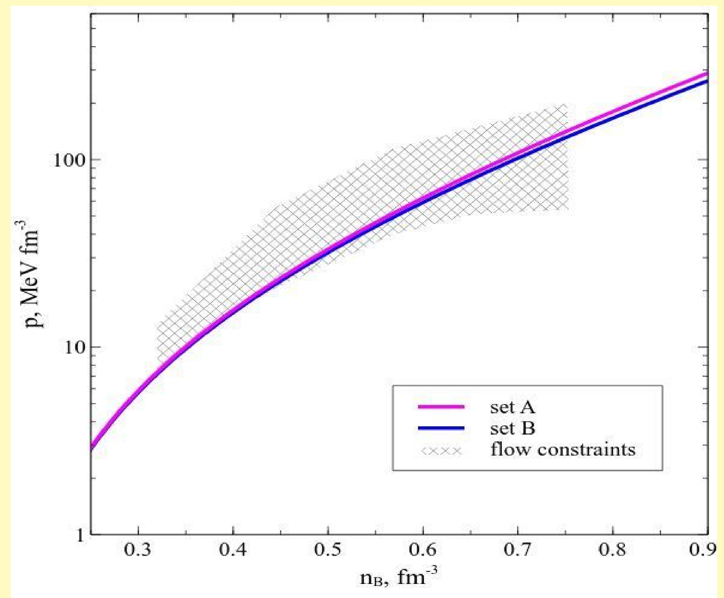

Fig.7. Dependence of the pressure p on the baryonic density n_B for the defined set of parameters of the IST model. The red and green curves correspond to the curves of the same colours in Table 1.

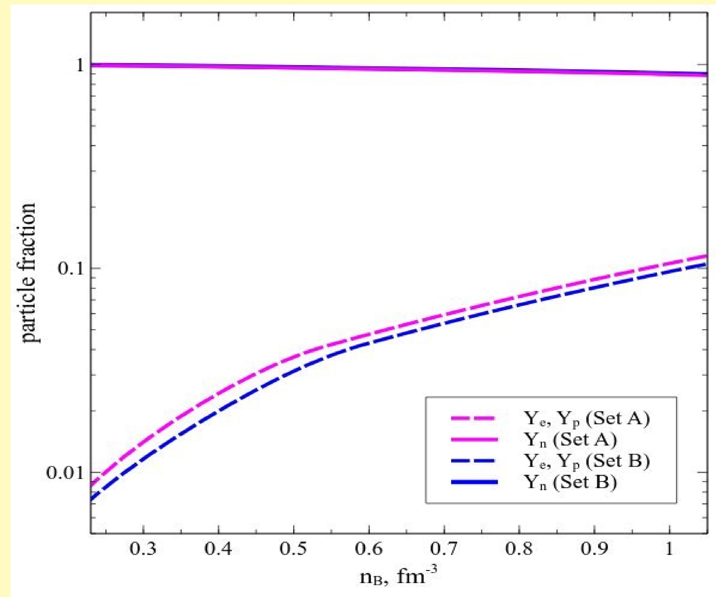

Fig.8. Fraction of the electrons, protons and neutrons for the A and B sets of parameters

VI Conclusions

- Using a novel thermodynamically self-consistent IST EoS the properties of the NS at zero-temperature limit were calculated;
- it was shown that the present EoS can be successfully applied to the description of the hadron multiplicities measured in A+A collisions, to studies of the nuclear matter phase diagram and to modelling of the NS interiors;
- IST EoS satisfies all astrophysical constraints, correctly reproduce properties of normal nuclear density, proton flow data, hadron multiplicities measured in A+A collisions and nuclear matter properties near the (3)CEP.

Model parameters

Sets	r fm	α –	$\varkappa$ –	B^{sym} fm^3	A^{sym} $MeV \cdot fm^3$	C_d^2 $MeV \cdot fm^{3\varkappa}$	U_0 MeV	K_0 MeV	J MeV	L MeV	M_{max} $M_\odot$
A (magenta curve on Fig. 5)	0.492	1.245	0.263	3.5	16.896	143.564	147.456	200.03	30.0	114.91	2.229
B (blue curve on Fig. 5)	0.484	1.245	0.26	4.5	14.762	144.042	150.97	200.00	30.0	113.28	2.189

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