Heavy quarks in medium (from the lattice)[1]

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overall summary

what are we talking about?

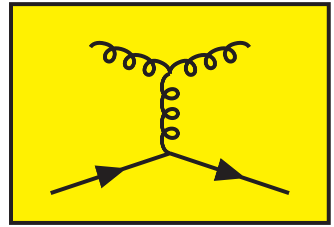
$\Rightarrow$ charm and bottom quarks at $T \sim 150...450$ MeV

$\Rightarrow$ non-equilibrium: gluons and N_{f} light quarks are thermalized, charm and bottom quarks are probes in this background

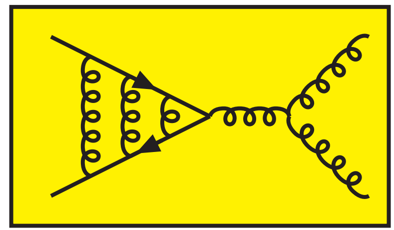
$\Rightarrow$ bottom quark is non-relativistic ($m_b \sim 10...30\ T$), charm quark is a borderline case ($m_c \sim 2...6\ T$)

conceptual motivations

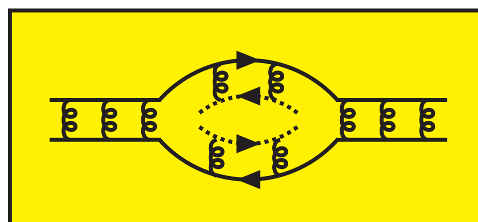
kinetic equilibration rate:
how fast does velocity adjust to hydrodynamic flow?



chemical equilibration rate:
how fast does number density adjust to Boltzmann weight?



quarkonium dissociation:
do $q\bar{q}$ states propagate as scattering or bound states?



phenomenological motivations

kinetic equilibration rate:
D mesons show large v_2 in heavy ion collisions

chemical equilibration rate:
would we have $N_\mathrm{f} = 2 + 2$ in future experiments?

bottomonium dissociation:
precision Υ studies in the LHC era

charmonium dissociation:
eternal questions on the fate of J/ψ

$\Rightarrow$ can we understand ingredients for these from lattice QCD?

general challenges

$\Rightarrow$ "usual" lattice systematics:

statistical signal for suppressed observables; finite-volume effects; continuum limit; topological freezing; non-perturbative renormalization; light dynamical sea quarks; ...

$\Rightarrow$ specific issues for thermal real-time rates:

- "easy": derivation of Kubo relations, i.e. expressing non-equilibrium physics in terms of equilibrium two-point correlators
- "exponentially hard": analytic continuation from euclidean to real time, particularly if spectral function contains narrow peaks[2]

[2] G. Cuniberti, E. De Micheli and G.A. Viano, *Reconstructing the thermal Green functions at real times from those at imaginary times*, cond-mat/0109175.

focus of this talk

- "exponentially hard": analytic continuation from euclidean to real time, particularly if spectral function contains narrow peaks[2]

[2] G. Cuniberti, E. De Micheli and G.A. Viano, *Reconstructing the thermal Green functions at real times from those at imaginary times*, cond-mat/0109175.

possible ways to facilitate analytic continuation

kinetic equilibration rate:

make use of HQET, reducing the observable to a gluonic correlator: the latter is believed to have a "flat" spectral shape

chemical equilibration rate:

make use of NRQCD, reducing the observable to a purely static measurement: no need for analytic continuation!

bottomonium dissociation:

make use of NRQCD or real-time potential models ($\sim$ pNRQCD), to remove contribution from a transport peak

charmonium dissociation:

look at the pseudoscalar (η_c) rather than vector channel (J/ψ), to remove contribution from a transport peak

kinetic equilibration rate

summary of recent developments

in full QCD there is a narrow $\sim \frac{\alpha^2 T^2}{M}$ transport peak[3]

accessible in HQET because can zoom inside the peak[4]

measurable with multilevel and other advanced techniques[5]

perturbative renormalization available up to NLO[6]

continuum limit can be taken within the quenched theory[7]

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[3] P. Petreczky and D. Teaney, *Heavy quark diffusion from the lattice*, hep-ph/0507318.

[4] J. Casalderrey-Solana and D. Teaney, *Heavy quark diffusion in strongly coupled $\mathcal{N} = 4$ Yang-Mills*, hep-ph/0605199; S. Caron-Huot, ML and G.D. Moore, *A Way to estimate the heavy quark thermalization rate from the lattice*, 0901.1195.

[5] H.B. Meyer, *The errant life of a heavy quark in the quark-gluon plasma*, 1012.0234; D. Banerjee, S. Datta, R. Gavai and P. Majumdar, *Heavy Quark Momentum Diffusion Coefficient from Lattice QCD*, 1109.5738.

[6] C. Christensen and ML, *Perturbative renormalization of ...*, 1601.01573.

[7] A. Francis *et al*, *Non-perturbative estimate of the heavy quark ...*, 1508.04543.

observables of interest

diffusion equation for conserved number density:

$$\partial_t n = D\nabla^2 n + \mathcal{O}(\nabla^4 n) \ .$$

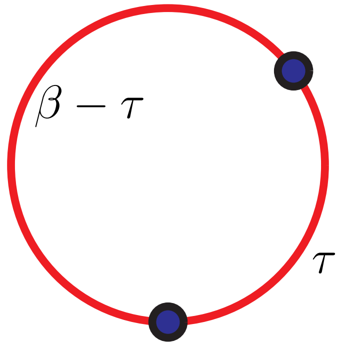
diffusion equation for single particle momentum:

$$\dot{k}_i = -\Gamma_{\rm kin}\, k_i + \xi_i \ , \quad \langle\langle \xi_i(t)\, \xi_j(t') \rangle\rangle = \kappa\, \delta_{ij}\, \delta(t-t') \ .$$

relations: $D = 2T^2/\kappa$, $\Gamma_{\text{kin}} = \kappa/(2MT)$.

purely gluonic formulation for κ

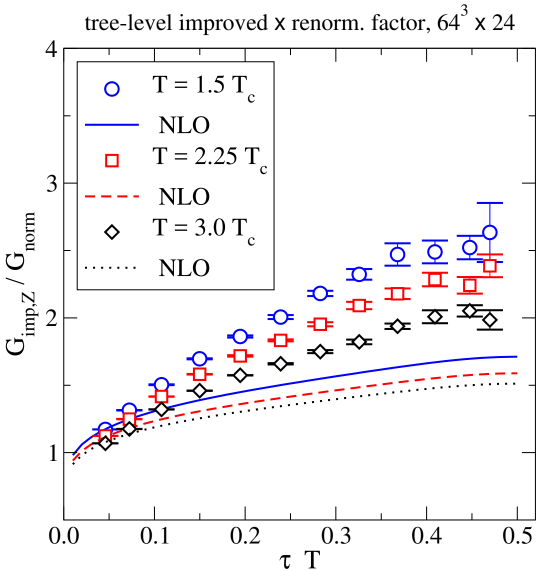
$$G_{\rm E}(\tau) = -\frac{1}{3}\sum_{i=1}^{3} \frac{\langle {\rm Re\,Tr}[U_{\beta;\tau}\, gE_i(\tau)\, U_{\tau;0}\, gE_i(0)]\rangle}{\langle {\rm Re\,Tr}[U_{\beta;0}]\rangle}\;.$$



$$\kappa \equiv \lim_{\omega\to 0} \frac{2T\rho_{\rm E}(\omega)}{\omega}\;.$$

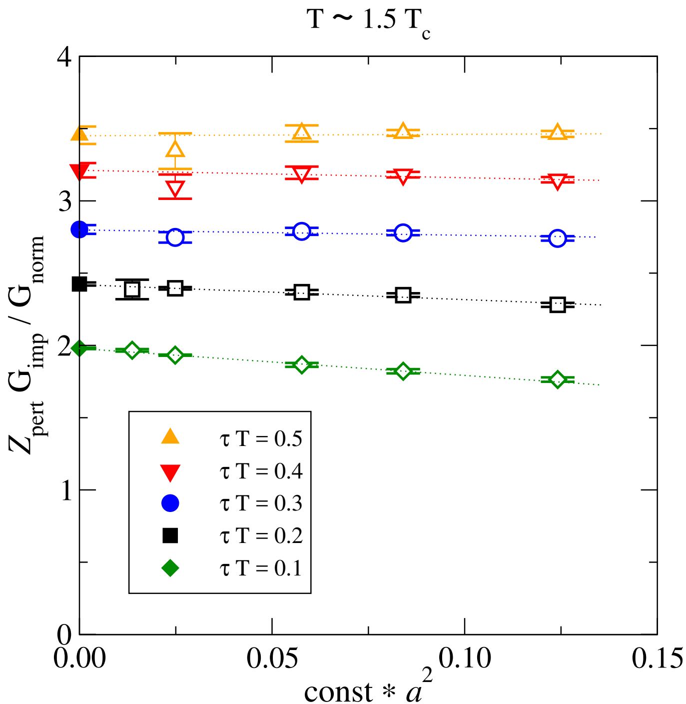
quenched measurements yield a signal for $G_{\mathrm{E}}(\tau)$

with "multilevel algorithm" and "tree-level improvement":



clear enhancement at large time separations

extrapolation to the continuum limit



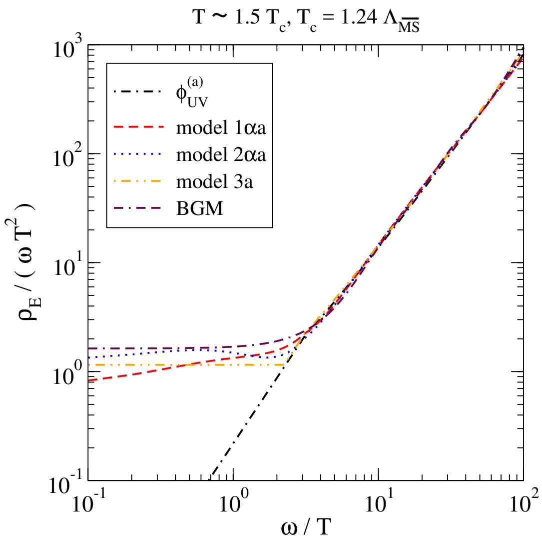
spectral shape is well constrained in the continuum limit

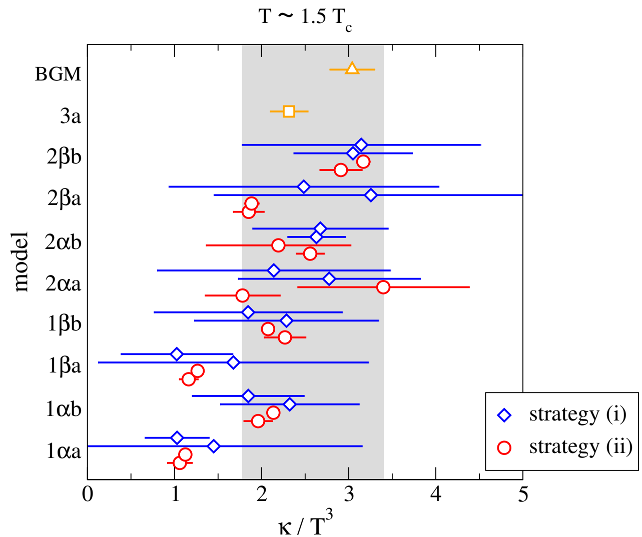
$$\phi_{\rm IR}(\omega) \;\equiv\; \frac{\kappa\omega}{2T} \;.$$

$$\phi^{(a)}_{\rm UV}(\omega) \;\equiv\; \frac{g^2(\bar\mu_\omega) C_{\rm F}\, \omega^3}{6\pi} \;, \quad \bar\mu_\omega \equiv \mathsf{max}(\omega, \pi T) \;.$$

$$\rho^{(2\mu i)}_{\rm E}(\omega) \;\equiv\; \Big[1 + \sum_{n=1}^{n_{\rm max}} c_n e^{(\mu)}_n(y)\Big] \sqrt{[\phi_{\rm IR}(\omega)]^2 + [\phi^{(i)}_{\rm UV}(\omega)]^2} \;.$$

fitting with such interpolations[8] yields $\kappa > T^3$





$$\kappa = (1.8 - 3.4)\, T^3 \,.$$

[8] final estimate based on models 2∗, 3a, BGM because 1∗ has wrong subleading UV-tail

convert to physical results at $T \approx 1.5T_{\rm c}$

$$
\begin{aligned}
2\pi D\, T &= 3.7\ldots6.9\ , \\
\tau_{\rm kin} = \frac{1}{\Gamma_{\rm kin}} &= (1.8\ldots3.4)\, \Big(\frac{T_{\rm c}}{T}\Big)^2 \Big(\frac{M}{1.5\ \text{GeV}}\Big)\ \text{fm/c}\ .
\end{aligned}
$$

$\Rightarrow$ close to $T_{\rm c}$, charm quark kinetic equilibration could be almost as fast as that of light partons

$\Rightarrow$ to be tackled: better statistical precision, non-perturbative renormalization, unquenching

chemical equilibration rate

summary of recent developments

in full QCD there is a very narrow $\sim e^{-M/T}$ transport peak[9]

easier through NRQCD because annihilation operators known[10]

in fact no issue with analytic continuation in NRQCD[11]

physics could be relevant for future colliders[12]

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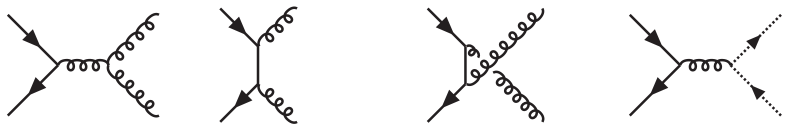
9 D. Bödeker and ML, *Heavy quark chemical equilibration rate as a transport coefficient*, 1205.4987; Y. Burnier and ML, *Charm mass effects in bulk channel correlations*, 1309.1573.

10 G.T. Bodwin, E. Braaten and G.P. Lepage, *Rigorous QCD analysis of inclusive annihilation and production of heavy quarkonium*, hep-ph/9407339; D. Bödeker and ML, *Sommerfeld effect in heavy quark chemical equilibration*, 1210.6153.

11 S. Kim and ML, *Rapid ... co-annihilation through bound states in QCD*, 1602.08105.

12 ML and K. Sohrabi, *Charm contribution to bulk viscosity*, 1410.6583.

physical picture (time runs in either direction)



energy released in the inelastic reaction is $2M \gg T \Rightarrow$ the "hard" annihilation process is effectively **local**

soft effects are encoded in the thermal expectation value of a 4-particle operator ("$\mathcal{M}^*\mathcal{M}$") describing the hard process[13]

[13] e.g. L.S. Brown and R.F. Sawyer, *Nuclear reaction rates in a plasma*, astro-ph/9610256.

the idea can be implemented within NRQCD

if θ, η annihilate q and $\bar{q}$ then, like in the optical theorem, decays are contained in an imaginary part of a 4-particle operator:

$$\mathcal{O} = \frac{i c_1 \alpha^2 \, \theta^\dagger \eta^\dagger \, \eta \theta}{M^2} + O(\alpha^3, v^2)$$

$\Rightarrow$ a linear response analysis yields

$$n_{\text{eq}} \Gamma_{\text{chem}} = \frac{8 c_1 \alpha^2}{M^2} \frac{1}{\mathcal{Z}} \sum_m e^{-E_m/T} \langle m | \theta^\dagger \eta^\dagger \, \eta \theta | m \rangle \ .$$

$\Rightarrow$ denote by $\bar{S}_i$ enhancement factor over pQCD in channel i

thermal expectation values in explicit form

G^θ = propagator, α, γ = colour indices, i, j = spin indices

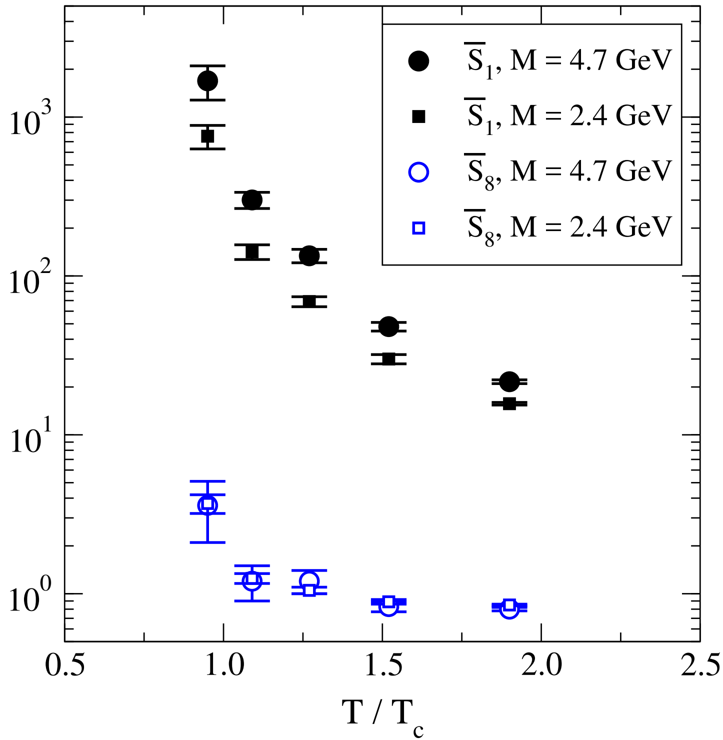
$$P_1 \equiv \frac{1}{2N_c} \mathrm{Re}\, \langle G^\theta_{\alpha\alpha;ii}(\beta, \mathbf{0}; 0, \mathbf{0}) \rangle \;,$$

$$P_2 \equiv \frac{1}{2N_c} \langle G^\theta_{\alpha\gamma;ij}(\beta, \mathbf{0}; 0, \mathbf{0}) G^{\theta\dagger}_{\gamma\alpha;ji}(\beta, \mathbf{0}; 0, \mathbf{0}) \rangle \;,$$

$$P_3 \equiv \frac{1}{2N_c^2} \langle G^\theta_{\alpha\alpha;ij}(\beta, \mathbf{0}; 0, \mathbf{0}) G^{\theta\dagger}_{\gamma\gamma;ji}(\beta, \mathbf{0}; 0, \mathbf{0}) \rangle$$

$$\Rightarrow \quad \bar{S}_1 = \frac{P_2}{P_1^2} \,, \quad \bar{S}_8 = \frac{N_c^2 P_3 - P_2}{(N_c^2 - 1) P_1^2} \,.$$

enhanced "singlet" decays, perhaps through bound states



implication for heavy ion collisions

the process splits into "colour-singlet" and "colour-octet" parts

$$\Gamma_{\text{chem}} \approx \frac{g^4 C_{\rm F}}{8\pi M^2}\left(\frac{MT}{2\pi}\right)^{3/2} e^{-M/T}$$

$$\times \left[\frac{1}{N_{\rm c}}\bar{S}_1 + \left(\frac{N_{\rm c}^2-4}{2N_{\rm c}} + N_{\rm f}\right)\bar{S}_8\right] .$$

for charm: $\bar{S}_8 \simeq 0.8$ is weighted more than $\bar{S}_1 \simeq 15$

$\Rightarrow \Gamma_{\text{chem}}^{-1} \sim 150$ fm/c at $T \approx 400$ MeV,
$\Gamma_{\text{chem}}^{-1} \sim 40$ fm/c at $T \approx 600$ MeV

quarkonium dissociation

summary of recent developments

bottomonium: detailed studies with effective theories

$\Rightarrow$ several lattice spacings with lattice NRQCD[14]

$\Rightarrow$ towards phenomenology with real-time potential models[15]

charmonium: look at pseudoscalar channel (no transport peak)[16]

$\Rightarrow$ unquenched lattice QCD at finite lattice spacing:[17] "*up to* $1.4T_\mathrm{c}$ *no significant variation is seen in the pseudoscalar channel.*"

$\Rightarrow$ continuum limit in quenched QCD:[18] no peaks above T_c?

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14 e.g. S. Kim *et al*, *Lattice NRQCD study of ... bottomonium states ...*, 1409.3630.

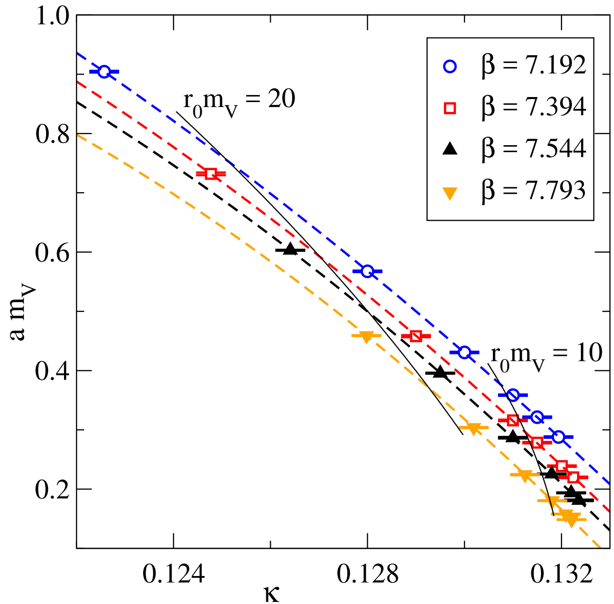
15 e.g. Y. Burnier *et al*, *... realistic phenomenology from first principles*, 1509.07366.

16 G. Aarts *et al*, *... meson spectral functions at ... high temperature*, hep-lat/0507004.

17 S. Borsányi *et al*, *Charmonium ... from 2+1 flavour lattice QCD*, 1401.5940.

18 A.-L. Kruse *et al*, *Thermal quarkonium ... in the pseudoscalar channel*, 1709.07612.

essential for continuum limit is vacuum mass measurement



$r_0 \simeq 0.5$ fm

$r_0 m_{\mathrm{V}} = 20$

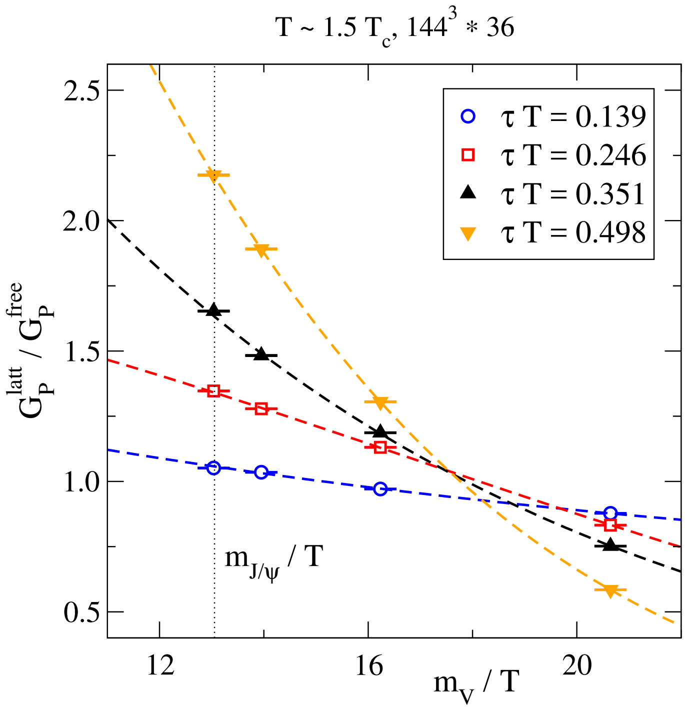
$\Leftrightarrow m_{\mathrm{V}} \simeq 8$ GeV

$r_0 m_{\mathrm{V}} = 10$

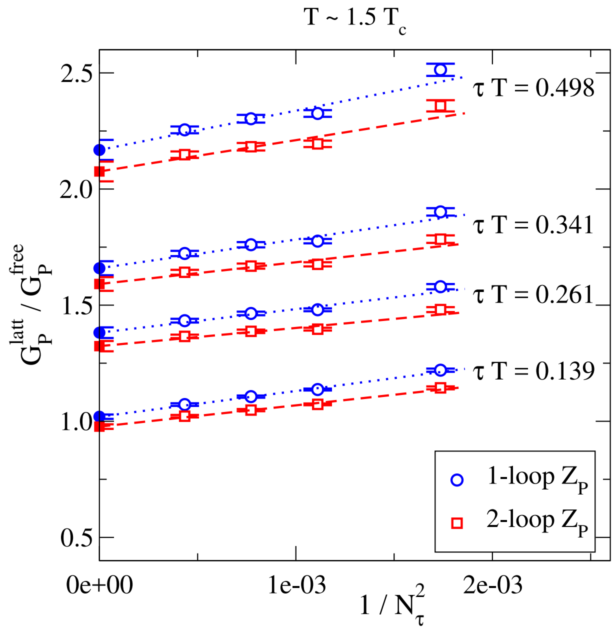
$\Leftrightarrow m_{\mathrm{V}} \simeq 4$ GeV

dependences are smooth $\Rightarrow$ obtain "lines of constant physics"

interpolate mass to physical point in thermal correlator



continuum limit after perturbative renormalization[19]



⇒ treat difference of 1- and 2-loop factors as an error estimate

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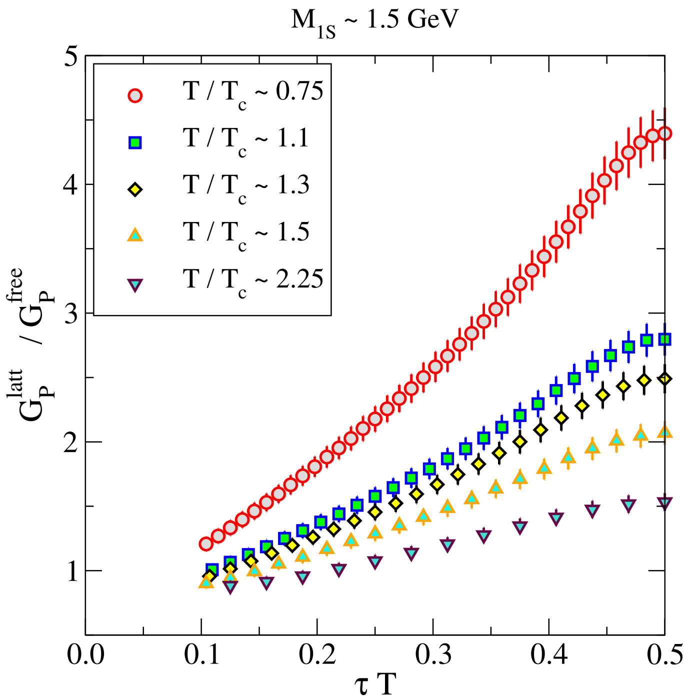
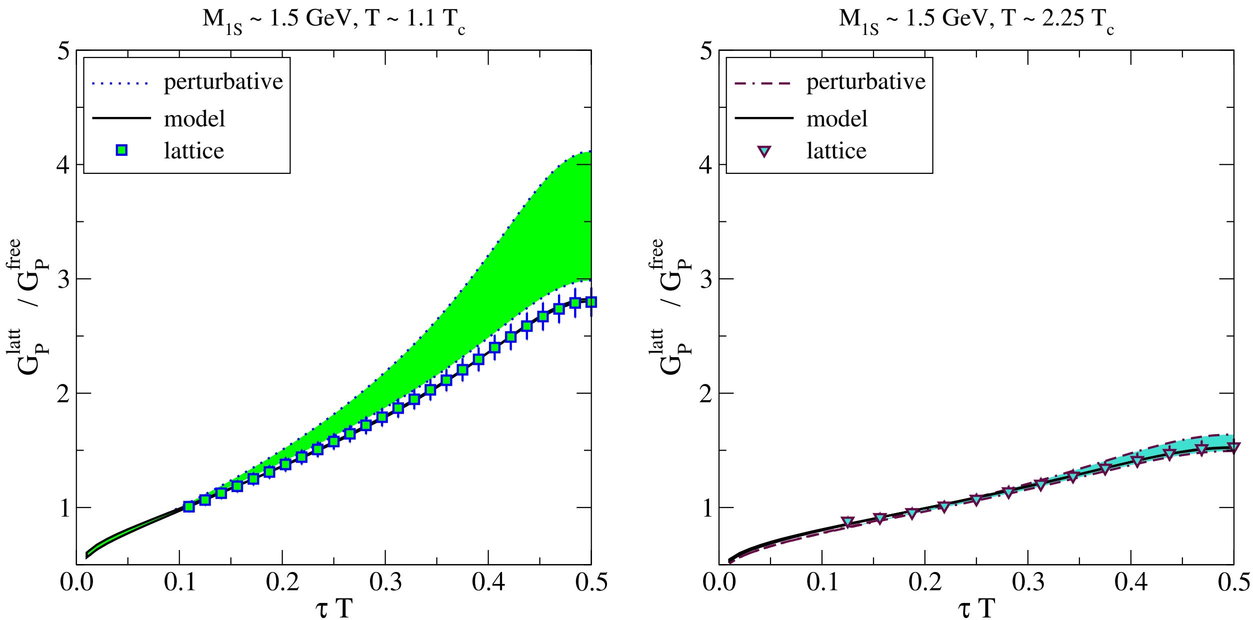
[19] S. Capitani *et al*, *Renormalization and off-shell improvement in lattice perturbation theory*, hep-lat/0007004; A. Skouroupathis and H. Panagopoulos, *Two-loop renormalization of scalar and pseudoscalar fermion bilinears on the lattice*, 0707.2906.

final results at different temperatures

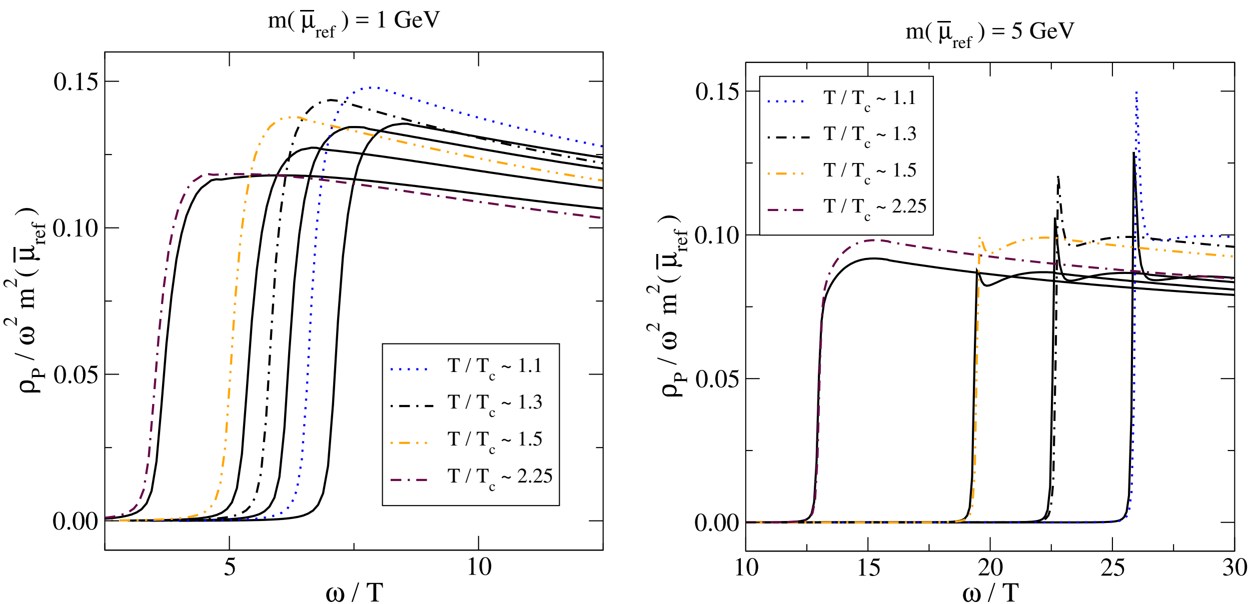


results differ from pQCD but this can be understood



$$\rho_{\mathrm{P}}^{\text{model}}(\omega) \;\equiv\; A\,\rho_{\mathrm{P}}^{\text{pert}}(\omega + B)\;.$$

best-fit spectral functions (pQCD vs model)



⇒ charmonium: threshold shifts compared with pQCD, no peak

⇒ bottomonium: one peak present up to $1.5T_c$

conclusions

⇒ great playground for theoretical and numerical progress

⇒ phenomenological comparisons are also possible[20]

⇒ it is worth identifying observables (perhaps through EFTs) for which problems related to analytic continuation are alleviated

⇒ lattice systematics remains to be scrutinized (finite-volume effects, continuum limit, unquenching, topological freezing)

⇒ there's room for progress, and it's worth going on!

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[20] for a review cf. e.g. G. Aarts *et al.*, *Heavy-flavor production and medium properties in high-energy nuclear collisions - What next?*, 1612.08032.