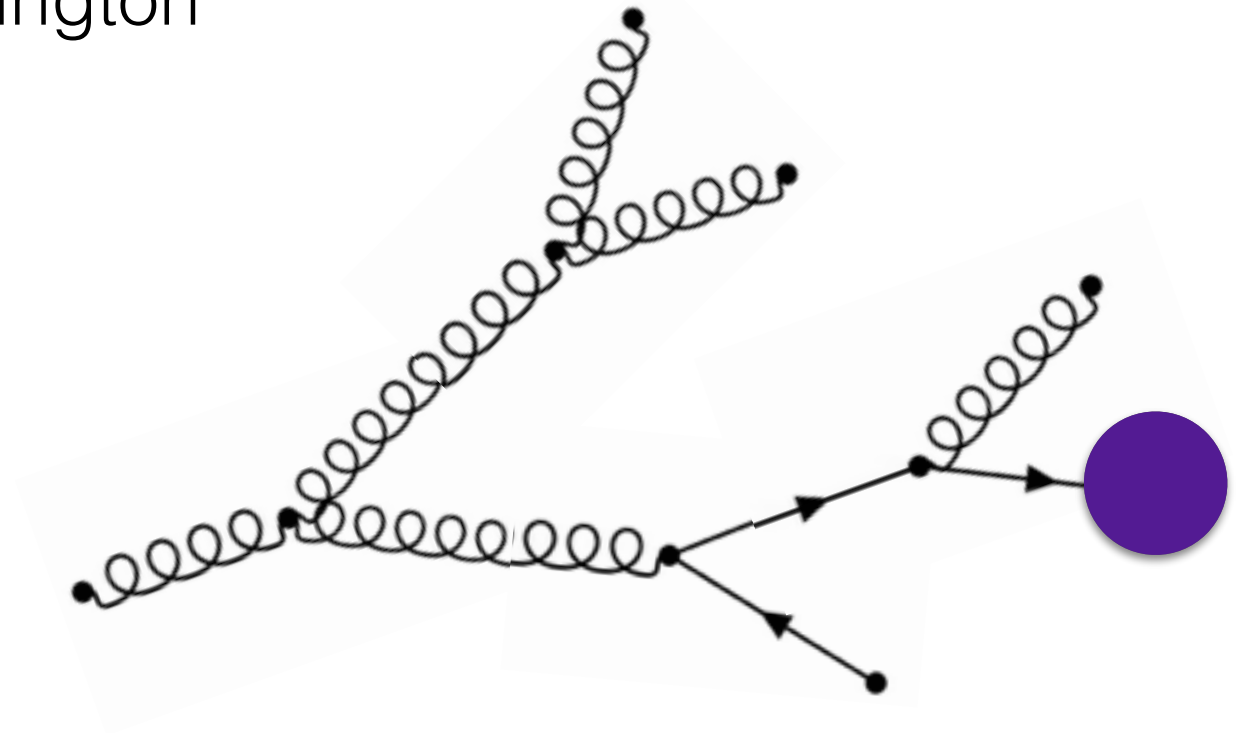


Jet fragmentation in a QCD medium: Universal q/g ratio and wave turbulence

Sören Schlichting | University of Washington

*Based on
SS, Y. Mehtar-Tani (in preparation)*



xQCD 2018
Frankfurt
May 2018

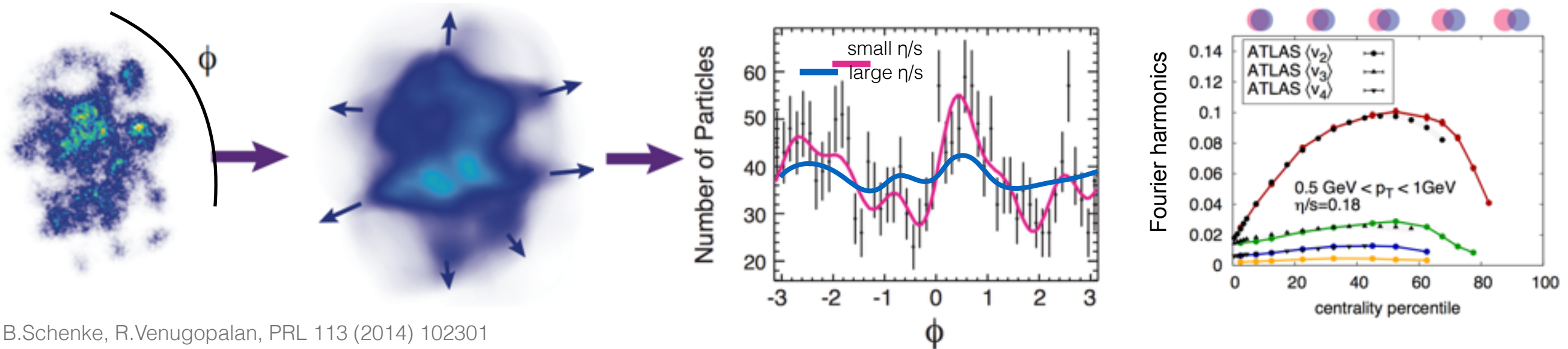
Probes of the QGP

High-energy heavy-ion collisions at RHIC, LHC produce a deconfined QGP

Goal: Characterize & understand properties of QGP phase

Experiments provide various ways to probe the QGP at different scales

Typical degrees of freedom ($p \sim T$) $\lesssim 1\text{GeV}$



B.Schenke, R.Venugopalan, PRL 113 (2014) 102301

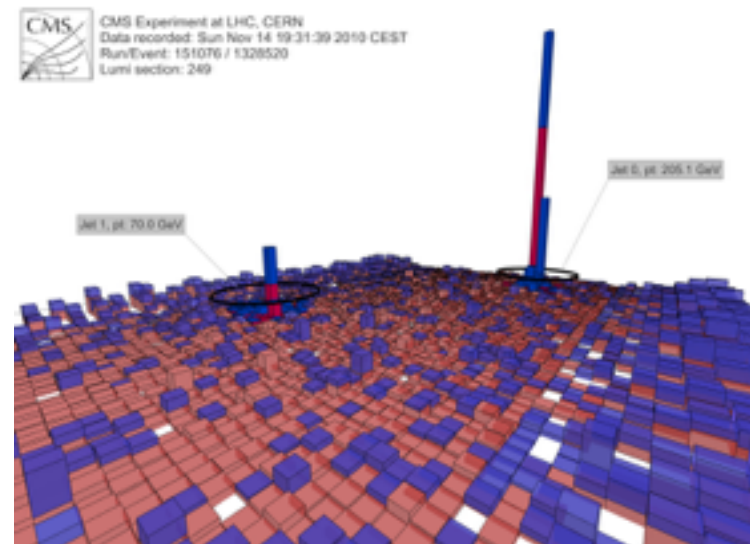
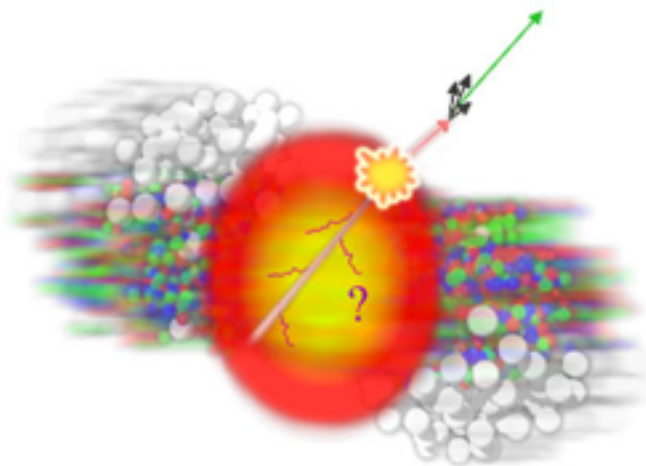
Phenomena: Elliptic flow, close to ideal fluidity ($\eta/s \ll 1$)

=> Extraction of near equilibrium/transport properties of QGP

Probes of the QGP

Experiments provide various ways to probe the QGP at different scales

Hard probes ($p \gg T$) $\sim 100\text{GeV}$



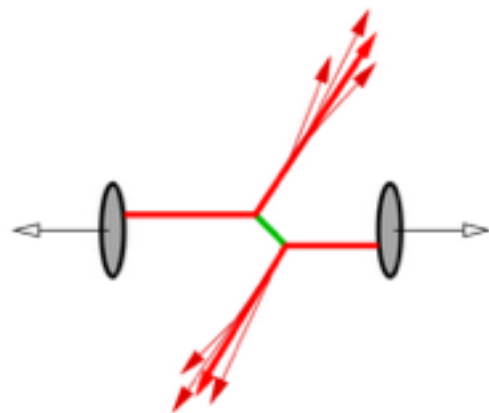
Hard scale in the problem allows for perturbative control of (at least) some aspects of jet-medium interaction \Rightarrow Calibrated probes of QGP

High- p_T objects provide non-equilibrium probes
 \Rightarrow Interesting in its own right to study their properties

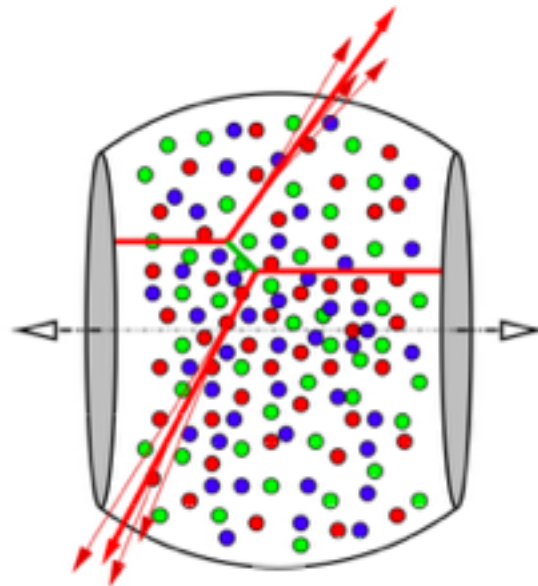
No sharp distinction between high p_T and low p_T
 \Rightarrow connection to non-equilibrium physics at intermediate scales

Jets/High- p_T probes

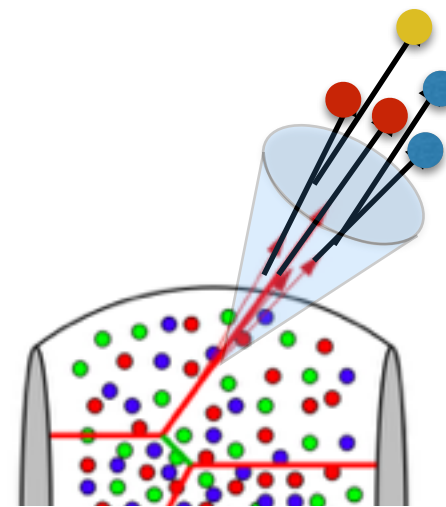
Description of hard probes in HIC involves at least three different processes



Initial production



Interaction with
medium



Vacuum shower/
Hadronization

Several MC generators for HIC (JEWEL, Martini, PHSD, CoLBT, Hybrid...), designed to describe this physics

Will follow focus on the evolution of the hard probe inside the medium

Develop (semi-) analytic insights into jet/medium interaction

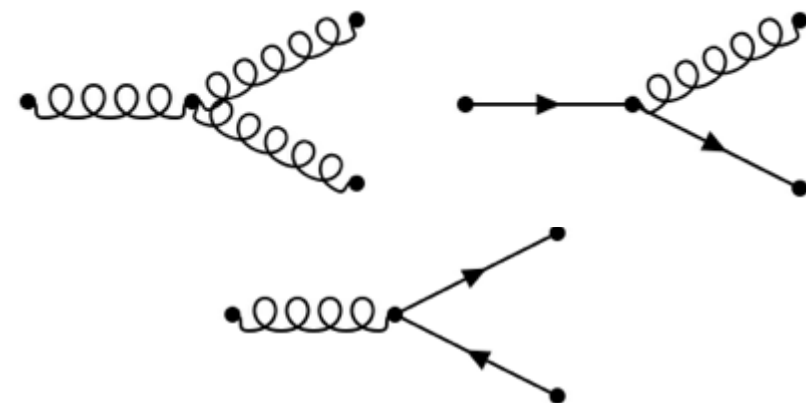
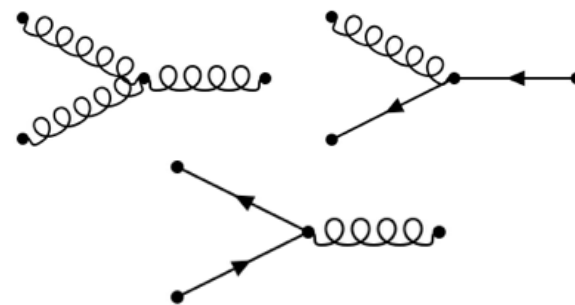
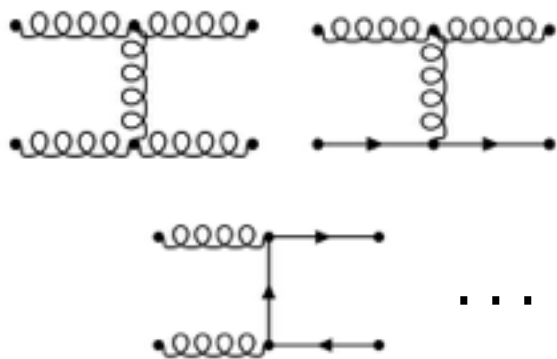
Jet-Medium interactions

Describe jet fragments as highly energetic on-shell quarks/gluons ($E \gg T$) propagating in a thermal QGP

=> Elastic & Inelastic interactions with the medium can modify properties

elastic processes & inelastic merging
(2->2; eff. 2->1)

radiative branching (eff. 1->2)



Characteristic
rates

$$\Gamma \sim \gamma_{Eq} \frac{T^2}{E^2}$$

$$\Gamma \sim \gamma_{Eq} \frac{T}{E}$$

$$\Gamma \sim \gamma_{Eq} \sqrt{\frac{T}{E}}$$

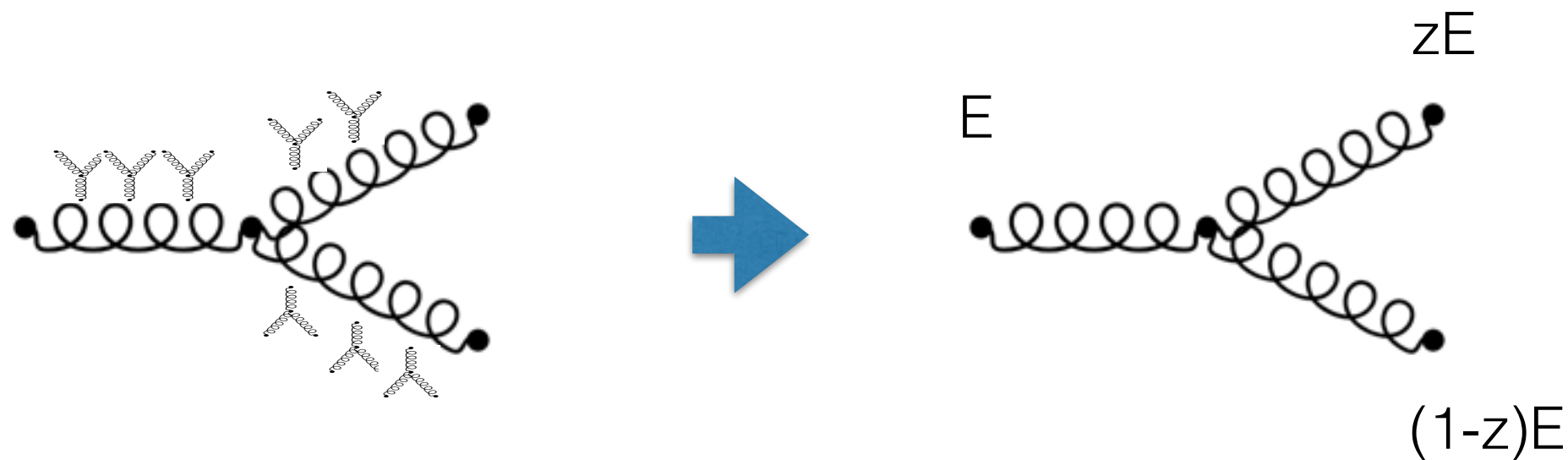
=> Evolution is dominated by radiative branching up to scales $E \sim T$

(c.f. Arnold, Moore, Yaffe (LO); Ghiglieri, Moore, Teaney (NLO))

Medium induced radiation

Splitting of the hard on-shell parton in a (thermal) medium is induced by elastic interactions with the medium

=> Clearly different from vacuum radiation from off-shell parton (DGLAP)



Since scatterings with small momentum transfer occur frequently inside the medium it is important to consider multiple scatterings and interference effects

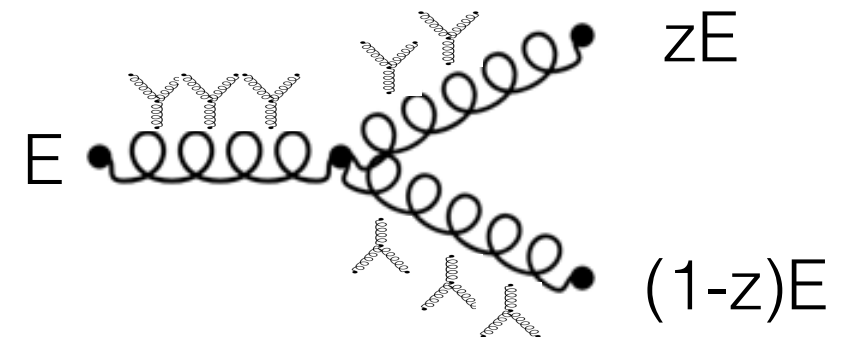
=> Coherence effects lead to suppression (LPM) of radiative emission

Medium induced radiation

Basic picture emerging from calculations

Emission rate is controlled by formation time

$$t_{form} \sim \frac{z(1-z)E}{k_T^2} \quad k_T^2 \sim \hat{q} t_{form}$$



$$t_{form} \sim \sqrt{\frac{z(1-z)E}{\hat{q}}} \quad \text{multiple scattering centers emit coherently during formation time}$$

Emission rates:

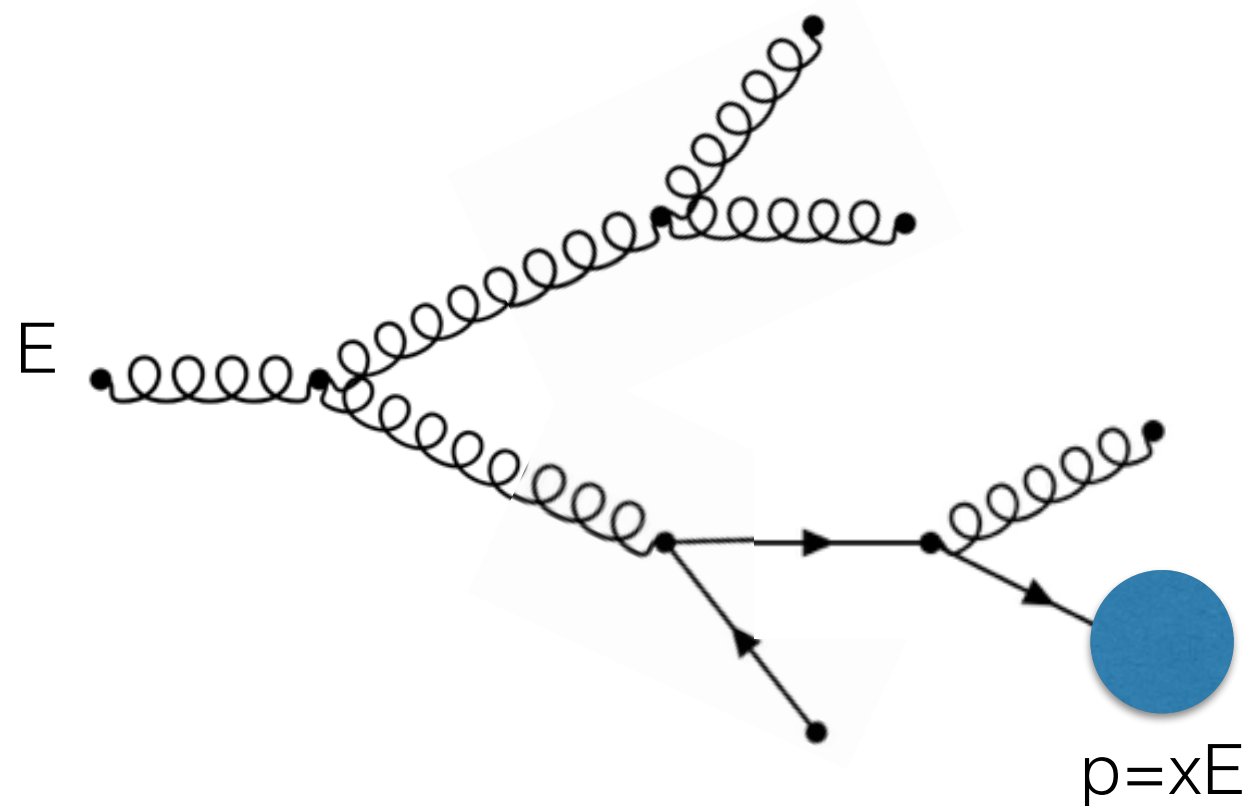
$$\Gamma_{fi}^{split}(E, zE, (1-z)E) = \sqrt{\frac{\alpha^2 \hat{q}}{\pi^2 E}} K_{fi}(z)$$

Will consider leading log E/T where splitting kernels K_{fi} are

$$\begin{aligned} \mathcal{K}_{gg}(z) &= \frac{1}{2} 2C_A \frac{[1 - z(1-z)]^2}{z(1-z)} \sqrt{\frac{(1-z)C_A + z^2 C_A}{z(1-z)}}, & \mathcal{K}_{qq}(z) &= \frac{1}{2} C_F \frac{1 + (1-z)^2}{z} \sqrt{\frac{(1-z)C_A + z^2 C_F}{z(1-z)}} \\ \mathcal{K}_{qg}(z) &= \frac{1}{2} 2N_f T_R (z^2 + (1-z)^2) \sqrt{\frac{C_F - z(1-z)C_A}{z(1-z)}}, & \mathcal{K}_{gq}(z) &= \frac{1}{2} C_F \frac{1 + z^2}{(1-z)} \sqrt{\frac{zC_A + (1-z)^2 C_F}{z(1-z)}}. \end{aligned}$$

Multiple branchings

Successive branchings



Will keep track of distribution of fragments in terms of in-medium fragmentation function

$$D_i(x, \tau) \equiv x \frac{dN_i}{dx}$$

which measures distribution of $i=q,g$ fragments after some evolution in the medium

Subsequent splittings are independent of each other and quasi-instantaneous => Effective kinetic equation for in-medium FF

In-Medium fragmentation

Decomposing into
flavor singlet/
non-singlet:

$$D_S \equiv \sum_{i=1}^{N_f} (D_{q_i} + D_{\bar{q}_i})$$

$$D_{NS}^{(i)} \equiv D_{q_i} - D_{\bar{q}_i},$$

Defining scaled
time variable:

$$\tau = \sqrt{\frac{\alpha^2 \hat{q}}{\pi^2 E}} t$$

Kinetic equations for in-medium fragmentation function:

$$\begin{aligned} \frac{\partial}{\partial \tau} D_g(x, \tau) &= \int_0^1 dz \mathcal{K}_{gg}(z) \left[\sqrt{\frac{z}{x}} D_g\left(\frac{x}{z}\right) - \frac{z}{\sqrt{x}} D_g(x) \right] - \int_0^1 dz K_{qg}(z) \frac{z}{\sqrt{x}} D_g(x) \\ &+ \int_0^1 dz K_{gq}(z) \sqrt{\frac{z}{x}} D_S\left(\frac{x}{z}\right), \end{aligned}$$

$$\frac{\partial}{\partial \tau} D_S(x, \tau) = \int_0^1 dz \mathcal{K}_{qq}(z) \left[\sqrt{\frac{z}{x}} D_S\left(\frac{x}{z}\right) - \frac{1}{\sqrt{x}} D_S(x) \right] + \int_0^1 dz \mathcal{K}_{qg}(z) \sqrt{\frac{z}{x}} D_g\left(\frac{x}{z}\right)$$

$$\frac{\partial}{\partial \tau} D_{NS}^{(i)}(x, \tau) = \int_0^1 dz \mathcal{K}_{qq}(z) \left[\sqrt{\frac{z}{x}} D_{NS}^{(i)}\left(\frac{x}{z}\right) - \frac{1}{\sqrt{x}} D_{NS}^{(i)}(x) \right]$$

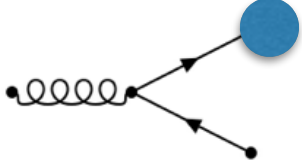
where $\sqrt{z/x}$ and $1/\sqrt{x}$ factors follow from shorter formation time for successive branchings

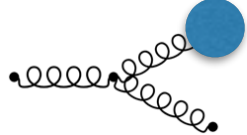
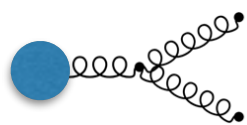
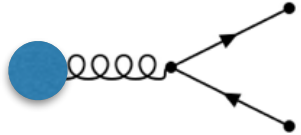
(c.f. Arnold, Moore, Yaffe)

In-Medium fragmentation

Solution to leading order τ in /single splitting:

gluon jet: $D_g(x, \tau) \simeq \delta(1-x) + \left[x\mathcal{K}_{gg}(x) - \int_0^1 dz z (\mathcal{K}_{gg}(z) + \mathcal{K}_{qg}(z)) \delta(1-x) \right] \tau$

 $D_S(x, \tau) \simeq xK_{qg}(x) \tau.$ $D_{NS}(x) = 0.$

However this is only meaningful in a very limited regime of (τ, x) :

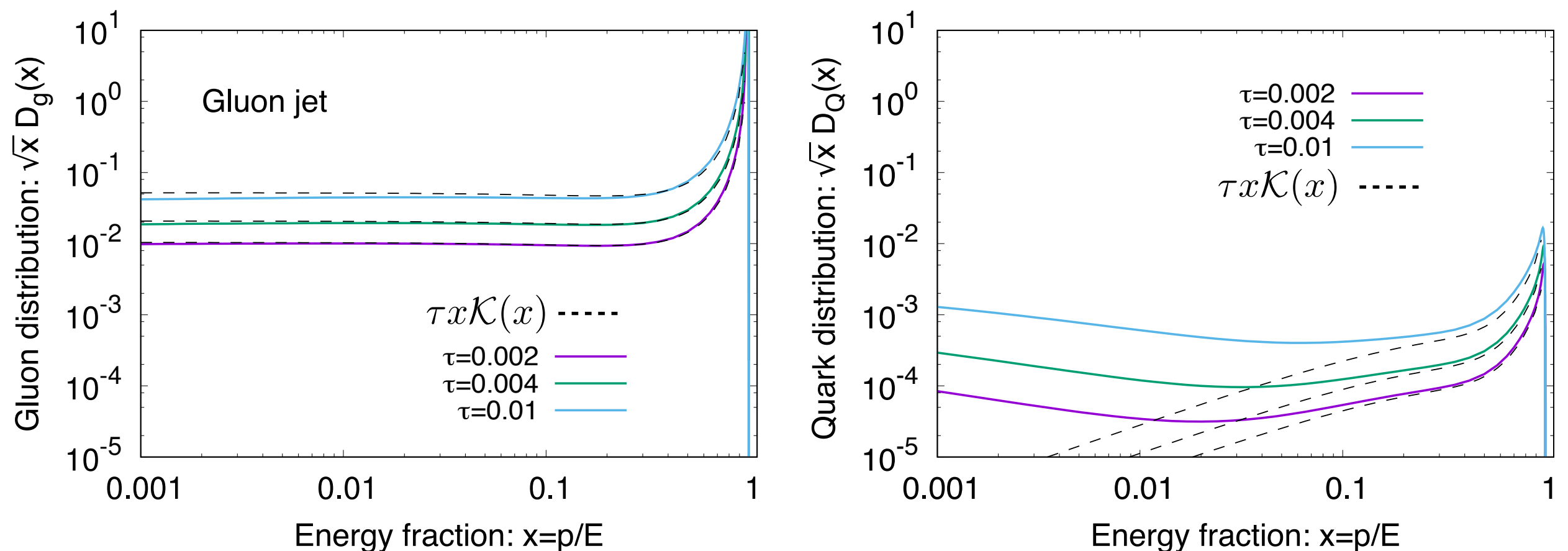
large x : Singularly at large x requires more elaborate treatment

small x : Emission rates at small x enhanced by $1/\sqrt{x}$

=> Dynamically generated scale $x_c \sim \tau^2$ below which probability for subsequent splitting is $O(1)$ for any $\tau > 0$

In-Medium fragmentation

Numerical solution of coupled evolution equations for gluon jet ($\tau \ll 1$)



Emergence of dynamically generated scale x_c
clearly visible in numerical solution of evolution equations

Stationary solution

Since splitting rates become of $O(1)$ for $x < x_c$, expect solution to become insensitive to initial conditions and approach fixed point of kinetic equation

Stationary non-equilibrium solution: $D_g(x) = \frac{G}{\sqrt{x}}$, $D_S(x) = \frac{2N_f Q}{\sqrt{x}}$

$$\begin{aligned} \frac{\partial}{\partial \tau} D_g(x, \tau) &= \int_0^1 dz \mathcal{K}_{gg}(z) \left[\sqrt{\frac{z}{x}} D_g\left(\frac{x}{z}\right) - \frac{z}{\sqrt{x}} D_g(x) \right] - \int_0^1 dz K_{qg}(z) \frac{z}{\sqrt{x}} D_g(x) \\ &+ \int_0^1 dz K_{gq}(z) \sqrt{\frac{z}{x}} D_S\left(\frac{x}{z}\right), \end{aligned}$$

Chemistry of fragments fixed by
balance of $g \rightarrow qq$ and $q \rightarrow gq$
processes

$$\frac{Q}{G} = \frac{\int_0^1 dz \, z \, \mathcal{K}_{qg}(z)}{2N_f \int_0^1 dz \, z \, \mathcal{K}_{gq}(z)} \approx 0.07$$

Existence of solution does not rely on detailed form of $K(z)$ but only
on the fact that emission rates behave as $1/\sqrt{E}$

Energy cascade

Solution is analogous to Kolmogorov-Zhakarov spectrum in weak wave turbulence

Even though the solution is stationary

$$\partial_\tau D_g(x) = \partial_\tau D_S(x) = 0,$$

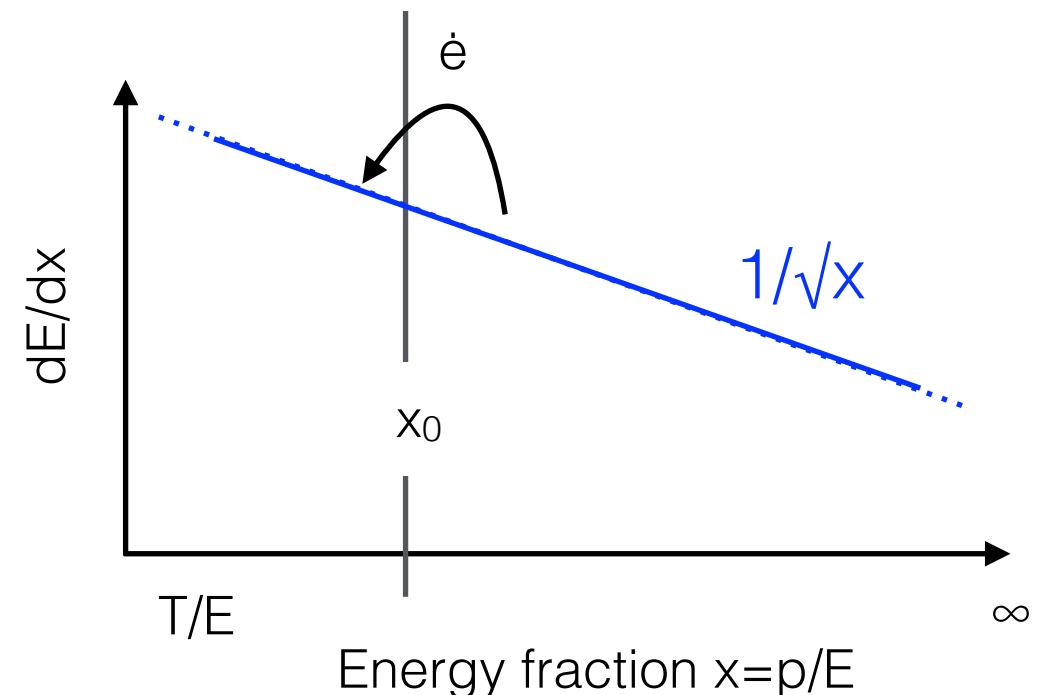
it is associated with a finite scale invariant energy flux

$$\begin{aligned} \dot{\epsilon}(x_0) &= \int_{x_0}^{\infty} dx [\partial_\tau D_g(x) + \partial_\tau D_S(x)] \\ &= -\gamma_g G - \gamma_q Q \end{aligned}$$

with flux constants

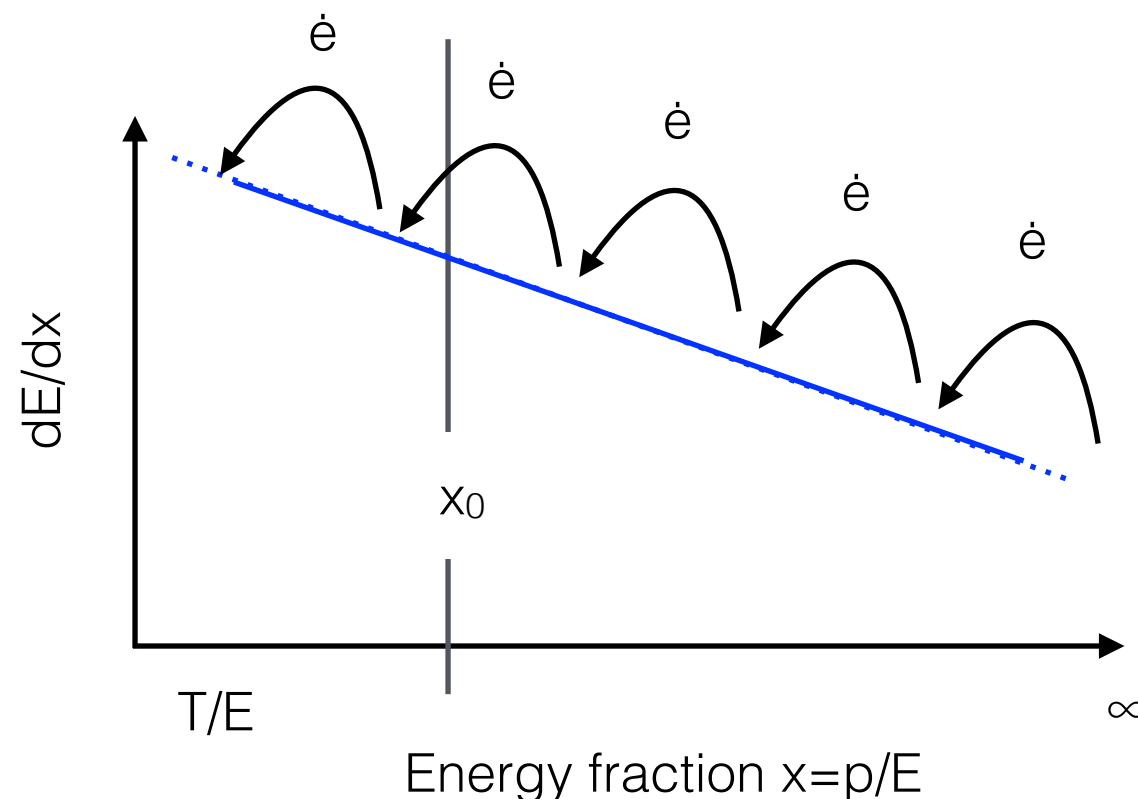
$$\gamma_g = \int_0^1 dz z \left(\mathcal{K}_{gg}(z) + \mathcal{K}_{qg}(z) \right) \log \left(\frac{1}{z} \right) \approx 25.78 + 0.177 N_f .$$

$$\gamma_q = 2N_f \int_0^1 dz z \left(K_{qq}(z) + K_{gq}(z) \right) \log \left(\frac{1}{z} \right) \approx 23.19 N_f$$



Energy cascade

Scale invariant energy flux associated with energy transfer from $x \gg 1$ to $x \sim T/E$ where it is absorbed by medium



-> analogous to Richardson cascade in wave-turbulence

Energy loss rate is dominated by $g \rightarrow gg$. Contributions from $q \rightarrow qg$ and $g \rightarrow qq$ give 15% (0.06%) correction per active flavor

(c.f. Blaizot, Mehtar-Tani; Blaizot, Mehtar-Tani, Iancu)

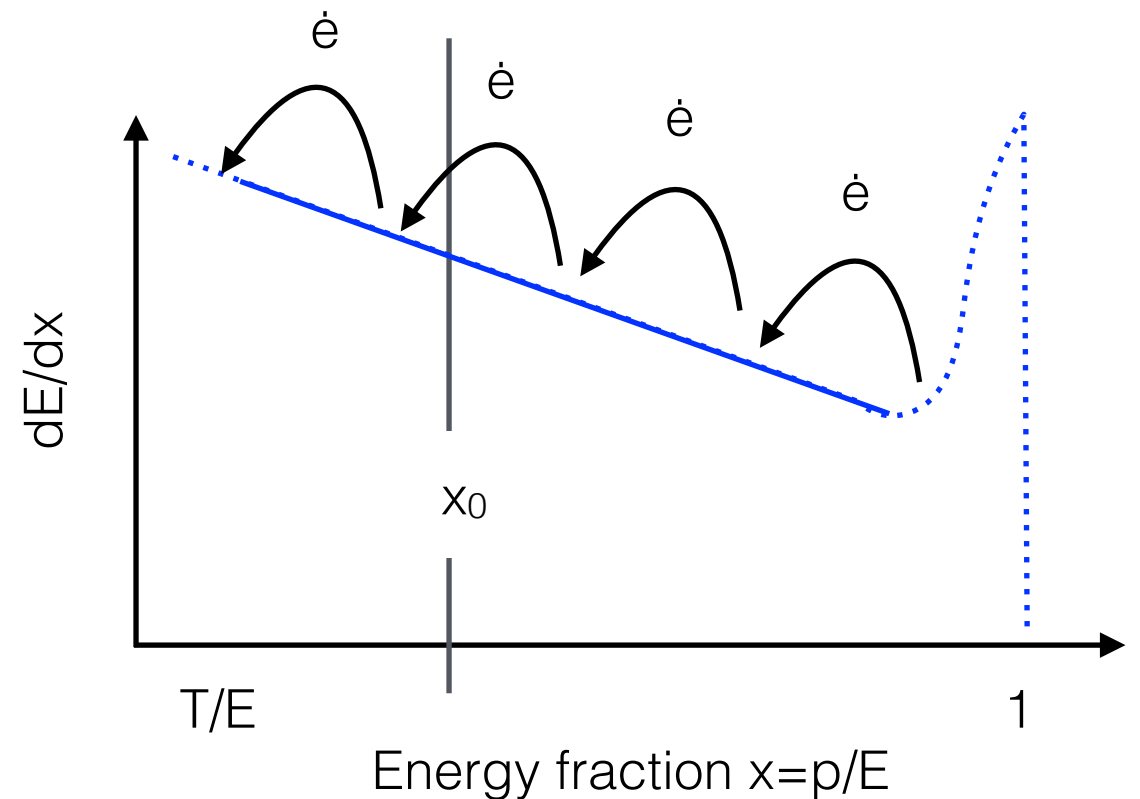
Energy cascade

Naive scaling solution requires $D(x) \sim 1/\sqrt{x}$ for all $x \in (0, \infty)$ to satisfy stationarity condition

-> necessary to provide infinite energy reservoir to realize stationarity

However even if we limit the support of the distribution to the physical range $(0, 1)$, the energy flux becomes scale invariant for $x \ll 1$

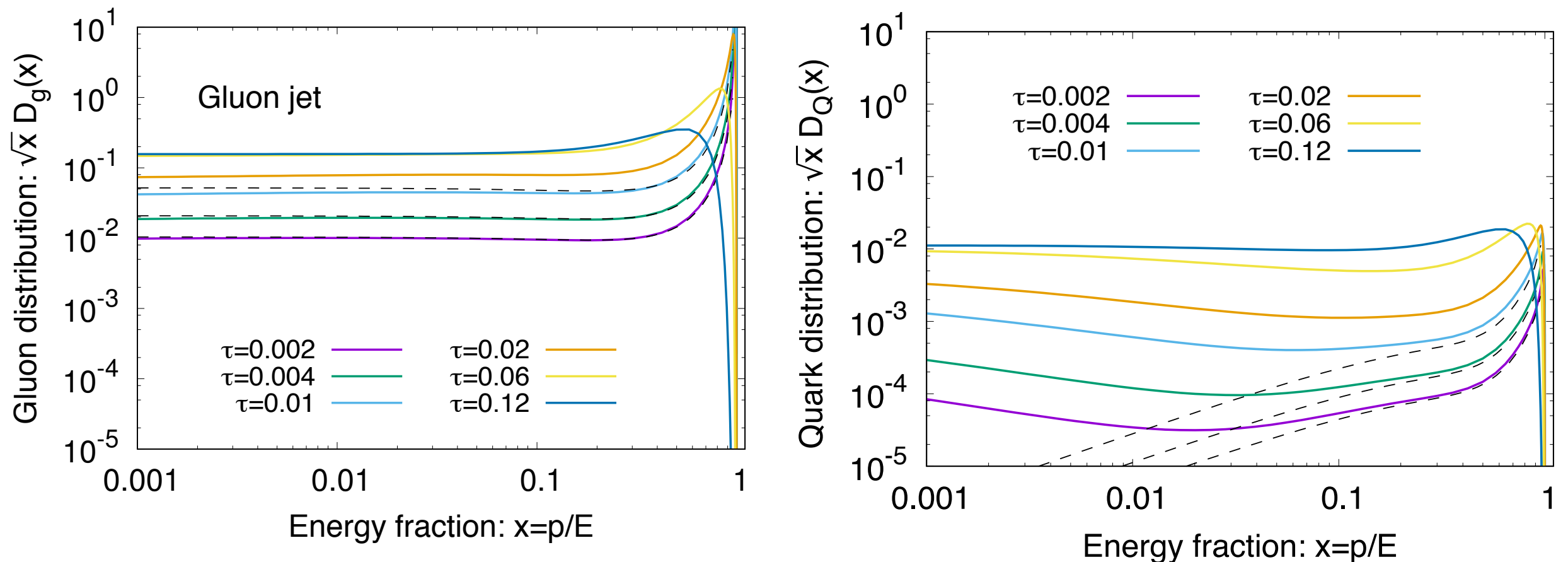
$$\dot{\epsilon}(x_0 \ll 1) \simeq -\gamma_g G - \gamma_q Q$$



=> locality of interactions implies that scaling solution can be realized within an inertial range of momenta $T/E \ll x \ll 1$

In-Medium fragmentation of g-jet

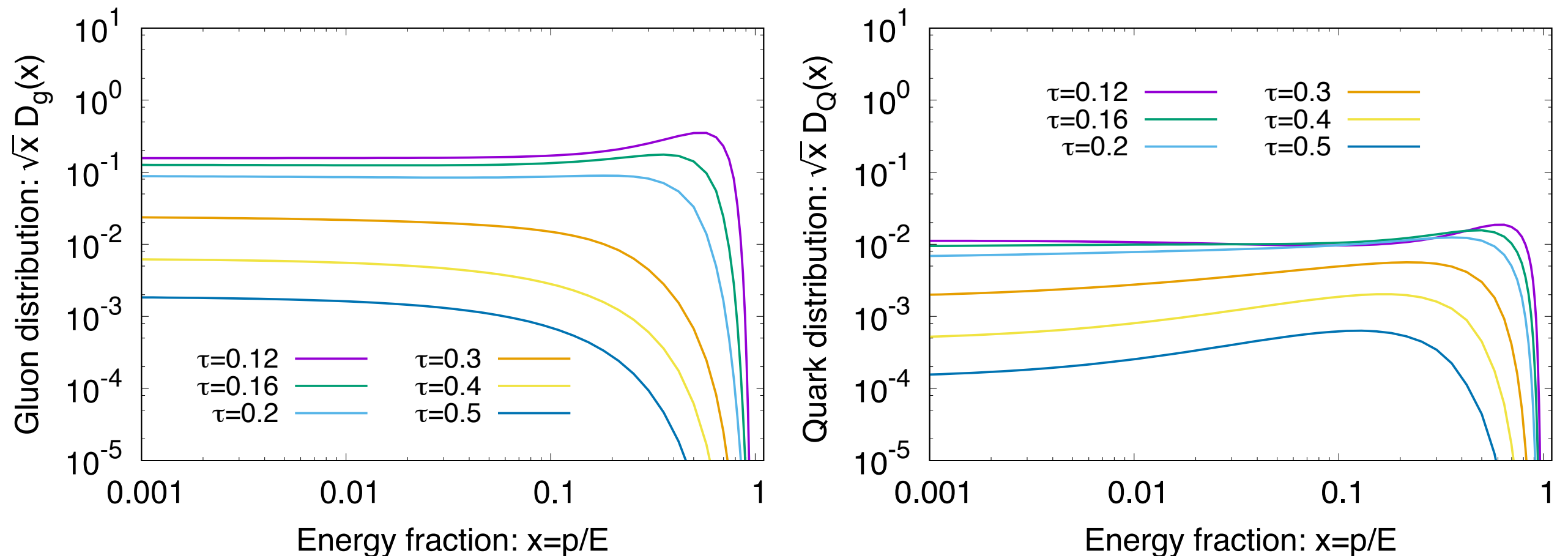
Numerical solution of coupled evolution equations for gluon jet



Enhanced splitting rates at small x lead to approach a non-equilibrium steady state, characterized by $D(x) \sim 1/\sqrt{x}$ behavior of gluon & quark distribution at small x

In-Medium fragmentation of g-jet

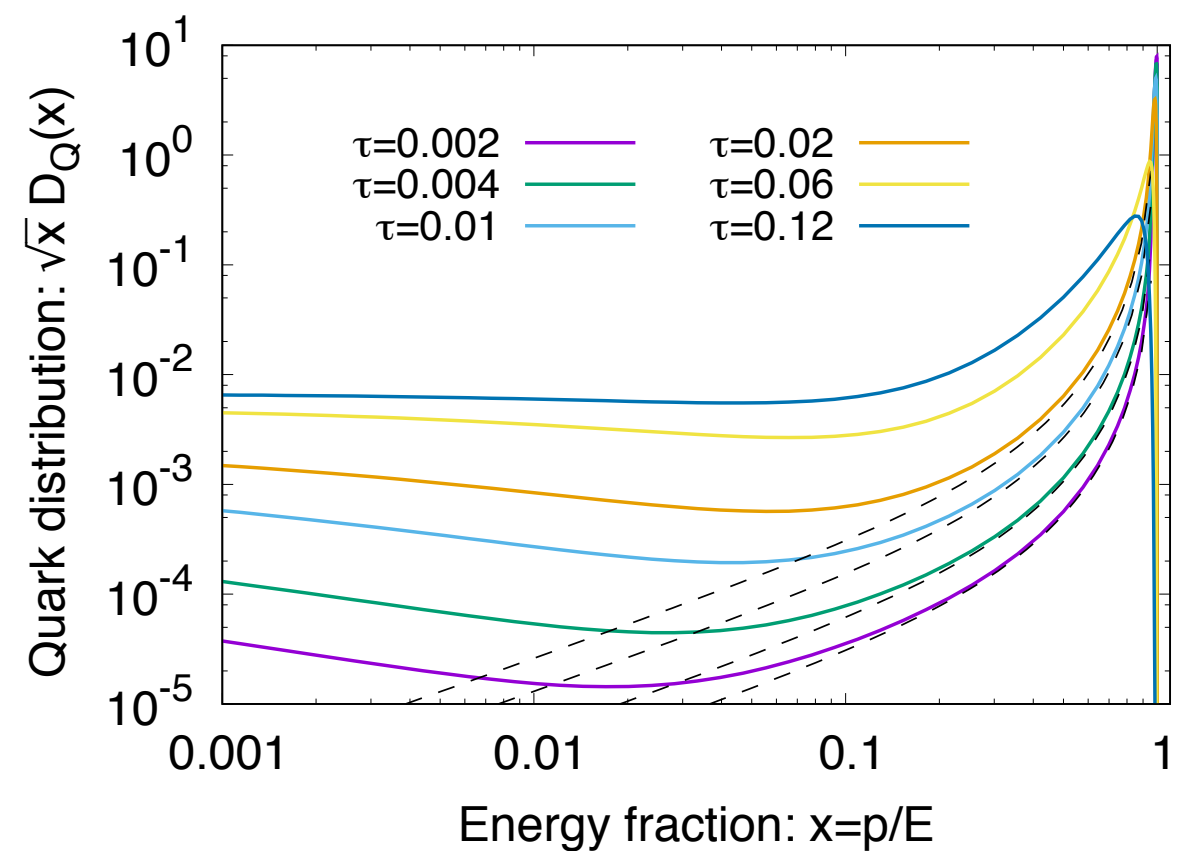
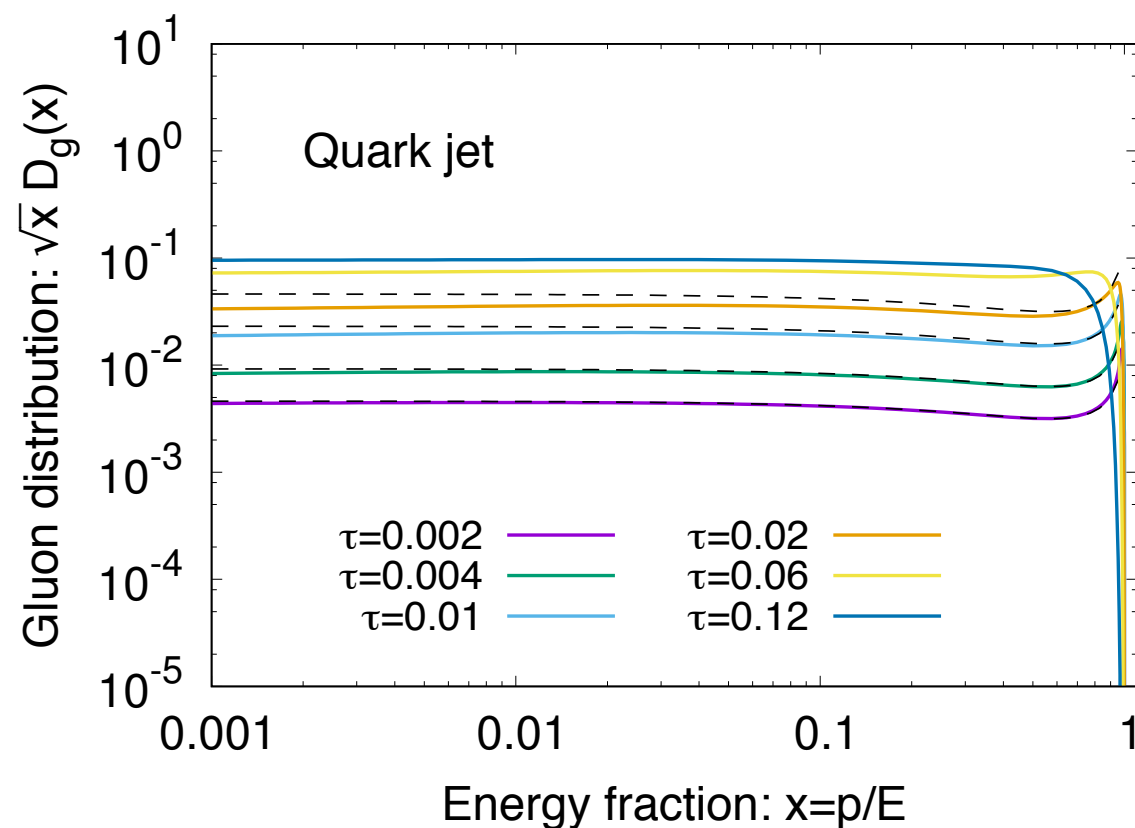
Numerical solution of coupled evolution equations for gluon jet



Kolmogorov spectrum at small x persists throughout the evolution, even when the jet has lost a significant amount of energy

In-Medium fragmentation of q-jet

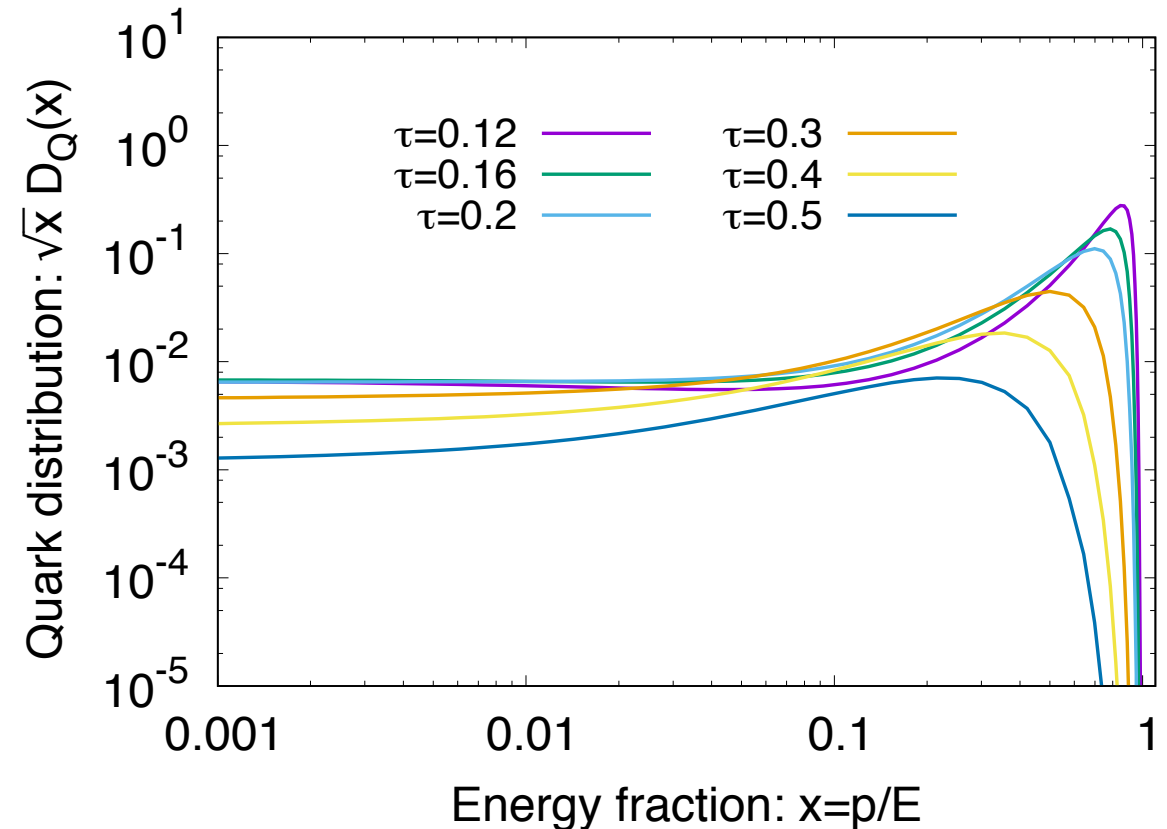
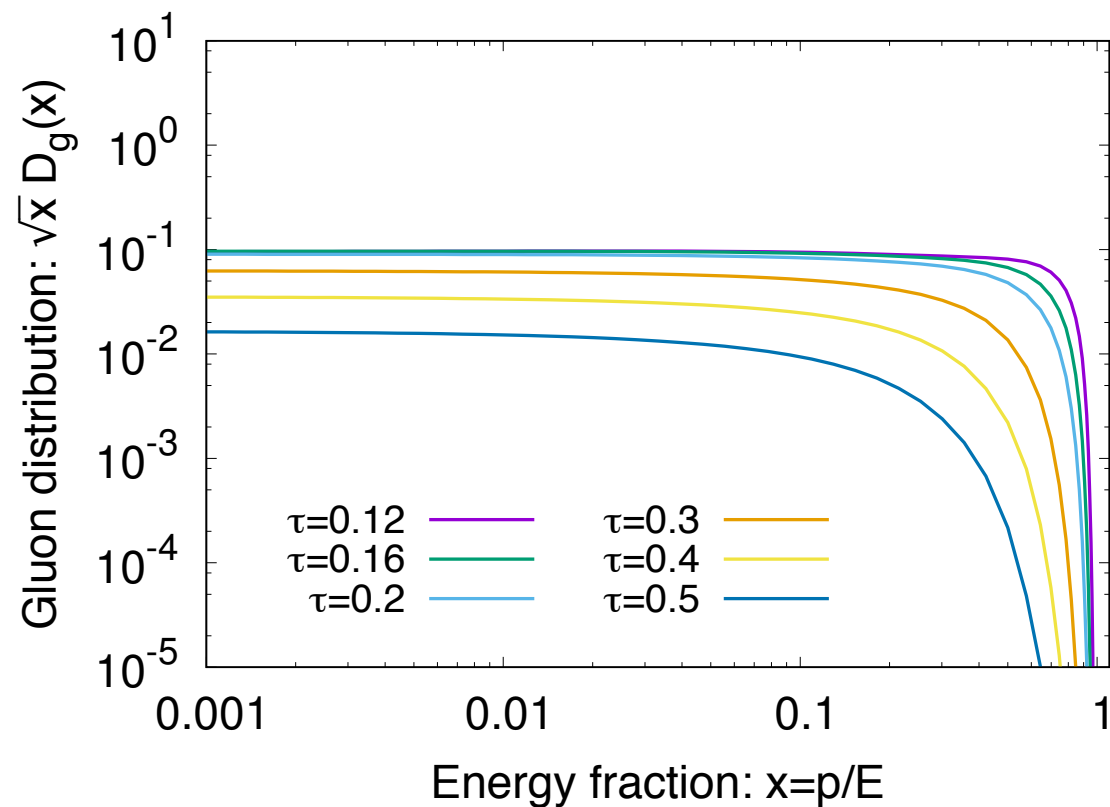
Numerical solution of coupled evolution equations for quark jet



Enhanced splitting rates at small x lead to approach a non-equilibrium steady state, characterized by $D(x) \sim 1/\sqrt{x}$ behavior of gluon & quark distribution at small x

In-Medium fragmentation of -qjet

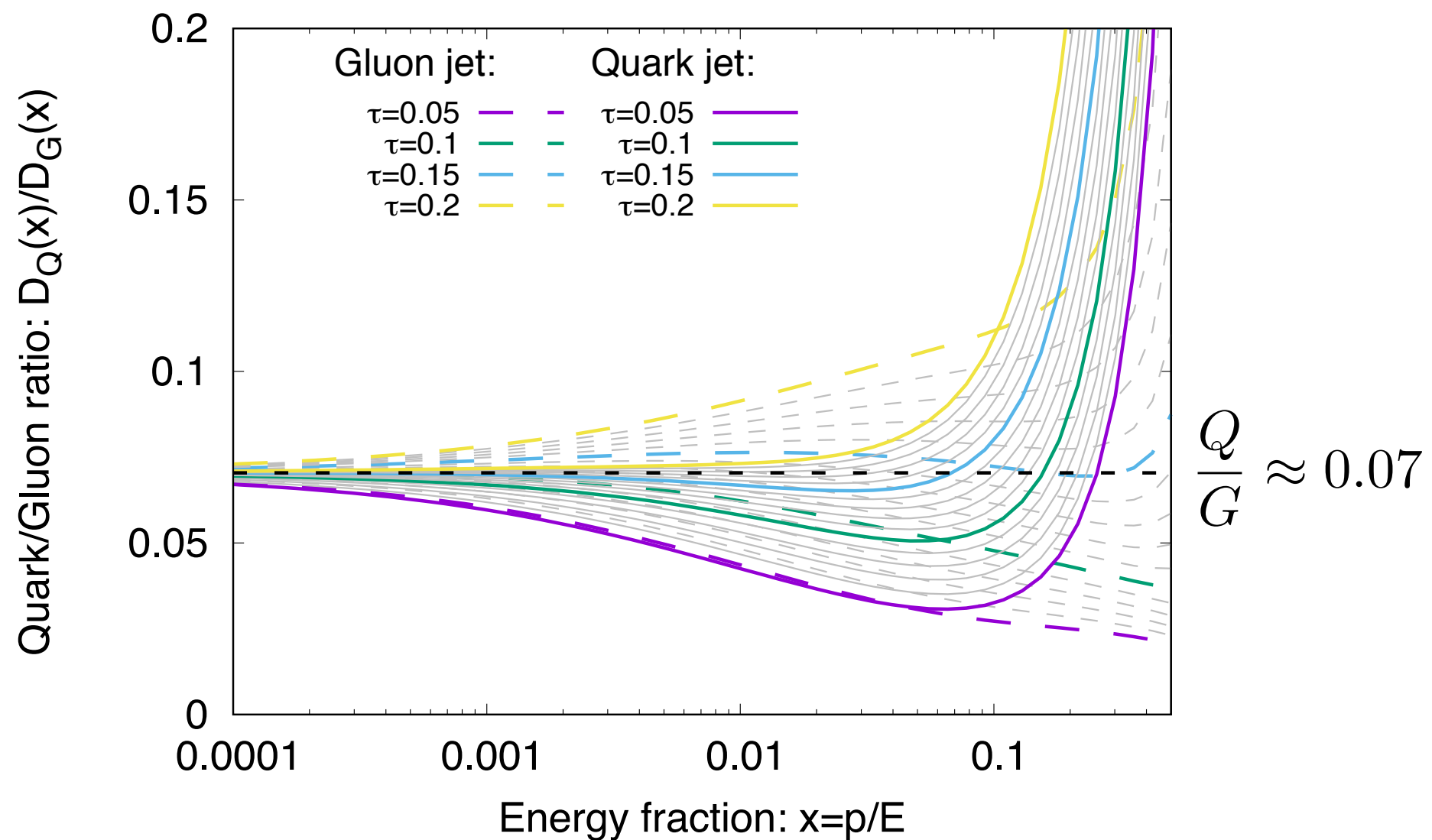
Numerical solution of coupled evolution equations for quark jet



Kolmogorov spectrum at small x persists throughout the evolution, even when the jet has lost a significant amount of energy

In-medium jet chemistry

Balance of the $g \rightarrow qq$ and $q \rightarrow qg$ processes at the non-equilibrium steady state ($x \ll 1$) uniquely determine chemistry



Universal Kolmogorov ratio approximately realized over a substantial range of momentum fractions x and evolution times τ

Energy loss

Single emission off the original hard parton create a $1/\sqrt{x}$ gluon spectrum at small x with amplitude

$$G \simeq \tau C_A^{1/2} C_R$$

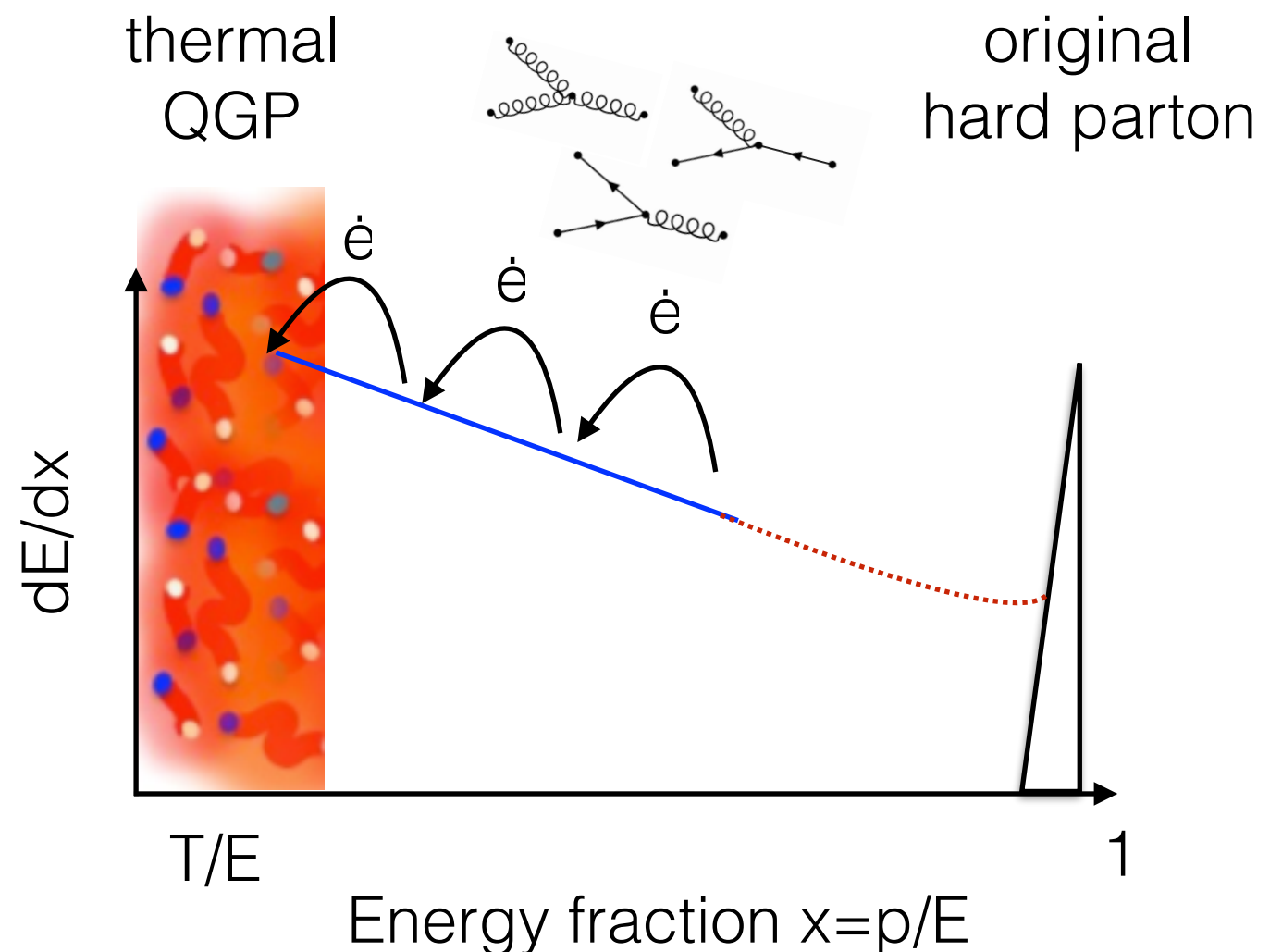
Multiple emissions off soft fragments create turbulent energy flux

$$\dot{\epsilon} = -\gamma_g G$$

all the way to T/E where energy is absorbed by the thermal QGP

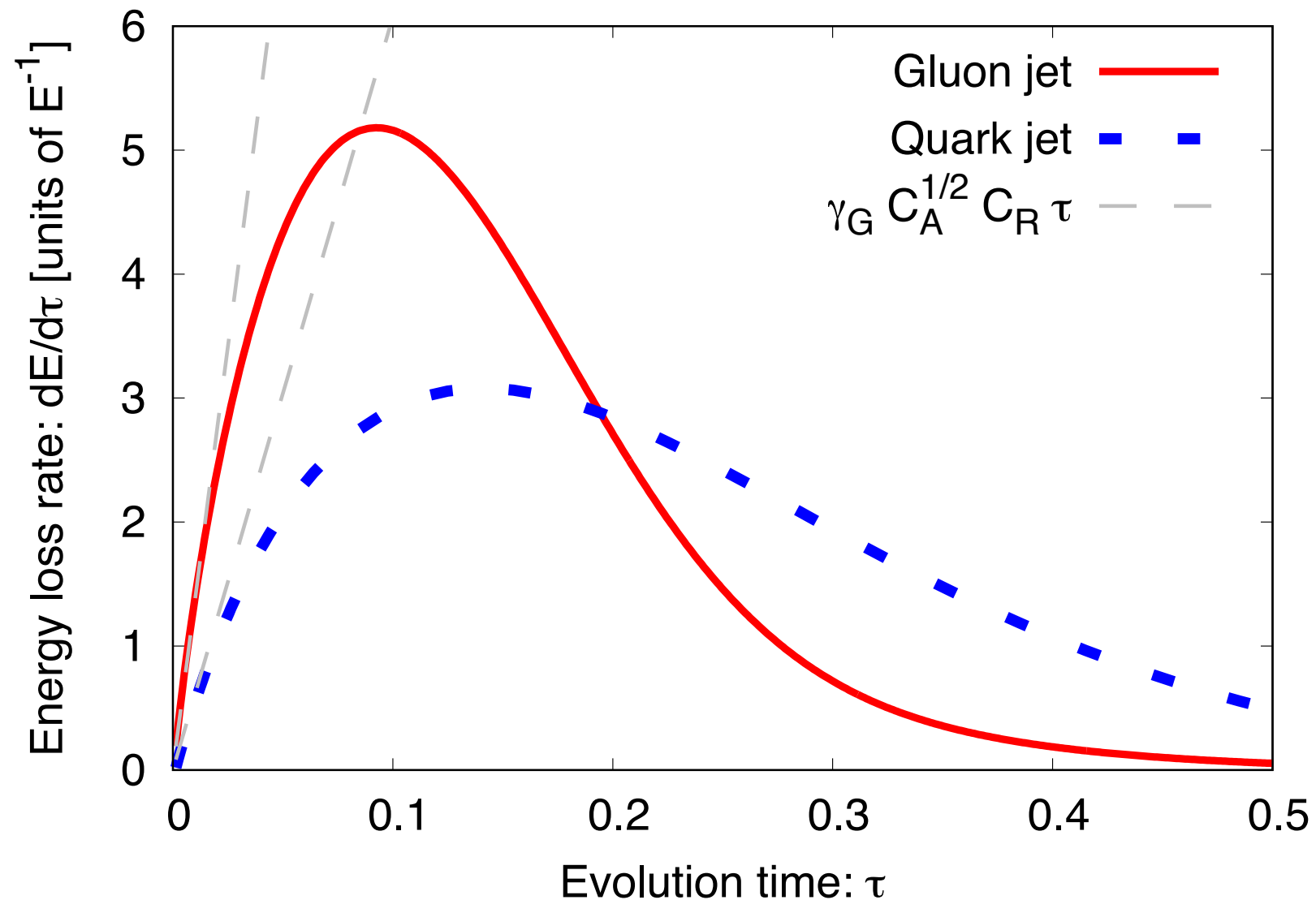
Energy loss rate at early times given by

$$\frac{1}{E} \frac{dE}{d\tau} \bigg|_{\tau \ll 1} \approx -\gamma_g C_A^{1/2} C_R \tau$$



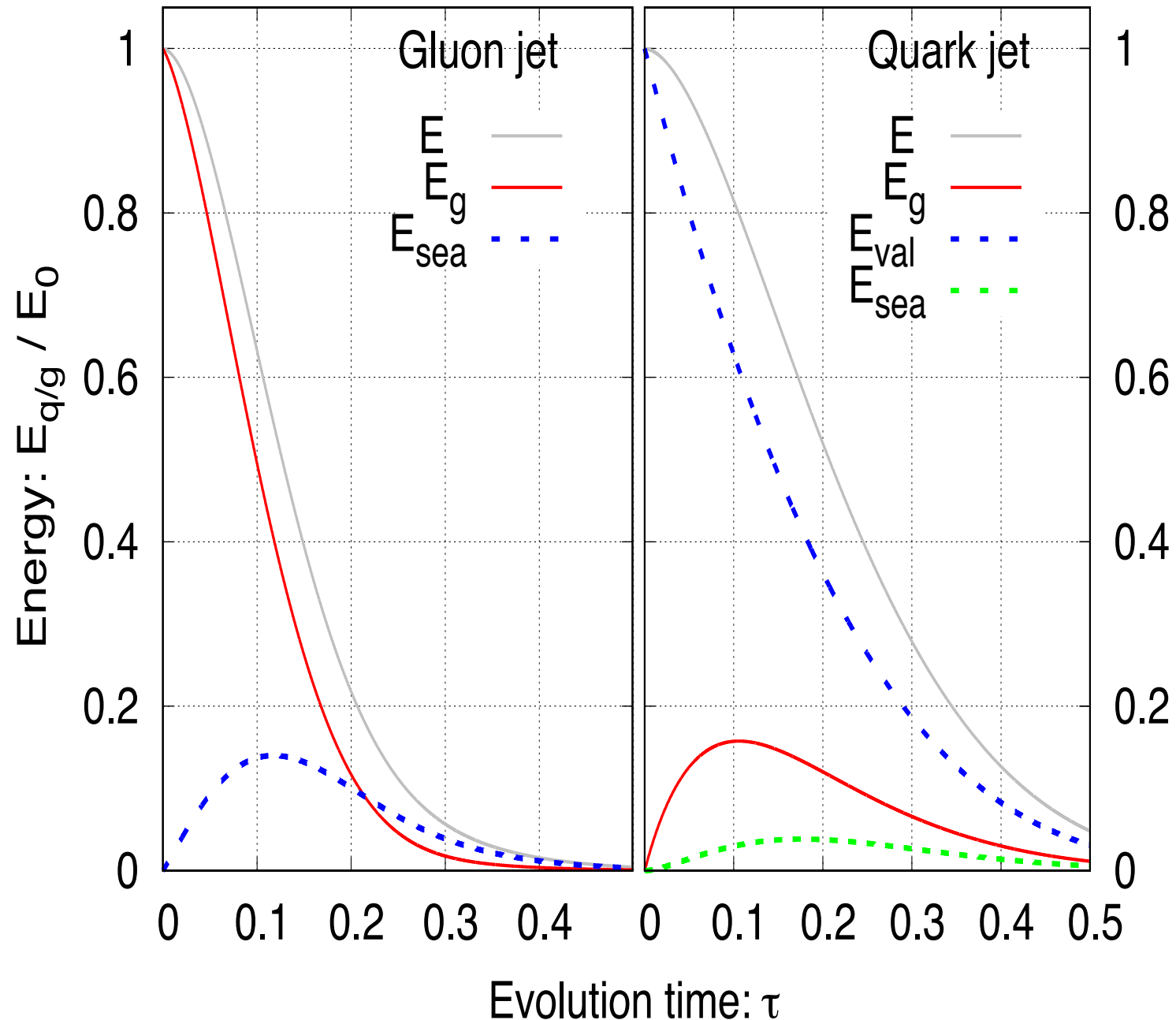
Energy loss of quark & gluon jets

Energy loss at early times follows expected behavior from turbulence analysis



Breakdown of Casimir scaling of energy loss at late times
as chemistry of fragments is strongly modified

Jet chemistry



medium filtering:

Since large x gluons lose energy faster than large x quarks, the large x distribution at late times is always dominated by quark d.o.f.

quark jet:

Energy always dominated by valence quarks

gluon jet:

Energy dominated by sea quarks once jet has lost $\sim 80\%$ of total energy

Phenomenological implications

So far connection to experiment is loose, but assuming factorization

$$d\sigma_{\gamma, h/jet} \sim f_a \times f_b \times H_{ab \rightarrow \gamma c} \times D_{c \rightarrow d}^{\text{Medium}} \times D_{d \rightarrow h/jet}^{\text{Vacuum}}$$

can speculate on phenomenological consequences of our findings

Universal q/g ratio for small x fragments & medium filtering

=> strangeness enhancement at small x (respectively large x)

Could be observable by looking at

- ratios of identified particles (e.g. K/ π or Λ/π) inside jets
- ratios of identified particles (e.g. K/ π or Λ/π) in backward hemisphere of high- p_T trigger particle

in A+A collisions relative to p+p reference

Ideally one would have an estimate of energy loss calibrate (x, τ)
e.g. by photon tagging

Detailed predictions will require to combine medium evolution with
initial production and hadronization (work in progress)

Conclusion & Outlook

In-Medium fragmentation of jets is governed by turbulent cascade, associated with scale independent energy flux from energy scales of jet ($p \sim E$) all the way to the energy scale of the medium ($p \sim T$)

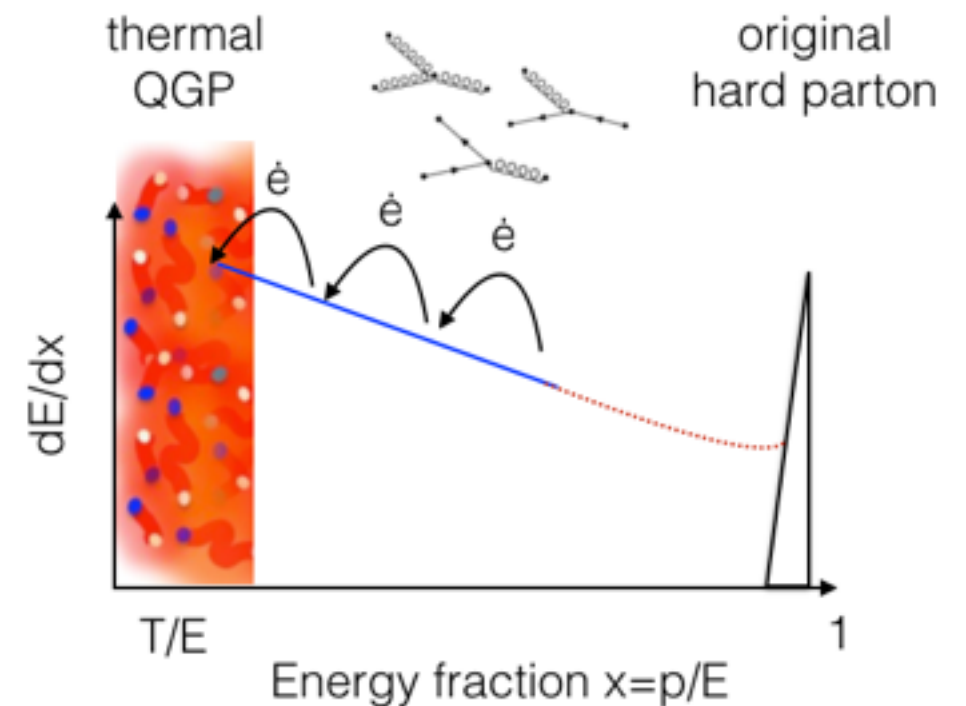
=> allows for analytic predictions of interesting features

$$\frac{D_S(x)}{D_G(x)} = \frac{\int_0^1 dz \, z \, K_{qg}(z)}{\int_0^1 dz \, z \, K_{gq}(z)} \approx 0.07 \times 2N_f$$

Interesting phenomenological consequences for jet chemistry

=> could be interesting to extend analysis to heavy (c,b) flavors

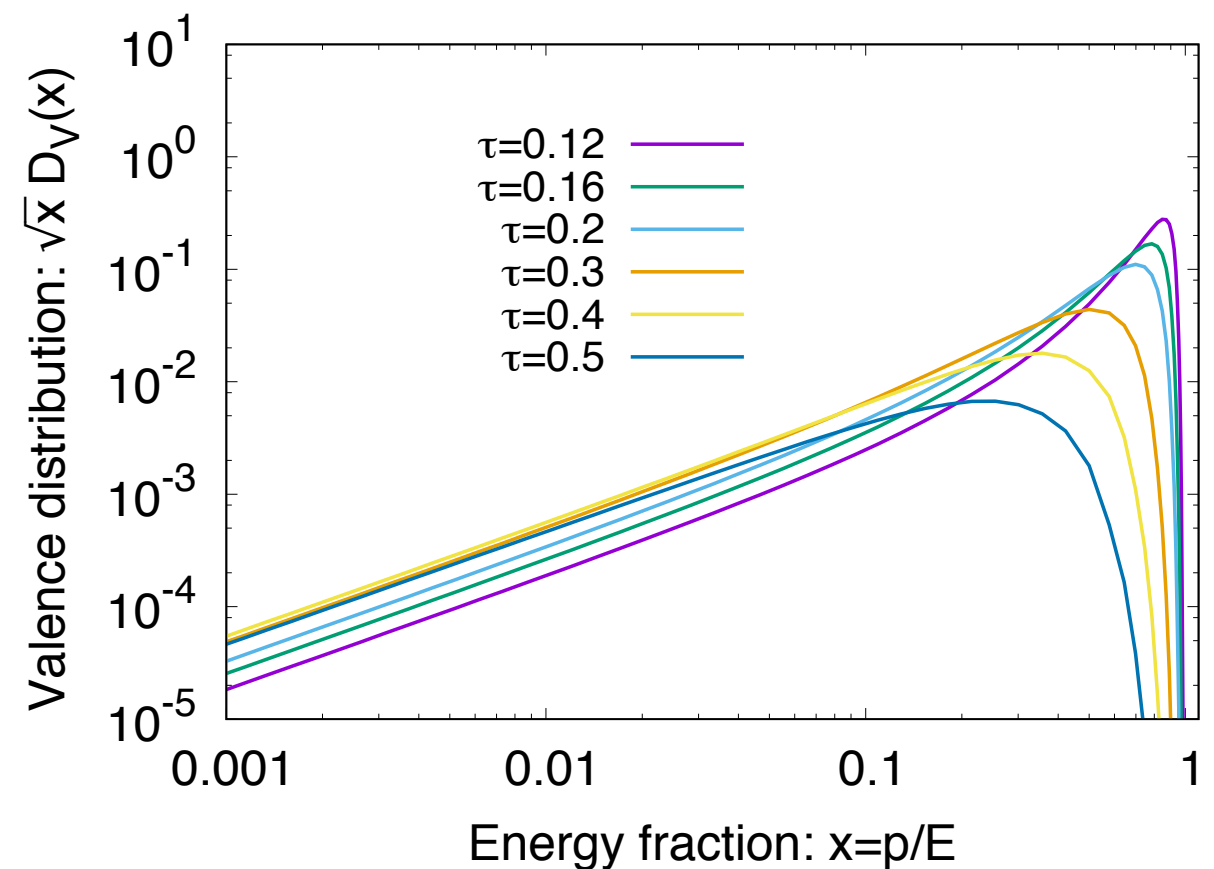
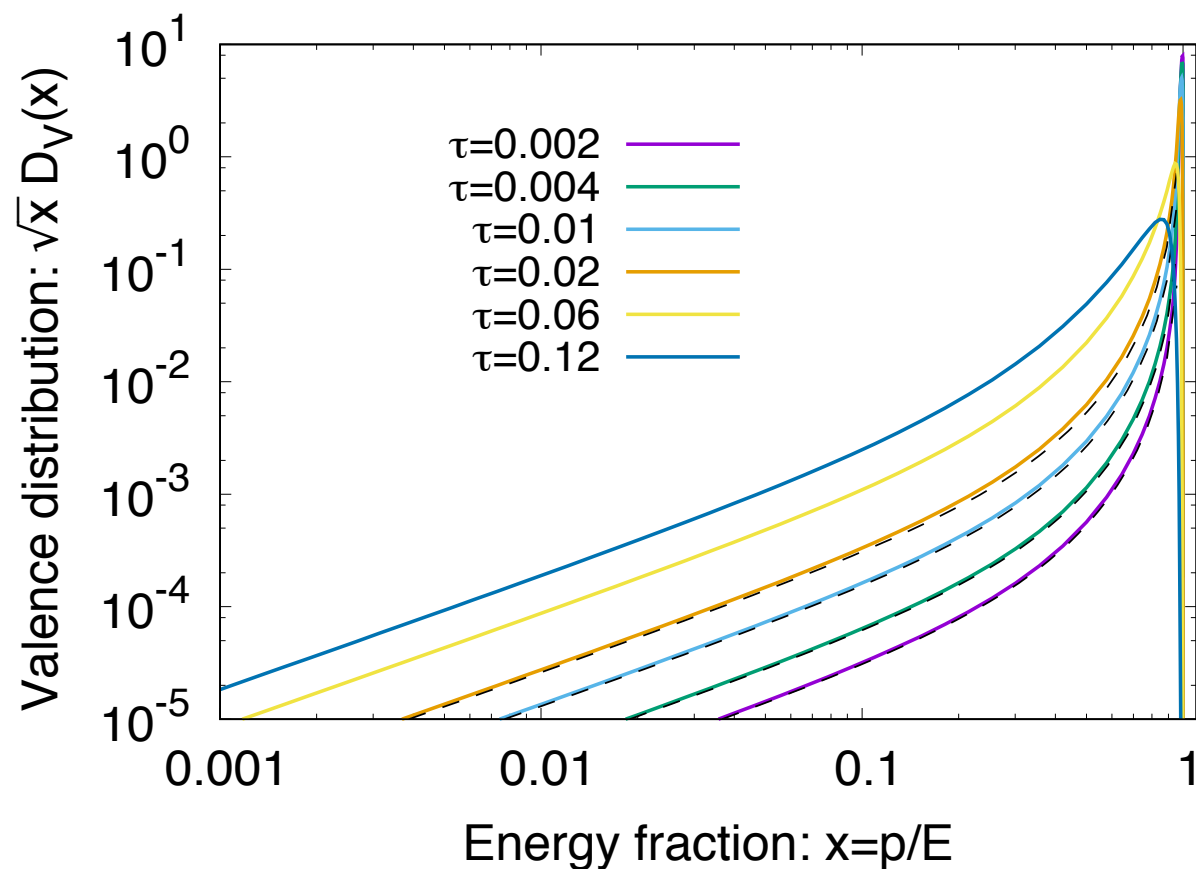
Ultimately, would like to extend study to include all processes relevant at lower scales $x \sim T/E$ to obtain a more complete picture of jet fragmentation in medium & address chemical equilibration of the QGP at early times



Backup

Valence flavor cascade

Beside energy quark jets also carry a valence flavor (u,d,s) which gives rise to a non-vanishing NS distribution



Energy cascade: gluon + flavor singlet quark channel $D_{g/s}(x) \sim 1/\sqrt{x}$

Valence particle number cascade: flavor non-singlet quark channel $D_{NS}(x) \sim \sqrt{x}$