# Exploring the phase structure and dynamics of QCD


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**Universität Heidelberg & ExtreMe Matter Institute**

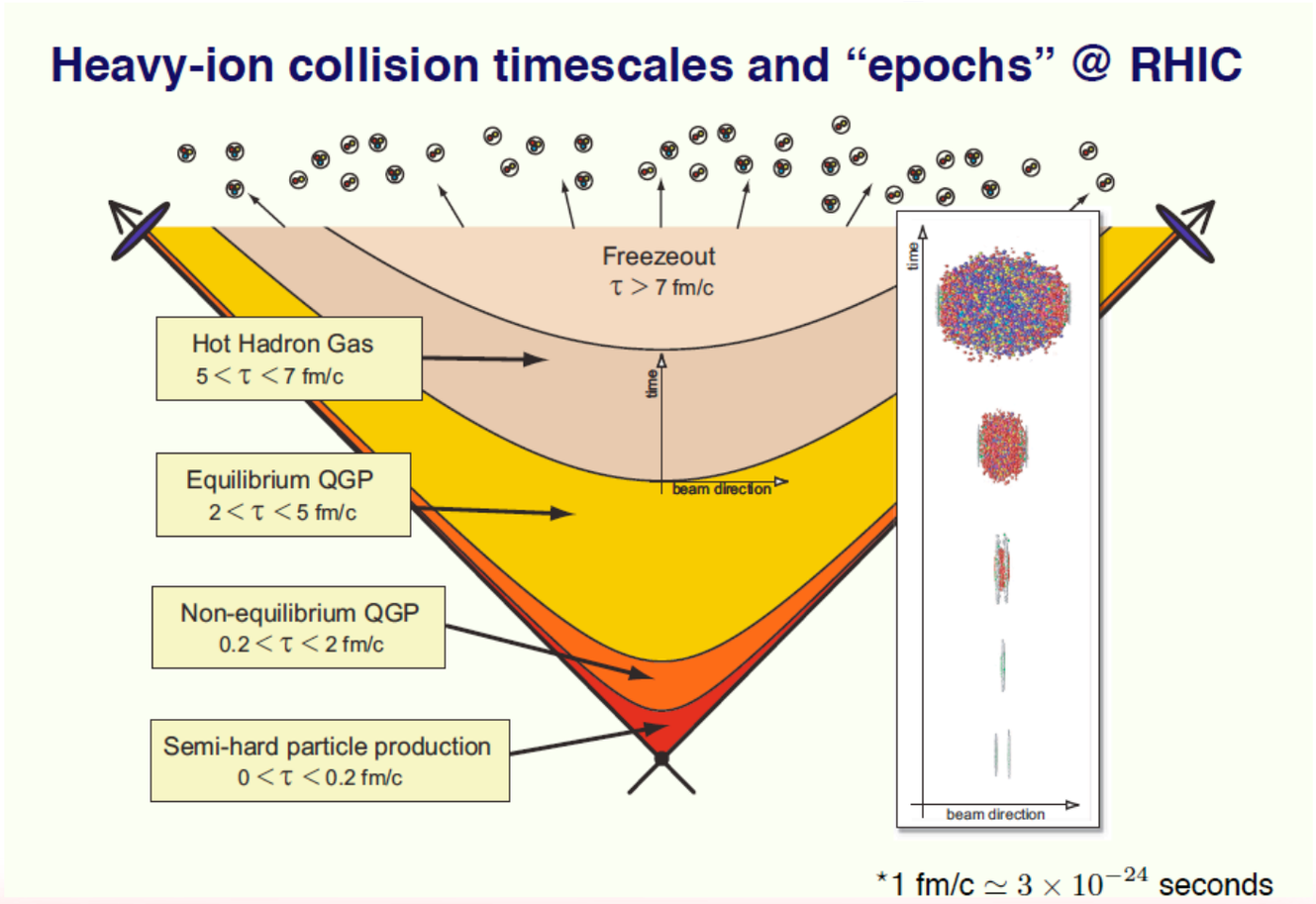
**Frankfurt, May 21st 2018**

GEFÖRDERT VOM

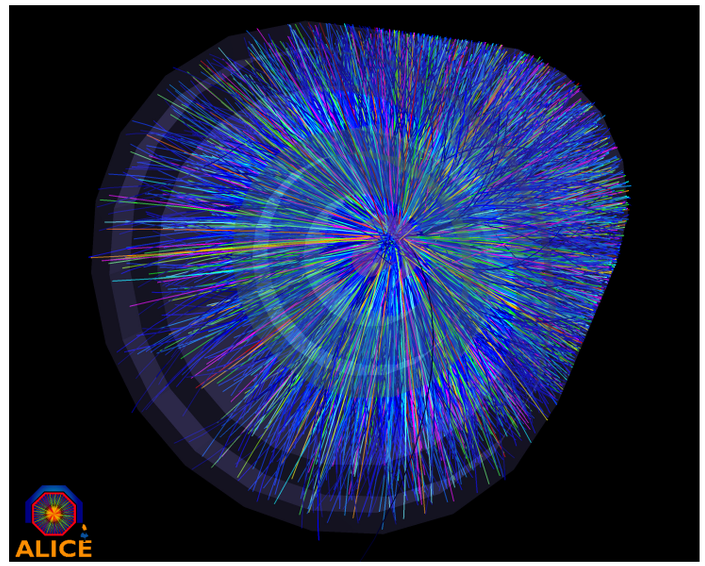
Bundesministerium
für Bildung
und Forschung

ISOQUANT
SFB1225

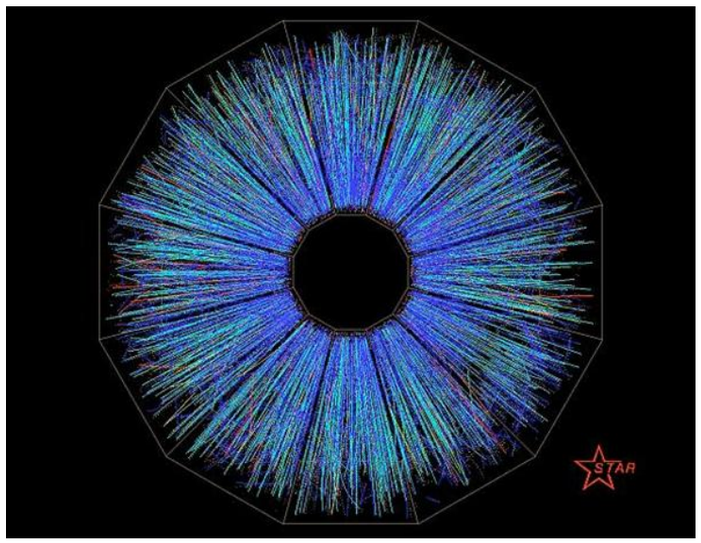
# Heavy ion collisions



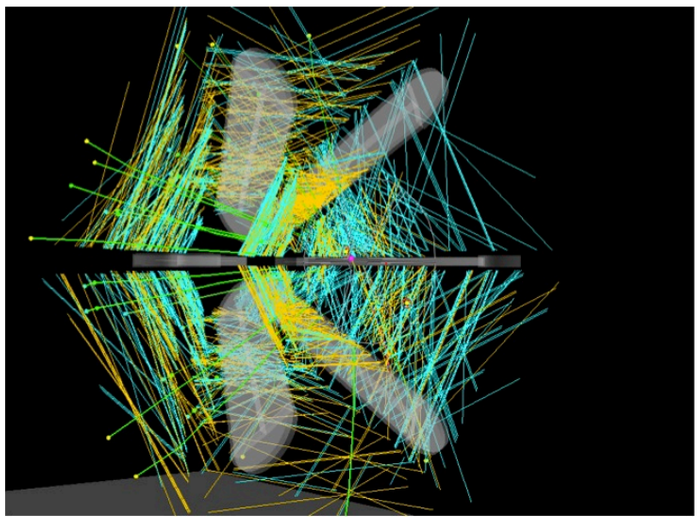

LHC

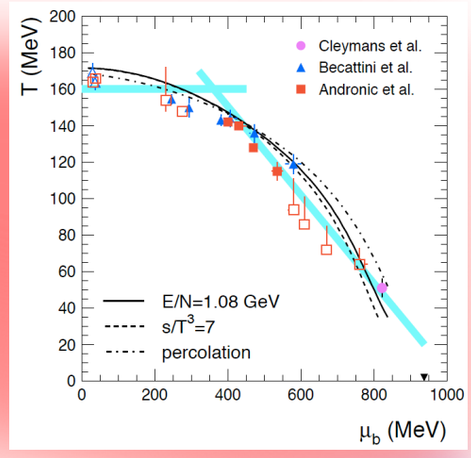



RHIC

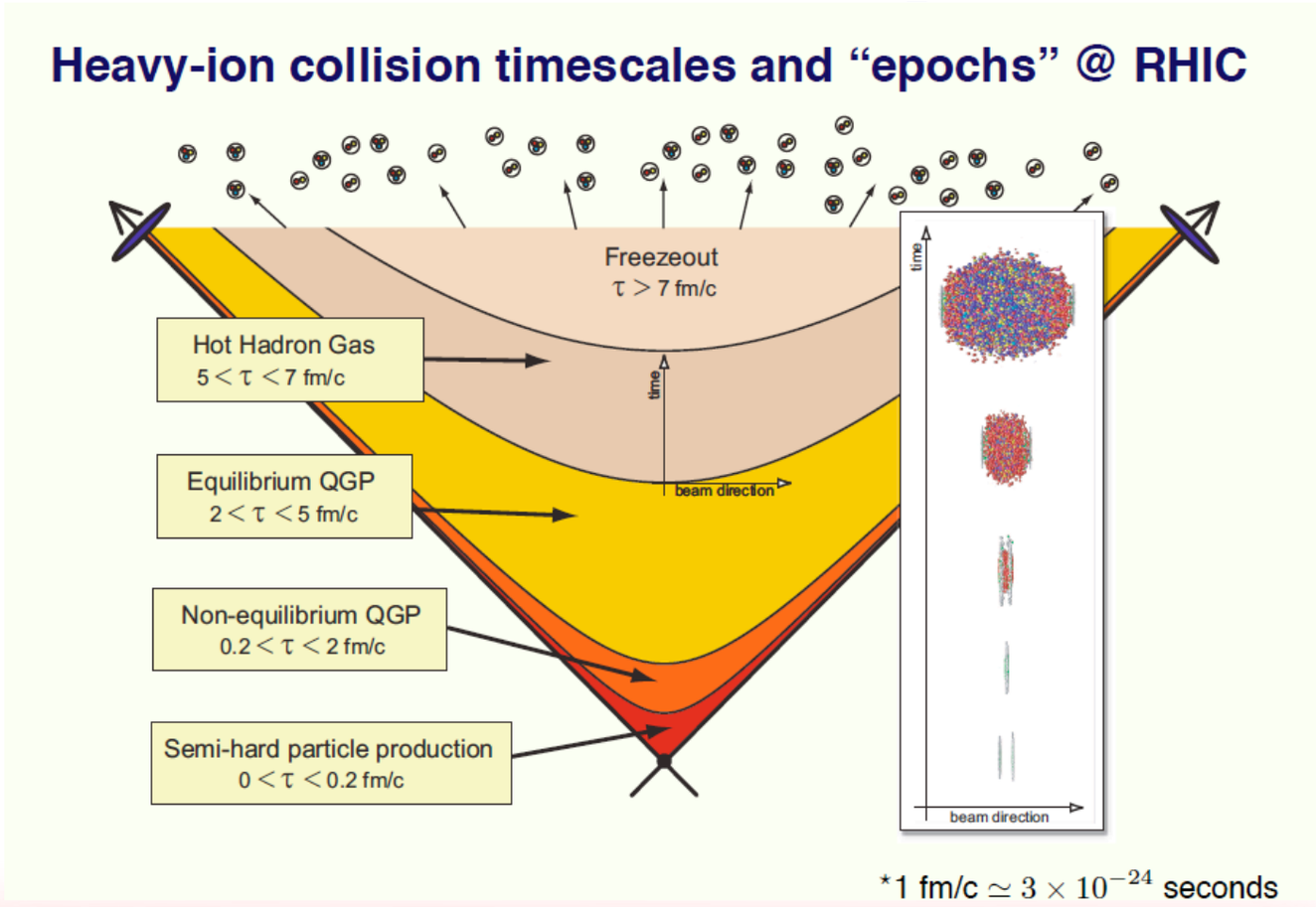


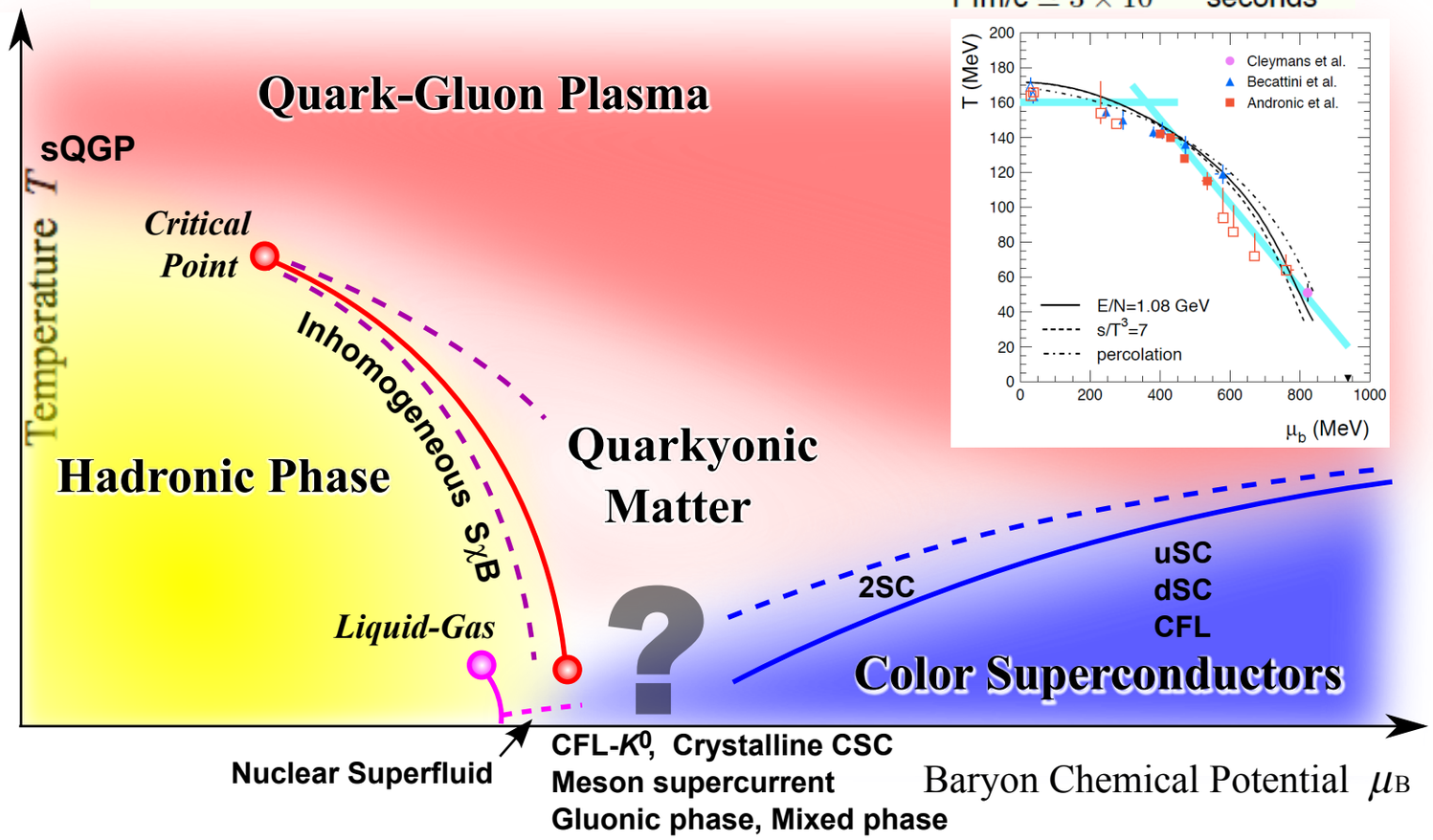

HADES





# Heavy ion collisions

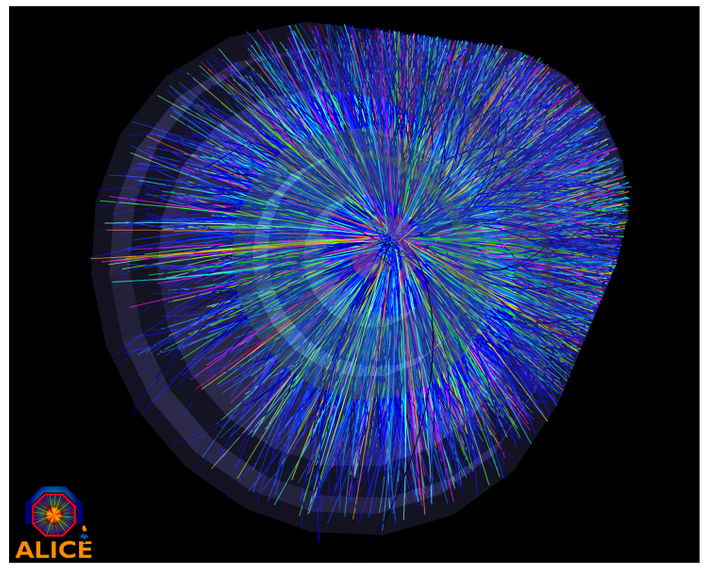


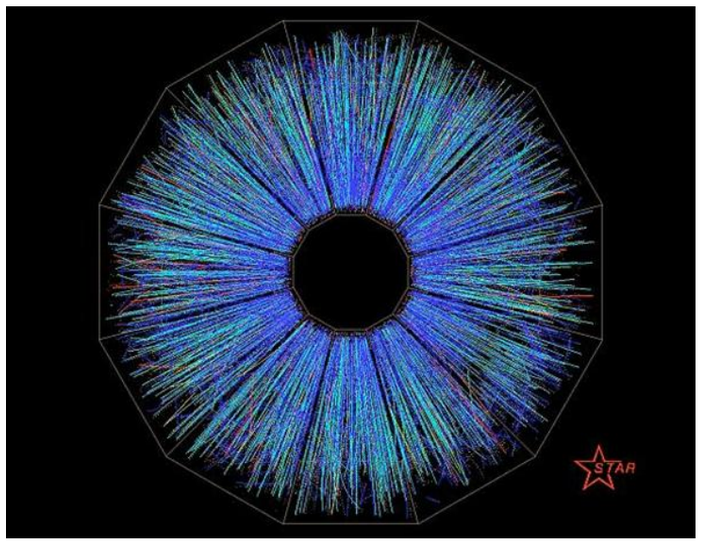




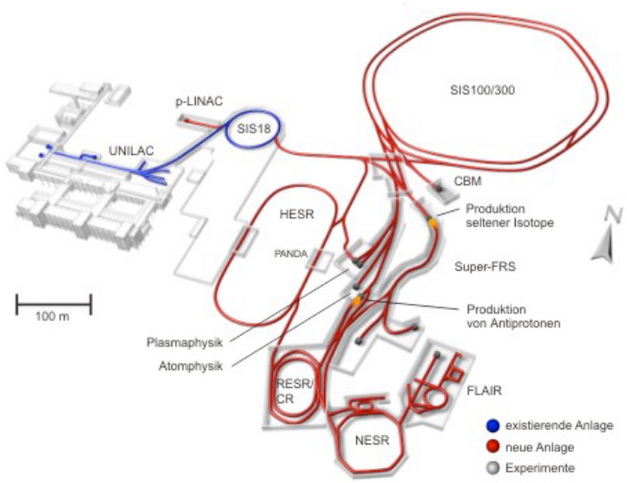
LHC




RHIC

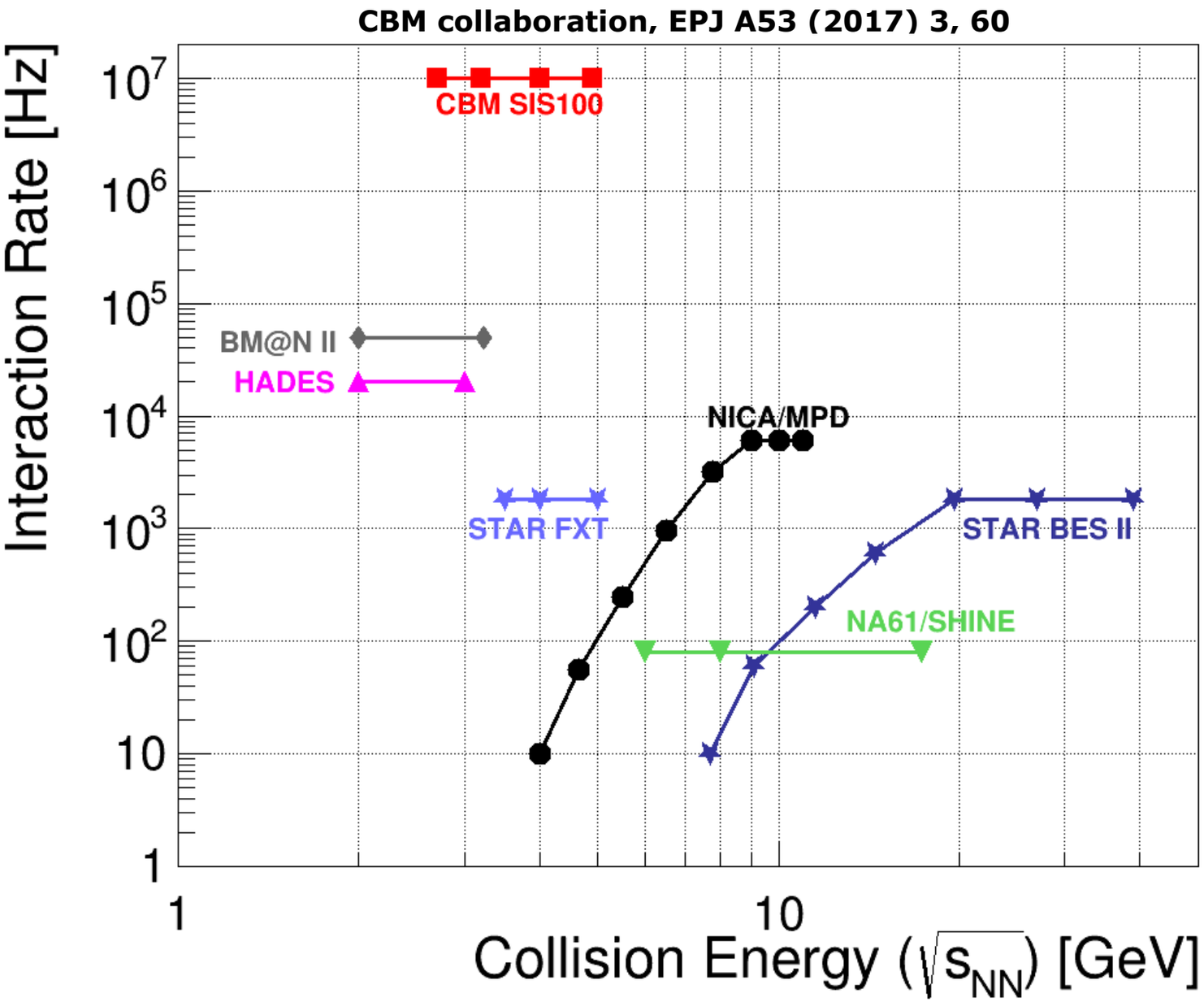



FAIR

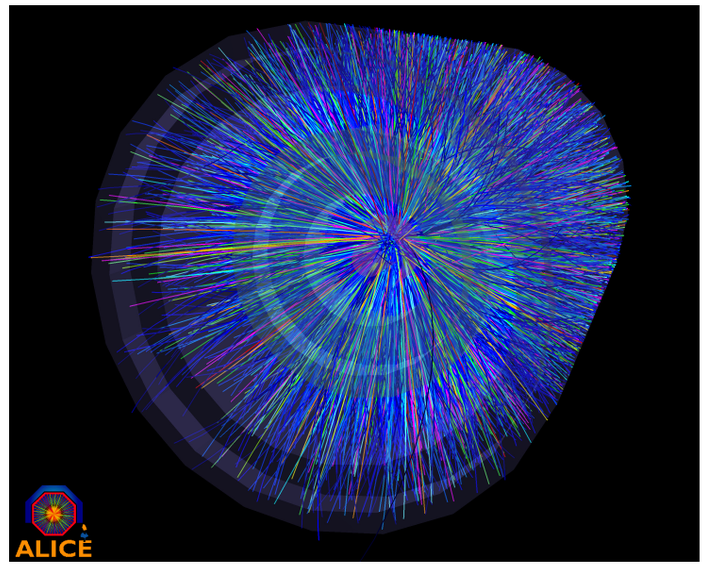



NICA

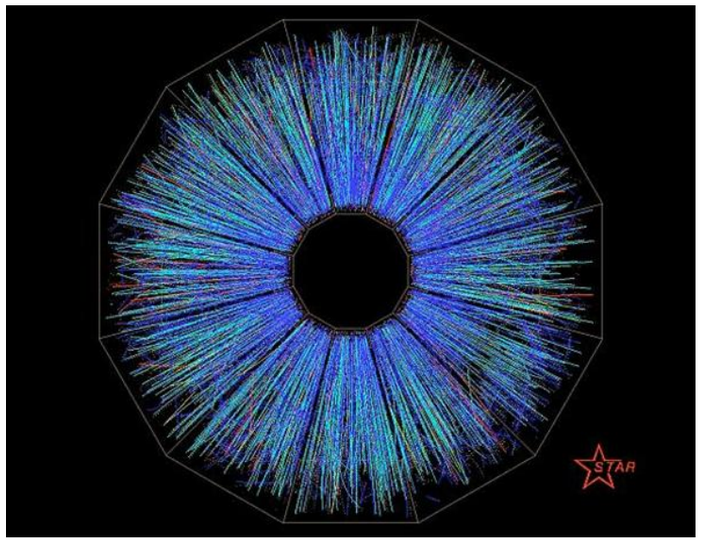
# Heavy ion collisions



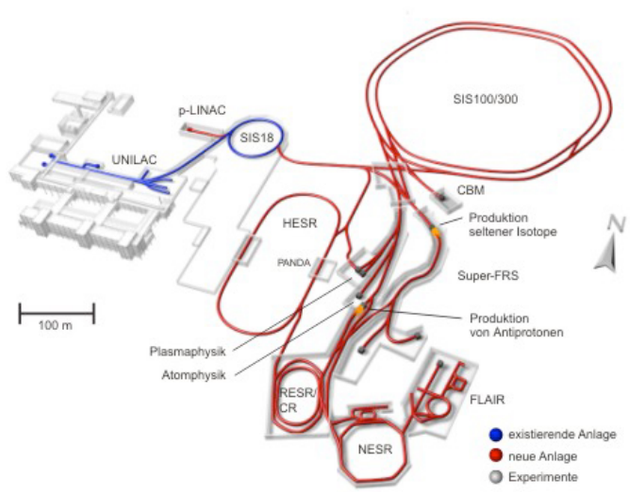

**LHC**




**RHIC**




**FAIR**




**NICA**

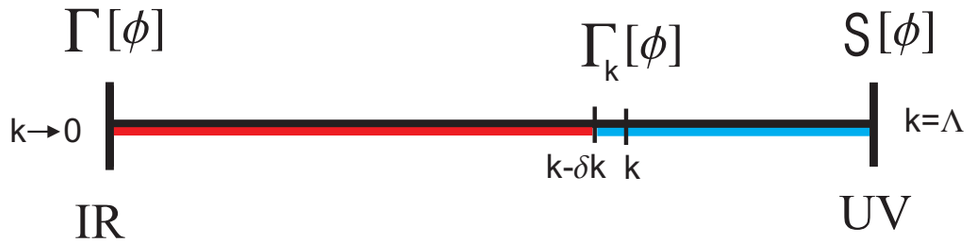
# Outline

- QCD from functional methods
- QCD-assisted hydrodynamics
- QCD-assisted transport
- Summary & outlook

# Functional RG for QCD

eg. JMP, AIP Conf.Proc. 1343 (2011)
NPA 931 (2014) 113

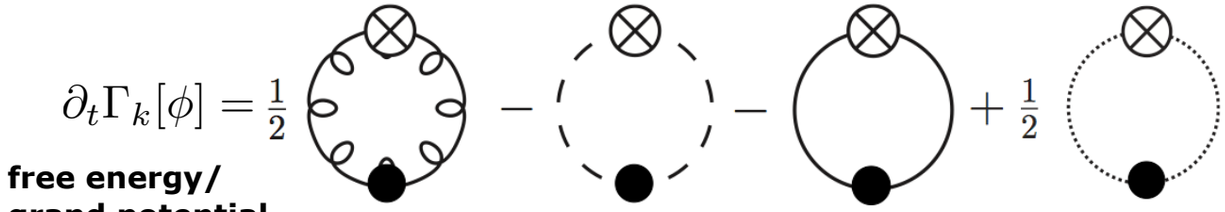
**free energy at momentum scale k**




**ab initio**

**glue**
**quantum fluctuations**

**hadronic**
**quantum fluctuations**




**free energy/**
**grand potential**

**quark**
**quantum fluctuations**

$\text{RG-scale k: } t = \ln k$

**closed form**

## properties

- **access to physics**

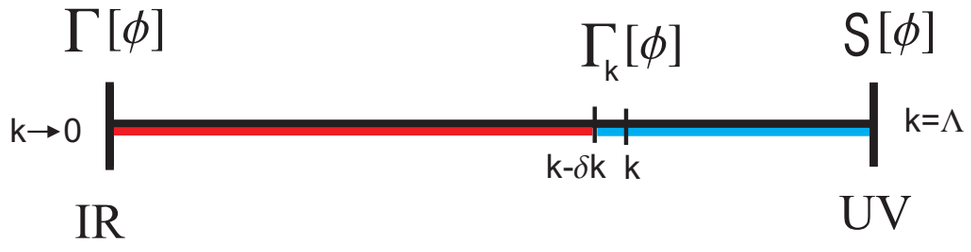
- **numerically tractable, also at real time**
  **no sign problem**
  **systematic error control via closed form**

- **low energy models naturally encorporated**

# Functional RG for QCD

eg. JMP, AIP Conf.Proc. 1343 (2011)
NPA 931 (2014) 113

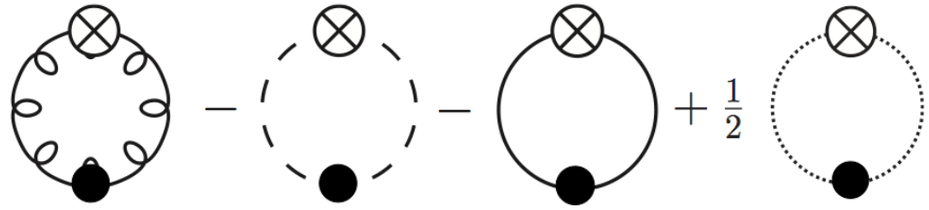
**free energy at momentum scale k**




**ab initio**

**glue**
**quantum fluctuations**

**hadronic**
**quantum fluctuations**

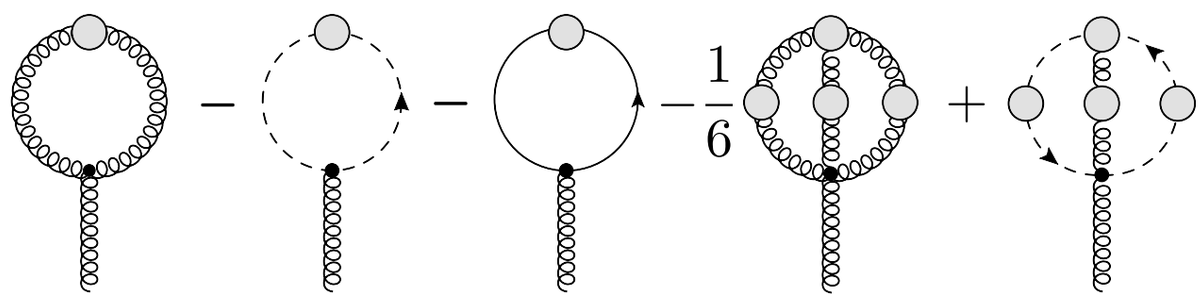
$$\partial_t \Gamma_k[\phi] = \frac{1}{2}$$  $$+ \frac{1}{2}$$

**free energy/**
**grand potential**

**quark**
**quantum fluctuations**

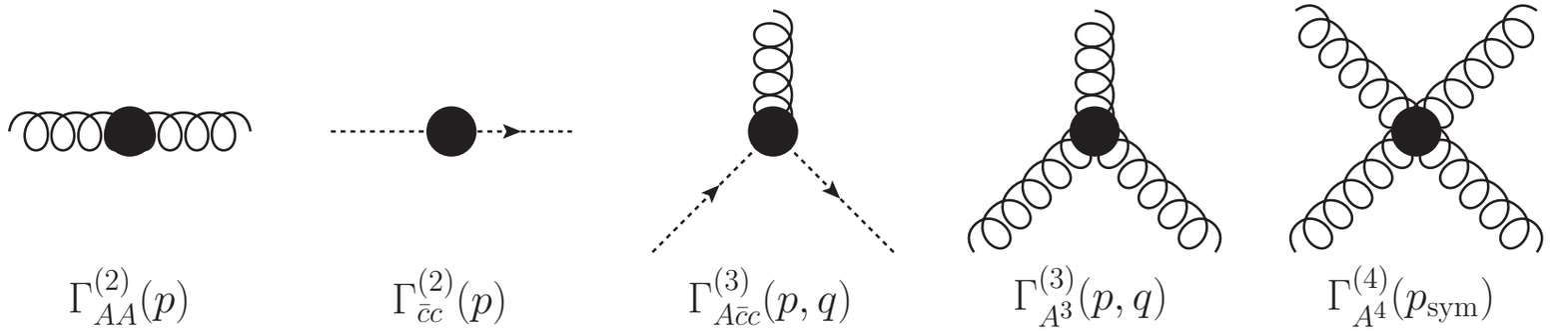
RG-scale k: $t = \ln k$

**closed form**

**functional DSE :** $\dfrac{\delta(\Gamma - S)}{\delta A_0} = \dfrac{1}{2}$  $-\dfrac{1}{6}$ 

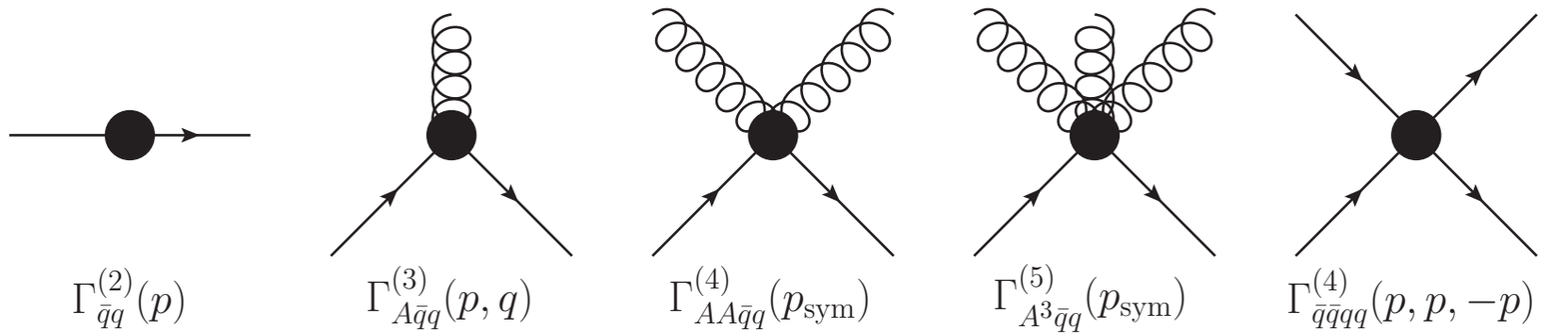
$A_0$ **: background field**

# QCD: current set of correlation functions



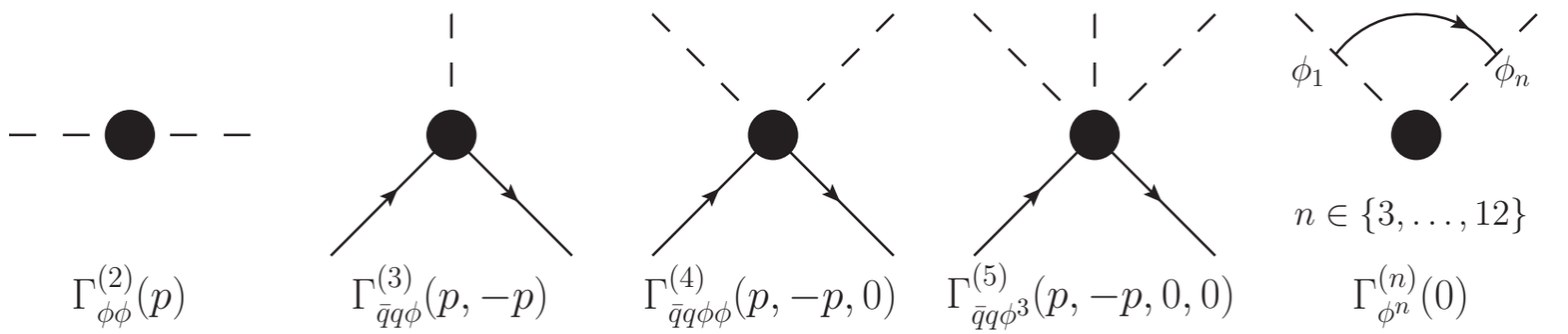
$\Gamma^{(2)}_{AA}(p)$ $\Gamma^{(2)}_{\bar{c}c}(p)$ $\Gamma^{(3)}_{A\bar{c}c}(p,q)$ $\Gamma^{(3)}_{A^3}(p,q)$ $\Gamma^{(4)}_{A^4}(p_{\rm sym})$

classical tensor classical tensor



$\Gamma^{(2)}_{\bar{q}q}(p)$ $\Gamma^{(3)}_{A\bar{q}q}(p,q)$ $\Gamma^{(4)}_{AA\bar{q}q}(p_{\rm sym})$ $\Gamma^{(5)}_{A^3\bar{q}q}(p_{\rm sym})$ $\Gamma^{(4)}_{\bar{q}\bar{q}qq}(p,p,-p)$

$\bar{q}\not{D}^n q$ complete, $n \leq 3$ mom.–ind. tensors




$\Gamma^{(2)}_{\phi\phi}(p)$ $\Gamma^{(3)}_{\bar{q}q\phi}(p,-p)$ $\Gamma^{(4)}_{\bar{q}q\phi\phi}(p,-p,0)$ $\Gamma^{(5)}_{\bar{q}q\phi^3}(p,-p,0,0)$ $\Gamma^{(n)}_{\phi^n}(0)$

$\phi \in \{\sigma, \vec{\pi}\}$ $n \in \{3,\ldots,12\}$

"classical" tensor "classical" tensor

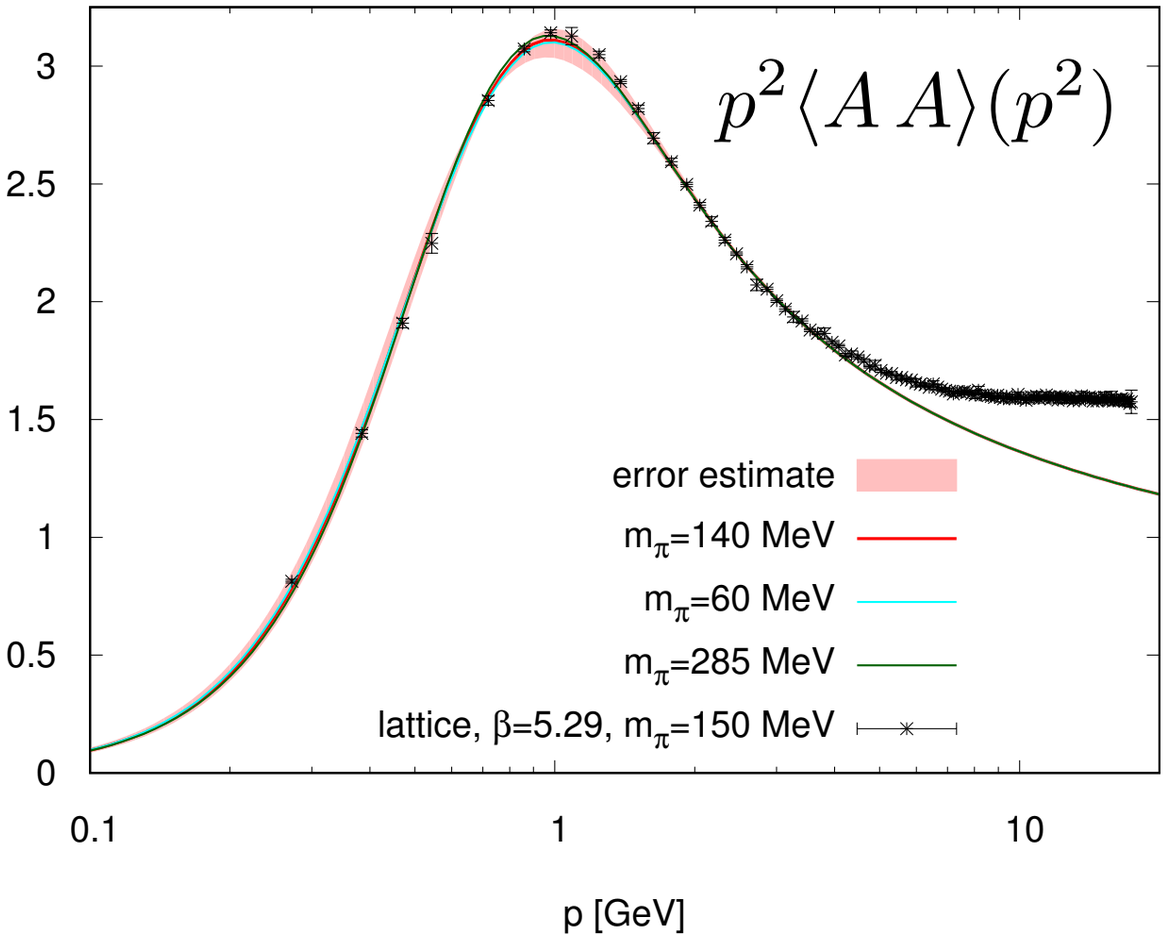
**Aiming at apparent convergence**


**Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006, PRD 97 (2018) 054015**

**Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005**

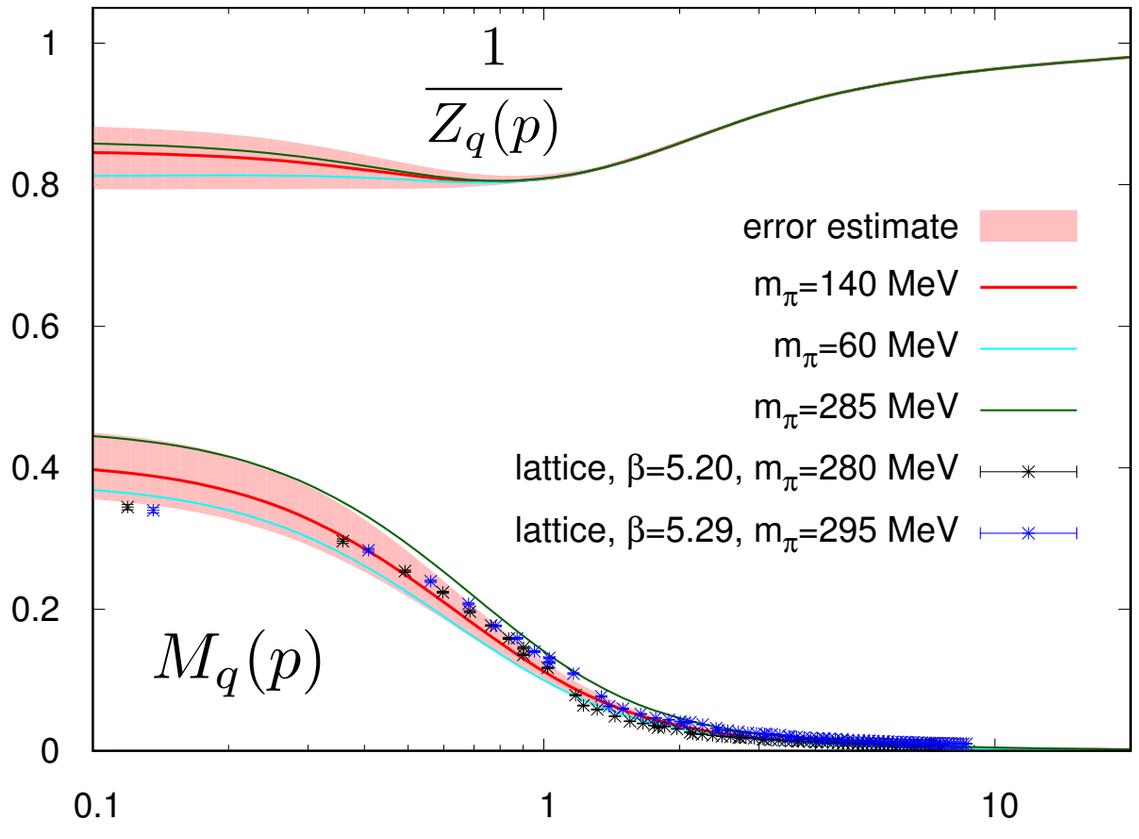
**Mitter, JMP, Strodthoff, PRD 91 (2015) 054035**

# QCD: Euclidean propagators




**lattice, e.g.: Oliviera et al, Acta Phys.Polon.Supp. 9 (2016) 363**
**Sternbeck et al, PoS LATTICE2016 (2017)**
**A. Athenodorou et al, PLB 761 (2016) 444**

$$\frac{1}{Z_q(p)}\;\frac{1}{i\,\not{p}+M_q(p)}$$



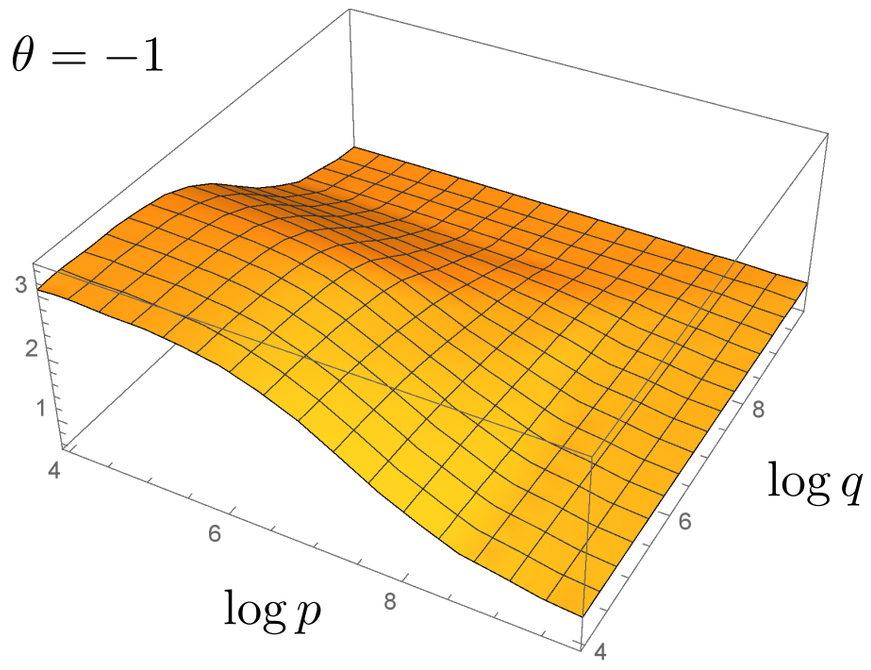

**Aiming at apparent convergence**

**Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006**


# QCD: Quark-gluon vertex

$$\theta = \frac{p \cdot q}{\sqrt{p^2\, q^2}}$$

**p,q in MeV**




$\lambda_1(p,q)$

**up-to-date 1st principles works:**

*Beware of BRST*

**FunMethods:** **Williams, EPJ A51 (2015) 57**
**Sanchis-Alepuz, Williams, PLB 749 (2015) 592**
**Williams, Fischer, Heupel, PRD 93 (2016) 034026**

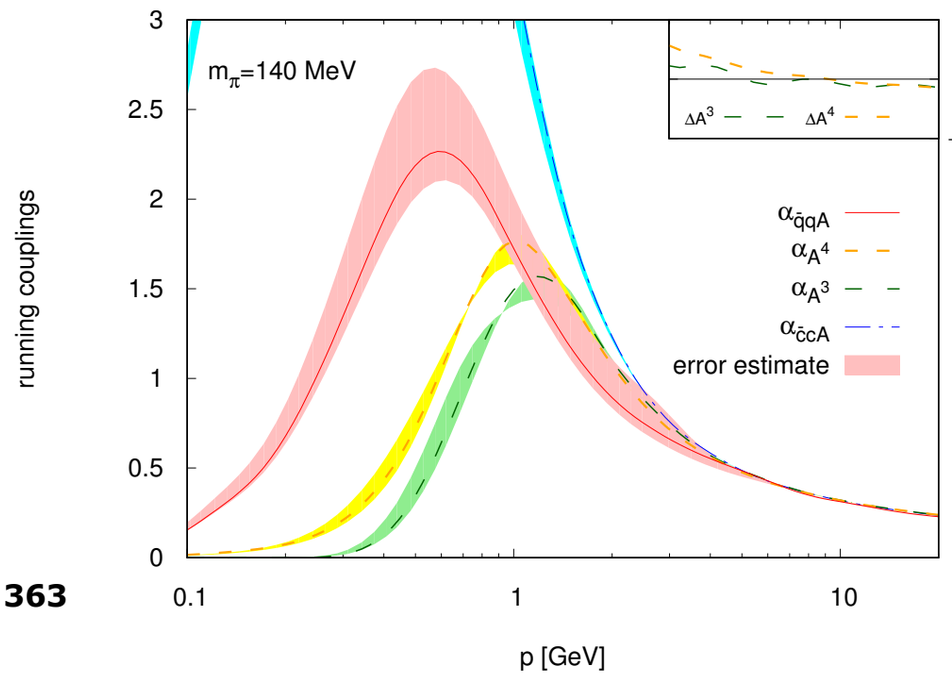
**Aguilar, Binosi, Ibanez, Papavassiliou, PRD 89 (2014) 065027**
**Binosi, Chang, Papavassiliou, Qin, Roberts, PRD 95 (2017) 031501**
**Aguilar, Cardona, Ferreira, Papavassiliou, arXiv:1610.06158**

**Mitter, JMP, Strodthoff, PRD 91 (2015) 054035**

**Pelaez, Tissier, Wschebor, PRD 92 (2015) 045012**

**Eichmann, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016) 1**

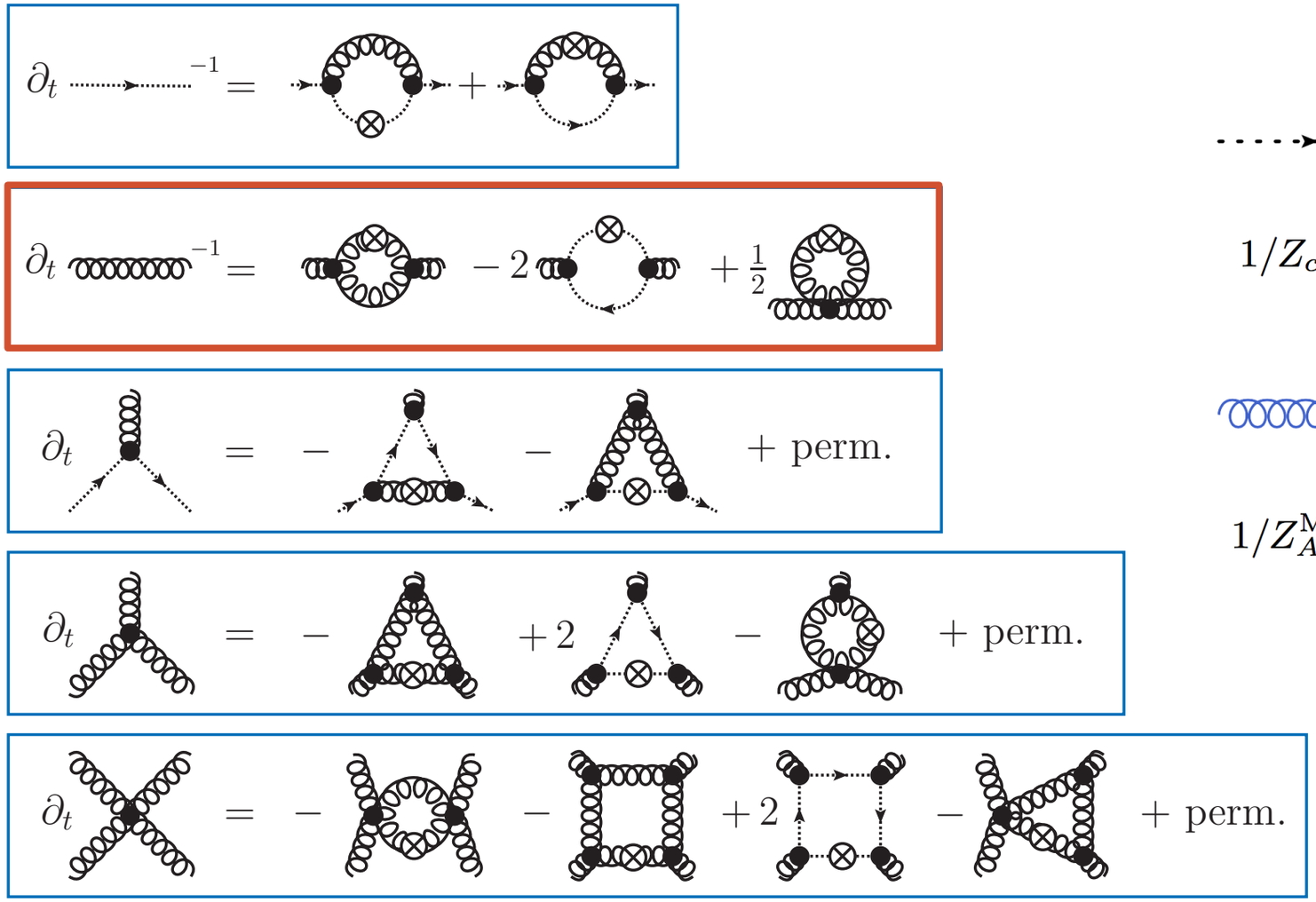
**lattice:** **Oliveira, Kizilersü, Silva, Skullerud, Sternbeck, Williams, APP Suppl. 9 (2016) 363**

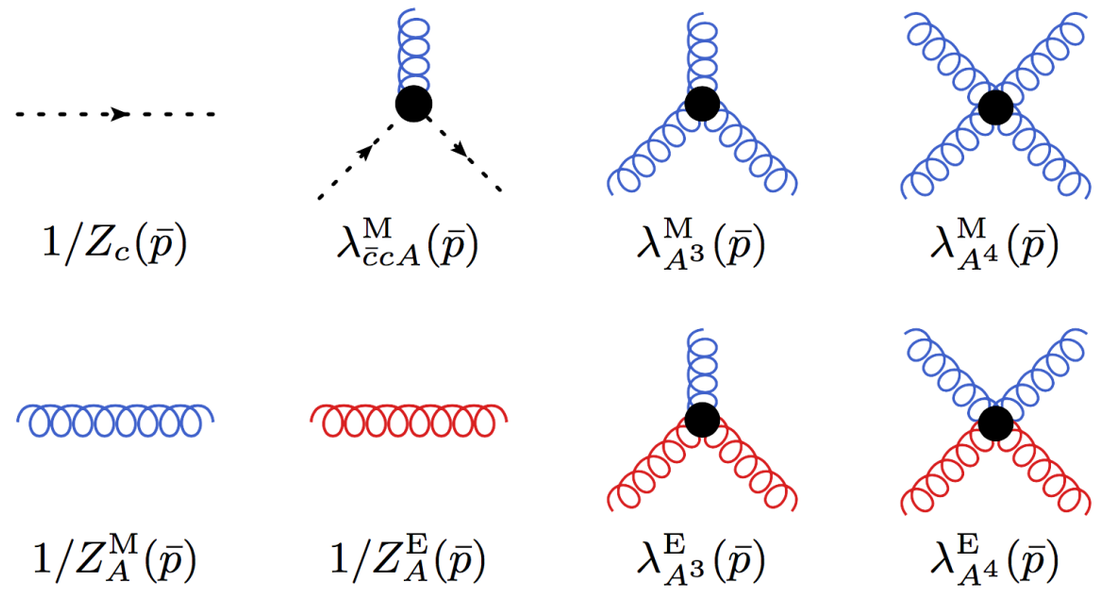



**Aiming at apparent convergence**

**Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006**

# YM-theory: gluonic correlation functions



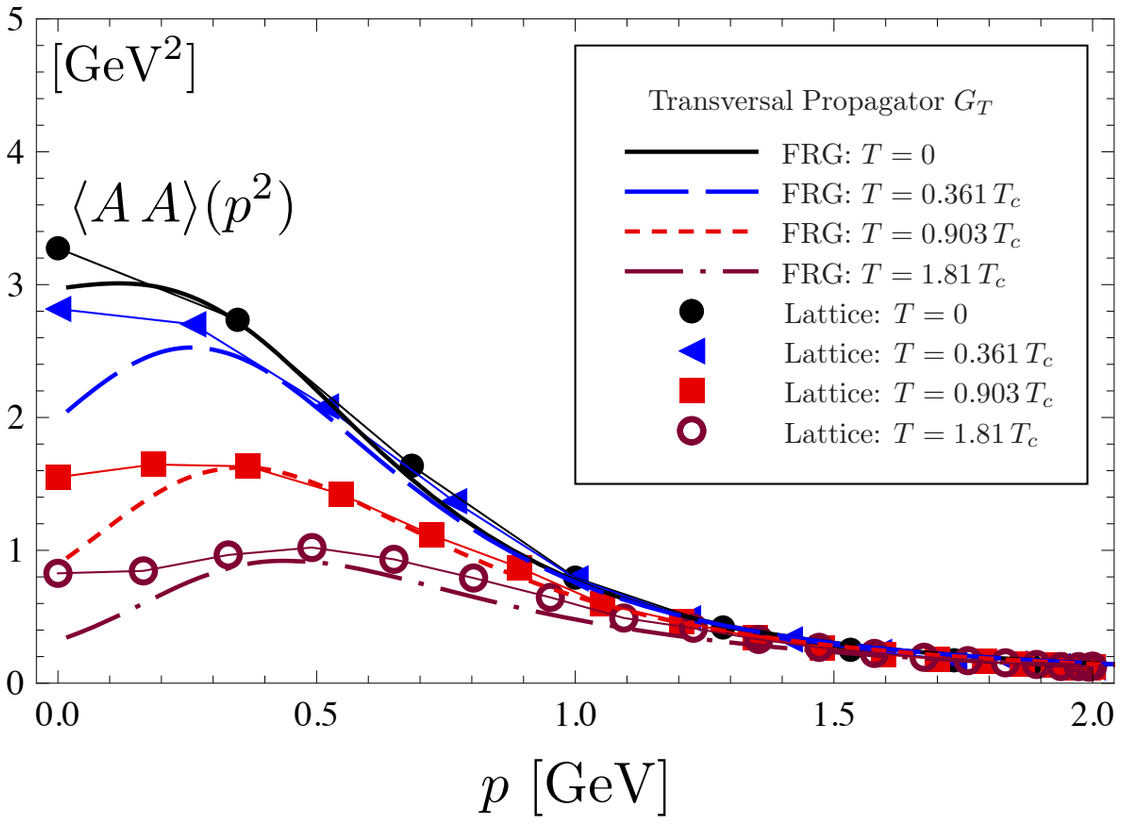

**Aiming at apparent convergence**

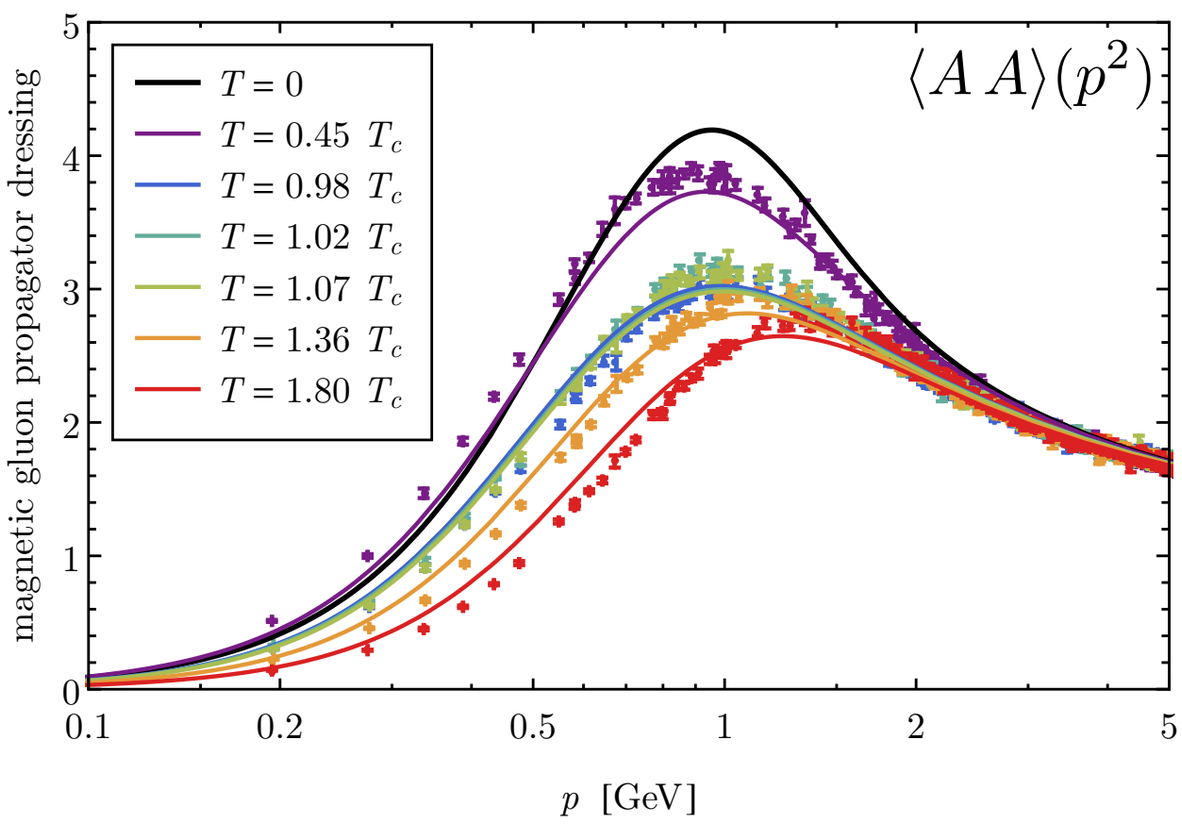
**Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 97 (2018) 054015**

# Euclidean gluon propagator at finite T

## Yang-Mills propagators, finite T

### chromo-magnetic propagator




**Fister, JMP, arXiv:1112.5440**

**Lattice: Maas, JMP, Smekal, Spielmann, PRD 85 (2012) 034037**

**CF model: Reinosa, Serreau, Tissier, Tresmontant, PRD 95 (2017) 045014**

**Lattice: Silva, Oliviera, Bicudo, Cardoso, PRD89 (2014) 7, 074503**

**Aiming at apparent convergence**

**Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 97 (2018) 054015**

# Outline

- QCD from functional methods
- QCD-assisted hydrodynamics
- QCD-assisted transport
- Summary & outlook

# QCD-assisted hydrodynamics

**Dubla, Masciocchi, JMP, Schenke, Shen, Stachel, arXiv:1805.02985**

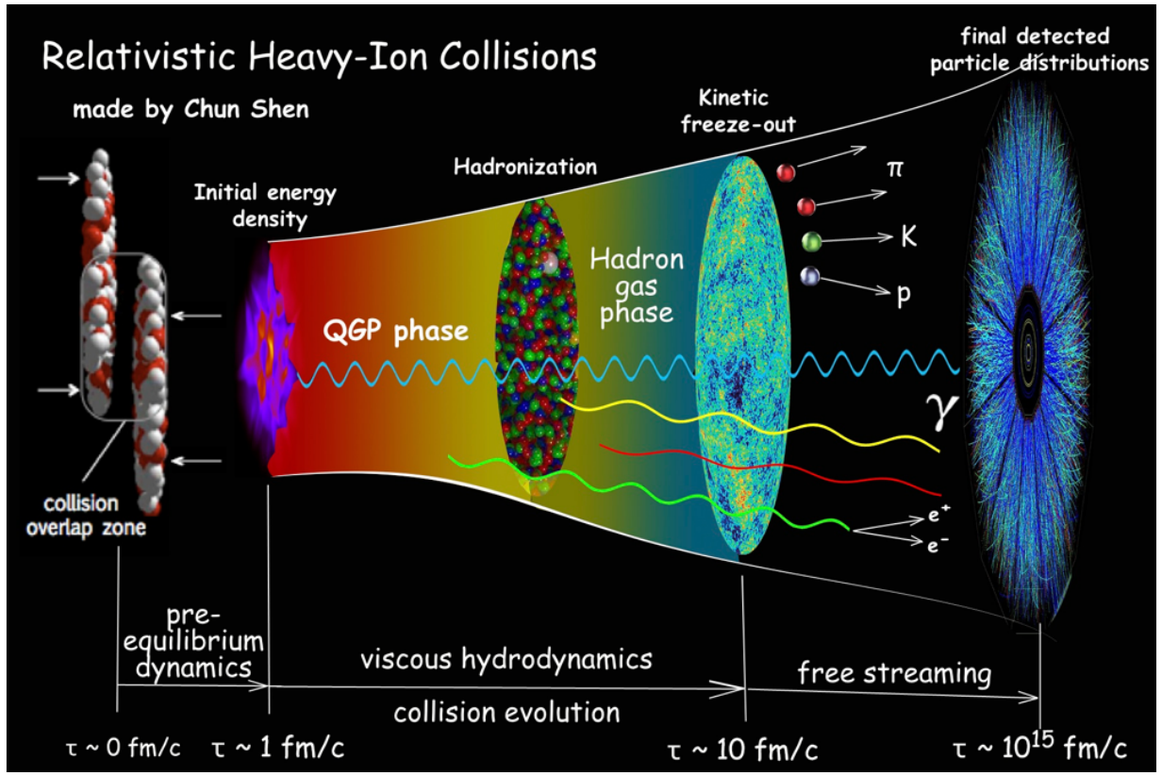
**HIC 'phases'**

**Far from equilibrium initial phase**

**Kinetic phase**

**Hydrodynamical phase**

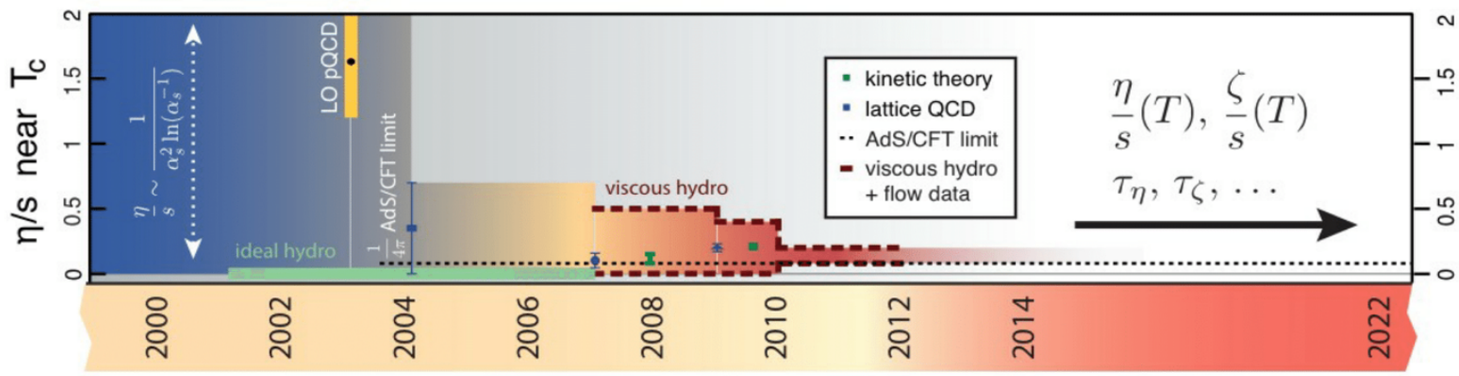
**Hadronisation & freeze out**




**QCD-assisted transport**

**Hydro with QCD transport coefficients**

**Equilibrium transport coefficients**

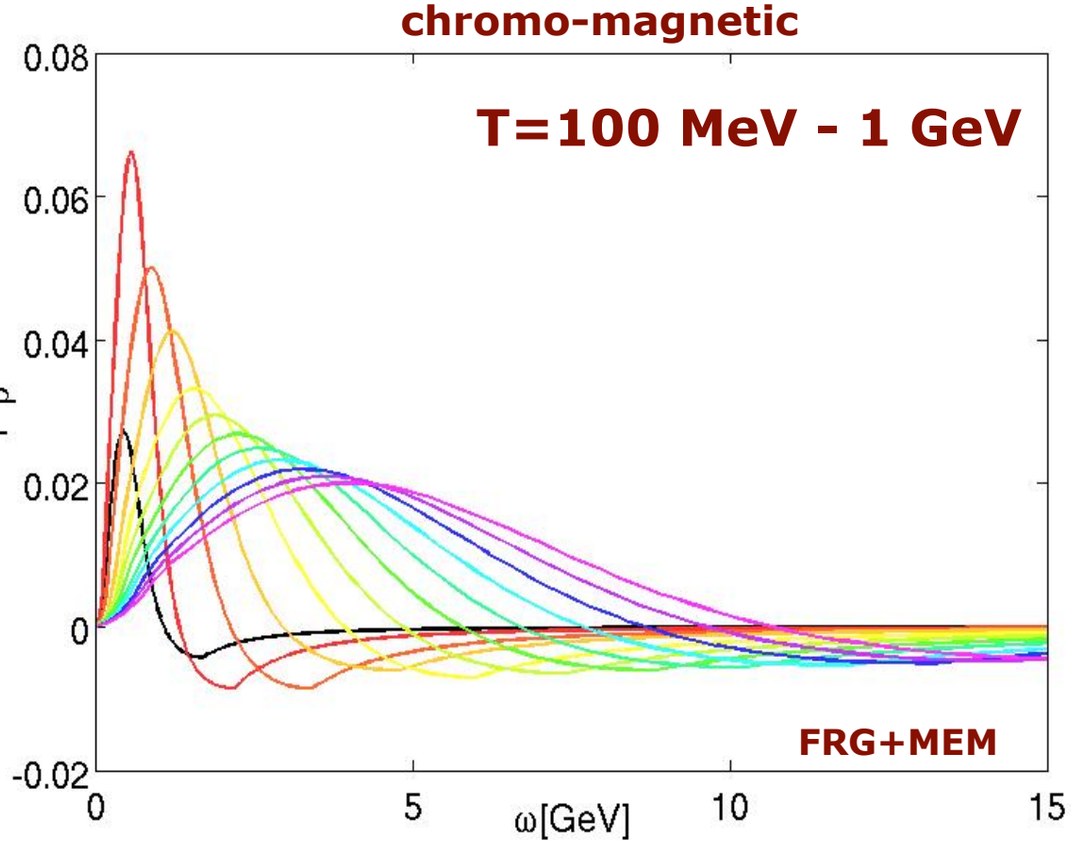



**'Steady-state' hydro**

**Constraints for the other phases**

$$\pi^{\mu\nu} = \eta(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha) - \frac{4}{3}\tau_\pi \pi^{\mu\nu}\partial_\alpha u^\alpha - \tau_\pi \Delta^\mu_\alpha \Delta^\nu_\beta u^\sigma \partial_\sigma \pi^{\alpha\beta} ,$$

# Single particle spectral functions

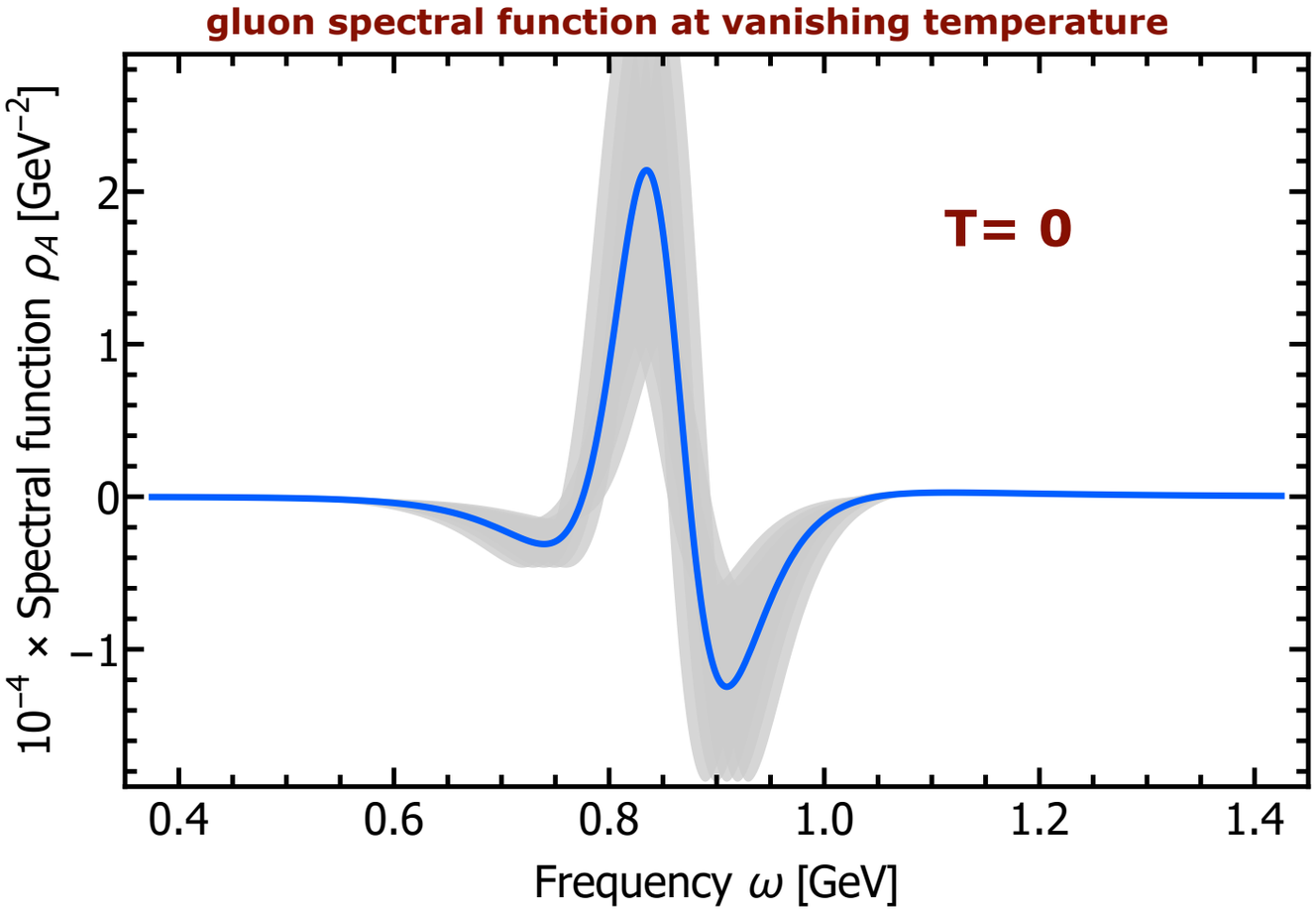



$$\rho(p) = 2\,\mathrm{Im}\langle A\,A\rangle_{\mathrm{ret}}(p)$$




Haas, Fister, JMP, PRD 90 (2014) 091501

# Single particle spectral functions




$$\rho(p) = 2\,\mathrm{Im}\langle A\,A\rangle_{\mathrm{ret}}(p)$$

**novel analytic IR (& UV) behaviour and qualitatively refined reconstruction**

**Cyrol, JMP, Rothkopf, Wink, arXiv:1804.00945**

# Transport coefficients

## viscosity over entropy ratio in Yang-Mills theory

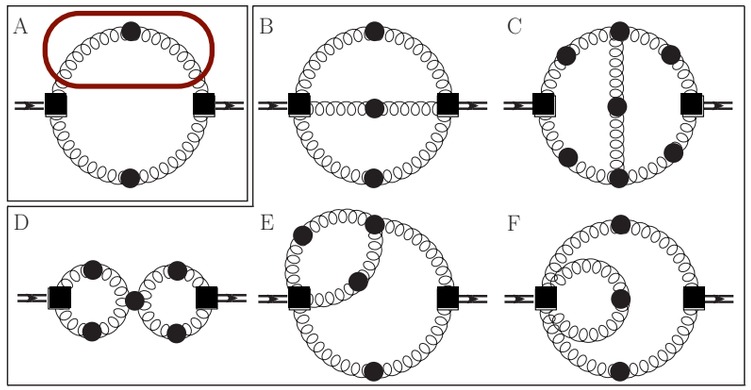
**Kubo relation**

$$\eta = \frac{1}{20} \left. \frac{d}{d\omega} \right|_{\omega=0} \rho_{\pi\pi}(\omega, 0)$$

**Gluon spectral function**

**'3-loop' exact functional relation for $\rho_{\pi\pi}$**

**1 & 2-loop terms**



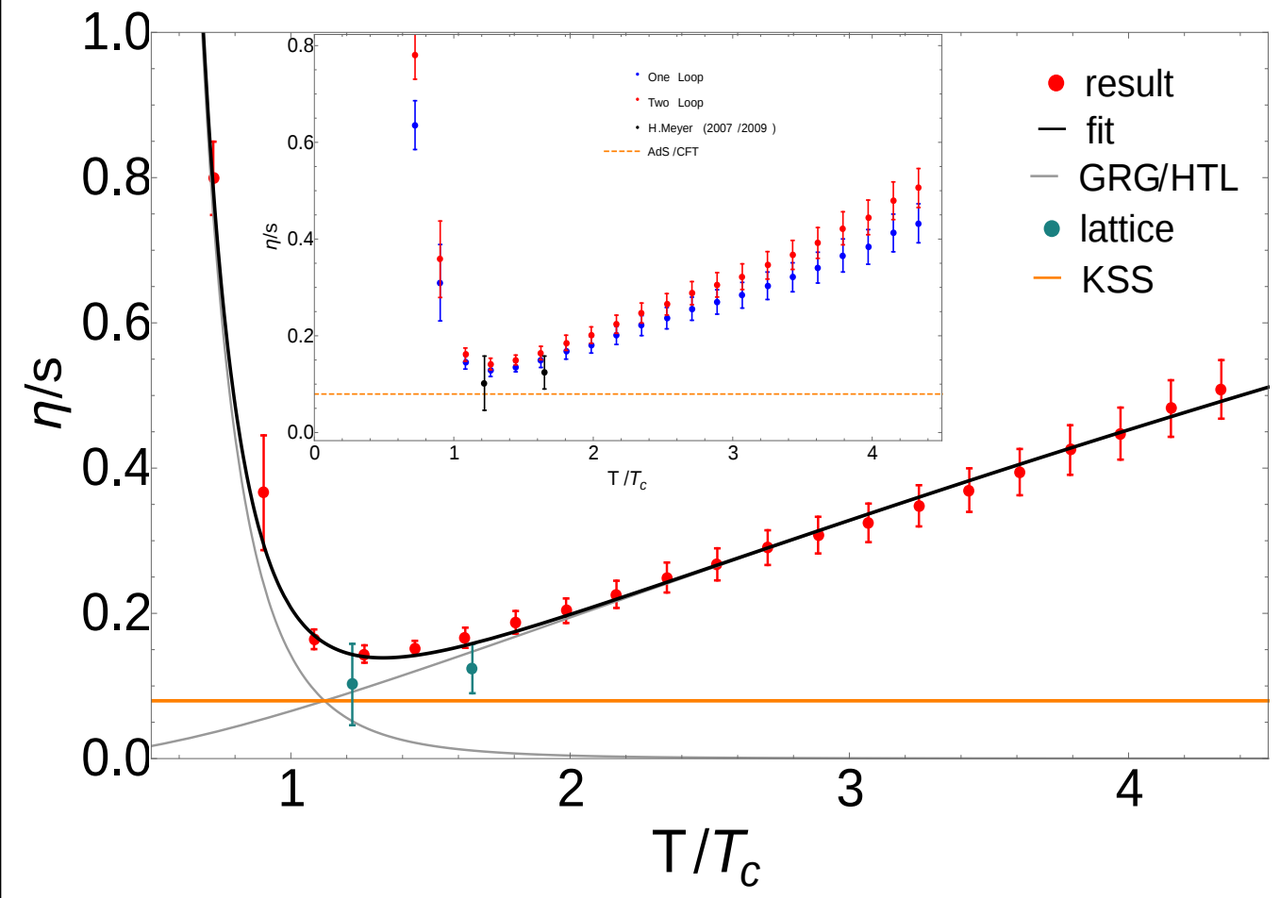

**Haas, Fister, JMP, PRD 90 (2014) 091501**

**Christiansen, Haas, JMP, Strodthoff, PRL 115 (2015) 112002**

# Transport coefficients

## viscosity over entropy ratio in Yang-Mills theory

### Yang-Mills viscosity over entropy ratio




recent lattice results: Astrakhantsev, Braguta, Kotov, JHEP 1704 (2017) 101
arXiv:1804.02382

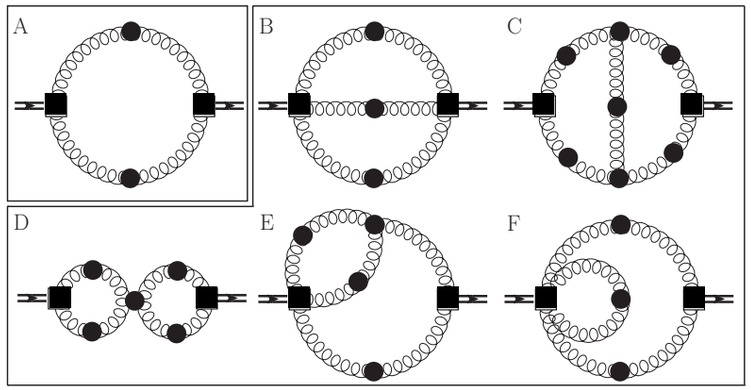
**Aiming at apparent convergence**

### Kubo relation

$$\eta = \frac{1}{20}\,\frac{d}{d\omega}\bigg|_{\omega=0} \rho_{\pi\pi}(\omega,0)$$

**'3-loop' exact functional relation for $\rho_{\pi\pi}$**

**1 & 2-loop terms**



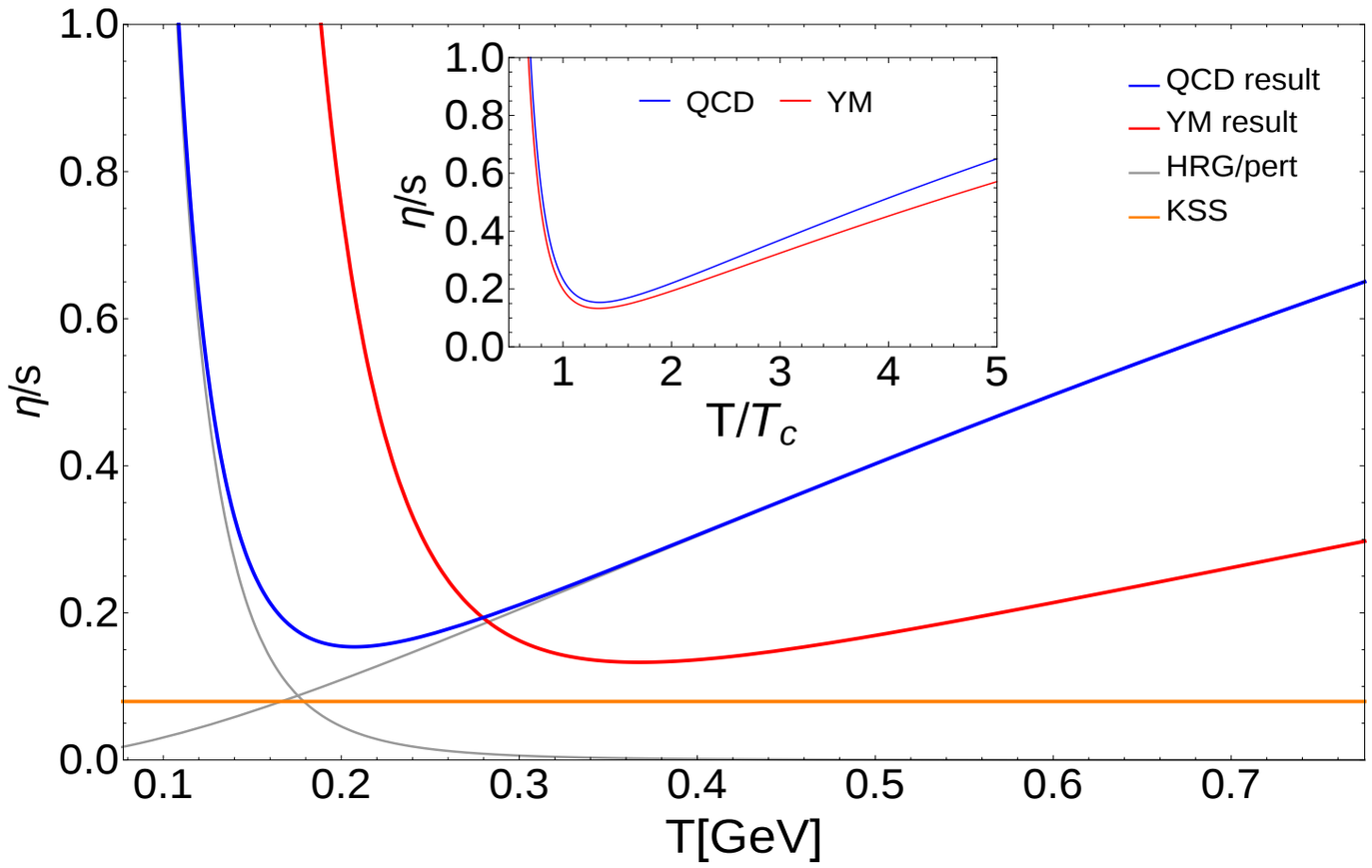

Haas, Fister, JMP, PRD 90 (2014) 091501

Christiansen, Haas, JMP, Strodthoff, PRL 115 (2015) 112002

# Transport coefficients

## QCD - estimate for viscosity over entropy ratio

**viscosity over entropy ratio**




$$a_{\rm qgp} \approx 0.2$$

$$a_{\rm hrg} \approx 0.16$$

$$c \approx 0.79$$

**QCD**

$$\gamma_{\rm grg} \approx 5$$

$$\gamma_{\rm qgp} \approx 1.6$$

**pure glue**

$$a_{\rm qgp} \approx 0.15$$

$$a_{\rm hrg} \approx 0.14$$

$$c \approx 0.66$$

$$\frac{\eta}{s}(T) = \frac{a_{\rm qgp}}{\alpha_s^{\gamma_{\rm qgp}}(cT/T_c)} + \frac{a_{\rm grg}}{(T/T_c)^{\gamma_{\rm grg}}}$$

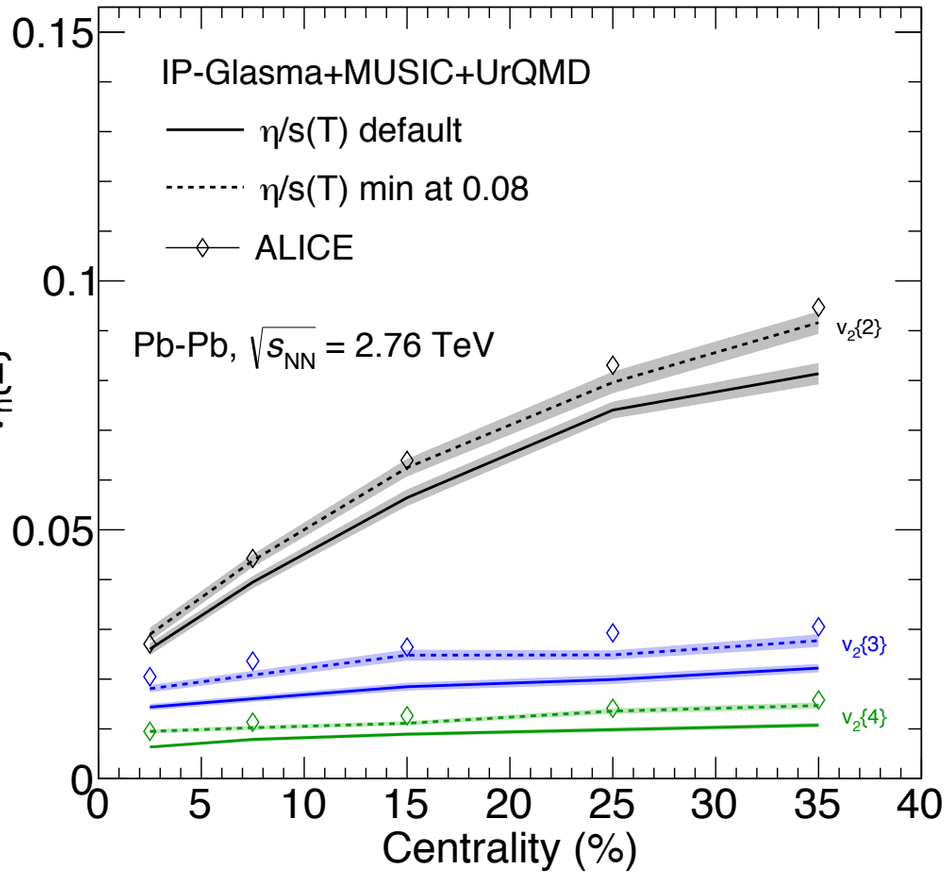
**Christiansen, Haas, JMP, Strodthoff, PRL 115 (2015) 112002**

# QCD-assisted hydrodynamics

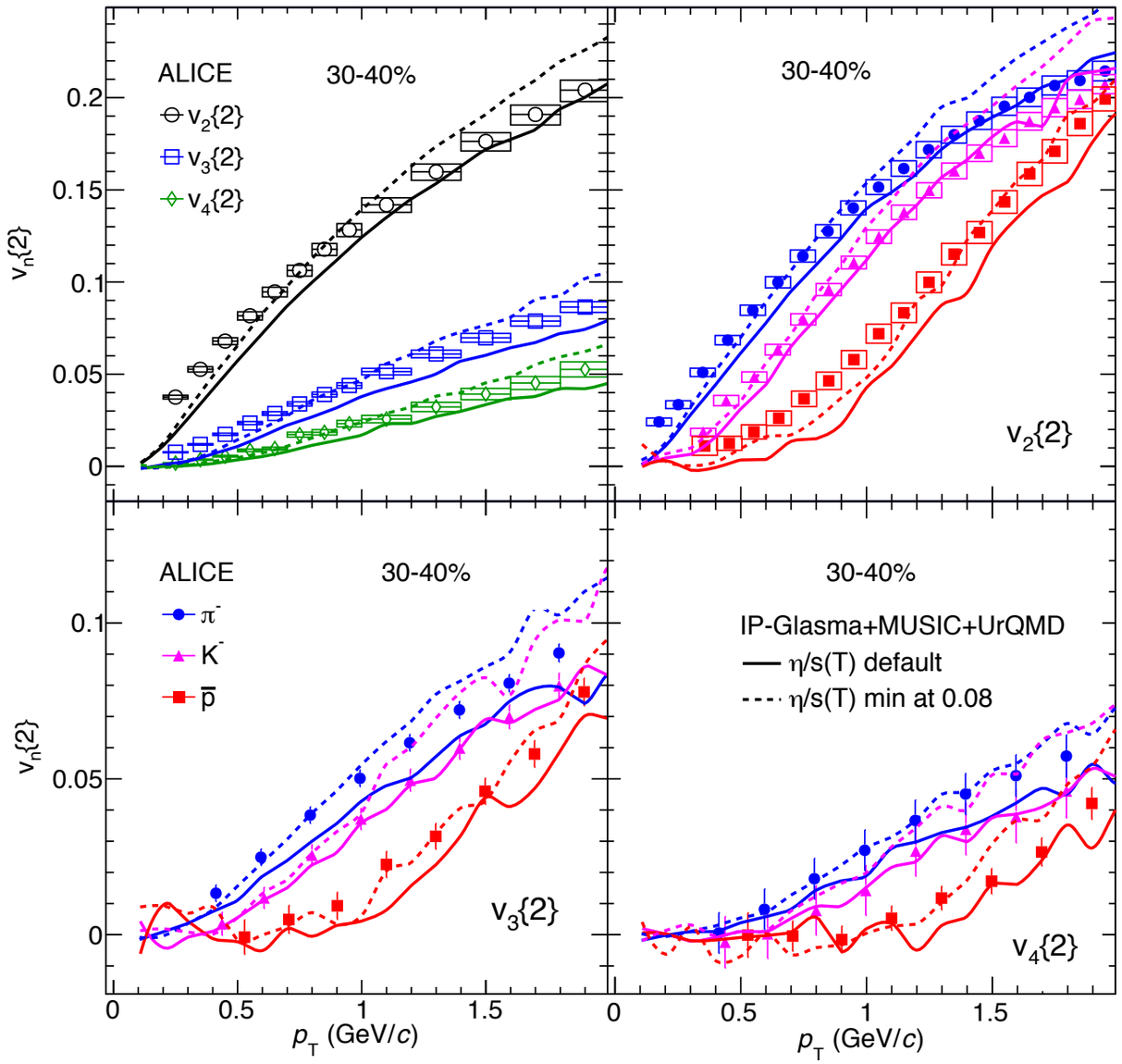
**Dubla, Masciocchi, JMP, Schenke, Shen, Stachel, arXiv:1805.02985**

## IP-Glasma - MUSIC - UrQMD

### $v_n$ as function of centrality




### $v_n$ as function of $p_T$




### Test of systematic error

$$\eta/s(T) \rightarrow \eta/s(T) + d$$

$$d \in [-0.06, 0]$$

**Initial state fluctuations?**

**'Steady-state' hydro?**

**Normalisation!?**

**Kinetic phase?**

**Hadronisation & freeze-out?**

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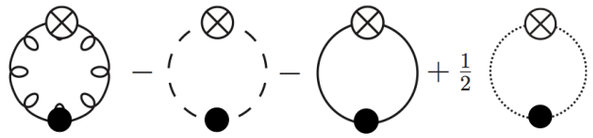
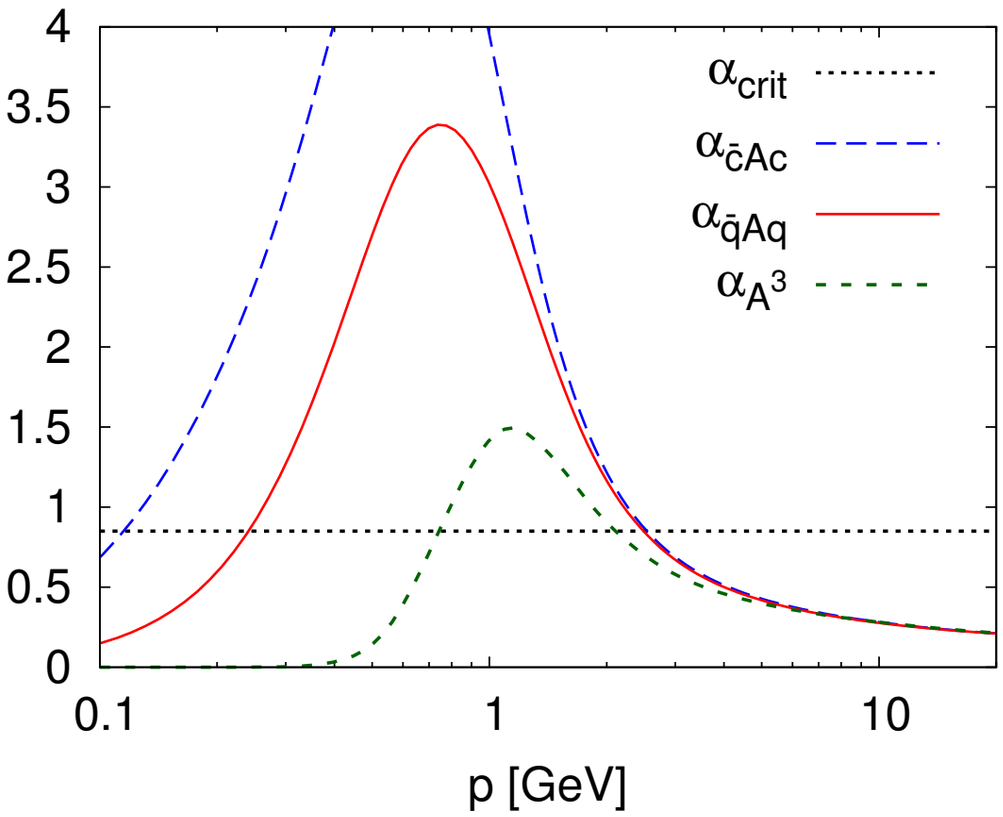
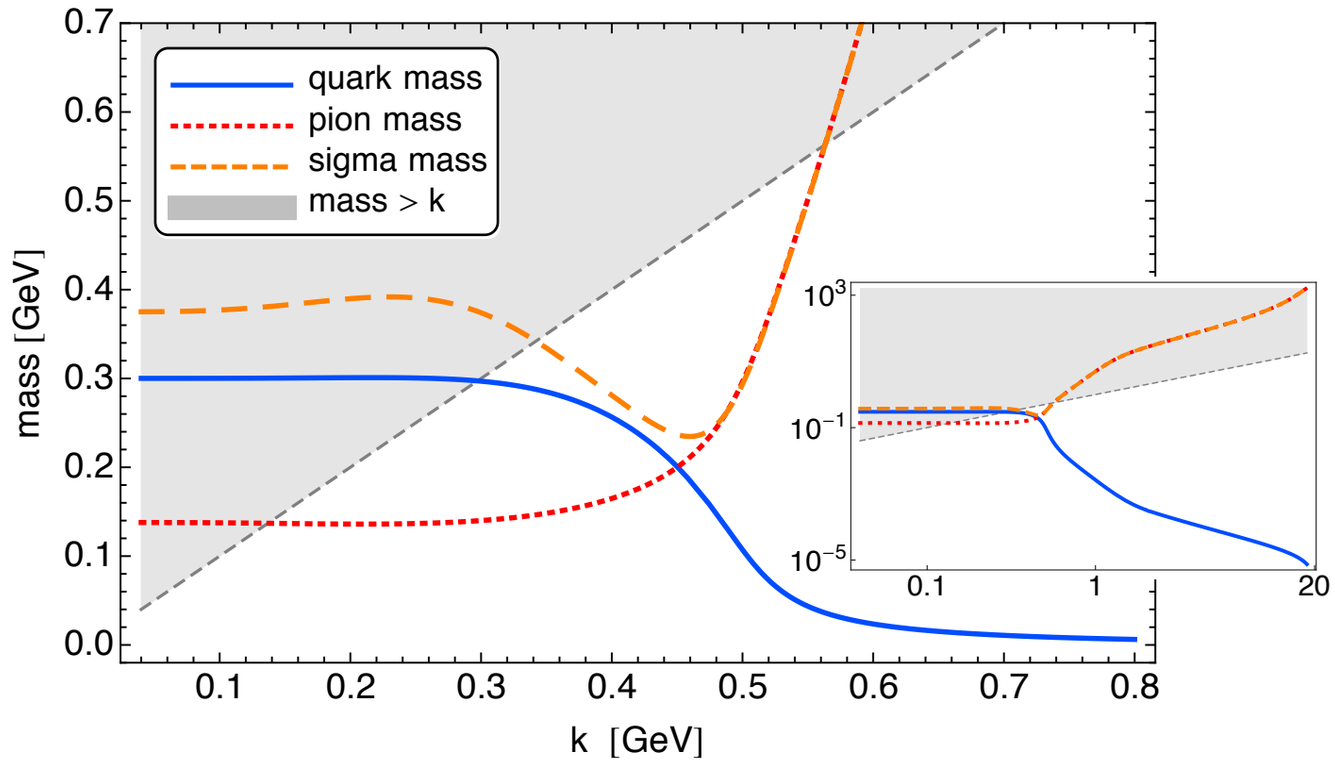
- Summary & outlook

# On the unreasonable effectiveness of low energy effective theories

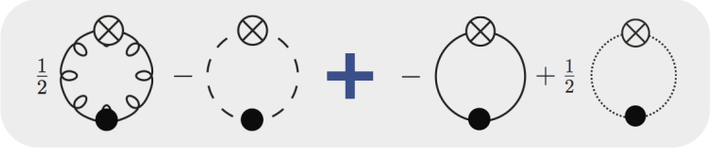
$$\partial_t \Gamma_k[\phi] = \tfrac{1}{2}$$  $$+ \tfrac{1}{2}$$

**Sequential decoupling of gluon, quark, sigma, pion fluctuations**



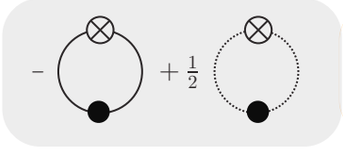




**PQM-model**

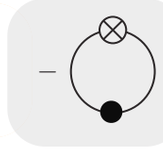


**PNJL-model**

**QM-model**

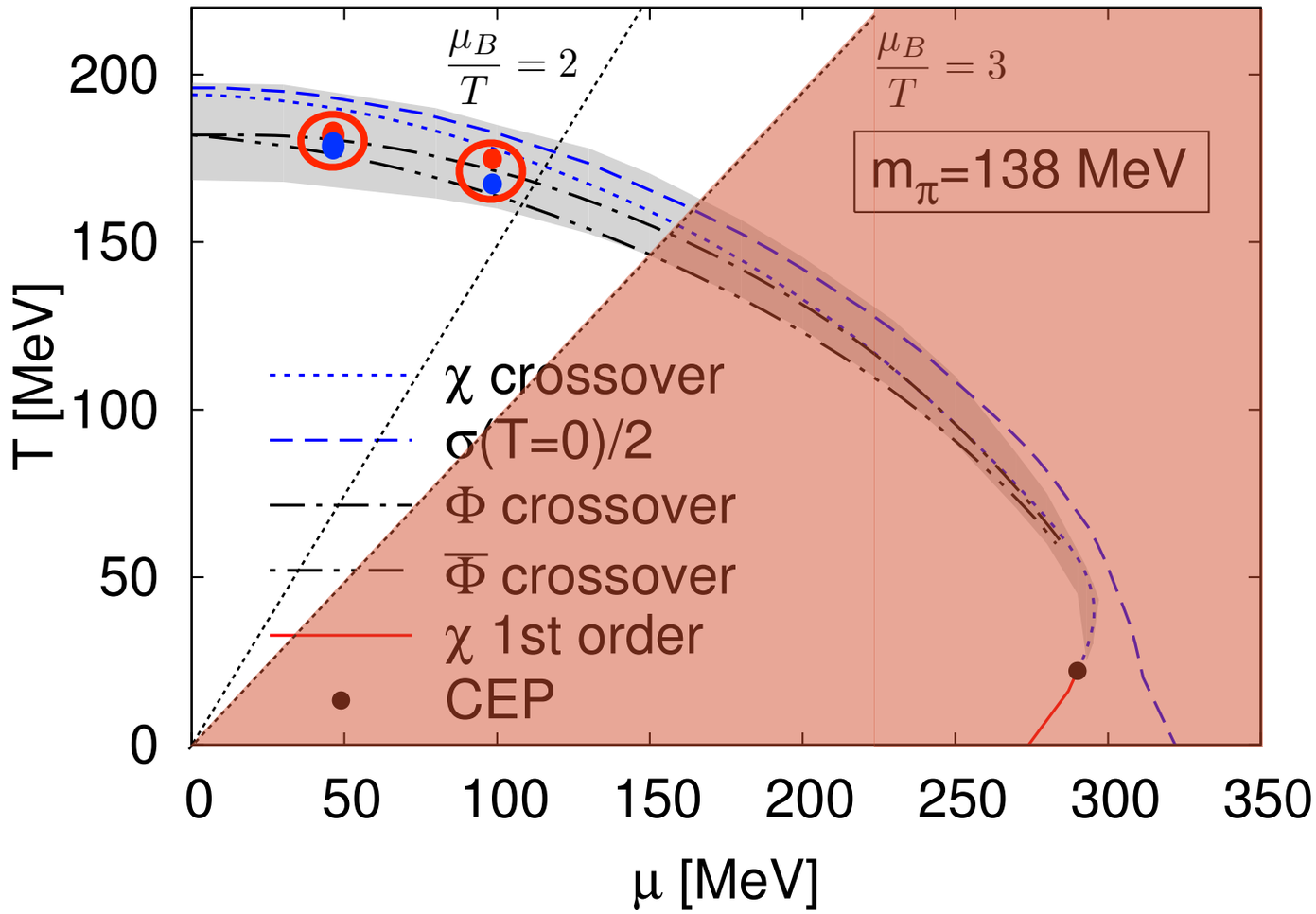


**NJL-model**



# QCD at finite density

## Phase diagram of the QCD-enhanced PQM




**Herbst, JMP, Schaefer, PLB 696 (2011) 58-67**
**PRD 88 (2013) 1, 014007**

**FRG QCD results at finite density**

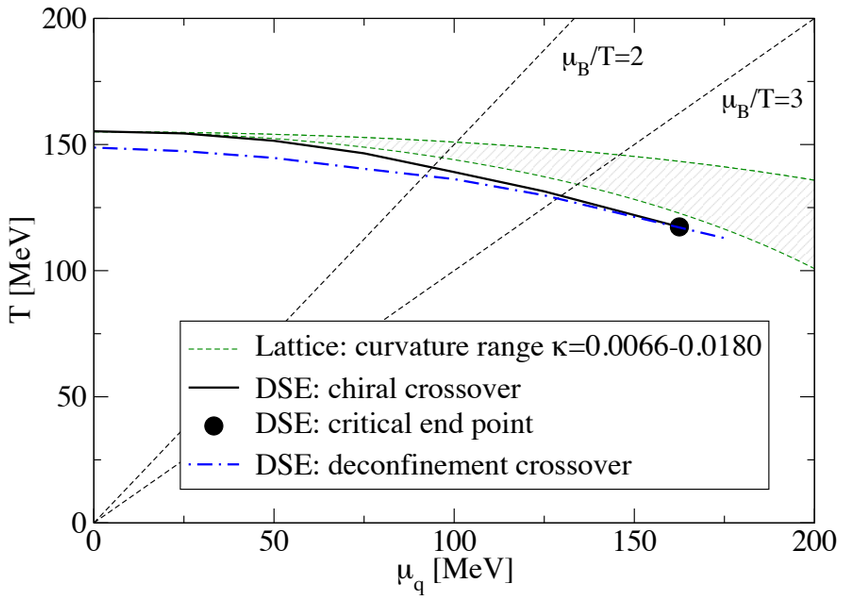
**Haas, Braun, JMP '09, unpublished**

**Extension of FRG QCD results at imaginary chemical potential**

**Braun, Haas, Marhauser, JMP, PRL 106 (2011) 022002**

# Phase structure at finite density

## Phase diagram of 2+1 flavor QCD




**Kaczmarek at al. '11**
**Endrodi, Fodor, Katz, Szabo '11**
**Cea, Cosmai, Papa '14**

**Fischer, Fister, Luecker, JMP, PLB732 (2014)**

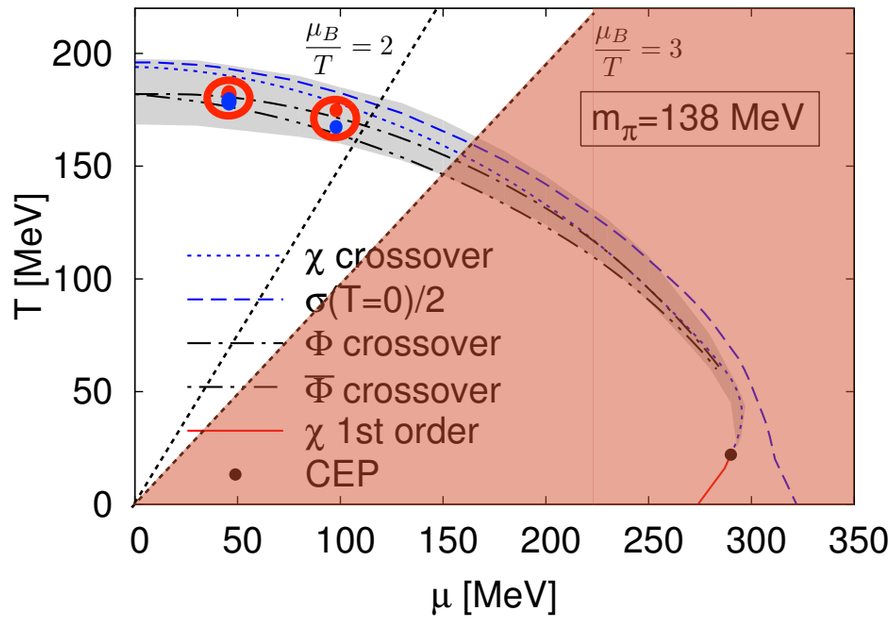
**Fischer, Luecker, Welzbacher, PRD 90 (2014) 034022**

**Eichmann, Fischer, Welzbacher, PRD 93 (2014) 034013**

## Chiral phase structure

**Qin, Chang, Chen, Liu, Roberts, PRL 106 (2011) 172301**

## Phase diagram of QCD-enhanced 2-flavor PQM-model




**Herbst, JMP, Schaefer, PLB 696 (2011) 58-67**
**PRD 88 (2013) 1, 014007**

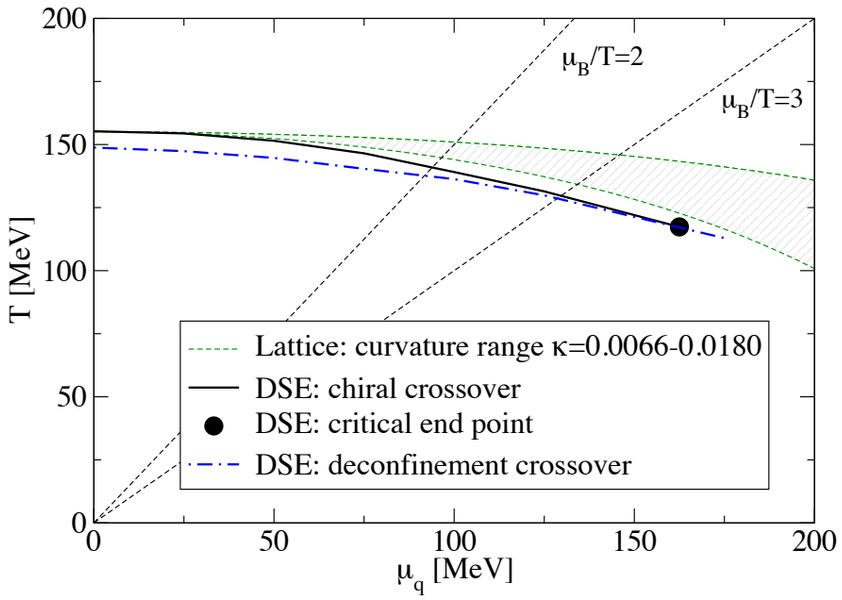
## FRG QCD results at finite density

**Haas, Braun, JMP '09, unpublished**

DSE-update: see talk of Christian Fischer

# Phase structure at finite density

## Phase diagram of 2+1 flavor QCD




**Kaczmarek at al. '11**
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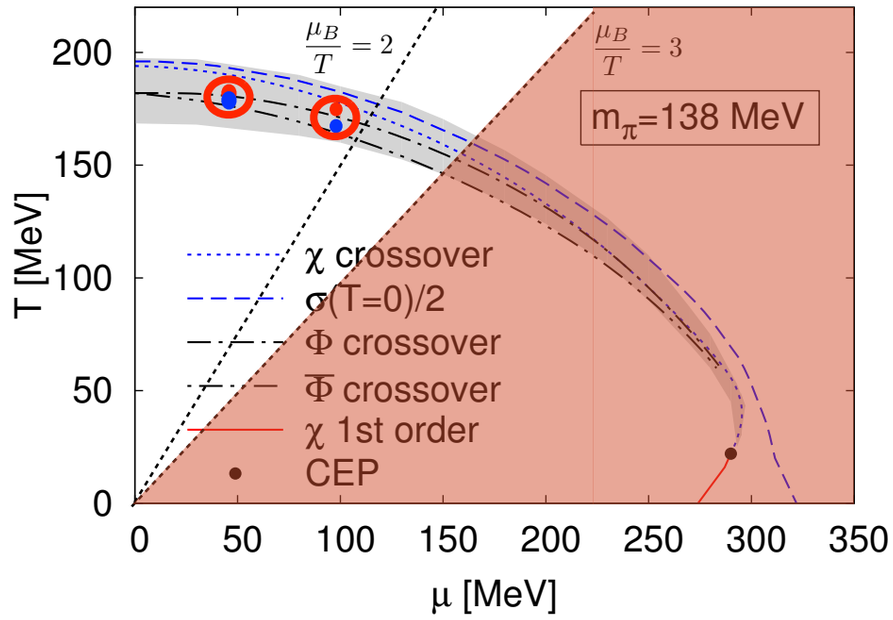
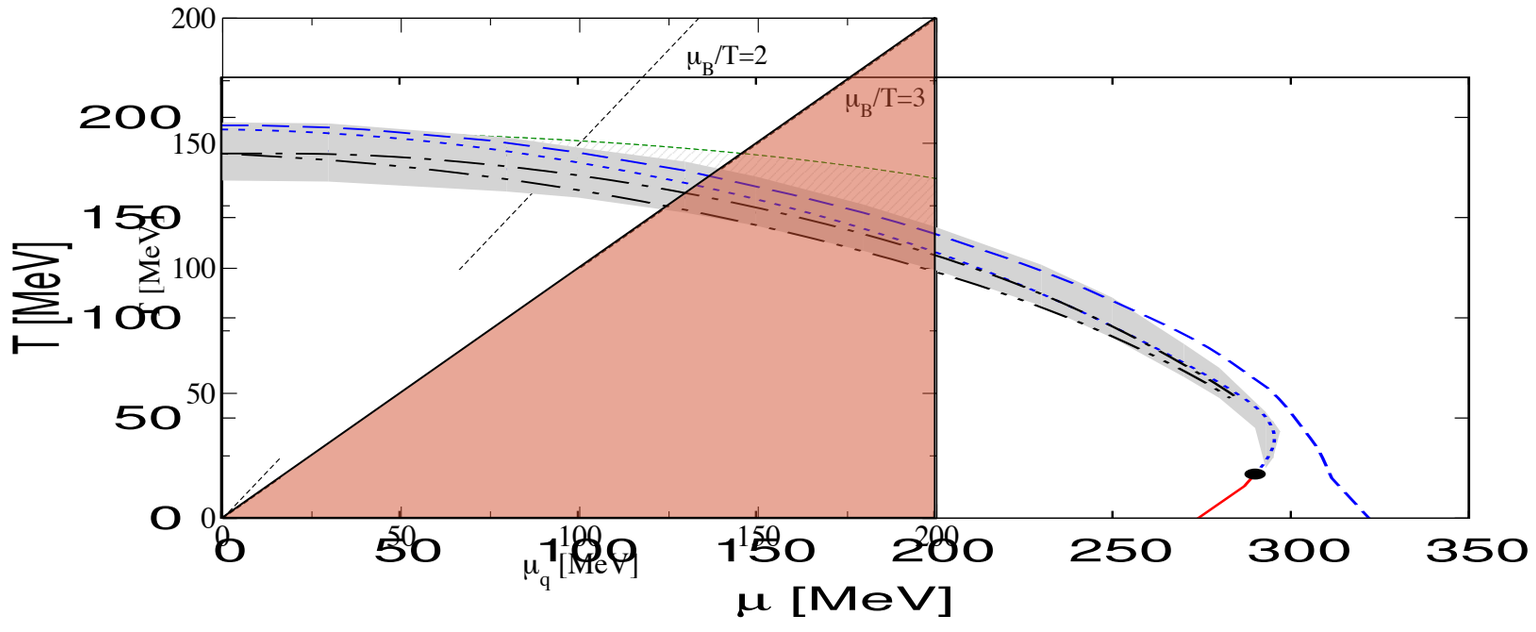



**Herbst, JMP, Schaefer, PLB 696 (2011) 58-67**
**PRD 88 (2013) 1, 014007**

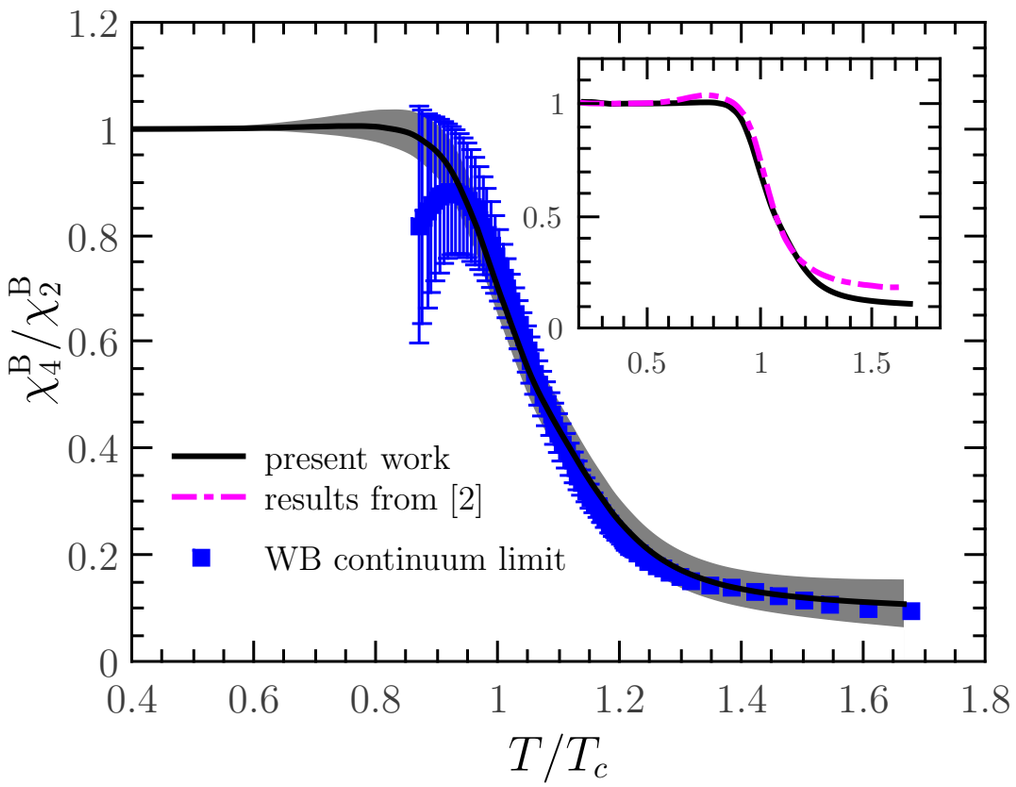
## FRG QCD results at finite density

**Haas, Braun, JMP '09, unpublished**

## Comparison with 2 flavor vs 2+1 flavor scale matching of $T_c$



# Fluctuations as a measure of confinement




$$\chi_n^{\rm B} = \frac{\partial^n}{\partial(\mu_{\rm B}/T)^n}\frac{p}{T^4}$$

**Skewness, Kurtosis**

$$\sigma^2 = VT^3\chi_2^{\rm B} \qquad S = \frac{\chi_3^{\rm B}}{\chi_2^{\rm B}\sigma} \qquad \kappa = \frac{\chi_4^{\rm B}}{\chi_2^{\rm B}\sigma^2}$$

**[2] Fu, JMP, PRD 92 (2015) 116006**

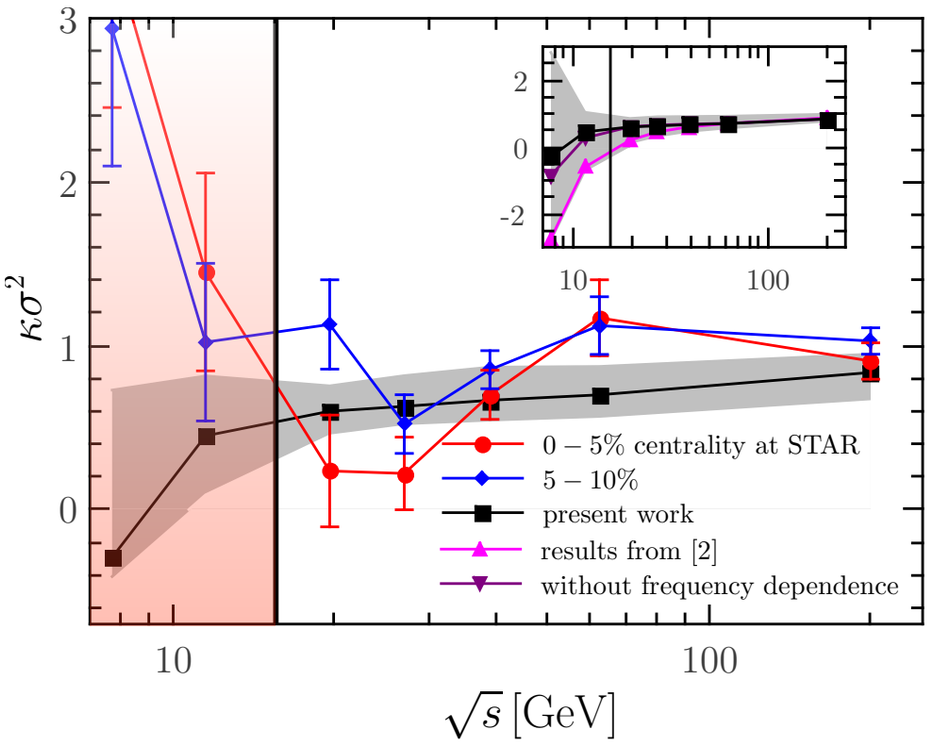
**Karsch, Schaefer, Wagner, Wambach, PLB 698 (2011) 256**

**Friman, Karsch, Redlich, Skokov, EPJ C71 (2011) 1694**

**Schaefer, Wagner, PRD 85 (2012) 034027**

**Skokov, Friman, Redlich, PRC 88 (2013) 034911**

**Almasi, Friman, Redlich, Nucl.Phys. A956 (2016) 356-359**




**[2] Fu, JMP, PRD 93 (2016) 091501**

**Fu, JMP, Schaefer, Rennecke, PRD 94 (2016) 116020**

# Transport approach to QCD

**Blum, Jiang, Mitter, Nahrgang, JMP, Rennecke, Wink**

**Time evolution of the critical (scalar) $\sigma$ mode**

$$\frac{\delta\Gamma}{\delta\sigma} = \xi$$

**quantum equation of motion** **noise field**

**Extension of mean-field version**

**Nahrgang, Leupold, Herold, Bleicher PRC84 (2011)**

**see also**

**Stephanov, Rajagopal, Shuryak PRL81 (1998)**

**Mukherjee, Venugopalan, Yin PRC92 (2015)**

**Herold, Nahrgang, Yan, Kobdaj PRC93 (2016)**

**Nahrgang, Bluhm, Schäfer, Bass arXiv:1804.05728**

# Transport approach to QCD

**Blum, Jiang, Mitter, Nahrgang, JMP, Rennecke, Wink**

**Time evolution of the critical (scalar) $\sigma$ mode**

$$\frac{\delta\Gamma}{\delta\sigma} = \xi$$

**quantum equation of motion** **noise field**

**Input from equilibrium low energy effective action of QCD**

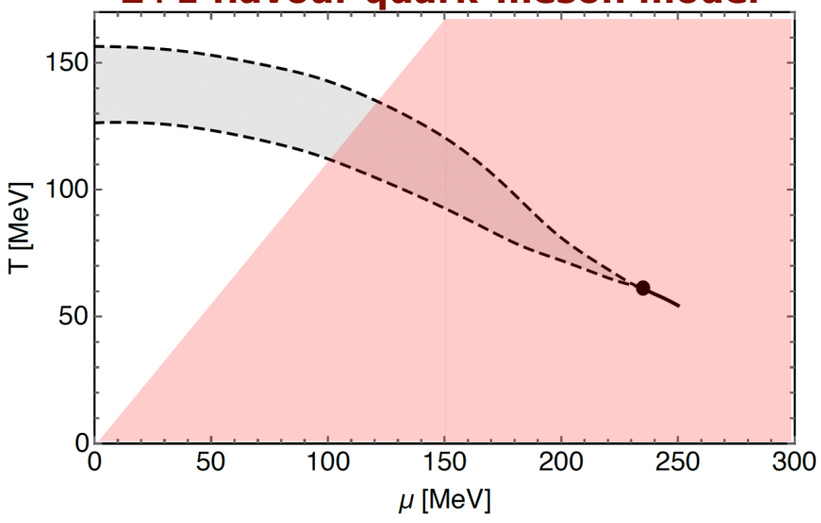
$\mathrm{Re}\,\Gamma^{(2)}_\sigma(\omega,\vec{p})$ **kinetic term**

$\mathrm{Im}\,\Gamma^{(2)}_\sigma(\omega,\vec{p})$ **diffusion term** $\eta\,\partial_t\sigma$

$U(\sigma)$ **effective potential**

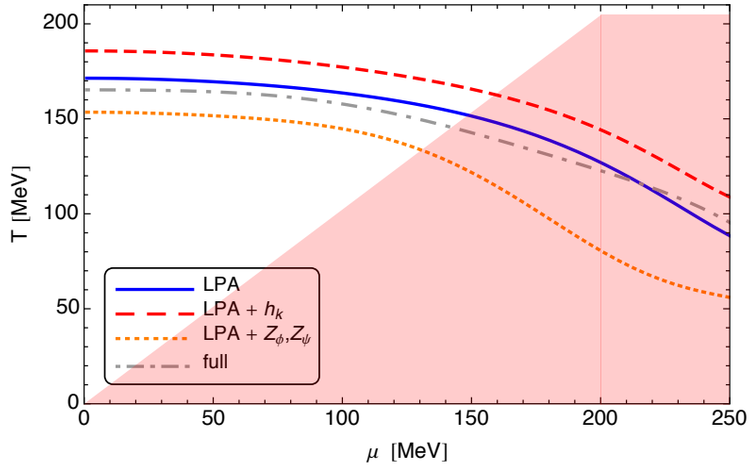
**Phase structure of low energy QCD**

**2+1 flavour quark-meson model**




**Schaefer, Rennecke, PRD 96 (2017) 1, 016009**

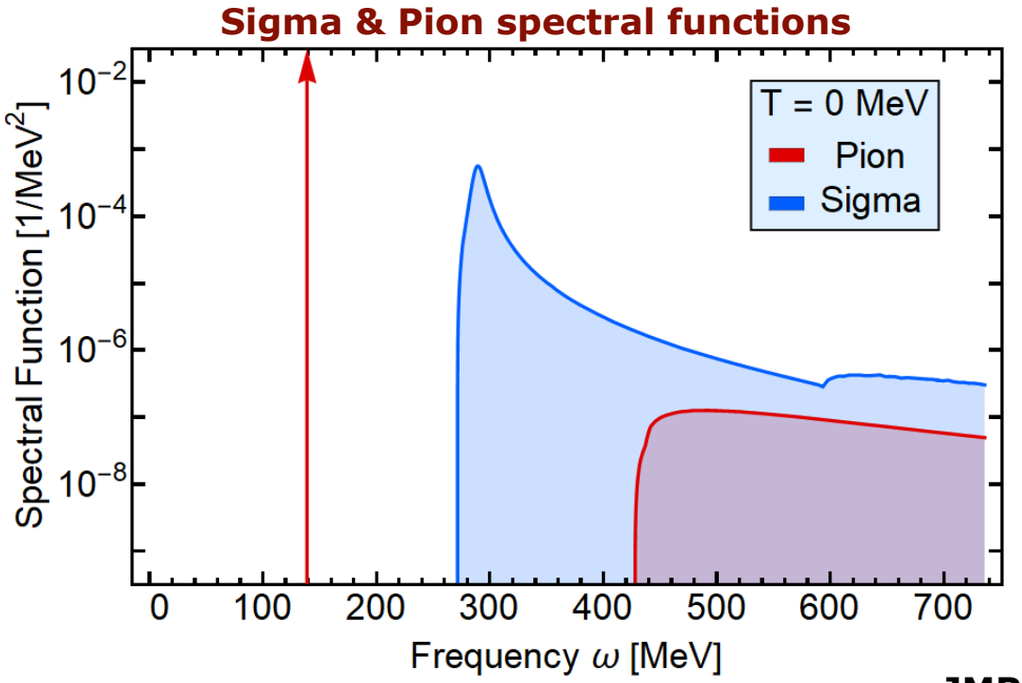
**Comparison of truncations (2 flavours)**




**JMP, Rennecke, PRD 90 (2014) 7, 076002**

# Pion & sigma spectral functions

**Show case in linear sigma model**




**JMP, Strodthoff, Wink, arXiv:1711.07444**

**Real-time FRG computations, e.g.**

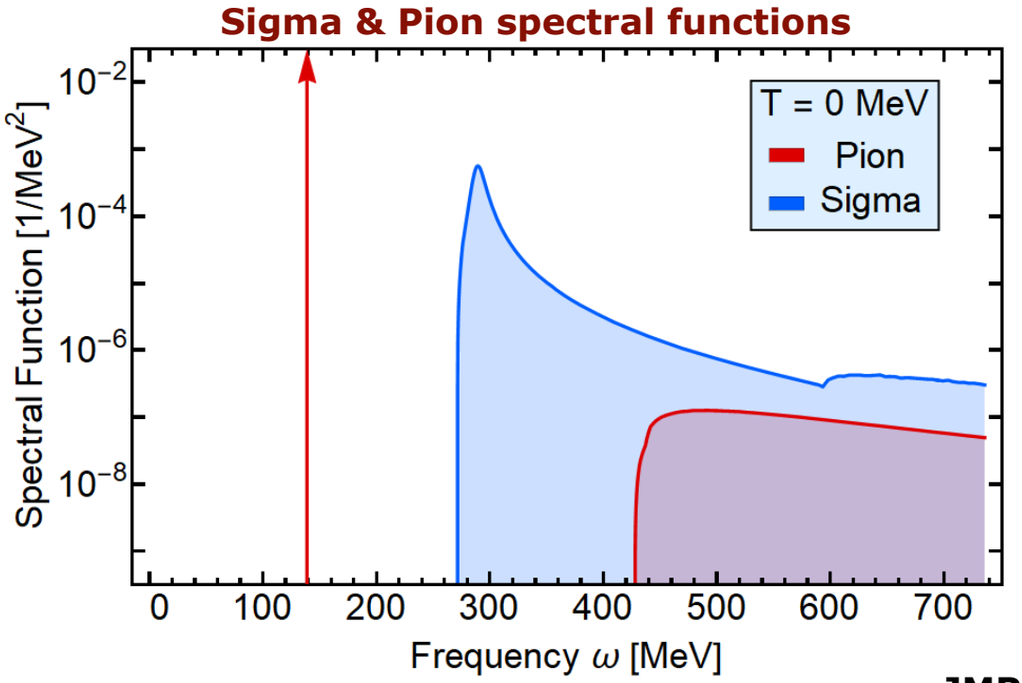
**Flörchinger JHEP 1205 (2012) 021**

**Kamikado, Strodthoff, von Smekal, Wambach, EPJC 74 (2014) 2806**

**JMP, Strodthoff, PRD 92 (2015) 094009**

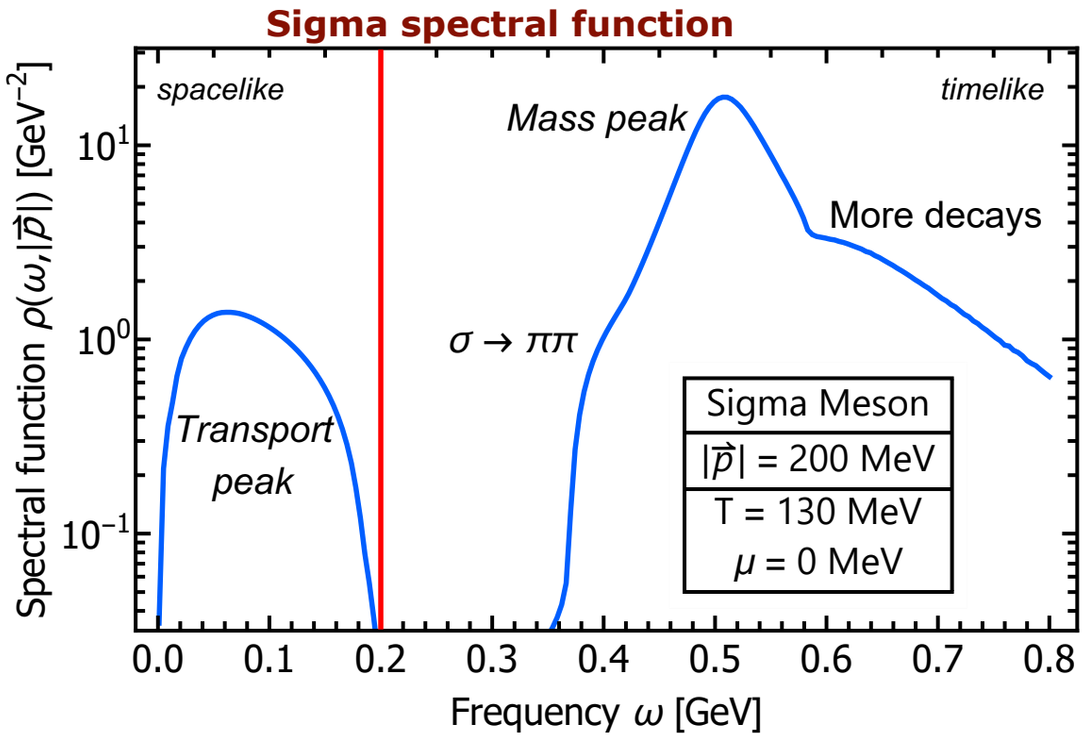
# Pion & sigma spectral functions

**Show case in linear sigma model**




**JMP, Strodthoff, Wink, arXiv:1711.07444**

**2+1 flavour quark-meson model sigma spectral function**

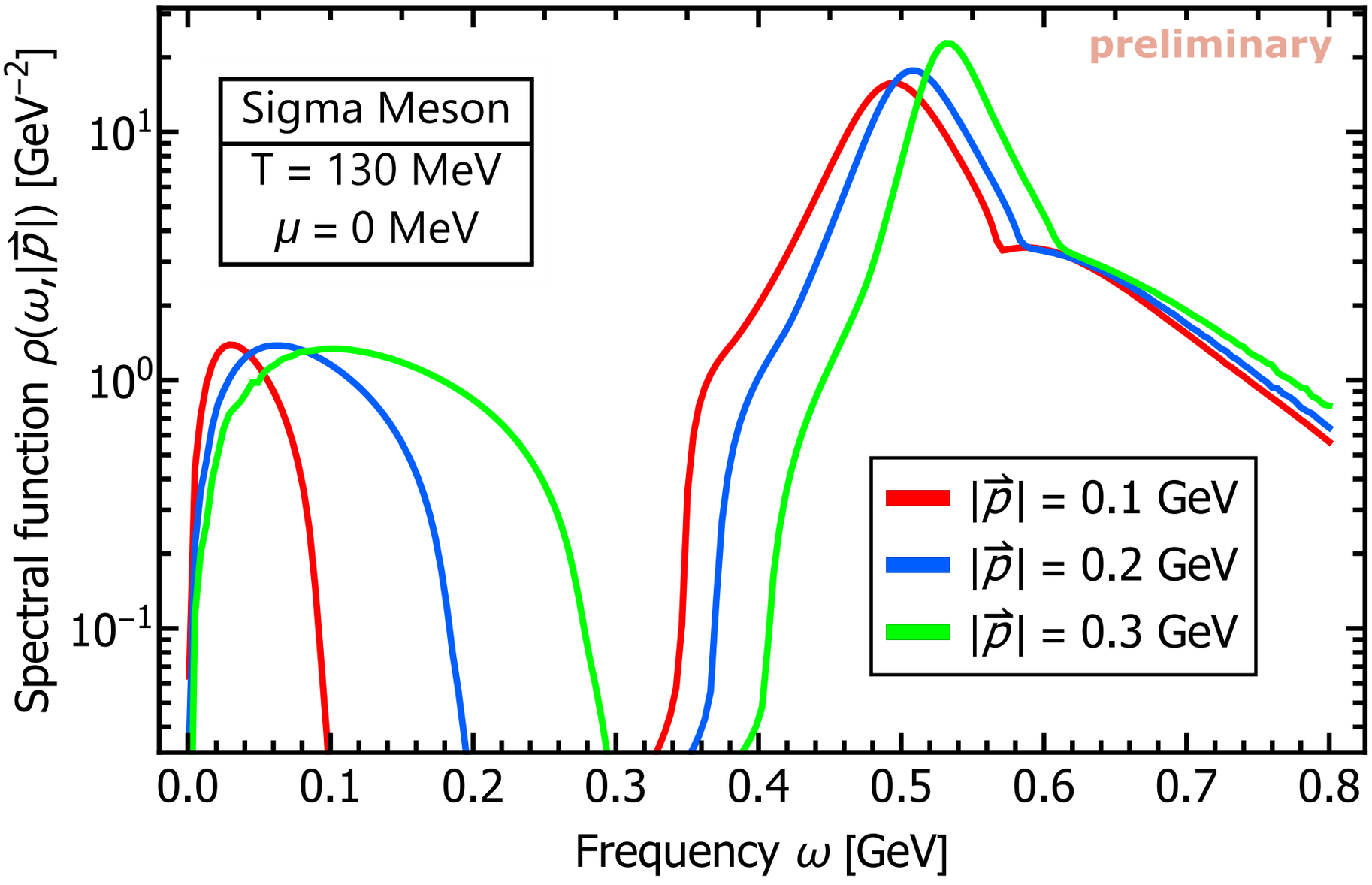



preliminary

**JMP, Rennecke, Wink, in prep**

# Pion & sigma spectral functions

**2+1 flavour quark-meson model sigma spectral function**

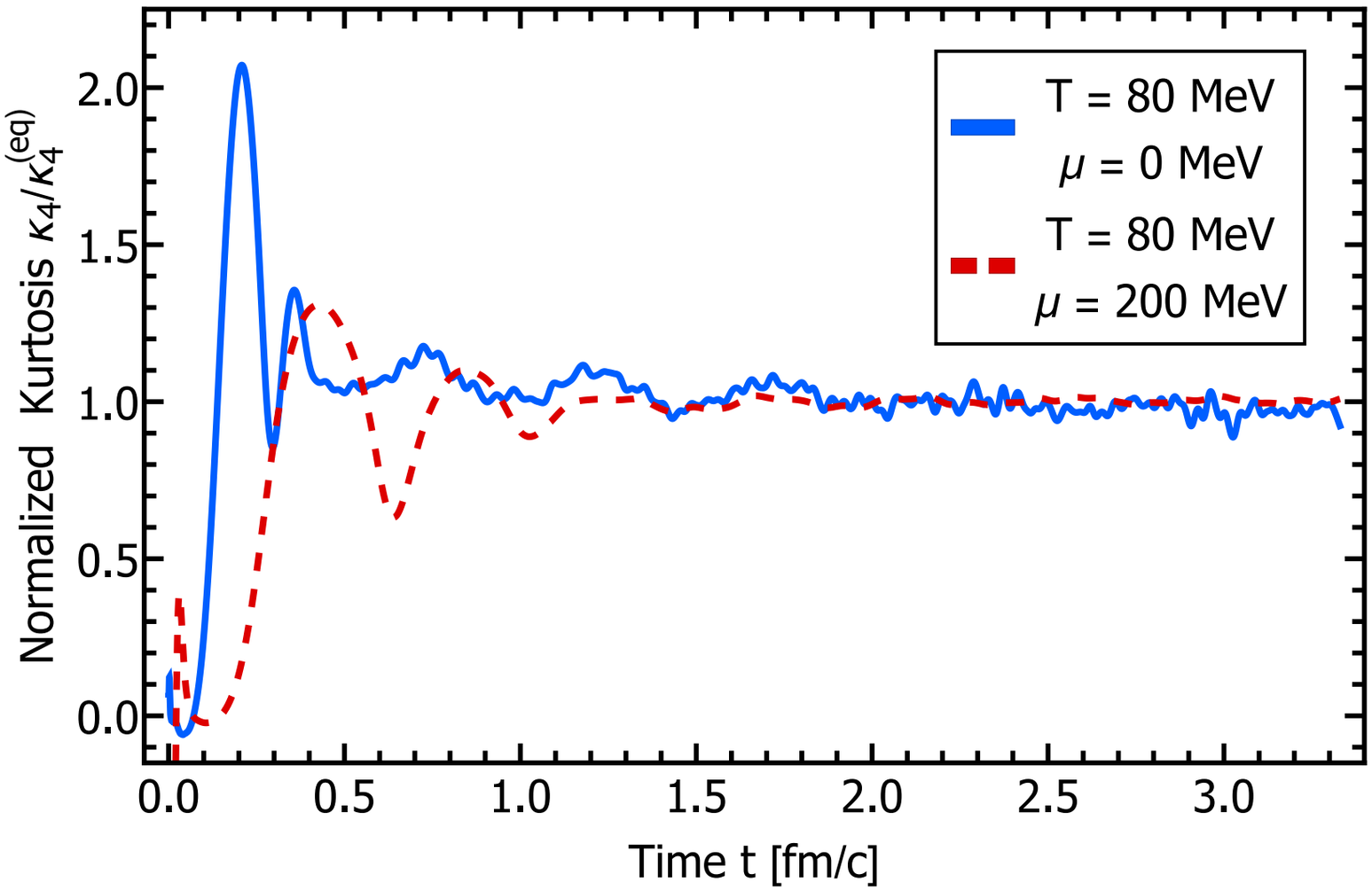



**JMP, Rennecke, Wink, in prep**

# Time evolution of cumulants

**Blum, Jiang, Nahrgang, JMP, Rennecke, Wink, in prep**

**Time evolution of kurtosis**




**nth central moment of the sigma field:** $\chi_n$

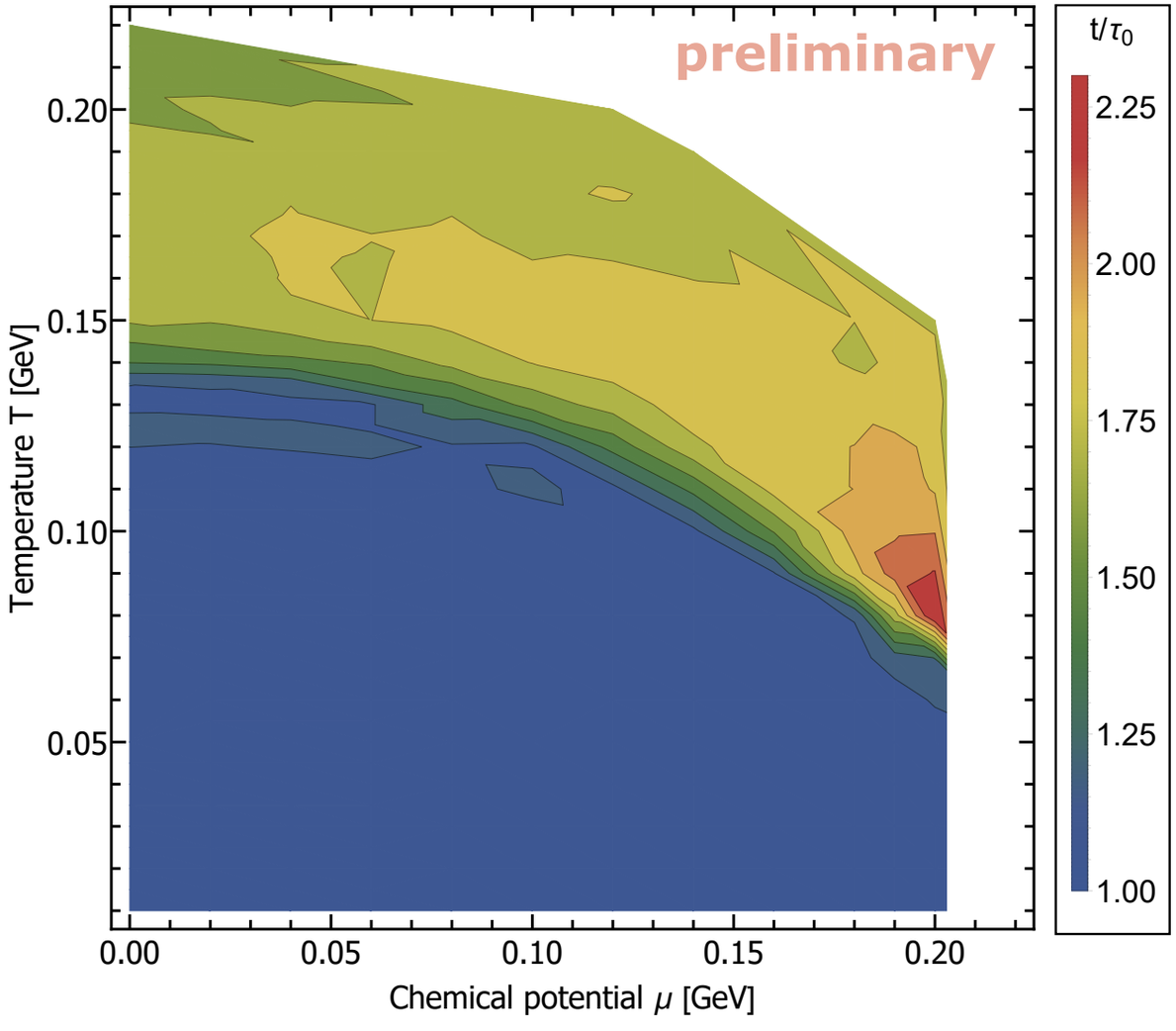
$$\chi_2 = \langle (\sigma - \langle \sigma \rangle)^2 \rangle$$

**kurtosis:** $\kappa = \dfrac{\chi_4}{\chi_2^2} - 3$

# Equilibration time phase structure

**Blum, Jiang, Nahrgang, JMP, Rennecke, Wink, in prep**

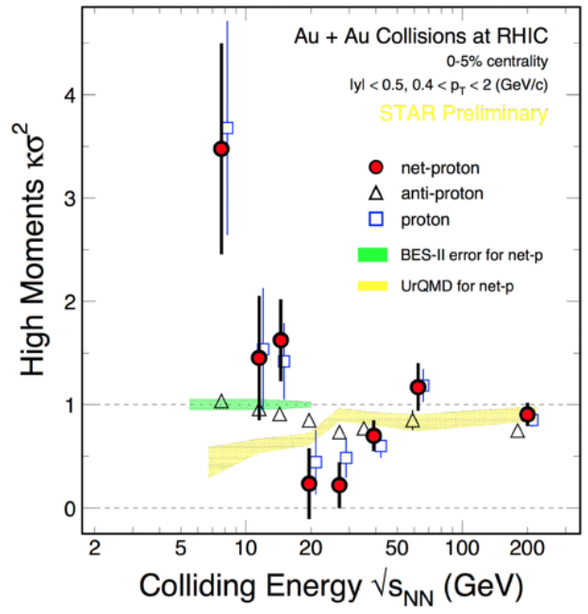
## Equilibration time of sigma-kurtosis




**nth central moment of the sigma field:** $\chi_n$

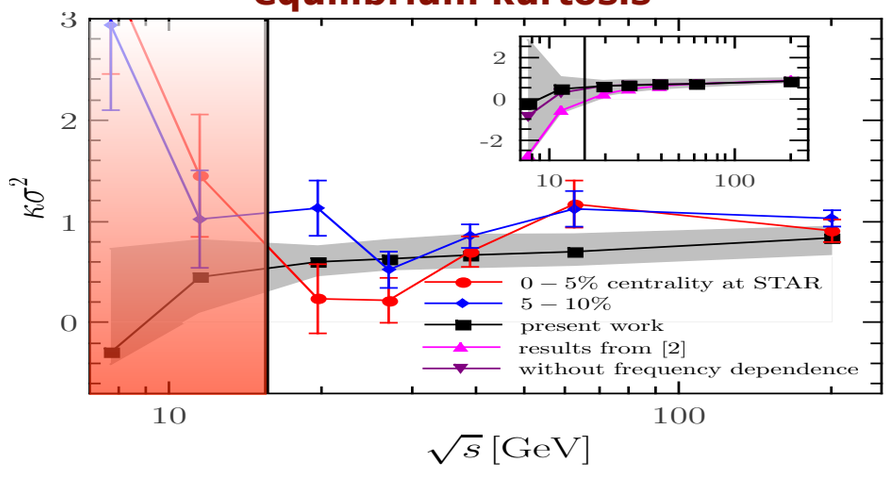
**variance:** $\chi_2 = \langle (\sigma - \langle \sigma \rangle)^2 \rangle$

## kurtosis of baryon number fluctuations




**Luo, Cu, NST 28 (2017)**

### equilibrium kurtosis




**Fu, JMP, Schaefer, Rennecke, PRD 94 (2016) 11, 116020**

**kurtosis:** $\kappa = \dfrac{\chi_4}{\chi_2^2} - 3$

# Outline

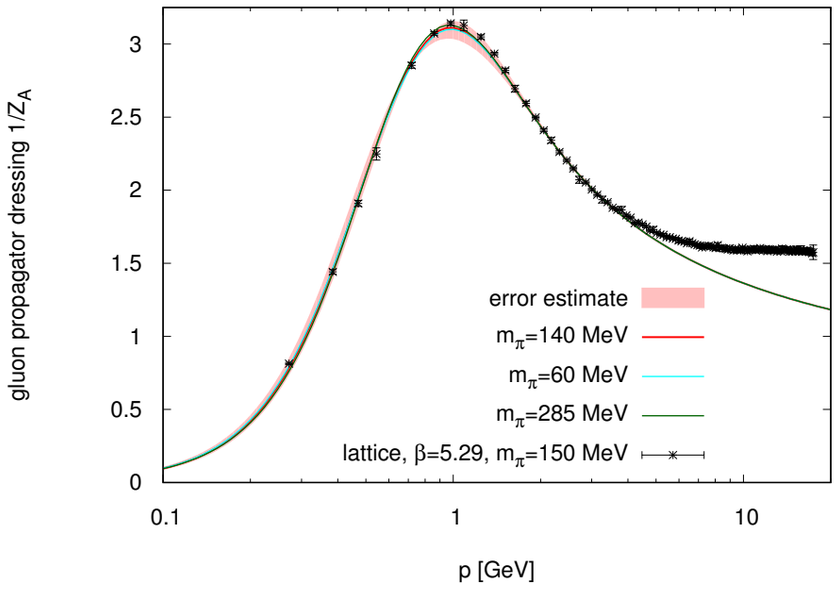
- QCD from functional methods
- QCD-assisted hydrodynamics
- QCD-assisted transport
- Summary & outlook

# Summary & Outlook

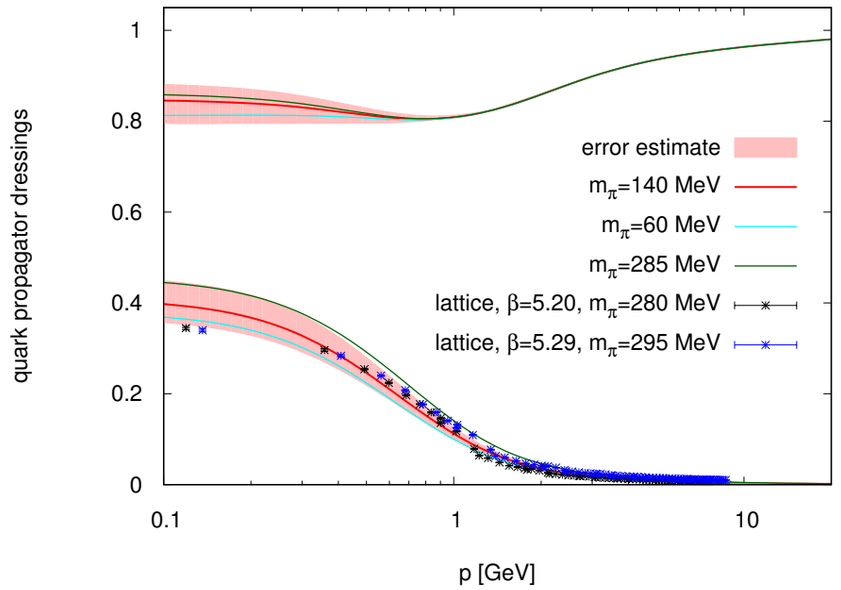
- **Phase structure of QCD**

$$\frac{f_{\pi,\mathrm{FRG}}}{f_{\pi,\mathrm{lattice}}} = 0.99$$

## Gluon correlations

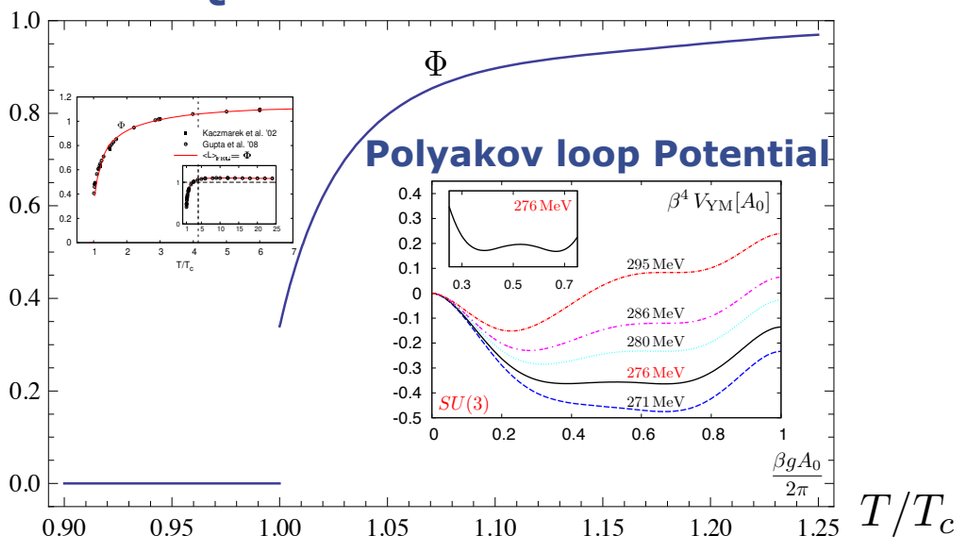



## chiral symmetry breaking

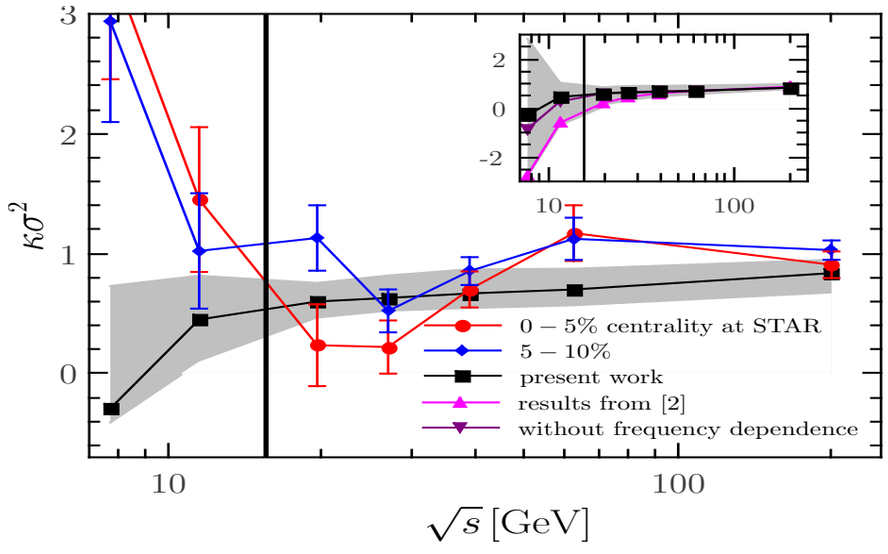



**fQCD**

## Quark Confinement



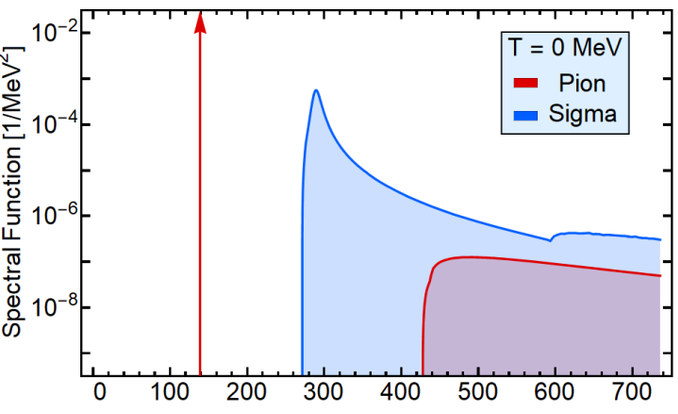

## fluctuations at finite density

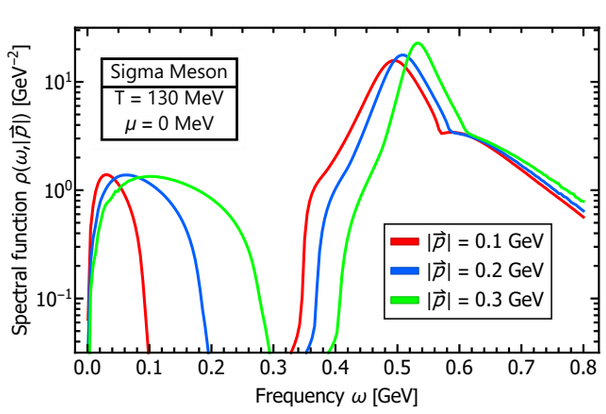


# Summary & Outlook

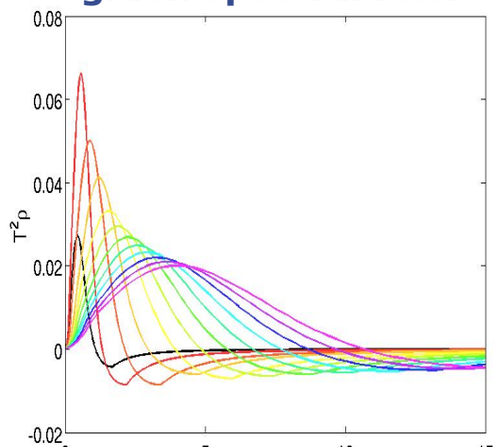
## ▪QCD Transport

### meson spectral fcts.

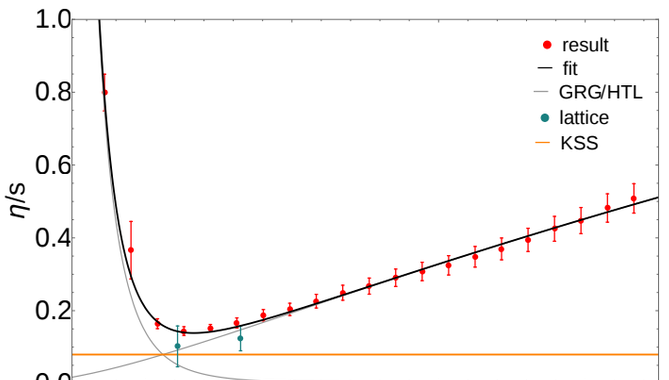


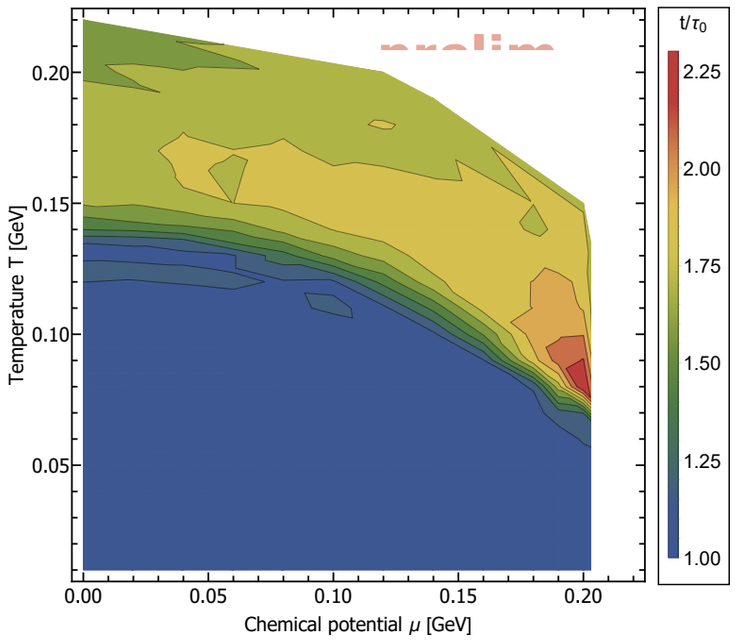

### gluon spectral fcts.



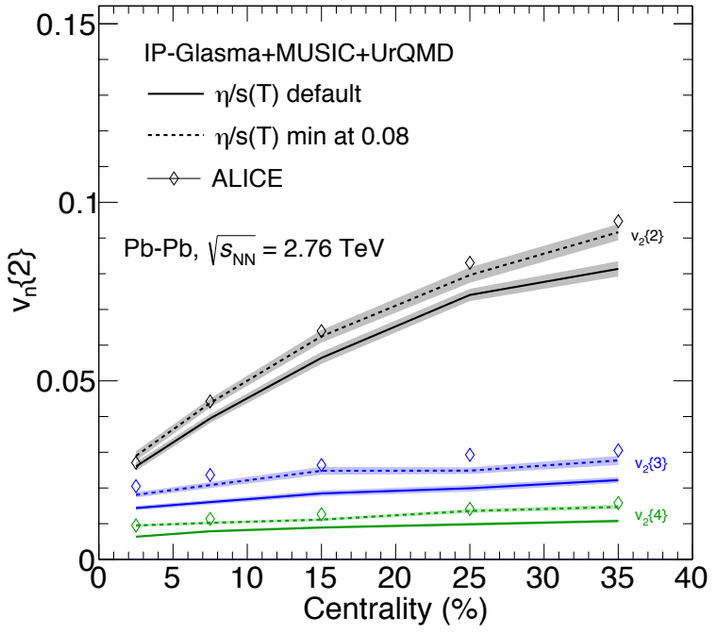

### glue viscosity/entropy




### Equilibration phase structure




### $V_n$ in QCD-assisted Hydro



# Summary & Outlook

- QCD phase structure at low density
- Transport in pure glue & low energy EFTs
- Towards quantitative precision
  - baryons, high density regime & CEP
- Threefold way to transport
  - imaginary time lattice simulations: Yang-Mills, QCD
  - real-time: Yang-Mills, QCD, finite density