QCD transition at zero and non-zero baryon densities

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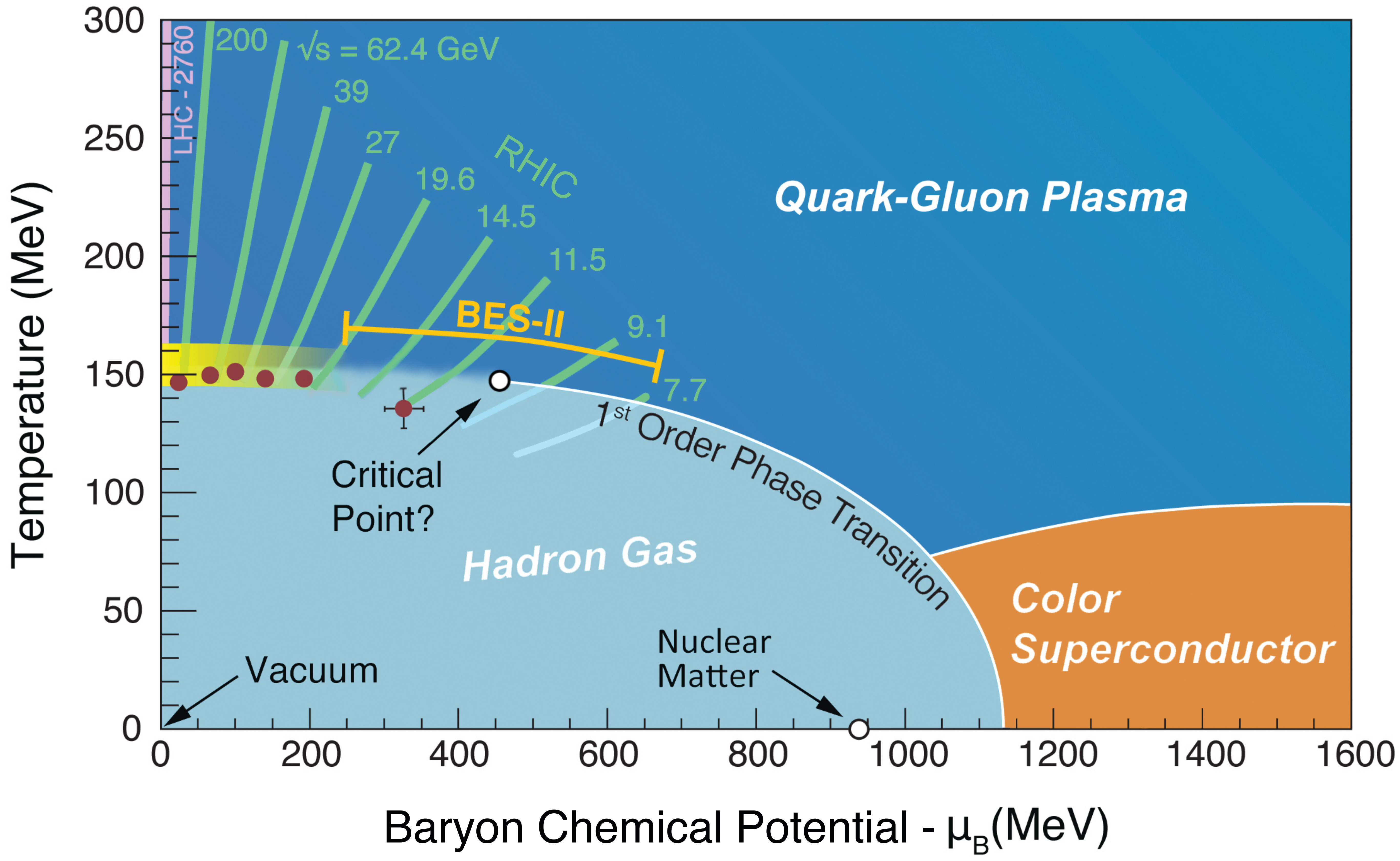
chiral crossover temperature

$$\frac{T_c(\mu_B)}{T_c(0)} = 1-\kappa_2^B\left(\frac{\mu_B}{T_c(0)}\right)^2-\kappa_4^B\left(\frac{\mu_B}{T_c(0)}\right)^4+\mathcal{O}\left(\mu_B^6\right)$$



Quark Matter 2018:
Patrick Steinbrecher (BNL)

HotQCD preliminary

order parameter: $$\Sigma_{\mathrm{sub}} \equiv m_s(\Sigma_u + \Sigma_d) - (m_u + m_d)\Sigma_s$$ $$\Sigma_f = \frac{T}{V}\frac{\partial}{\partial m_f} \ln Z$$

susceptibility: $$\chi_{\mathrm{sub}} \equiv \frac{T}{V} m_s \left(\frac{\partial}{\partial m_u} + \frac{\partial}{\partial m_d}\right) \Sigma_{\mathrm{sub}}$$

and it's quark-line disconnected part: χ_{disc}

Taylor's expansion: $$\frac{\Sigma_{\mathrm{sub}}}{f_K^4} = \sum_{n=0}^{\infty} \frac{c_n^{\Sigma}}{n!} \hat{\mu}_B^n$$ $$\frac{\chi_{\mathrm{disc}}}{f_K^4} = \sum_{n=0}^{\infty} \frac{c_n^{\chi}}{n!} \hat{\mu}_B^n$$

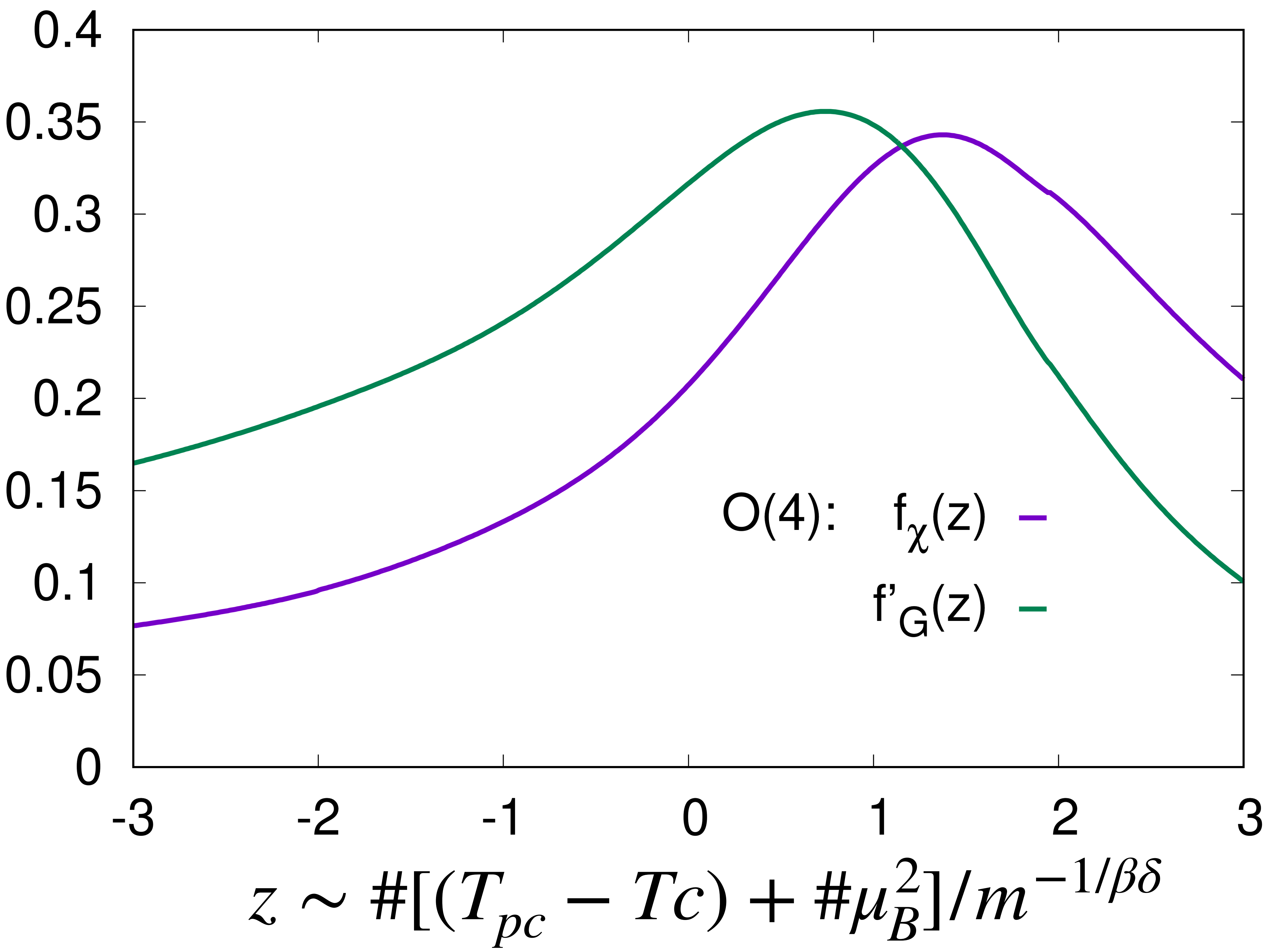
crossover temperature: $$\frac{d^2}{dT^2} \frac{\Sigma_{\mathrm{sub}}(T, \mu_B)}{f_K^4} \equiv 0$$ $$\frac{d}{dT} \frac{\chi_{\mathrm{disc}}(T, \mu_B)}{f_K^4} \equiv 0$$

$$\chi_t \sim m^{(\beta-1)/\beta\delta} f'_G(z)$$

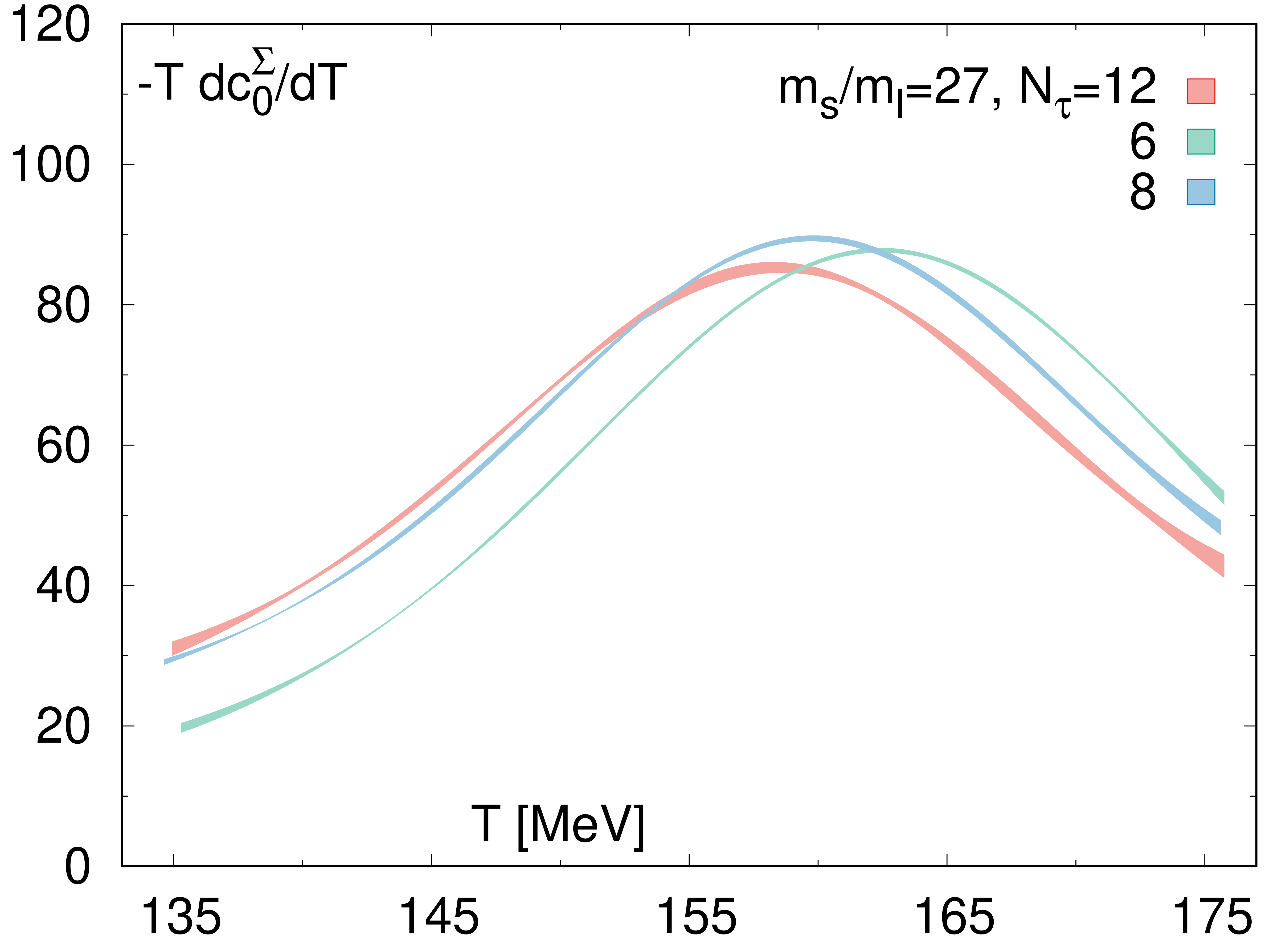
$$f'_G: \;\; \partial_T \Sigma_{sub}, \; \partial^2_{\mu_B} \Sigma_{sub}$$

$$\chi_m \sim m^{1/\delta - 1} f_\chi(z)$$

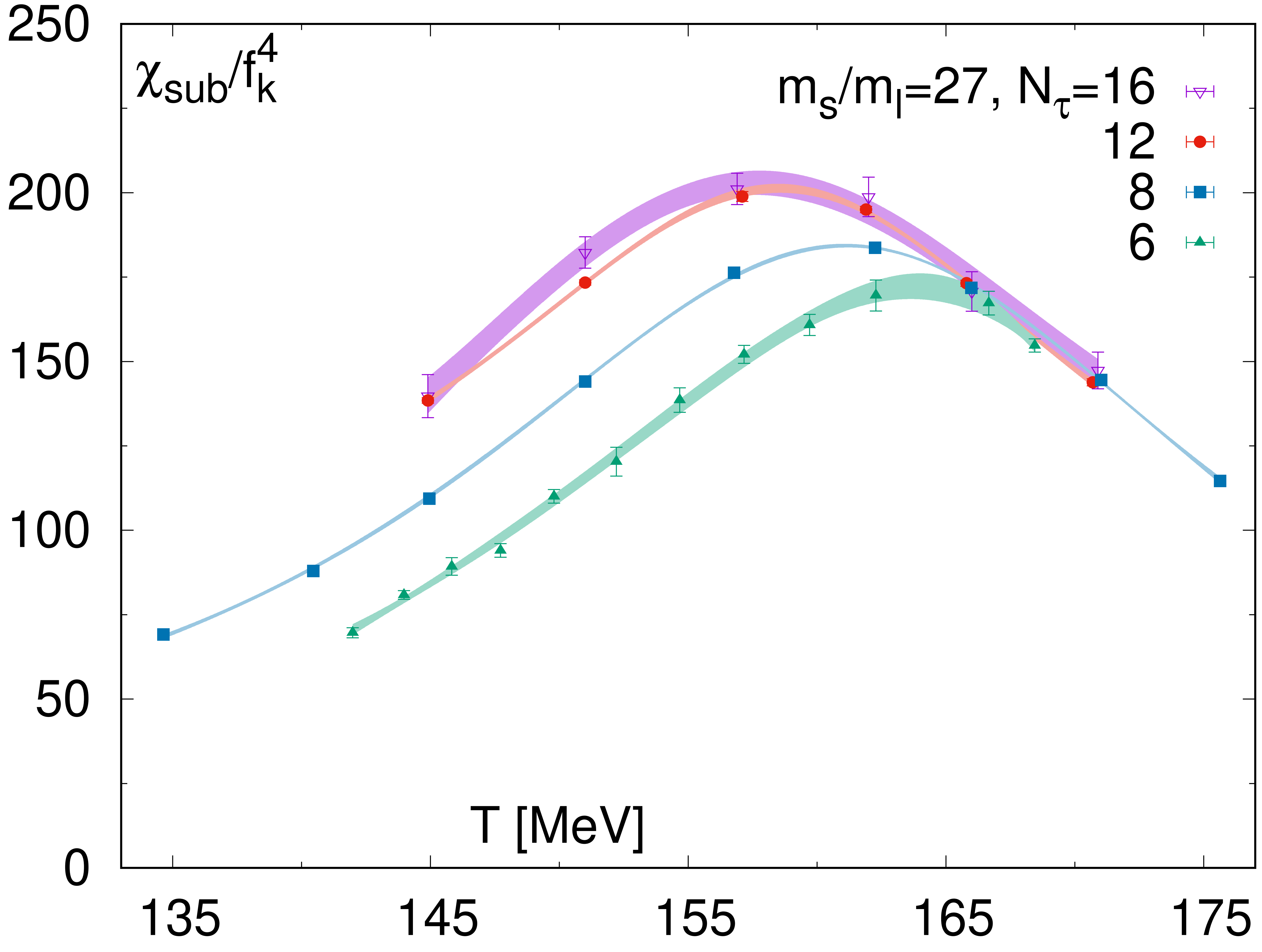
$$f_\chi(z): \;\; \chi_{sub}, \chi_{disc}, \partial^2_{\mu_B} \chi_{disc}$$



- m = 0: all these susceptibilities will diverge at a unique transition temperature
- m > 0: crossover, different susceptibilities can lead to different crossover temperatures

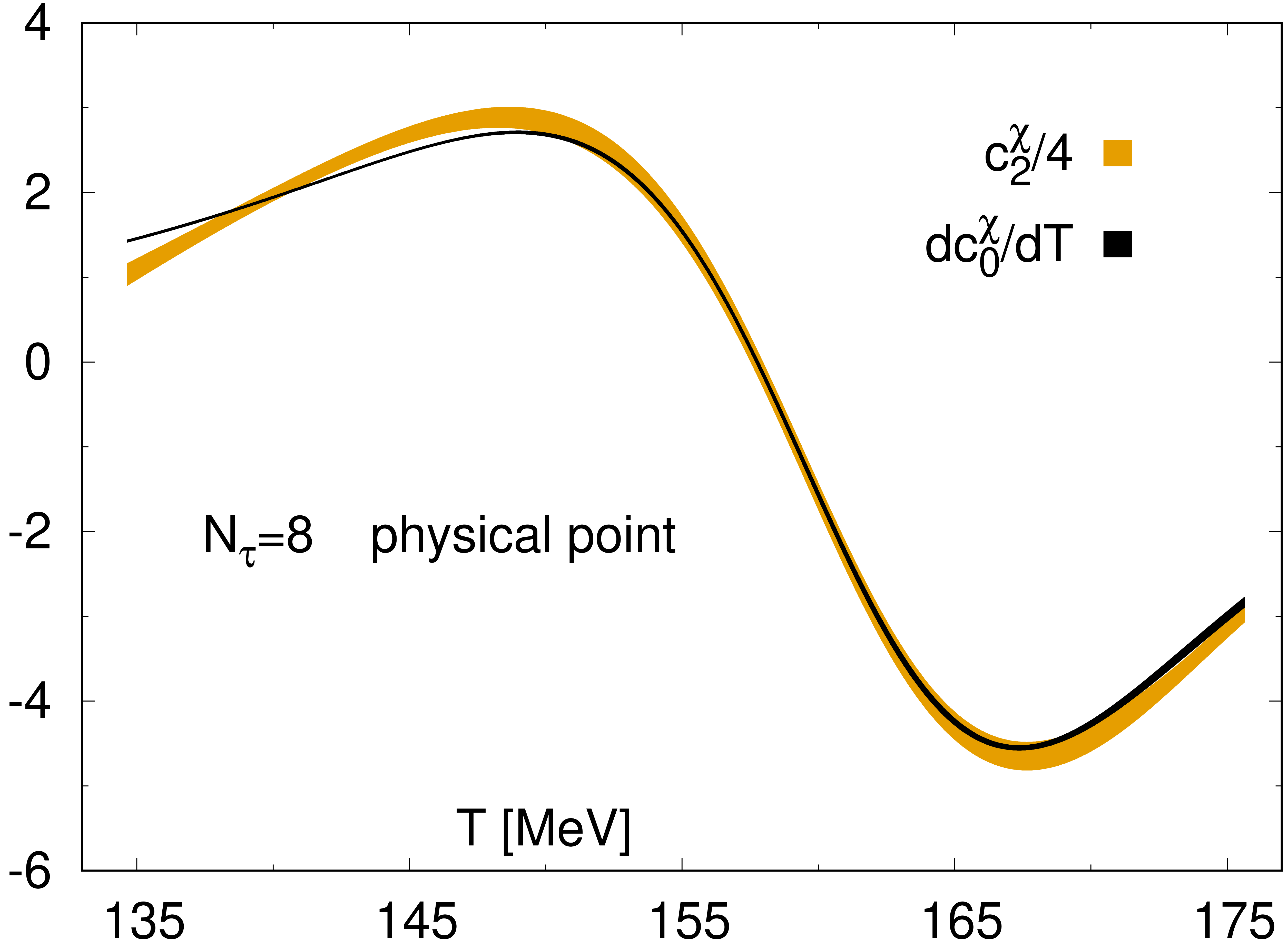


$f'_G : \ \partial_T \Sigma_{sub}$



$f_\chi(z) : \ \chi_{sub}$

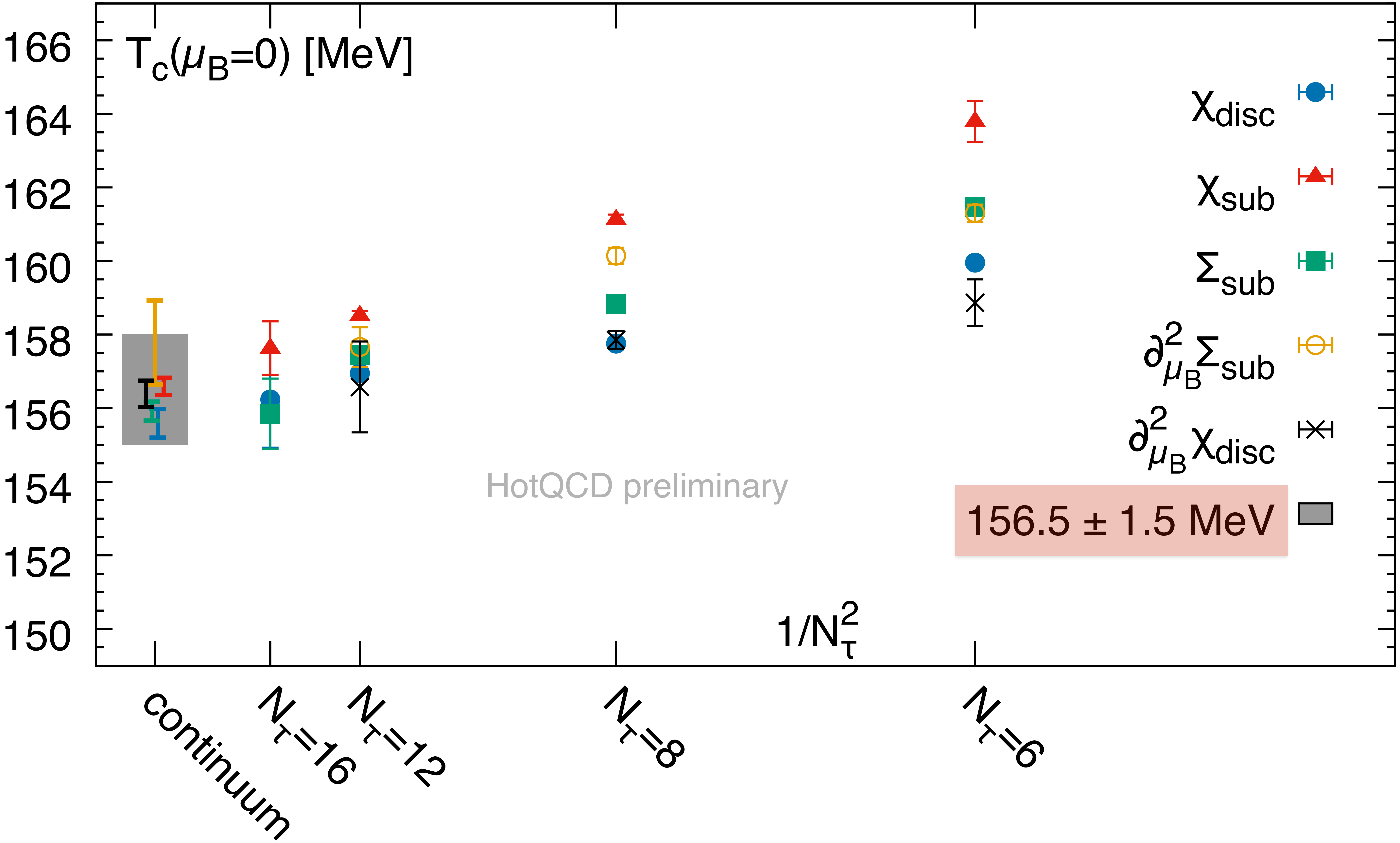
$$\partial_T \chi_{disc} \sim \partial^2_{\mu_B} \chi_{disc}$$

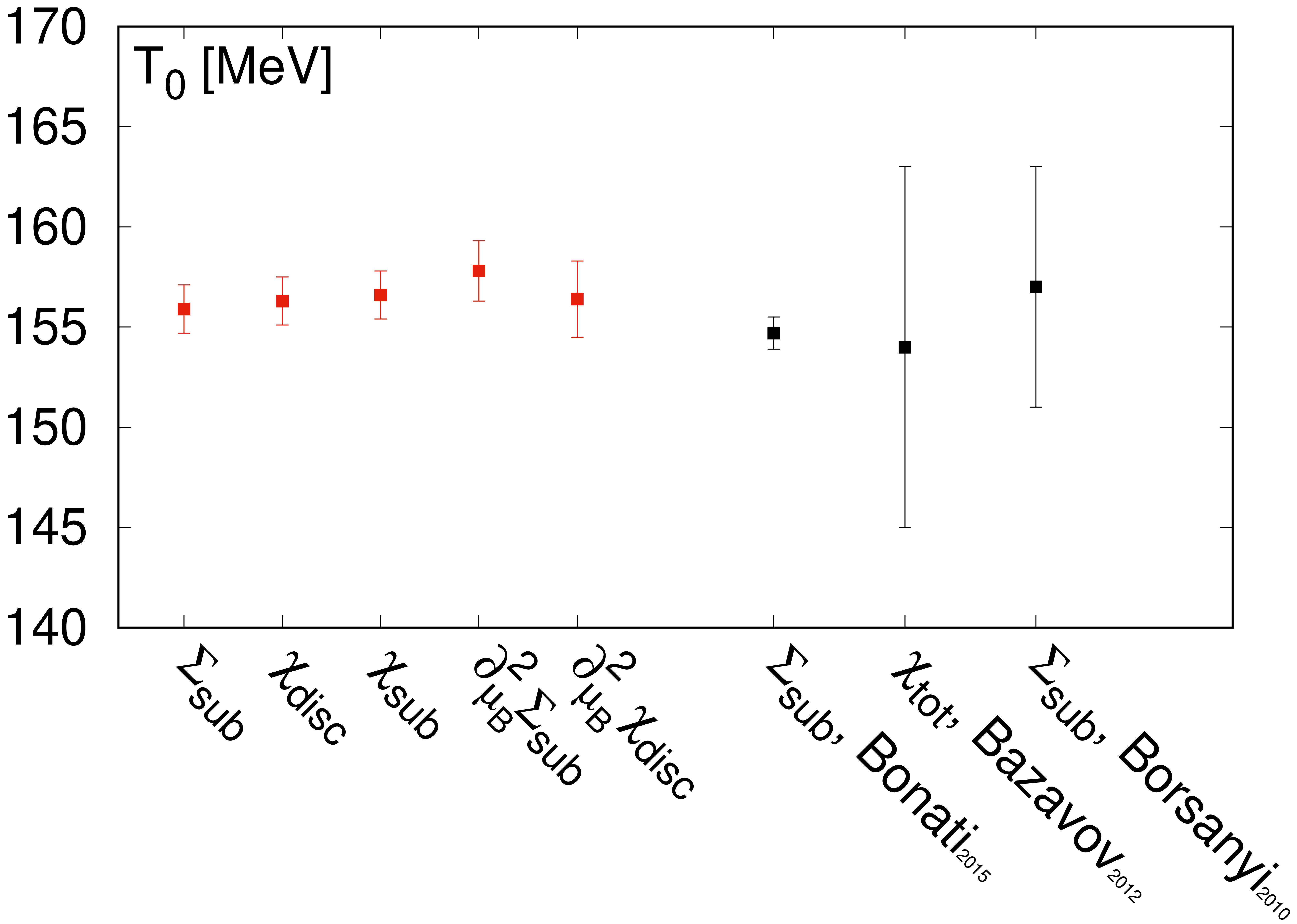


$$z \sim \#[(T_{pc} - Tc) + \#\mu_B^2]/m^{-1/\beta\delta}$$

$$\chi_{disc} \sim m^{1/\delta - 1} f_\chi(z)$$

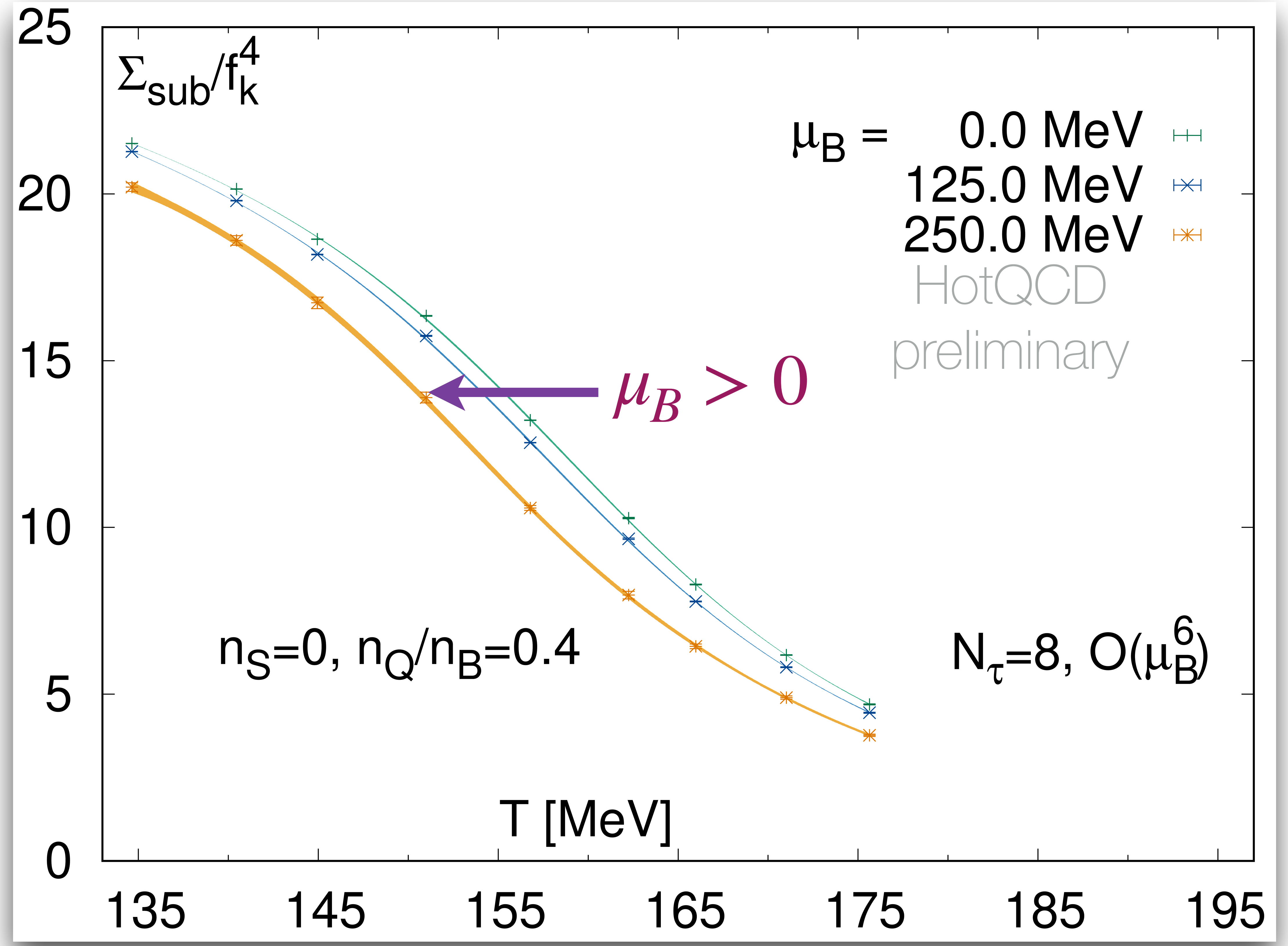
$T_c(\mu_B = 0)$ revamped

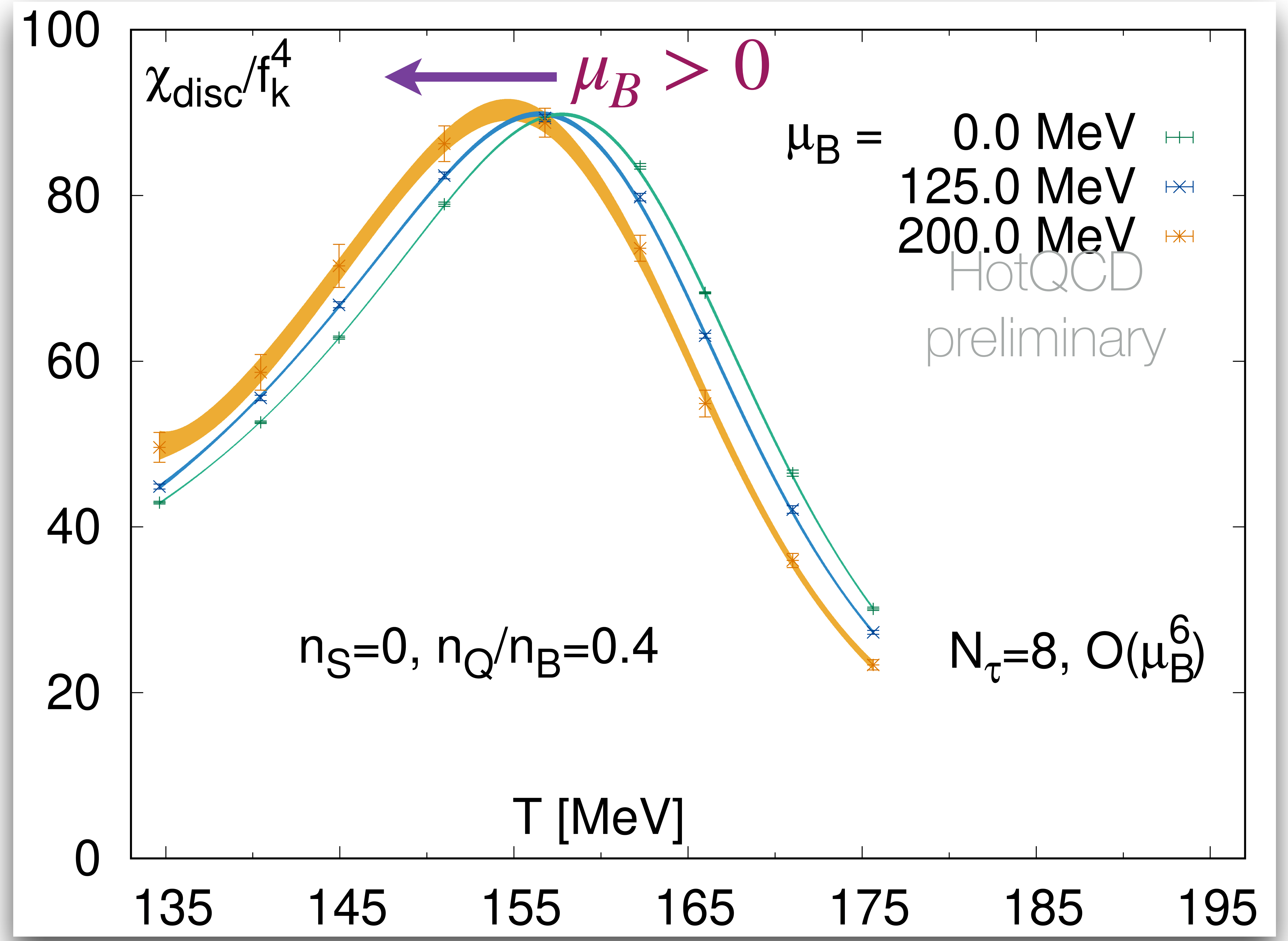




T_0 [MeV]
170
165
160
155
150
145
140
Σ_{sub}
χ_{disc}
χ_{sub}
$\partial^2_{\mu_B}\Sigma_{sub}$
$\partial^2_{\mu_B}\chi_{disc}$
Σ_{sub}, Bonati$_{2015}$
χ_{tot}, Bazavov$_{2012}$
Σ_{sub}, Borsanyi$_{2010}$

$T_c(\mu_B)$



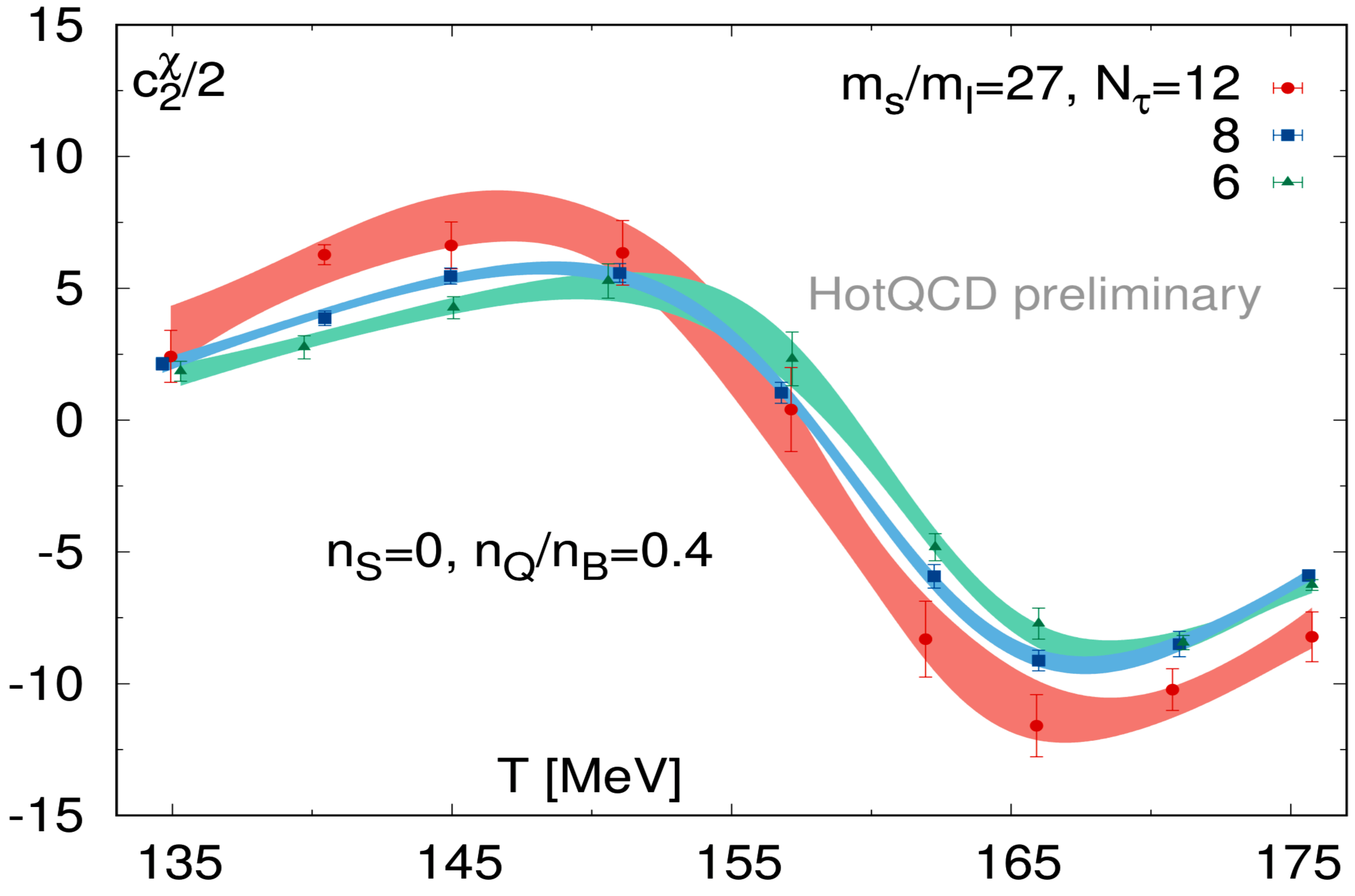


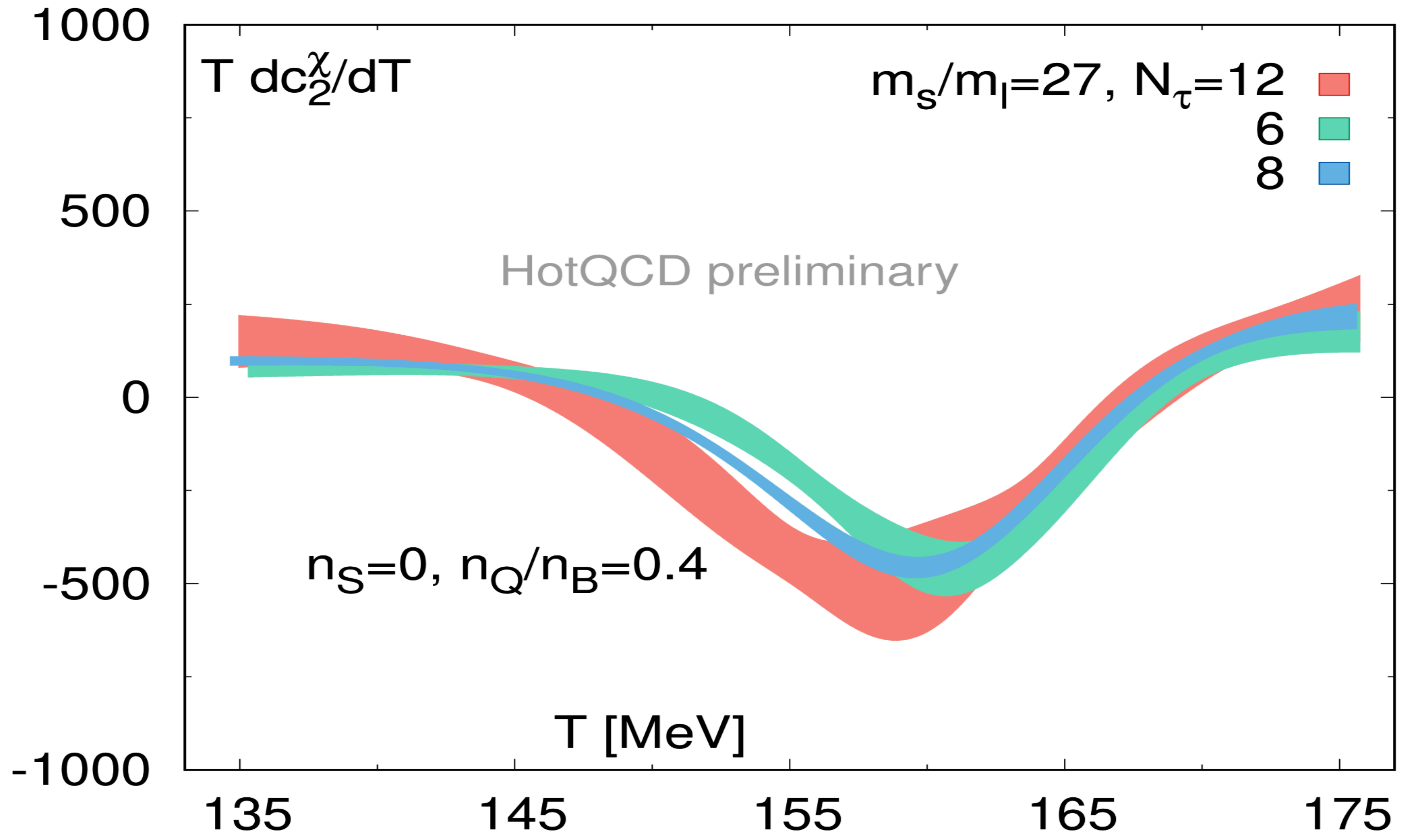
$n_S = 0,\ n_Q/n_B = 0.4$: heavy-ion collision like strangeness neutrality & charge-to-baryon ratio

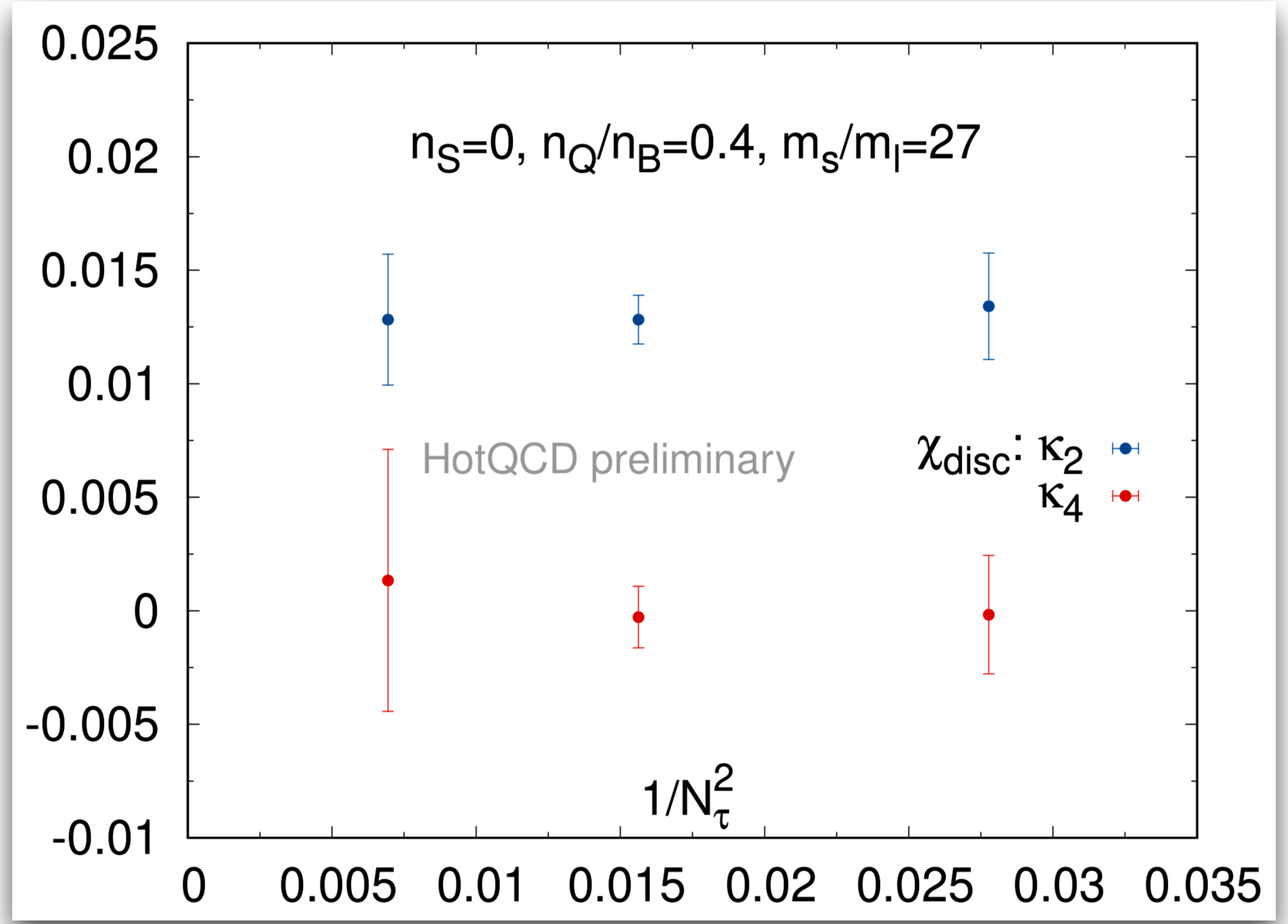
$$\frac{T_c(\mu_B)}{T_0} = 1 - \kappa_2 \left(\frac{\mu_B}{T_0}\right)^2 - \kappa_4 \left(\frac{\mu_B}{T_0}\right)^4 + \mathcal{O}(\mu_B^6)$$

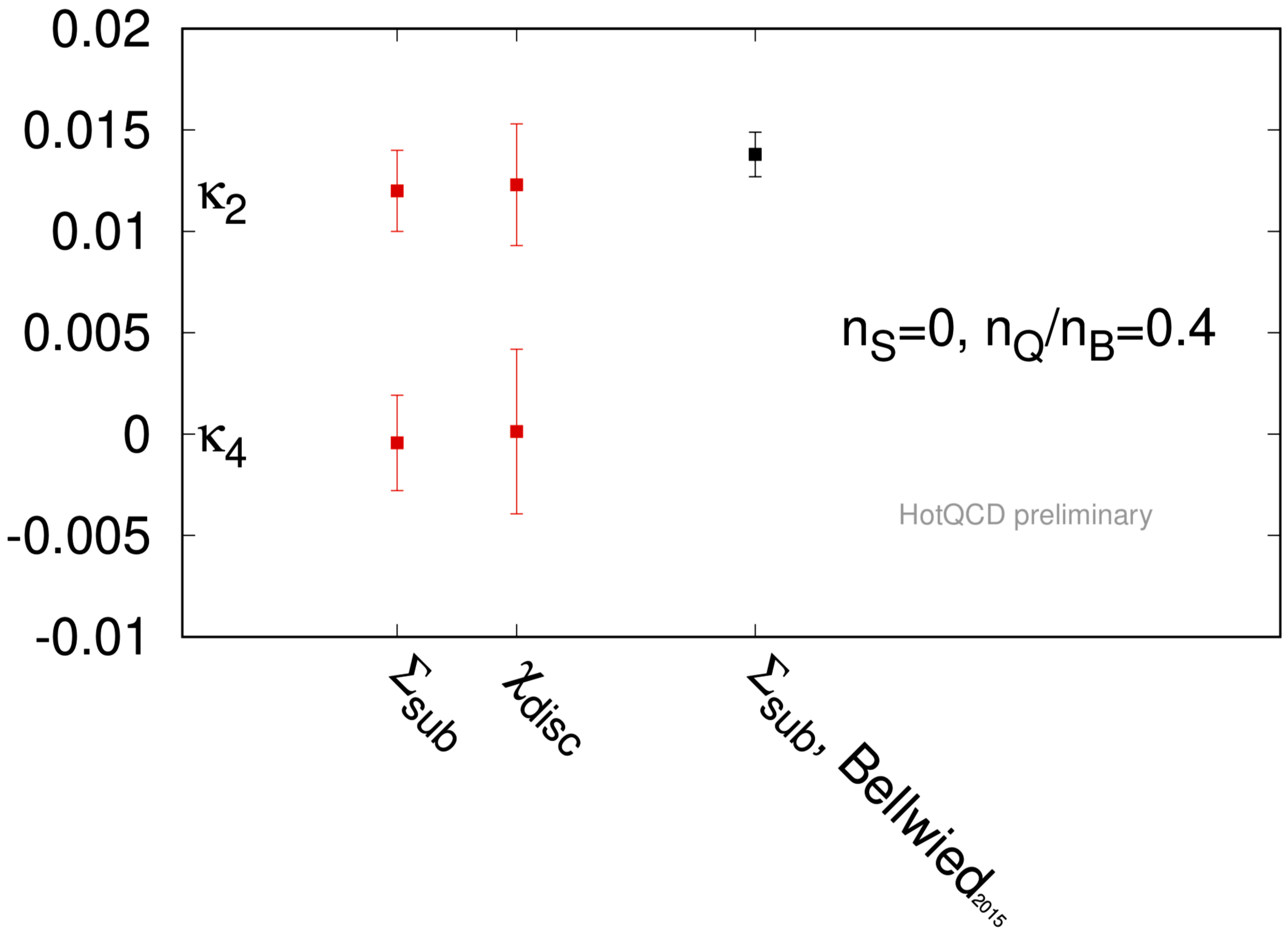
$$\frac{d}{dT}\frac{\chi_{\rm disc}(T,\mu_B)}{f_K^4} = (\ldots)\mu_B^2 + (\ldots)\mu_B^4 + \ldots = \mathbf{0}$$

$$\kappa_2 = \frac{1}{2T_0^2}\frac{T_0\left.\frac{\partial c_2^\chi}{\partial T}\right|_{(T_0,0)} - 2\left. c_2^\chi\right|_{(T_0,0)}}{\left.\frac{\partial^2 c_0^\chi}{\partial T^2}\right|_{(T_0,0)}}$$





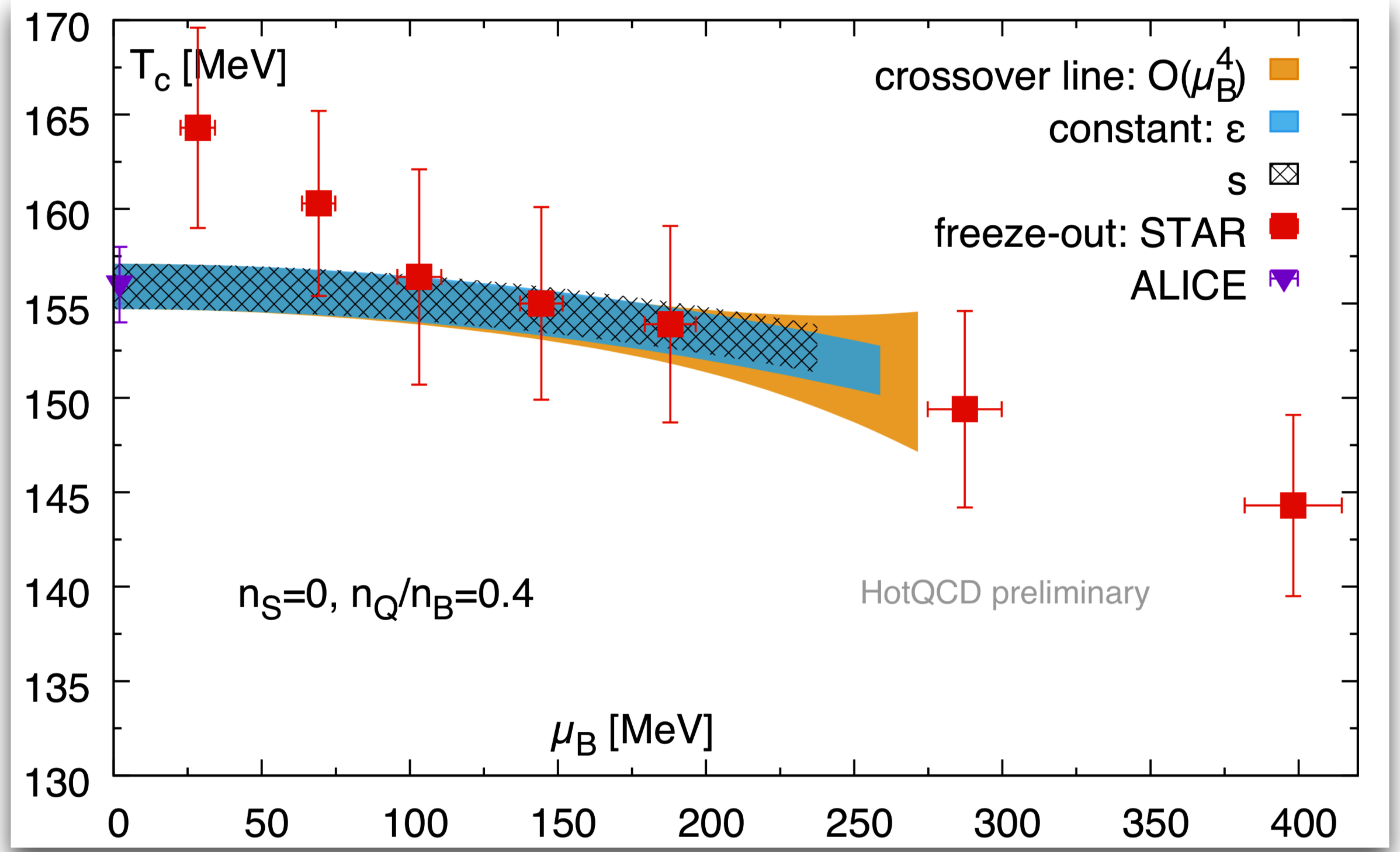




4th order corrections orders of magnitude smaller than 2nd

$$\frac{T_c(\mu_B)}{T_c(0)} = 1-\kappa_2^B\left(\frac{\mu_B}{T_c(0)}\right)^2-\kappa_4^B\left(\frac{\mu_B}{T_c(0)}\right)^4+\mathcal{O}\left(\mu_B^6\right)$$

- along the chiral crossover energy density & entropy density remains constant
- freeze-out line coincides with the chiral crossover

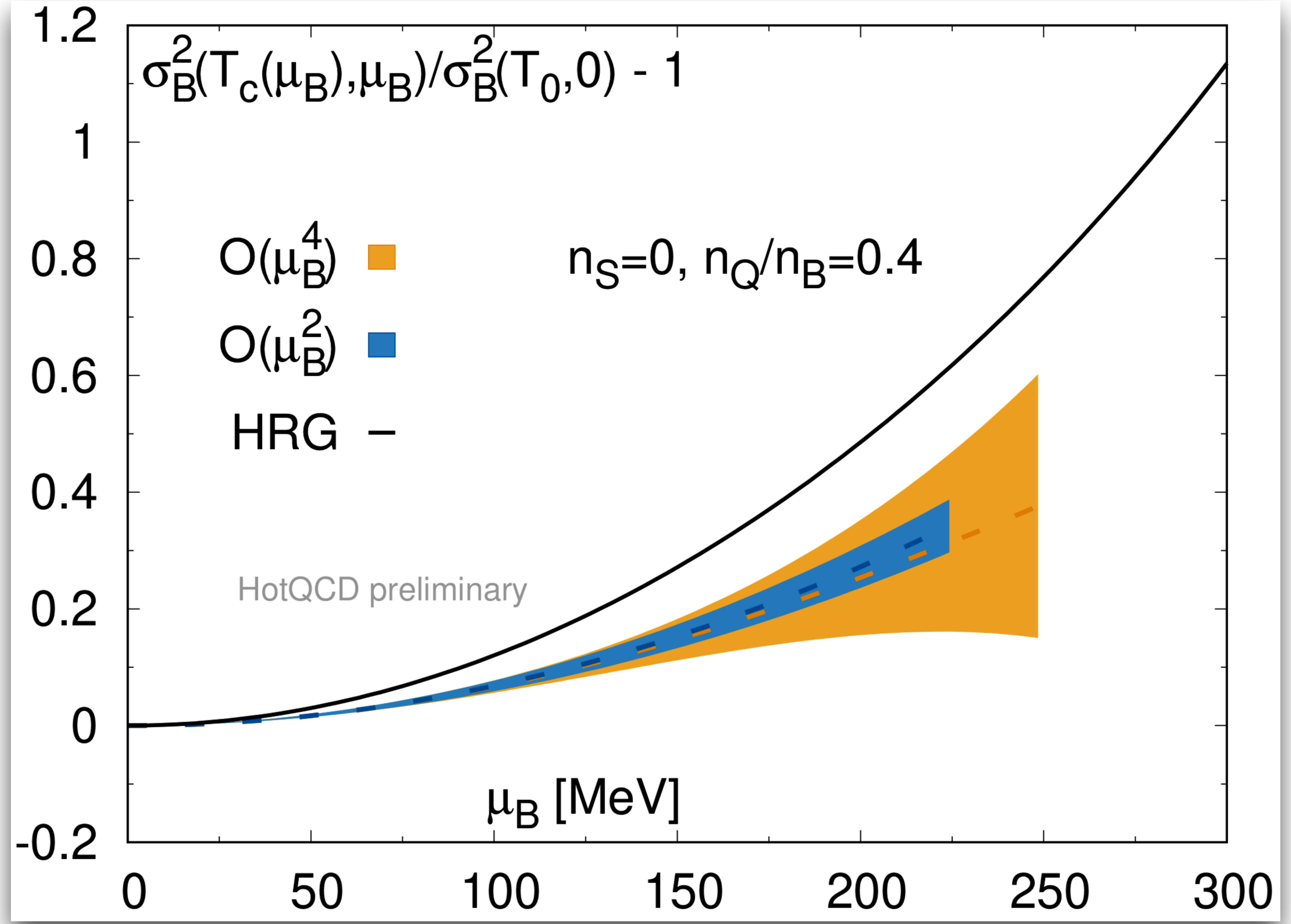


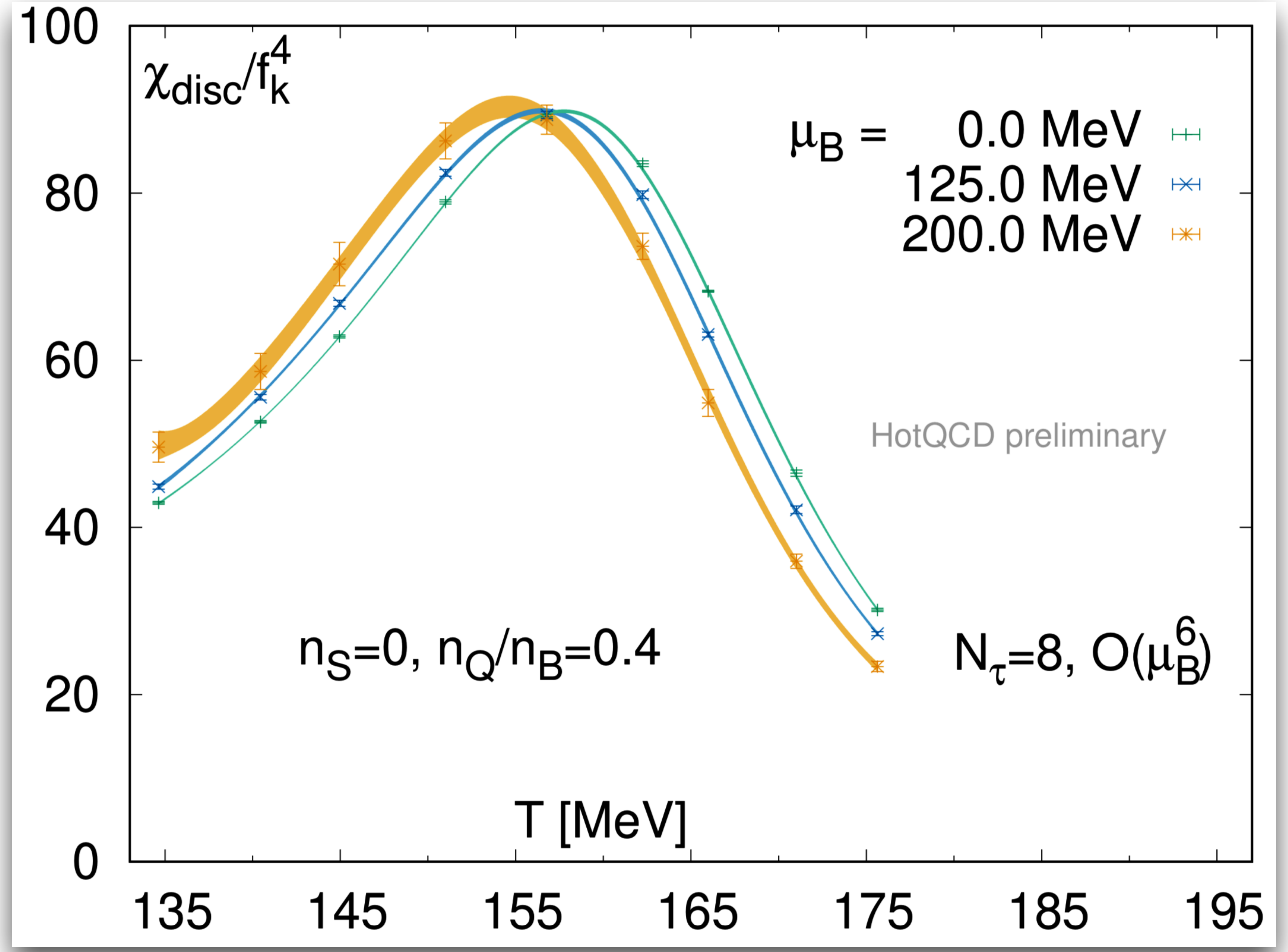
net-baryon number fluctuations along the crossover line

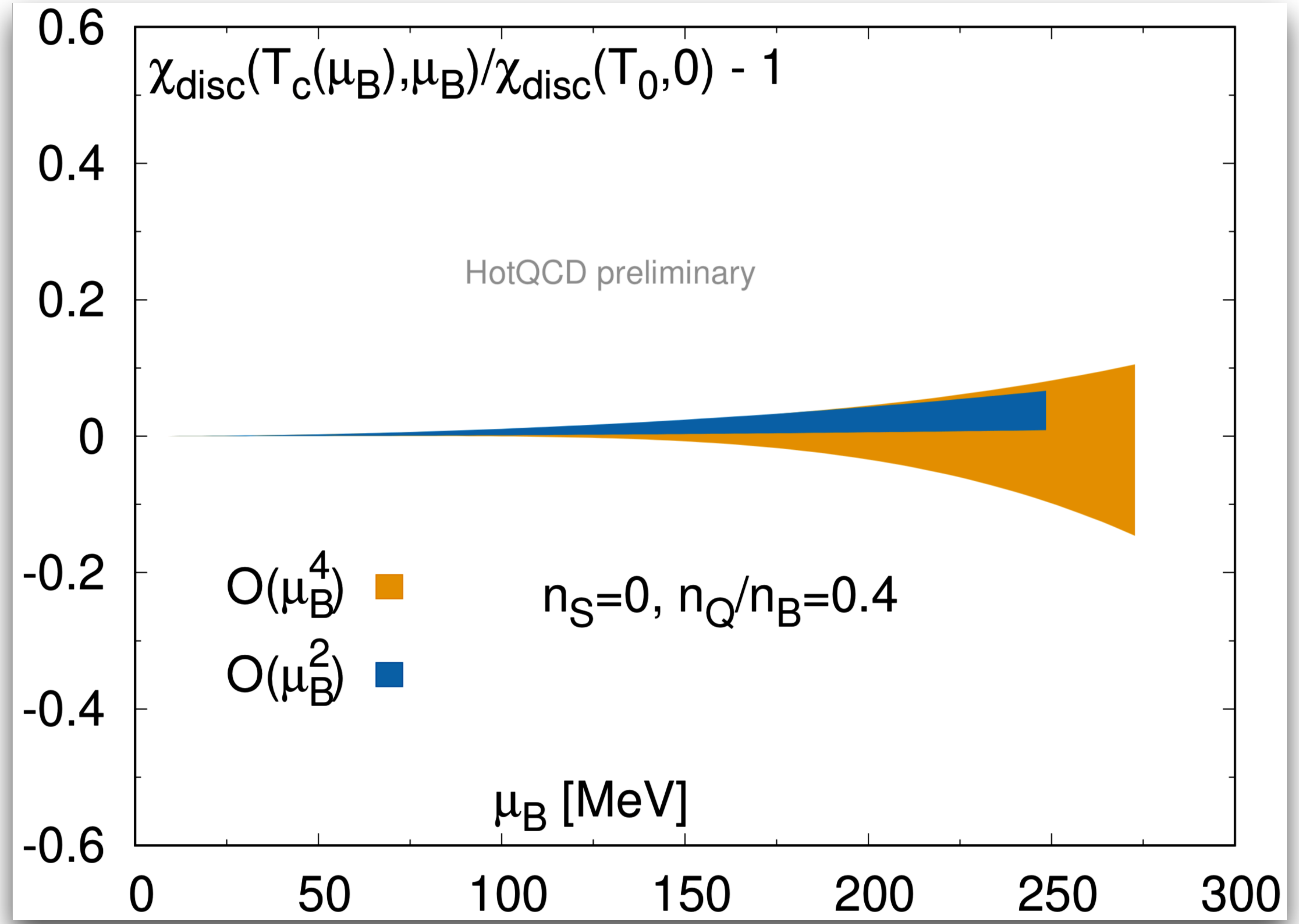
- increase remains less than (ideal) hadron gas resonance gas model

$$\frac{\sigma_B^2}{Vf_K^3} = \frac{1}{Vf_K^3}\frac{\partial \ln Z}{\partial \hat{\mu}_B^2} = \sum_{n=0}^{\infty} \frac{c_n^B}{n!}\hat{\mu}_B^n$$

$$\frac{\sigma_B^2(T_c(\mu_B),\mu_B) - \sigma_B^2(T_0,0)}{\sigma_B^2(T_0,0)} = \lambda_2\left(\frac{\mu_B}{T_0}\right)^2 + \lambda_4\left(\frac{\mu_B}{T_0}\right)^4 + \cdots$$





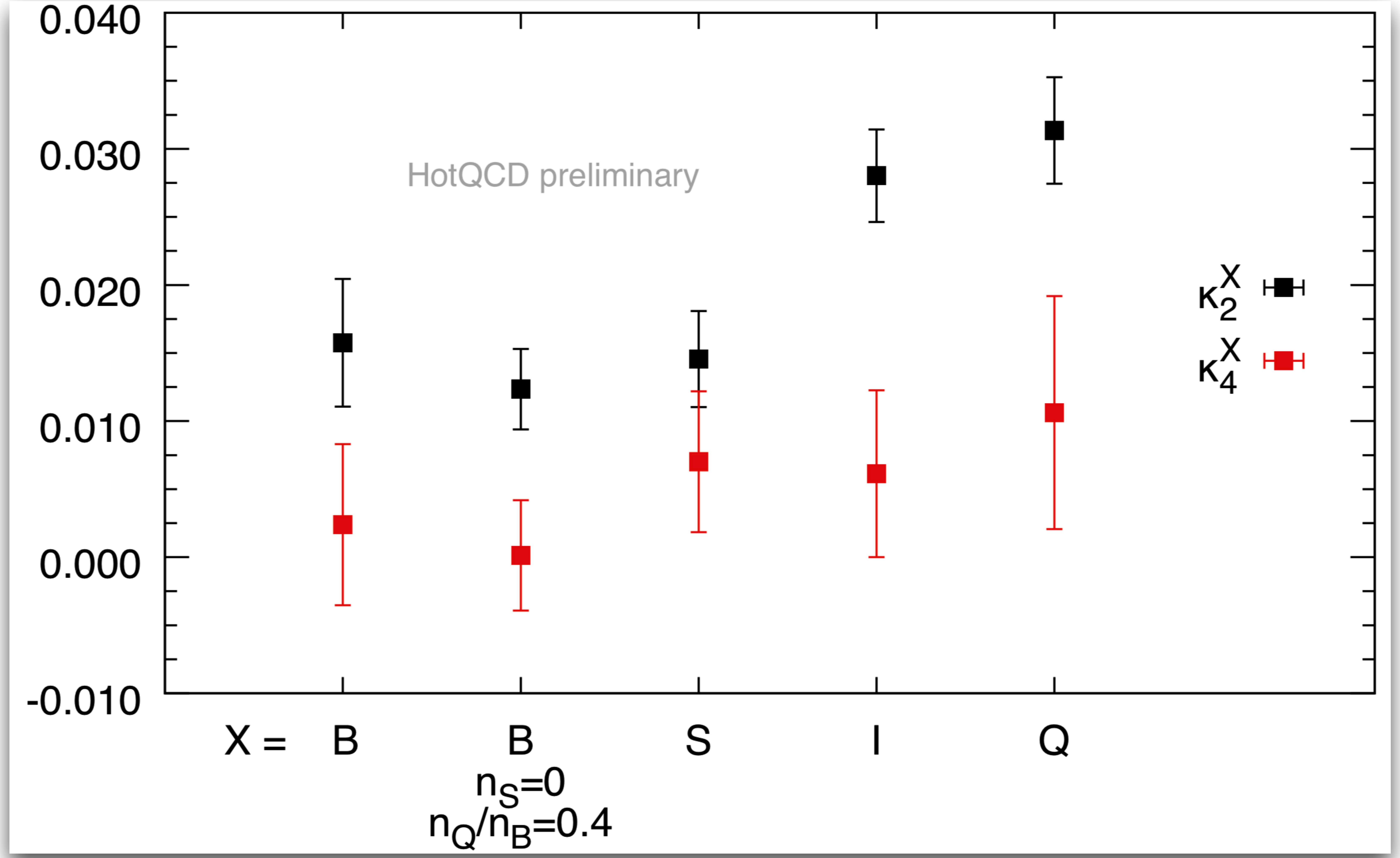


- little change in peak-height & width with increasing baryon chemical potential: no indication of a stronger transition becoming stronger

$T_c(\mu_X)$

$$\frac{T_c(\mu_X)}{T_0} = 1 - \kappa_2^X \left(\frac{\mu_X}{T_0}\right)^2 - \kappa_4^X \left(\frac{\mu_X}{T_0}\right)^4 + \mathcal{O}(\mu_X^6)$$

- X=B: baryon
- X=Q: electric charge
- X=S: strangeness
- X=I: isospin



Summary

- chiral crossover temperature: $\mu_B \lesssim 250\ MeV$

$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa_2^B \left(\frac{\mu_B}{T_c(0)}\right)^2 - \kappa_4^B \left(\frac{\mu_B}{T_c(0)}\right)^4 + \mathcal{O}\left(\mu_B^6\right)$$

$$T_c(\mu_B{=}0) = 156.5 \pm 1.5\ \mathrm{MeV}$$

$n_S = 0,\ n_Q/n_B = 0.4:$

$$\kappa_2^B = 0.012 \pm 0.003$$

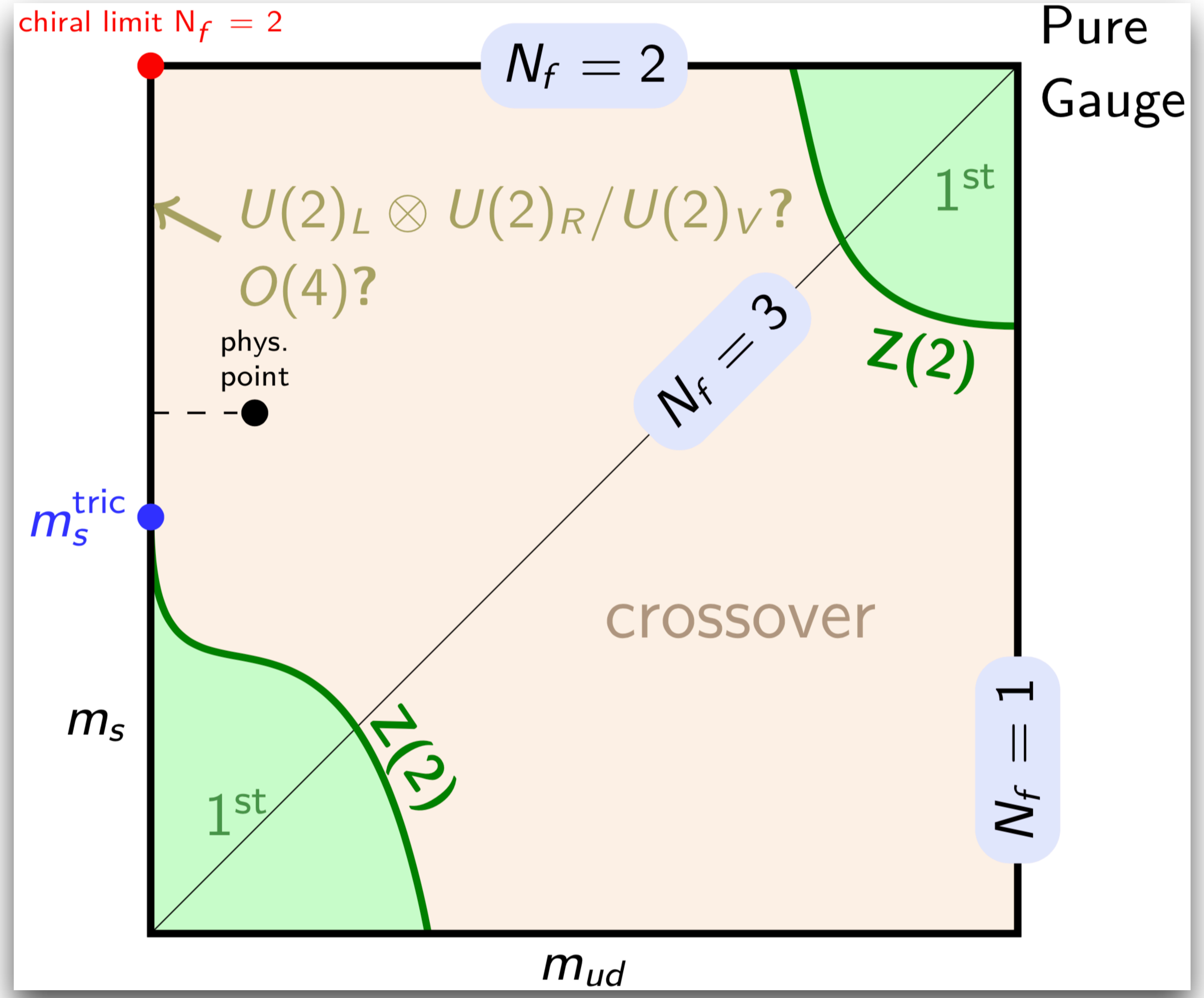
$$\kappa_4^B = 0.0001 \pm 0.004$$

- no indication of increased fluctuations & transition becoming stronger

- along the chiral crossover energy density & entropy density remains constant

- freeze-out line coincides with the chiral crossover

QCD transition: towards chiral limit



- u/d quark masses: $m_u = m_d \equiv m_l \to 0$
- s quark mass: m_s^{phys}

$$H \equiv m_l / m_s^{phys}$$

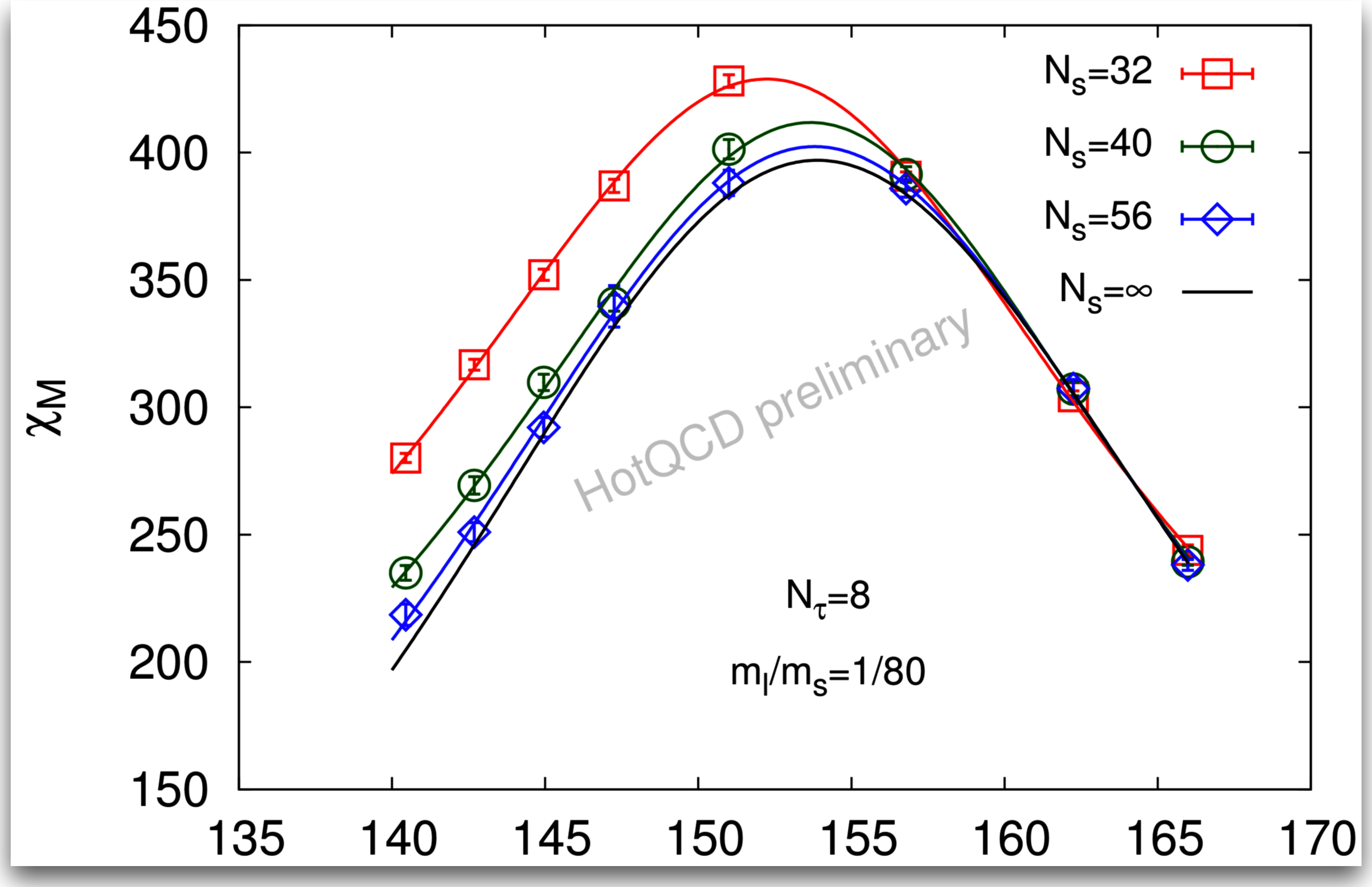
$$M \equiv \Sigma_{sub} \sim H^{1/\delta} f_G(z)$$

$$\chi_M \equiv \chi_{sub} \sim H^{1/\delta - 1} f_\chi(z)$$

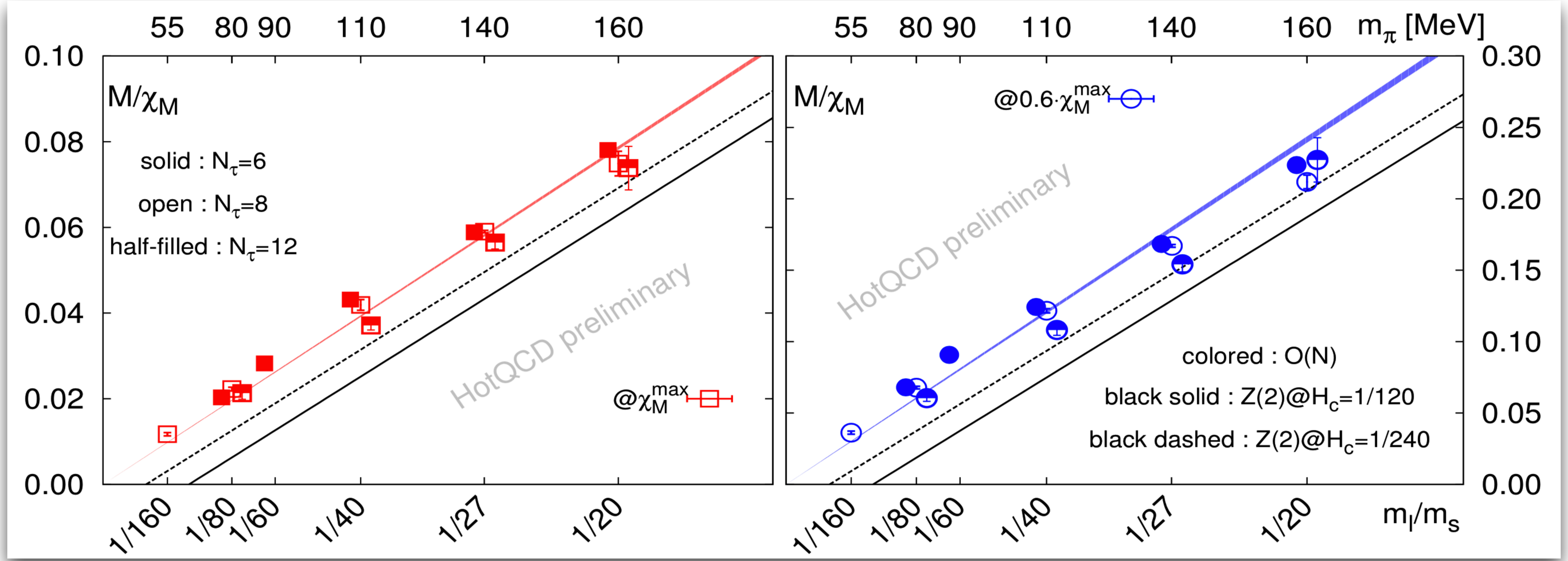
HotQCD preliminary

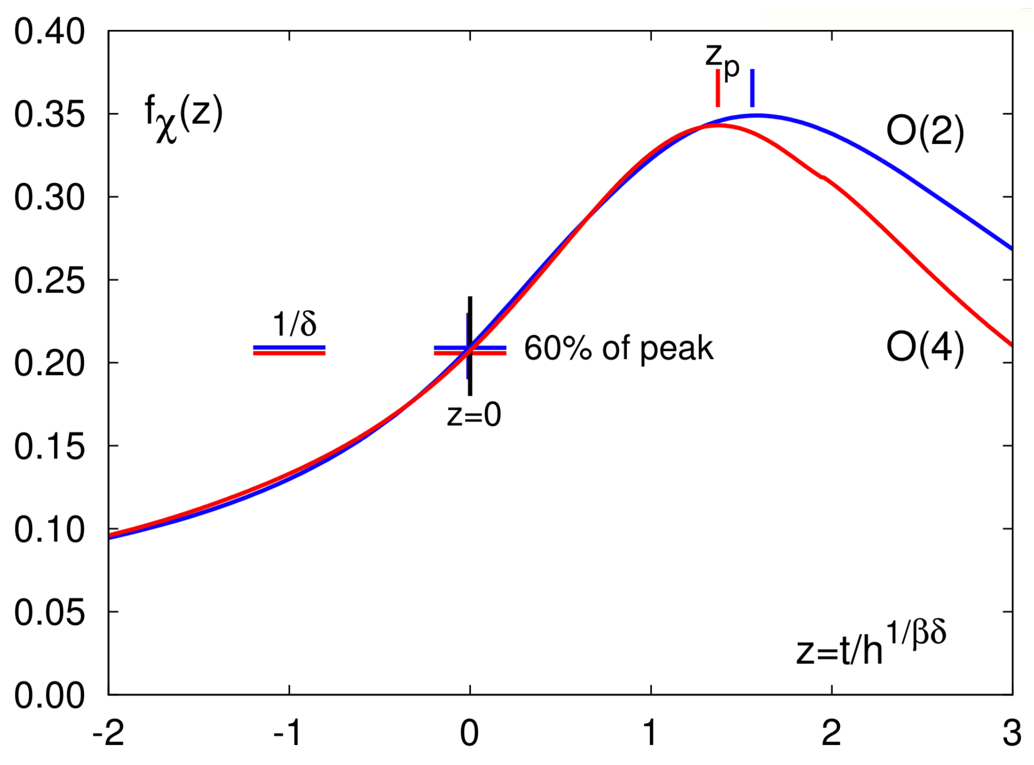
$m_\pi \approx 80\ MeV$

- no direct evidence of 1st order transition



volume dependence of chiral susceptibility





$$\frac{H\chi_M}{M} = \frac{f_\chi(z)}{f_G(z)} + \text{regular terms.}$$

- consistent with O(N) universality