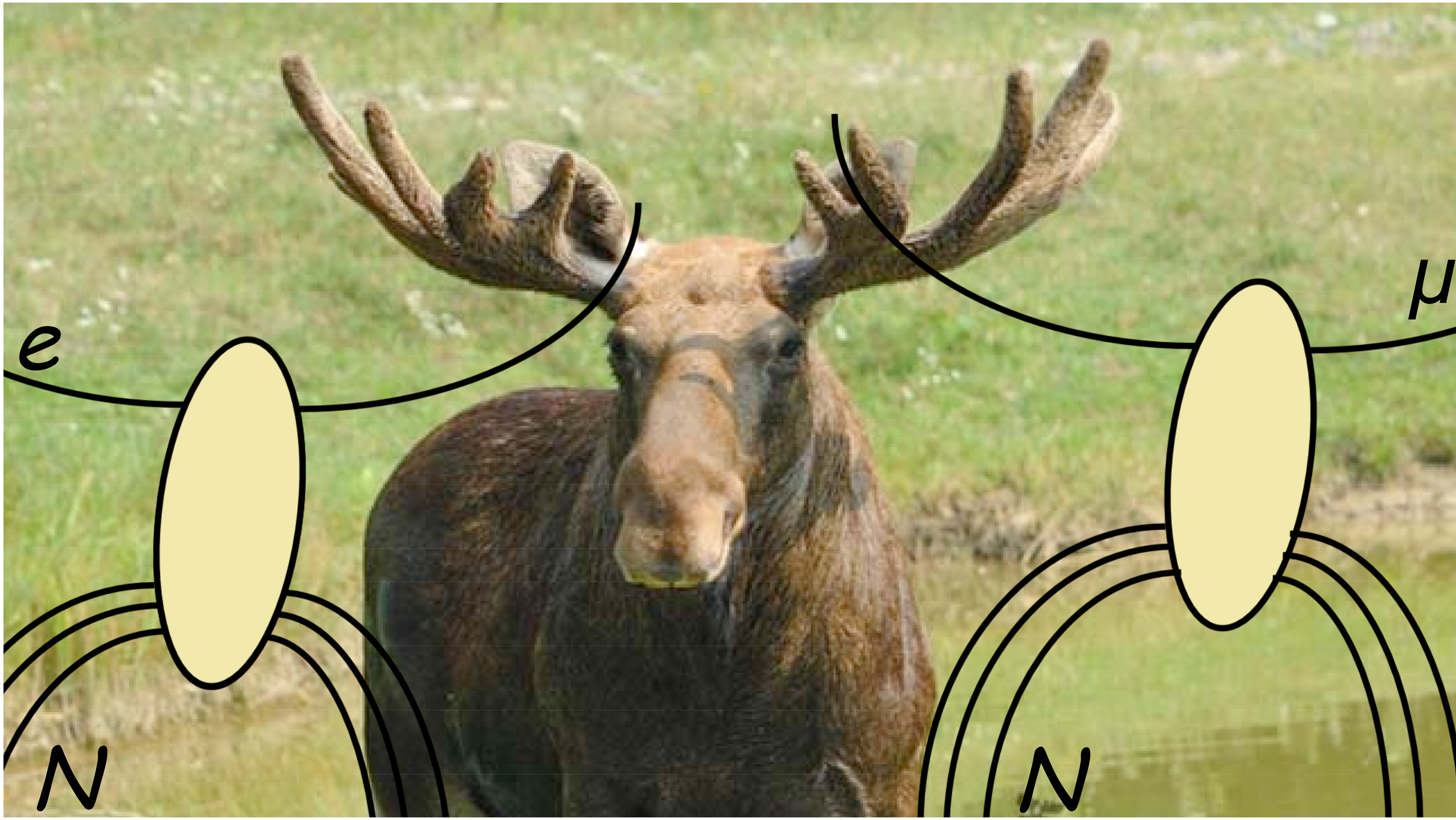
Physics of bound electrons and muons

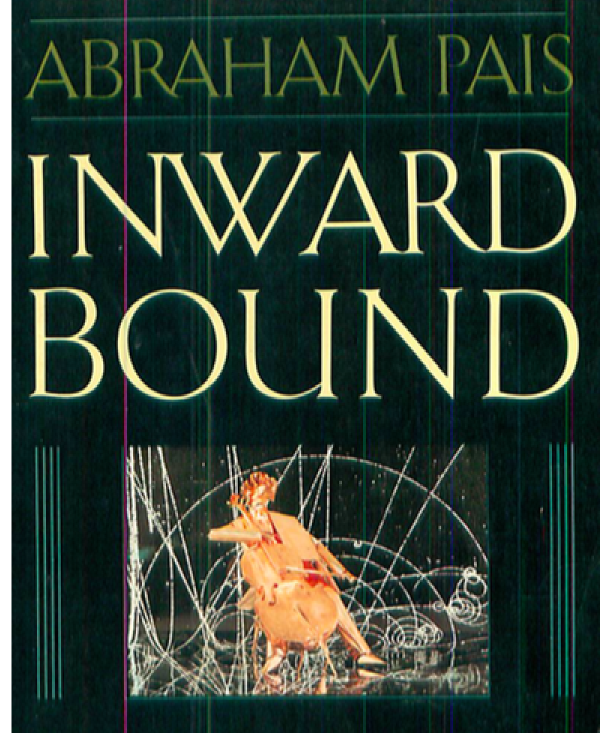


Colloquium, GSI Darmstadt

Andrzej Czarnecki 🍁 University of Alberta

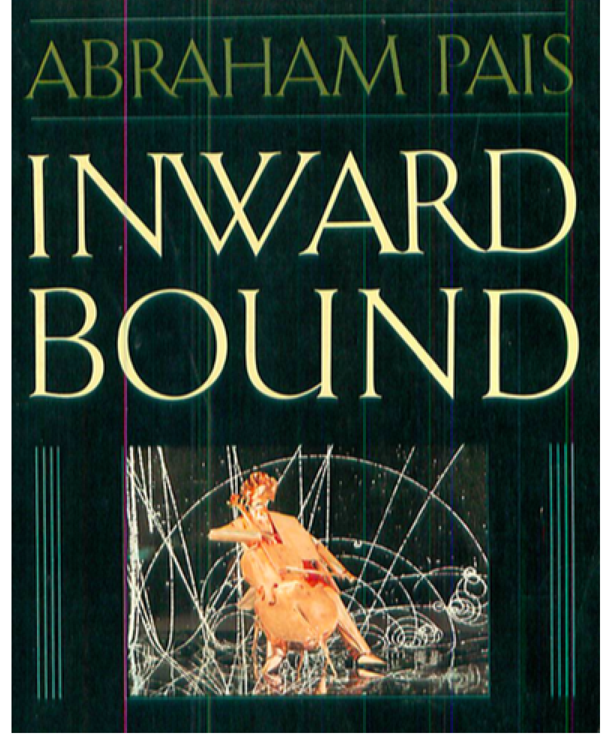
November 8, 2016

Purpose and plan



(a) From X to Z

It was the afternoon of 8 November ...



Purpose and plan

(a) From X to Z

It was the afternoon of 8 November 1895. Wilhelm Conrad Roentgen was all by himself in his laboratory in Würzburg, experimenting once again with

Today: anniversary of Röntgen's discovery

November 8, 1895

Outline

Puzzles and perspectives in muon physics

Muon decay in orbit
- context: search for muon-electron conversion
- energy regions and radiative corrections

g-factor of a bound electron
- best way to determine the atomic mass of the electron
- possible source of the fine structure constant

New era of experiments with muons

PSI (Switzerland):

- muonic atoms
- mu -> e + gamma
- mu + p scattering
- mu -> eee

Fermilab (USA):

- g-2
- Mu2e

J-PARC (Japan):

- g-2
- DeeMe
- COMET
- muonium HFS

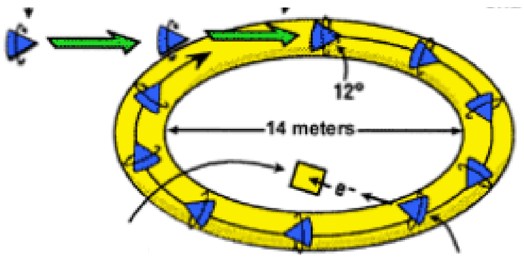
Muons are indeed a great tool for New Physics searches:
long-lived, just massive enough, easy to produce, with convenient spin properties.

They are also mysterious. Some precise measurements disagree with expectations:
g-2, proton radius, Pierre Auger Observatory, B-decays.

$$B^0 \to K^{*0}\mu^+\mu^-$$
$$R_K = \mathrm{BR}(B^+ \to K^+\mu^+\mu^-)/\ \mathrm{BR}(B^+ \to K^+e^+e^-)$$

The puzzle of the muon magnetic moment

The 3.6 sigma discrepancy,



$$a_\mu^{\rm exp} - a_\mu^{\rm SM} = 287(80) \times 10^{-11}$$

PRD 86, 095009 (2012)

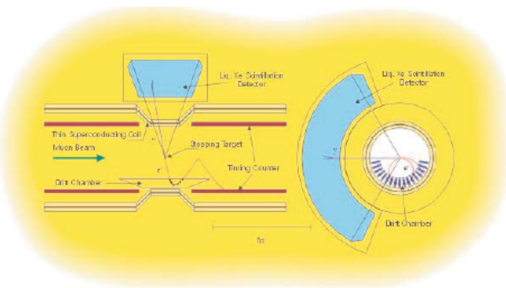
is rather large when compared with other bounds on New Physics.

Lepton flavor violation: $\mu \rightarrow e\gamma$

New bound (MEG @ Paul Scherrer Institute)

$$\mathrm{BR}\,(\mu \to e\gamma) < 4.2 \cdot 10^{-13}$$

arXiv:1605.05081



This corresponds to the transition dipole moment

$$d_{\mu\to e} \lesssim 3.5 \cdot 10^{-27}\ e \cdot \mathrm{cm}$$

Sensitive to the heaviest "new physics"

For comparison: electron EDM $d_e < 0.87 \cdot 10^{-28}\, e \cdot \mathrm{cm}$ 10.1126/science.1248213

muon g-2 $d_\mu < 3 \cdot 10^{-22}\, e \cdot \mathrm{cm}$

How to check g_μ-2?

Electron g-2 is likely sensitive to the same New Physics; but at present it is used to determine the fine-structure constant.

A new source of alpha is needed.

Note:

Centenary!
First introduced by Sommerfeld 1916

How to check g_μ-2?

Nature 442, 516 (2006)
PRA 89, 052118 (2014)

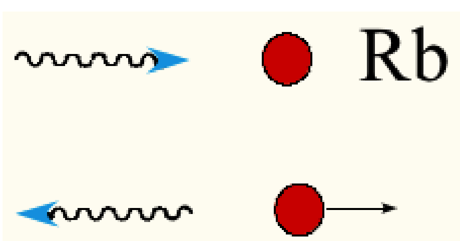
The second best determination of alpha:
from atomic spectroscopy

$$R_\infty = \frac{m_e c \alpha^2}{2h}$$

Needed precision:

$$\alpha^2 = \frac{2R_\infty}{c} \cdot \frac{u}{m_e} \cdot \frac{M_X}{u} \cdot \frac{h}{M_X}$$

- α^2: $14 \cdot 10^{-11}$
- $\frac{2R_\infty}{c}$: $7 \cdot 10^{-12}$ (but is it for sure?)
- $\frac{u}{m_e}$: $8 \cdot 10^{-11}$ Nature 2014 Sturm et al
- $\frac{M_X}{u}$: $12 \cdot 10^{-11}$ for Rb (better for He)
- $\frac{h}{M_X}$: $124 \cdot 10^{-11}$ improvement needed by factor ~10



gives h/m

$$\alpha(\mathrm{Rb}) = 1/137.035\,999\,049(90) \quad [66 \cdot 10^{-11}]$$

PRL 106, 080801 (2011)

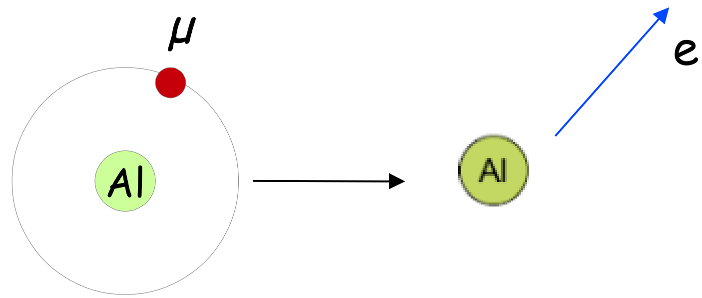
Today: Rydberg's birthday (November 8, 1854)

1897: struggle for professorship at Lund:

"Doctor Rydberg's works are
of great scientific value
and prove great diligence
and interest for the treatment
of an often ungrateful problem,
but cannot completely establish
his competency for the appointment in question,
as they are not based on his own measurements"

$$\frac{1}{\lambda} = R_\infty \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

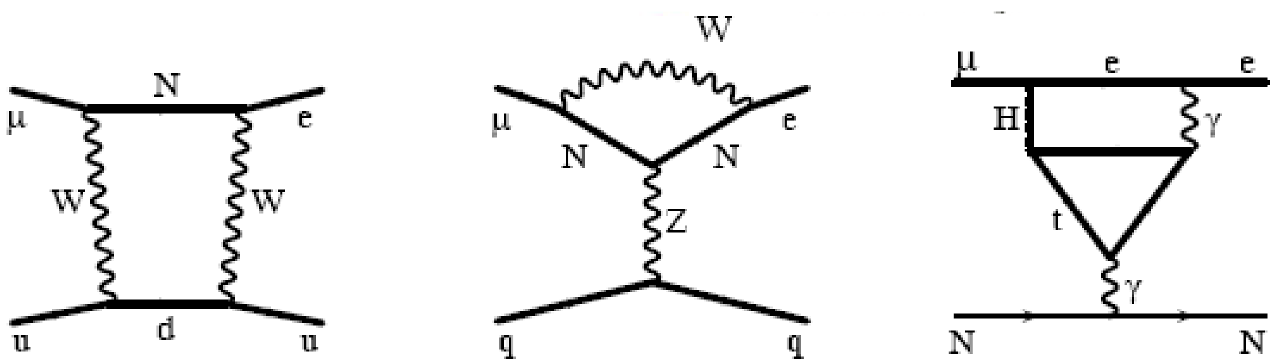
Muon-electron conversion: probes various types of interactions



New process: muon-electron conversion

(as well as mu --> eee)

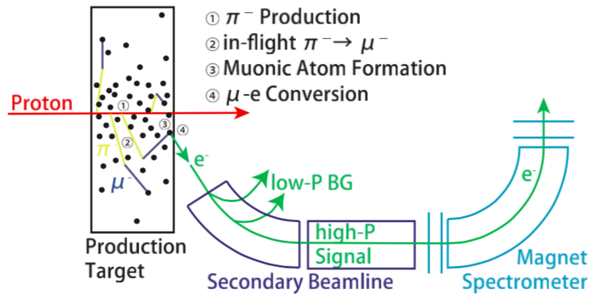
Variety of mechanisms:



Probes also non-dipole interactions that are not (directly) probed by processes with external photons, by gauge invariance requirements.

Muon-electron conversion plans (The Next Big Thing in muon physics)

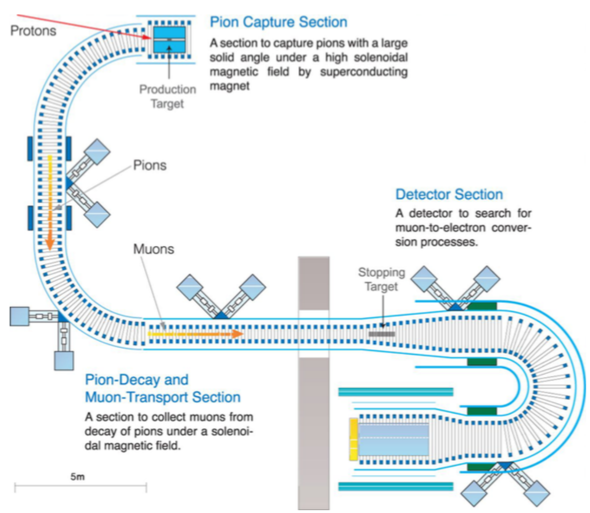
DeeMe
J-PARC



starts 2016;
aims for 1e-13 (graphite target),
followed by 1e-14 (SiC target)

COMET
Phase 1
J-PARC

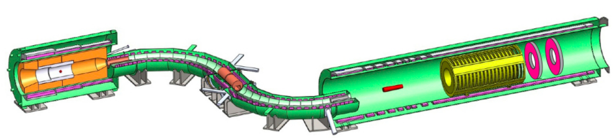
7e-15



COMET
Phase 2
J-PARC

2.6e-17

Mu2e
Fermilab

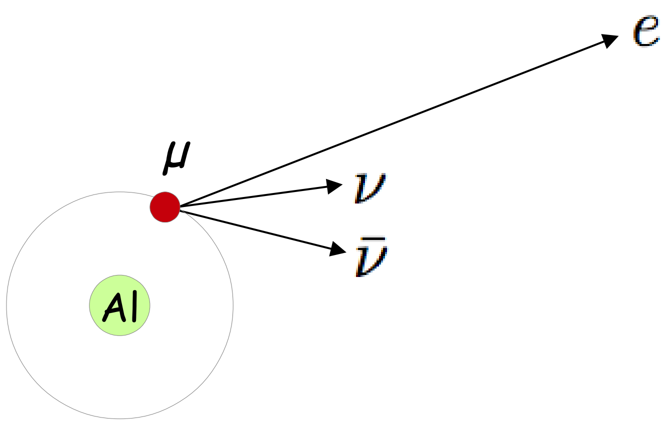


2e-17

from Hiroaki Natori JPS Conf. Proc. 8, 025017 (2015)

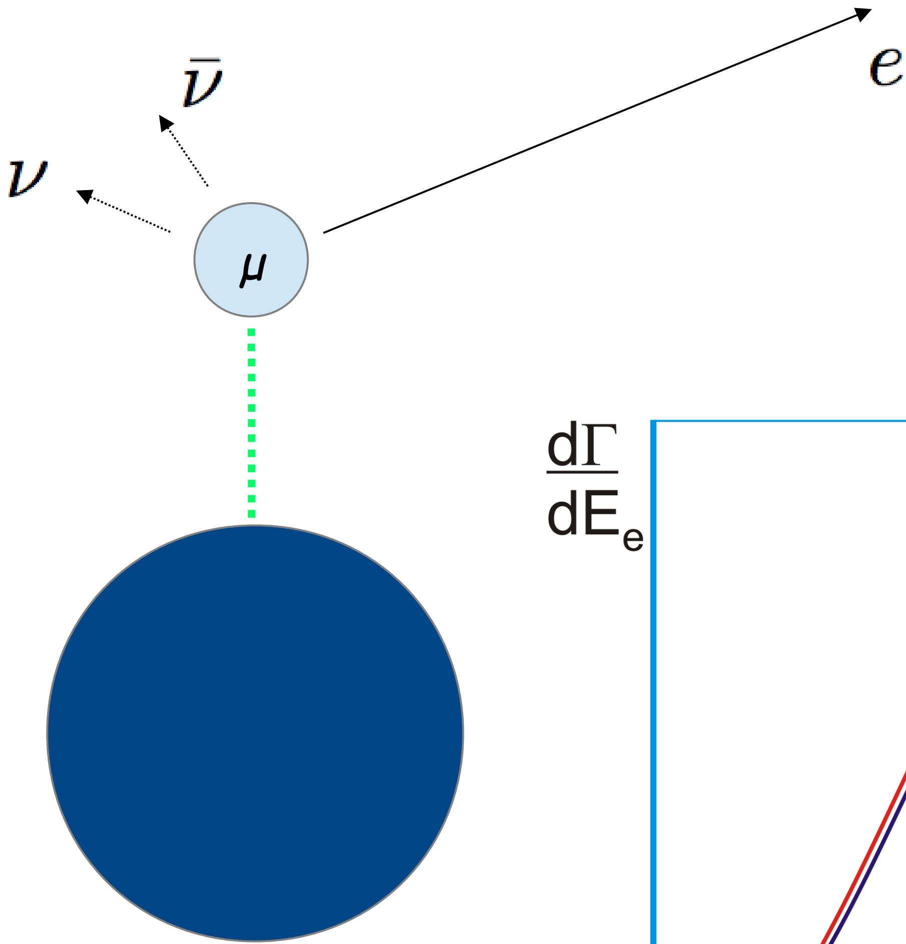
Background for the conversion search

Normal decay of the muon bound in the atom
can produce high-energy electron,



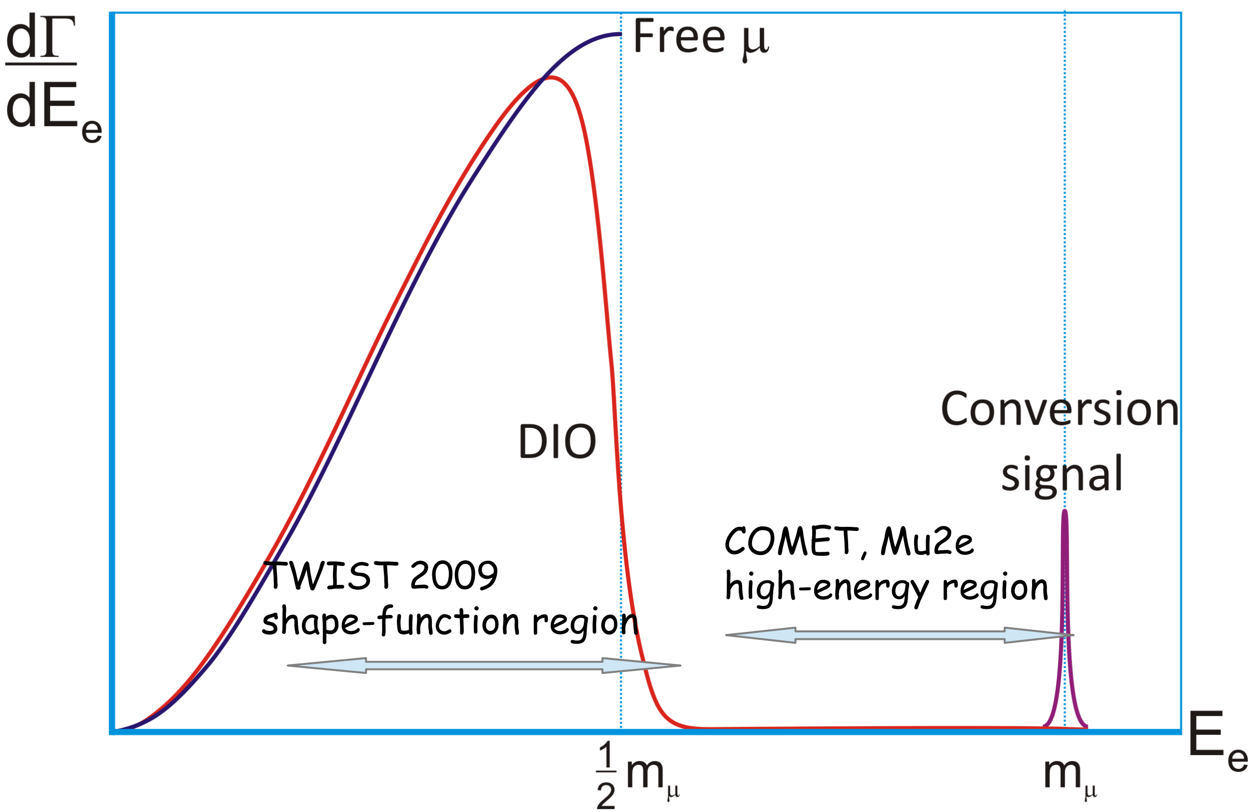
Spectrum has to be well understood.

Electron spectrum in a bound muon decay



Electron energy can be as large as the whole muon mass

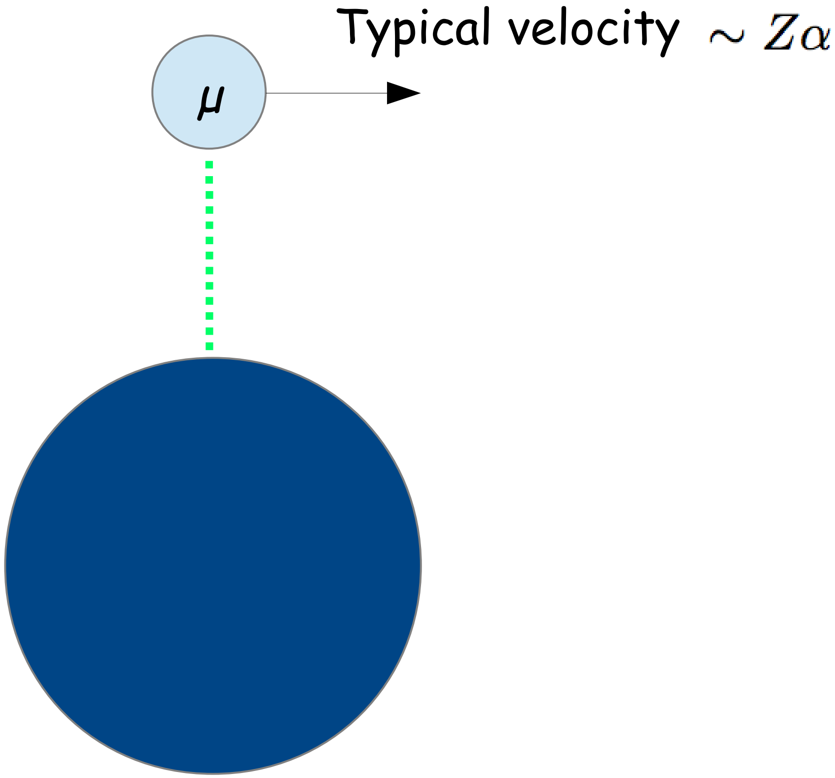
Similar to semileptonic heavy quark decays



Muon decay-in-orbit spectrum:
the shape-function region

Experiment: TWIST

Two effects: muon motion & Coulomb attraction



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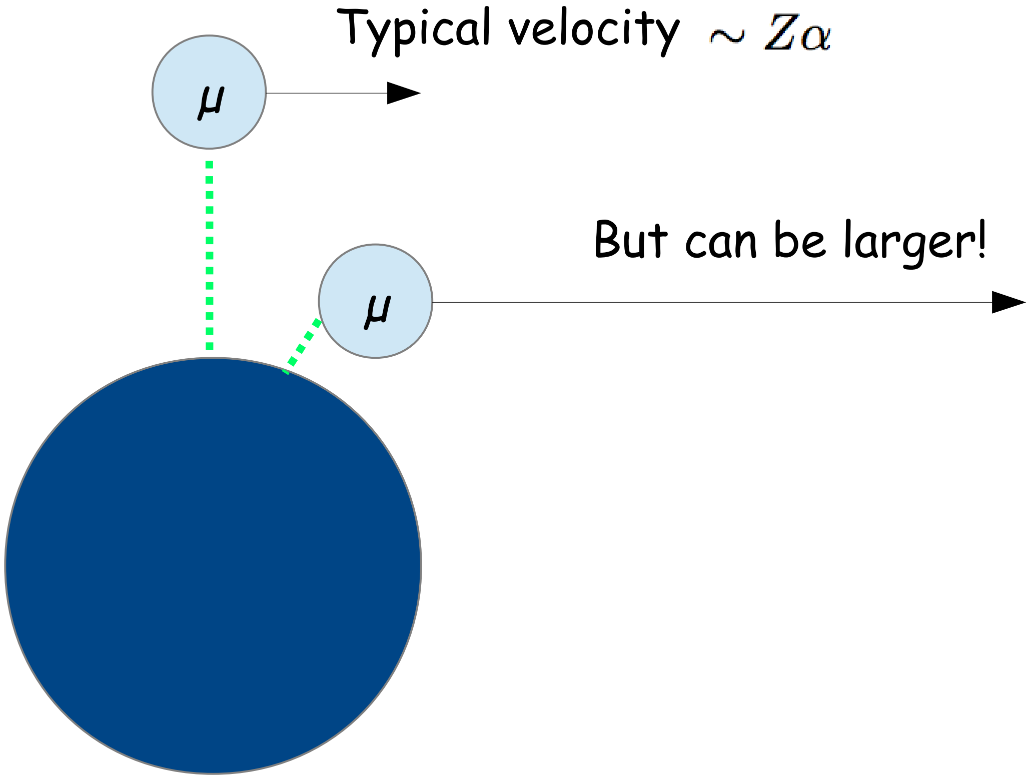

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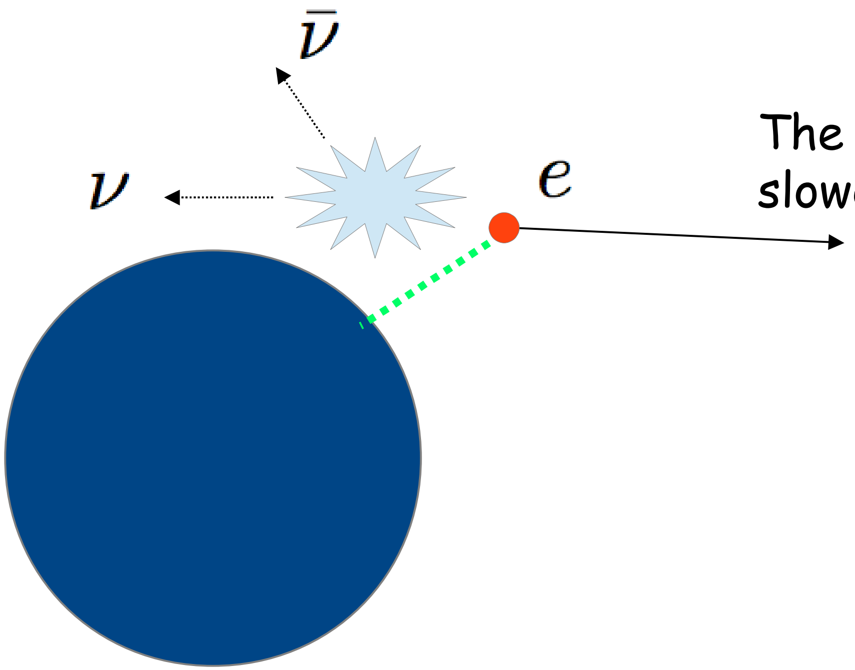
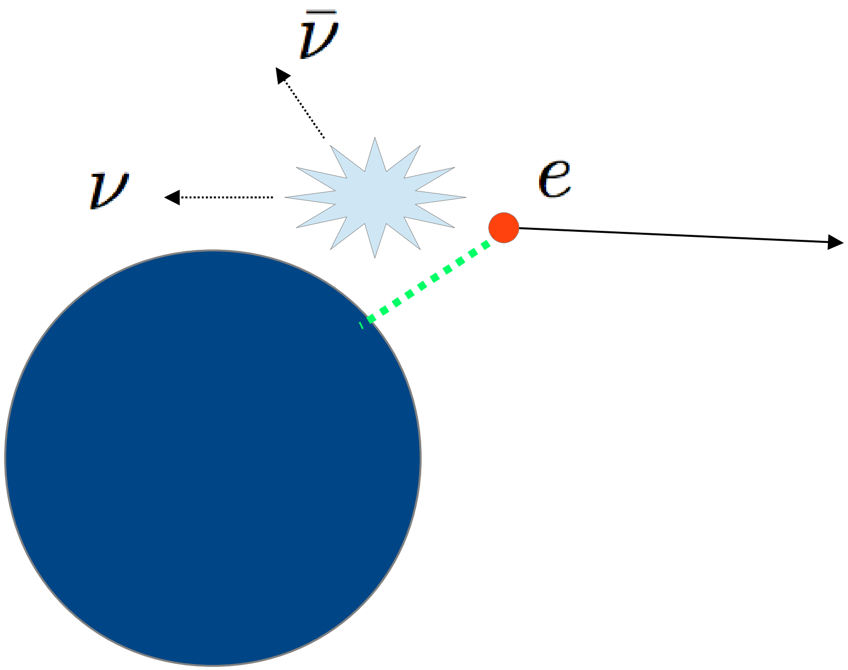


The boosted daughter electron
slowed down by the Coulomb attraction

Two effects: muon motion & Coulomb attraction



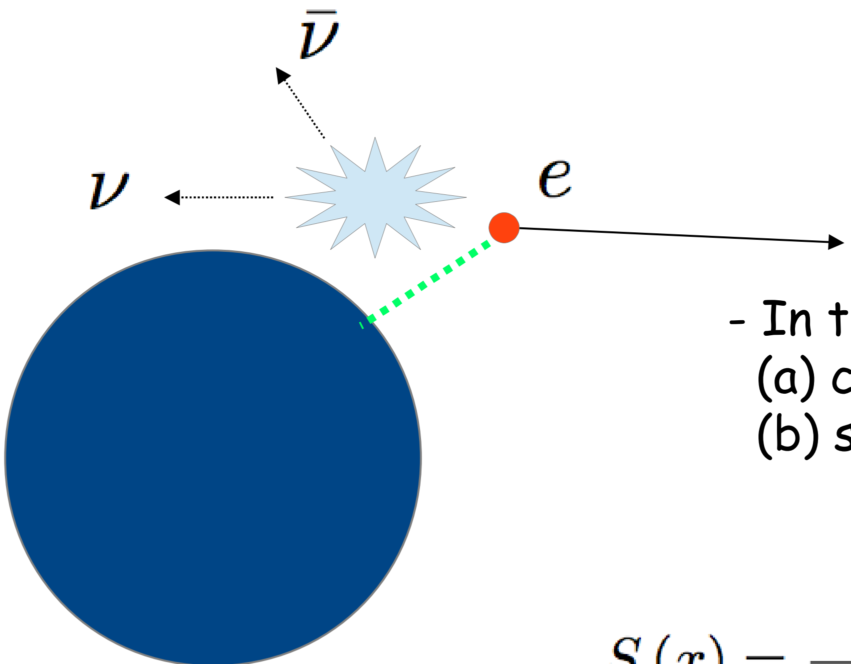
Net effect:
- In the decay rate: almost none;
  only time dilation

$$\Gamma \to \left(1 - \frac{(Z\alpha)^2}{2}\right)\Gamma$$

Überall, Phys. Rev. 119, 365 (1960)

Two effects: muon motion & Coulomb attraction

Szafron, AC, PRD 92 (2015) 053004



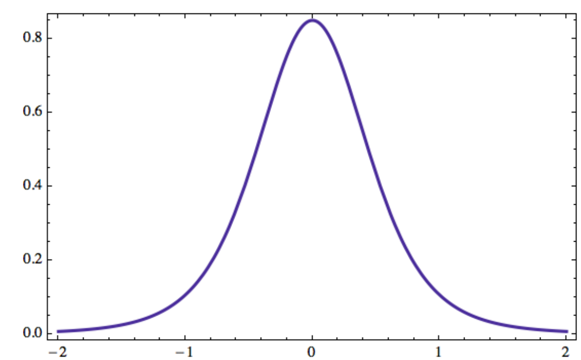
Net effect:

- In the decay rate: almost none; only time dilation

$$\Gamma \to \left(1 - \frac{(Z\alpha)^2}{2}\right)\Gamma$$

- In the electron energy spectrum:
  (a) computable shift
  (b) smearing -> "shape function"

$$S(x) = \frac{8}{3\pi\left[1+x^2\right]^3}$$



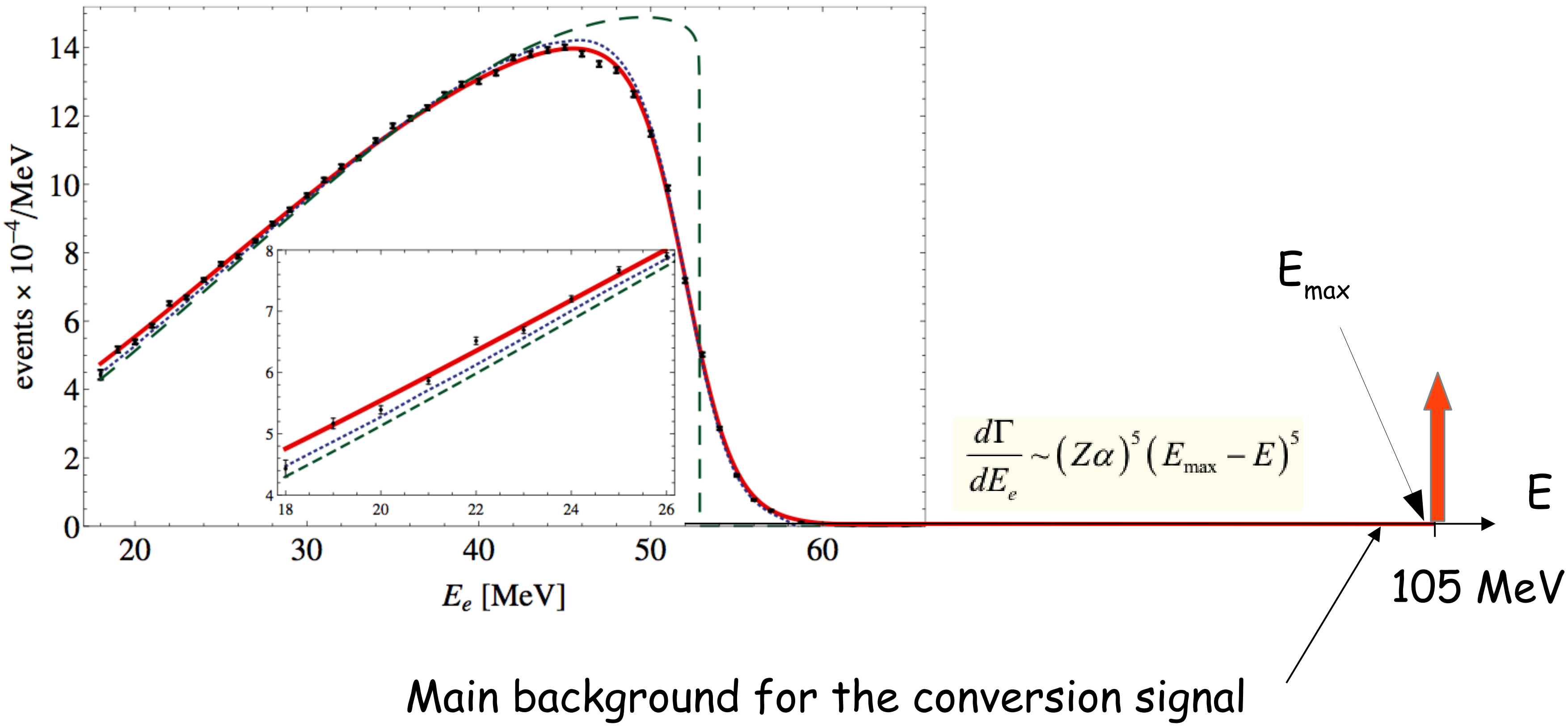
Previously used in heavy mesons, where it cannot be computed from first principles, but can be experimentally accessed.

Mannel, Neubert,
Bigi, Shifman, Uraltsev, Vainshtein

Muon decay-in-orbit spectrum: the high-energy region

Experiments: Mu2e and COMET

Spectrum of the bound muon decay

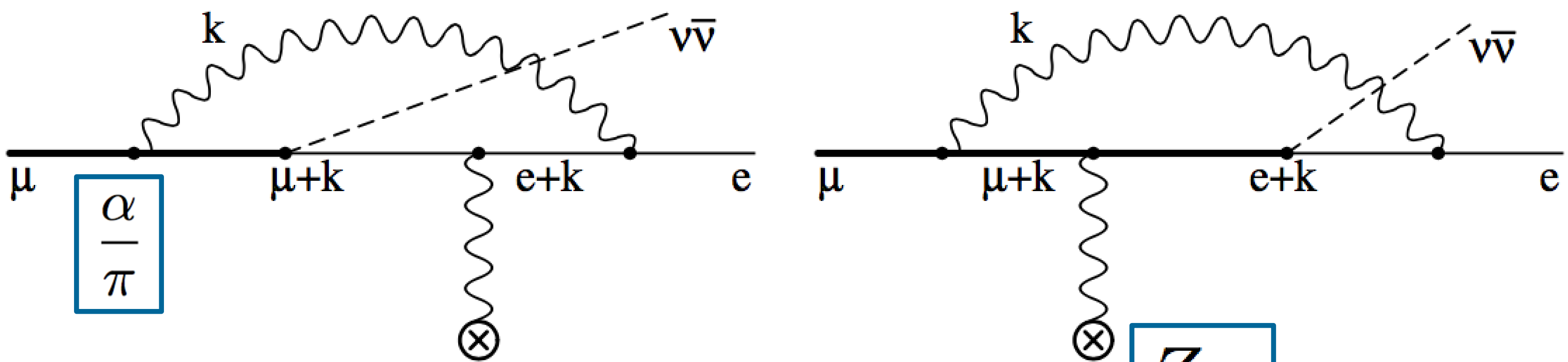


AC, M. Dowling, X. Garcia i Tormo, W. Marciano, R. Szafron
R. Szafron, AC

Radiative corrections to the electron spectrum

Expansion near the end-point $\frac{m_\mu}{\Gamma_0}\frac{d\Gamma}{dE} = \sum_{ijk} B_{ijk}\Delta^i (\pi Z\alpha)^j \left(\frac{\alpha}{\pi}\right)^k$

Three "small" parameters: $\Delta = \frac{E_{\max} - E}{m_\mu}$



$Z\alpha$

$$B_{550} + \frac{\alpha}{\pi}B_{551} \to B_{550}\left[\Delta^{\frac{\alpha}{\pi}\delta_S} + \frac{\alpha}{\pi}\delta_H\right]$$

$$\delta_S = 10.1$$

number of electrons in the end-point bin of 1 (0.1) MeV is reduced by 11% (16%)

Szafron, AC, PLB753, 61 (2016)

Part 2/2: magnetic moment (bound electron)

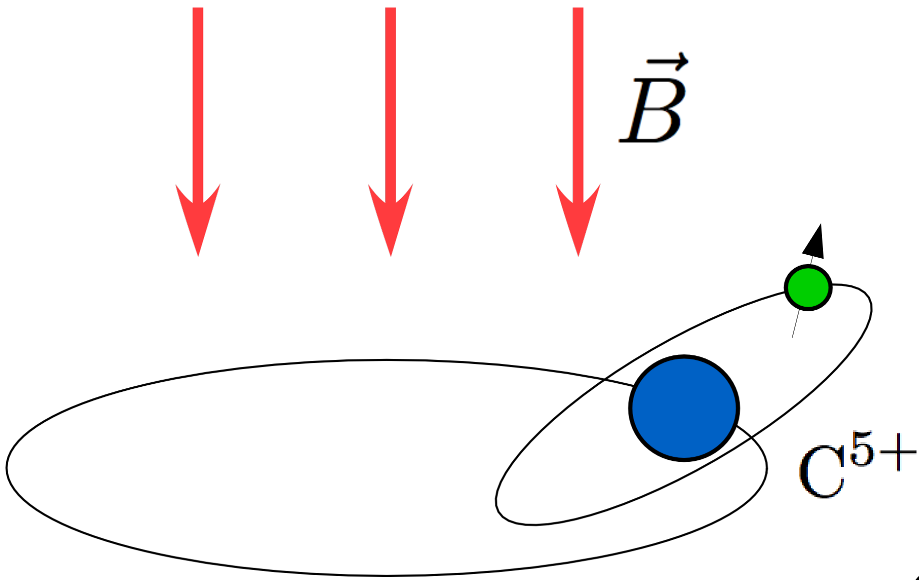
Why useful?

- determination of the electron mass
- future determination of alpha
- indirectly related to muon *g*-2 (muonium)

Why interesting?

- quantum effects in external field
- simple system, model for more complex ones
- numerical estimates exist for large Z
- should be analytically feasible for small Z (many have tried)

To avoid thermal motion: anchor the electron in an ion

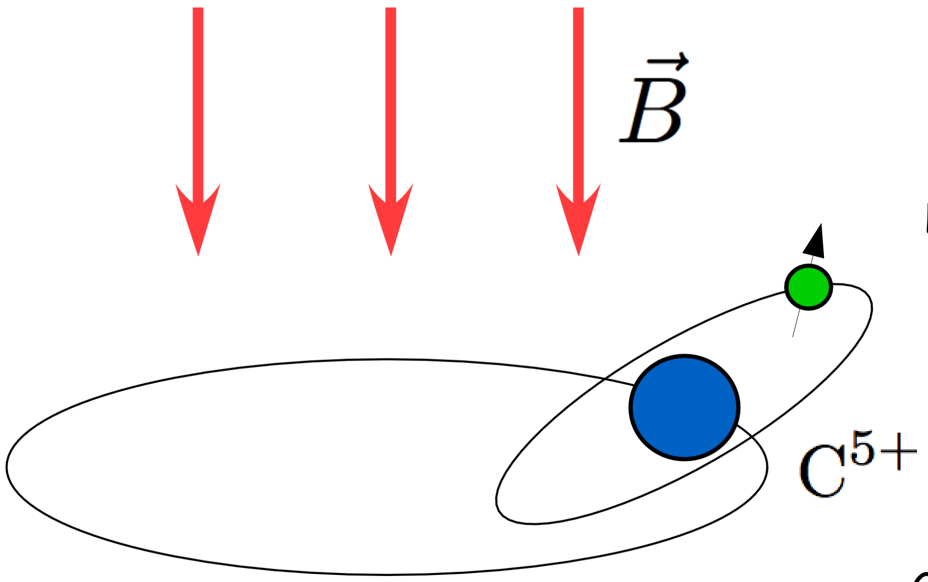


Larmor frequency $\omega_L = \dfrac{geB}{2m_e}$

Cyclotron frequency $\omega_{\rm cycl} = \dfrac{ZeB}{M}$

$$m_e = \frac{g}{2Z}\frac{\omega_{\rm cycl}}{\omega_L}M$$

To avoid thermal motion: anchor the electron in an ion



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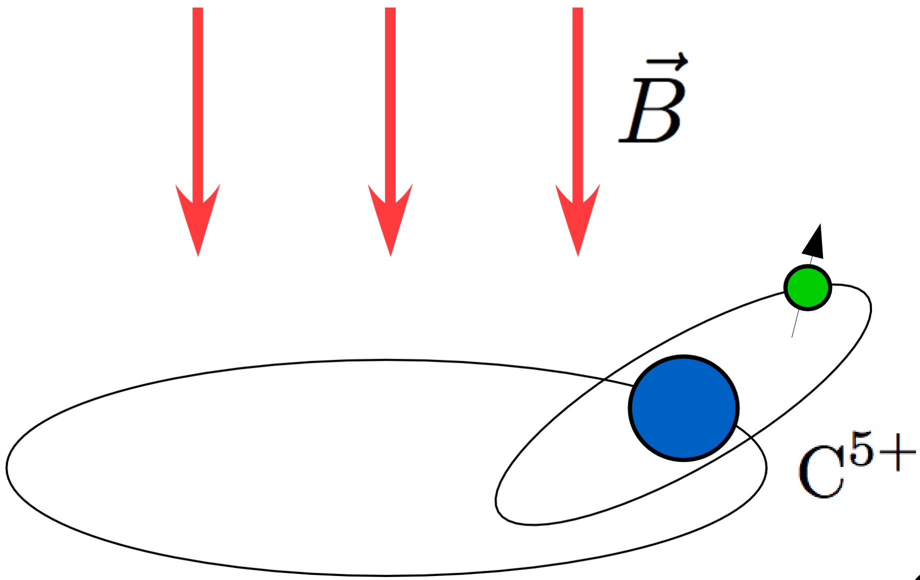
$$m_e = \frac{g}{2Z}\frac{\omega_{\rm cycl}}{\omega_L}M$$

Side remark: M and m_e have different origins! (QCD vs. Higgs)
Opportunity for searching for time variation.

To avoid thermal motion: anchor the electron in an ion



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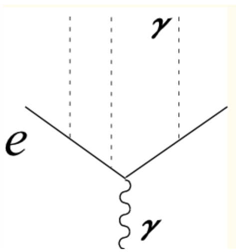
Cyclotron frequency $\omega_{\rm cycl} = \frac{ZeB}{M}$

$$m_e = \frac{g}{2Z} \frac{\omega_{\rm cycl}}{\omega_L} M$$

Interesting complication:
this g-factor is modified by the binding

Bound-electron g-2: the leading effect

Breit 1928: energy correction due to magnetic field
in the hydrogen ground state.



$$\delta E = e \int d^3x f^2 v^* \left[1 - i\gamma \boldsymbol{\Sigma} \cdot \hat{\boldsymbol{r}} \gamma^5\right] \gamma^5 \boldsymbol{A} \cdot \boldsymbol{\Sigma} \left[1 + i\gamma \boldsymbol{\Sigma} \cdot \hat{\boldsymbol{r}} \gamma^5\right] v$$

$$g = 2 \cdot \frac{1}{3} \left(1 + 2\sqrt{1 - (Z\alpha)^2}\right) \simeq 2 \left(1 - \frac{(Z\alpha)^2}{3}\right)$$

Bound-electron g-2: the leading effect

Breit 1928: energy correction due to magnetic field
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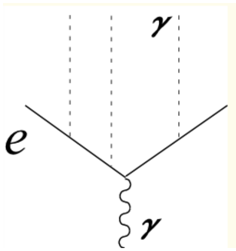
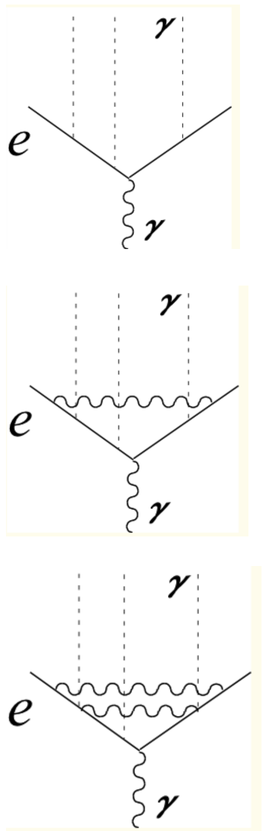


$$\delta E = e \int d^3x f^2 v^* \left[1 - i\gamma \boldsymbol{\Sigma} \cdot \hat{\boldsymbol{r}} \gamma^5\right] \gamma^5 \boldsymbol{A} \cdot \boldsymbol{\Sigma} \left[1 + i\gamma \boldsymbol{\Sigma} \cdot \hat{\boldsymbol{r}} \gamma^5\right] v$$

$$g = 2 \cdot \frac{1}{3} \left(1 + 2\sqrt{1-(Z\alpha)^2}\right) \simeq 2\left(1 - \frac{(Z\alpha)^2}{3}\right)$$

Important: dependence
on alpha; may be exploited
to determine its value.
(Use ions with various Z)

Bound-electron g-2: binding and loops



$$
\begin{aligned}
g = 2 &- \frac{2\,(Z\alpha)^2}{3} - \frac{(Z\alpha)^4}{6} + \dots \\
&+ \frac{\alpha}{\pi}\left[1 + \frac{(Z\alpha)^2}{6} + (Z\alpha)^4\,(a_{41}\ln Z\alpha + a_{40}) + \dots\right] \\
&+ \underbrace{\left(\frac{\alpha}{\pi}\right)^2\left[-0.65..\left(1 + \frac{(Z\alpha)^2}{6}\right) + (Z\alpha)^4\,(b_{41}\ln Z\alpha + b_{40}) + \dots\right]}_{\text{two-loop corrections}}
\end{aligned}
$$

Pachucki,
AC
Jentschura,
Yerokhin
(2005)

A series of beautiful measurements

VOLUME 92, NUMBER 9 PHYSICAL REVIEW LETTERS week ending 5 MARCH 2004

Electronic g Factor of Hydrogenlike Oxygen $^{16}O^{7+}$

J. Verdú, S. Djekić, S. Stahl, T. Valenzuela, M. Vogel, and G. Werth
Institut für Physik, Johannes-Gutenberg-Universität, D-55099 Mainz, Germany

T. Beier, H.-J. Kluge, and W. Quint
Gesellschaft für Schwerionenforschung, D-64291 Darmstadt, Germany

VOLUME 88, NUMBER 1 PHYSICAL REVIEW LETTERS 7 JANUARY 2002

New Determination of the Electron's Mass

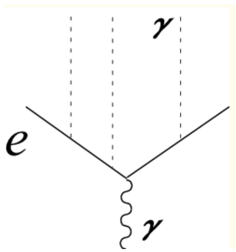
Thomas Beier,[1] Hartmut Häffner,[1,2] Nikolaus Hermanspahn,[2] Savely G. Karshenboim,[3,4] H.-Jürgen Kluge,[1] Wolfgang Quint,[1] Stefan Stahl,[2] José Verdú,[1,2] and Günther Werth[2]
[1]*Gesellschaft für Schwerionenforschung, 64291 Darmstadt, Germany*

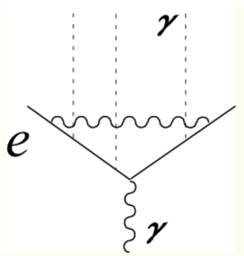
VOLUME 85, NUMBER 25 PHYSICAL REVIEW LETTERS 18 DECEMBER 2000

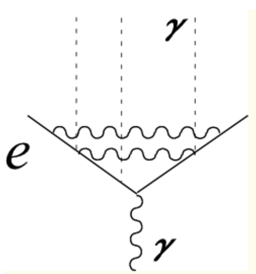
High-Accuracy Measurement of the Magnetic Moment Anomaly of the Electron Bound in Hydrogenlike Carbon

H. Häffner,[1,2] T. Beier,[1,3] N. Hermanspahn,[2] H.-J. Kluge,[1] W. Quint,[1] S. Stahl,[2] J. Verdú,[1] and G. Werth[2]
[1]*Gesellschaft für Schwerionenforschung, 64291 Darmstadt, Germany*

Recent experimental improvement







$$g = 2 - \frac{2\,(Z\alpha)^2}{3} - \frac{(Z\alpha)^4}{6} + \ldots$$
$$+\frac{\alpha}{\pi}\left[1 + \frac{(Z\alpha)^2}{6} + (Z\alpha)^4\,(a_{41}\ln Z\alpha + a_{40}) + \ldots\right]$$
$$+\left(\frac{\alpha}{\pi}\right)^2\left[-0.65..\left(1 + \frac{(Z\alpha)^2}{6}\right) + (Z\alpha)^4\,(b_{41}\ln Z\alpha + b_{40}) + \ldots\right]$$

$$b_{41} = \frac{28}{9}$$
$$b_{40} = \sout{-16.4}$$
-18.0

Together, new experiments in Mainz and this theory improved the accuracy of m_e by about a factor 3,

$$\frac{m_e}{u} = 0.000\;548\;579\;909\;32\,(29)\,(1)$$

The latest electron's atomic mass: 2014

LETTER

doi:10.1038/nature13026

High-precision measurement of the atomic mass of the electron

S. Sturm[1], F. Köhler[1,2], J. Zatorski[1], A. Wagner[1], Z. Harman[1,3], G. Werth[4], W. Quint[2], C. H. Keitel[1] & K. Blaum[1]

RESEARCH NEWS & VIEWS

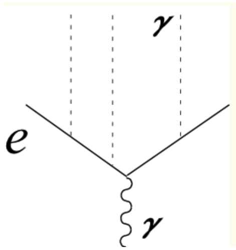
FUNDAMENTAL CONSTANTS

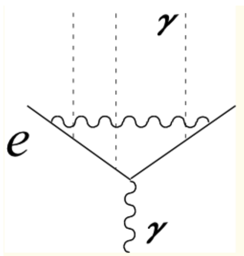
The teamwork of precision

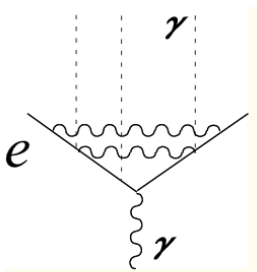
A new value for the atomic mass of the electron is a link in a chain of measurements that will enable a test of the standard model of particle physics with better than part-per-trillion precision. SEE LETTER P.467

EDMUND G. MYERS

Recent experimental improvement







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$$b_{41} = \frac{28}{9}$$

$b_{40} =$ ~~-16.4~~

-18.0

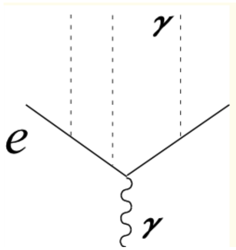
Together, new experiments in Mainz and this theory improved the accuracy of m_e by about a factor 3,

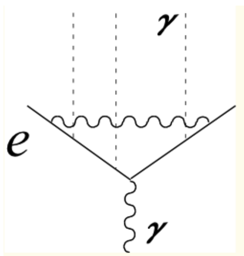
$$\frac{m_e}{u} = 0.000\ 548\ 579\ 909\ 32\,(29)\,(1)$$

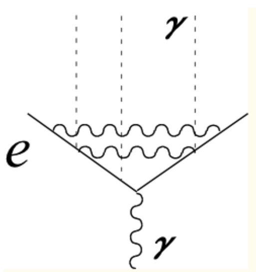
$$\frac{m_e}{u} = 0.000\ 548\ 579\ 909\ 067\,(17)$$

Nature 2014
Sturm et al

Recent experimental improvement







$$g = 2 - \frac{2\,(Z\alpha)^2}{3} - \frac{(Z\alpha)^4}{6} + \dots$$
$$+\frac{\alpha}{\pi}\left[1 + \frac{(Z\alpha)^2}{6} + (Z\alpha)^4\,(a_{41}\ln Z\alpha + a_{40}) + \dots\right]$$
$$+\left(\frac{\alpha}{\pi}\right)^2\left[-0.65..\left(1+\frac{(Z\alpha)^2}{6}\right) + (Z\alpha)^4\,(b_{41}\ln Z\alpha + b_{40}) + \dots\right.$$

$$\frac{m_e}{u} = 0.000\;548\;579\;909\;32\,(29)\,(1)$$

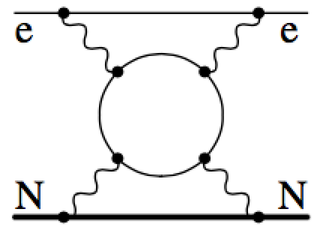
$$\frac{m_e}{u} = 0.000\;548\;579\;909\;067\,(17)$$

Nature 2014
Sturm et al

Next theory challenge:
$(Z\alpha)^5$ effects.

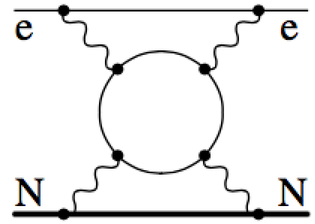
Light-by-light contribution to the Lamb shift

We consider two momentum regions in

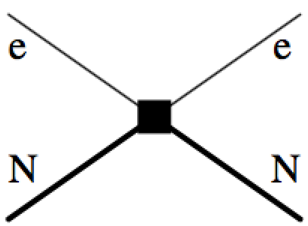


Light-by-light contribution to the Lamb shift

We consider two momentum regions in



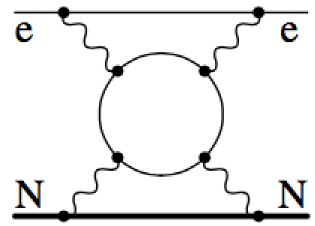
If all loops are
short-distance (hard)



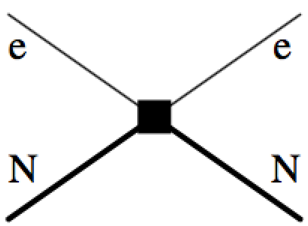
$$\alpha^2 (Z\alpha)^2 \cdot \psi_0^2 \to \alpha^2 (Z\alpha)^5$$

Light-by-light contribution to the Lamb shift

We consider two momentum regions in



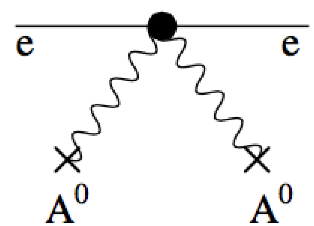
If all loops are short-distance (hard)



$$\alpha^2 (Z\alpha)^2 \cdot \psi_0^2 \to \alpha^2 (Z\alpha)^5$$

Pachucki; Eides+Shelyuto
Dowling, Mondejar, Piclum, AC

If the lowest loop is soft



$$\alpha^2 \langle e^2 \boldsymbol{E}^2 \rangle \sim \alpha^2 \left\langle \frac{(Z\alpha)^2}{r^4} \right\rangle$$

$$\to \alpha^2 (Z\alpha)^6 \ln Z\alpha$$

AC, Szafron
(Jentschura, AC, Pachucki)

How large is the resulting Lamb shift?

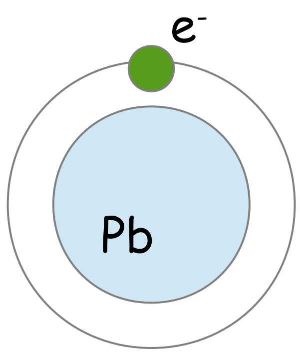
Decreases 1S-2S splitting by about 280 Hz.

For comparison, the experimental error is 10 Hz (~ 10^{-15} relative error)

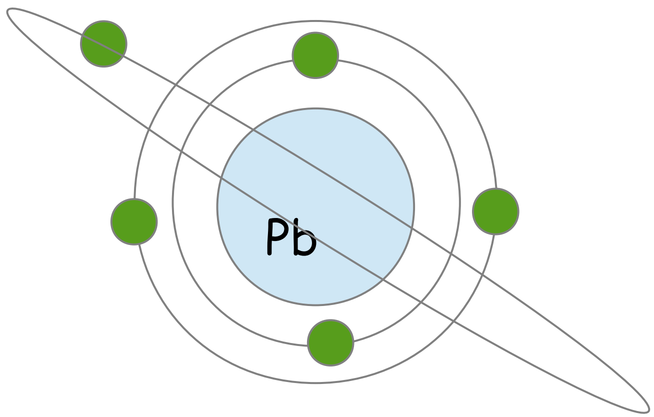
A new source of alpha: highly-charged ions

$$g \simeq 2 - \frac{2\,(Z\alpha)^2}{3} \longrightarrow \frac{\delta\alpha}{\alpha} \sim \frac{1}{(\alpha Z)^2}\sqrt{(\delta g_{\rm exp})^2 + (\delta g_{\rm th})^2}$$

large Z favorable



Hydrogen-like lead



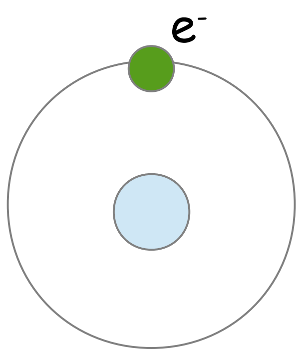
Boron-like lead

There is a combination of g-factors in both ions where the sensitivity to the nuclear structure largely cancels, but the sensitivity to alpha remains.

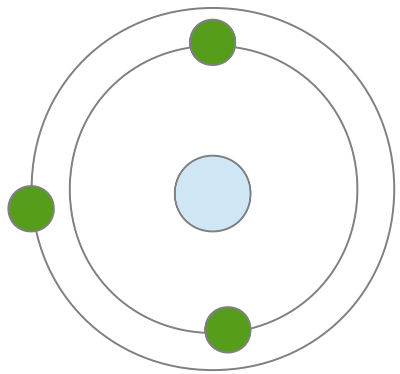
Shabaev, Glazov, Oreshkina, Volotka, Plunien, Kluge, Quint

New idea: medium-charged ions

$$g \simeq 2 - \frac{2\,(Z\alpha)^2}{3}$$



Hydrogen-like ion



Lithium-like ion

Combine H-like and Li-like to remove nuclear dependence;
then combine with a different nucleus, to remove free-g dependence!

Much interesting theoretical work remains to be done!

Yerokhin, Berseneva, Harman, Tupitsyn, Keitel: PRL (2016)

Summary

* Binding modifies the muon decay and the electron *g*-factor

* Theory of both effects is more fun than for free particles

* Synergy with beautiful experiments: lepton-flavor violation, mass of the electron and, in future, the fine structure constant.

* For *g*: $\alpha(Z\alpha)^5$ effects almost finished; $\alpha^2(Z\alpha)^5$ hopefully soon.

* Opportunities for more theoretical improvement...