Heavy quark diffusion on the lattice

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- Deconfinement and properties of QGP: lattice QCD vs. weak coupling
- Lattice determination of heavy quark diffusion in quenched approximation:
  a) electric field correlator method
  b) comments current-current correlators
- Charm correlations and fluctuations and charmed hadrons above T_c
- Summary

EMMI Rapid Reaction Task Force: Extraction of heavy flavor Transport coefficients in QCD mattrer , July 18-22, 2016

Deconfinement and color screening

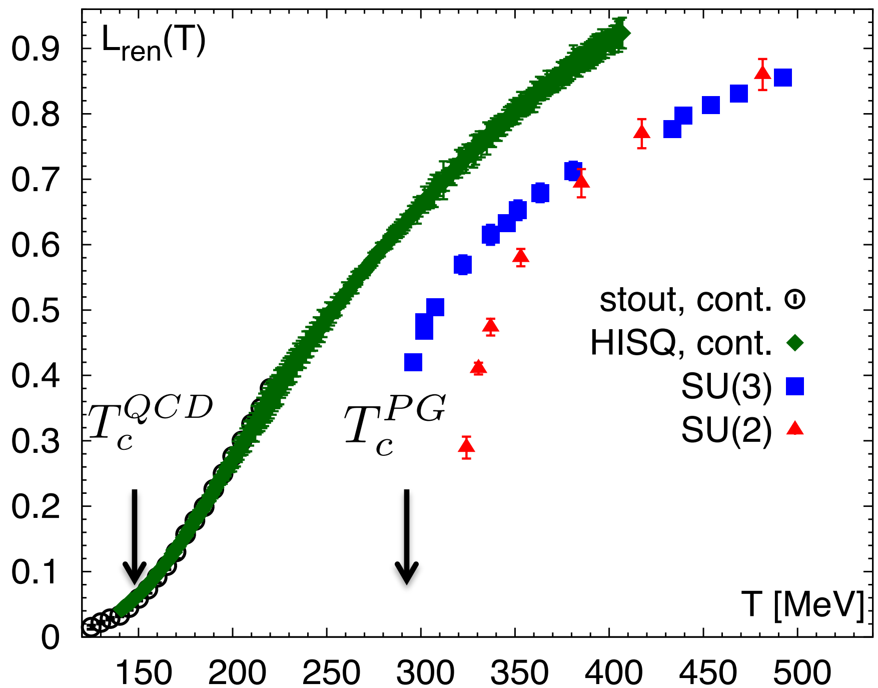
Onset of color screening is described by Polyakov loop (order parameter in SU(N) gauge theory)

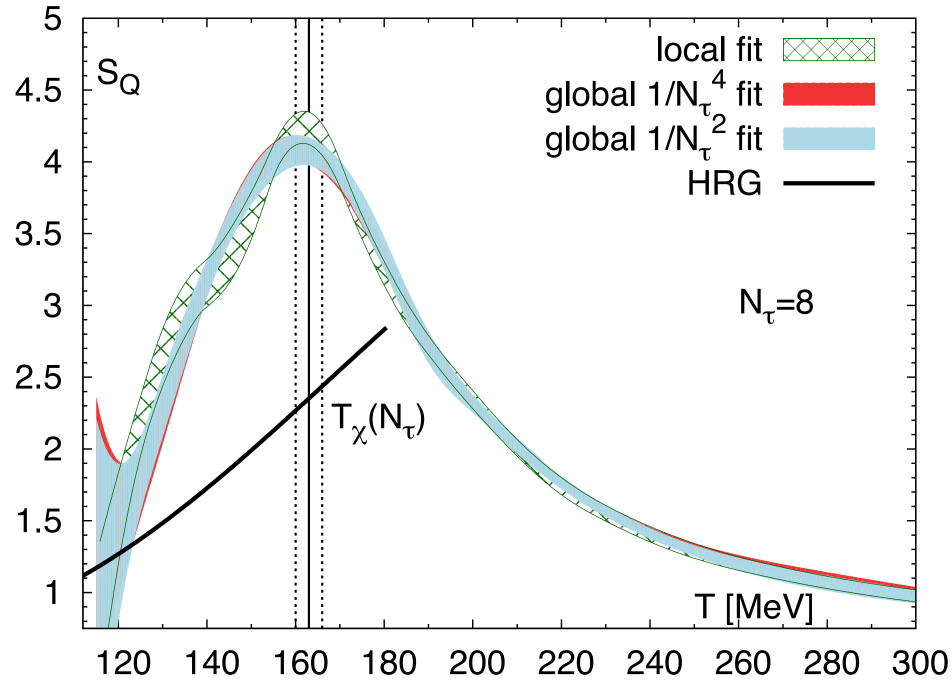
$$L = \mathrm{tr}\mathcal{P}e^{ig\int_0^{1/T} d\tau A_0(\tau,\vec{x})}$$

Bazavov et al, PRD 93 (2016) 114502

$$\Rightarrow \quad L_{ren} = \exp(-F_Q(T)/T)$$

$$S_Q = -\frac{\partial F_Q}{\partial T}$$





The screening properties of SU(3) gauge and QCD are similar Only for *T>300* MeV

The onset of screening corresponds to peak is S_Q and its position coincides with T_c

QCD thermodynamics at non-zero chemical potential

Taylor expansion :

$$\frac{p(T,\mu_B,\mu_Q,\mu_S,\mu_C)}{T^4} = \sum_{ijkl} \frac{1}{i!j!k!l!} \chi_{ijkl}^{BQSC} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k \left(\frac{\mu_C}{T}\right)^l$$ hadronic

$$\frac{p(T,\mu_u,\mu_d,\mu_s,\mu_c)}{T^4} = \sum_{ijkl} \frac{1}{i!j!k!l!} \chi_{ijkl}^{udsc} \left(\frac{\mu_u}{T}\right)^i \left(\frac{\mu_d}{T}\right)^j \left(\frac{\mu_s}{T}\right)^k \left(\frac{\mu_c}{T}\right)^l$$ quark

$$\chi_{ijkl}^{abcd} = T^{i+j+k+l} \frac{\partial^i}{\partial \mu_b^i} \frac{\partial^j}{\partial \mu_b^j} \frac{\partial^k}{\partial \mu_c^i} \frac{\partial^l}{\partial \mu_d^l} \ln Z(T,V,\mu_a,\mu_b,\mu_c,\mu_d) \,|_{\mu_a=\mu_b=\mu_c=\mu_d=0}$$

Taylor expansion coefficients give the fluctuations and correlations of conserved charges, e.g.

$$\chi_2^X = \chi_X = \frac{1}{VT^3}(\langle X^2\rangle - \langle X\rangle^2) \qquad \chi_{11}^{XY} = \frac{1}{VT^3}(\langle XY\rangle - \langle X\rangle\langle Y\rangle)$$

⬇

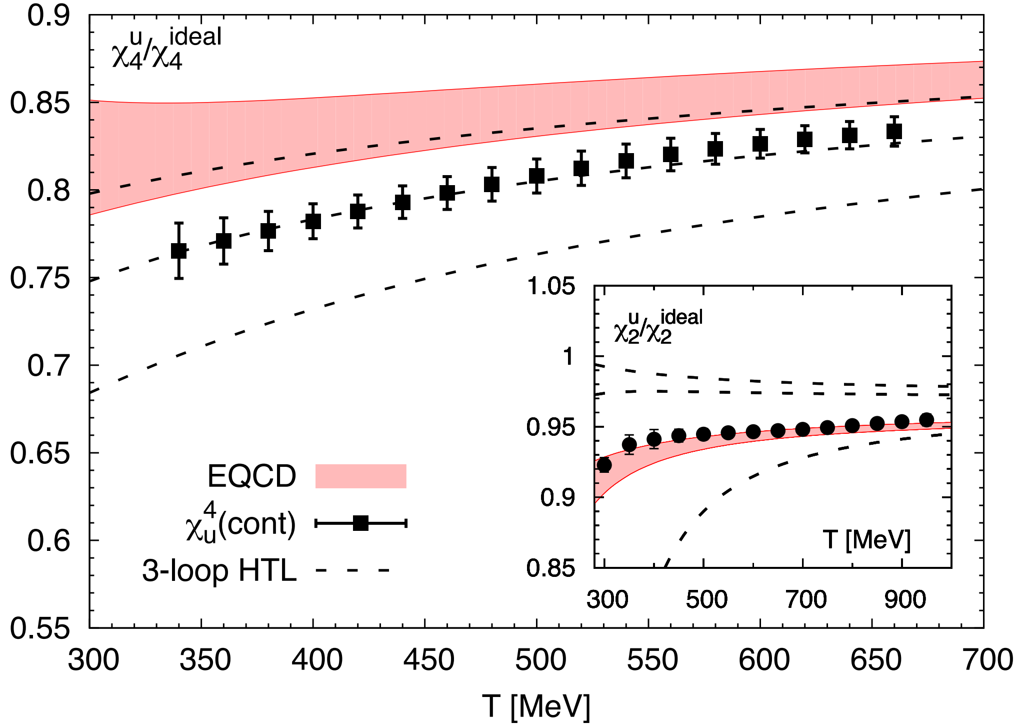
information about carriers of the conserved charges (hadrons or quarks)

⬇

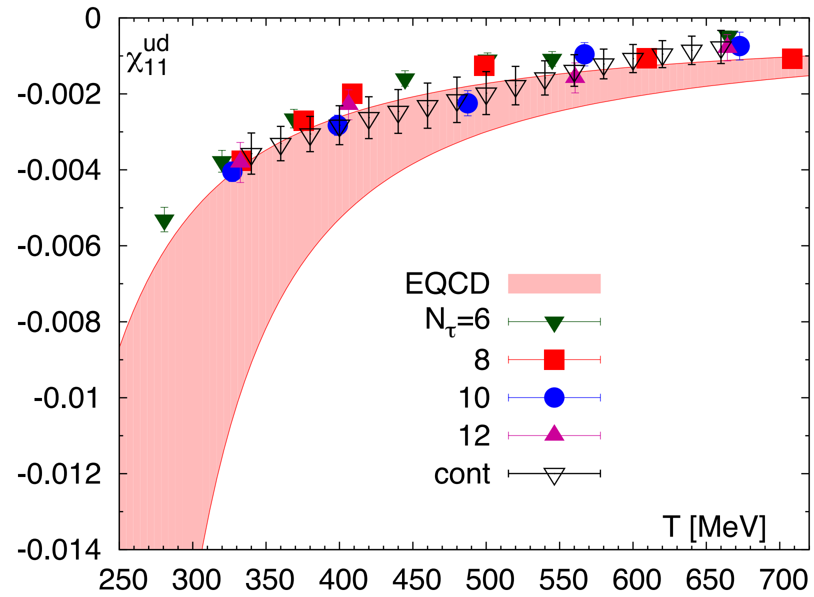
probes of deconfinement

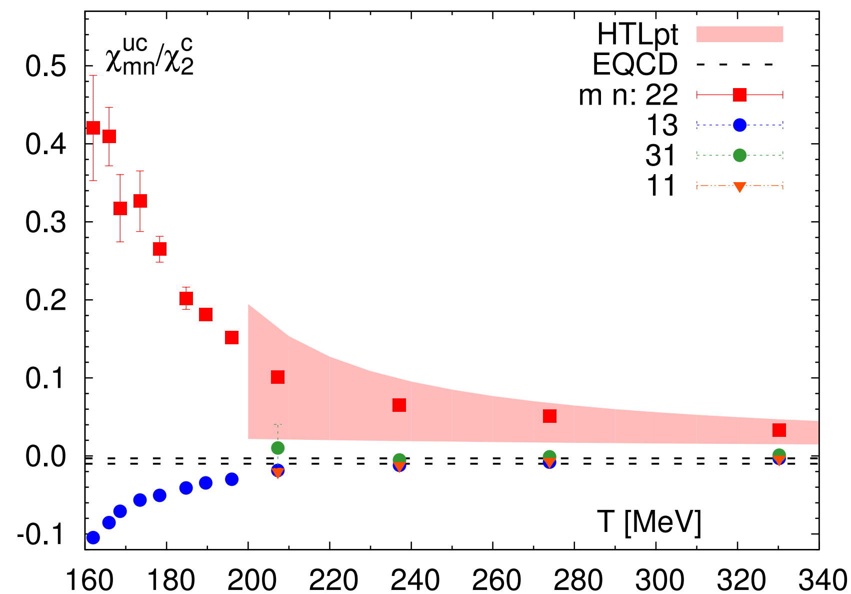
Quark number fluctuations at high T

quark number fluctuations



quark number correlations





Good agreement between lattice and the weak coupling approach for 2nd and 4th order quark number fluctuations

Bazavov et al, PRD88 (2013) 094021, Ding et at, PRD92 (2015) 074043

Correlations are large for *T<200* MeV but agree with weak coupling expectations for *T>300* MeV, e.g.

$$\chi_{22}^{uc} \gg \chi_{13}^{uc} \sim \chi_{31}^{uc} \sim \chi_{11}^{uc}$$

Current-current correlators and heavy quark diffusion

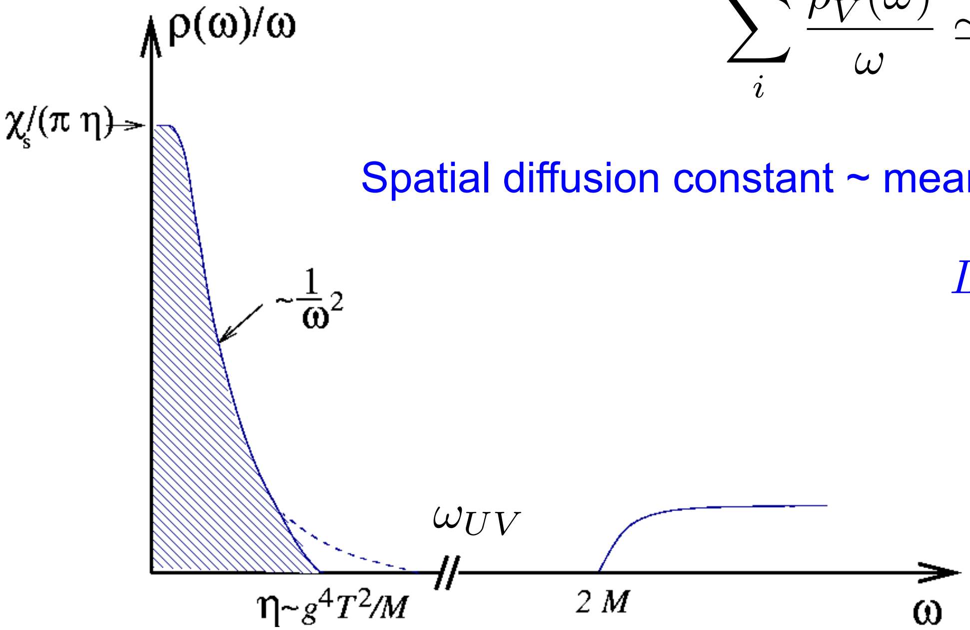
$$\rho_V^{\mu\nu}(\omega) \equiv \int_{-\infty}^{\infty} \mathrm{d}t e^{i\omega t} \int \mathrm{d}^3x \left\langle [\hat{J}^\mu(t,\vec{x}), \hat{J}^\nu(0,\vec{0})] \right\rangle$$

$$\sum_i \frac{\rho_V^{ii}(\omega)}{\omega} \simeq 3\chi_2^q D \frac{\eta^2}{\eta^2+\omega^2}, \quad \omega < \omega_{UV}, \quad \eta = \frac{T}{M}\frac{1}{D}$$

Spatial diffusion constant ~ mean free path (weak coupling)

drag constant

$$D \sim \frac{1}{g^4 T}$$



Momentum diffusion coefficient

$$\kappa^{(M)} \equiv \frac{M^2\omega^2}{3T\chi_2^q} \sum_i \frac{2T\rho_V^{ii}(\omega)}{\omega}\Bigg|_{\eta \ll \omega < \omega_{UV}}$$

$$\kappa^{(M)} = 2T^2/D$$

area under the peak ~ χ_2^q

heavy quark coefficient ~ width of the peak

For large quark mass the transport peak is very narrow even for strong coupling and its difficult to reconstruct it accurately from Euclidean correlator calculated on the lattice

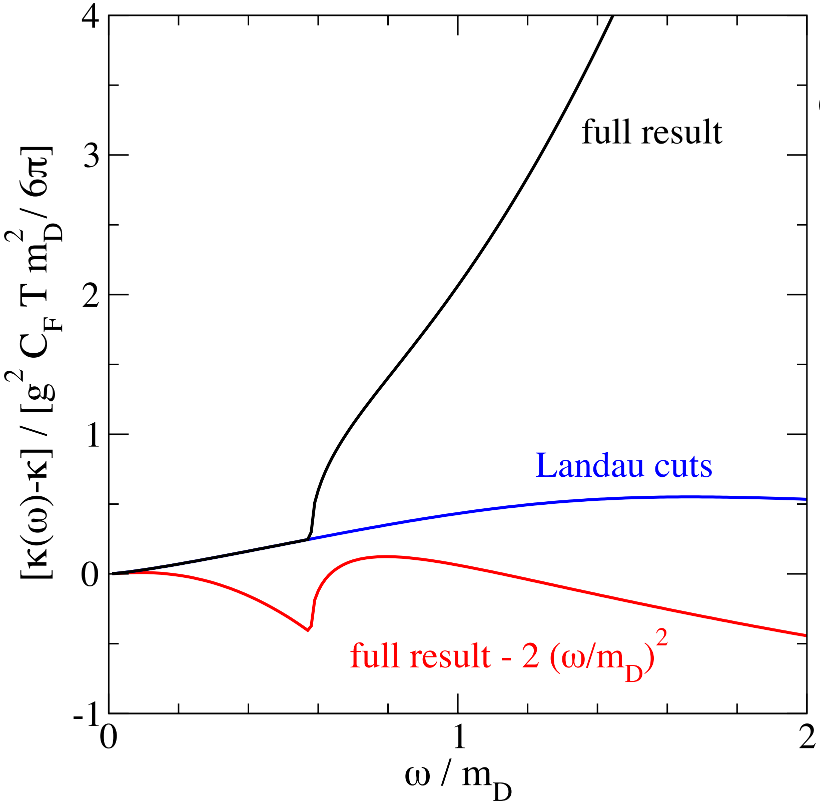
Current-current correlators in the heavy quark limit

$$\kappa = \frac{1}{3T}\sum_{i=1}^{3}\lim_{\omega\to 0}\left[\lim_{M\to\infty}\frac{M^2}{\chi_2^q}\int_{-\infty}^{\infty}\mathrm{d}t\, e^{i\omega(t-t')}\int \mathrm{d}^3\vec{x}\left\langle \frac{1}{2}\left\{\frac{\mathrm{d}\hat{J}^i(t,\vec{x})}{\mathrm{d}t},\frac{\mathrm{d}\hat{J}^i(t',\vec{0})}{\mathrm{d}t'}\right\}\right\rangle\right]$$

$$\frac{\mathrm{d}\hat{J}^i}{\mathrm{d}t} = \frac{1}{M}\left\{\hat{\phi}^\dagger gE^i\hat{\phi} - \hat{\theta}^\dagger gE^i\hat{\theta}\right\} + \mathcal{O}\left(\frac{1}{M^2}\right)$$

$t \to i\tau$

$$G_E(\tau) = \frac{1}{3\chi_2^q T}\sum_i\int \mathrm{d}^3x\left\langle\left[\phi^\dagger gE_i\phi - \theta^\dagger gE_i\theta\right](\tau,\vec{x})\left[\phi^\dagger gE_i\phi - \theta^\dagger gE_i\theta\right](0,\vec{0})\right\rangle$$



Integrate out ϕ, θ

$$G_E(\tau) = -\frac{1}{3}\sum_{i=1}^{3}\frac{\left\langle \mathrm{ReTr}\left[U(\beta,\tau)\, gE_i(\tau,\vec{0})\, U(\tau,0)\, gE_i(0,\vec{0})\right]\right\rangle}{\left\langle \mathrm{ReTr}[U(\beta,0)]\right\rangle}$$

$$G_E(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\rho(\omega)\frac{\cosh\left(\tau-\frac{1}{2T}\right)\omega}{\sinh\frac{\omega}{2T}}$$

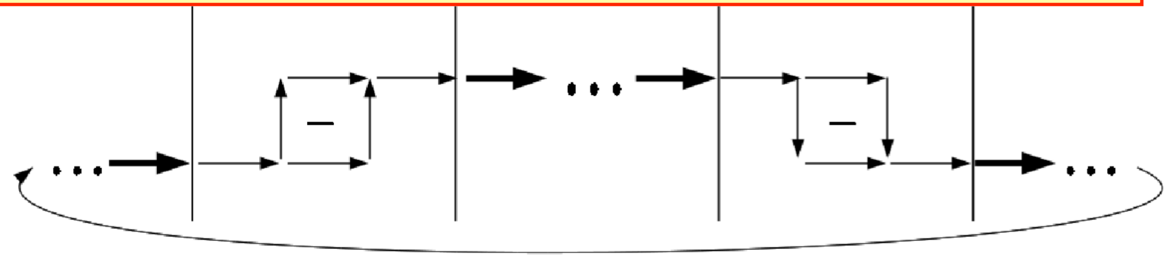
Transport coefficient ~ intercept of the spectral function not its width

$$\kappa = \lim_{\omega\to 0}\frac{2T}{\omega}\rho(\omega)$$

Caron-Huot, Laine, Moore, JHEP 0904 (2009) 053

Calculating the electric field strength correlator on the lattice

Straightforward to discretize by deforming the path of the Wilson lines to spatial direction

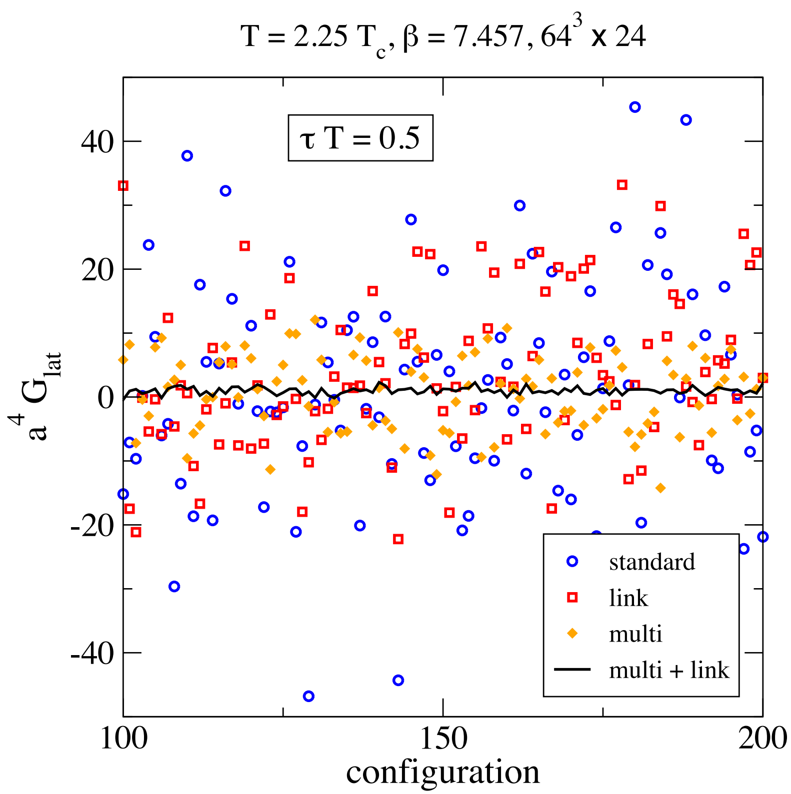


Challenge : MC noise

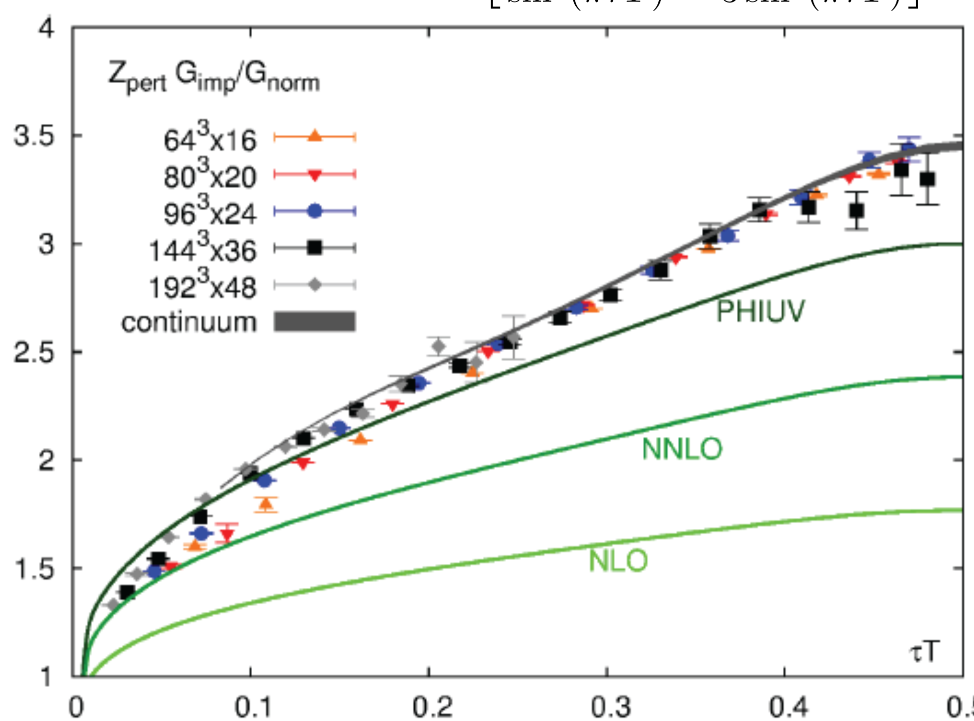
→ multilevel algorithm + link integration (only works for pure glue theory)

Luscher, Weisz, JHEP 0109 (2010), 010; Parisi, Petronzio, Rapuano, PLB 128 (1983) 418

Francis, Kaczmarek, Laine, et al, arXiv:1109.3941, arXiv:1311.3759, PRD 92 (2015) 116003



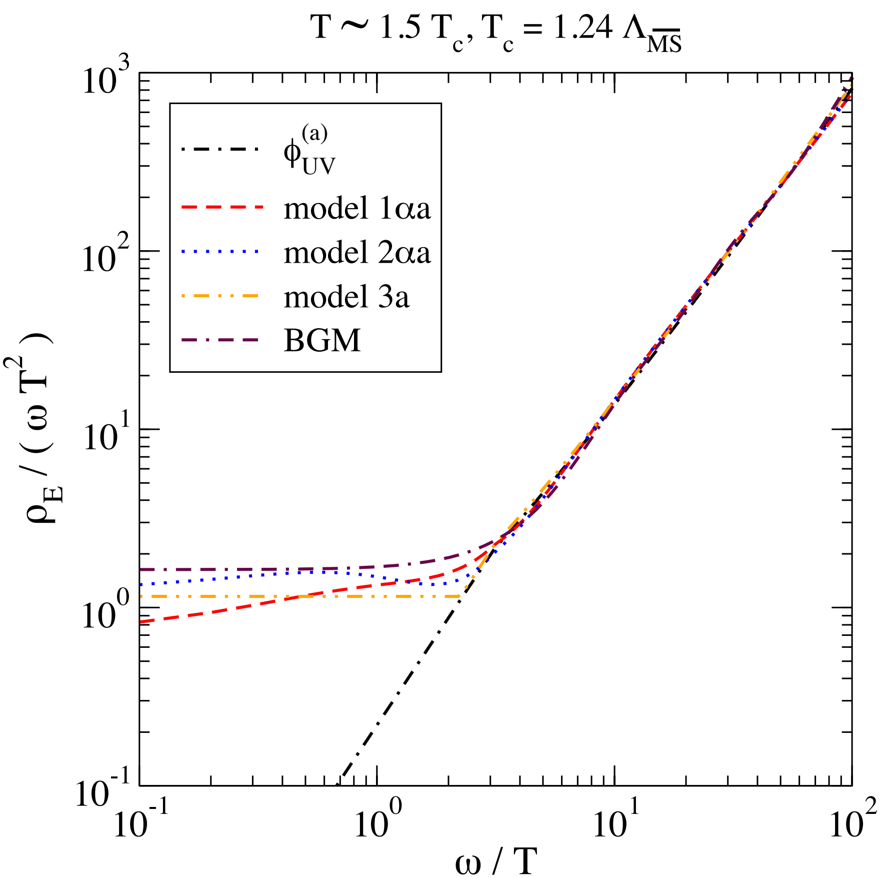
$$G_{norm}(\tau) = g^2 C_F\, \pi^2 T^4 \left[\frac{\cos^2(\pi\tau T)}{\sin^4(\pi\tau T)} + \frac{1}{3\sin^2(\pi\tau T)} \right]$$

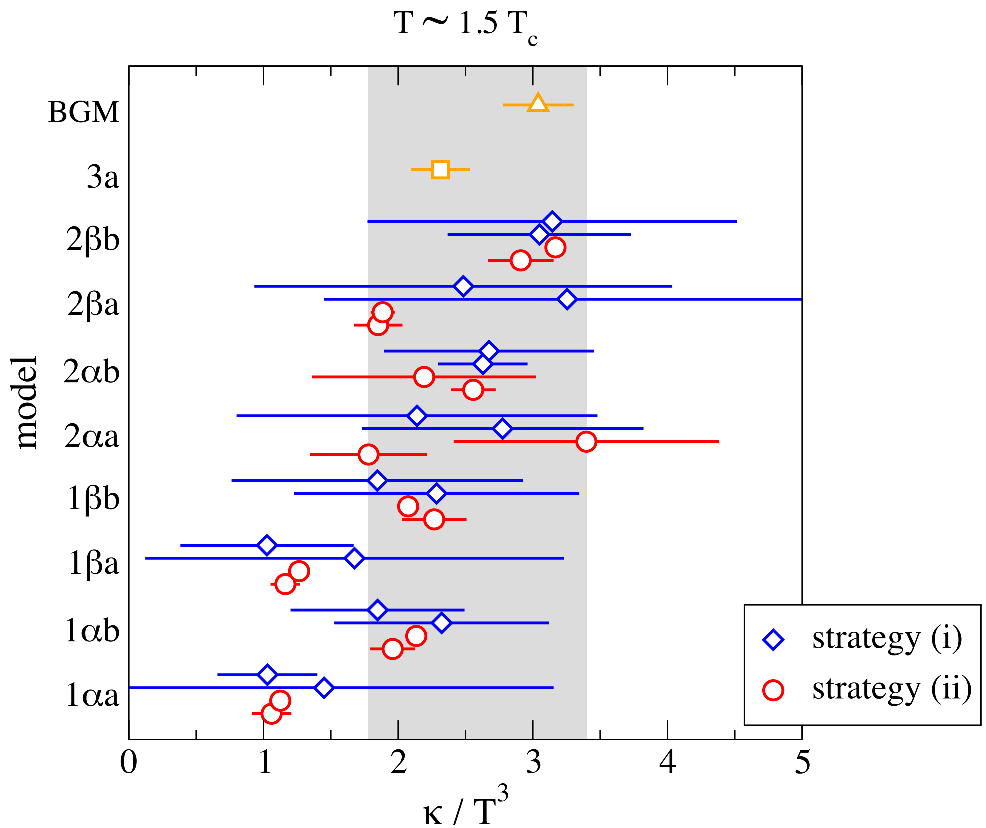


Extracting the spectral function and the diffusion constant

Fit the lattice using a forms of the spectral function constrained by low and high energy asymptotic behavior + corrections

$$\rho^{low}(\omega) = \frac{\kappa\omega}{2T} \qquad \rho^{high}(\omega) = \frac{g^2(\mu_\omega)C_F}{6\pi}\omega^3, \mu_\omega = max(\omega, \pi T)$$

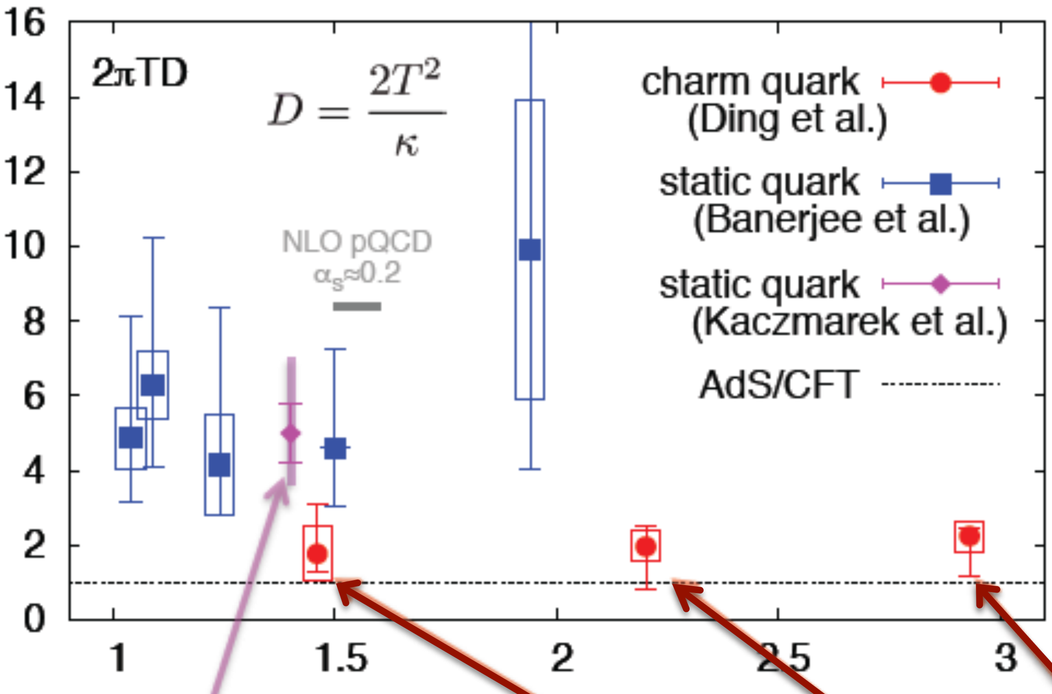




Francis, Kaczmarek, Laine, et al, PRD 92 (2015) 116003

$$\kappa = \lim_{\omega\to 0} \frac{2T}{\omega}\rho(\omega) = (1.8 - 3.4)$$

Comparison with other lattice approaches



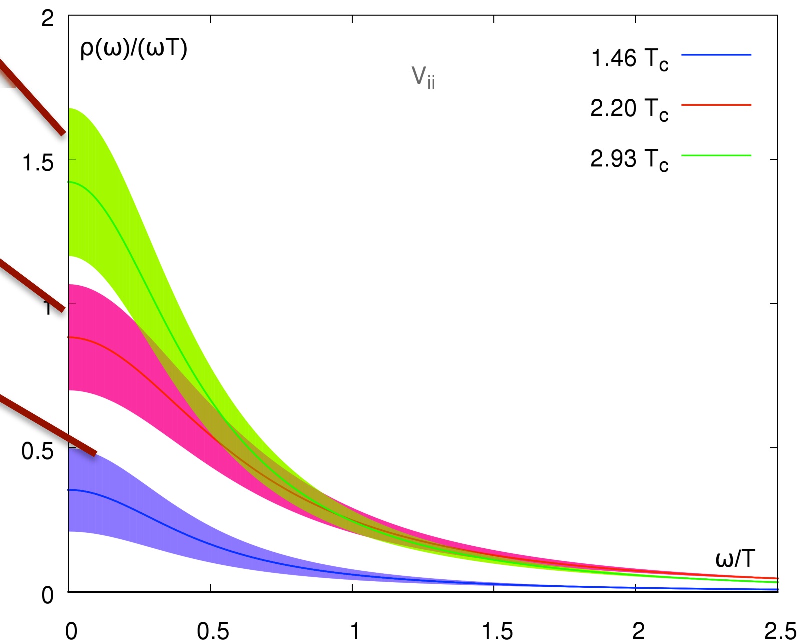
Electric correlator metho with multi-level algorigthm but no continuum limit

Banerjee et al, PRD 85 (2012) 014510

D is slightly smaller than the pQCD result

Determination from vector charmonium correlators
Ding et al, PRD 86 (2012) 014509

The width of the transport peak is dominated by Systematic effects, it too broad because of limited resolution => *D* is too small



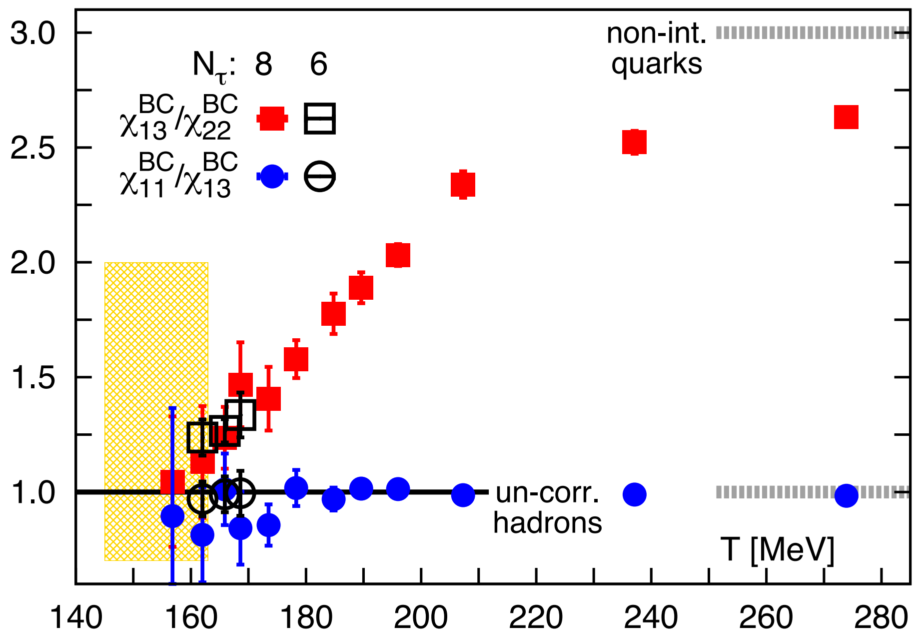
Deconfinement of charm

$$\chi_{nml}^{XYC} = T^{m+n+l} \frac{\partial^{n+m+l} p(T,\mu_X,\mu_Y,\mu_C)/T^4}{\partial\mu_X^n \partial\mu_Y^m \partial\mu_C^l}$$

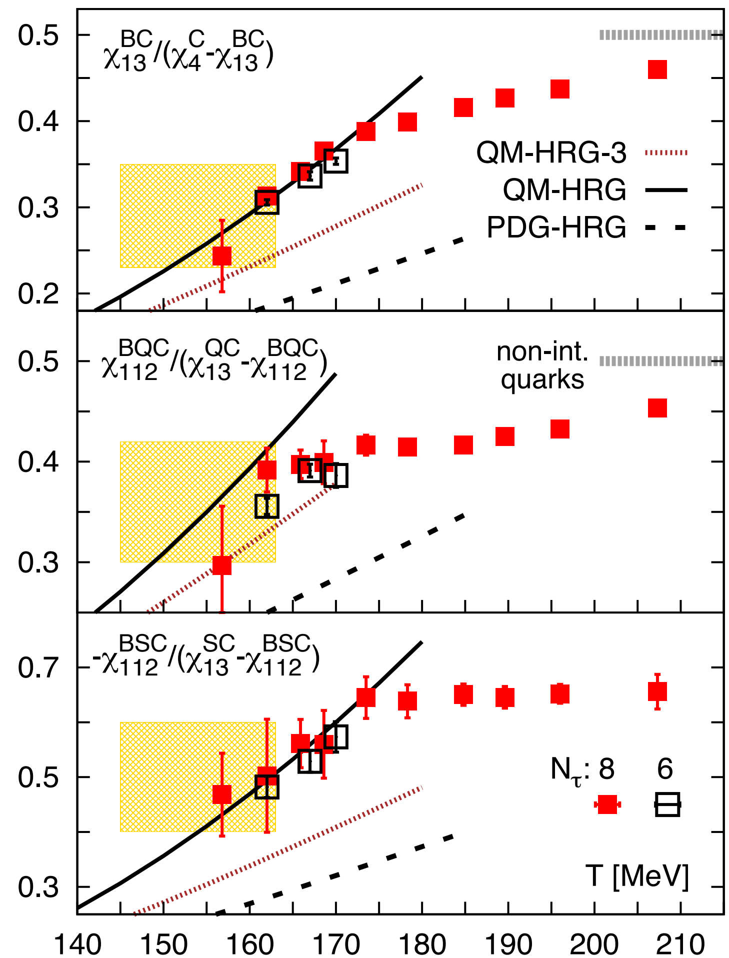
Bazavov et al, PLB 737 (2014) 210

$m_c \gg T$ ⇨ only |*C*|=*1* sector contributes

In the hadronic phase all *BC*-correlations are the same !



Charm baryon to meson pressure



Hadronic description breaks down just above T_c
⇒ open charm deconfines above T_c

The charm baryon spectrum is not well known (only few states in PDG), HRG works only if the “missing” states are included

Quasi-particle model for charm degrees of freedom

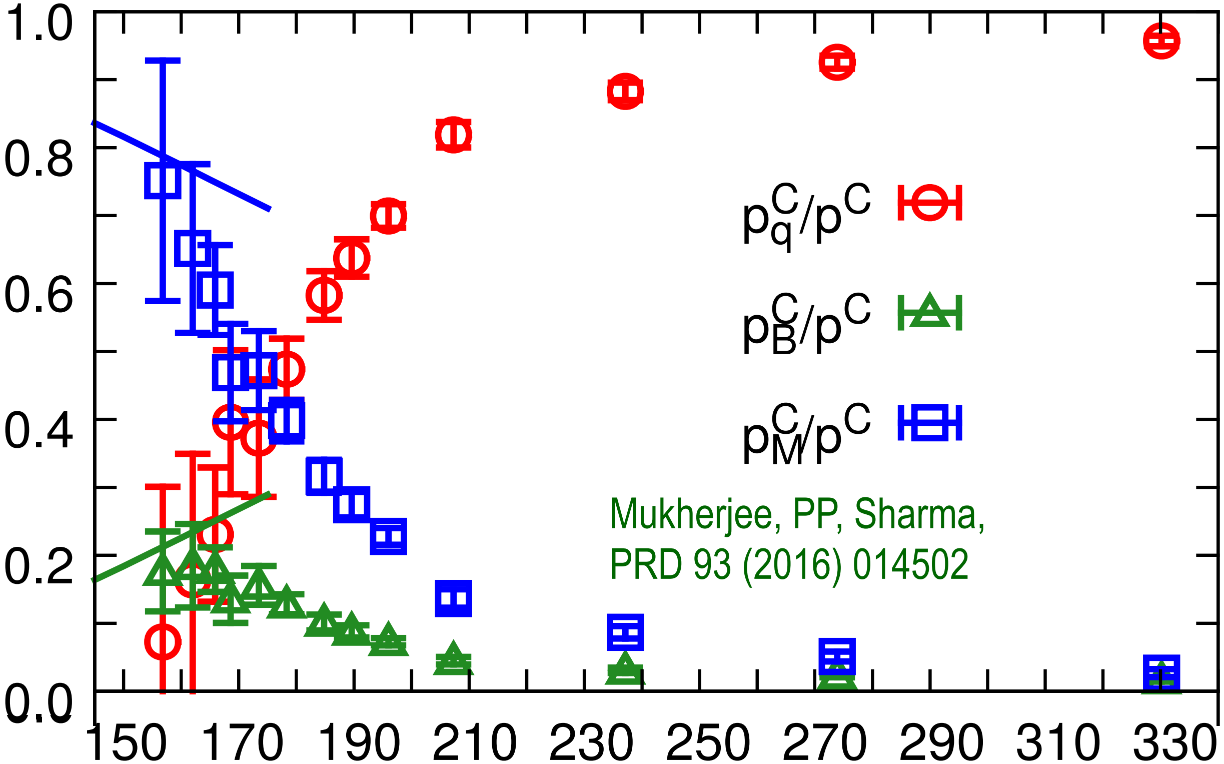
Charm dof are good quasi-particles at all T because M_c>>T and Boltzmann approximation holds

$$p^C(T,\mu_B,\mu_c) = p_q^C(T)\cosh(\hat{\mu}_C+\hat{\mu}_B/3) + p_B^C(T)\cosh(\hat{\mu}_C+\hat{\mu}_B) + p_M^C(T)\cosh(\hat{\mu}_C)$$

$$\hat{\mu}_X = \mu_X/T$$

$$\chi_2^C,\ \chi_{13}^{BC},\ \chi_{22}^{BC} \Rightarrow p_q^C(T), p_M^C(T), p_B^C(T)$$

Partial meson and baryon pressures described by HRG at T_c and dominate the charm pressure then drop gradually, charm quark only dominant dof at T>200 MeV or $\varepsilon > 6$ GeV/fm^3



Partial pressures drop because hadronic excitations become broad at high temperatures (bound state peaks merge with the continuum)

See
Jakovác, PRD88 (2013), 065012
Biró, Jakovác, PRD(2014)065012

Vice versa for quarks

Summary

- Chiral symmetry restoration and deconfinement in terms of appearance of quark dof and screening happen at the same temperature

- QCD with dynamical quarks and quenched QCD behave similarly for $T>300$ MeV in terms of color screening => quenched can be used for qualitative guidance

- Heavy quark diffusion coefficient can be determined in quenched approximation and at present is the best known transport coefficient on the lattice:
  $2\pi T=1.8-3.4$

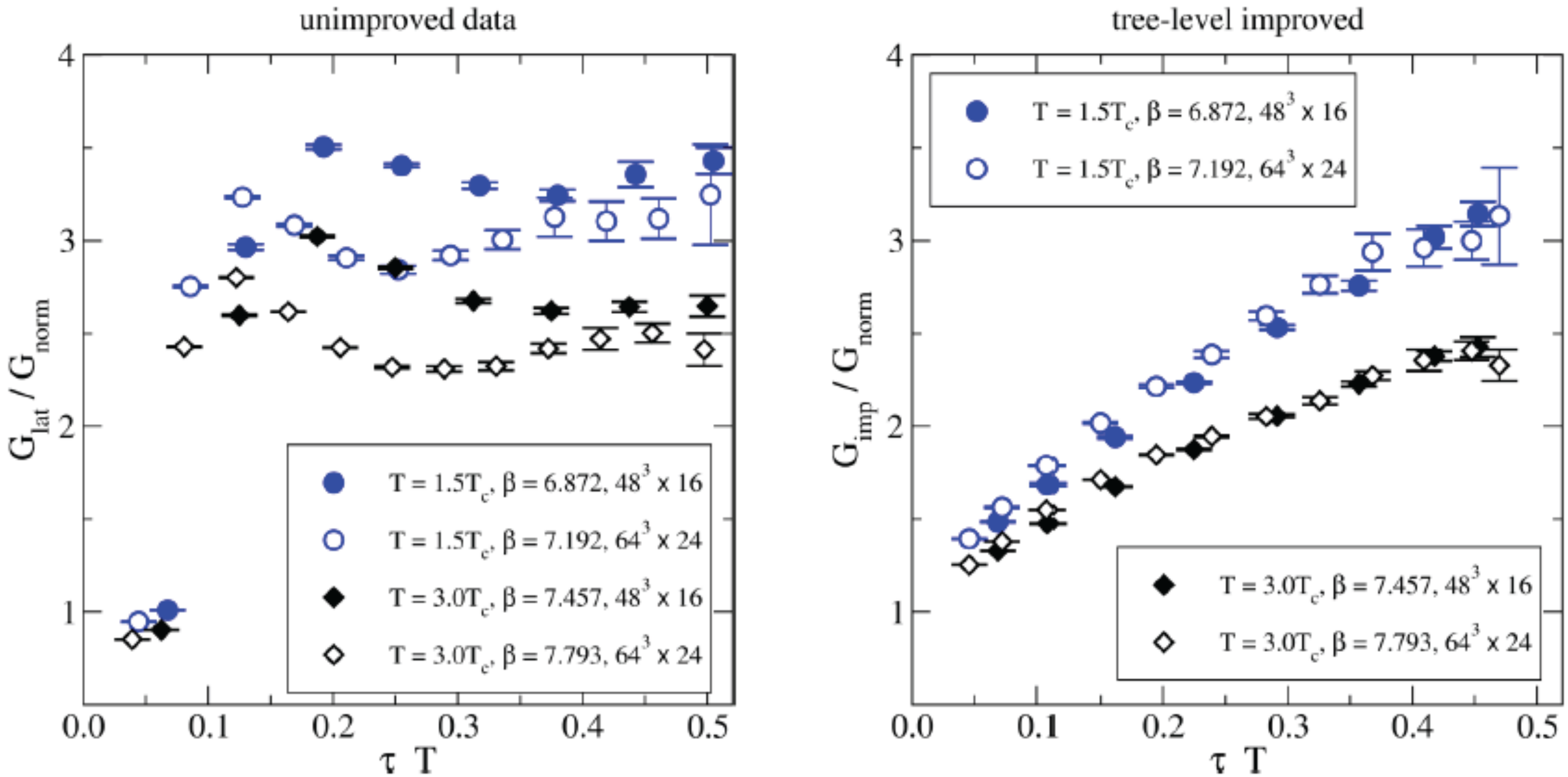
- It is not known how to do the calculations of heavy quark diffusion coefficient in QCD with dynamical quarks

- It would be of interest to extend the lattice study of heavy diffusion coefficient to higher temperatures to make contact with weak coupling calculations

- For $T< 200$ MeV the deconfined matter is strongly interacting and charm hadrons may exist. Nothing is known about transport coefficients in this region

Back-up:

[A.Francis,OK,M.Laine,J.Langelage, arXiv:1109.3941 and arXiv:1311.3759]



lattice cut-off effects visible at small separations (left figure)

→ **tree-level improvement** (right figure) to reduce discretization effects

$$G^{\rm LO}_{\rm cont}(\overline{\tau T}) = G^{\rm LO}_{\rm lat}(\tau T)$$

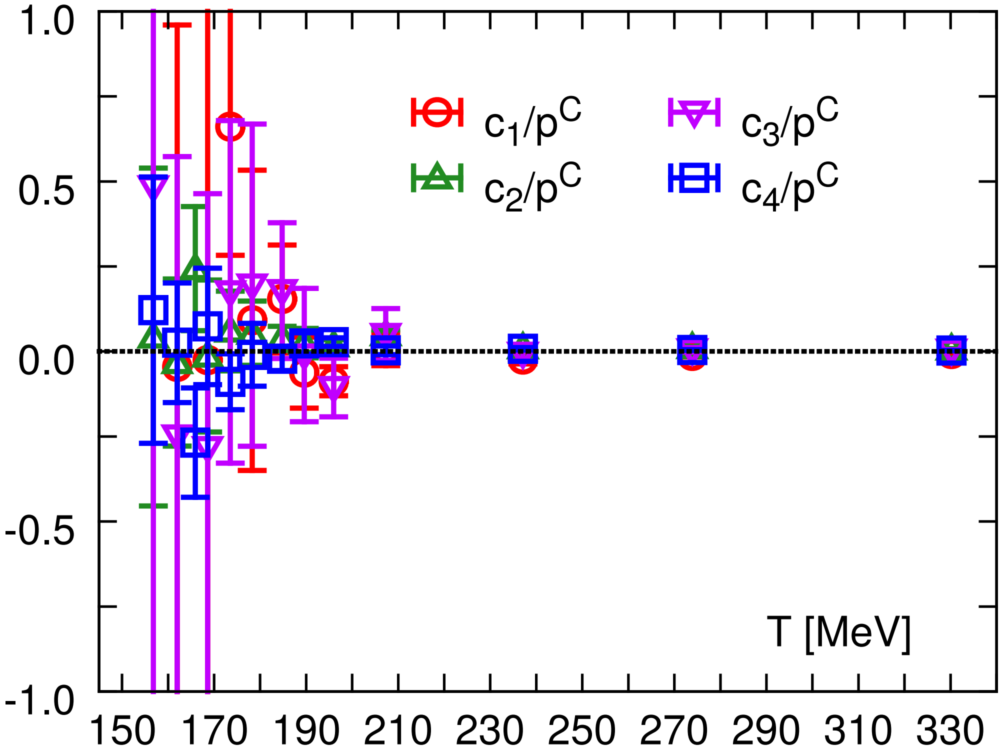
From Kaczmarek

Does the quasi-particle model makes sense ?

4 non-trivial constraints on the model provided by : $\chi_{31}^{BC}, \chi_{31}^{SC}, \chi_{121}^{BSC}, \chi_{211}^{BSC}$

$$c_1 \equiv \chi_{13}^{BC} - 4\chi_{22}^{BC} + 3\chi_{31}^{BC} = 0,$$
$$c_2 \equiv 2\chi_{121}^{BSC} + 4\chi_{112}^{BSC} + \chi_{22}^{SC} + 2\chi_{13}^{SC} - \chi_{31}^{SC} = 0$$
$$c_3 \equiv 6\chi_{112}^{BSC} + 6\chi_{121}^{BSC} + \chi_{13}^{SC} - \chi_{31}^{SC},$$
$$c_4 \equiv \chi_{211}^{BSC} - \chi_{112}^{BSC}.$$

← Diquark pressure is zero !



Models with charm quark only:
correlations from an effective mass

$$m_c = m_c(T, \mu_C, \mu_S, \mu_B)$$

Taylor expand the effective mass in chemical potential

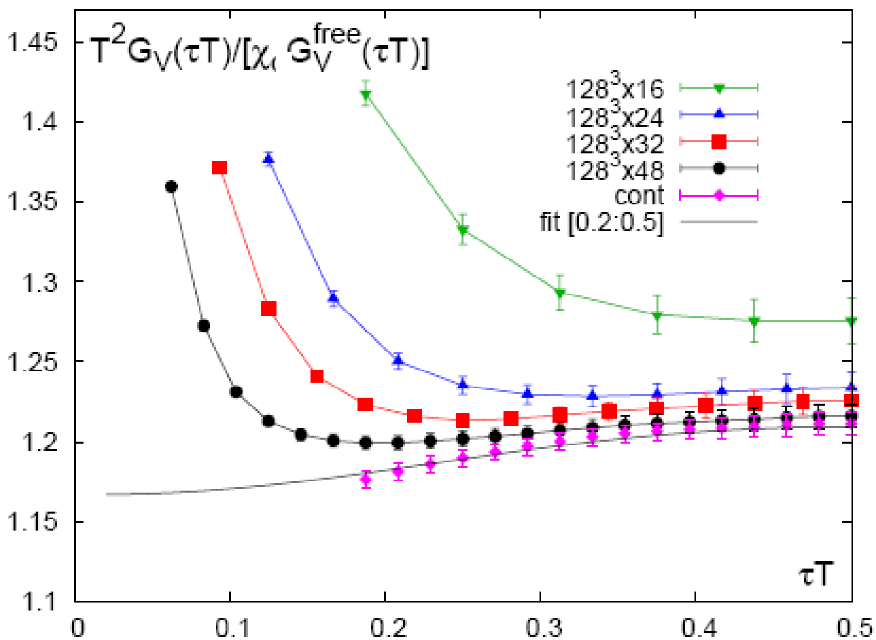
c_n

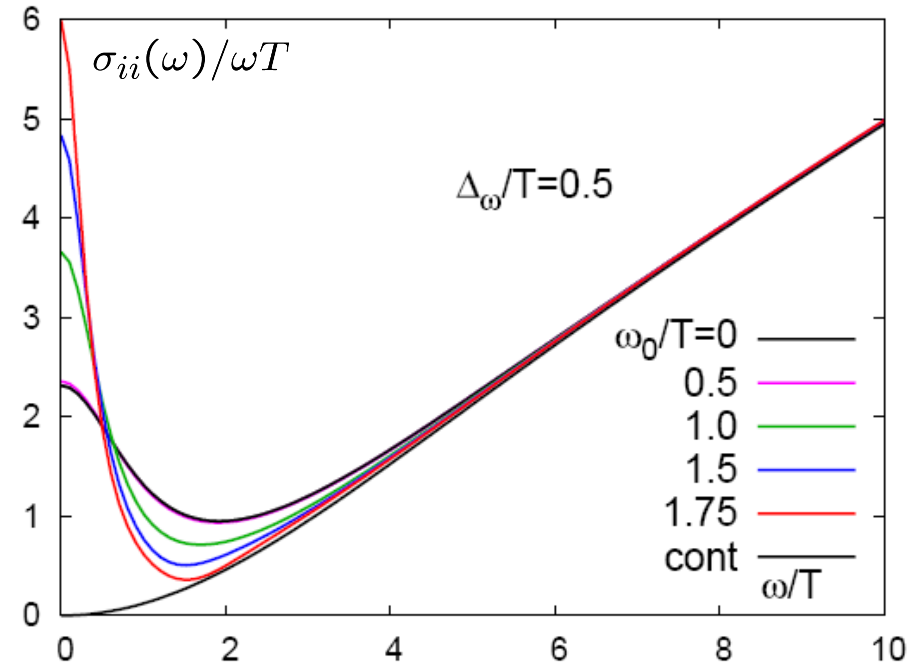
⇒ Un-natural fine tuning of the expansion coefficients

Lattice calculations of the vector spectral functions:

Ding et al, PRD 83 (11) 034504

Isotropic Wilson gauge action, quenched non-perturbatively improved clover fermion action on $128^3 \times N_\tau$ lattices, $T = 1.45T_c$, $m_q^{\overline{MS}}(2\text{GeV}) = 0.1/T$, $N_\tau = 24,\ 32, 48$ $(a^{-1} = 9.4 - 18.8\text{GeV})$





$$\sigma_{ii}(\omega) = \chi c_{BW} \frac{1}{\pi} \frac{\omega\Gamma/2}{\omega^2 + (\Gamma/2)^2} + \frac{3}{4\pi^2}(1+k)\omega^2 \tanh(\omega/4T)\Theta(\omega_0, \Delta_\omega),$$

$$\Theta(\omega_0, \Delta_\omega) = (1 + e^{(\omega_0^2 - \omega^2)/\omega\Delta_\omega})^{-1}$$

Fit parameters : c_{BW}, Γ, k

Different choices of : ω_0, Δ_ω