Theoretical Methods in Hadron Spectroscopy

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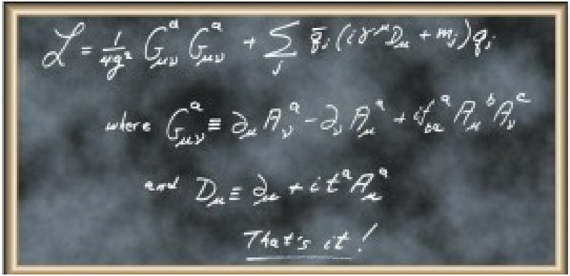
HADRON SPECTROSCOPY: WHY?

- Many recently discovered hadrons have unexpected properties.
- Understand the hadron spectra to separate EW physics from strong-interaction effects
- Techniques for non-perturbative physics useful for physics at LHC energies.
- Understanding EW symmetry breaking may require nonperturbative techniques at TeV scales, similar to spectroscopy at GeV.
- Better techniques may help understand the nature of masses and transitions

The theory of the strong force:
Quantum Chromodynamics

Quantum Chromodynamics (QCD)

The quantum field theory of the strong interaction that binds quarks and gluons to form hadrons.



from F.A. Wilczek

- this doesn't look too bad - a bit like QED which we have a well-developed toolkit to deal with

SOME MORE DETAILS

QCD is a gauge-invariant quantum field theory

$$\mathcal{L} = \bar{q}\left(i\gamma^\mu \partial_\mu - m\right) q + g\bar{q}\gamma^\mu t_a q A^a_\mu - \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a$$

- Actually not easy at all! an enormous challenge!
- One way to see this is to note that g is not a small number so perturbation theory (an expansion in a small parameter) that works so well for QED will not be so useful for **QCD** .
  - There are some small numbers around - the quark masses $m_{u,d} \sim \mathcal{O}(1) MeV$.
- Matter: quark fields the building blocks; quark mass is input parameter in $\mathcal{L}$

$$q_i^f \begin{cases} i & \in \{\text{red}, \text{blue}, \text{green}\} \\ f & \in \{\text{u}, \text{d}, \text{s}, \text{c}, \text{b}, \text{t}\} \end{cases}$$

spin = 1/2; charge = 2/3, -1/3

- the t_a are the generators (matrices) of the group $SU(3)$
  $[t_a, t_b] = if_{abc} t_c$
- interaction (force) carriers: 8 massless spin-1 gluons in the 8-dim representation of $SU(3)$.
- hadrons are color-singlet (ie not colored) combinations of quarks, anti-quarks and gluons

QCD vs QED

QED

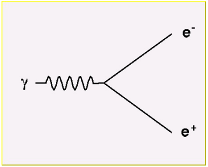
Quantum theory of electromagnetic interactions, mediated by exchange of photons.
Photon couples to electric charge e
Coupling strength $\propto e \propto \sqrt{\alpha}$

QCD

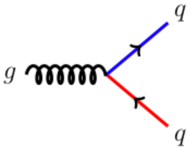
Quantum theory of strong interactions, mediated by exchange of gluons between quarks.
Gluon couples to colour charge of quark
Coupling strength is $\propto \sqrt{\alpha_s}$

Fundamental vertices

QED



QCD



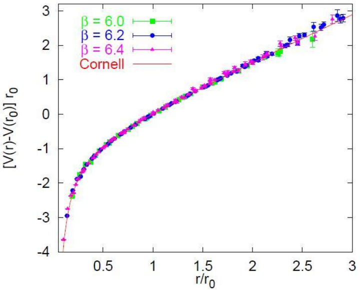
Coupling constants: coupling strength of QCD $\gg$ QED

COLOR FORCE AND QUARK POTENTIALS

Between 2 quarks at distance $r \sim O(1)\mathrm{fm})$ define a **string** with **tension** k and a potential $V(r) = kr$.

Stored energy/unit length is constant and separation of quarks requires infinite amount of energy.

QCD Potential QED-like at short distance $r \leq 0.1fm$. String tension - potential increases linearly at large distance $r \geq 1fm$.



Force between 2 quarks at large distance is $|dV/dr| = k = 1.6 \times 10^{-10}\mathrm{J}/10^{-15}\mathrm{m} = 16000\mathrm{N}$ or equivalent to the weight of a car!

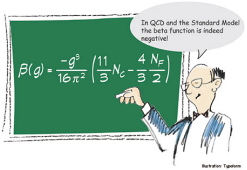
This stored energy gives the proton its mass (and not the Higgs as you sometimes hear)! Recall $m_u + m_u + m_d \sim 9\mathrm{MeV}$ but $m_{\mathrm{proton}} = 938\mathrm{MeV}$

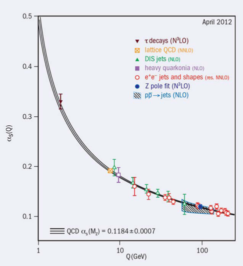
THE RUNNING QCD COUPLING

In QED, α varies with distance - running and the bare e^{-} is *screened* at large distances - reducing.
The same but different in QCD where *anti-screening* dominates!
$\Rightarrow$ At large distances (low energies) $\alpha_s \sim 1$ i.e. large. Higher-order diagrams - α_s increasingly larger, summation of diagrams diverges ... perturbation theory fails.

Asymptotic freedom

Coupling constant is small at high energies i.e. energetic quarks are (almost) free. QCD perturbation theory works!





Nobel prize 2004 for Gross, Politzer and Wilczek.

QCD: MAKING CALCULATIONS

There are two regimes:

Deep inside the proton

- at short distances quarks behave as free particles
- weak coupling

⇒ perturbation theory works

At "observable" (hadronic) distances

- at long distance (1fm) quarks confined
- strong coupling

⇒ perturbation theory fails: nonperturbative approach needed.

Consequences of strong dynamics

The strong-coupling and nature of gluons ⇒ interesting particles can appear

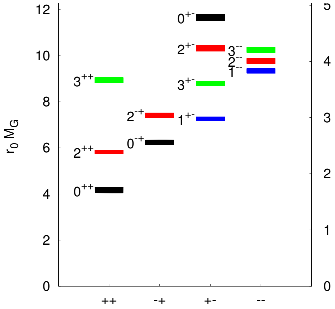
- quark condensates
- glueballs
- hybrids

GLUEBALLS

Gluons couple strongly to each other

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a, \quad F_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

- expect a spectrum of gluonic excitations
- possible even in a theory without quarks i.e. "pure Yang-Mills"
- particles are called glueballs
- lattice predictions ...



Morningstar & Peardon

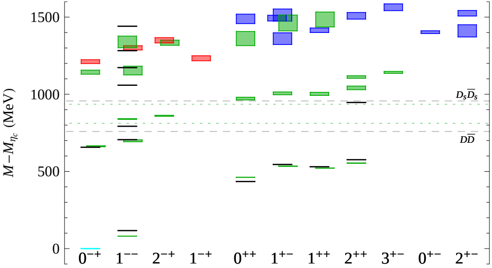
In full QCD glueballs much more complicated.

- same quantum numbers as isospin 0 mesons
- mix with lots of things!

Hybrid mesons

States with quarks and excited gluonic field content $[q\bar{q}g]$.

- a better chance to see gluonic excitations at experiments
- the signal is exotic: $J^{PC}_{q\bar{q}} \otimes J^{PC}_{\text{glue}} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, \ldots$
- lattice is providing model-independent simulations now ...
- on the shopping list at GlueX and PANDA



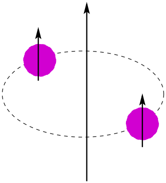
HadSpec Collab

Quark Models

CLASSIFYING STATES: MESONS

- Recall that continuum states are classified by J^{PC} multiplets (representations of the poincare symmetry):
  - Recall the naming scheme: $n^{2S+1}L_J$ with $S = \{0, 1\}$ and $L = \{0, 1, \ldots\}$
  - J, hadron angular momentum, $|L-S| \leq J \leq |L+S|$
  - $P = (-1)^{(L+1)}$, parity
  - $C = (-1)^{(L+S)}$, charge conjugation. Only for $q\bar{q}$ states of same quark and antiquark flavour. So, not a good quantum number for eg heavy-light mesons $(D_{(s)}, B_{(s)})$.

MESONS



- two spin-half fermions $^{2S+1}L_J$
- $S = 0$ for antiparallel quark spins and $S = 1$ for parallel quark spins;

- States in the natural spin-parity series have $P = (-1)^J$ then $S = 1$ and $CP = +1$:
  - $J^{PC} = 0^{-+}, 0^{++}, 1^{--}, 1^{+-}, 2^{--}, 2^{-+}, \ldots$ allowed
- States with $P = (-1)^J$ but $CP = -1$ forbidden in $q\bar{q}$ model of mesons:
  - $J^{PC} = 0^{+-}, 0^{--}, 1^{-+}, 2^{+-}, 3^{-+}, \ldots$ forbidden (by quark model rules)
  - These are **EXOTIC** states: not just a $q\bar{q}$ pair ...

Methods for calculating in QCD

EFT SUMMARY

- The basic ideas underpinning EFTs: separate physics at different scales; identify approprite degrees of freedom
- Implement the consequences of symmetries
- EFT allows you to compute using dimensional analysis - even if the underlying theory is unknown
- EFT a powerful tool for probing QCD and hadron spectroscopy

Keep in mind ...

- in some cases the full theory (QCD) cannot be formally recovered i.e. the EFT is nonrenormalisable e.g. lattice NRQCD .
- the effective theory is a good description of some regime in QCD of interest but cannot predict/describe beyond that.
- accuracy/precision physics needs a robust expansion as well as a reliable estimate of systematic uncertainties.

POTENTIAL MODELS

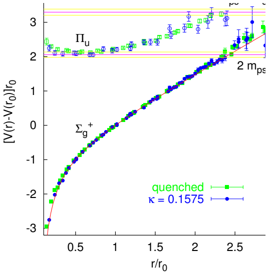
$$V(r) = \underbrace{\frac{4}{3}\frac{\alpha_s}{r}}_{vectorpart} + \underbrace{kr}_{scalarpart} + \underbrace{V_{LS}}_{spin-orbit} + \underbrace{V_{SS}}_{spin-spin} + \underbrace{V_T}_{tensorterm}$$

Many models exist, most have a similar set of ingredients:

The (confining) potl assumed from phenomenological arguments and might be extracted from data or a lattice.

With EFTs gives a useful tool.

Particularly effective for understanding particular regimes (e.g. quarkonia) or states (e.g. XYZ)



Keep in mind

Relies on an assumed potential. There are many choices and some discrimination is needed.

Not a systematic approach to full QCD

In QCD

$$Z_{\mathbf{QCD}} = \int \mathcal{D}\bar{q}\mathcal{D}q\mathcal{D}A_\mu e^{i\int d^4x \bar{q}(i\gamma^\mu\partial_\mu - m)q + g\bar{q}\gamma^\mu t_a q A^a_\mu - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a}$$

and now $\mathcal{D}\bar{q}\mathcal{D}q\mathcal{D}A_\mu$ represent an infinite number of d.o.f. that is the field strength at every point in continuous spacetime.

- make the number of degrees of freedom finite then the integral is tractable
  - this is Lattice QCD
  - discretise spacetime on a grid of points of finite extent (L), with finite grid spacing (a).

What symmetries are lost and what is the effect?

In QCD

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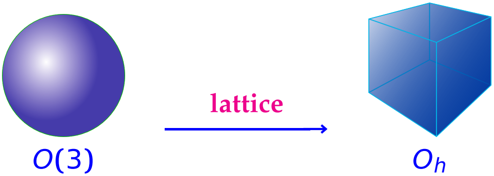
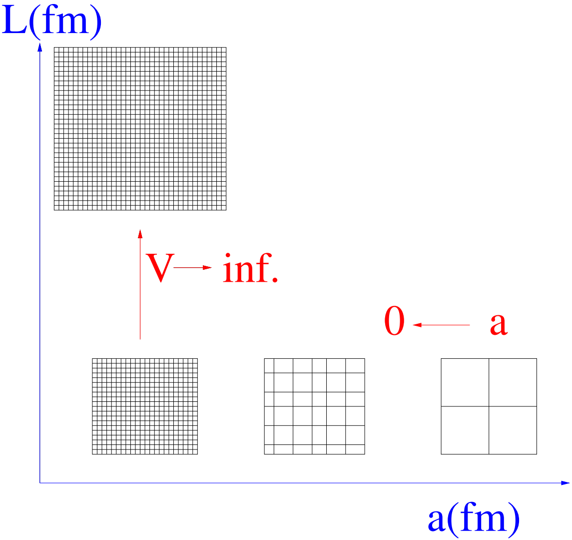


Recovering continuum QCD



PRACTICAL LQCD

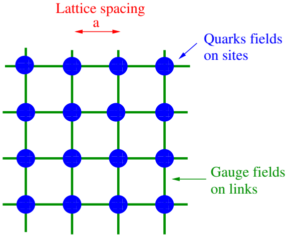
- Consider gluons on links of the lattice i.e. $U_\mu(x) = e^{-aA_\mu(x)}$. Quark fields on sites.
- Discretise derivatives with finite differences e.g. in 1-dim

$$\frac{df}{dx} = \frac{f(x+a) - f(x-a)}{2a} + \mathcal{O}(a^2)$$

Exercise: Write a 1-dim derivative correct to $\mathcal{O}(a^4)$.

- Many ways to discretise fermions and you will hear many philosophies ...

- Wilson, Clover
- Staggered, asqtad, HISQ
- Domain wall, overlap



MAKING CALCULATIONS

- If $e^{i\int d^4x\mathcal{L}}$ real then treat as a probability and use stochastic estimation (Monte Carlo) to estimate the integral
- Rotate to Euclidean time: $t \to i\tau; i\int d^4x\mathcal{L} \to -i\int d^4x\tilde{\mathcal{L}}$
- An observable looks like

$$\langle\mathcal{O}\rangle = \int \mathcal{D}\bar{q}\mathcal{D}q\mathcal{D}U\mathcal{O}e^{-S[q,\bar{q},U]}$$

- Fermion fields integrate exactly, $\int \mathcal{D}\bar{q}\mathcal{D}qe^{-\bar{q}_iQ_{ij}q_j} = \det Q$ leaving something like

$$\langle\bar{q}_x(t')\Gamma' q_x(t')\cdot\bar{q}_y(t)\Gamma q_y(t)\rangle = \int \mathcal{D}UQ^{-1}_{x,y}\Gamma' Q'_{y,x}\Gamma \det Q[U]e^{-S_{\text{gauge}}[U]}$$

- Notice $\det Q[U]e^{-S_{\text{gauge}}[U]}$ looks like a probability weight so generate gauge field configurations according to this and save them.
- An observable (two point function) is then $\sum_{\{U\}} Q^{-1}_{xy}\Gamma' Q^{-1}_{y,x}\Gamma$

WHY DOES LQCD NEED BIG COMPUTERS??

- need $detQ$ for gauge field ensembles. What does Q look like?
  - a lattice might have $24 \times 24 \times 24 \times 128 = 1.8 \times 10^6$ sites
  - a fermion (quark) has 4 Dirac components and 3 colours in $SU(3)$
  - $\Rightarrow$ a sparse matrix of size $(2 \times 10^7) \times (2 \times 10^7)$
  - storage space alone = 6.4 PetaBytes!
- once the gauge configurations are generated just have to invert the Dirac matrix Q to get the fermion propagators ...

Keep in mind in addition to statistical errors:

- Lattice artefacts

$$\left.\frac{m_N}{m_\Omega}\right|_{lat} = \left.\frac{m_N}{m_\Omega}\right|_{cont} + \mathcal{O}(a^p),\ p \geq 1$$

  requires extrapolation to the continuum limit, $a \to 0$
- Finite volume effects
  - Energy measurements can be distorted by the finite box
  - Rule of thumb: $m_\pi L > 3$ ok for many things ...
- Unphysically heavy pions
  - Simulations at physical pion mass started but most calculations rely on chiral extrapolation to reach physical m_u, m_d
  - Use Chiral Perturbation Theory to guide the extrapolations. Are chiral corrections reliably described by ChPT?
- Fitting
  - Uncertainties from the choice of fit range, t_0 etc.

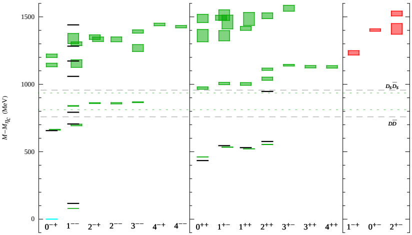
LQCD and Spectroscopy

Huge progress in the last 5 years. (With the caveats mentioned)

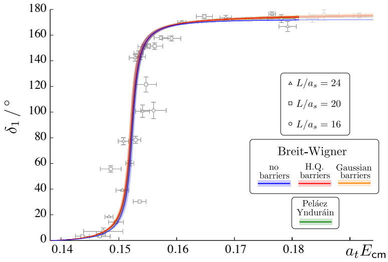
- Understood how to determine the excited and exotic (hybrid) spectra of states from light to heavy; including isoscalars and up to spin 4.
- First results from studies of the XYZ states in charmonium and $D\pi$, DK scattering.
- Huge strides made on scattering and resonance calculations. $\rho \to \pi\pi$ phase shift determined; partial wave mixing analyses ...
- Understood how to tackle coupled-channels: results for two coupled channels, theory and proof of concept for three ...

Why was this such a problem?

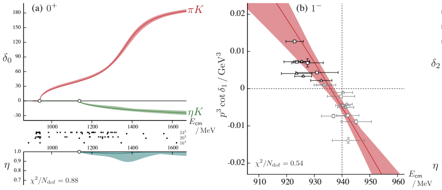
$t \to i\tau$ allows computation but loses direct info on scattering. New theoretical ideas mean now know how to retrieve this.



charmonium



$\rho \to \pi\pi$



$k\pi$ scattering

HadSpec results

I hope this has been useful
THANKS FOR LISTENING!

Last comment on single-hadron spectrum

- Disconnected diagrams a remaining uncertainty in most $c\bar{c}$ calculations.
- Distillation allows precision determination. BUT it's a can of worms!

LAST COMMENT ON SINGLE-HADRON SPECTRUM

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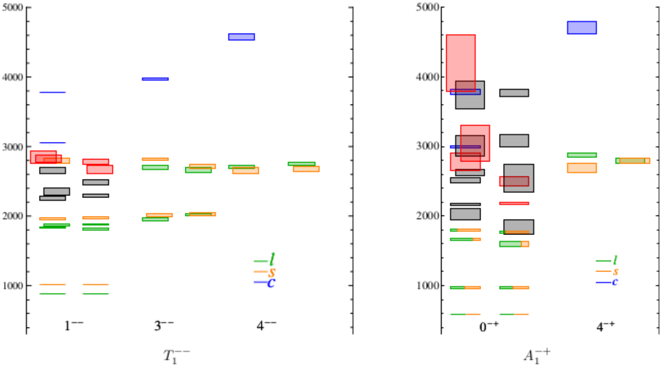

from HadSpec

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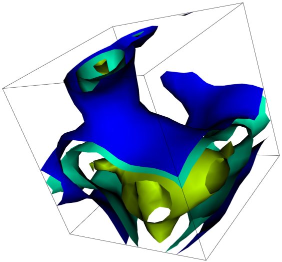
What's the plot?



Distillation

- A new approach to quark propagation by redefining smearing as a projection operator
- Basis vectors of the distillation operator (lattice laplacian) look like confining blobs