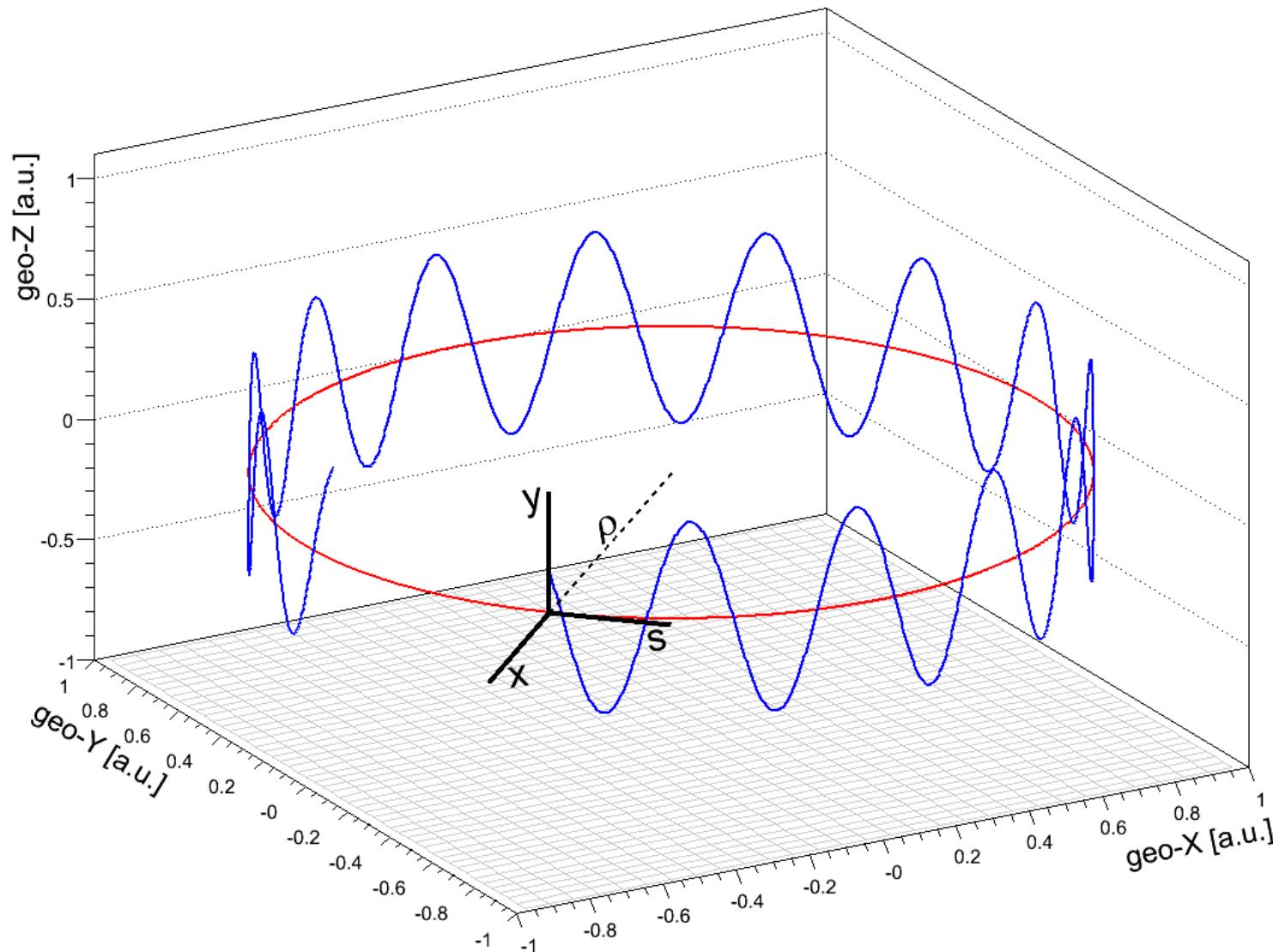


Transverse Accelerator Dynamics

Ralph J. Steinhagen



- Special acknowledgements and credits to: B. Goddard, B. Holzer & R. Steerenberg

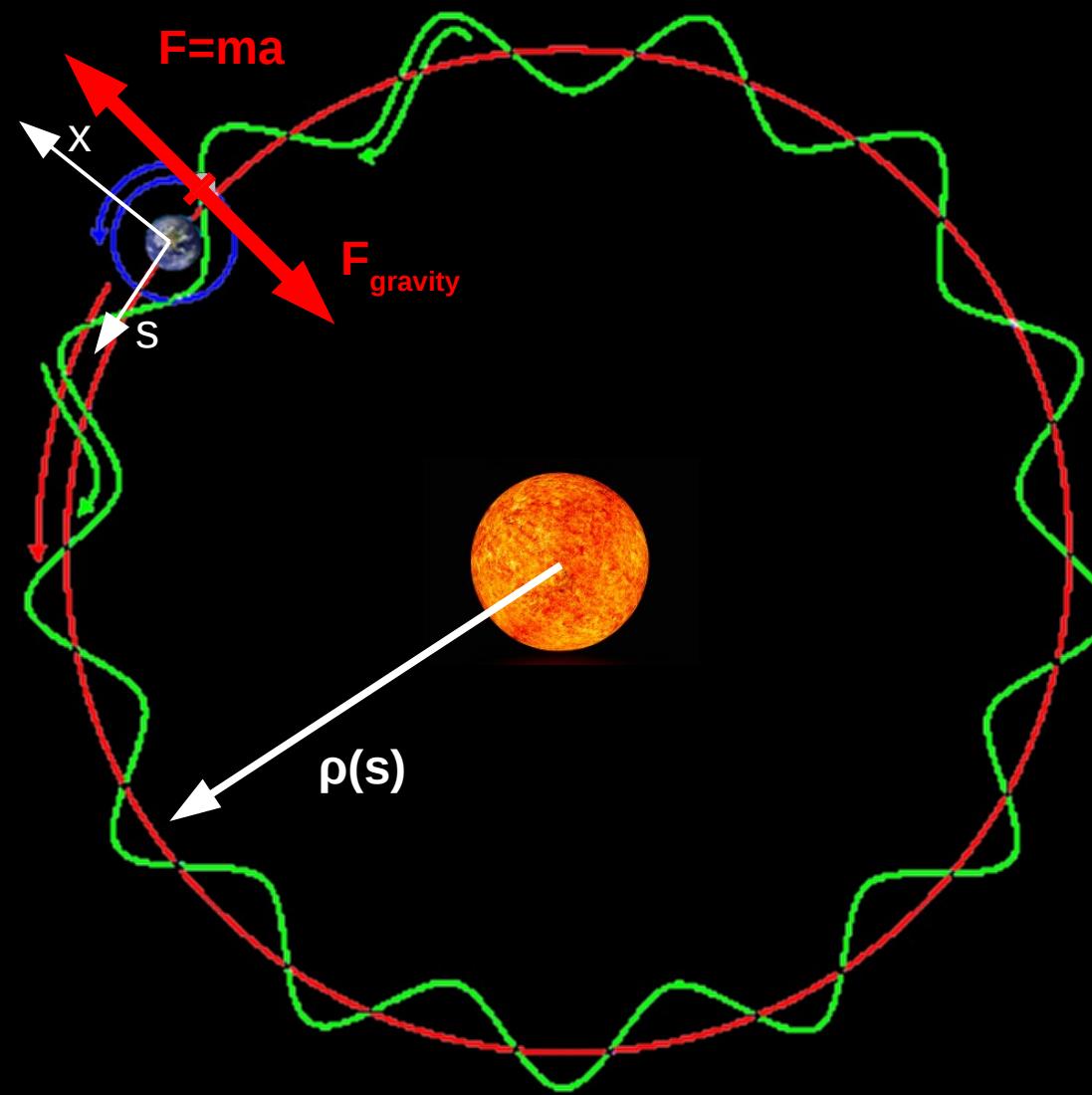
Outline

- Part I – Linear Beam Dynamics & Hill's Equation
 - Periodic Focusing System in Circular Accelerators
 - Phase Space Ellipse
 - Emittance & Acceptance
 - Machine Imperfections
 - Betatron Tune & Beam Stability
- Part II – Non-Linear Dynamics & Injection/Extraction
 - Non-linear dynamics:
 - limits of stable motion – Separatrix
 - Dispersion & Chromaticity
 - Space charge effects
 - Injection & Extraction:
 - Fast extraction, Multiturn Injection (phase-space painting)
 - Basics of resonance-, KO-extraction

Question:

Does the Moon revolve around the Earth or the Sun?

Moon's Trajectory around Sun



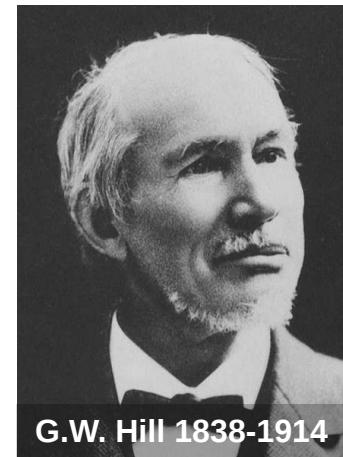
not to scale!*

Transverse Beam Dynamics

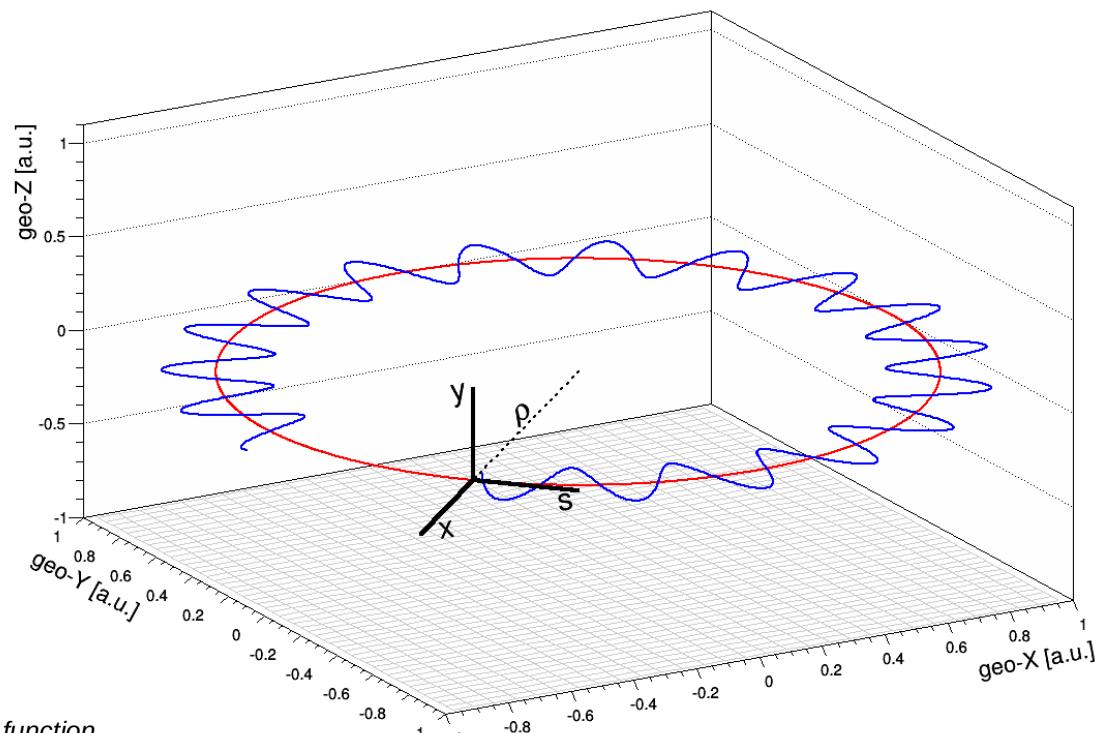
- Hill's equation^{1,2}:

$$z'' + K(s) \cdot z = f(s, t)$$

$$K(s) = \underbrace{\left(\frac{q}{p} B_{dipole} \right)^2}_{\text{weak focusing: } \frac{1}{\rho^2}} - \underbrace{\frac{q}{p} \frac{\partial B_y}{\partial x}}_{\text{strong focusing: } k(s)}$$



- $k(s)$: focusing strength, defines:
 - betatron function $\beta(s)$ → envelope of the oscillation
 - dispersion function $D(s)$ → trajectory for off-momentum $\Delta p/p_0$ particles
- $f(s,t)$: external driving force



¹George William Hill, "On the part of the motion of the lunar perigee which is a function of the mean motions of the sun and moon", Acta Mathematica, 8:1–36, 1886

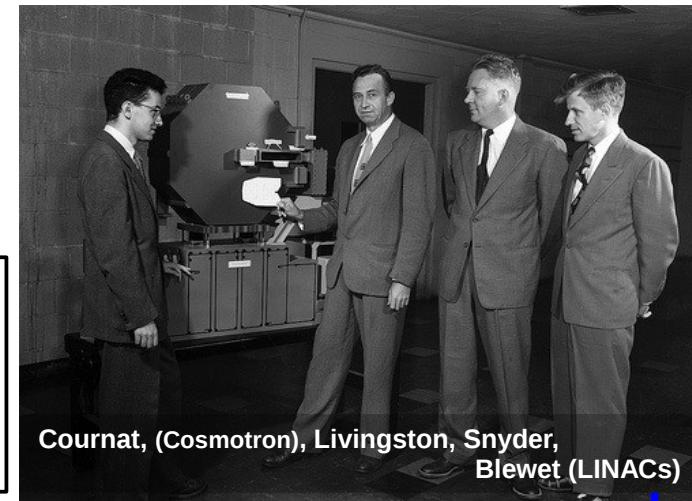
²coordinate 'z' being place holder for either x,y

shorthand: $x' = \frac{dx}{ds}$ & $z := 'x' \text{ or } 'y'$

Hill's Equation I/II

- Simplified Hill's equation^{1,2}:

$$(I) \quad z'' + K(s) \cdot z = 0 \quad \wedge \quad K(s) = -\underbrace{\frac{q}{p} \frac{\partial B_y}{\partial x}}_{\text{strong focusing}}$$



- If the restoring force $K(s)$ would be constant in 's' → Simple Harmonic Oscillator
 - usually $K(s)$ varies strongly with 's' (discrete magnets, FODO arrangement, ..)

How to solve?

- Try the following Ansatz^{1,2}:

$$(II) \quad z_\beta(s) = \sqrt{\epsilon_i \beta(s)} \cdot \sin(\mu(s) + \phi_i)$$

ϵ_i, ϕ_i : initial particle state

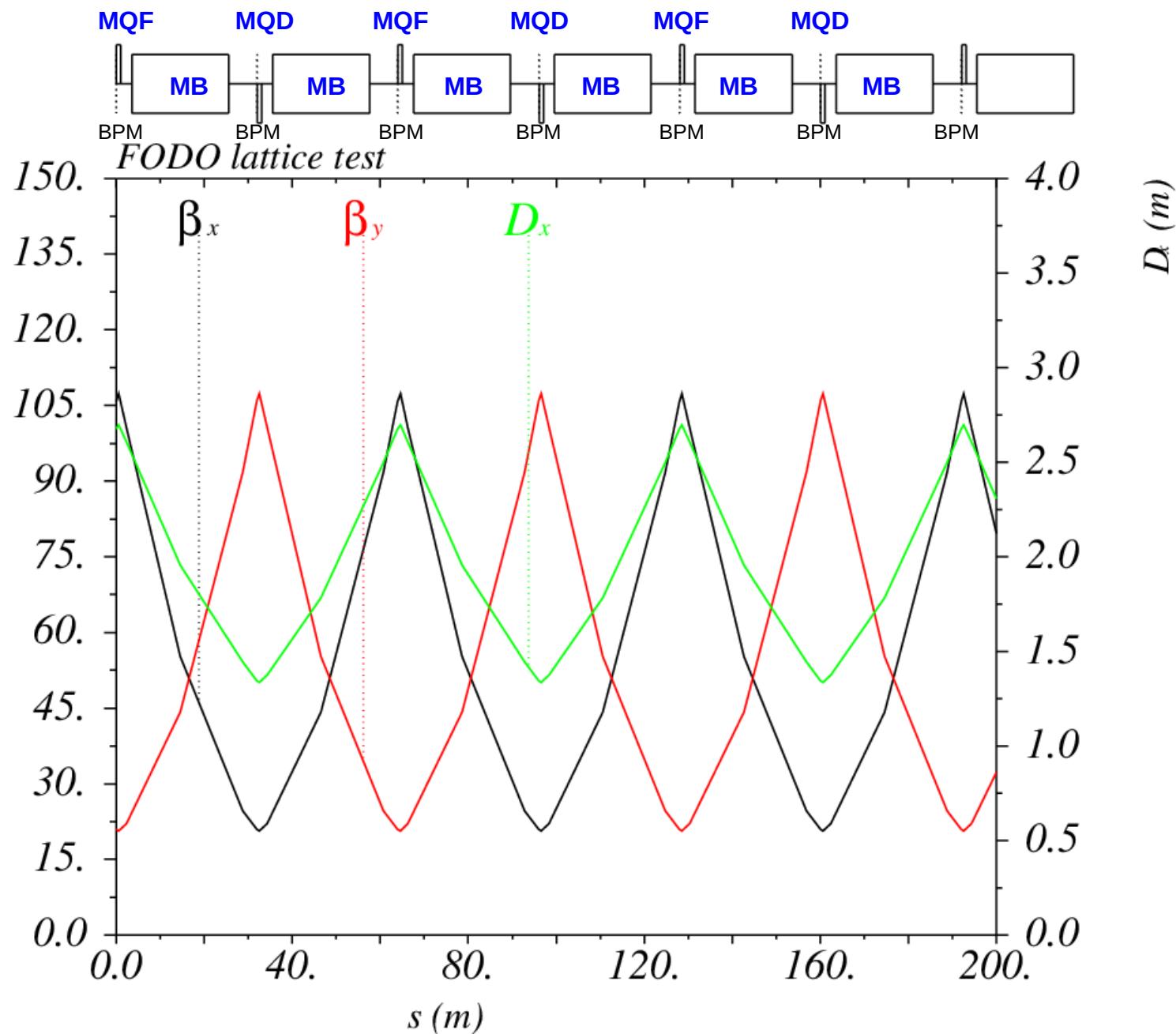
- derive (II) twice to obtain z'' and insert into (I)
- Don't worry, KISS → solution typically (nearly always) done numerically using tools like: MAD-X, ORBIT, TRANSPORT, ELEGANT, MIRKO, ...

¹Richard Q. Twiss and N. H. Frank, "Orbital stability in a proton synchrotron", Rev. Sci. Instr., 20(1):1–17, January 1949.

² E. D. Courant and H. S. Snyder, "Theory of the Alternating-Gradient Synchrotron", Annals of Physics, 3, 1, 1958.

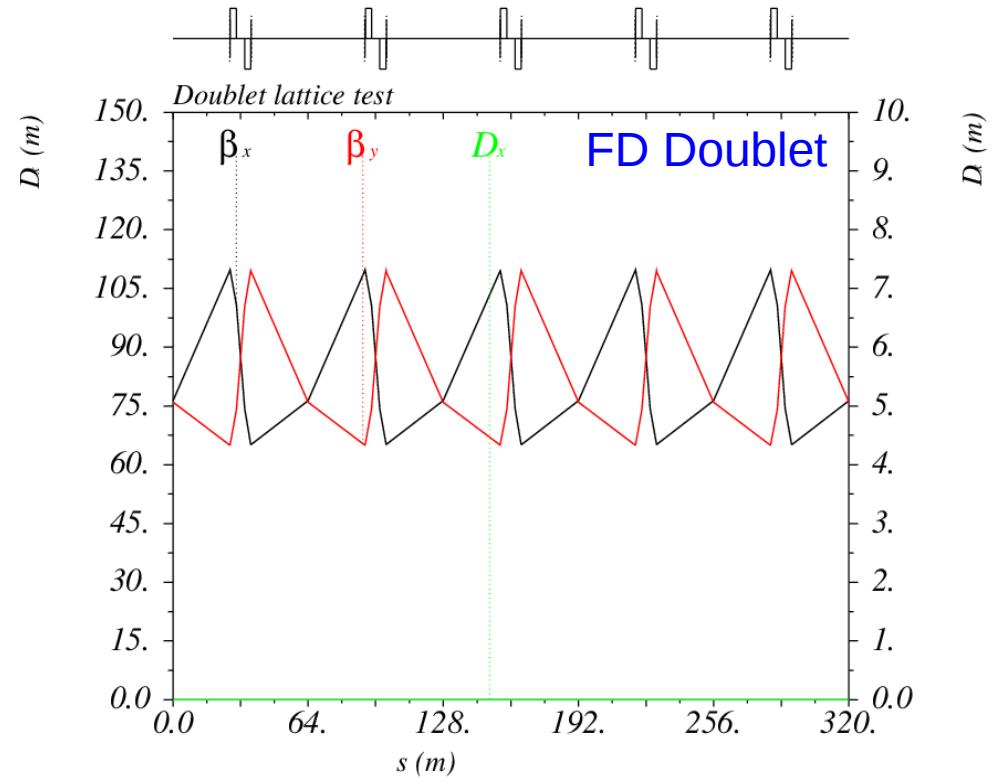
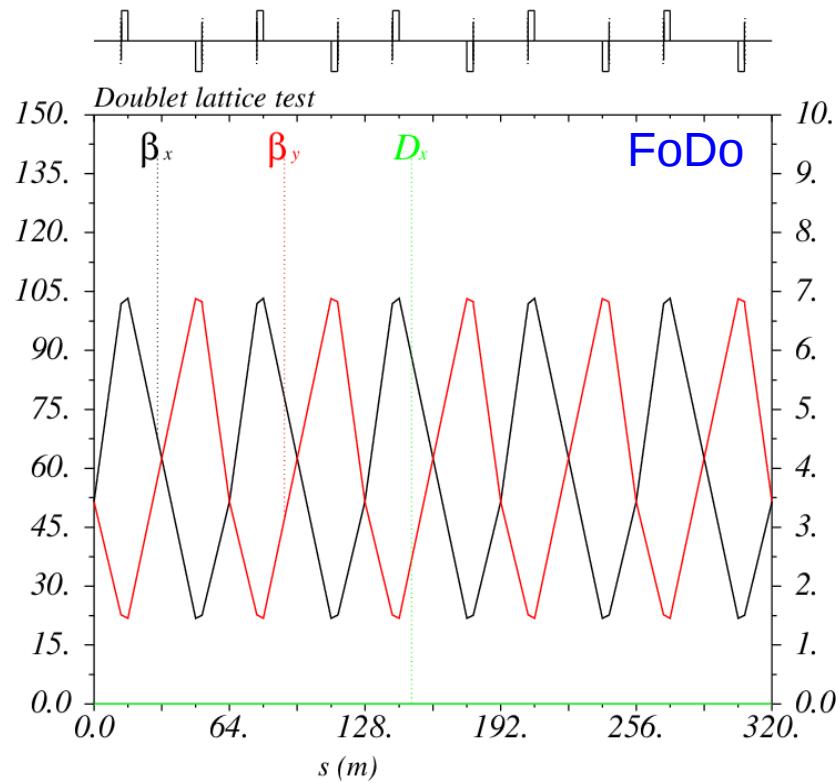
shorthand: $x' = \frac{dx}{ds}$

MAD-X Typical Output

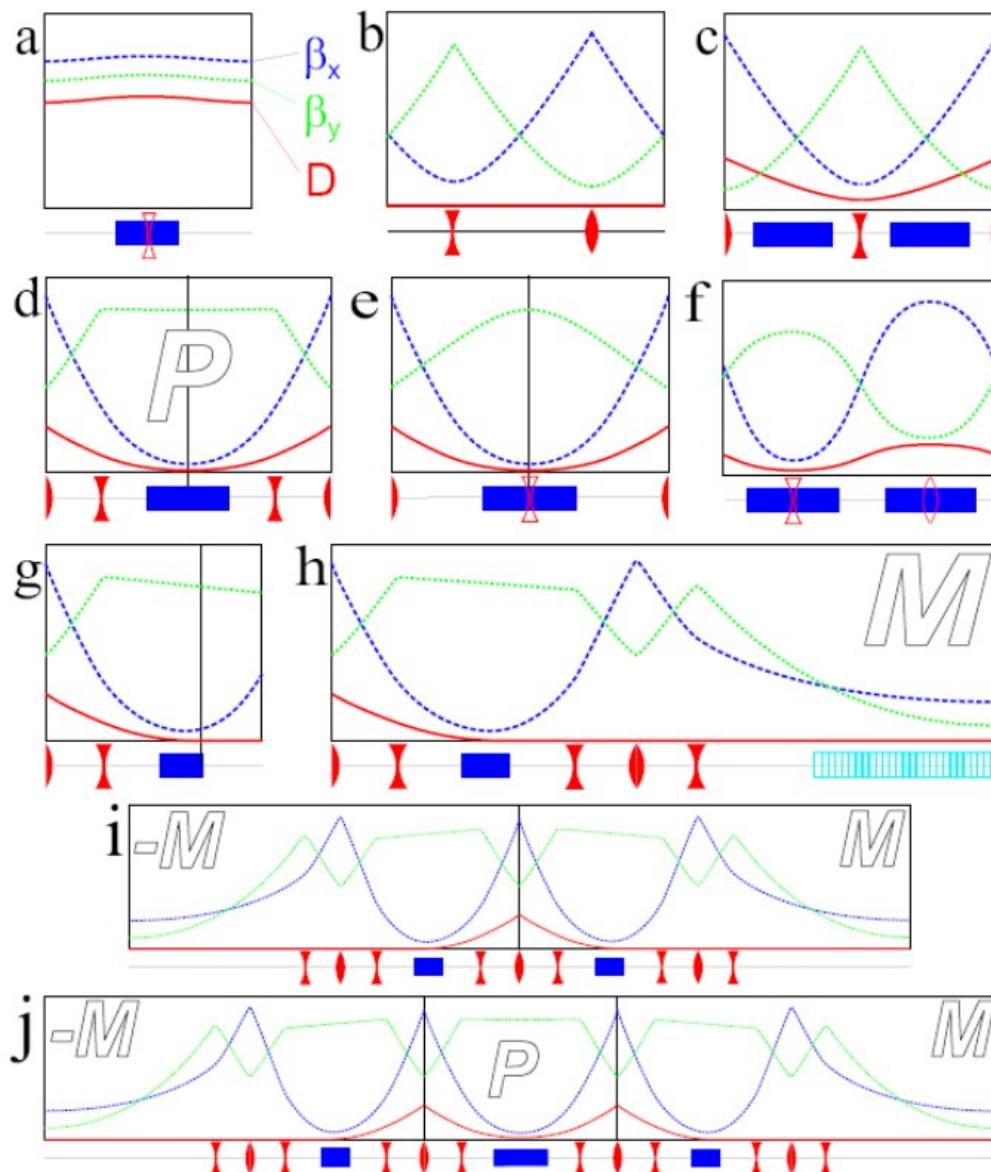


FoDo alternatives → FD Doublet Lattice

- More space between quads
- Stronger quad strengths
- Round beams
- Used e.g. in CTF3 linac



Many Alternatives



- a) Weak focusing (dipole only)
- b) FODO line (w/o dipoles)
- c) FODO cell
- d) Low-emittance cell
- e) CF low-emittance cell
- f) Low-emittance FODO
- g) Dispersion match
- h) Periodic dispersion match
- i) Double-bend achromat
- j) Triple-bend achromat
- k) ...

Very good course on low-emittance lattice design: A.Streun, PSI

Hill's Equation II/II

- Usually define add. 'Twiss' functions¹:

- betatron phase advance $\mu(s)$, $\alpha(s)$ & $\gamma(s)$

$$\Delta \mu(s) := \int_0^s \frac{1}{\beta(s')} ds' \quad \alpha(s) := -\frac{\beta'(s)}{2} \quad \gamma(s) := \frac{1+\alpha^2(s)}{\beta(s)}$$

- typically stored in look-up tables (e.g. LSA) and re-used for other computations
- More general first-order solution to Hill's equation:

$$z(s) = \underbrace{z_{co}(s)}_{closed\ orbit} + \underbrace{D(s) \cdot \frac{\Delta p}{p_0}}_{dispersion\ orbit} + \underbrace{z_\beta(s)}_{betatron\ oscillations}$$

→ sinusoidal particle motion in accelerators:

$$z_\beta(s) = \sqrt{\epsilon_i \beta(s)} \cdot \sin(\mu(s) + \phi_i)$$

- N.B. discussed later: Dispersion function $D(s)$

- ↔ trajectory dependence for off-momentum particles

¹Richard Q. Twiss and N. H. Frank, "Orbital stability in a proton synchrotron", Rev. Sci. Instr., 20(1):1–17, January 1949.

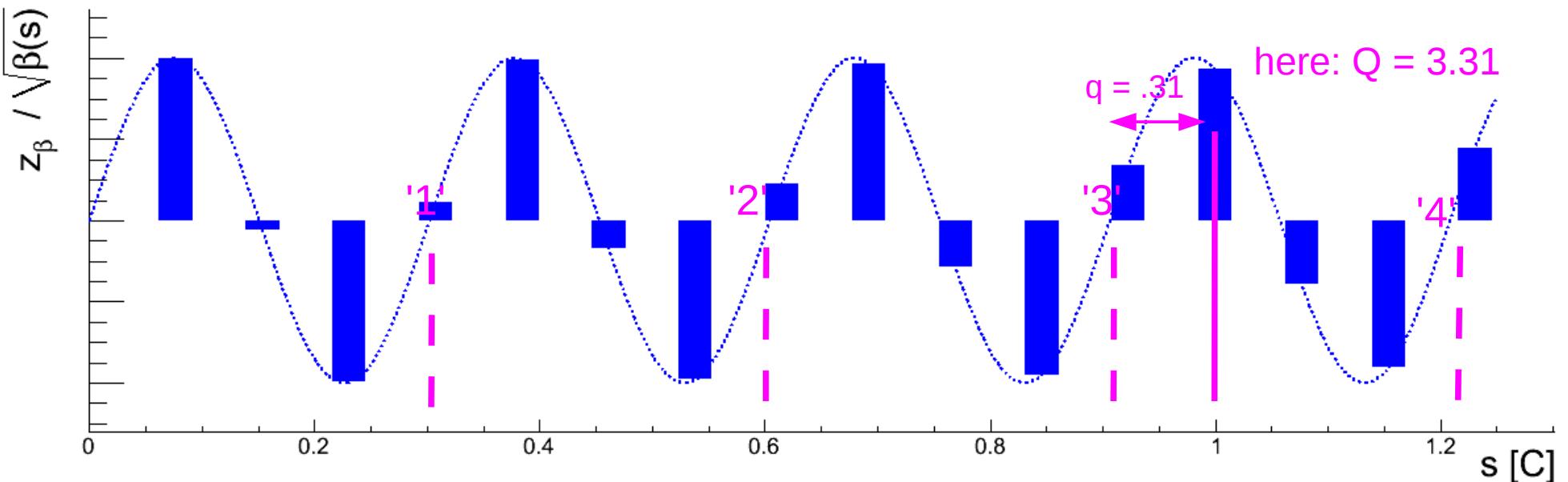
²E. D. Courant and H. S. Snyder, "Theory of the Alternating-Gradient Synchrotron", Annals of Physics, 3, 1, 1958.

shorthand: $x' = \frac{dx}{ds}$

Free Betatron Oscillations

- Free Betatron Oscillations:

$$z_\beta(s) = \sqrt{\epsilon_i \beta(s)} \cdot \sin(\mu(s) + \varphi_i)$$



- Betatron Phase Advance: $\Delta\mu(s)$

Tune defined as betatron phase advance over one turn:

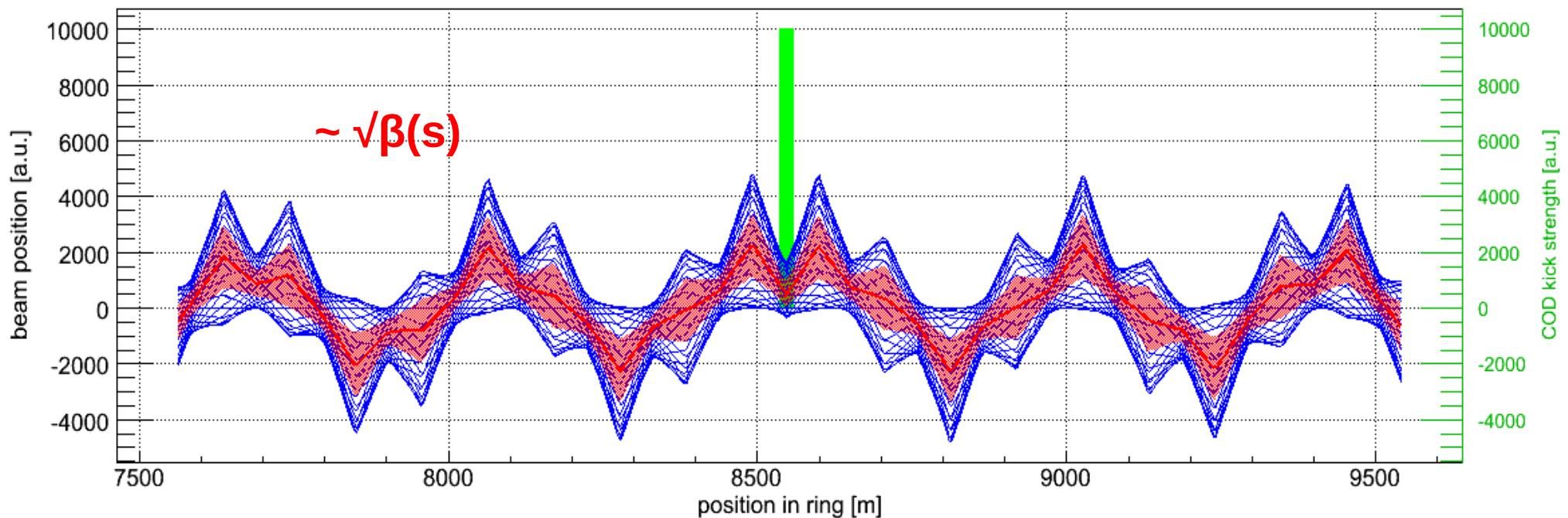
$$Q := \frac{1}{2\pi} [\mu(C) - \mu(0)]$$

common: $Q = \underbrace{Q_{int}}_{\text{integer tune}} + \underbrace{q_{frac}}_{\text{fractional tune}}$

Free Betatron Oscillations

- Example: LHC Betatron Oscillations

$$z_\beta(s) = \sqrt{\varepsilon_i \beta(s)} \cdot \sin(\mu(s) + \varphi_i)$$



- Beam size*:

$$\sigma \approx \sqrt{\varepsilon_i \beta(s)}$$

Emittance (beam quality) ε_i

constant around ring → discussed later

Beta-function $\beta(s)$

specific for accelerator lattice/
magnet arrangement

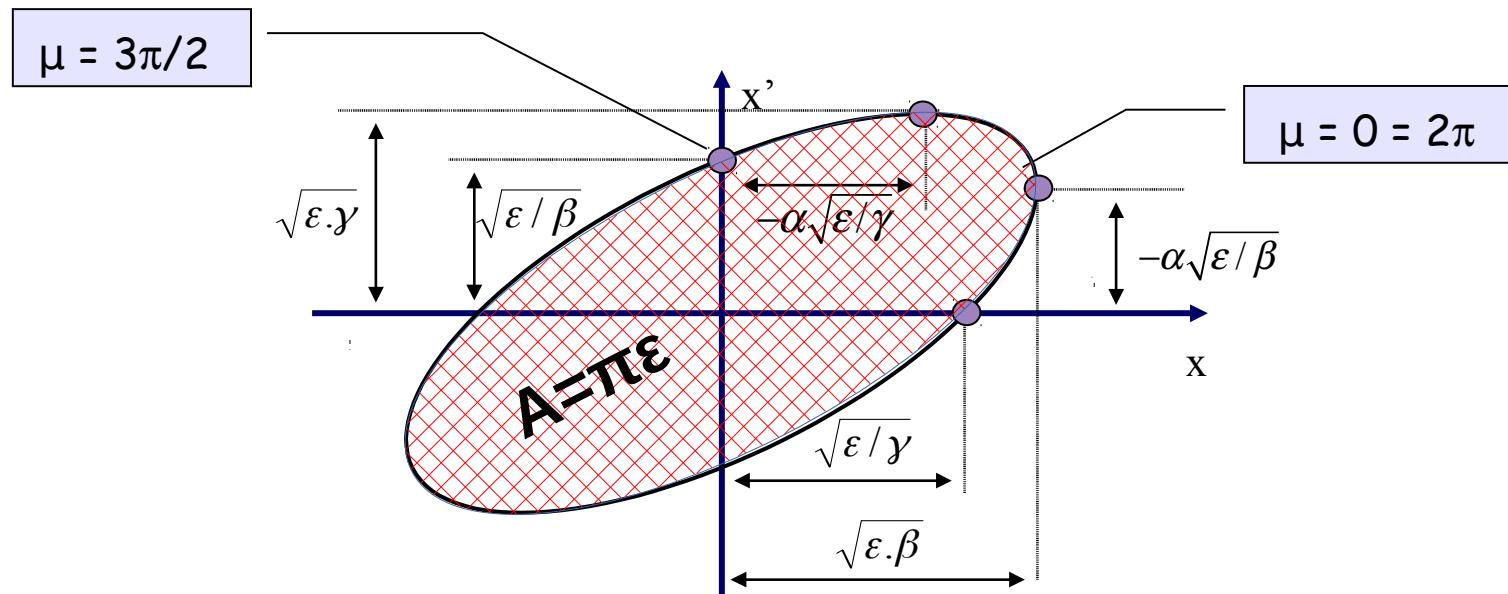
*ignores momentum & dispersion dependence

Phase Space & (Single-Particle) Emittance

- Additional result from our Ansatz: Courant-Synder invariant of motion

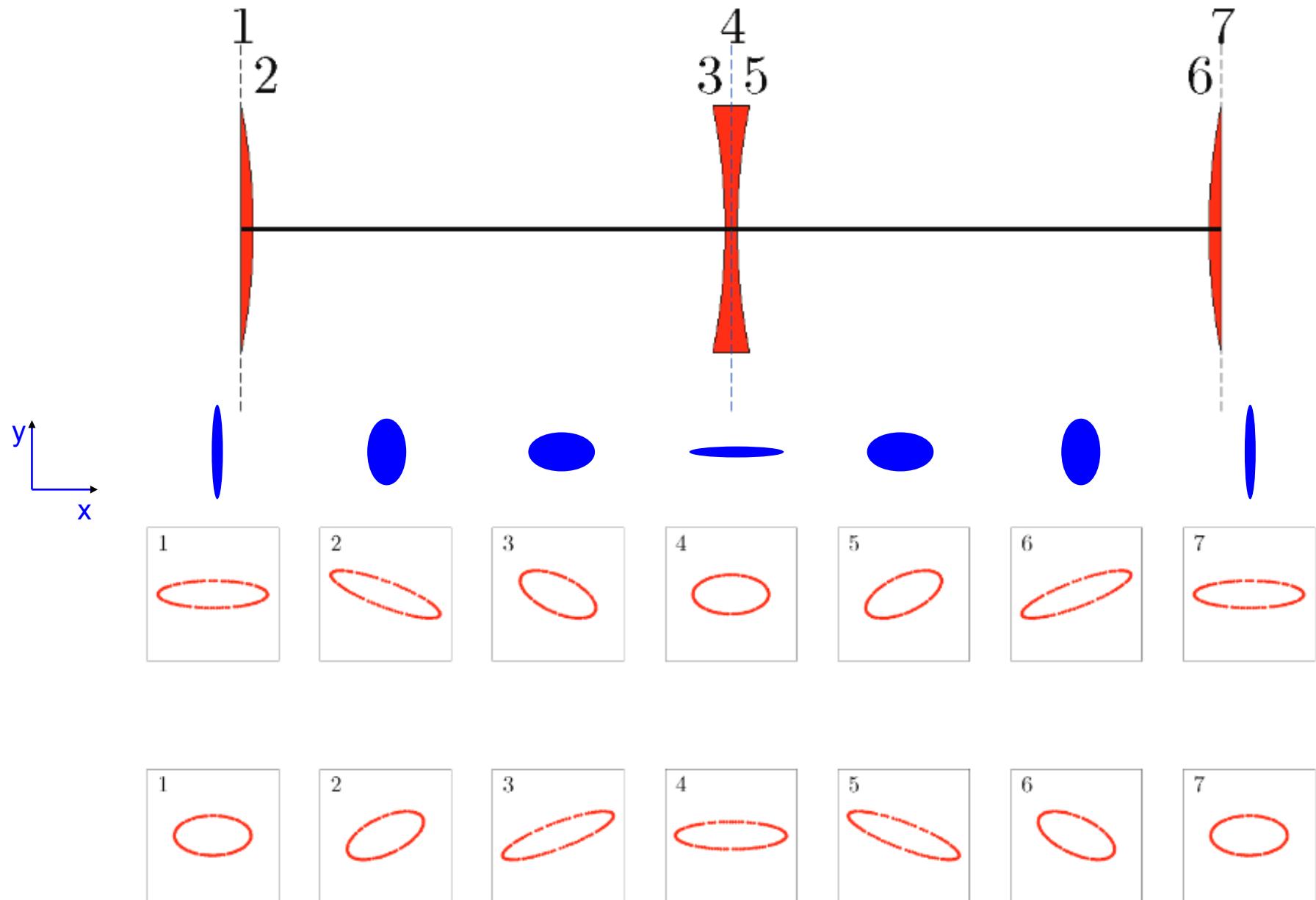
$$\epsilon = \beta(s) \cdot x'^2 + 2\alpha(s) \cdot xx' + \gamma(s) \cdot x^2$$

- Interpretation – particle motion describe ellipse in phase space:



- Liouville's Theorem: 'conservative system' (no 'friction', i.e. no energy loss/gain), the phase-space area is invariant/preserved (\leftrightarrow energy conservation)
 - N.B. if energy changes 'normalised emittance' ϵ^* is preserved: $\epsilon^* = \epsilon \cdot \beta_{\text{rel}} \gamma_{\text{rel}}$

Phase-Space & Twiss-Parameter inside a FoDo Lattice



Courtesy A. Wolski

Emittance & Acceptance II/II

- E.g. circular beam pipe of radius r_{ap}

- By analogy with emittance

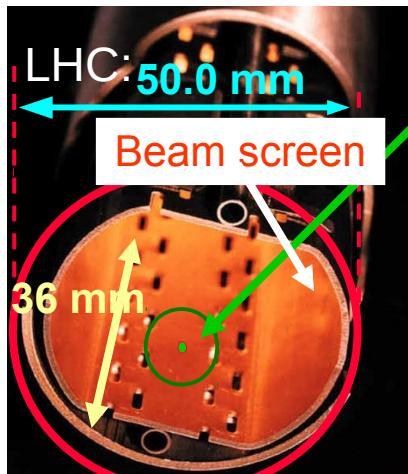
– N.B. beam size: $\sigma \approx \sqrt{\epsilon \beta(s)}$

$$A := \frac{r_{ap}^2}{\beta(s)}$$

- Acceptance

– N.B. sometimes given in units of ' σ ' (beam widths)

- Acceptance chosen such that: $A \gg \epsilon$ or $\sigma < \sqrt{A \beta(s)}$
 - ie. "beam pipe needs to be larger than beam – duh"



LHC:

Beam 3σ envel. ~ 1.8 mm @ 7 TeV

Beam 6σ envel. ~ 12 mm @ 450 TeV

\sim third of aperture if filled @ injection

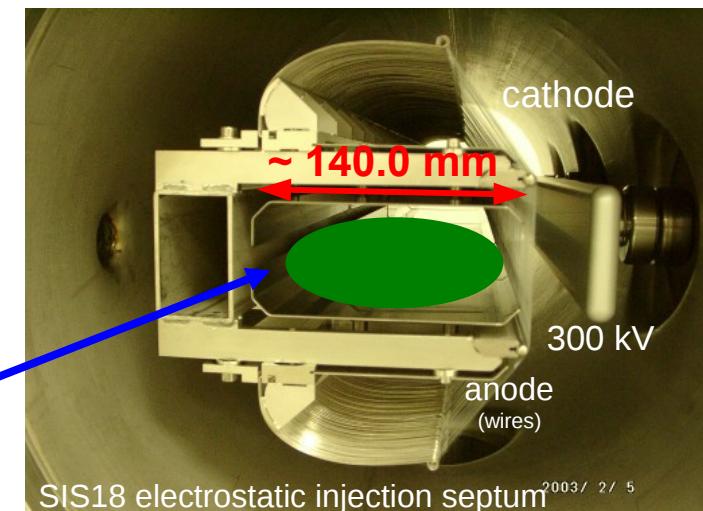
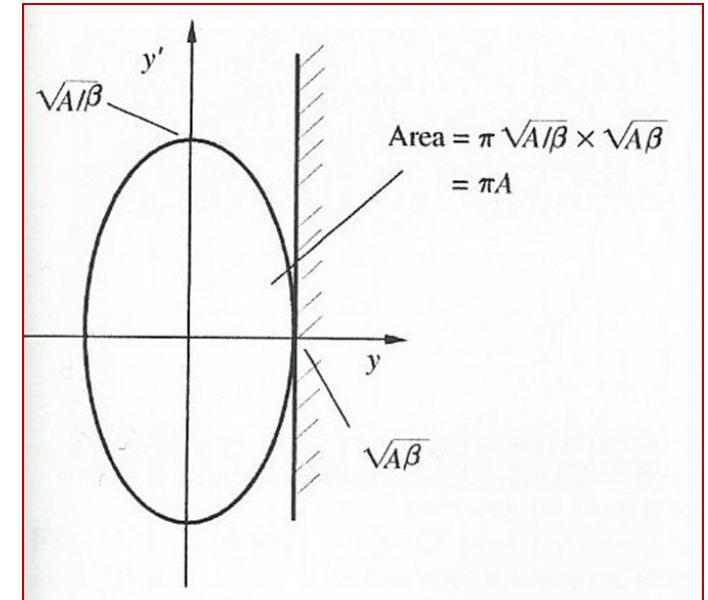
(rest kept for orbit/optics/injection error uncertainties)

SIS18:

design acceptance 325 umrad @ injection

design emittance < 299 umrad @ injection

($\pm \sim 14$ mm margin for injection, optics and orbit errors)

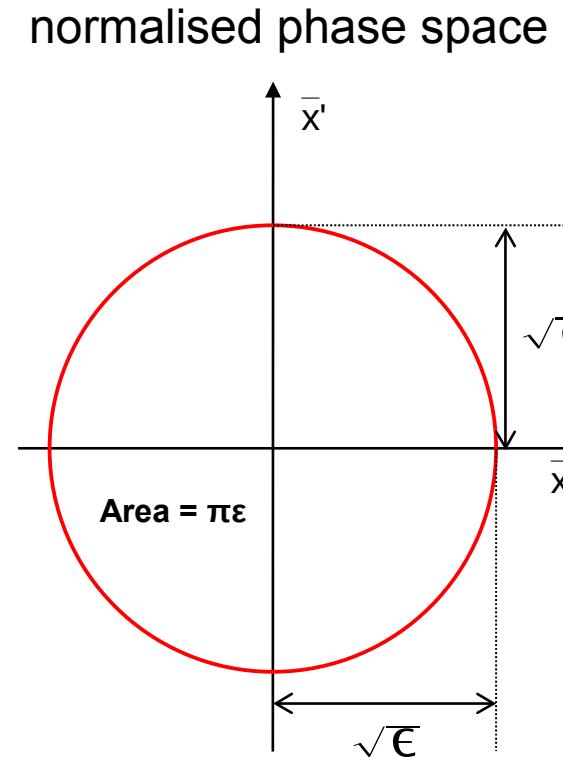
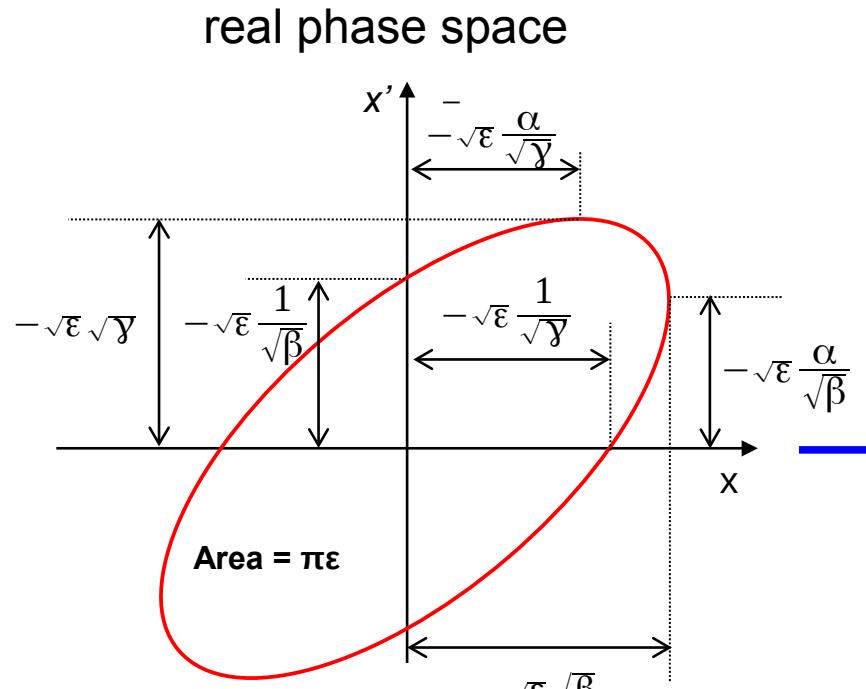


SIS18 electrostatic injection septum^{2003/ 2 / 5}

Normalised Phase Space

- often more conveniently one defines a 'normalised phase space'

$$\begin{pmatrix} \bar{x} \\ \bar{x}' \end{pmatrix} = \mathbf{N} \cdot \begin{pmatrix} x \\ x' \end{pmatrix} = \sqrt{\frac{1}{\beta}} \begin{pmatrix} 1 & 0 \\ \alpha & \beta \end{pmatrix} \cdot \begin{pmatrix} x \\ x' \end{pmatrix}$$



$$\epsilon = \beta(s) \cdot x'^2 + 2\alpha(s) \cdot xx' + \gamma(s) \cdot x^2$$

$$\epsilon = \bar{x}^2 + \bar{x}'^2$$

Recap: Transfer of Optics Parameter

- Conservation of emittance

$$\epsilon = \beta_0 x_0'^2 + 2\alpha_0 x_0 x_0' + \gamma_0 x_0^2$$

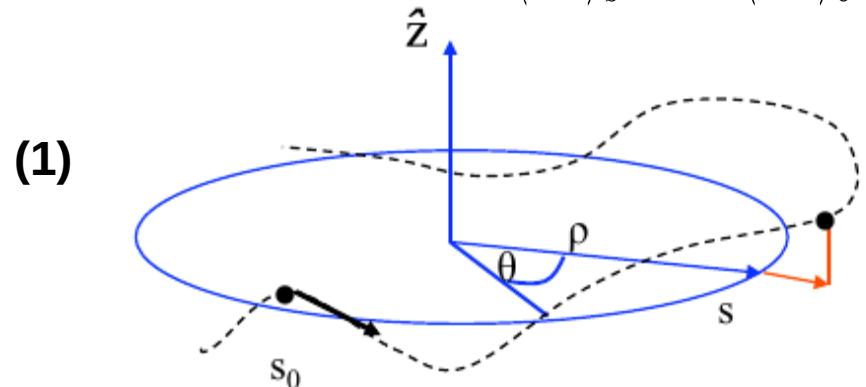
$$\epsilon = \beta_1 x_1'^2 + 2\alpha_1 x_1 x_1' + \gamma_1 x_1^2$$

- Express x_0, x_0' as a function of x, x'

$$\begin{pmatrix} x \\ x' \end{pmatrix}_0 = M^{-1} \begin{pmatrix} x \\ x' \end{pmatrix} \rightarrow \begin{aligned} x_0 &= m_{22}x - m_{12}x' \\ x_0' &= -m_{21}x + m_{11}x' \end{aligned} \quad (2)$$

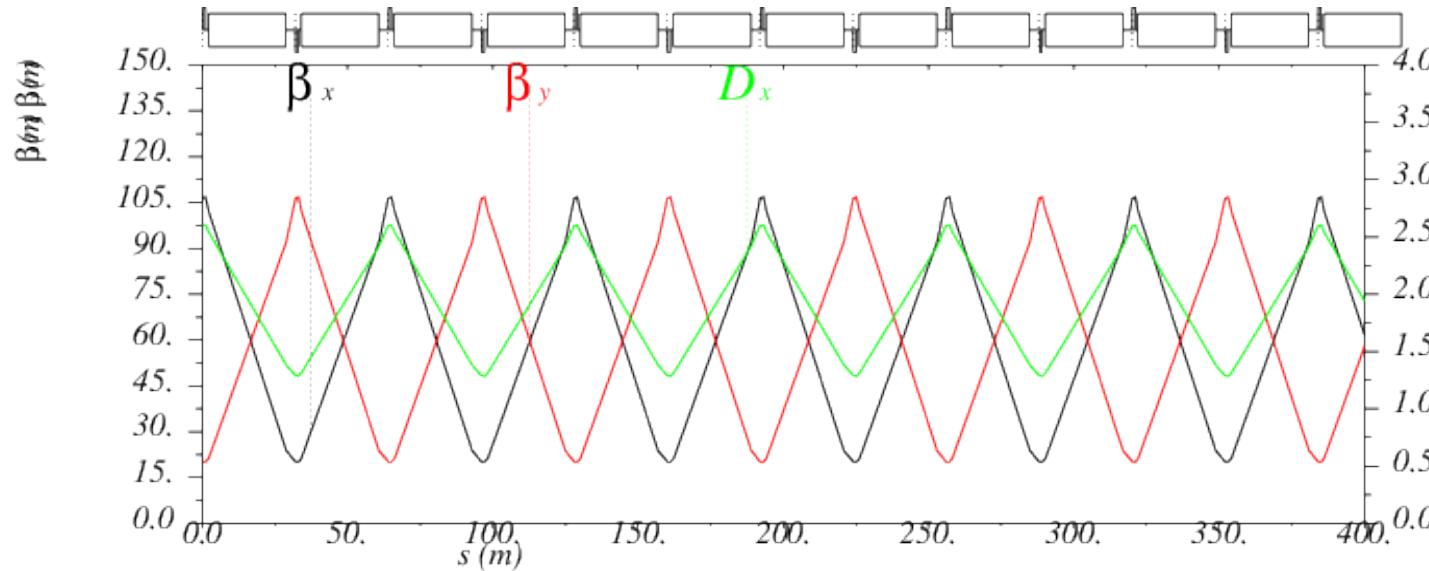
- Inserting (2) into (1), sorting via x, x' , the rest is math ...

$$\begin{pmatrix} x \\ x' \end{pmatrix}_s = M \begin{pmatrix} x \\ x' \end{pmatrix}_0$$



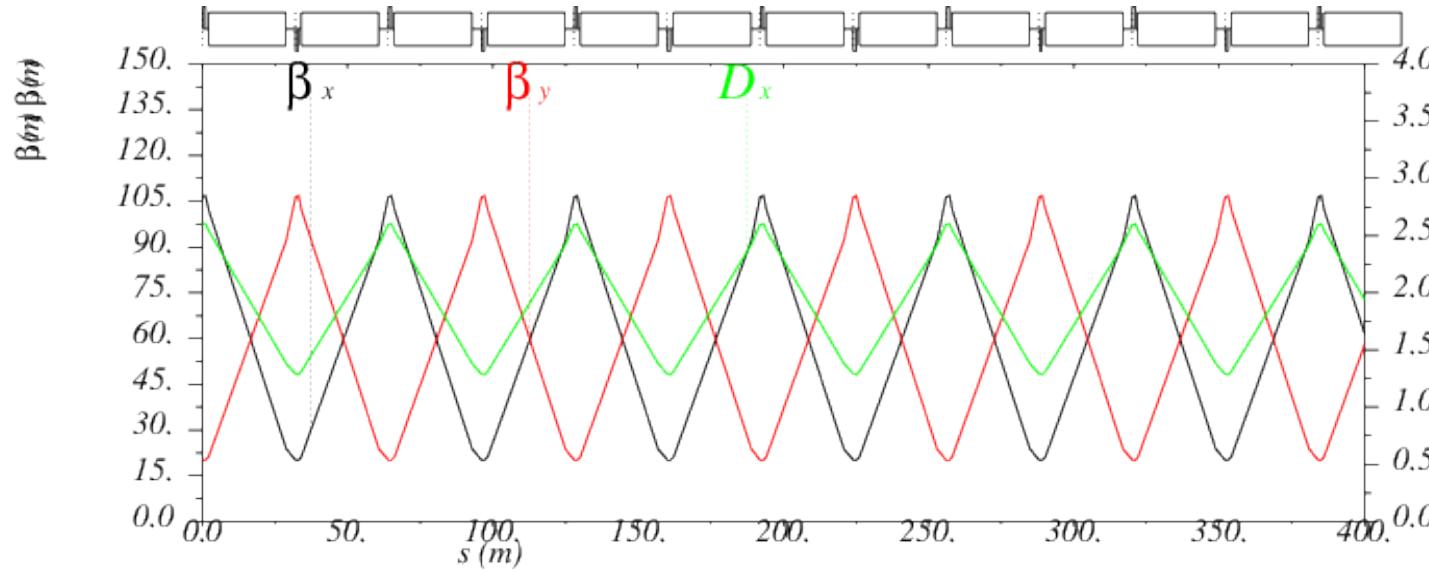
$$\begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix} = \begin{pmatrix} m_{11}^2 & -2m_{11}m_{12} & m_{12}^2 \\ -m_{11}m_{21} & m_{12}m_{21} + m_{22}m_{11} & -m_{12}m_{22} \\ m_{12}^2 & -2m_{22}m_{21} & m_{22}^2 \end{pmatrix} \cdot \begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_0$$

Difference: Transfer-Line (LINAC) vs. Ring I/II



Circular Machine (ring):

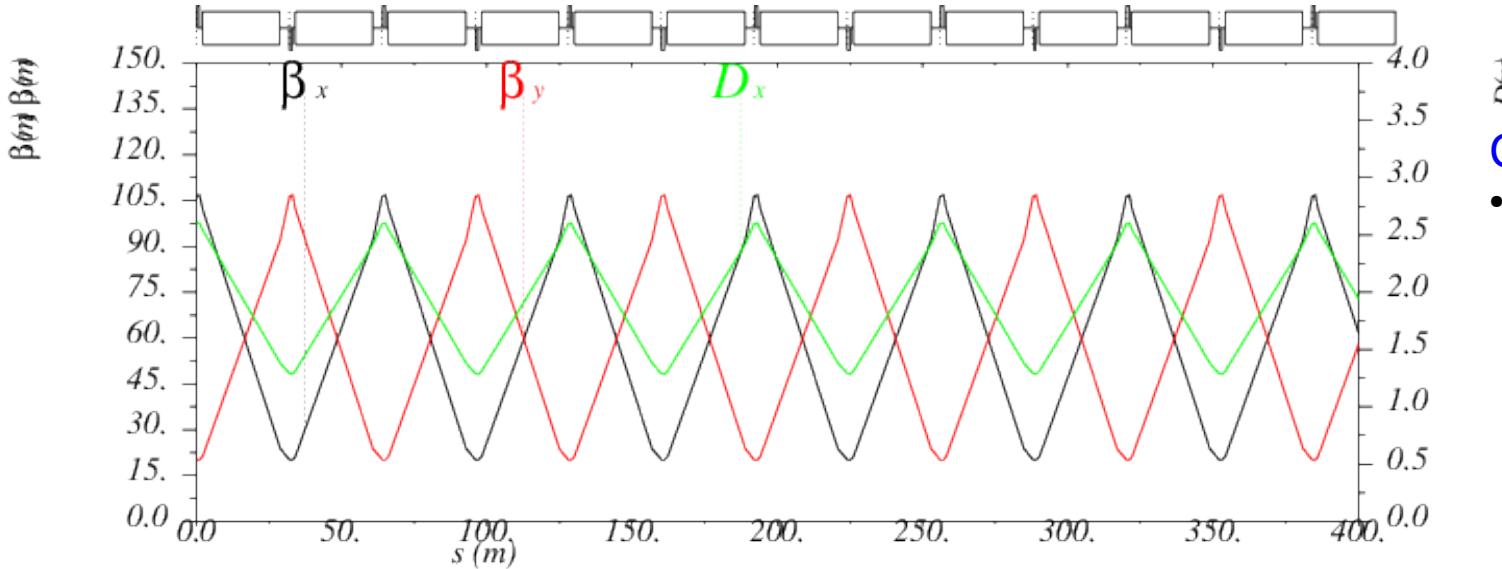
- β , α & γ are not free parameter but need to fulfill special periodic boundary condition:
 - $\beta(s+C) = \beta(s)$
 - $\alpha(s+C) = \alpha(s)$
 - $\gamma(s+C) = \gamma(s)$



One-pass Machine (LINAC/transferring lines):

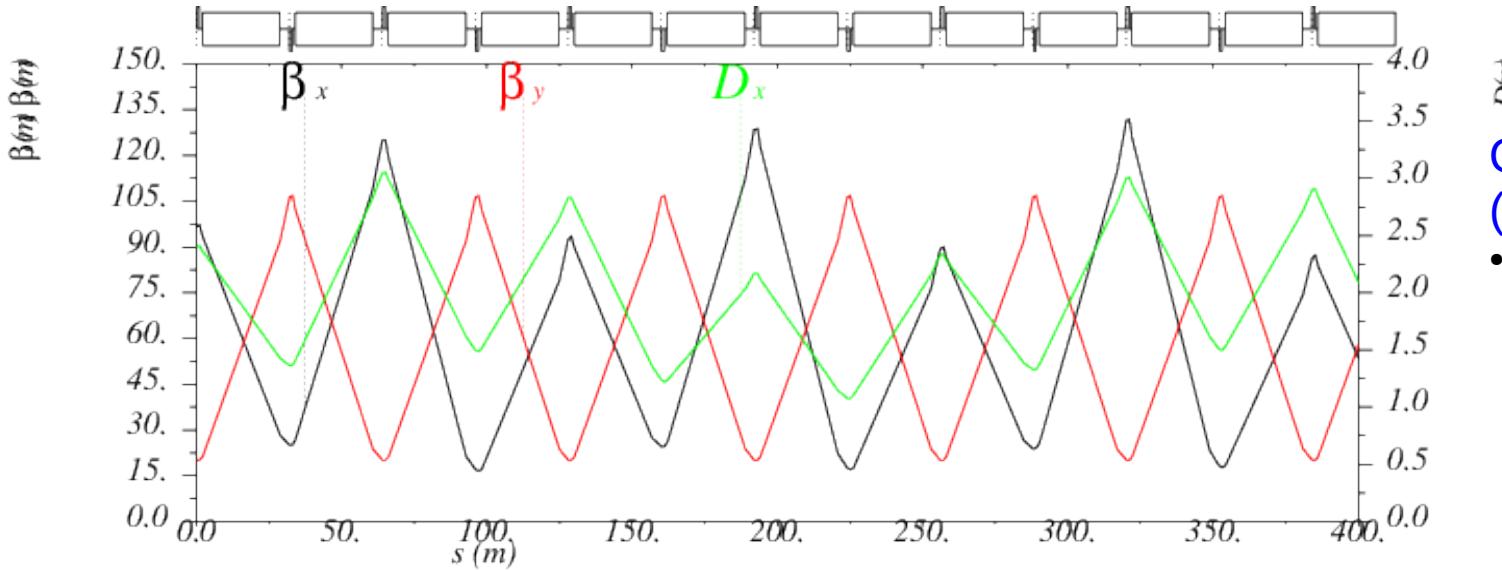
- Causality!
- Need to provide initial parameter for β , α & γ :
 - $\beta(s=0) = \beta_0$
 - $\alpha(s=0) = \alpha_0$
 - $\gamma(s=0) = \gamma_0$
- otherwise, trans. particle transfer the same

Difference: Transfer-Line (LINAC) vs. Ring II/II



Circular Machine (ring):

- as before



One-pass Machine
(LINAC/transfer-lines):

- 10% different initial conditions → β -beating
→ causes problems with matching and emittance preservation at injection (later more)

Most Simple Example – Drift Space

$$M_{drift} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$$

- Particle coordinates:

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_0$$

$$\begin{aligned} x(L) &= x_0 + L \cdot x'_0 \\ x'(L) &= x'_0 \end{aligned}$$

- Transformation of Twiss parameters

$$\begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix} = \begin{pmatrix} 1 & -2L & L^2 \\ 0 & 1 & -L \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_0$$

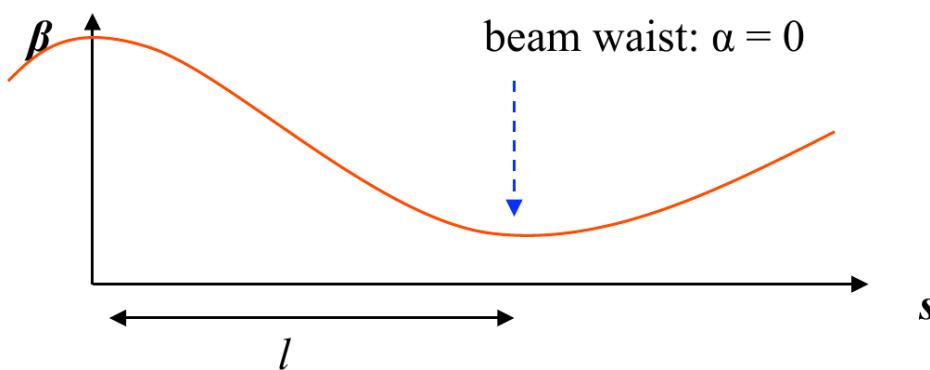
$$\beta(s) = \beta_0 - 2\alpha_0 \cdot L + \gamma_0 \cdot L^2$$

→ equation being important for
Low-beta insertions/final focus

- Stability?

$$\text{trace}(M) = 1 + 1 = 2$$

Most Simple Example – Drift Space II/II



$$(I) \beta(s) = \beta_0 - 2\alpha_0 \cdot L + \gamma_0 \cdot L^2$$

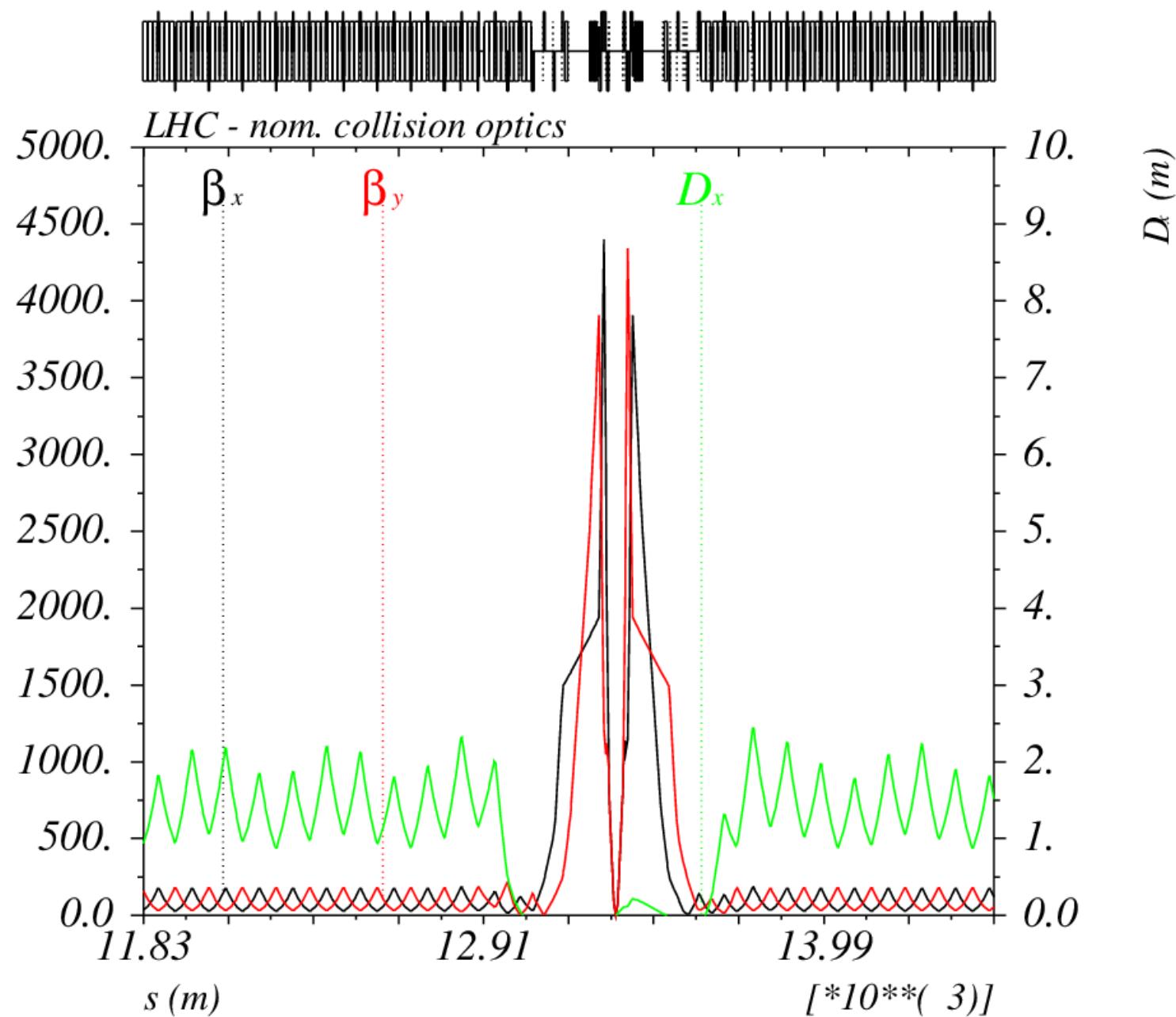
$$(II) \alpha(s) = \alpha_0 - L \cdot \gamma_0$$

- Beam size is smallest at $\alpha(s) = 0 \rightarrow \alpha_0 = \gamma_0 \cdot s$ $\rightarrow L = \alpha_0 / \gamma_0$
- Beam size at that point: $\gamma(L) = \gamma_0$ & $\alpha(L) = 0$ $\rightarrow \beta(L) = 1 / \gamma_0$
- Inserting in (I):

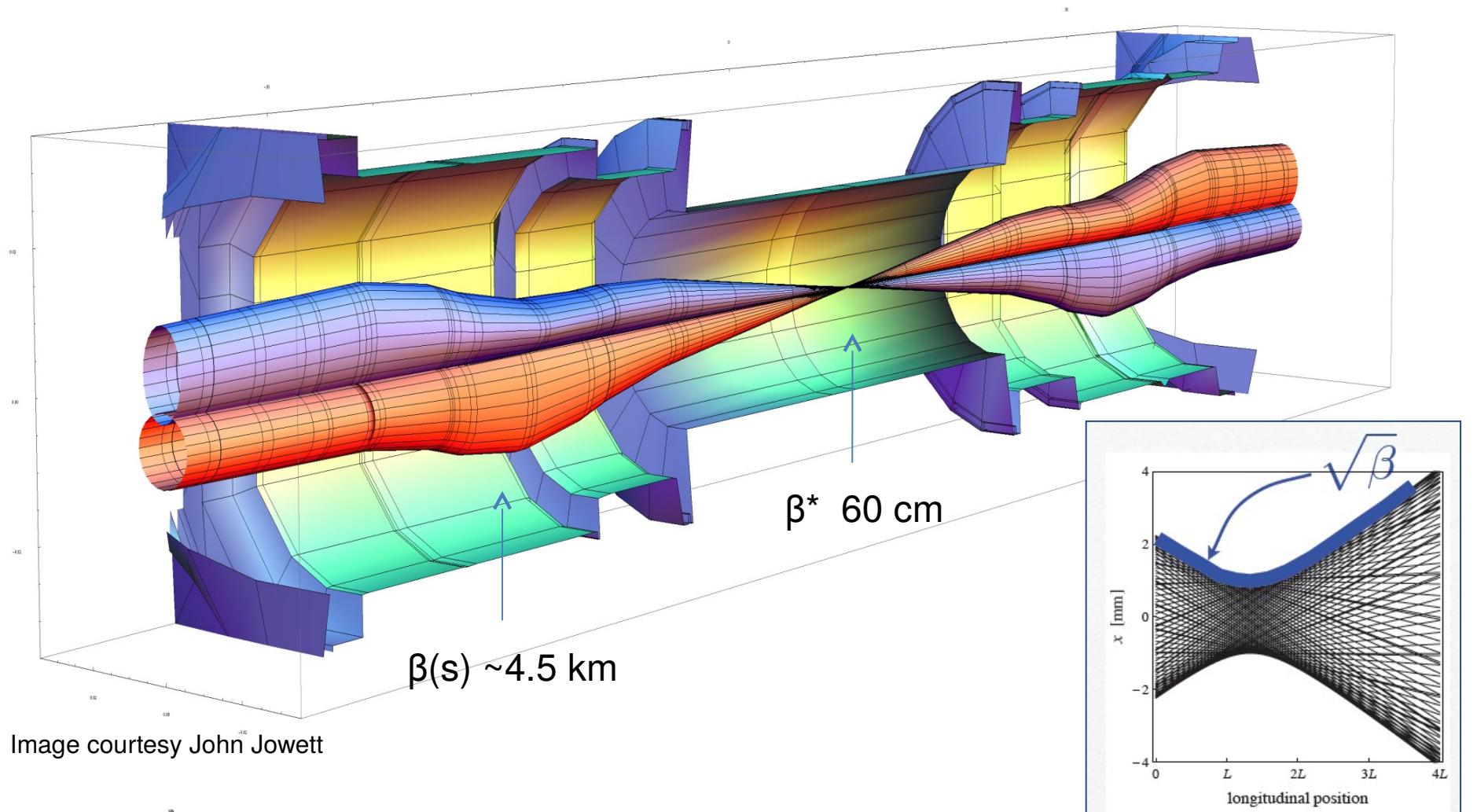
$$\beta(s) = \beta_0 + \frac{s^2}{\beta_0}$$

- Phase advance: $\Delta\mu(s) := \int_{-L}^{+L} \frac{1}{\beta(s')} ds' \approx \pi$

Example LHC



LHC: Squeezing in ATLAS – Beam Envelope



- Main take-away: need to magnify the beam in the focusing elements before being able to focus on a tiny spot coordinates (N.B. equally applies for focus on targets)
 - aperture requirements/constraints in focusing quads → don't focus too much

Reference: FoDo Lattice – Useful Equations

- FoDo cell transfer matrix (\rightarrow tutorial)

$$M_{FoDo} = \begin{pmatrix} 1 - \frac{L^2}{2f^2} & L\left(1 + \frac{L}{2f}\right) \\ \left(\frac{L^2}{2f^3} - \frac{L}{f^2}\right) & 1 - \frac{L^2}{2f^2} \end{pmatrix}$$

- Phase advance per cell

- N.B. also correct for non-FoDo cells

$$\cos \mu_{cell} = \frac{1}{2} \operatorname{trace}(M)$$

- Stability criterion $|\operatorname{trace}(M)| < 2$

$$|f| \approx \left| \frac{1}{kl_q} \right| > \frac{L}{4} !$$

- For stability the focal length has to be larger than a quarter of the cell length \rightarrow don't focus too strong!

for FODO:

$$f = \pm \frac{L}{4 \sin \frac{\mu}{2}} = (k l_q)^{-1}$$

$$\mu_{cell} = 2 \arcsin \left(\frac{L}{4f} \right)$$

$$\beta^\pm = \frac{L(1 \pm \sin \frac{\mu}{2})}{\sin \frac{\mu}{2}}$$

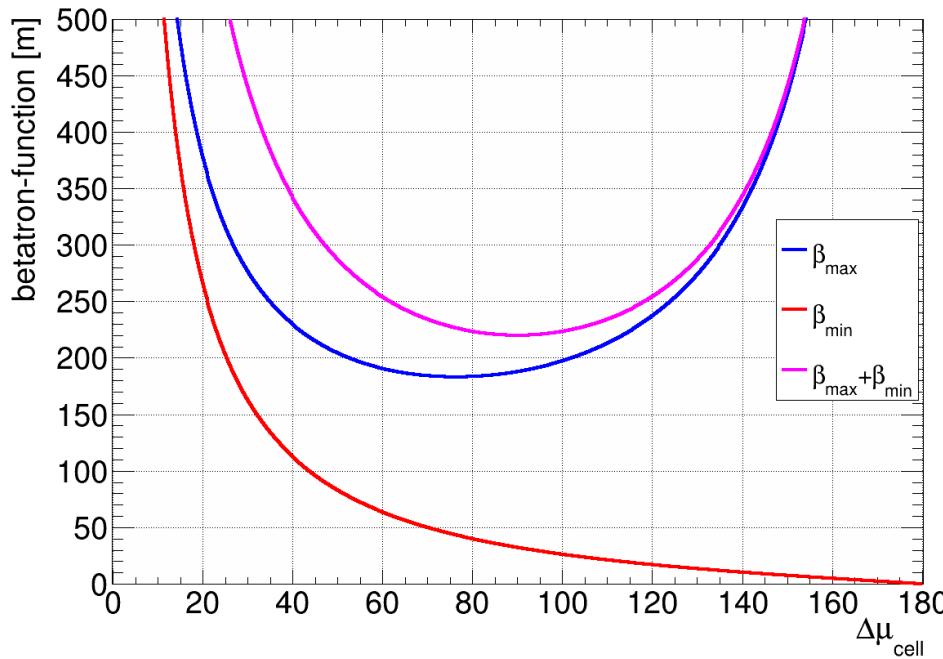
$$\alpha^\pm = \frac{\mp 1 - \sin \frac{\mu}{2}}{\cos \frac{\mu}{2}}$$

$$D^\pm = \frac{L \varphi(1 \pm \frac{1}{2} \sin \frac{\mu}{2})}{4 \sin^2 \frac{\mu}{2}}$$

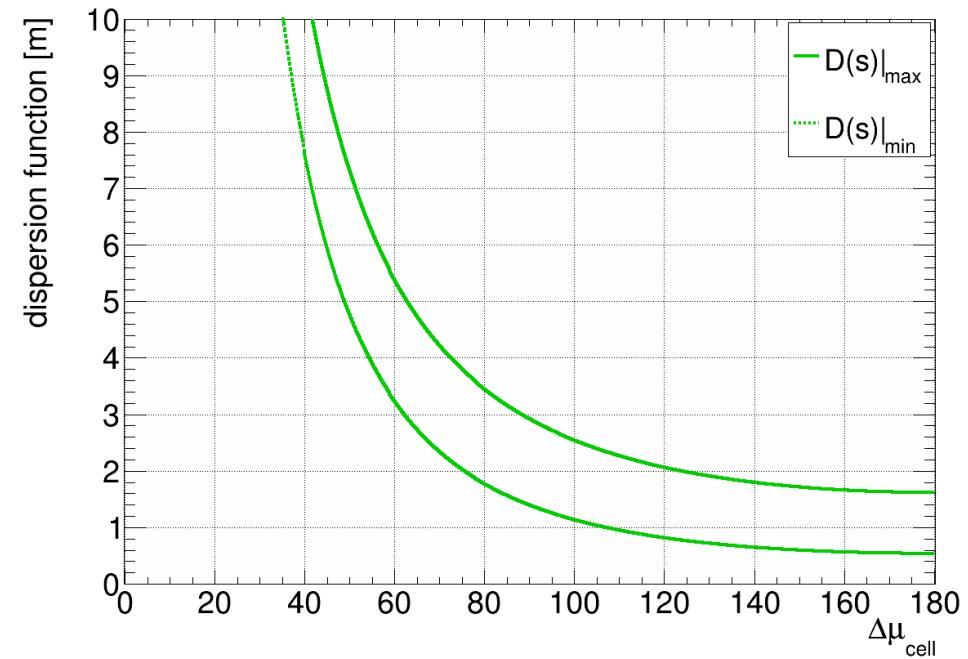
$$\xi_{FODO} = -\frac{1}{\pi} \tan \frac{\mu}{2}$$

Recap: FoDo Lattice – Summary

- Beam size optimisation:
 - Hadrons prefer $\mu=90^\circ$
 - minimises: $\epsilon_x \approx \epsilon_y$ & $a = \sqrt{\sigma_x^2 + \sigma_y^2} \sim \beta_x + \beta_y$
 - Leptons prefer $\mu \sim 137^\circ$
 - vertical emittance very small
→ optimise mainly β_x & $D_x|_{\max}$
- Dispersion minimisation



→ don't focus too strong!



Down the rabbit hole ...



... and welcome to Wonderland!

Machine Imperfections

- It is not possible to construct a perfect machine.
 - Magnets can have imperfections
 - The alignment in the machine has non-zero tolerances
 - etc...
- So, we have to ask ourselves:
 - What will happen to the betatron oscillation due to the different field errors
 - need to consider errors in dipoles, quadrupoles, sextupoles, etc...
- We will have a look at the beam behaviour as a function of ‘Q’
- How is it influenced by these resonant conditions?

Tune Diagnostics - Primer

- Importance of tune:
 - defines beam life-time
 - strong impact on beam physics experiments:

Laymen/Musician's view

(Beethoven's 5th):

in tune (**good**):



off-tune (**bad**):



Audience will leave the concert

↔ Beam will leave the vacuum pipe

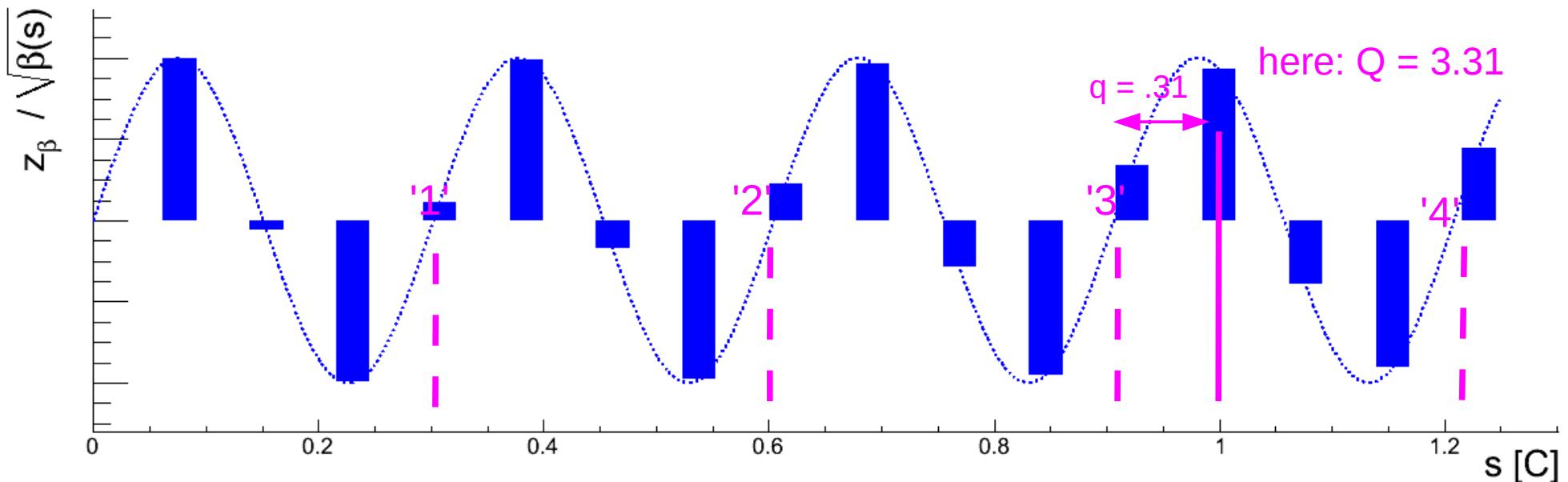


"I don't think we've quite repeated the experiment -
last time we did it, the glass gave out a middle 'c'."

Recap: Transverse Beam Dynamics

- Free Betatron Oscillations:

$$z_\beta(s) = \sqrt{\epsilon_i \beta(s)} \cdot \sin(\mu(s) + \phi_i)$$



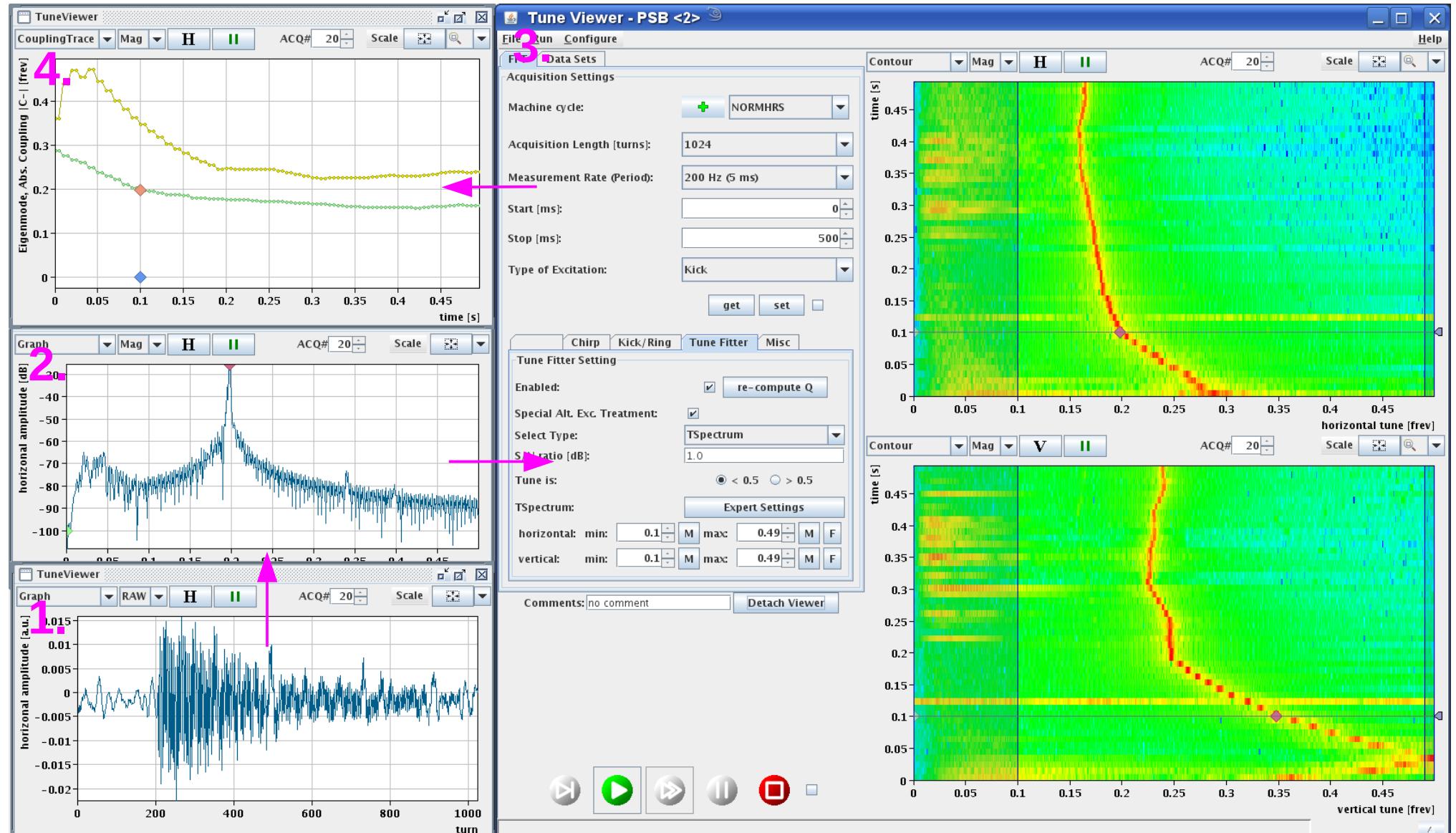
- Betatron Phase Advance: $\Delta\mu(s)$

Tune defined as betatron phase advance over one turn:

$$Q := \frac{1}{2\pi} [\mu(C) - \mu(0)]$$

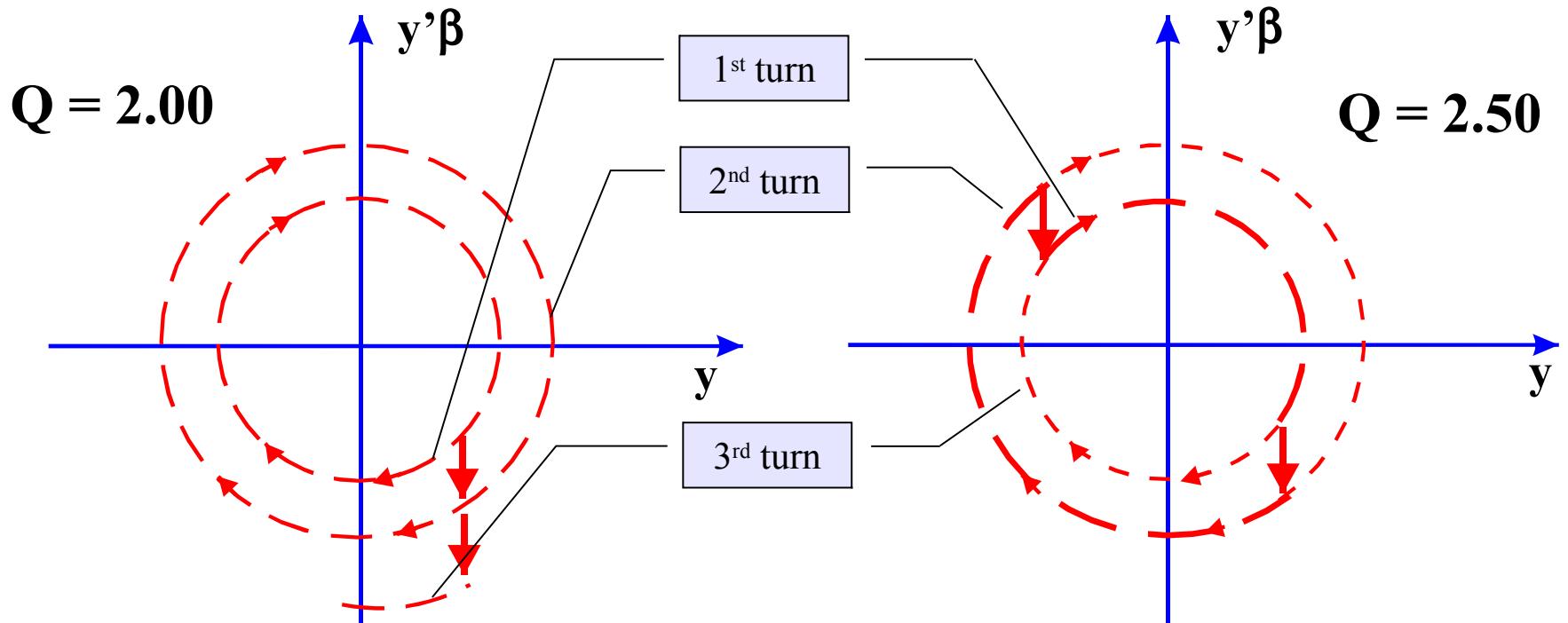
common: $Q = \underbrace{Q_{int}}_{\text{integer tune}} + \underbrace{q_{frac}}_{\text{fractional tune}}$

Example: BBQ Spectra CERN-PSB, $f_{rev} \approx 2$ MHz



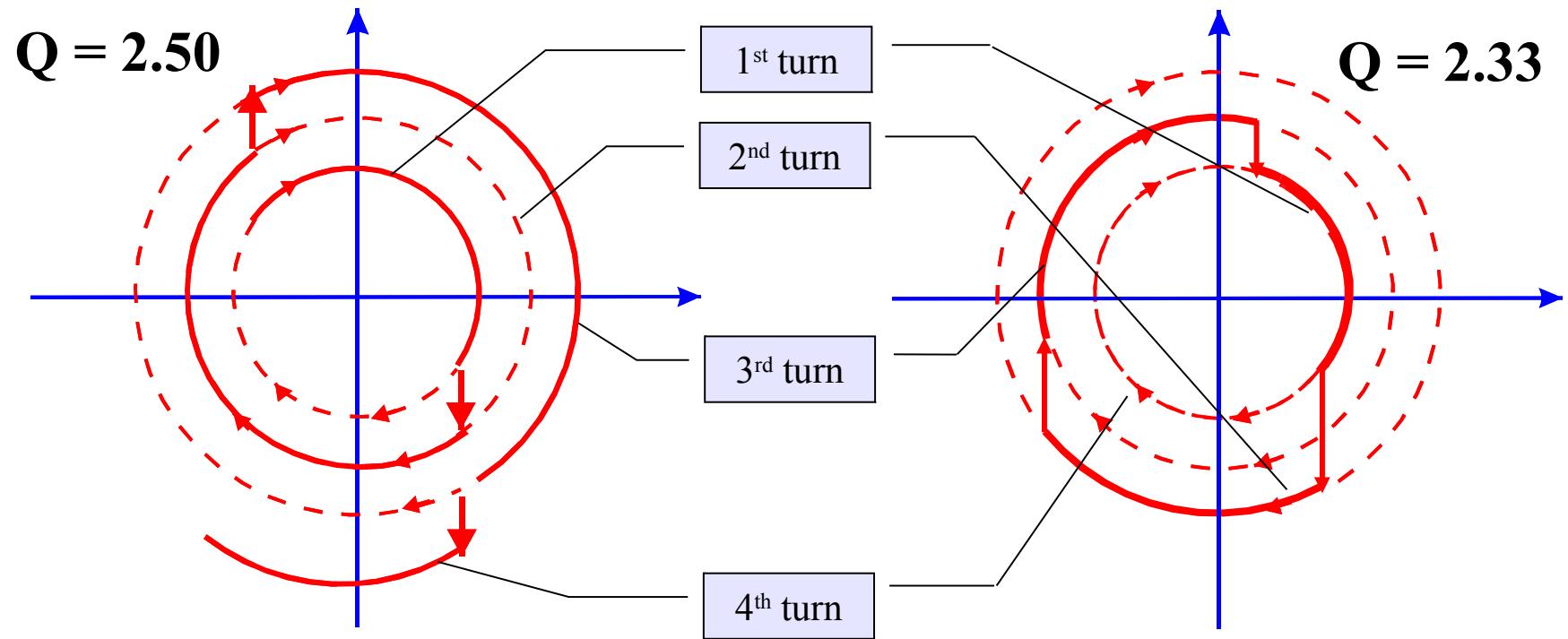
- BBQ → fast ADC → FPGA based digital signal processing chain, FFTs @ 500 – 1 kHz!

Dipole (deflection independent of position)



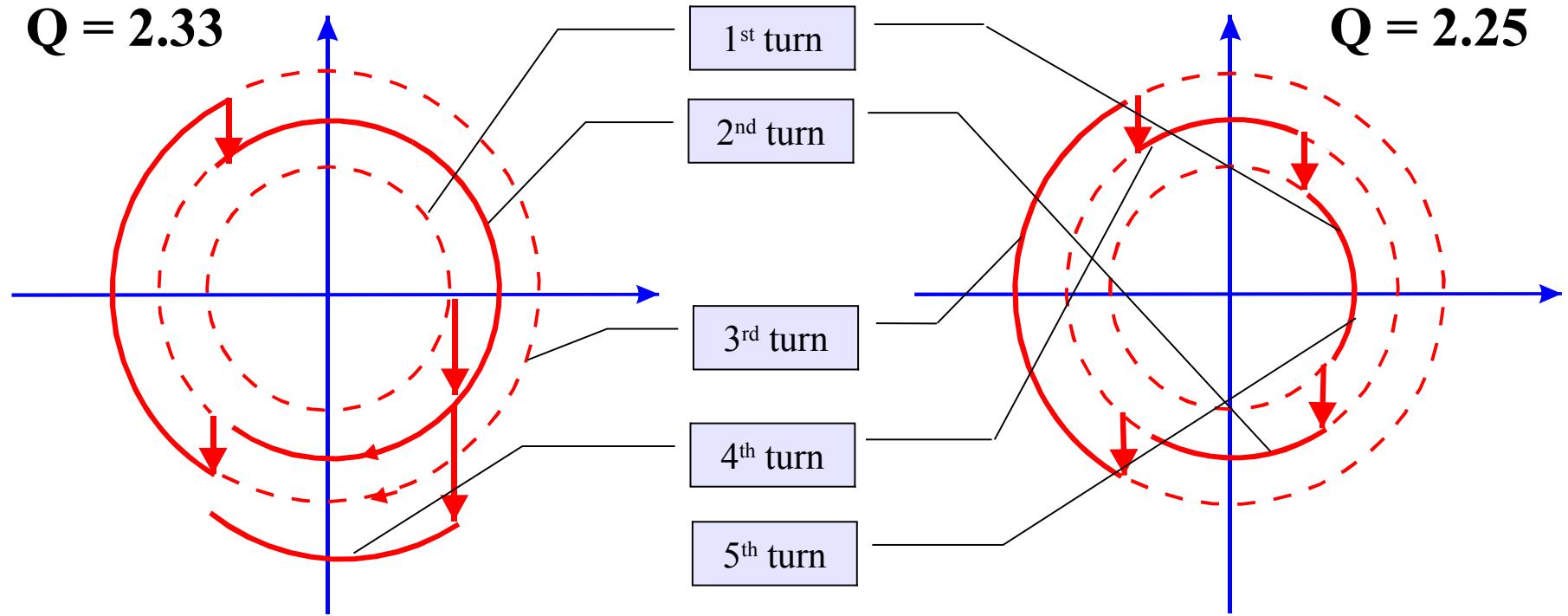
- For $Q = 2.00$: Oscillation induced by the dipole kick grows on each turn and the particle is lost (1st order resonance $Q = 2$).
- For $Q = 2.50$: Oscillation is cancelled out every second turn, and therefore the particle motion is stable.

Quadrupole (deflection \propto position)



- For $Q = 2.50$: Oscillation induced by the quadrupole kick grows on each turn and the particle is lost (2nd order resonance $2Q = 5$)
- For $Q = 2.33$: Oscillation is cancelled out every third turn, and therefore the particle motion is stable.

Sextupole (deflection \propto position²)



- For $Q = 2.33$: Oscillation induced by the **sextupole kick** grows on each turn and the particle is lost (**3rd order resonance $3Q = 7$**)
- For $Q = 2.25$: Oscillation is cancelled out **every fourth turn**, and therefore the particle **motion is stable**.

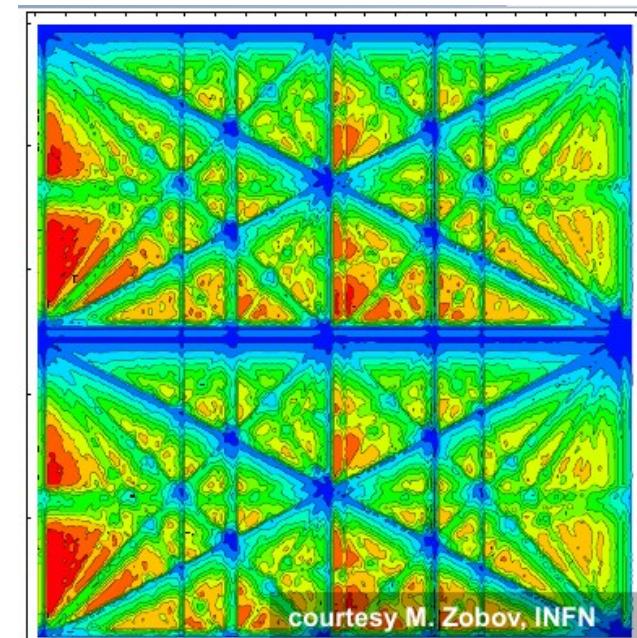
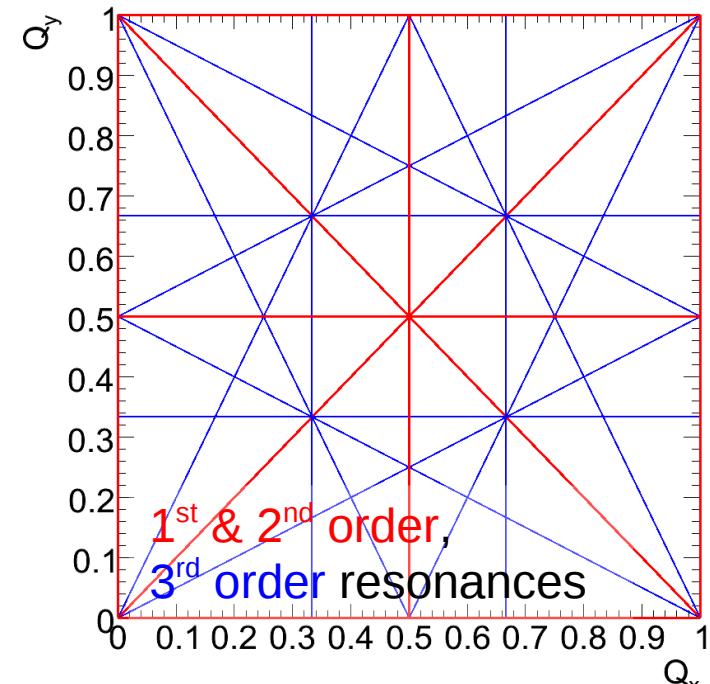
Tune Stability Requirements & Constraints

- Unstable particle motion if:

$$p = m \cdot Q_x + n \cdot Q_y \quad \wedge \quad m, n, p \in \mathbb{Z}$$

- similar relation also in between Q_x & Q_s
(important for lepton accelerators)
- Resonance order: $O = |m| + |n|$
 - Lepton accelerator: avoid up to ~ 3
 - Hadron colliders:
 - negligible synchrotron radiation damping
 - need often to avoid up to the 12th order

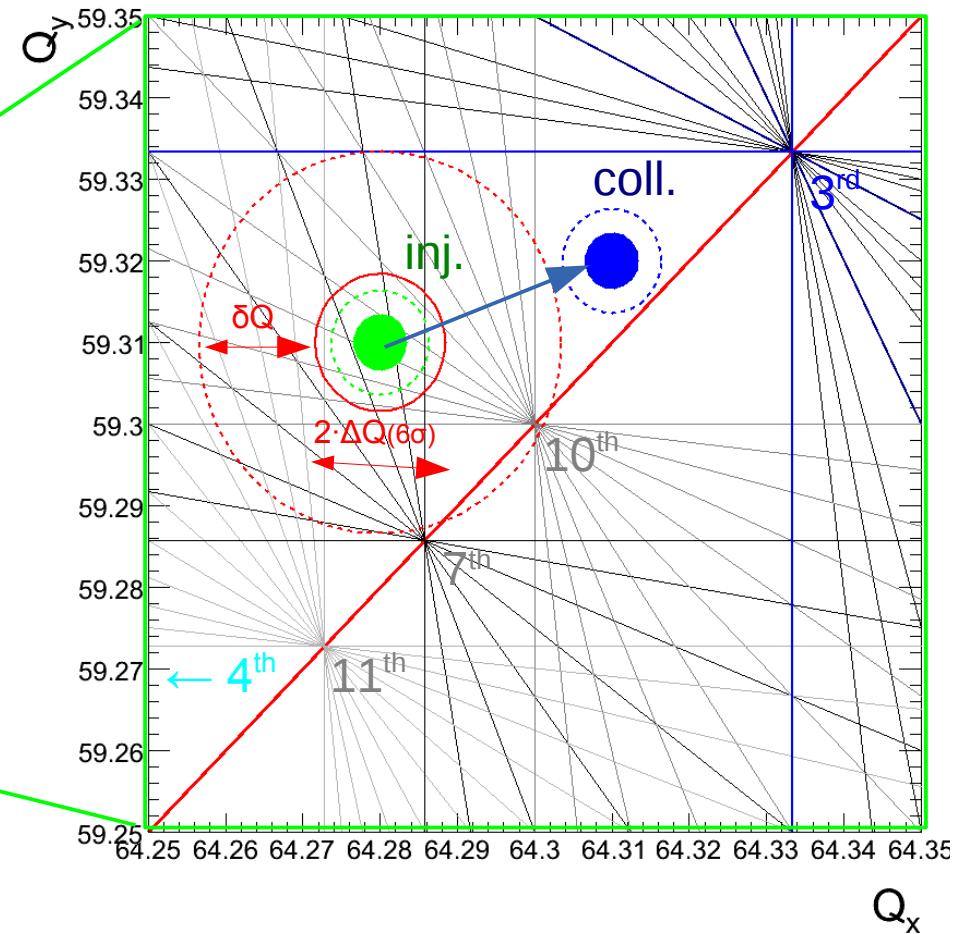
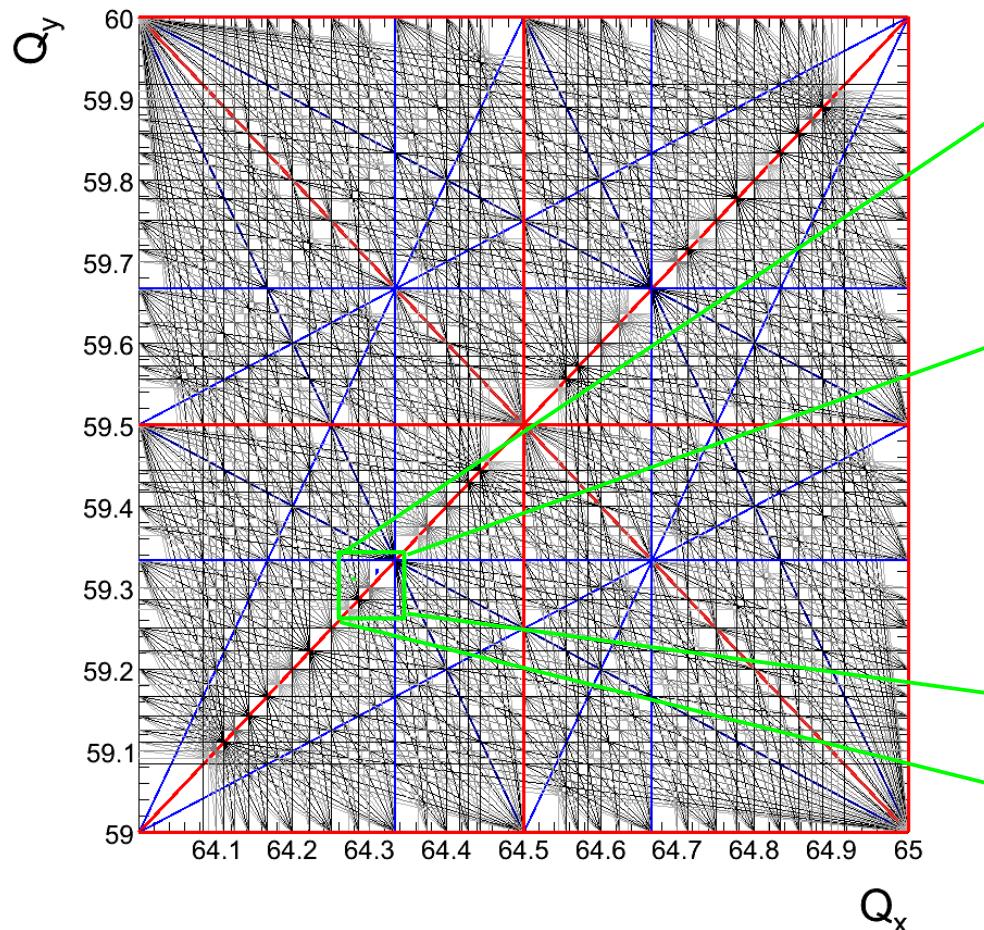
“Hadron beams are like elephants –
treat them bad and they'll never forgive you!”



courtesy M. Zobov, INFN

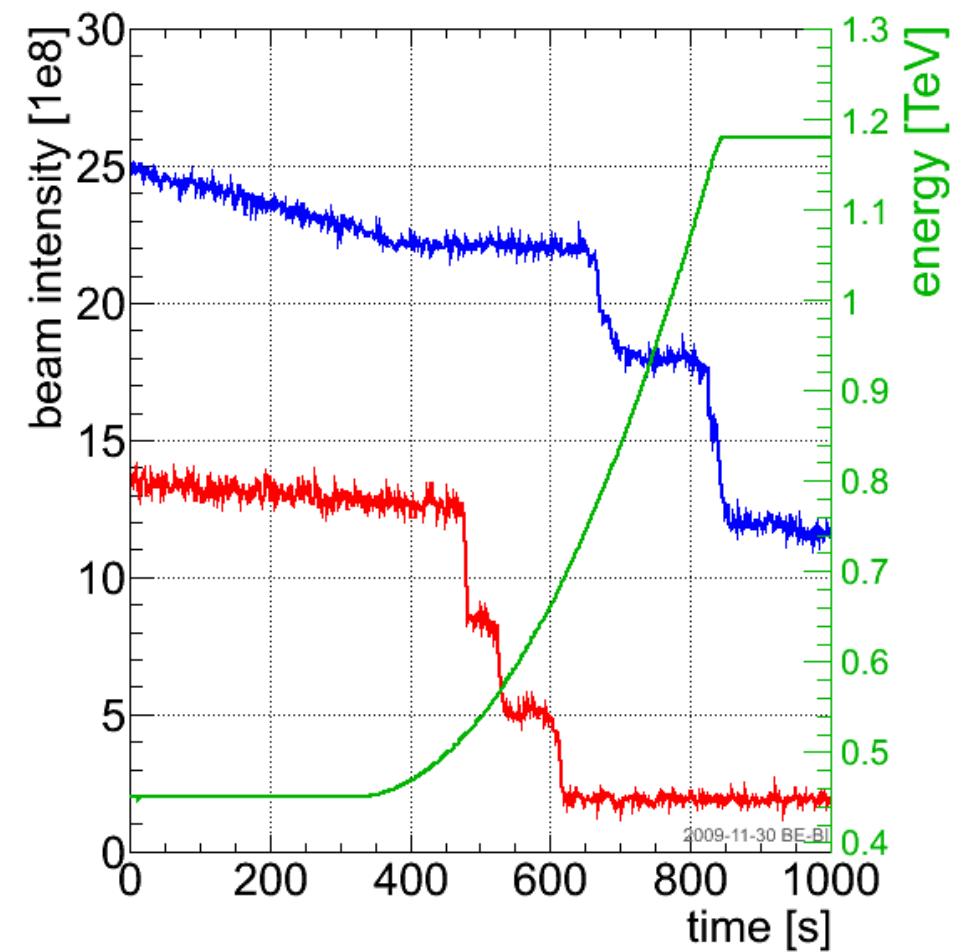
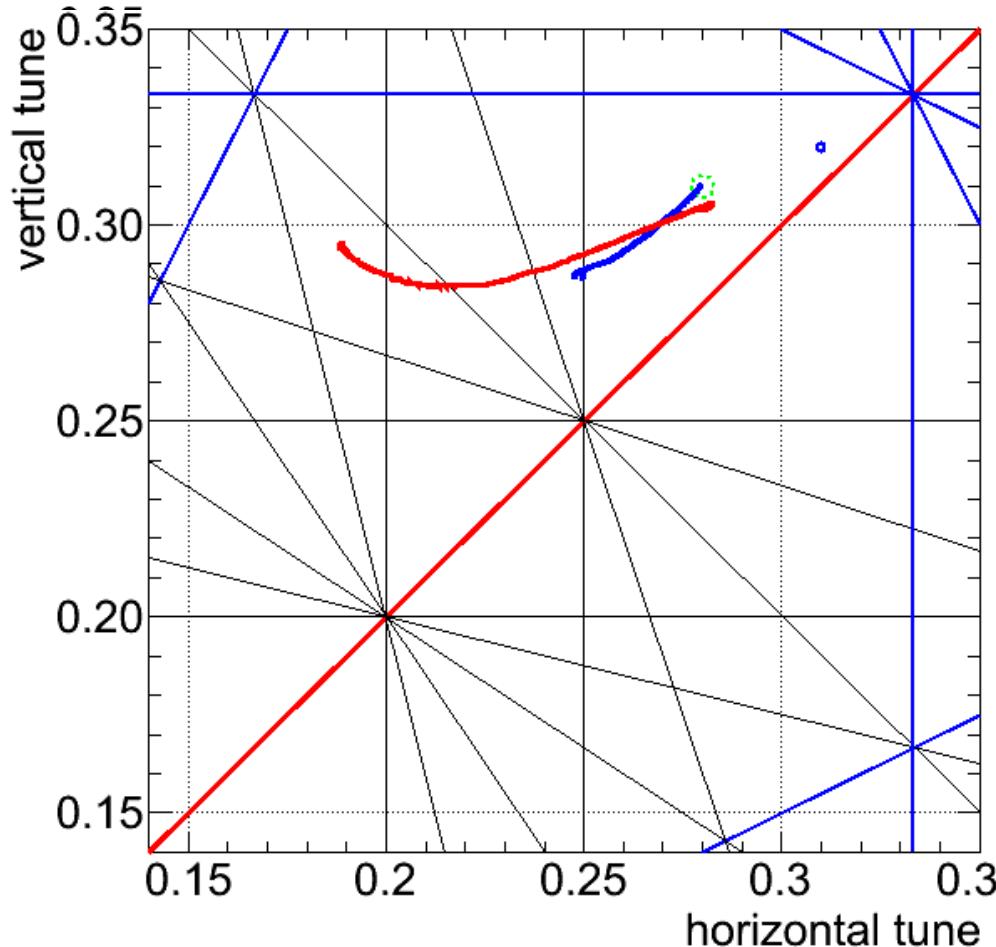
Tune Stability Requirements & Constraints

- Example LHC: stability requirement: $\Delta Q \approx 0.001$ vs. exp. drifts ~ 0.06

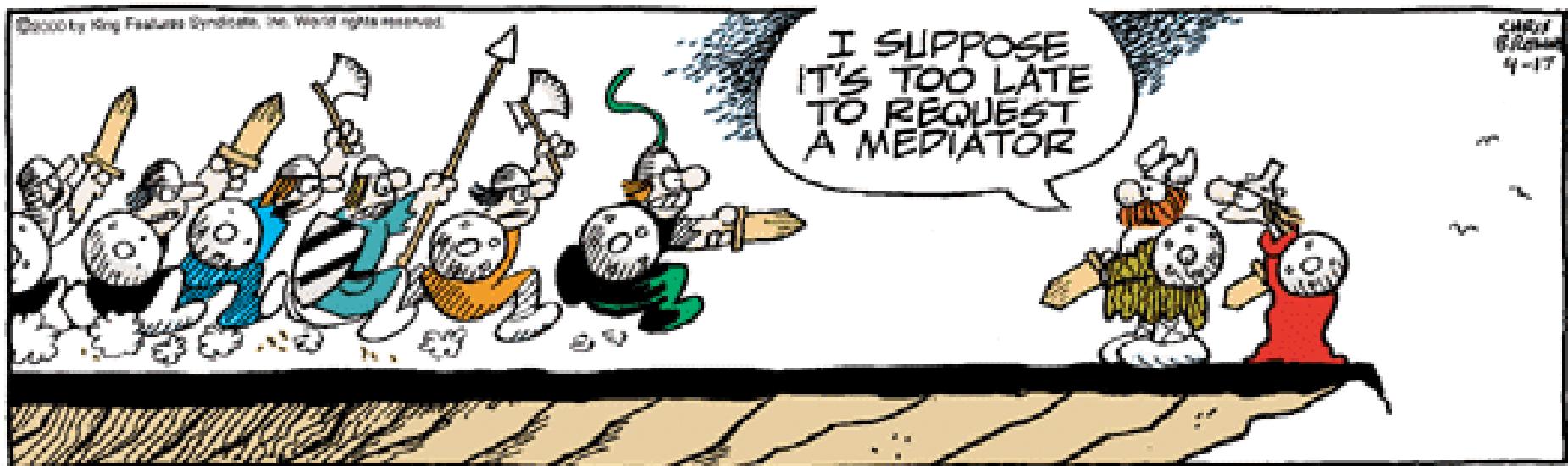


- Storage rings have even a tighter requirement on control of Q
 - typically: the longer the storage time \leftrightarrow better the control of Q (& Q', ...)

Example: Tune During LHC Ramp



That's all – Questions?

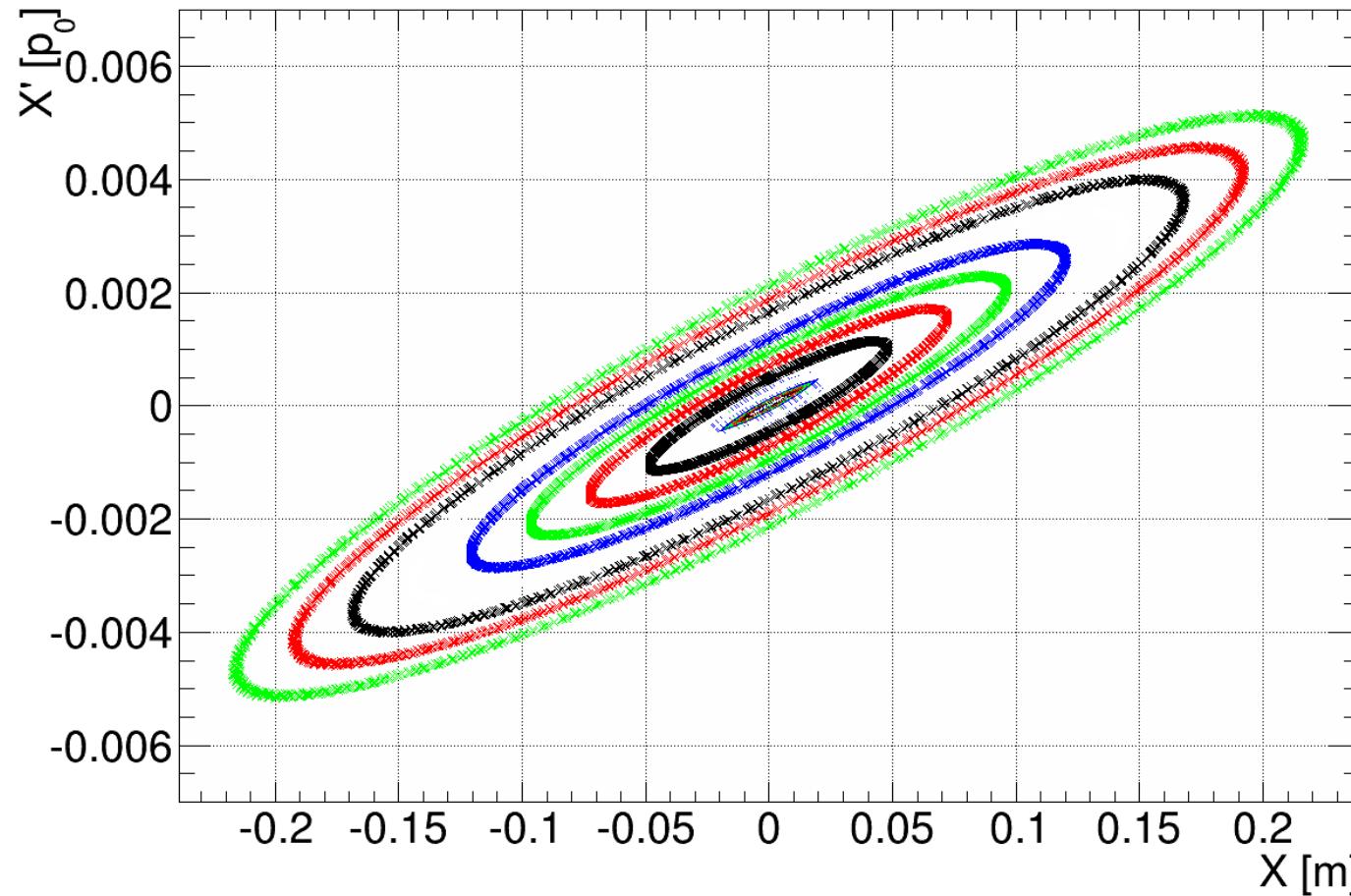


Outline

- Part I – Linear Beam Dynamics & Hill's Equation
 - Periodic Focusing System in Circular Accelerators
 - Phase Space Ellipse
 - Emittance & Acceptance
 - Machine Imperfections
 - Betatron Tune & Beam Stability
- Part II – Non-Linear Dynamics & Injection/Extraction
 - Non-linear dynamics:
 - limits of stable motion – Separatrix
 - Dispersion & Chromaticity
 - Space charge effects
 - Injection & Extraction:
 - Fast extraction, Multiturn Injection (phase-space painting)
 - Basics of resonance-, KO-extraction

Phase-Space-Plot – Linear Optics

- Lattice with only dipoles and quadrupoles
→ particles describe ellipses in phase-space plots



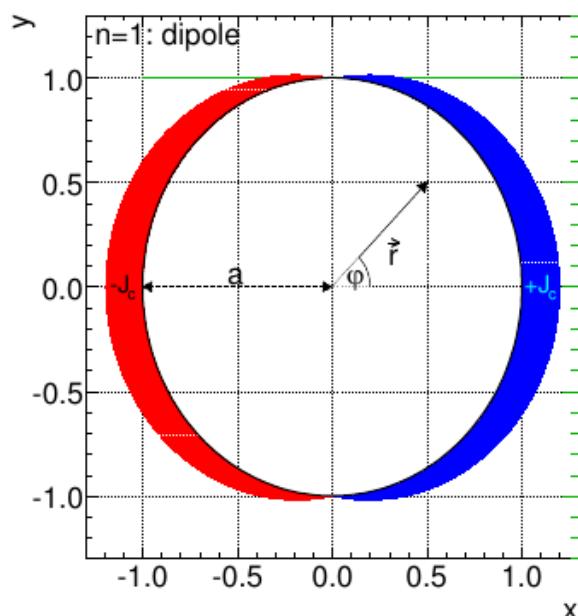
- Linear optics (dipoles & quads only)
→ particle motion independent on particle amplitude

Magnets – Basic Arsenal

- Hill's Equation

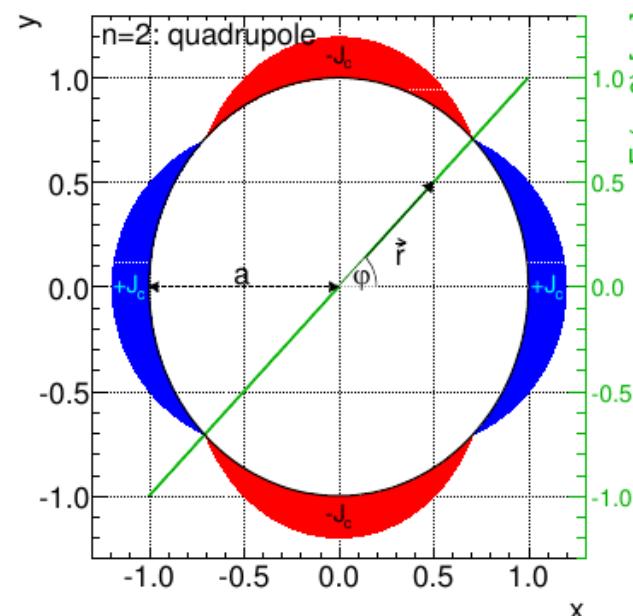
$$z'' + k(s) \cdot z = f(s, t)$$

Dipole:
constant field



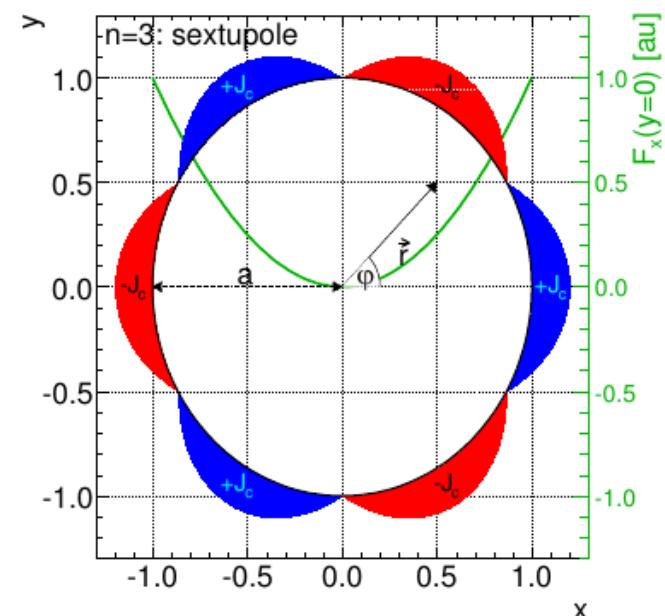
→ defines circular
trajectory/orbit

Quadrupole:
linear field



→ defines transverse
focusing and
periodic betatron
oscillation

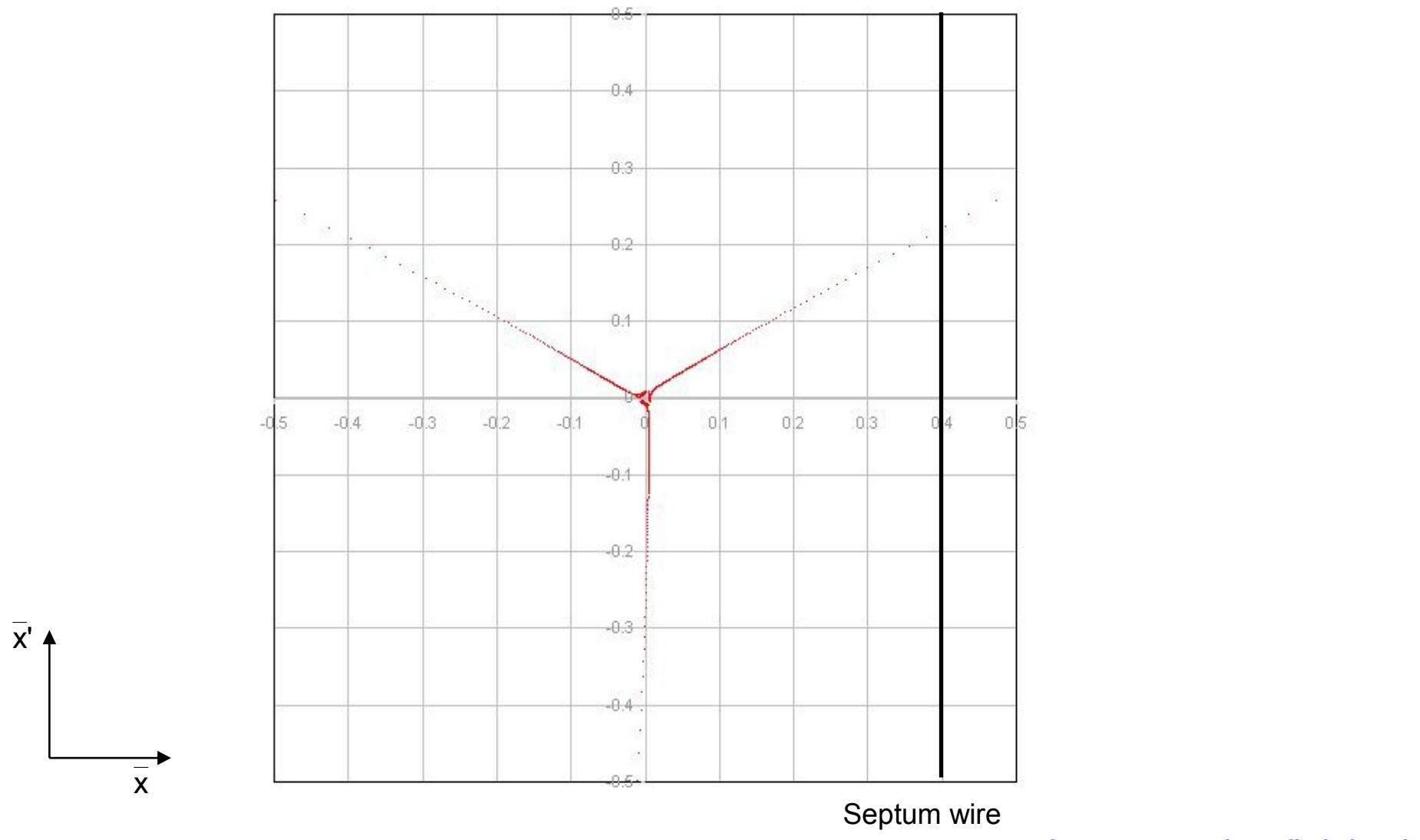
Sextupole:
quadratic field



→ corrects for linear
/chromatic effects
→ defines dynamic aperture

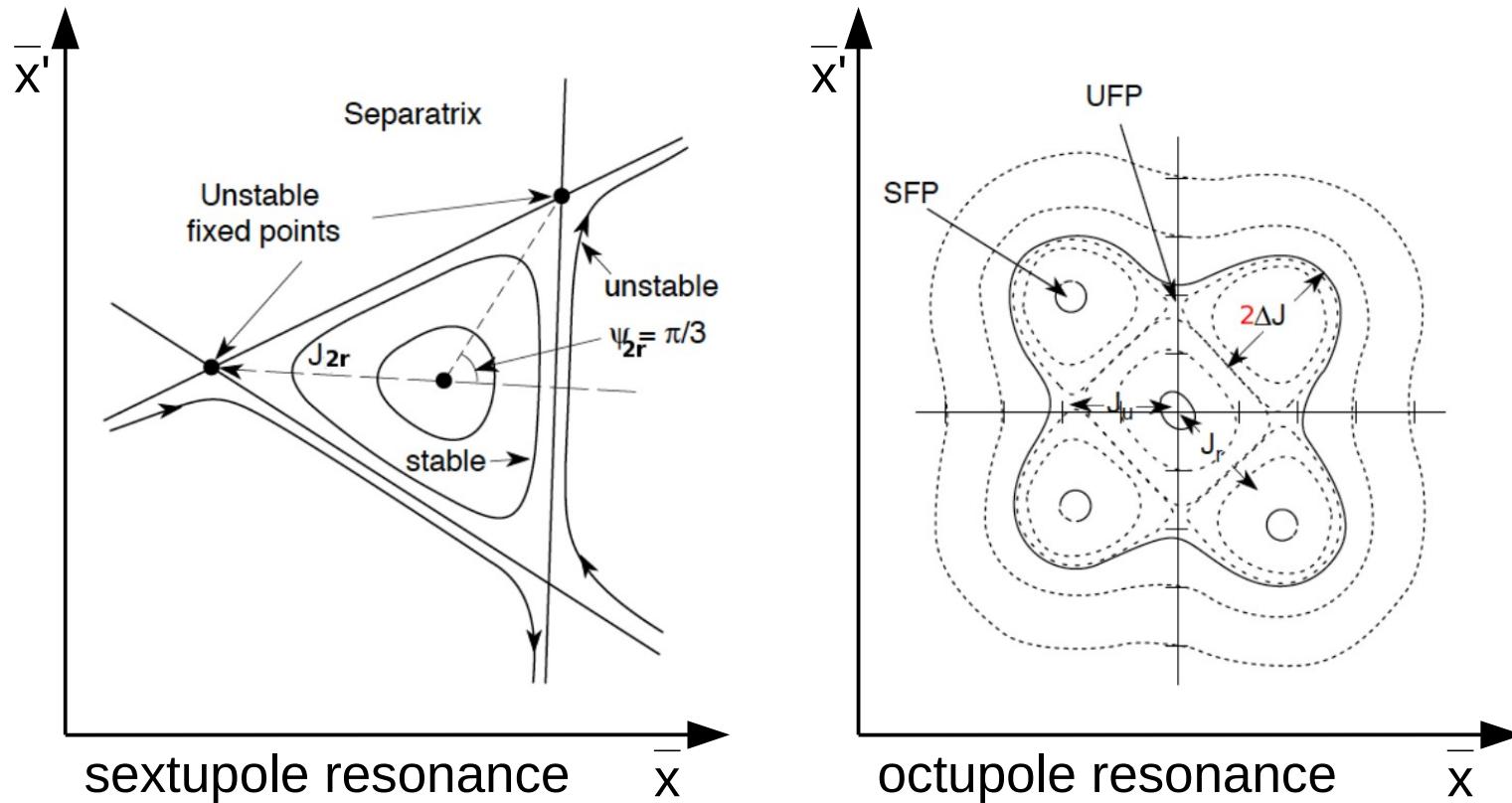
Third-order resonant extraction

- Effect of strong sextupoles on the particle motion:



Phase Space III/II

- What happens if you add strong non-linear sextupole or octupole-components
 - 'separatrix' (aka. 'dynamic aperture') being the border between stable and unstable beam motion regime

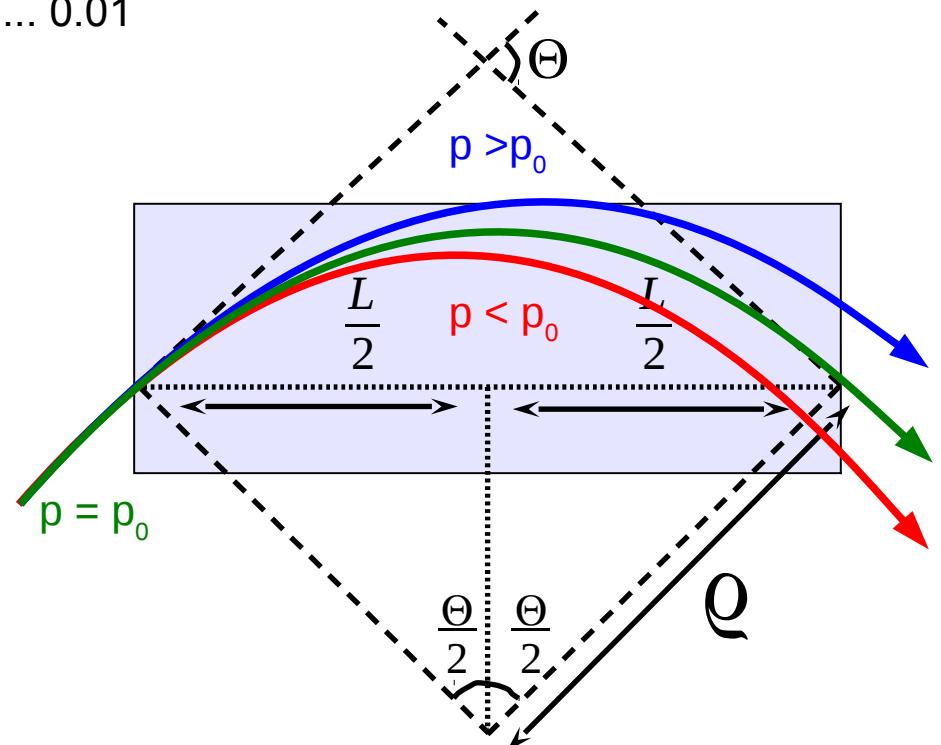


- You typically want to avoid this, but this also builds the basis of the slow resonant extraction technique

Dispersion

- Not all circulating article have the same energy
 - finite momentum spread $\Delta p/p$ typically in the range of
 - cooled beams $\Delta p/p < 10^{-4}$
 - low intensity beams $\Delta p/p \sim 10^{-4}$
 - high intensity beams $\Delta p/p \sim 10^{-3} \dots 0.01$
- Causes (small) energy dependent deflections
 - Remember dipole deflection relation:

$$\Theta = \frac{1}{(BQ)} \cdot LB = \frac{q}{p} \cdot LB$$



Dispersion

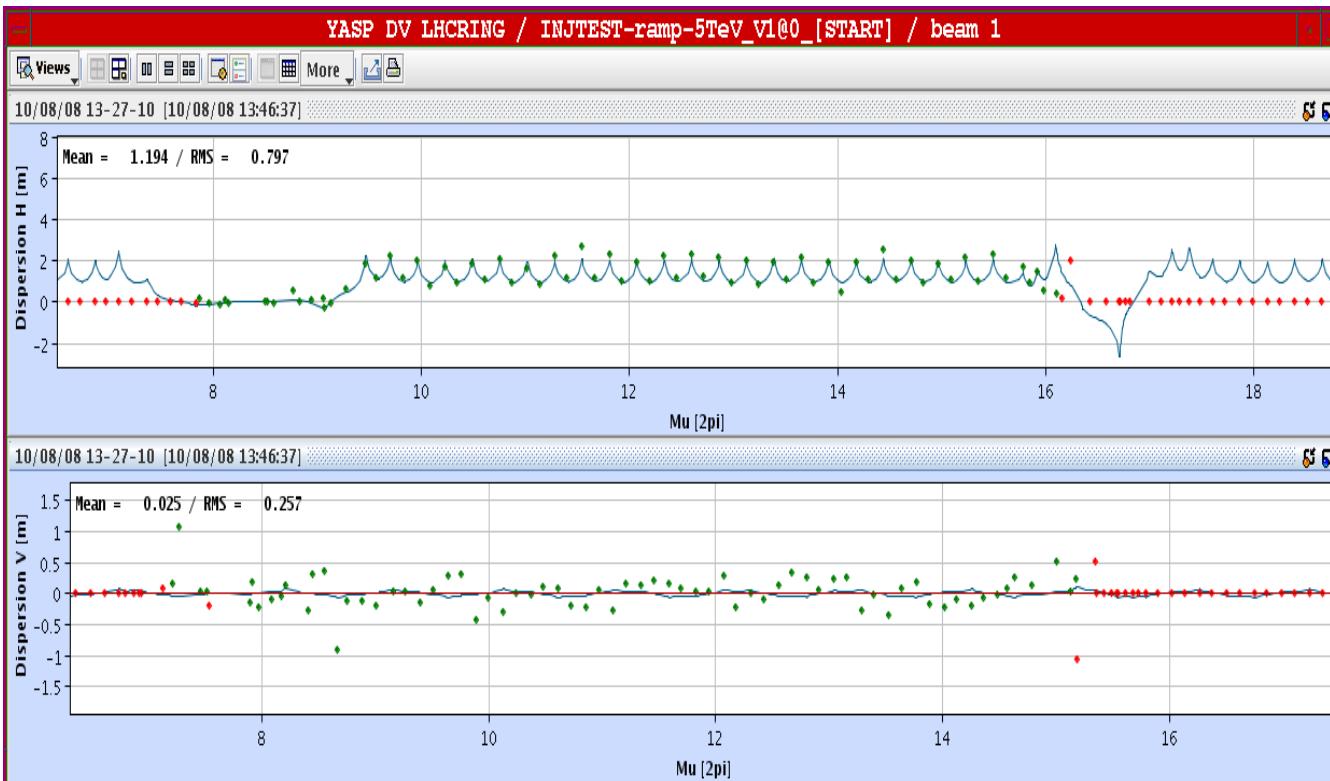
- Recap: Hill's equation:

$$z'' + K(s) \cdot z = f(s, t)$$

$$K(s) = \underbrace{\left(\frac{q}{p} B_{dipole} \right)^2}_{\text{weak focusing: } \frac{1}{\rho^2}} - \underbrace{\frac{q}{p} \frac{\partial B_y}{\partial x}}_{\text{strong focusing: } k(s)}$$

- Yields the more general (linear) solution:

$$z(s) = \underbrace{z_{co}(s)}_{\text{closed orbit}} + \underbrace{D(s) \cdot \frac{\Delta p}{p_0}}_{\text{dispersion orbit}} + \underbrace{z_\beta(s)}_{\text{betatron oscillations}}$$

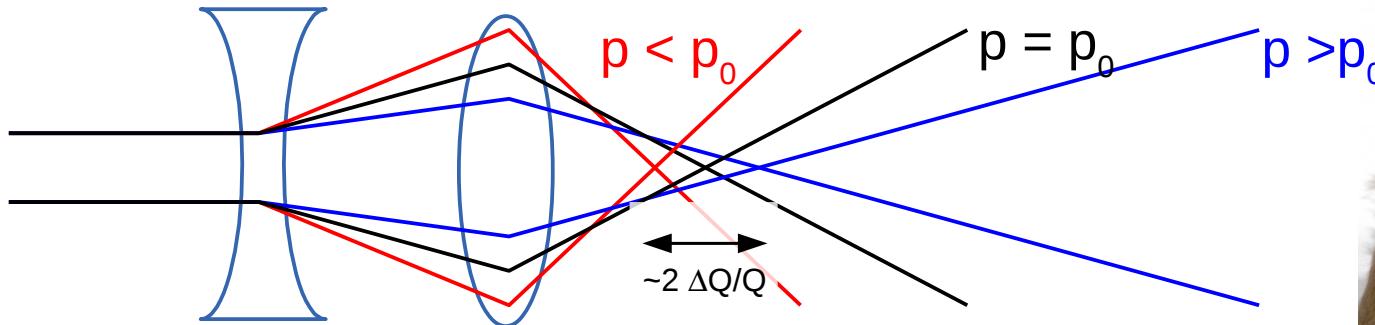


- modifies eff. beam size:
 - N.B. important for high-intensity primary beams

$$\sigma = \sqrt{\epsilon \beta + D^2(s) \left(\frac{\Delta p}{p} \right)^2}$$

Beam Chromaticity - Primer

- Light optics analogue: chromatic error



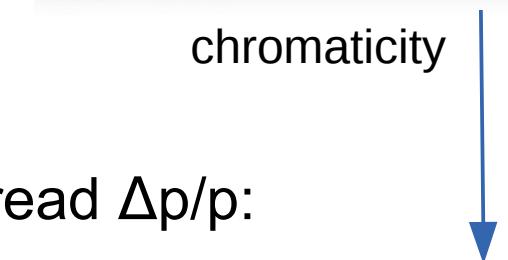
$$k(s) = \frac{q}{p} \frac{\partial B}{\partial x}$$



- Tune spread $\Delta Q/Q$ dependence on momentum spread $\Delta p/p$:

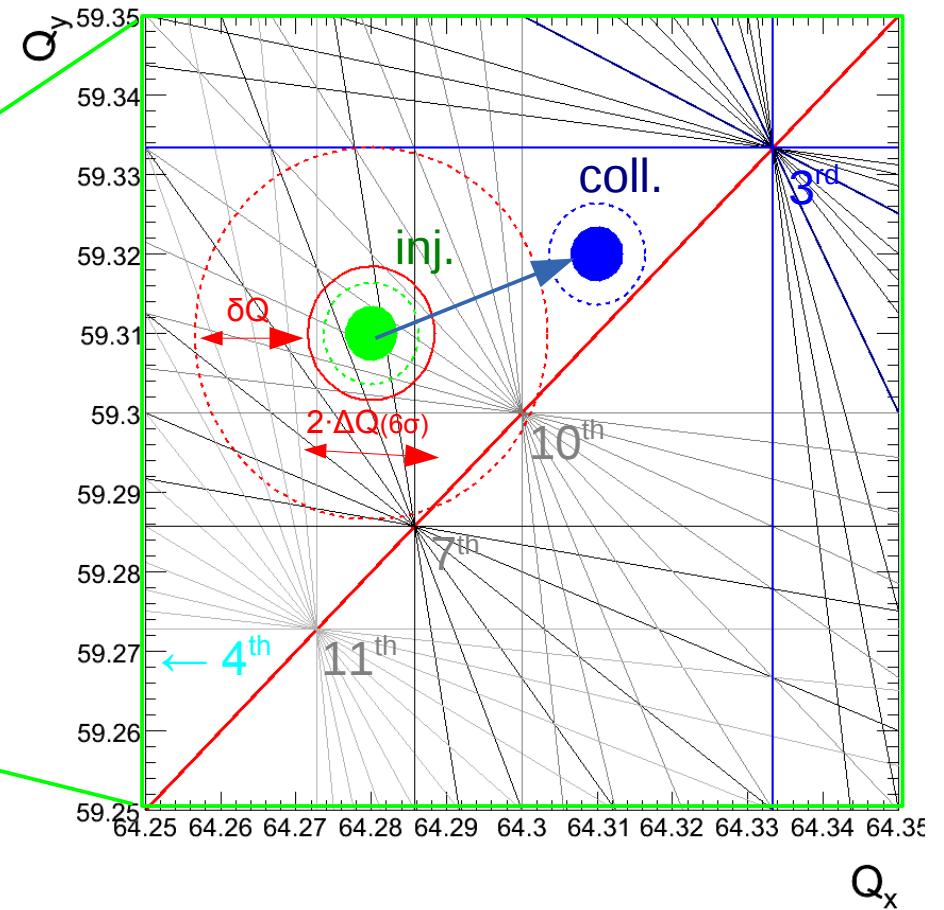
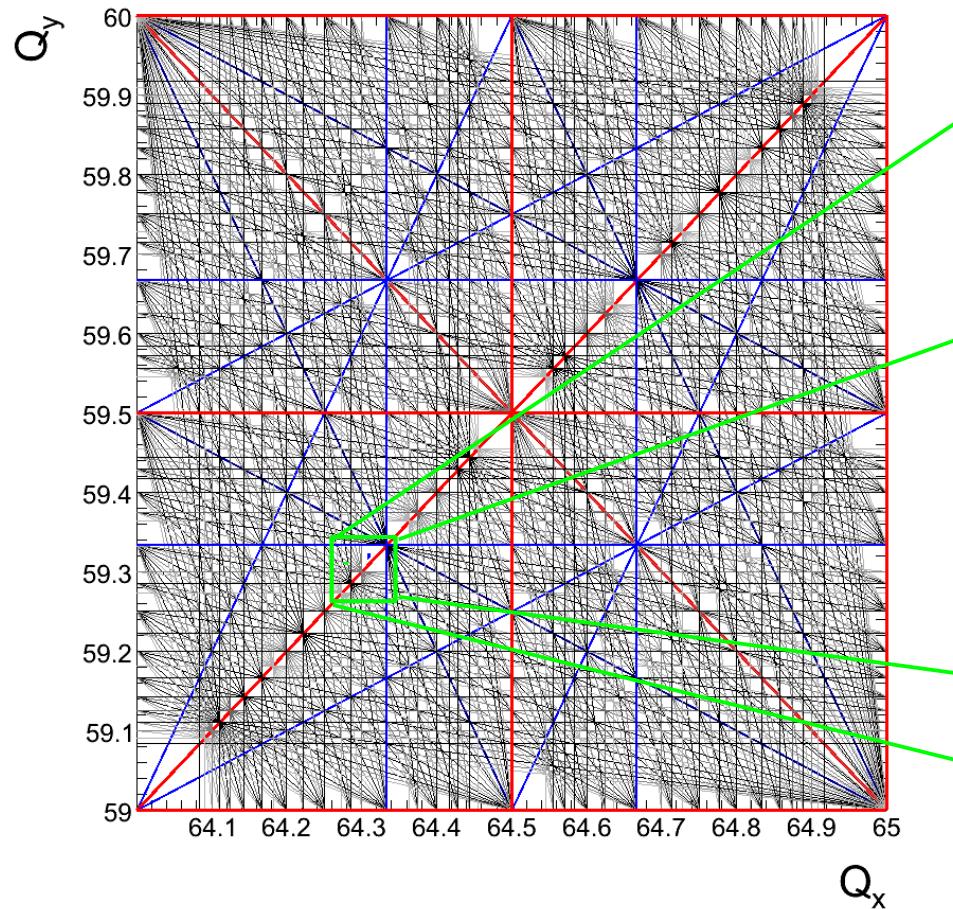
$$\Delta Q := Q' \cdot \frac{\Delta p}{p} \quad \text{or:} \quad \frac{\Delta Q}{Q} := \xi \cdot \frac{\Delta p}{p}$$

- defines: (normalised) 'chromaticity' Q' (ξ)
→ also 1st order measurement principle



Why bother about measurement, stability & control of Q', Q'', ...?

- Increases footprint in Q diagram and causes resonances for off-momentum particles
- Example LHC (RF cavities 'off'):



- need to obey this if we want to have more than one particle in the machine.
- Head-Tail instability → requires positive chromaticity for machines above transition
 - practically all lepton accelerators (e^+e^- collider, light sources, ...)
 - high-energy proton accelerator (Tevatron, RHIC, SPS, LHC, ...)

Non-Linear Chromaticity

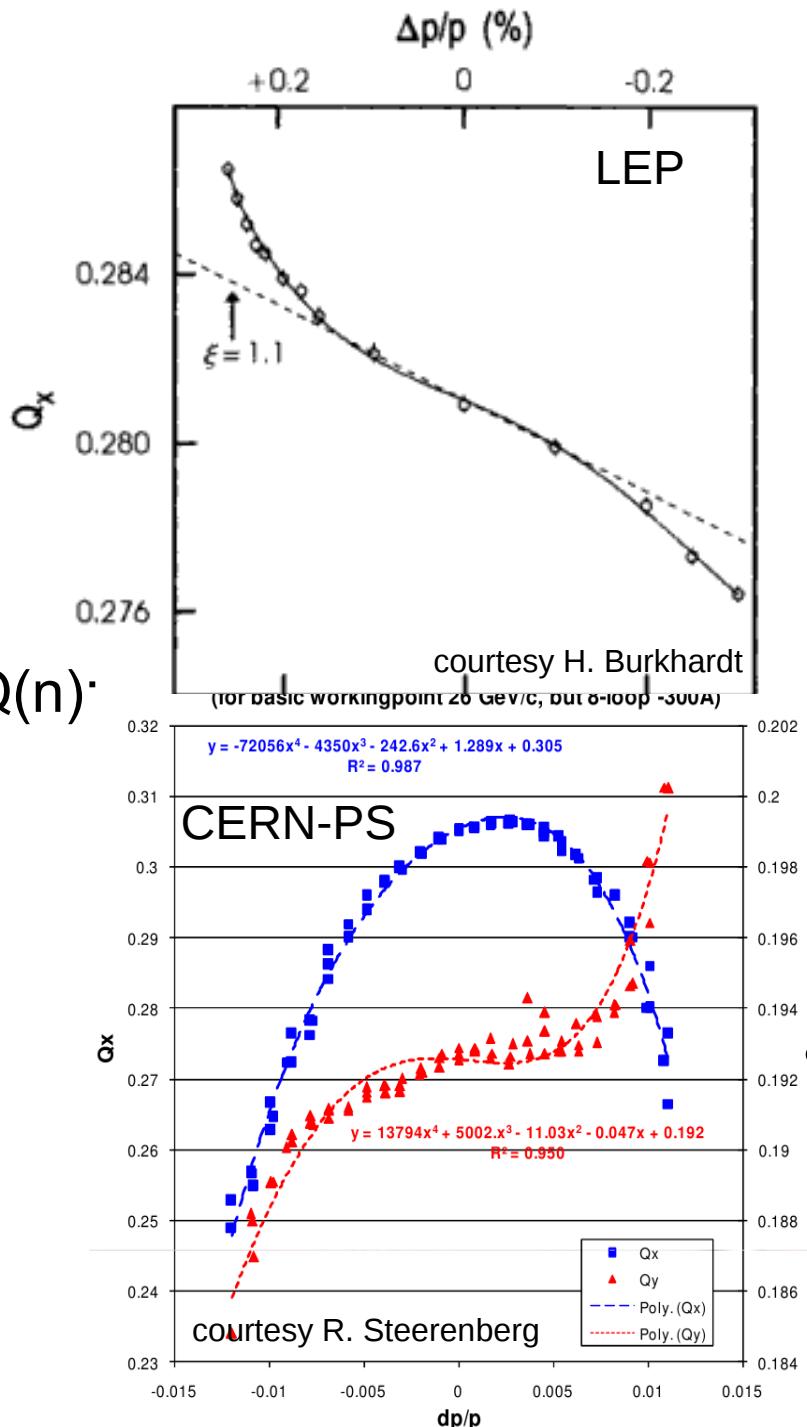
- Tune-shifts may depends not only linearly but also quadratically on $\Delta p/p$
→ Second order Chromaticity Q''

$$\Delta Q = Q'' \cdot \left(\frac{\Delta p}{p} \right)^2$$

- Can be generalised to higher orders $Q''' \dots Q(n)$

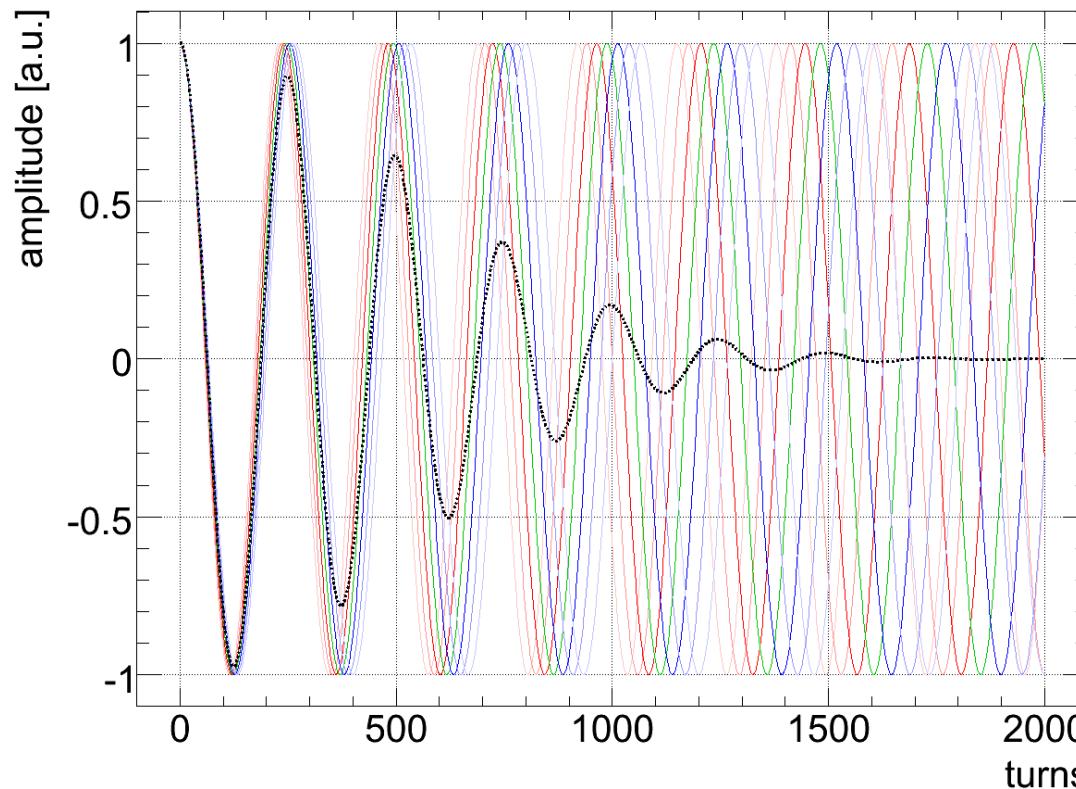
$$Q^{(n)} = \frac{\partial^{(n)} Q}{\partial \delta^{(n)}} \quad \text{with} \quad \delta := \frac{\Delta p}{p}$$

- Principle stays the same:
 - Measure Q as a function of $\Delta p/p$
 - Fit n -th order polynomial to the tune shift
 - returns: Q, Q', Q'', Q''', \dots
- However: correction is highly non-trivial!!



Recap: “Landau Damping”

- Individual bunch particles usually differ slightly w.r.t. their individual tune
→ Literature: “Landau Damping” (Historic misnomer: particle energy is preserved!)



- E.g. if $f(\Delta Q)$ is a narrow Gaussian distribution with $\sigma Q \ll Q$:

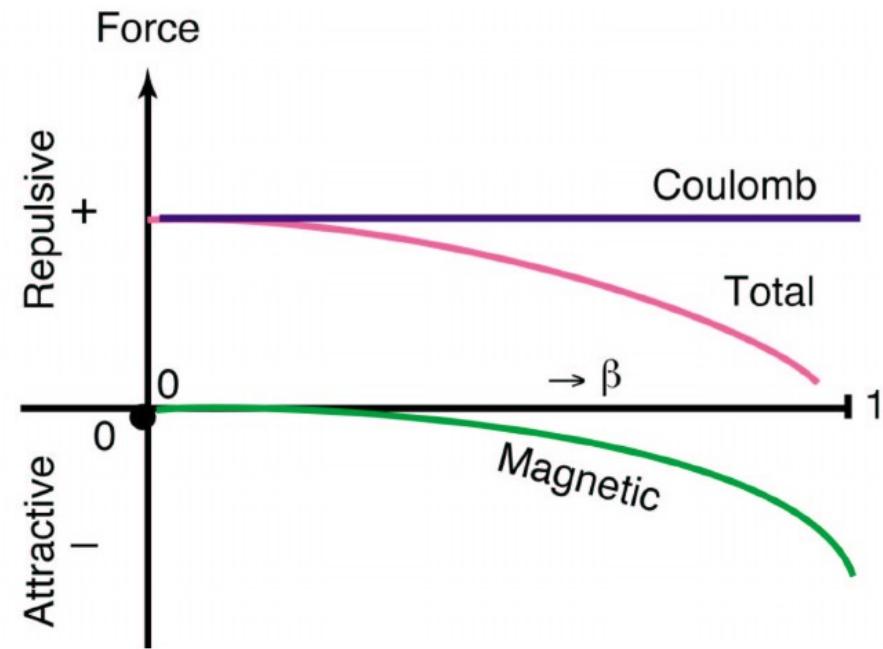
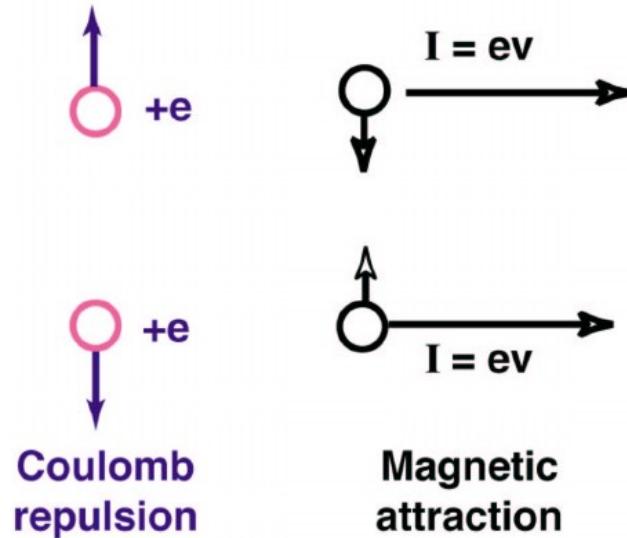
$$\bar{z}(t) = \bar{z}_0 \cdot e^{-\frac{1}{2} \cdot \sigma_Q^2 n^2} \cdot \cos(2\pi Q \cdot n)$$

dampening tune oscillations

→ large tune spread ↔ fast damping of e.g. head-tail instabilities

→ Tune oscillations are usually damped

Space Charge – First Principles



- Resulting force:

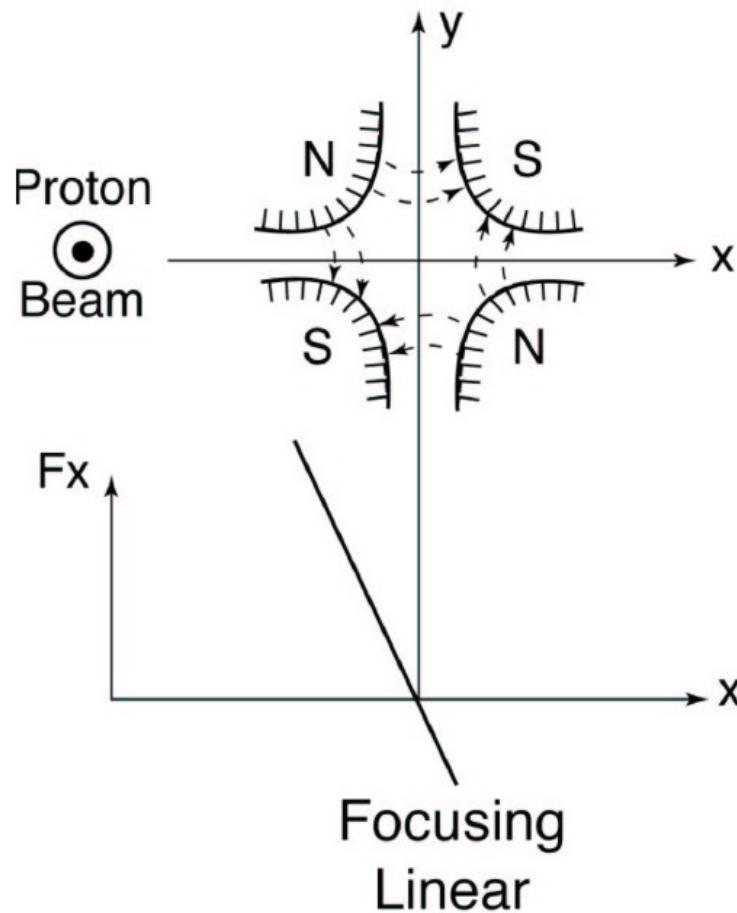
$$F(\vec{r}) = \frac{eI}{2\pi\epsilon_0\beta c} (1 - \beta^2) \cdot \frac{\vec{r}}{a^2} = \frac{eI}{2\pi\epsilon_0} \frac{1}{\gamma^2} \cdot \frac{\vec{r}}{a^2}$$

(a : radius of uniformly charged cylinder, $\beta := v/c$ relativistic velocity, c : speed-of-light)

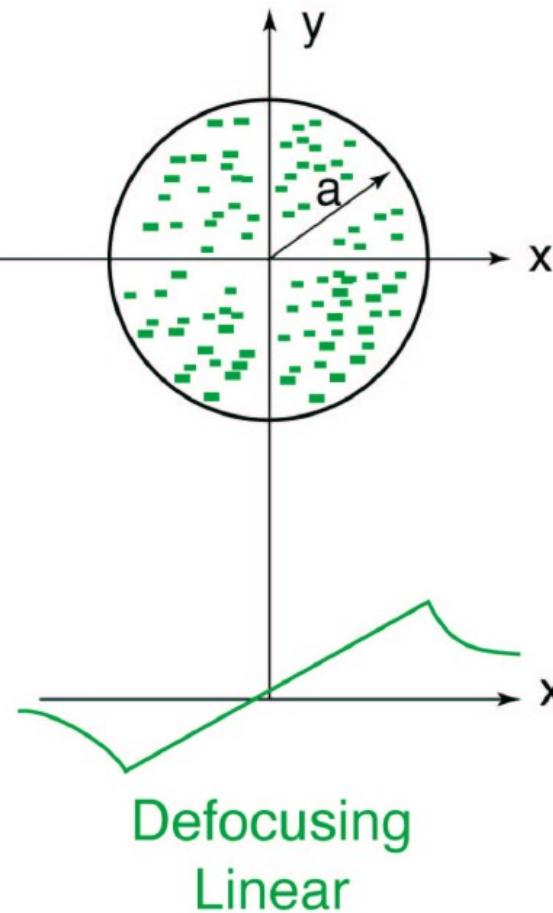
- important limitation for non(ie. ultra)-relativistic machines $\beta < 1$,
e.g. CERN-PS, GSI/FAIR-SIS18/100: $\beta \approx 0.1$

Space Charge – First Principles

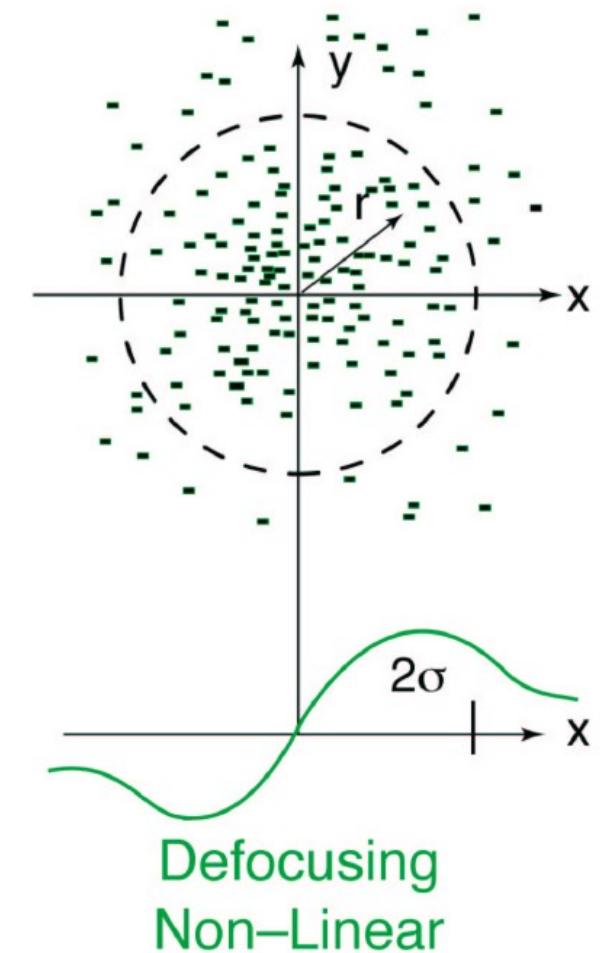
Quadrupole
(Hor. Foc.)



Uniform



Gaussian

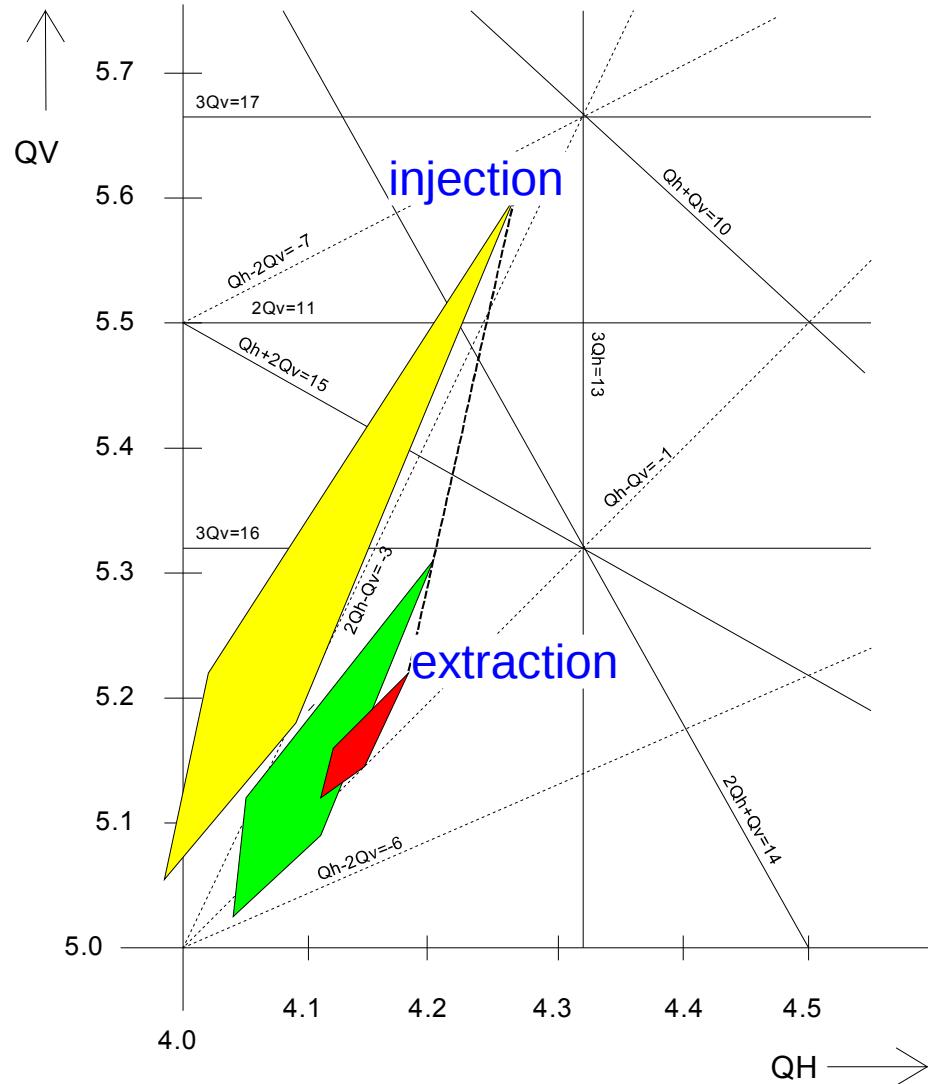


$$k_{s.c.} = \frac{-2r_0 I}{e a^2 (\beta \gamma)^3}$$

r_0 = proton radius
 I = beam intensity
 a = cyl. beam size

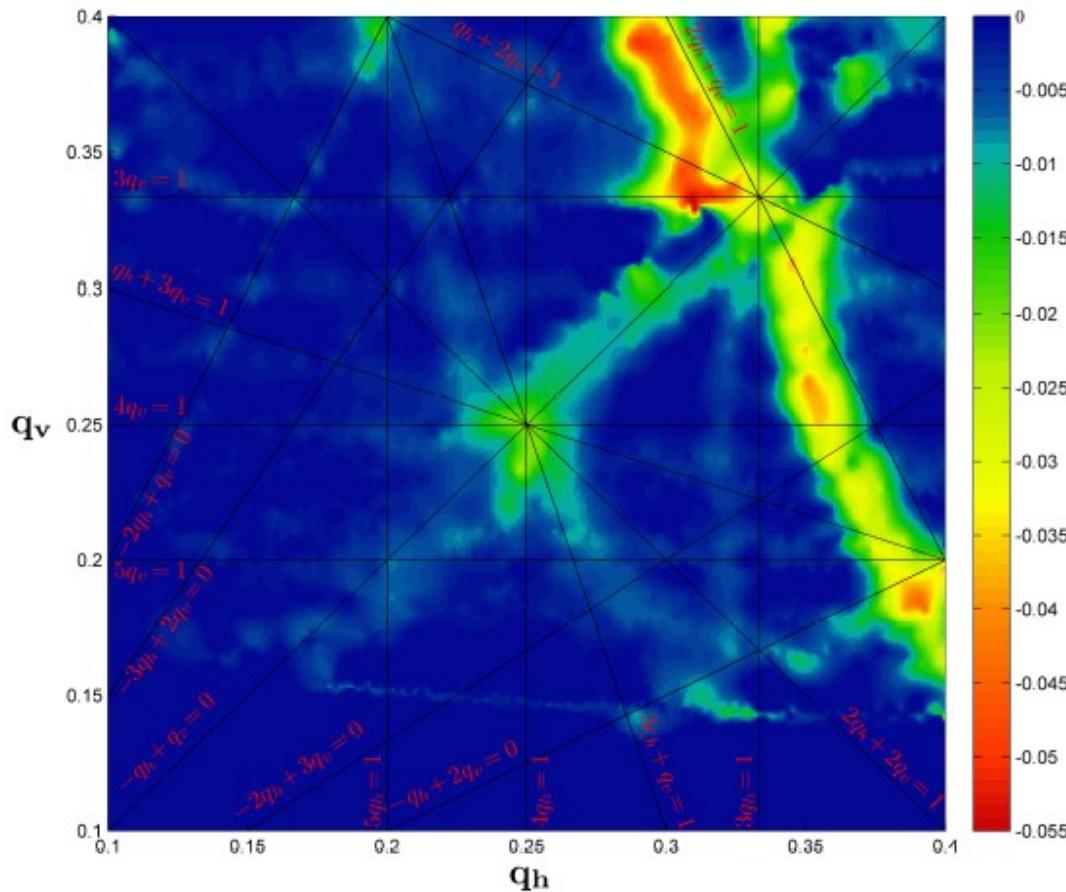
figures courtesy K. Schindl

Tune Diagram with Space Charge



during acceleration Q_H and Q_V are shifted to place where the beam is the least influenced by resonances

Measured Tune Diagram



- Move a large emittance beam around in this tune diagram and measure the beam losses.
- Important:
 - not all resonance lines are harmful!
 - some can be compensated
 - control of optics
 - control of driving terms

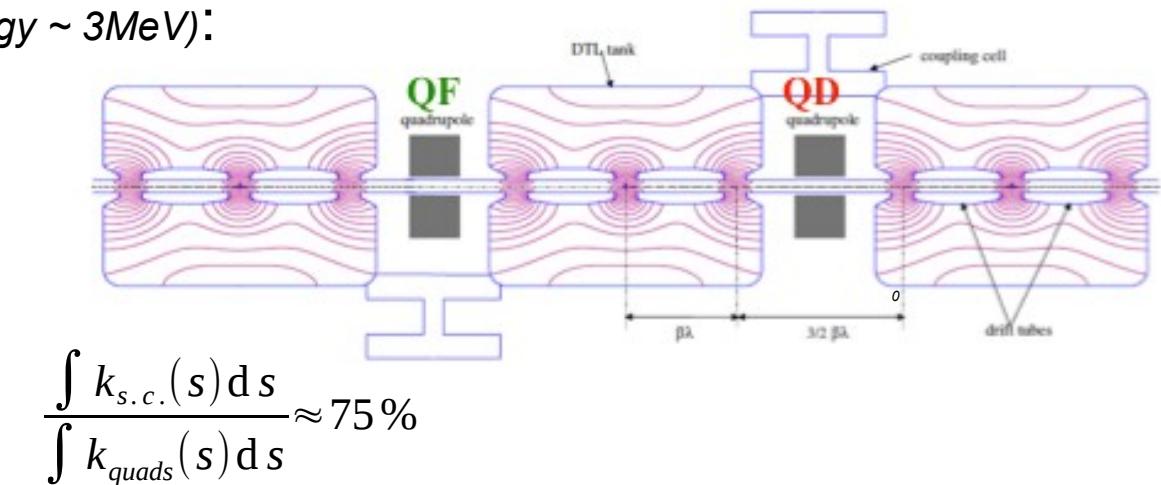
Effect of Space Charge on Optics

- Example LINAC4 TL (*Kin. Energy* $\sim 3\text{MeV}$):

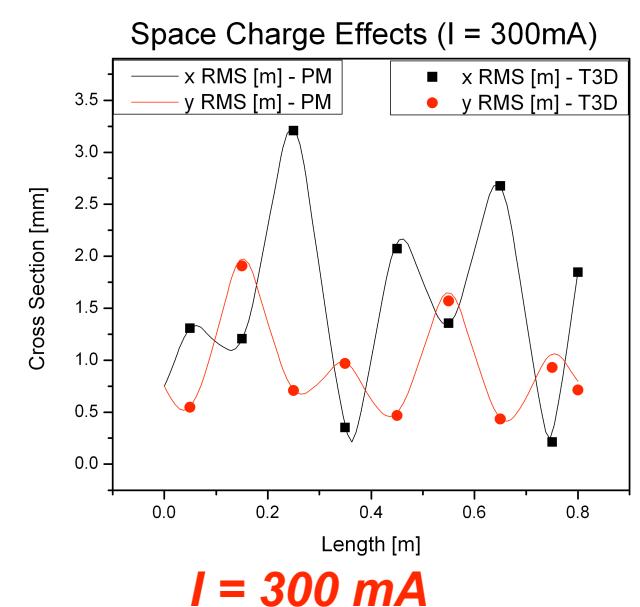
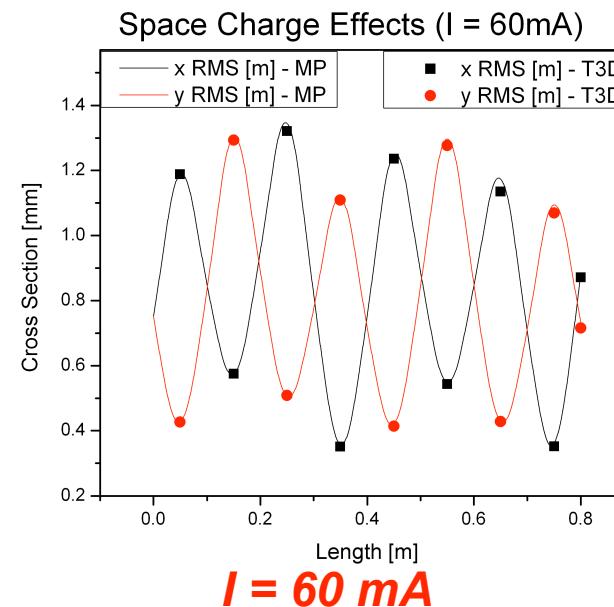
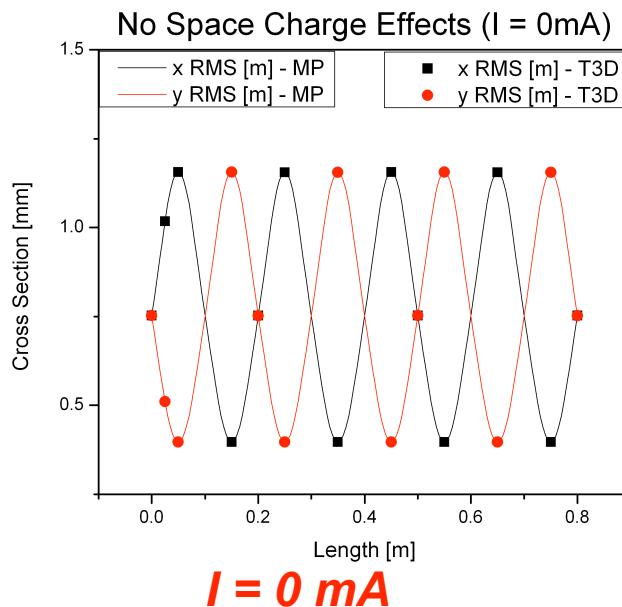
$$k_{s.c.} \approx 3 \cdot 10^{-4} \frac{1}{\text{m}^2}$$

$$k_{quads} \approx 5 \cdot 10^{-1} \frac{1}{\text{m}^2}$$

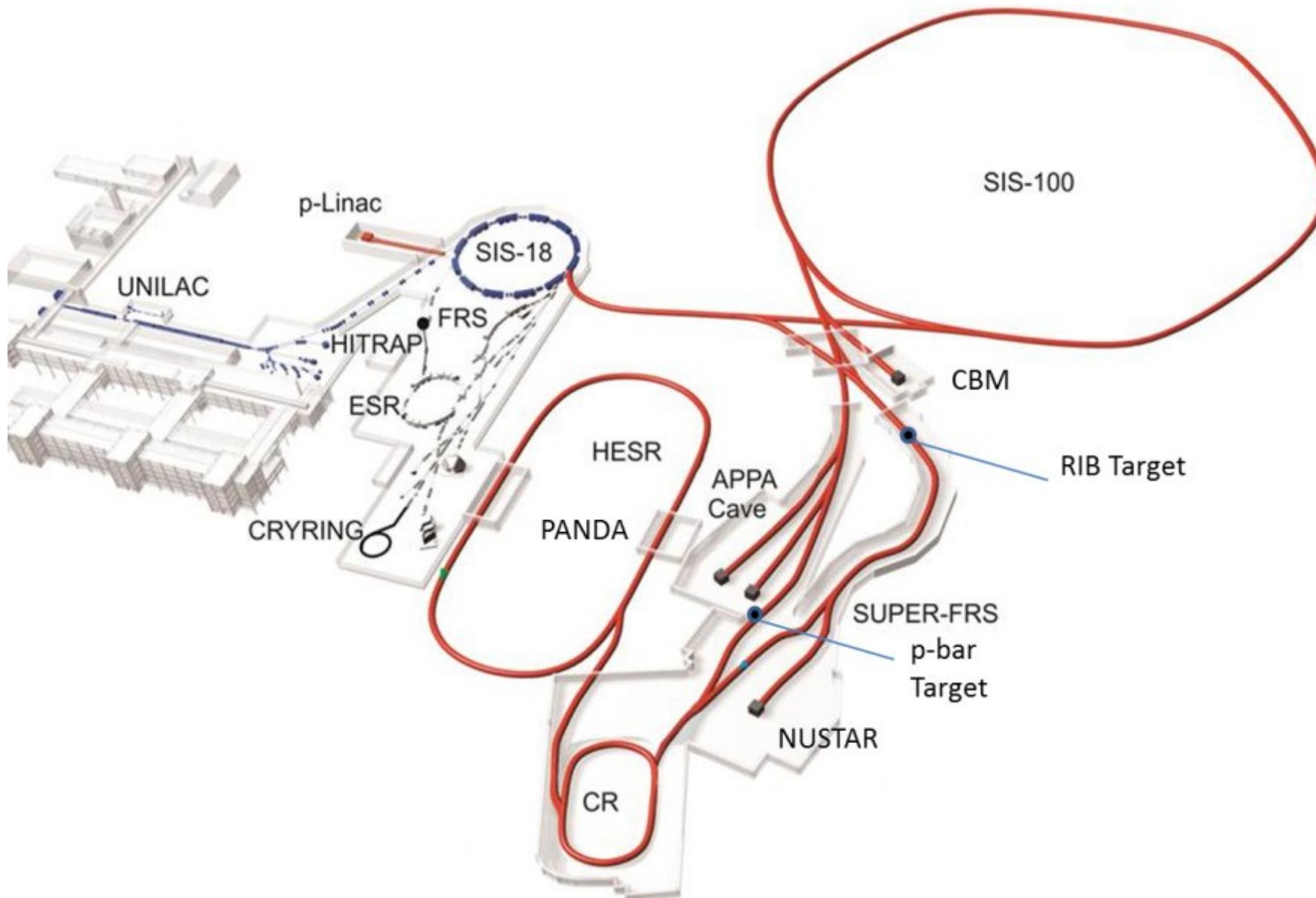
– integrating along LINAC



- Space-charge introduces enormous optical error → needs compensation*



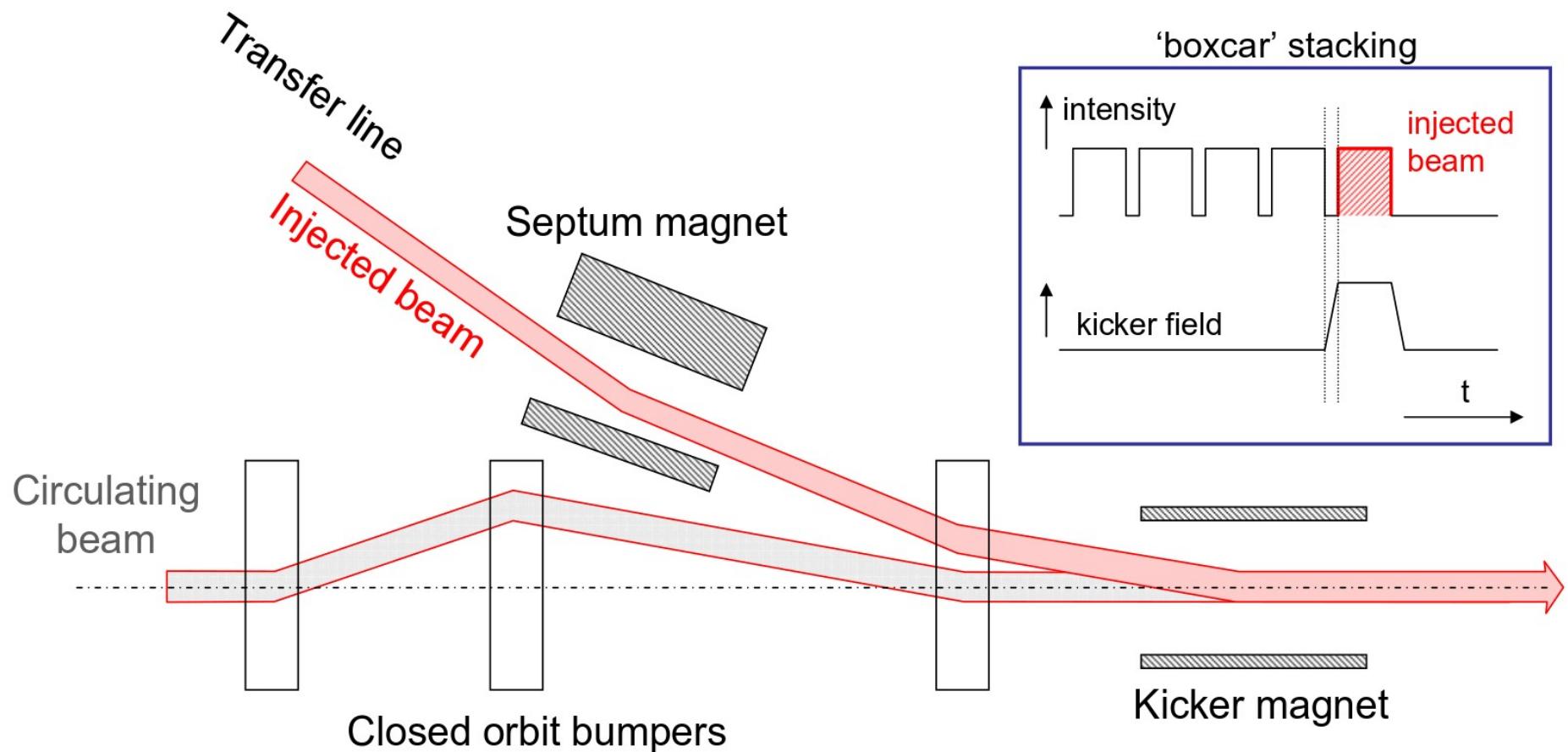
A FAIR amount of Injection & Extraction



- Several different injection techniques
 - Single-turn injection for hadrons
 - Boxcar stacking: transfer between machines in accelerator chain
 - Angle / position errors \Rightarrow injection oscillations
 - Optics errors \Rightarrow betatron mismatch oscillations
 - Oscillations \Rightarrow filamentation \Rightarrow emittance increase
 - Multi-turn injection for hadrons
 - Phase space painting to increase intensity
 - H^- injection allows injection into same phase space area
 - Lepton injection: take advantage of synchrotron radiation damping
 - Less concerned about injection precision and matching
 - Rare-isotopes and anti-proton beam stacking using electron cooler

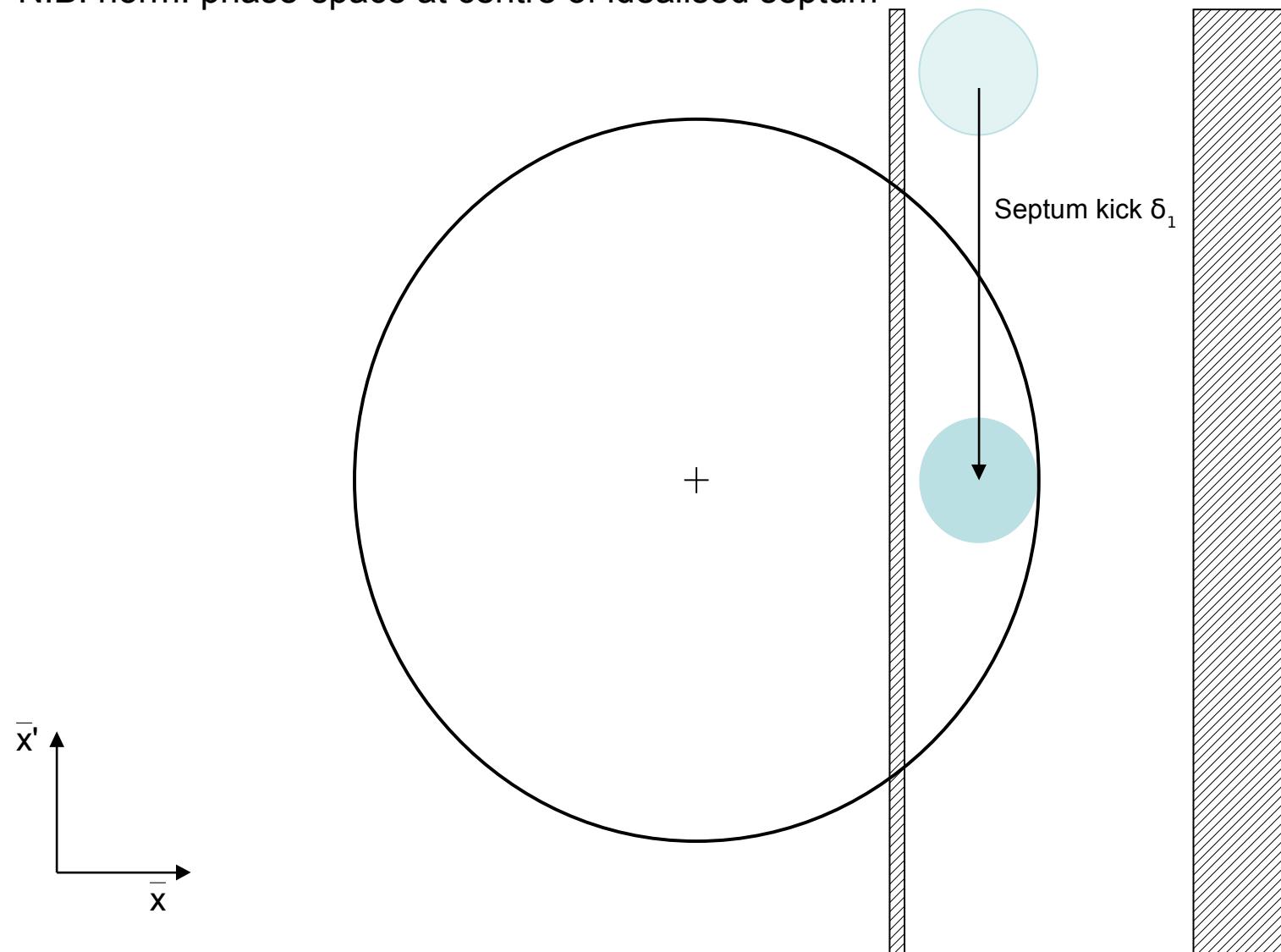
Single-Turn Injection (e.g. SIS100)

- Septum deflects the beam onto the closed orbit at the centre of the kicker \rightarrow kick δ_1
- Fast kicker magnet compensates for the remaining angle \rightarrow kick δ_2
 - N.B. 4-corrector orbit bumps sometimes used to minimise septa/kicker strengths
 - N.B. septa could be in the same (as shown here) or opposite planes (e.g. LHC)



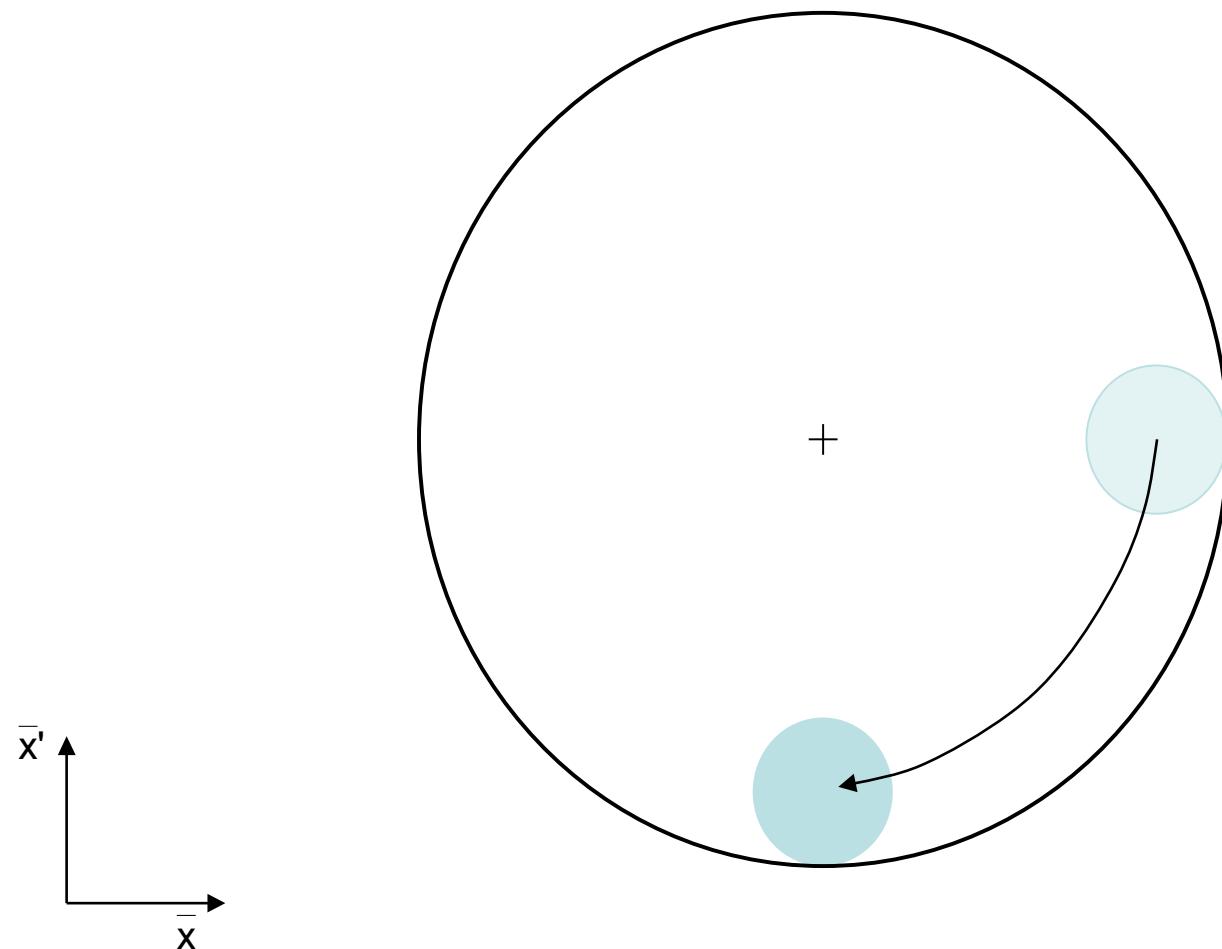
Single-Turn Injection I/III

- Large deflection by septum
 - N.B. norm. phase-space at centre of idealised septum



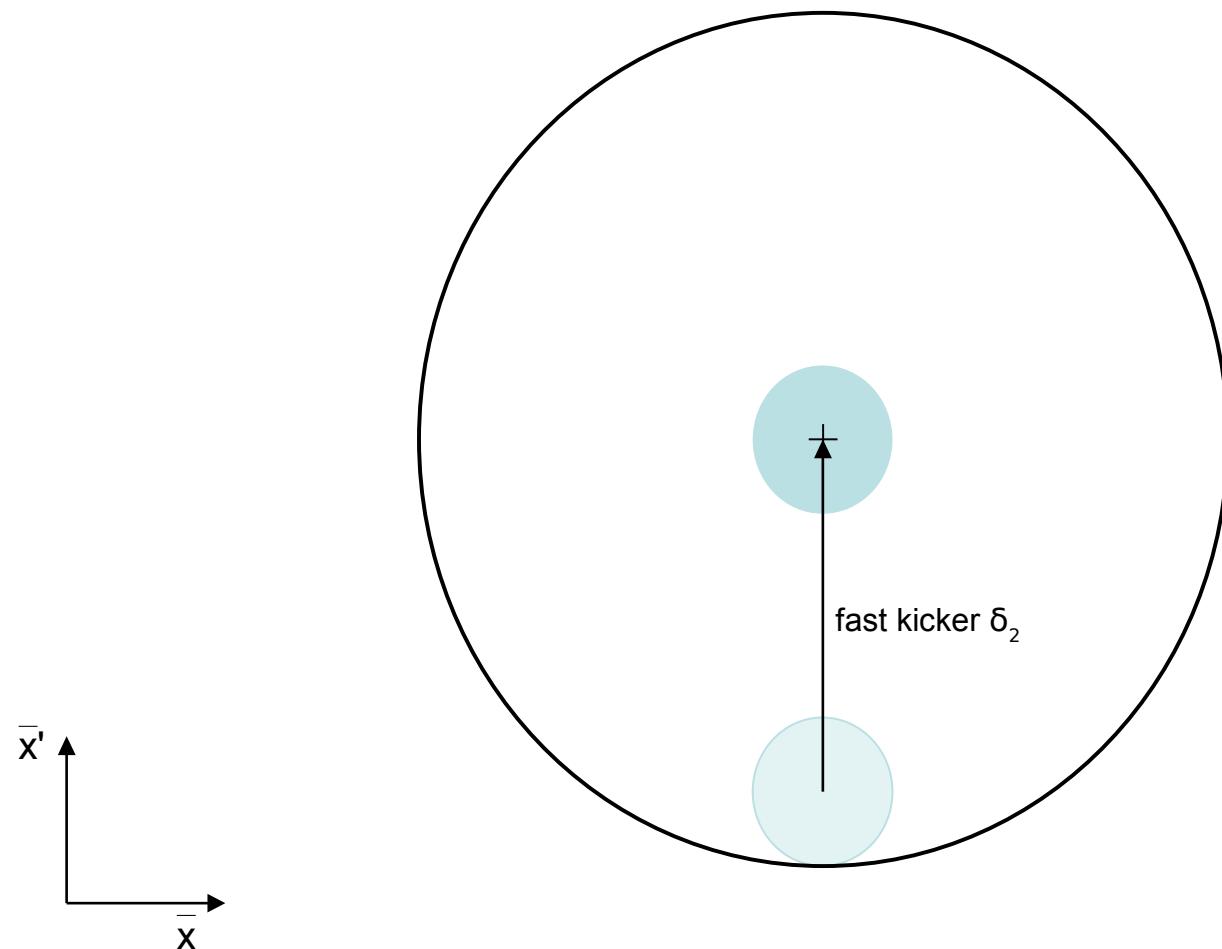
Single-Turn Injection II/III

- $\pi/2$ phase advance to kicker location



Single-Turn Injection III/III

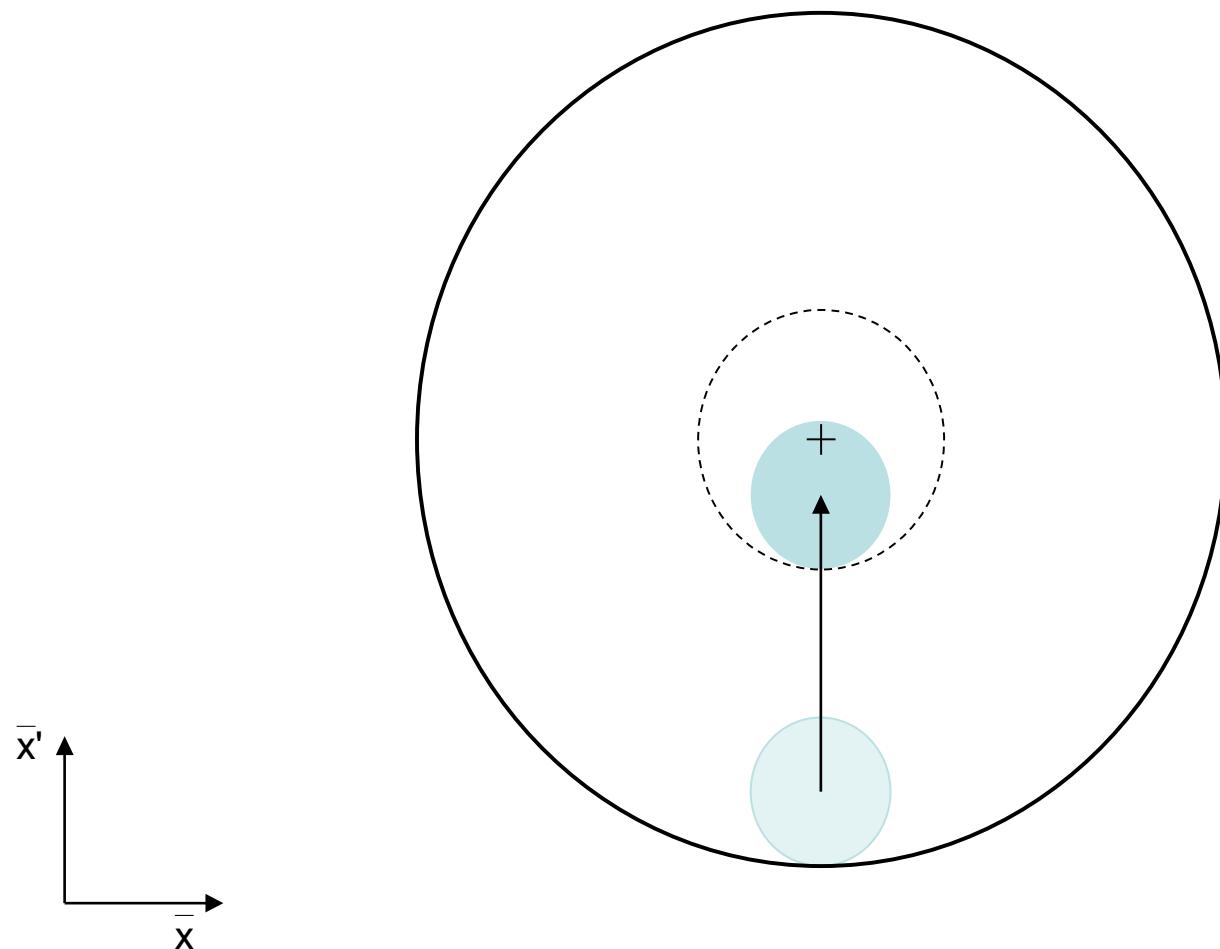
- Kicker deflection places beam on central orbit



Single-Turn Injection Errors I/IV

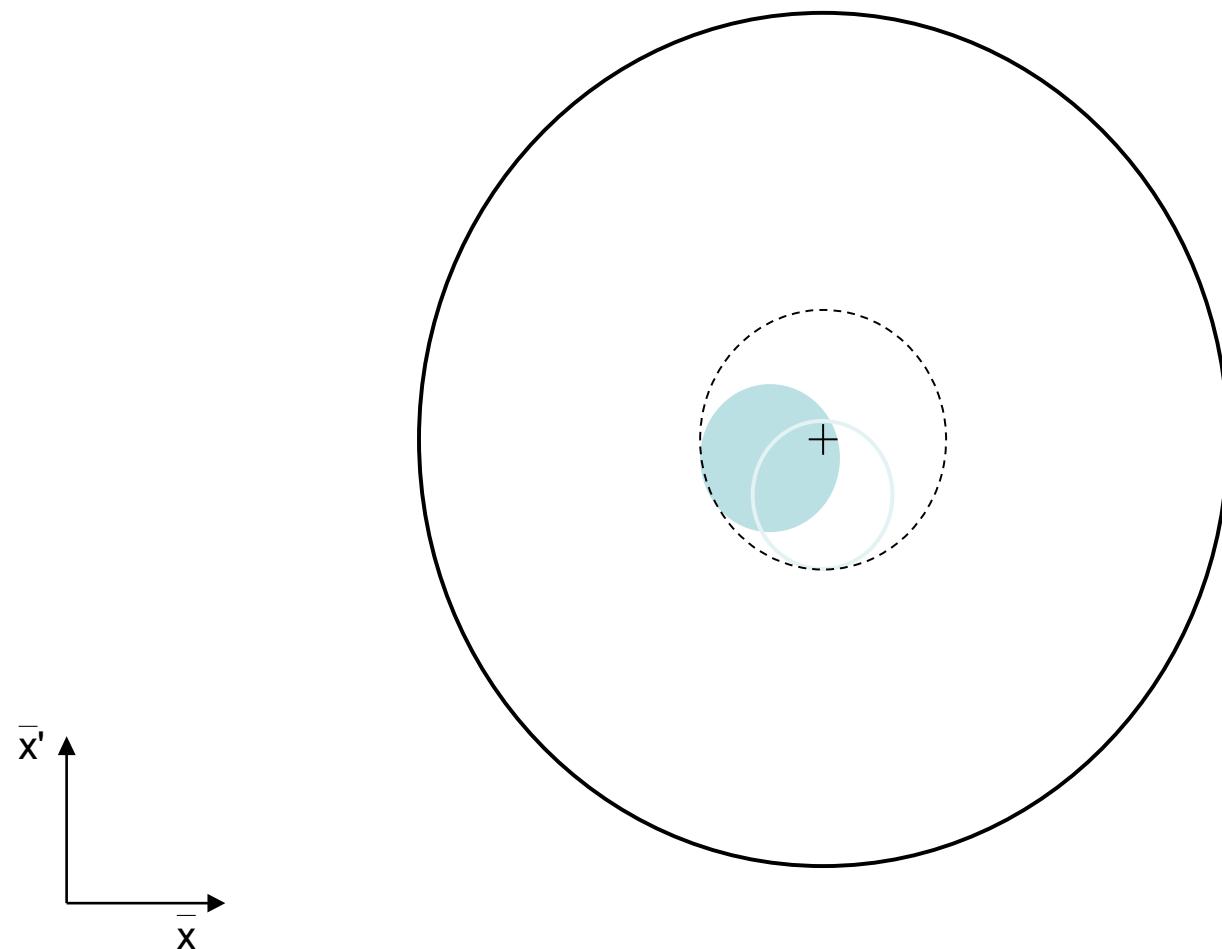
- For imperfect injection the beam oscillates around the central orbit.

kicker error $\Delta\delta_2$



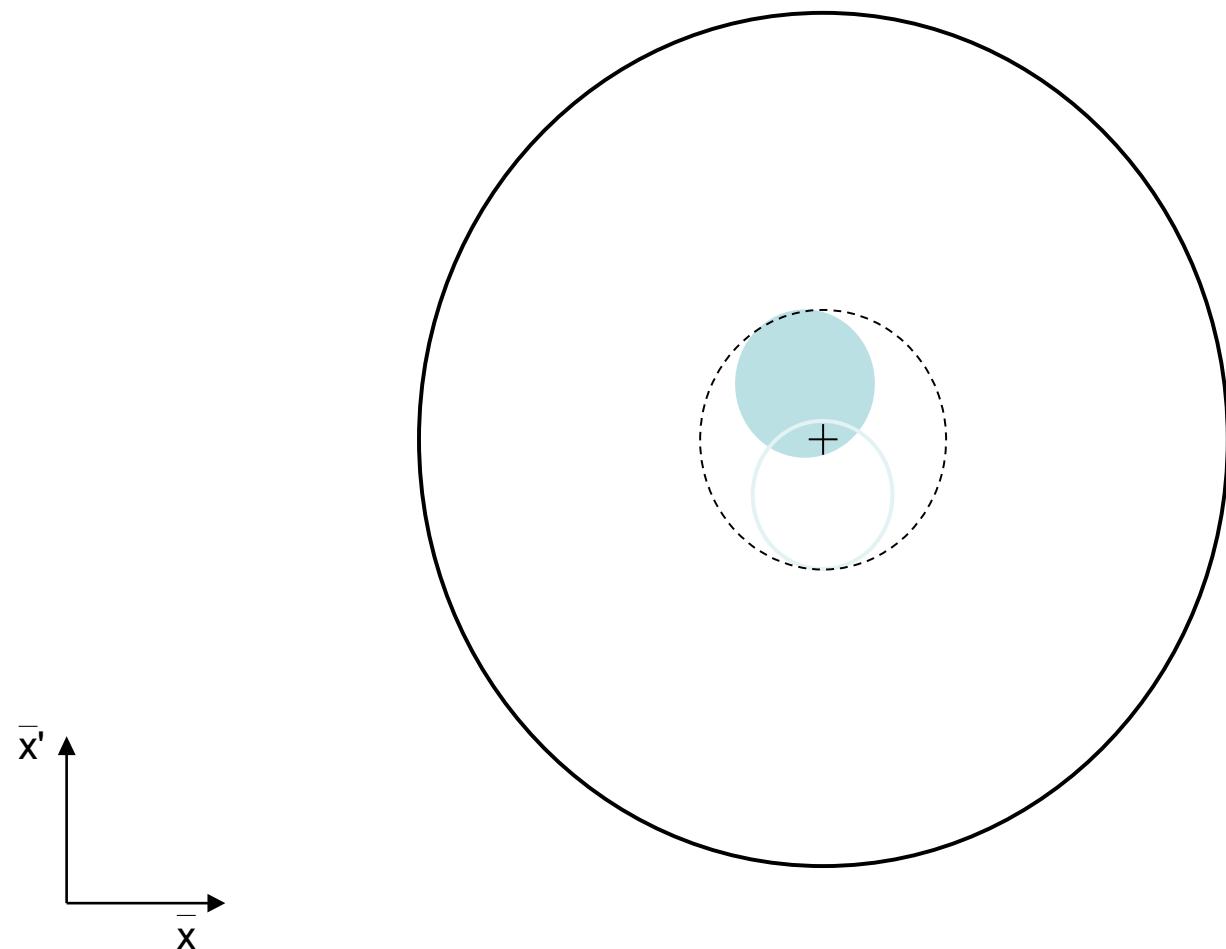
Single-Turn Injection Errors II/IV

- For imperfect injection the beam oscillates around the central orbit.



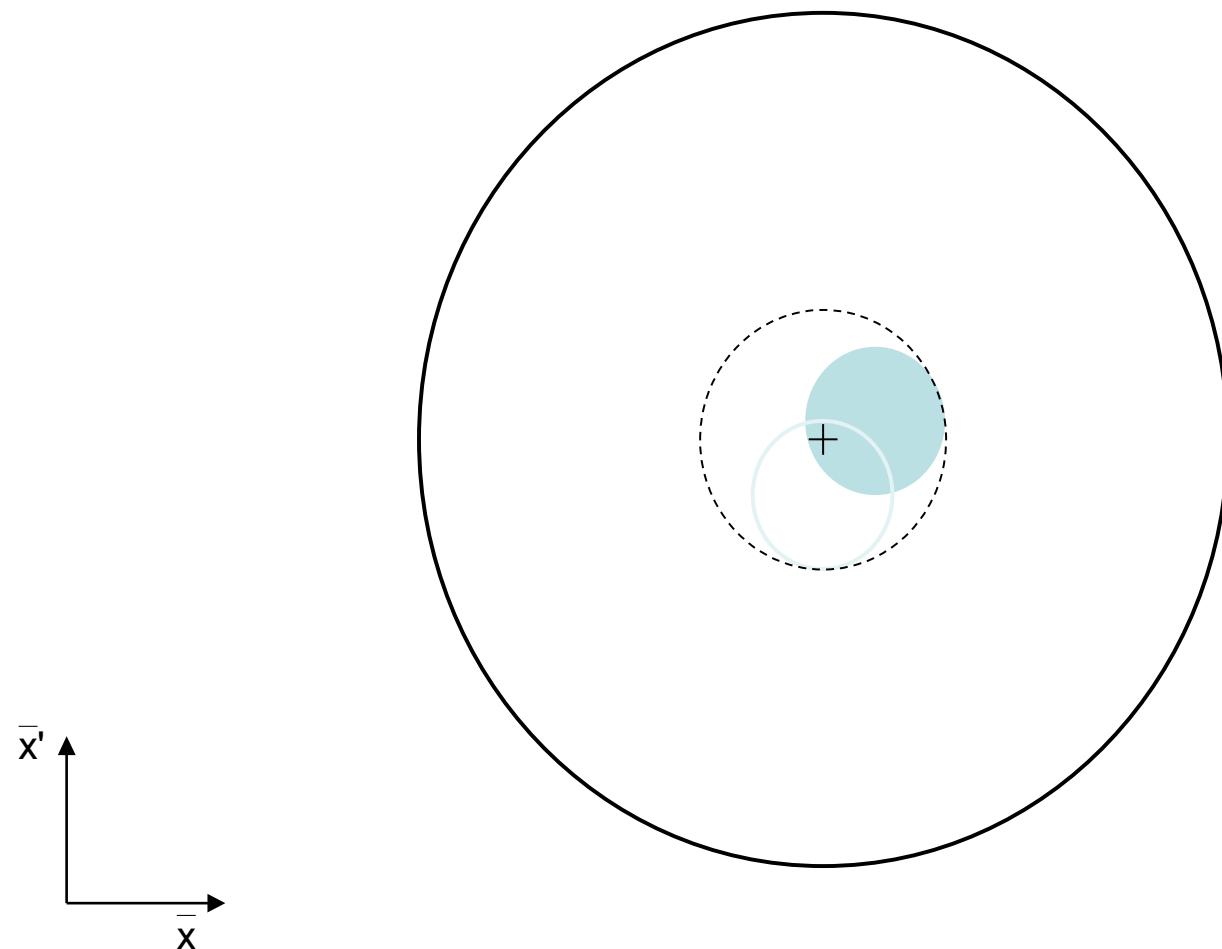
Single-Turn Injection Errors III/IV

- For imperfect injection the beam oscillates around the central orbit.

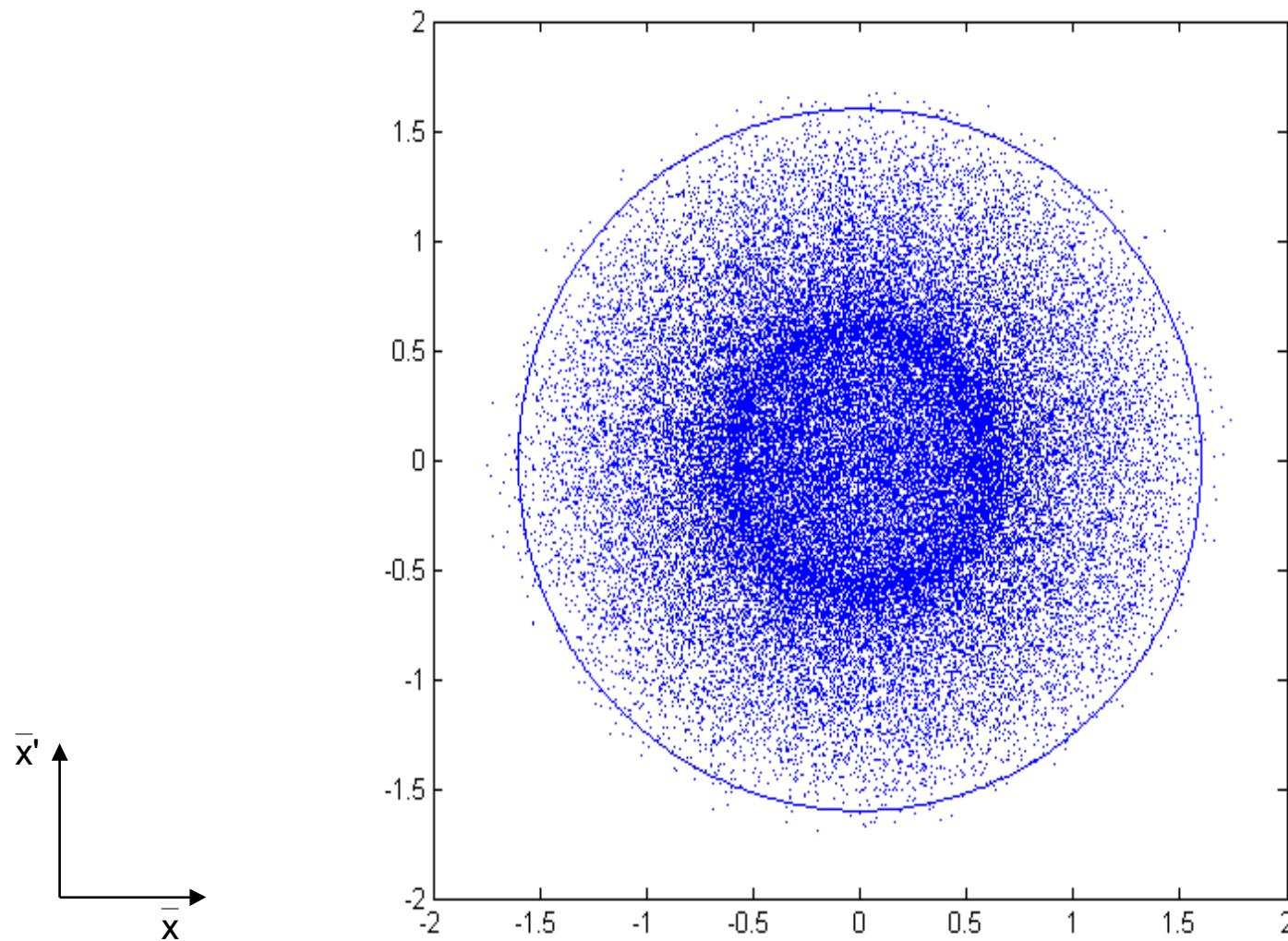


Single-Turn Injection Errors IV/IV

- For imperfect injection the beam oscillates around the central orbit.



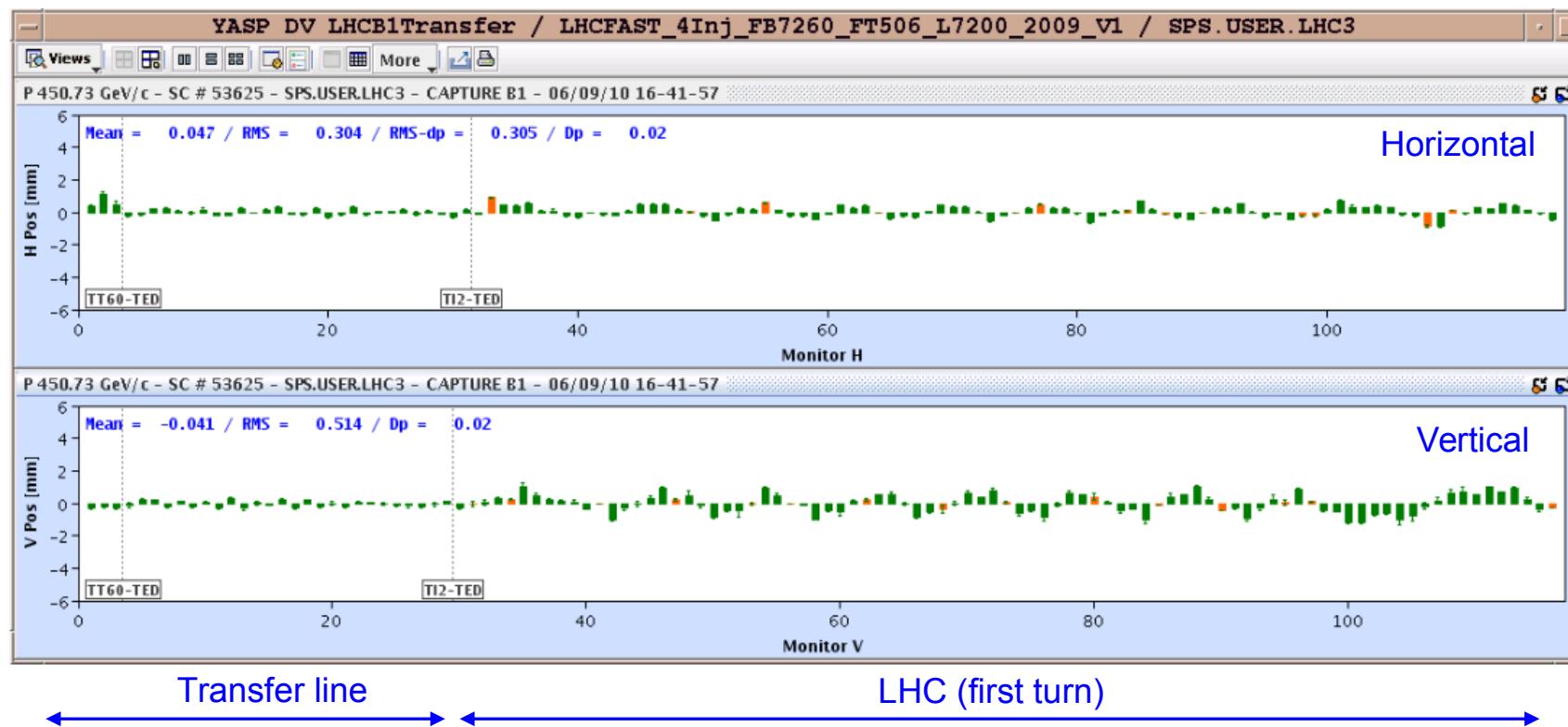
Filamentation



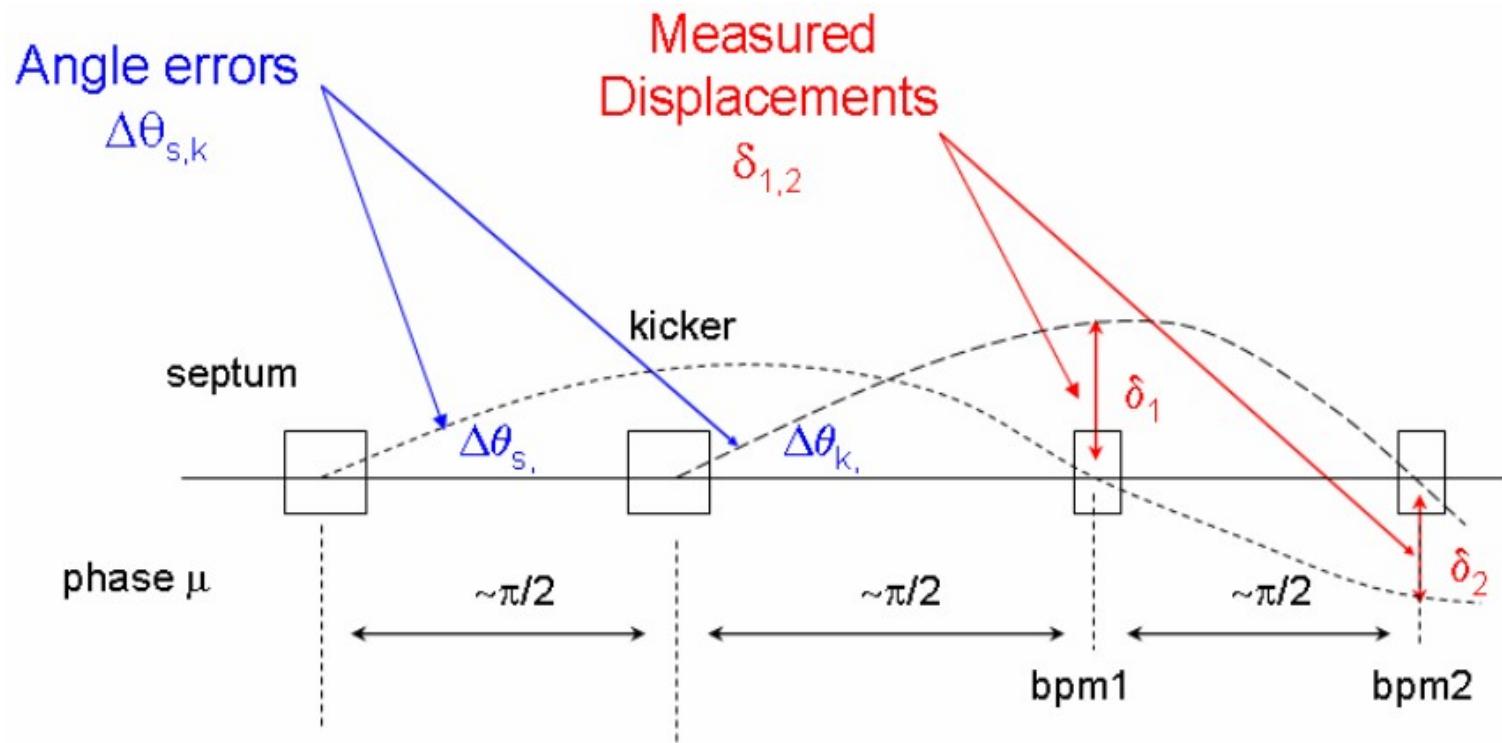
- Residual transverse oscillations lead to an emittance blow-up through filamentation
- “Transverse damper” systems used to damp injection oscillations -bunch position measured by a pick-up, which is linked to a kicker

Single-Turn Injection Error Correction I/II

- Betatron oscillations with respect to the Closed Orbit
 - Example LHC: monitored for every injection and corrected when necessary



Single-Turn Injection Error Correction II/II

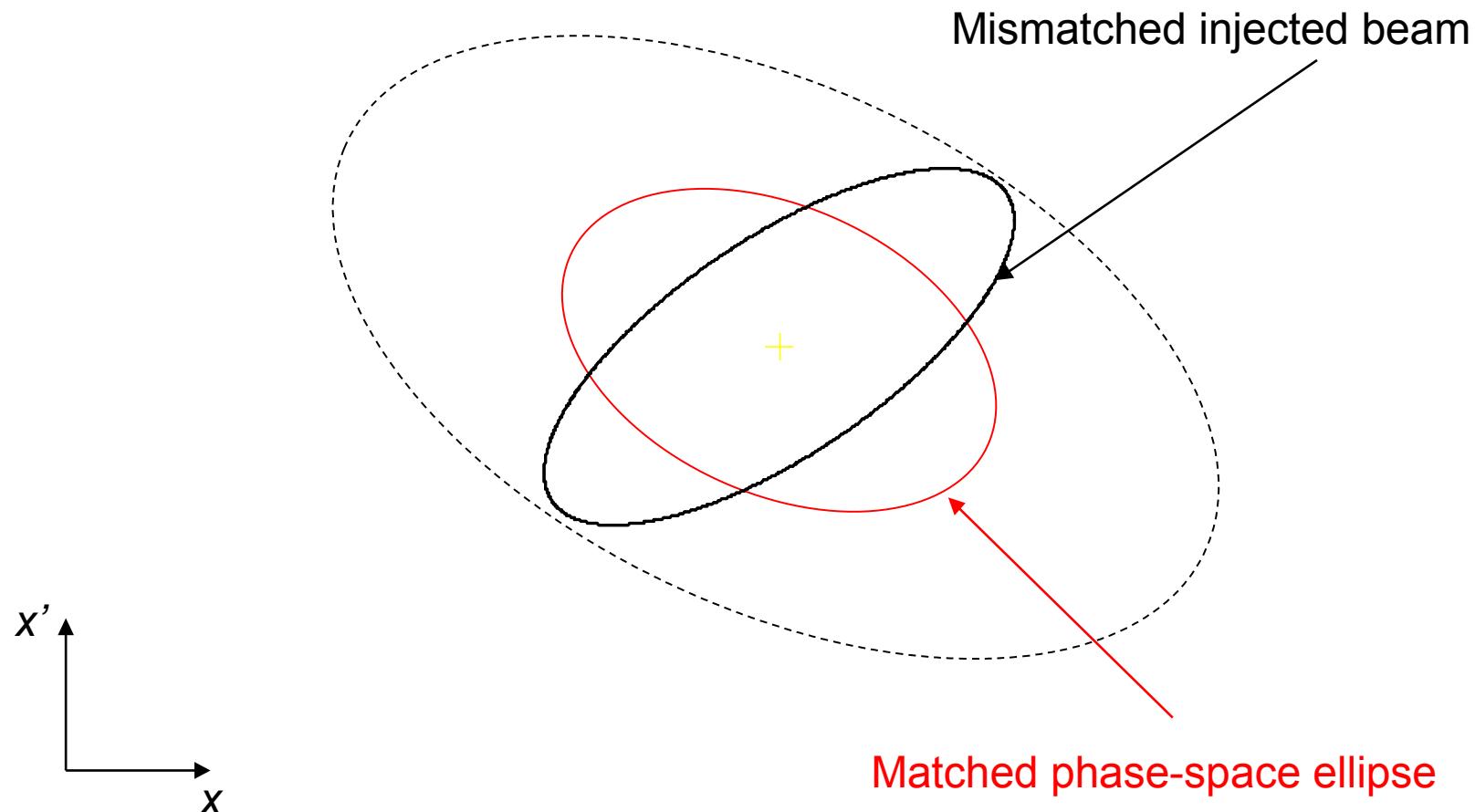


$$\begin{aligned}\delta_1 &= \Delta\theta_s \sqrt{(\beta_s \beta_1)} \sin (\mu_1 - \mu_s) + \Delta\theta_k \sqrt{(\beta_k \beta_1)} \sin (\mu_1 - \mu_k) \\ &\approx \Delta\theta_k \sqrt{(\beta_k \beta_1)}\end{aligned}$$

$$\begin{aligned}\delta_2 &= \Delta\theta_s \sqrt{(\beta_s \beta_2)} \sin (\mu_2 - \mu_s) + \Delta\theta_k \sqrt{(\beta_k \beta_2)} \sin (\mu_2 - \mu_k) \\ &\approx -\Delta\theta_s \sqrt{(\beta_s \beta_2)}\end{aligned}$$

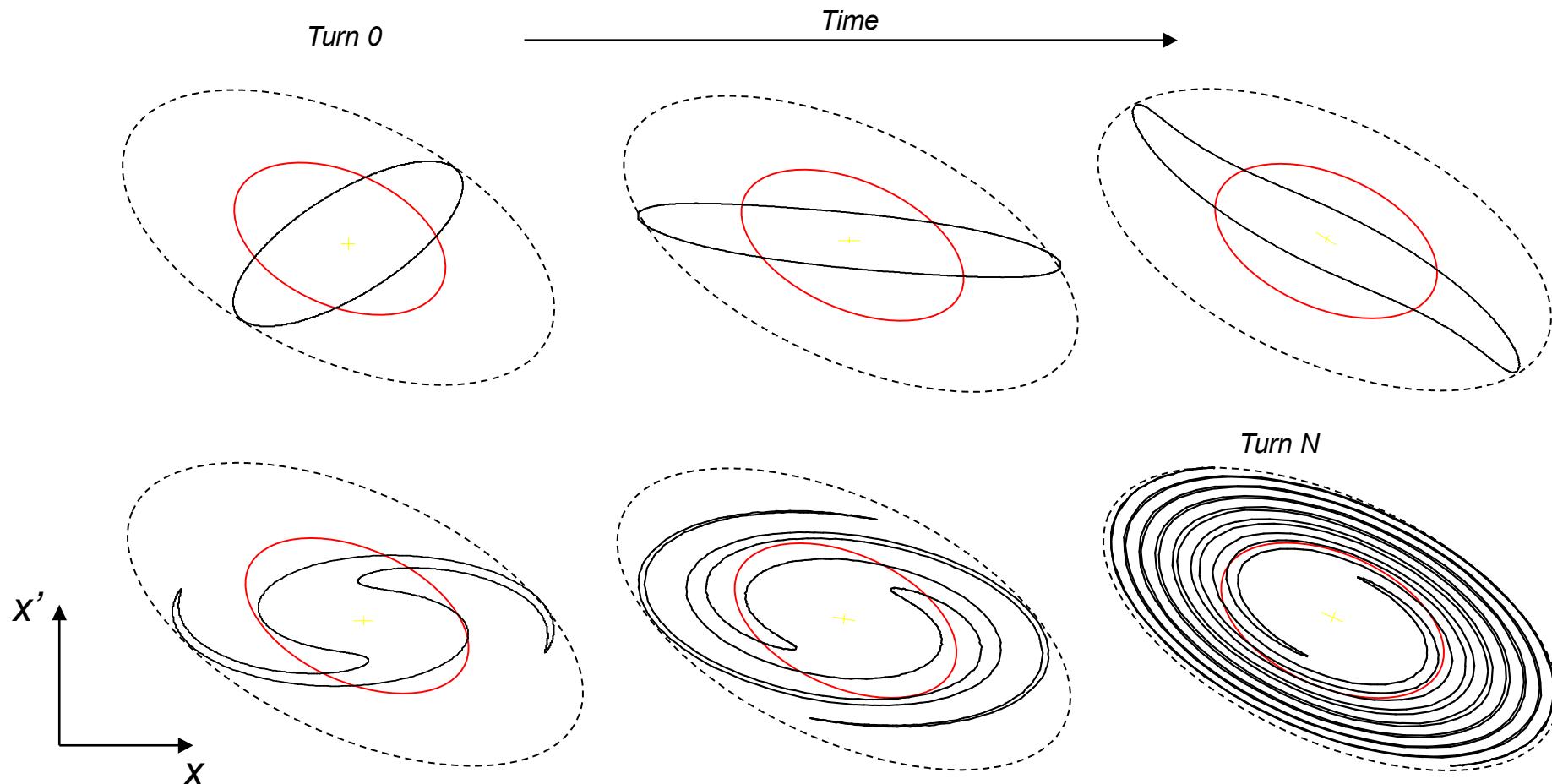
Optical Mismatch at Injection I/II

- Can also have an emittance blow-up through optical mismatch
- Individual particles oscillate with conserved CS invariant:



Optical Mismatch at Injection II/II

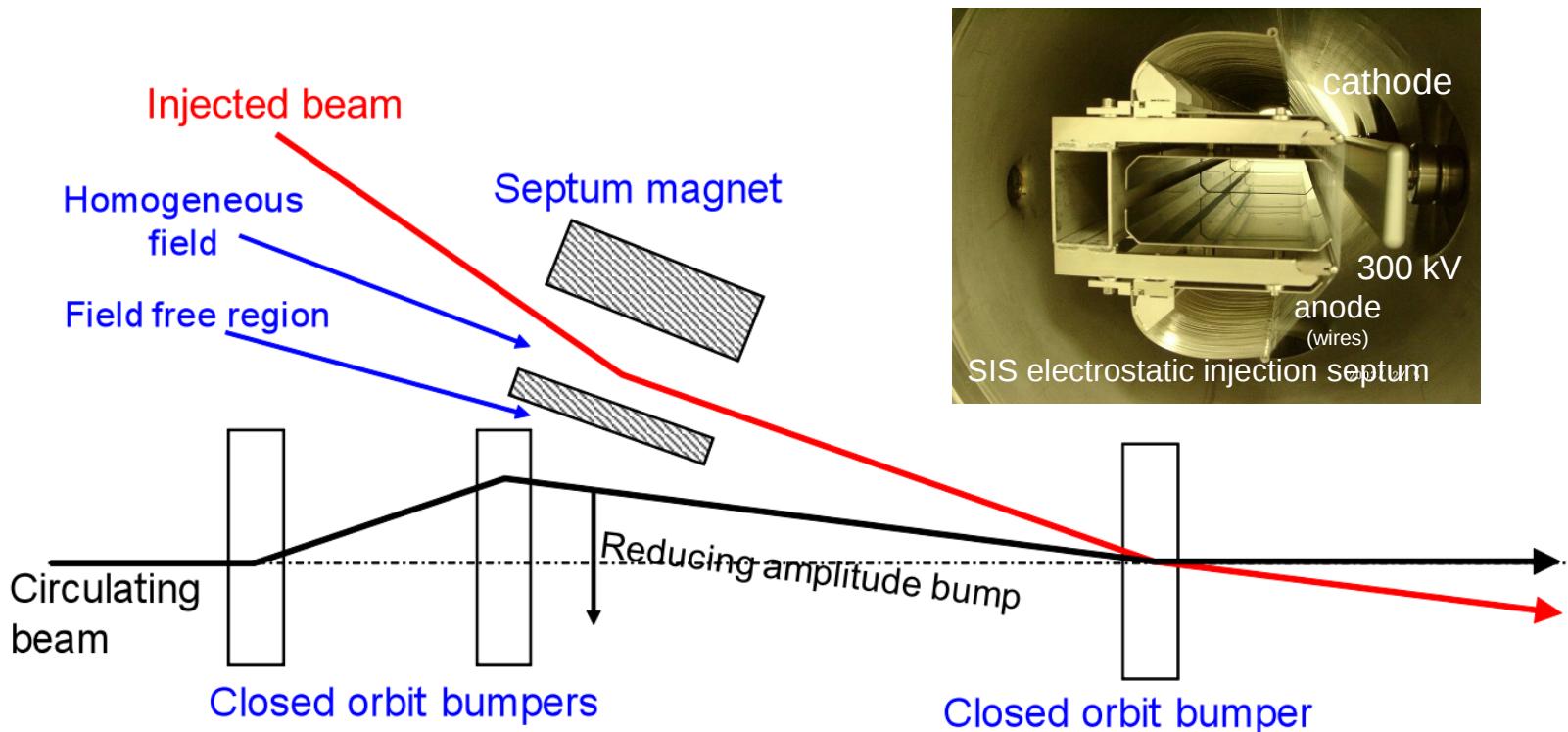
- Filamentation fills larger ellipse with same shape as matched ellipse



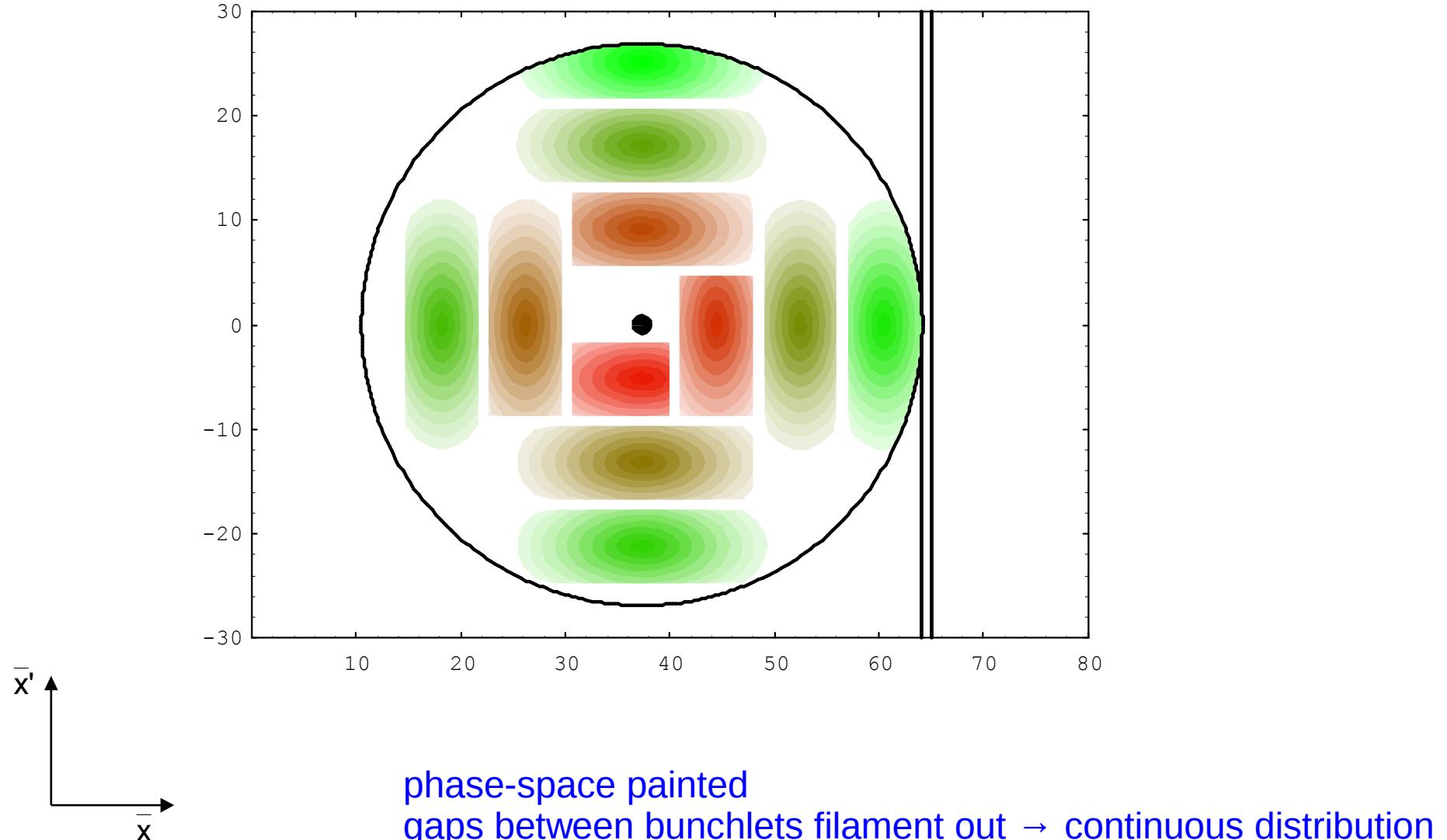
- Importance of optics correction for high-intensity beam transfers

Multi-turn injection

- For hadrons the beam density at injection may be limited by space charge effects or the injector capacity (beam brilliance)
- If we cannot increase charge density, we can sometimes fill the e.g. horizontal phase-space to increase overall injected intensity
 - Condition: acceptance of receiving machine > delivered beam emittance

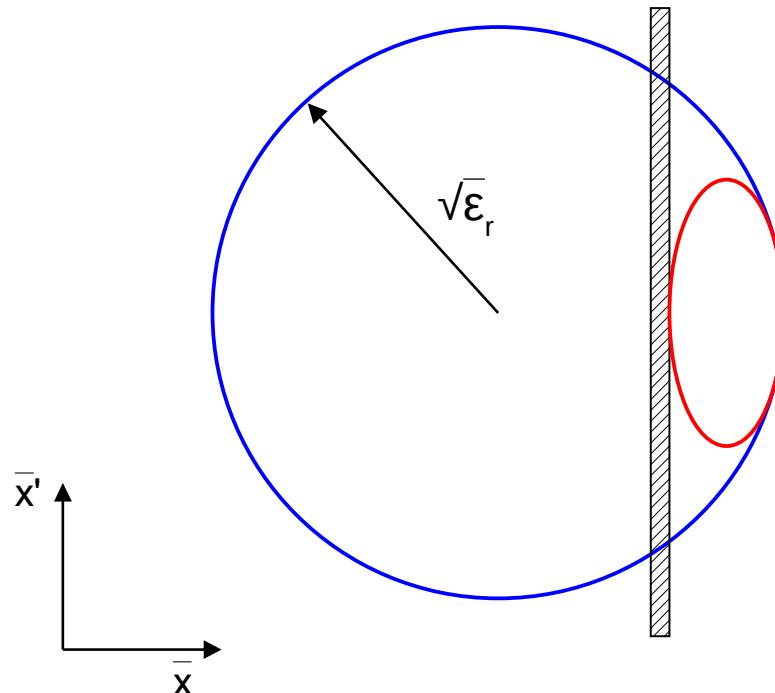


Multi-Turn Injection for Hadrons



Multi-Turn Injection – Mismatch

- In order to reduce losses for multiturn injection over n turns:
 - injected beam ellipse is deliberately mismatched to circulating beam
 - emittance distribution tails 'clipped' in up-stream transfer-line using collimators



- Optimum for: $\frac{\beta_i}{\beta_r} \approx \frac{\alpha_i}{\alpha_r} \approx \left(\frac{\varepsilon_i}{\varepsilon_r} \right)^{\frac{1}{3}}, \quad \frac{\alpha_r}{\beta_r} = \frac{\alpha_i}{\beta_i} = -\frac{x'}{x_r}, \quad n \approx 0.5 \dots 0.7 \cdot \frac{\varepsilon_r}{\varepsilon_i}$

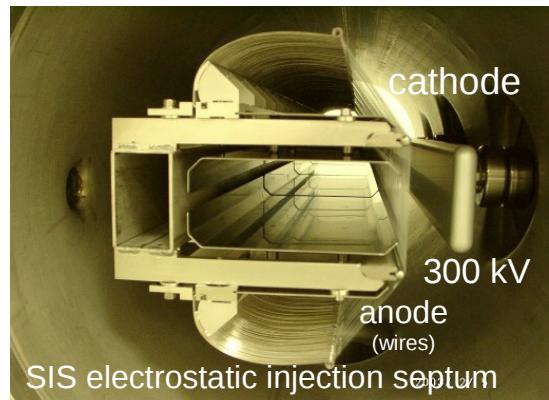
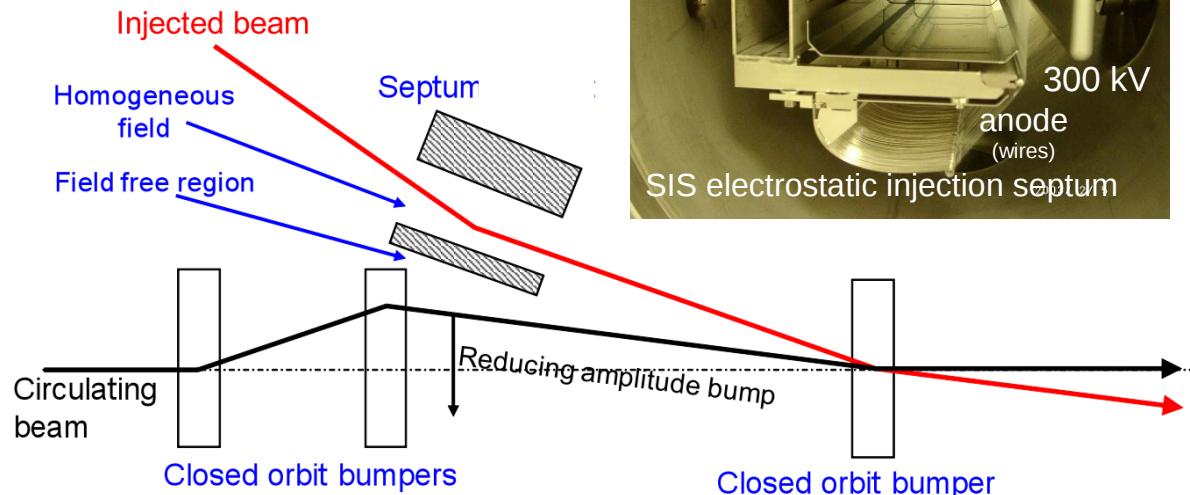
SIS18 Multi-Turn Injection (H-Phase-Space Painting)

P. Spiller, Y. El-Hayek, U. Blell et al., IPAC'12, 2012

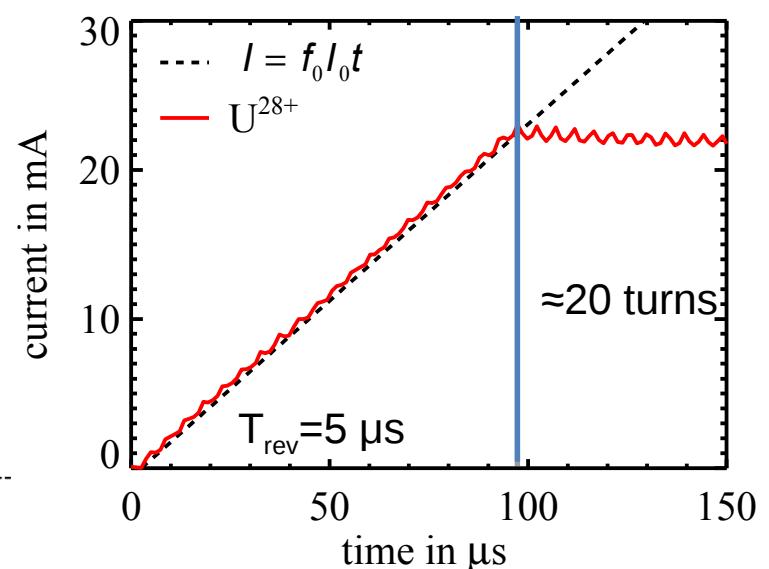
From a linac

e.g. SIS-18, CERN PSB

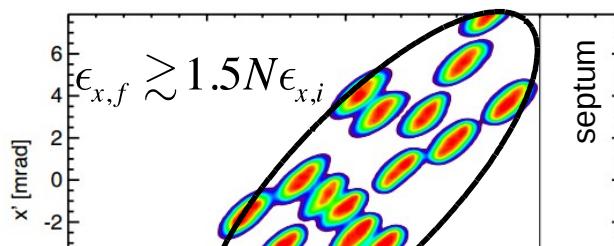
courtesy Mike Barnes



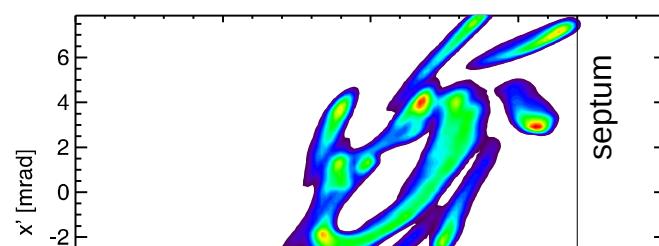
Measured MTI performance in SIS-18



Simulation: without space charge

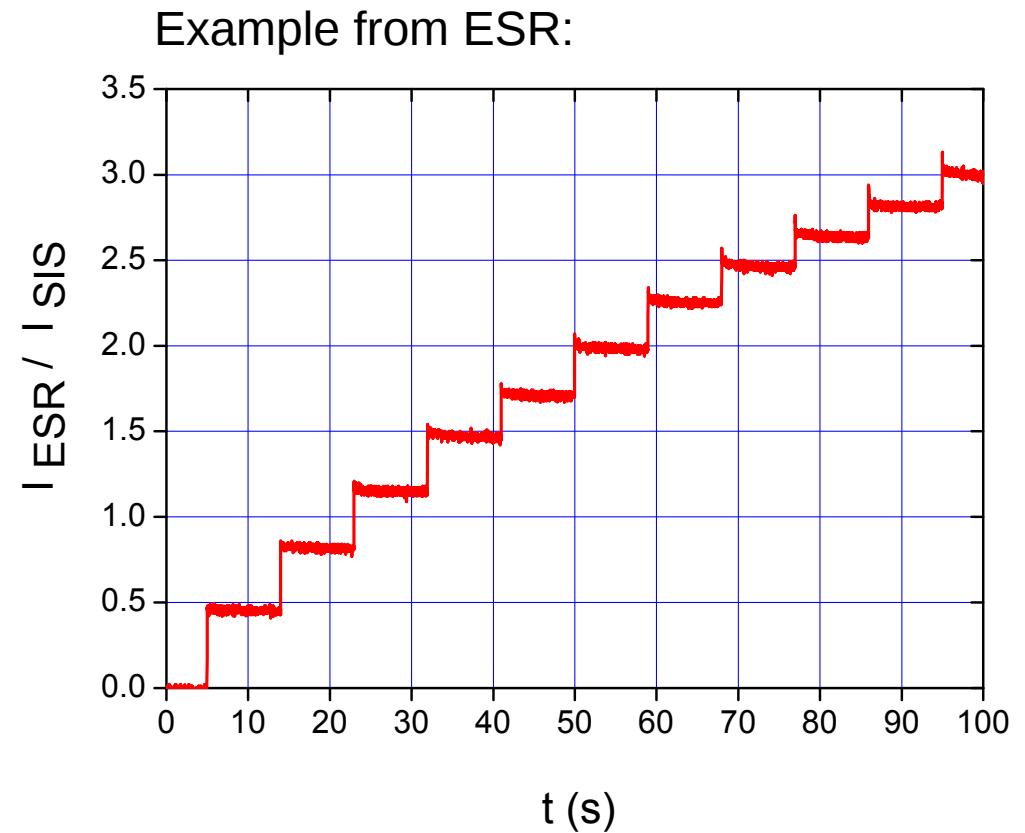
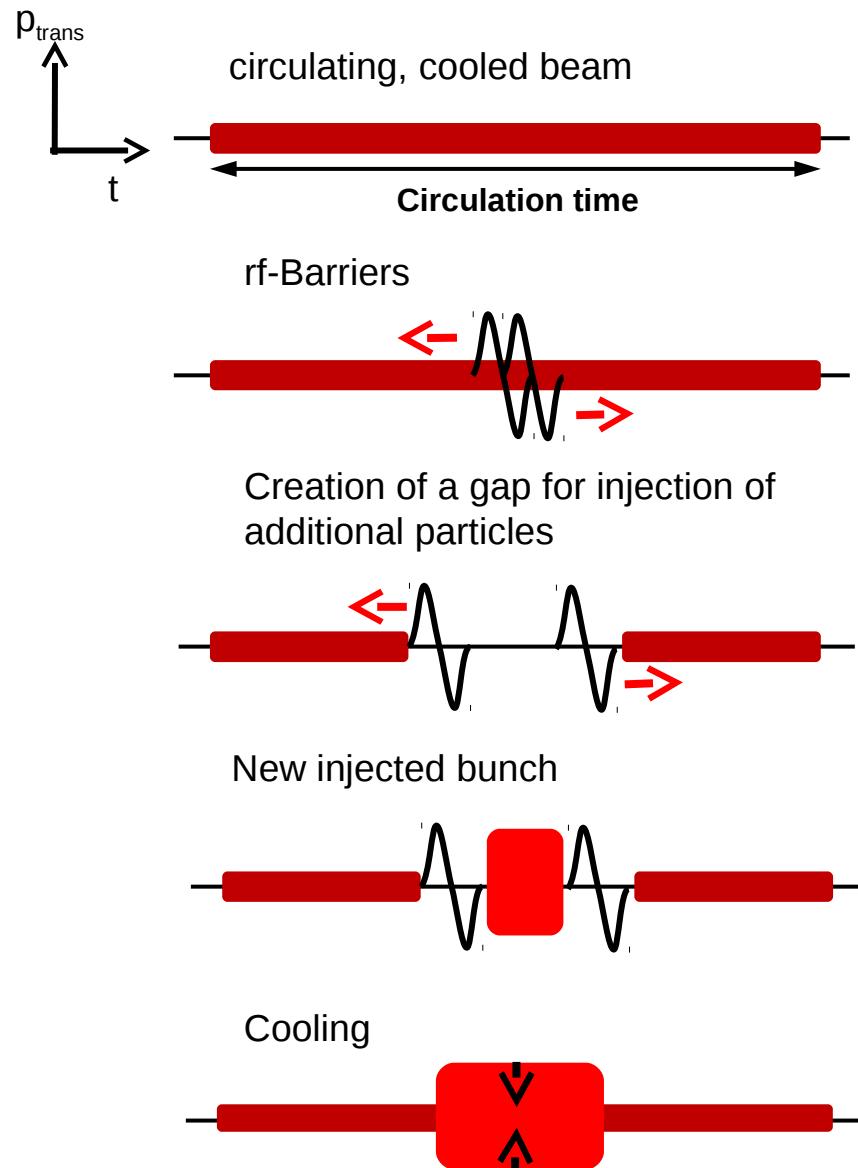


Simulation: with space charge



Injection losses \rightarrow dynamic vacuum pressure rise
(highly complex: easy to simulate \leftrightarrow hard to measure/tune with beam)
looking forward to: injection steering (BPMs) & turn-by-turn profiles (IPMs)

FAIR Storage Rings: Particle-Stacking & Cooling



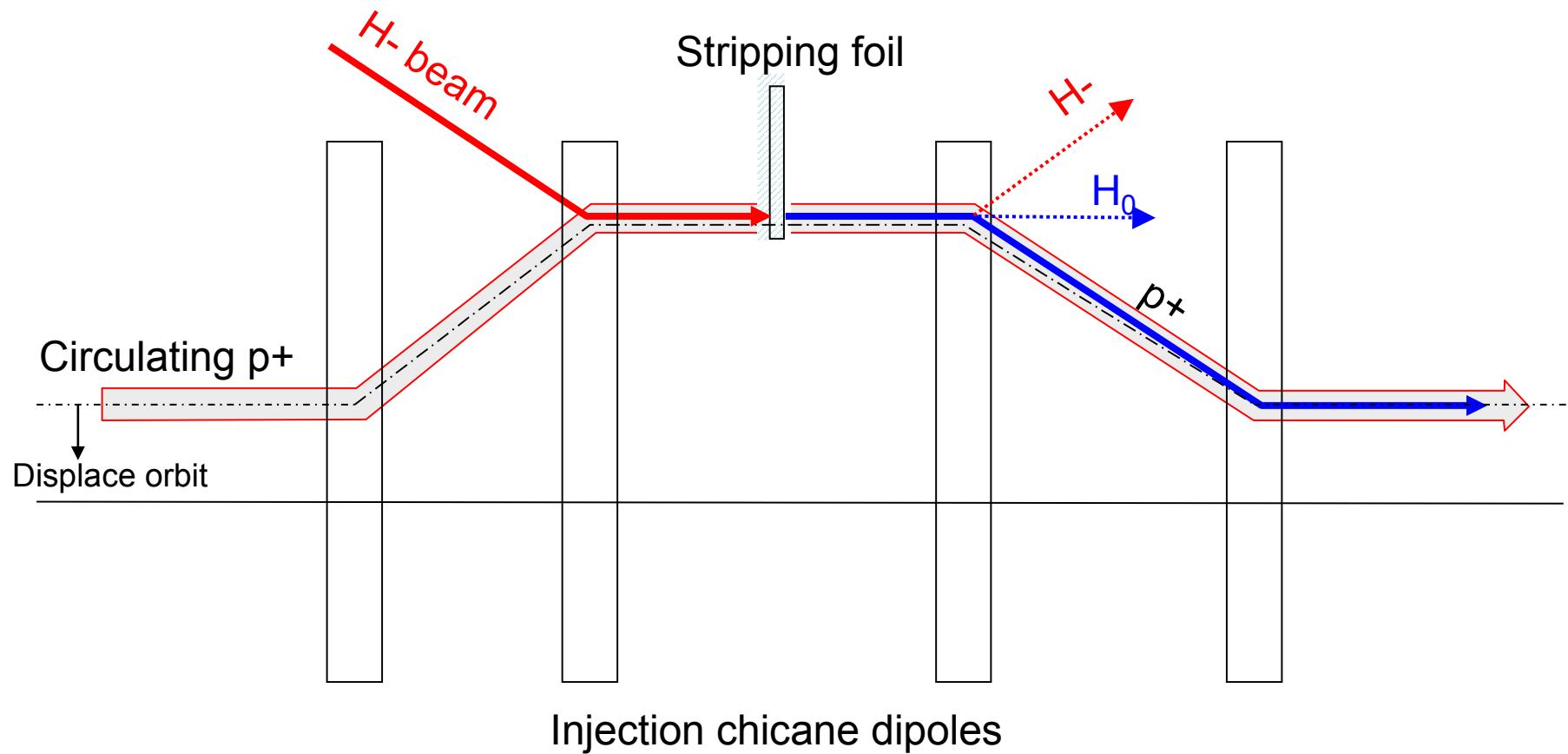
similar barrier-bucket schemes proposed also for SIS100!

Charge exchange H⁻ Injection

- Multi-turn injection is essential to accumulate high intensity
- Disadvantages inherent in using an injection septum
 - Width of several mm reduces aperture
 - Beam losses from circulating beam hitting septum
 - Limits number of injected turns to 10-20
- Charge-exchange injection used as an alternative (e.g. LINAC4)
 - “Beats” Liouville’s theorem, (conservation of emittance ...)
 - makes space-charge & emittance blow-up in LINAC less critical
 - Convert H⁻ to p⁺ using a thin stripping foil → injection into the same phase-space area

Charge exchange H⁻ Injection

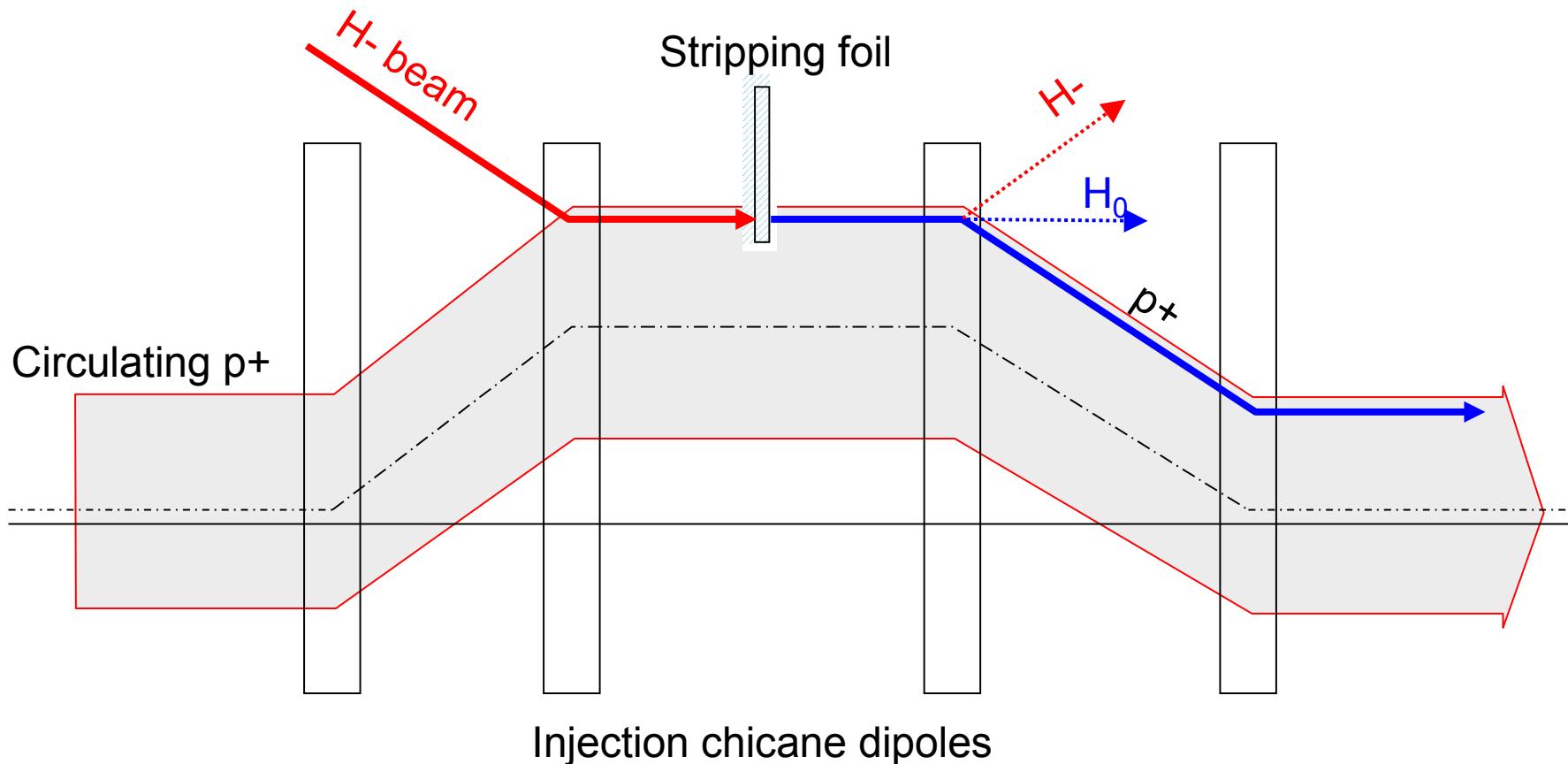
Start of injection process



Injection chicane dipoles

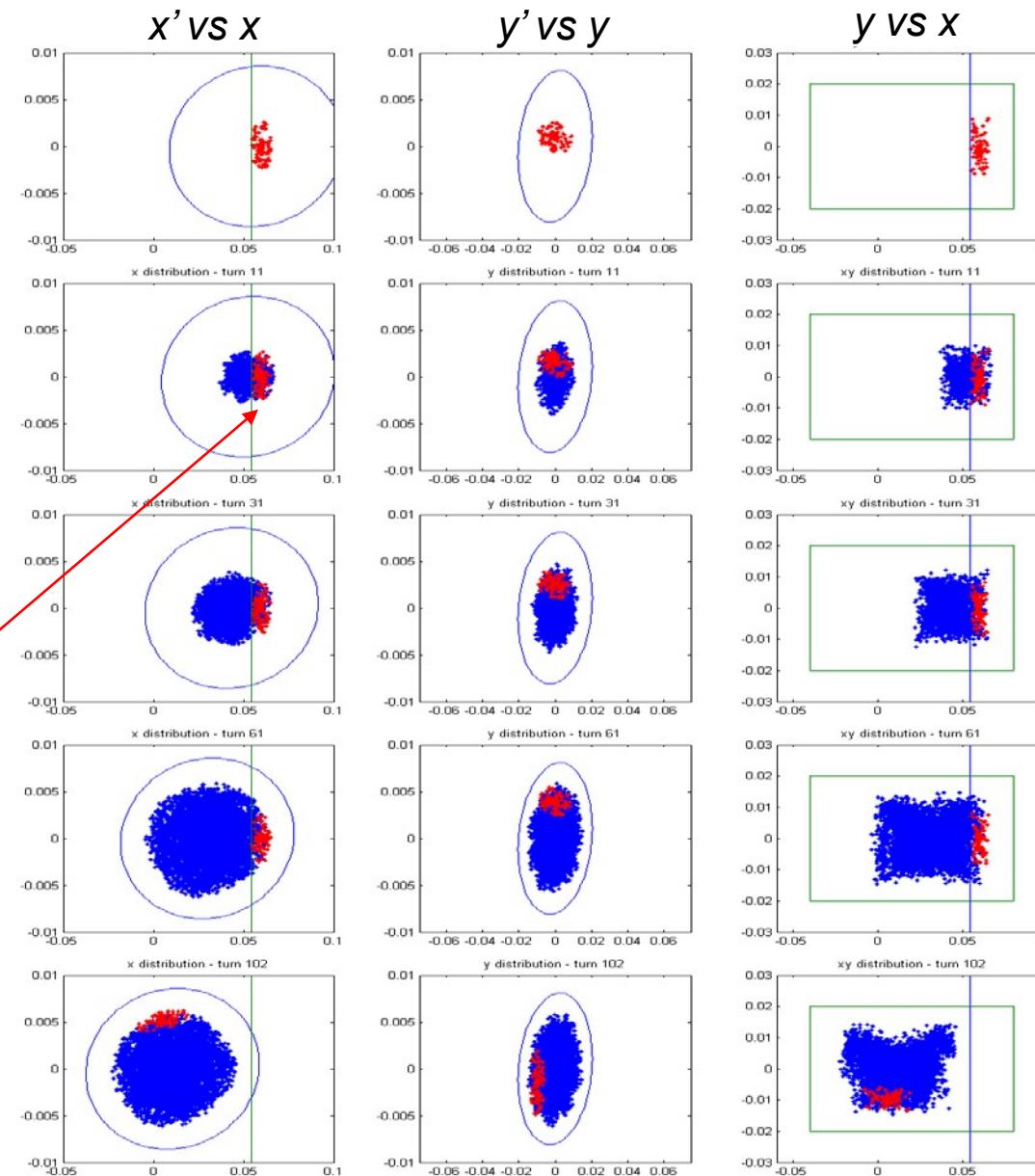
Charge exchange H⁻ Injection

End of injection process



Charge exchange H⁻ Injection

Note injection into
same phase
space area as
circulating beam

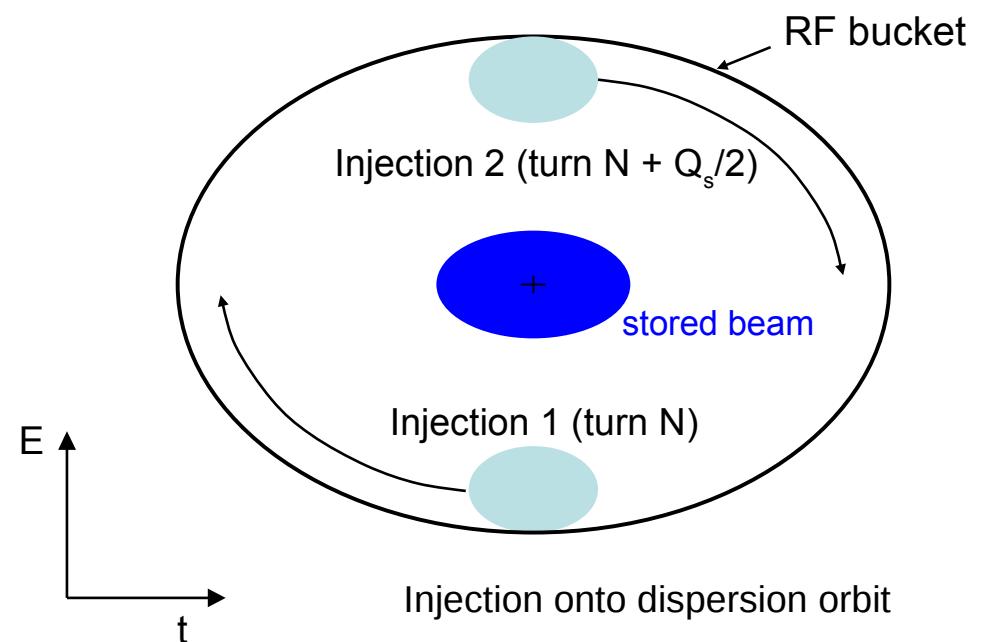
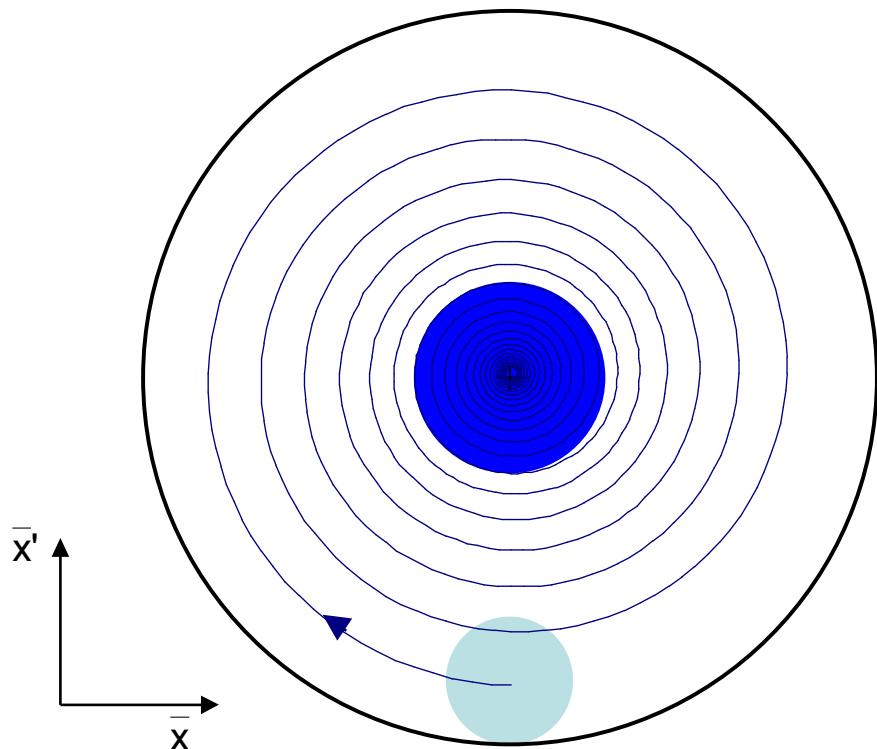


Time

~100 turns

Lepton Injection

- Single-turn injection can be used as for hadrons; however, lepton motion is strongly damped (different with respect to proton or ion injection)
 - synchrotron radiation makes lepton machine operators' life easy
- Can use transverse or longitudinal damping:
 - Transverse betatron accumulation
 - Longitudinal synchrotron accumulation

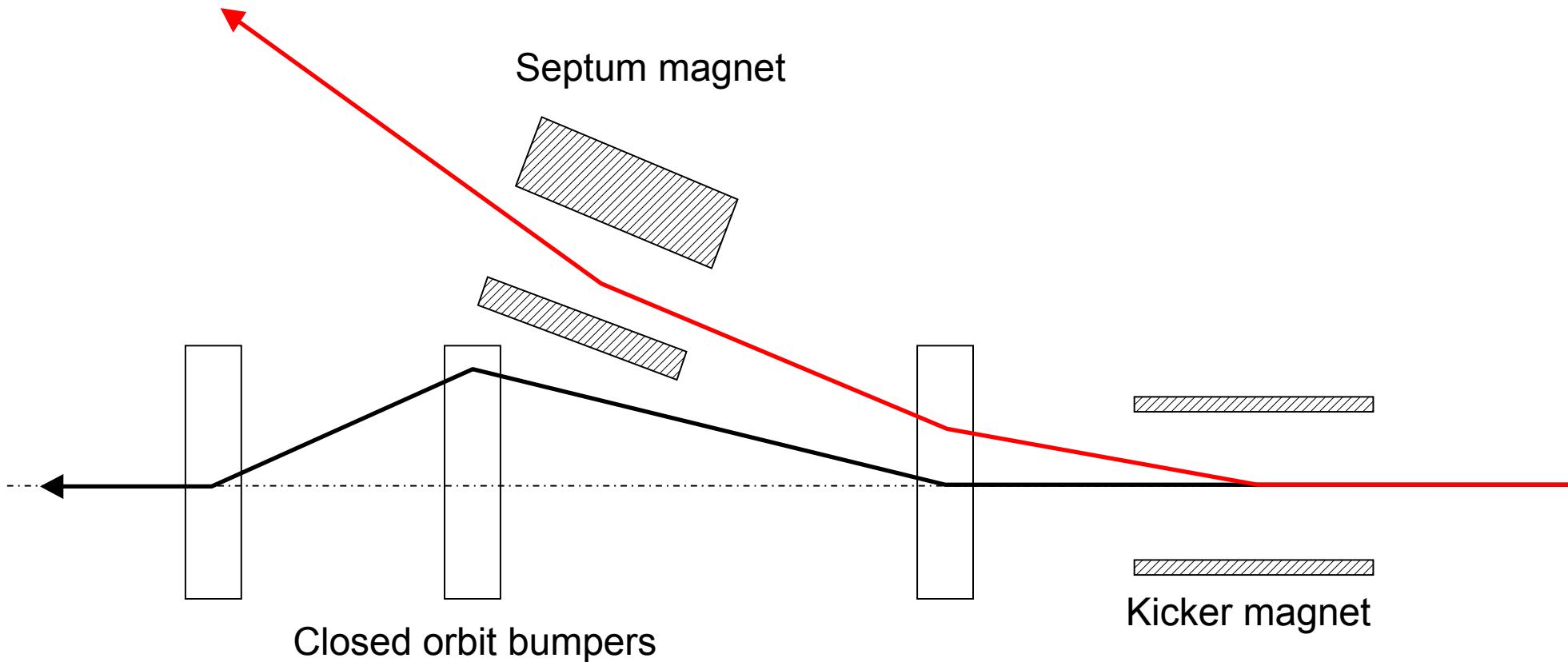


Extraction

- Usually at higher energy than injection – needs more $\int B \cdot dl$
- Different extraction techniques exist, depending on requirements
 - Fast extraction: ≤ 1 turn
 - Whole beam kicked into septum gap and extracted.
 - Non-resonant multi-turn extraction: few turns
 - Beam kicked to septum; part of beam ‘shaved’ off each turn.
 - Resonant multi-turn extraction: many thousands of turns
 - Non-linear fields excite resonances which drive the beam slowly across the septum.
 - Resonant low-loss multi-turn extraction: few turns
 - Non-linear fields used to trap ‘bunchlets’ in stable island. Beam then kicked across septum and extracted in a few turns
- To reduce kicker and septum strength, beam can be moved near to septum by closed orbit bump

Fast Single Turn Extraction

Whole beam kicked into septum gap and extracted.



- Kicker deflects the entire beam into the septum in a single turn
- Septum deflects the beam entire into the transfer line
- Most efficient (lowest deflection angles required) for $\pi/2$ phase advance between kicker and septum

Non-resonant Multi-Turn extraction I/II

Beam bumped to septum; part of beam ‘shaved’ off each turn.

Extracted beam

Septum

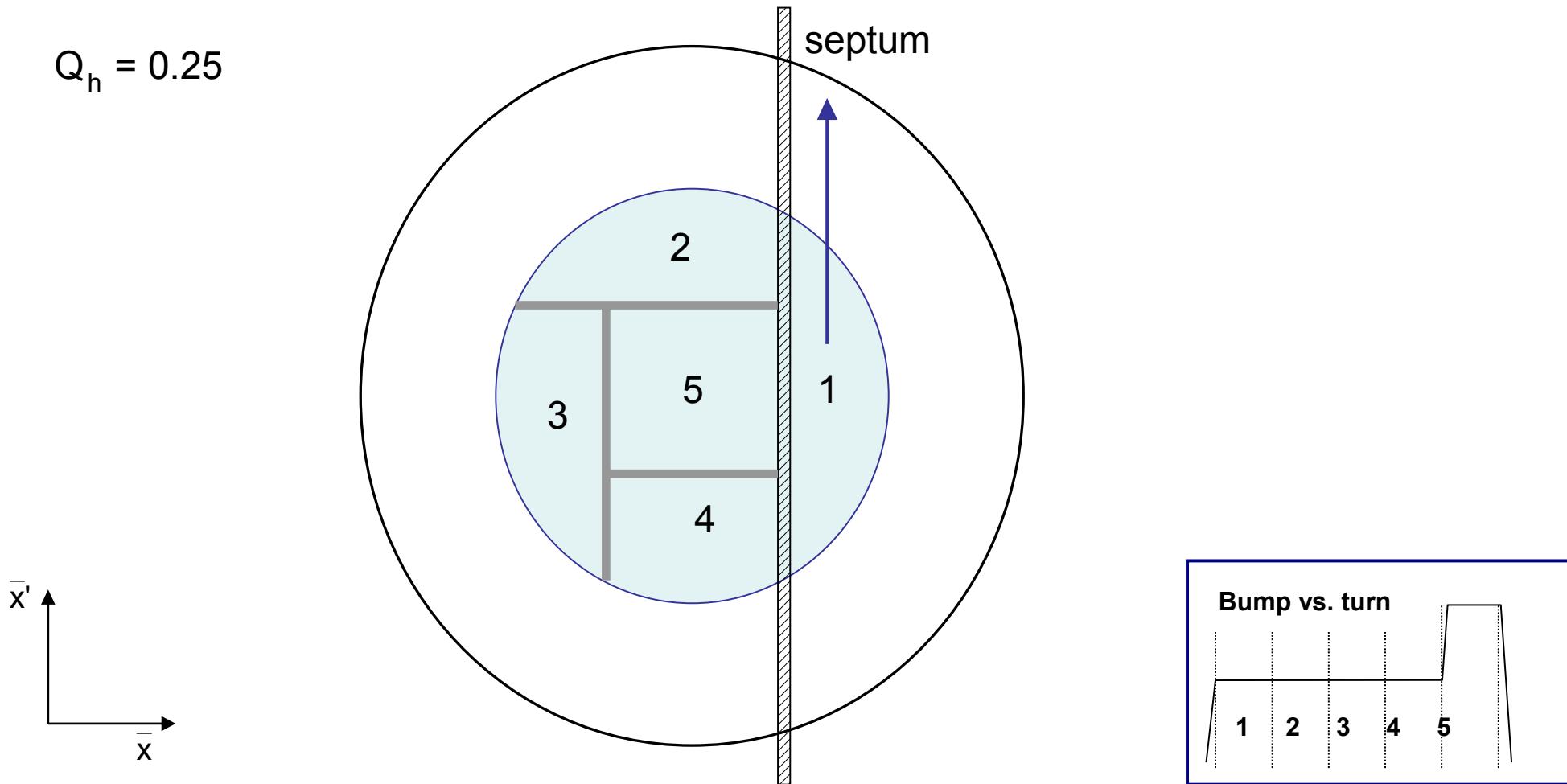
Bumped circulating beam

Fast closed orbit bumpers

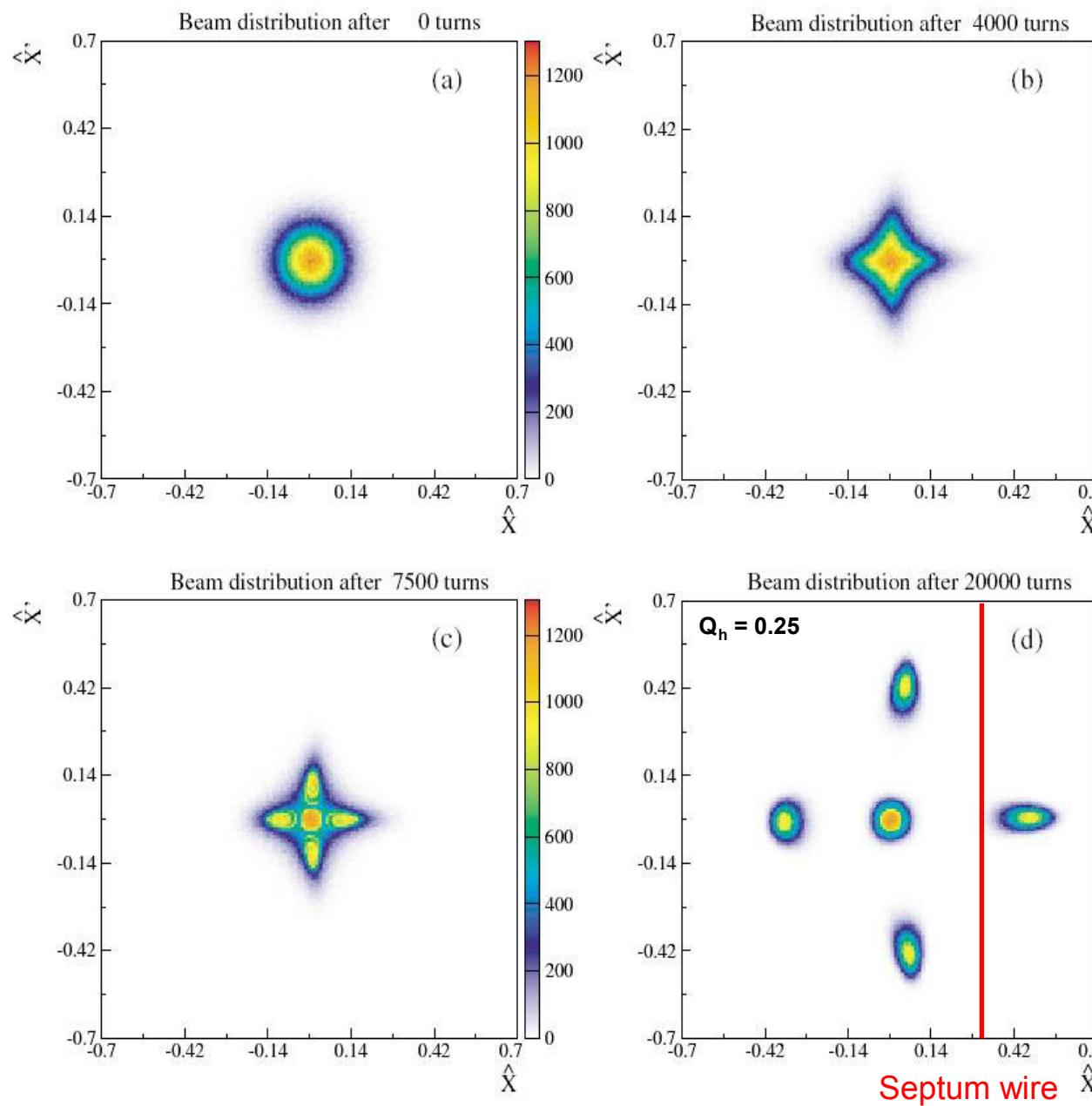
- Fast bumper deflects the whole beam onto the septum
- Beam extracted in a few turns, with the machine tune rotating the beam
- Intrinsically a high-loss process – thin septum essential

Non-resonant Multi-Turn extraction II/II

- Basically the reverse of the multi-turn injection (phase painting)
 - Example: CERN PS to SPS: 5-turn continuous transfer

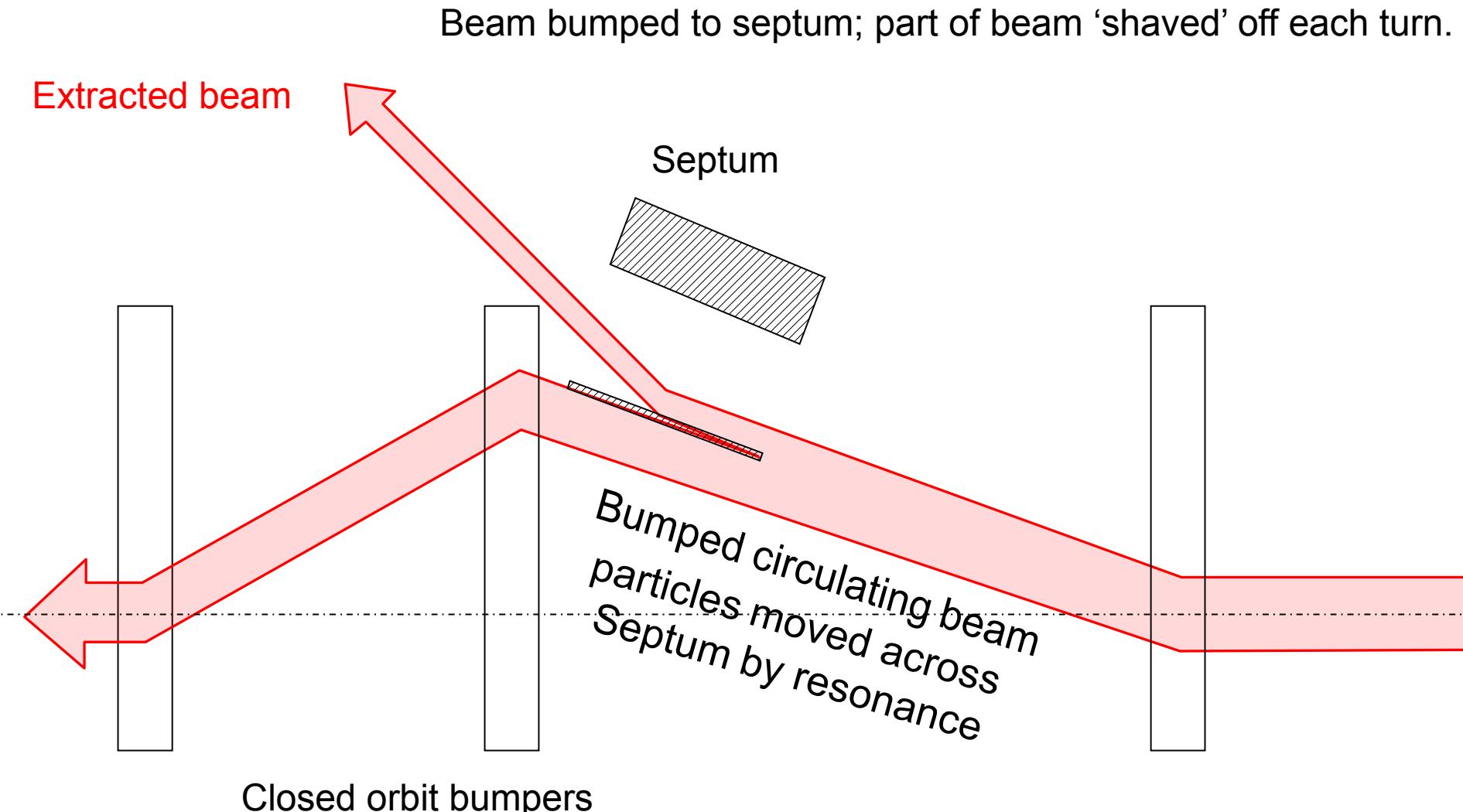


Resonant Low-Loss Multi-Turn Extraction



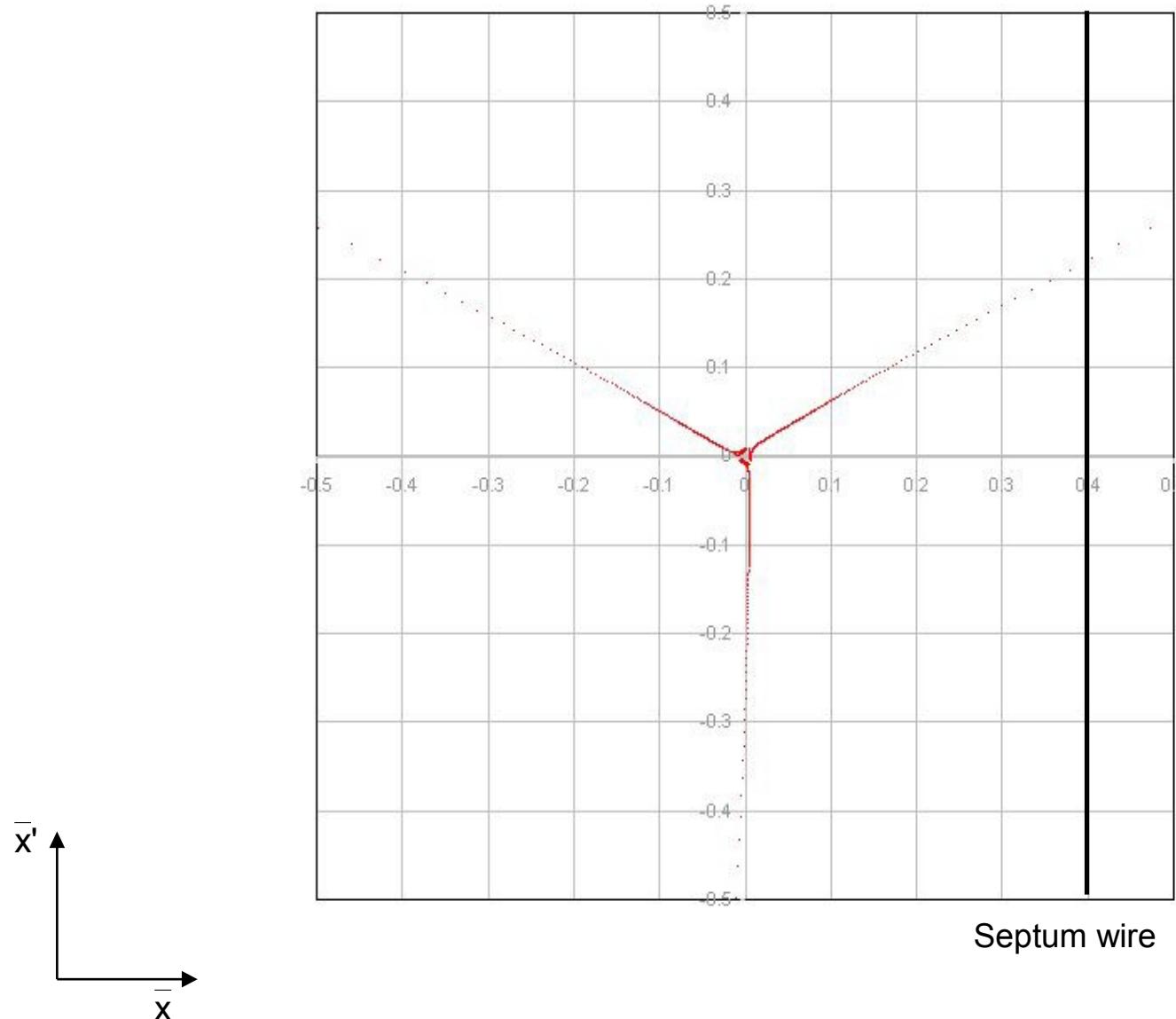
- a. Unperturbed beam
- b. Increasing non-linear fields
- c. Beam captured in stable islands
- d. Islands separated and beam bumped across septum – extracted in 5 turns

Resonant multi-turn extraction



- Slow bumpers move the beam near the septum
- Tune adjusted close to n^{th} order betatron resonance
- Multipole magnets excited to define stable area in phase-space, size depends on $Q = Q - Q_r$

Third-order resonant extraction

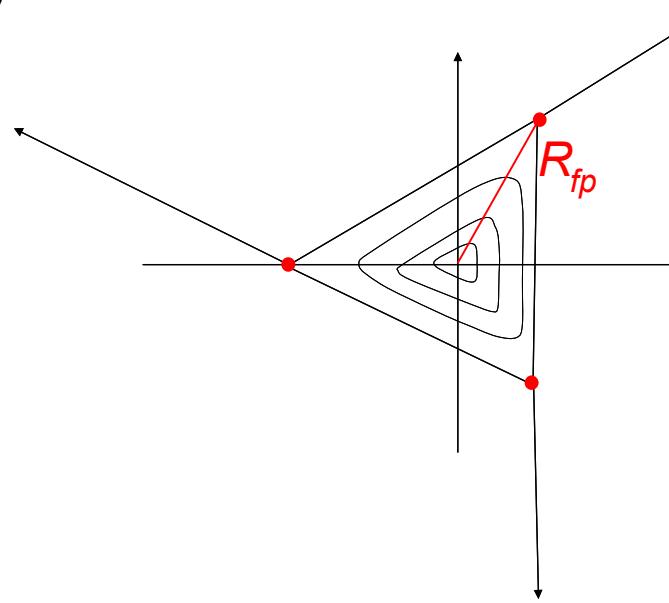


slow extraction finished

Resonant Multi-Turn Extraction

- 3rd order resonances
 - Sextupole fields distort the circular normalised phase space particle trajectories.
 - Stable area defined, delimited by unstable Fixed Points.

$$\sqrt{R_{fp}} \propto \frac{\Delta Q}{m_{sext}}$$

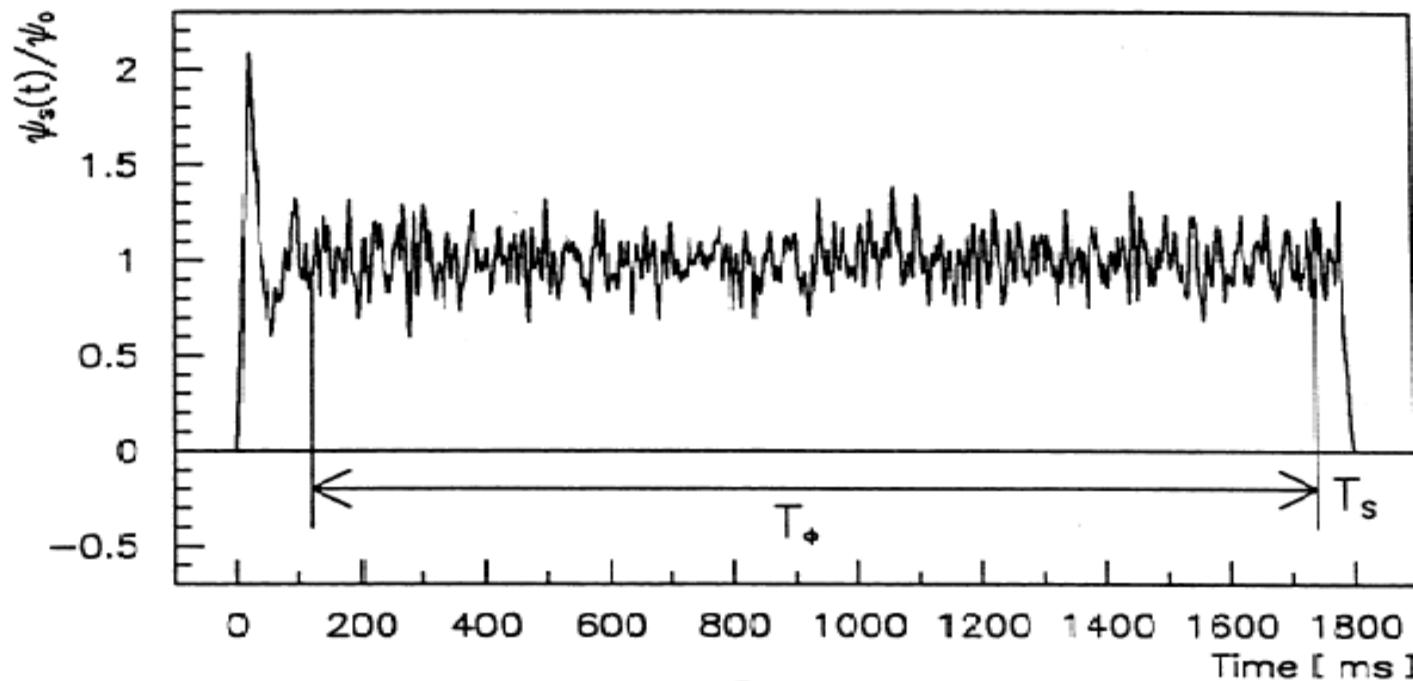


- Sextupoles families arranged to produce suitable phase space orientation of the stable triangle at thin electrostatic septum
- Stable area can be reduced by increasing the sextupole strength, or (easier) by approaching machine tune Q to resonant 1/3 integer tune
- Reducing ΔQ with main machine quadrupoles can be augmented with a ‘servo’ quadrupole, which can modulate ΔQ in a servo loop, acting on a measurement of the spill intensity

Third-order resonant extraction

Example – SPS slow extraction at 450 GeV/c.

$\sim 3 \times 10^{13}$ p+ extracted in a 2-4 second long spill ($\sim 200,000$ turns)



Intensity vs time:
 $\sim 10^8$ p+ extracted per turn

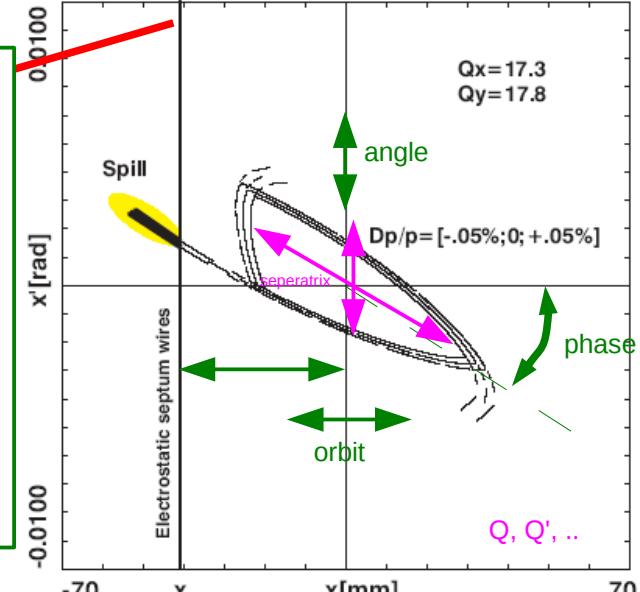
Slow Extraction from SIS-100

Intense Heavy-Ion Beams for NuSTAR & CBM

Ion	Energy	N/s	spill	Power
U^{28+}	1.5 GeV/u	5E11	> 1 s	10 kW



Optics, $Q/Q'(","")$ drive uncertainties on slow-extraction performance
→ remedy: control of the machine optics, Q/Q' , linearisation prior to s.e., ...
(highly complex, a lot of work ongoing)

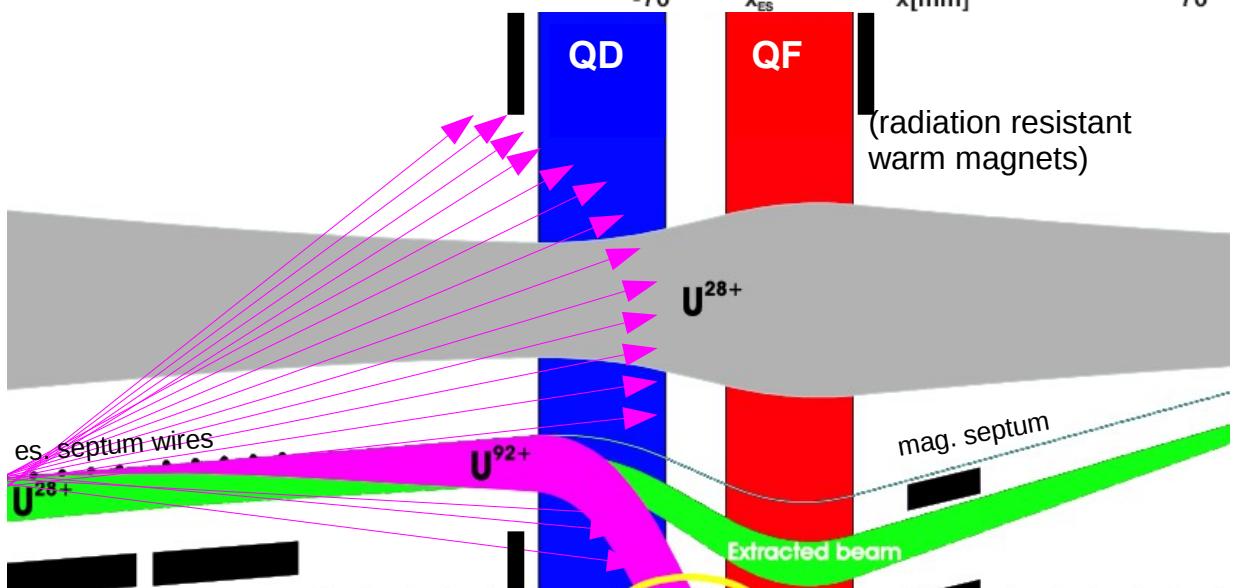


Tracking simulations:

5 % (approx. 500 W) loss in the septum wires
 U^{92+} beam loss in warm magnet > 5 W/m

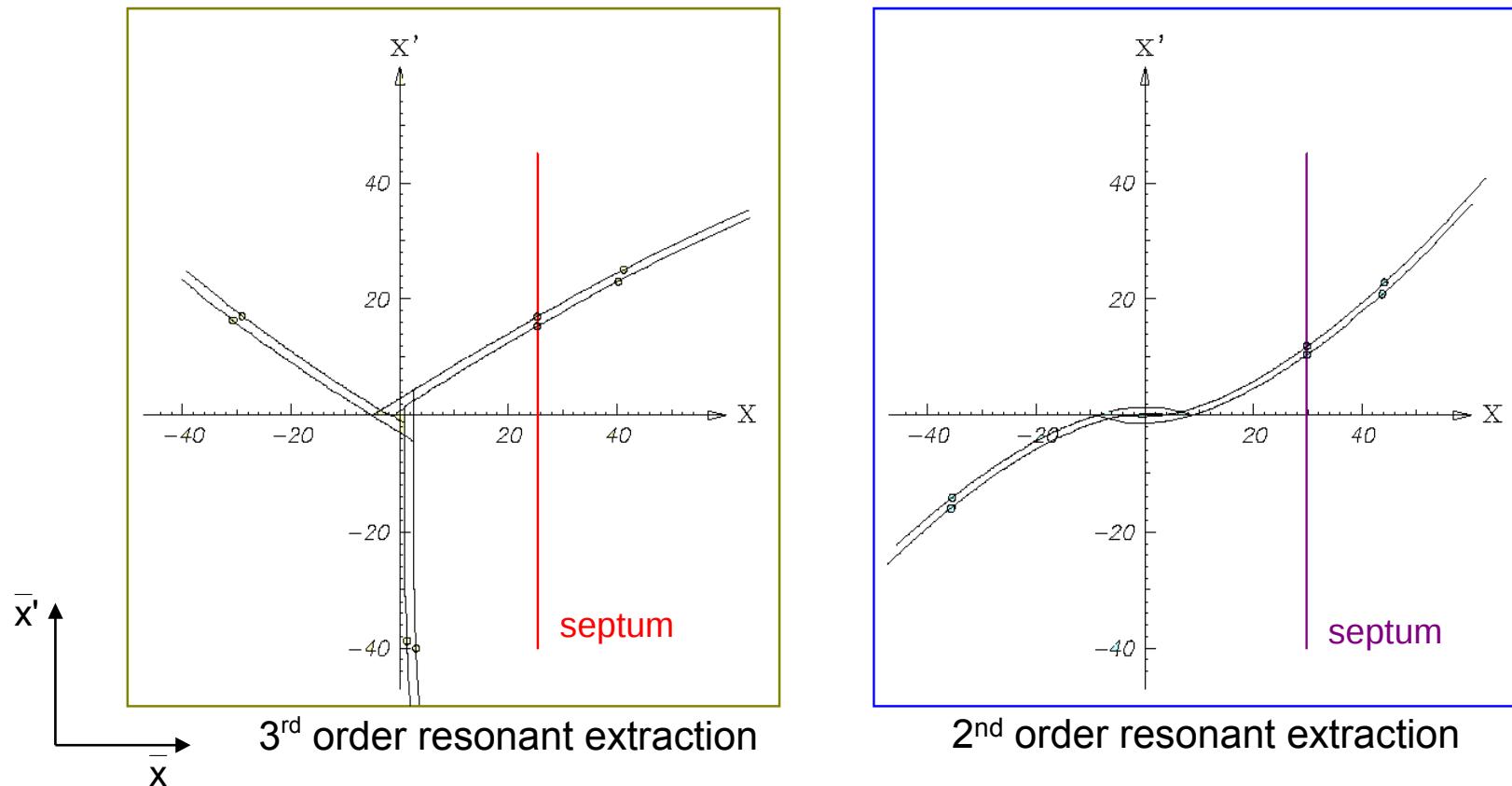
Non-trivial machine protection:

protection of septa wires
down-stream absorbers setup
activation minimisation



Second-order resonant extraction

- An extraction can also be made over a few hundred turns using 2nd or 4th order resonances:

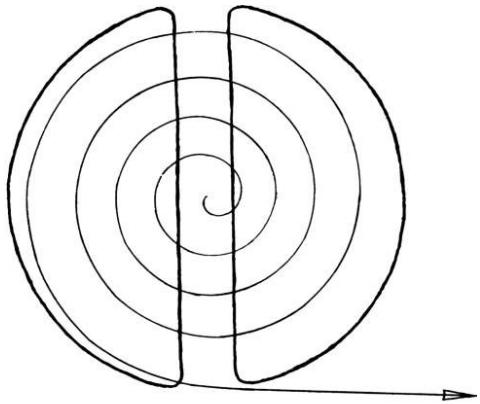


- Amplitude growth for 2nd order resonance much faster than 3rd – shorter spill
- Used where intense pulses are required on target – e.g. neutrino production

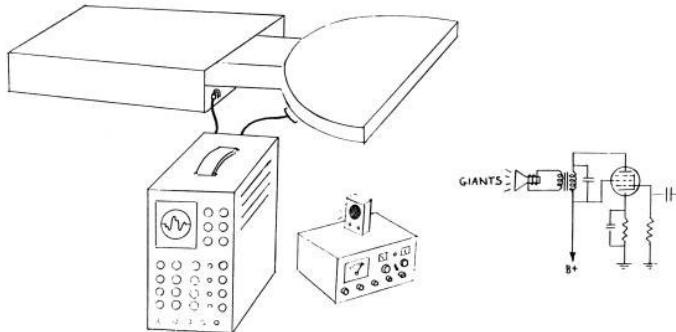
That's all – Questions?



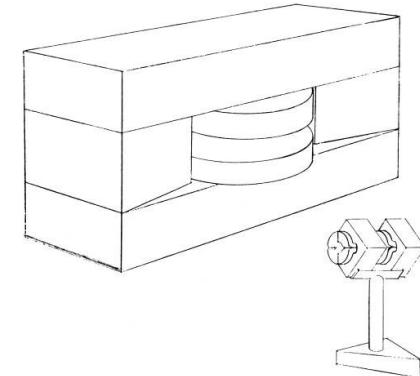
The Accelerator seen by ...



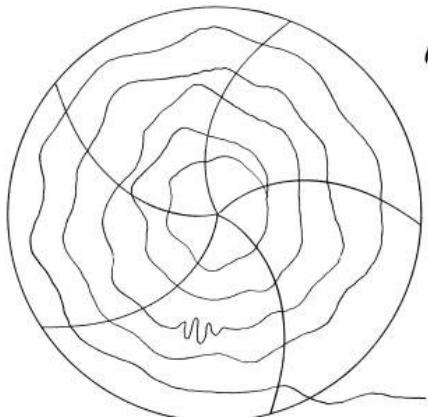
... the inventor



... the electrical engineer



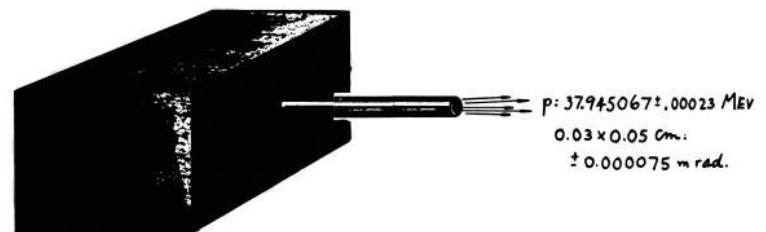
... the mechanical engineer



... the theoretical physicist

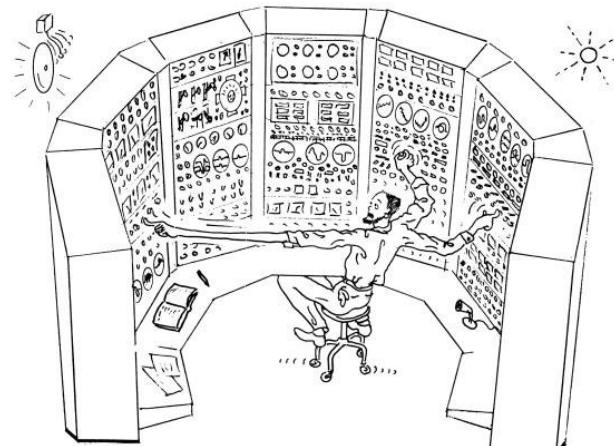
$$r = r_0 \left[1 + \left(\frac{fr\omega}{c} \right) \cos(3\theta + \delta_0 + \delta_s r) + \left(\frac{fr\omega}{c} \right)^2 \cos(5\theta + \delta_s - \delta_{s'} r^2) + \left(\frac{fr\omega}{c} \right)^3 \cos(7\theta + \delta_s - \delta_{s'} r^3) + \dots \right] \times \left\{ \frac{e^{2\pi r^2 \ln Z}}{1 + \left(\frac{r}{r_0} \right)^{2\alpha}} \right\}$$

$$\frac{d\theta}{dt} = \left[\sin(\omega t - \delta\theta) - \sin(\delta\theta - \frac{3}{2}ff_s f_s') \right] \frac{eV_0}{2\pi\omega}$$

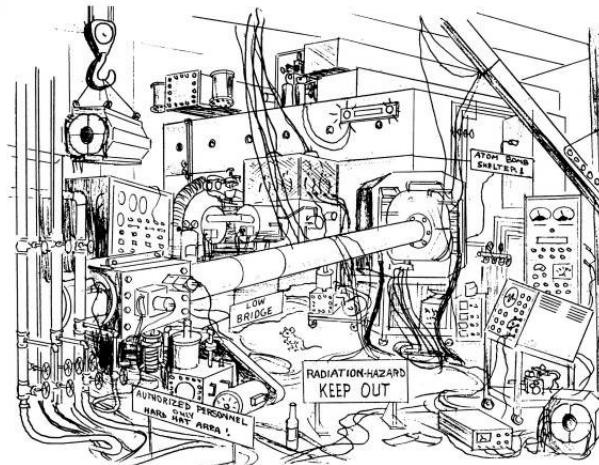


... the experimental physicist

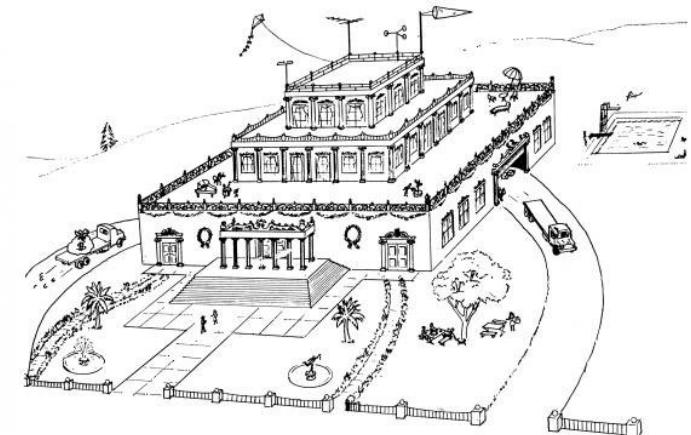
The Accelerator seen by ...



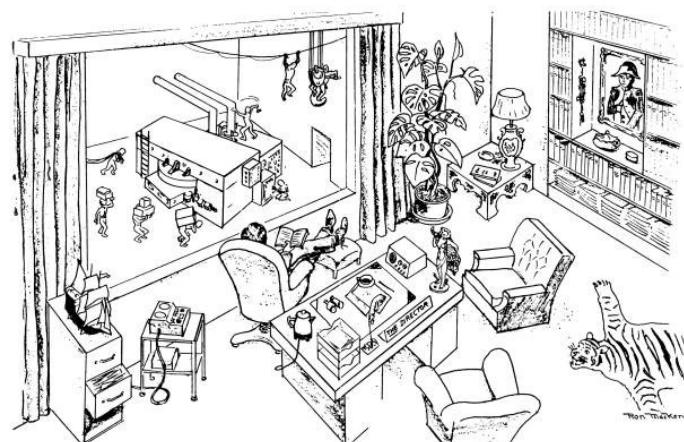
... the operator



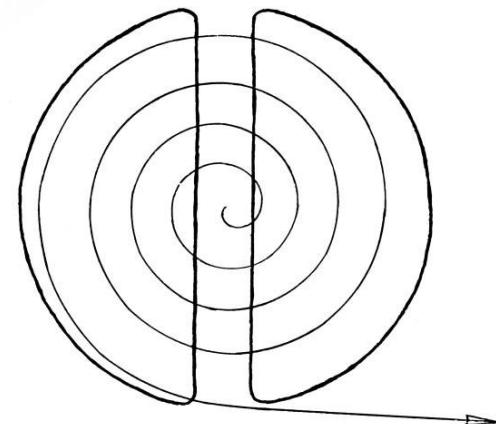
... the visitor



... the governmental funding agency



... the laboratory director



... the student

Cartoons: Dave Judd and Ronn MacKenzie