Introduction to Accelerator Physics

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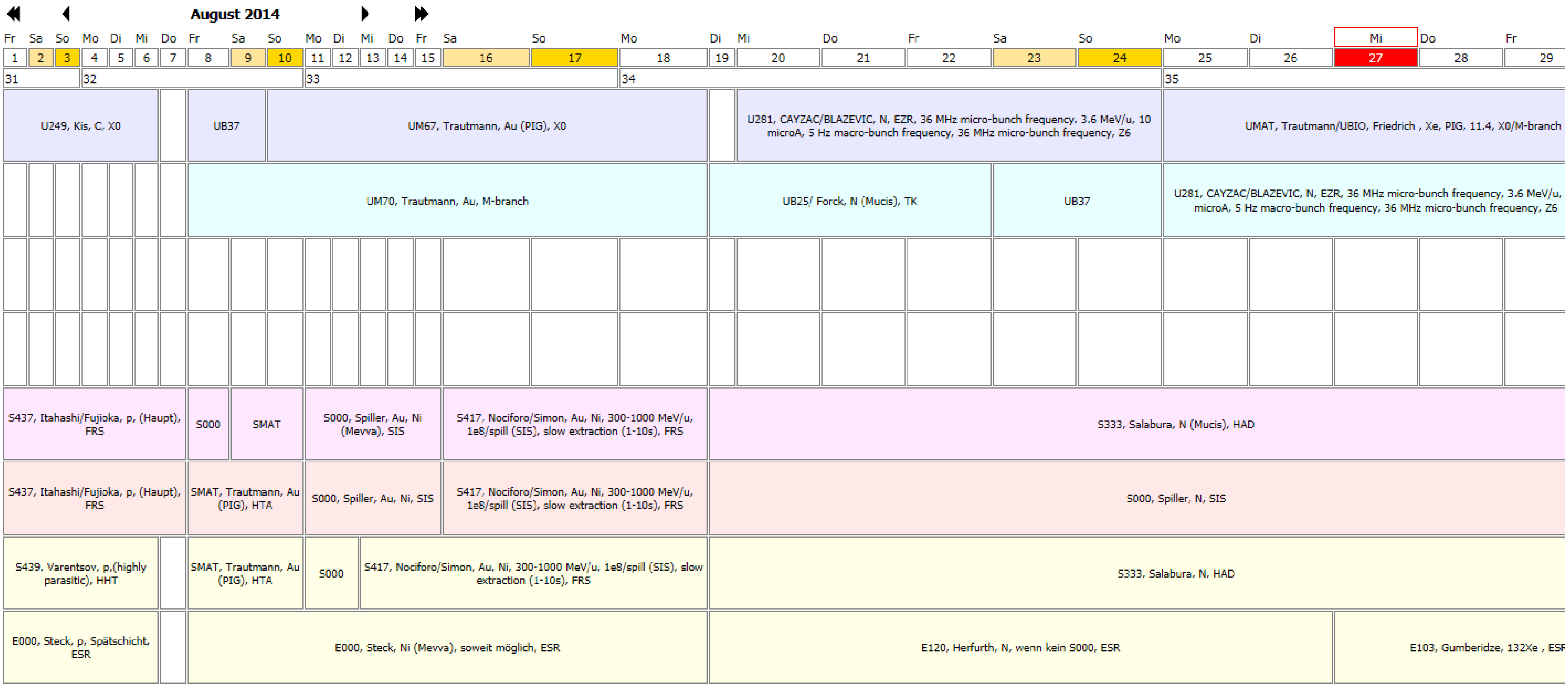
Outline

- Introduction
- Particles and units
- Relativity
- Charged particles in fields
- Magnets
- Charged particle dynamics
- Emittance and acceptance

Experimenters wishes



- What are these fancy symbols here about: p, Au, N, Ni ?
- What do these funny numbers mean: 3.6MeV/u, 300-1000MeV/u, 1e8/spill ?
- What do our accelerators have to do with all this?

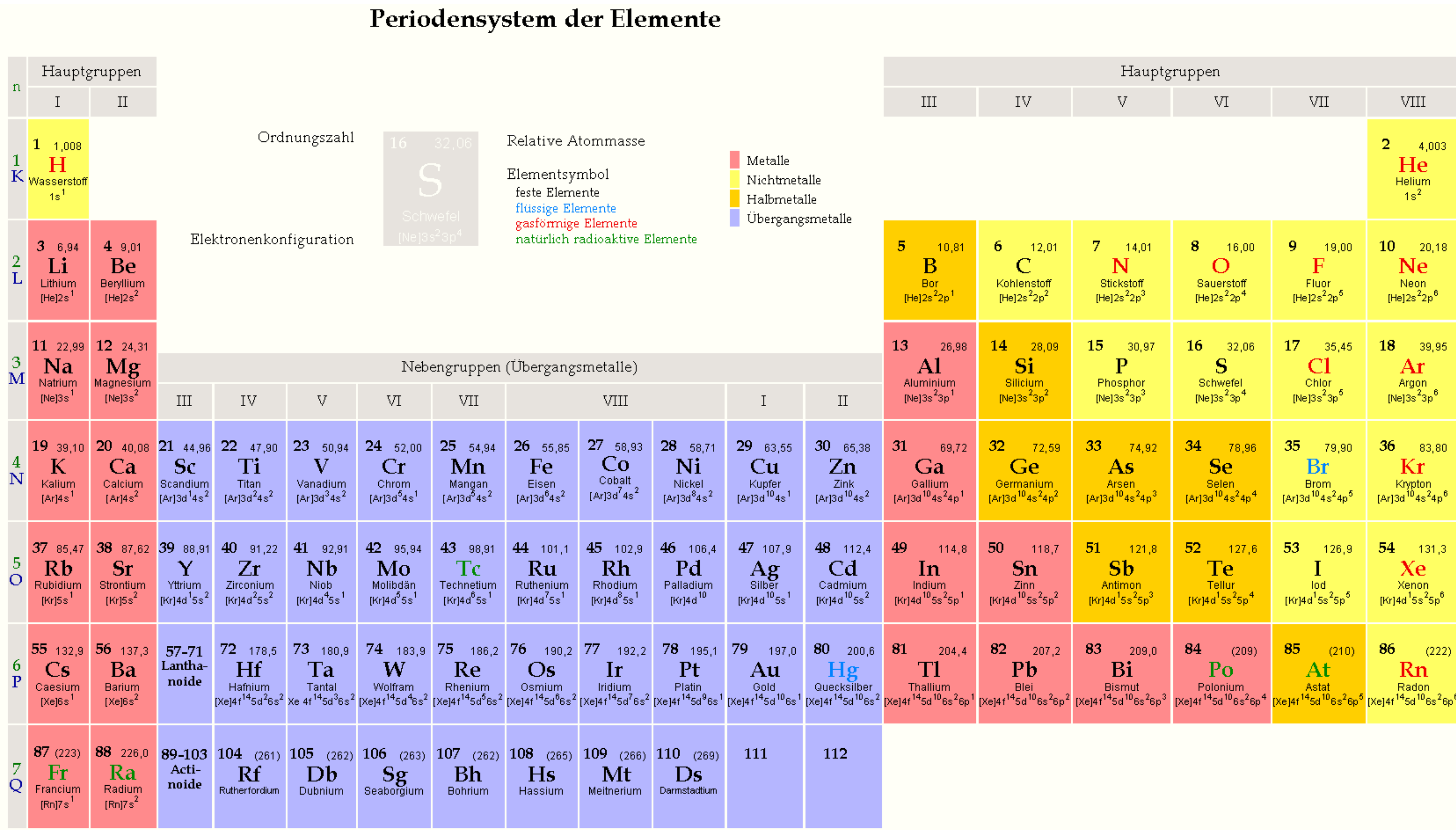
Elements and Isotopes

- Elements
  - atomic building blocks of matter
  - characterized by atomic number Z
    - number of protons in nucleus and electrons in neutral atom
    - one-to-one correspondence to chemical symbol X
- Isotopes
  - variants of the same element with different number N of neutrons
  - identified by mass number A = Z + N
  - same electron configuration, hence same chemical properties
  - only few isotopes stable
- Notation ^{A}X

	Z	A	N	Abundance
^{40}Ca	20	40	20	96.9%
^{48}Ca	20	48	28	0.2%

neutron-rich nucleus, often used for SHE experiments

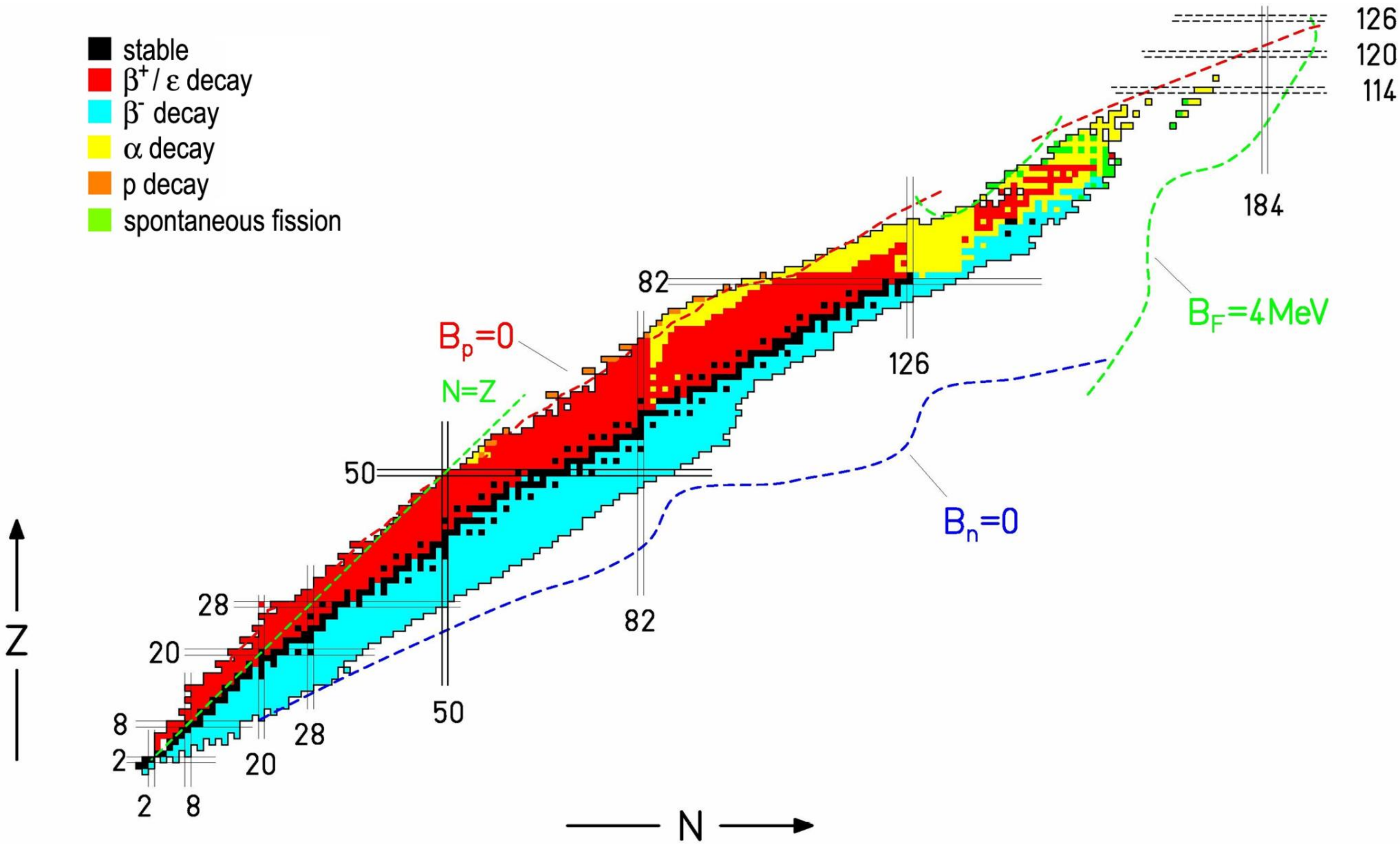
Elements



(Source: http://www.pctheory.uni-ulm.de/didactics/quantenchemie/html/PSE-F.html)

Ordering of elements according to atomic number Z and electron configuration

Isotopes



Source: http://moriond.in2p3.fr/radio/moriond-huyse.ppt

Ordering of elements according to proton number Z and neutron number N

Ions

- Acceleration requires charged particles (we'll come to that...)
- Ions
  - created by removing electrons from atoms (or molecules)
  - characterized by their charge state Q
  - notation for ions of atoms (i.e. isotopes)

    $^{A}X^{Q}$

	Z	Q	#electrons
^{48}Ca	20	0	20
$^{48}Ca^{10+}$	20	10	10

  - notation for molecules similar
    - examples: H_2^+, H_3^+, CH_3^+
  - creation of ions from neutral particles in ion sources
  - increasing of charge state by stripping in gas or foils

Ion Masses

- Acceleration of ions depends on mass of ion
  - Need to know the masses of ions
- SI units impractical:
  - $m_p \approx m_n \approx 1.7 \cdot 10^{-27}$ kg
- Atomic mass units (AMU)
  - definition of atomic mass unit:

    $m(^{12}C) = 12u = A \cdot u$
  - for general isotopes we define a mass number M:

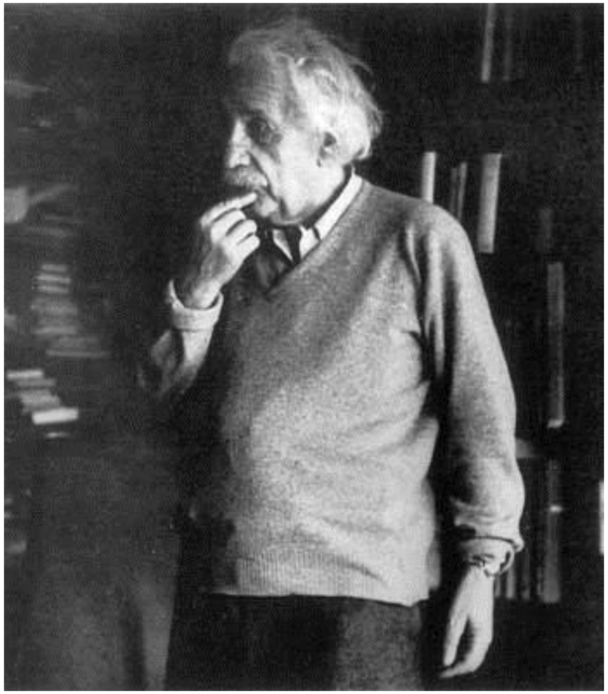
    $m(^{A}X) = M \cdot u \approx A \cdot u$
  - small difference between A and M:
    - mass excess ME: $M = A + ME$ ME/M < 0,1% (except e.g. p, d) *but this is the origin of nuclear power...*
    - missing electrons for ions: $m(^{A}X^{Q}) = (M - Q \cdot A_e) \cdot u$ $Z \cdot A_e / M < 0{,}05\%$
    - significant for high precision exp. (e.g. mass measurements in storage rings)

	SI units	AMU	ME
u	$1.661 \cdot 10^{-27}$ kg	1u	0
m_p	$1.672 \cdot 10^{-27}$ kg	1.007u	0.007
m_n	$1.675 \cdot 10^{-27}$ kg	1.009u	0.009
$m(^{12}C)$	$1.993 \cdot 10^{-26}$ kg	12u	0
$m(^{48}Ca)$	$7.965 \cdot 10^{-26}$ kg	47.953u	-0.047
m_e	$9.109 \cdot 10^{-31}$ kg	$5.5 \cdot 10^{-4}$u	n.d.

Ion Energies

- Experimentalists require ions with a certain kinetic energy
  - usually specified as kinetic energy per atomic mass unit
  - example: $^{238}U^{73+}$, E=1GeV/u, $N_{ions}=10^9$ particles
  - what's this mysterious 'GeV'?
    - elementary charge: $e = 1.602 \cdot 10^{-19}C$ charge of proton and electron (up to sign)
    - when pushed by a voltage of 1V, a particle with charge e gains energy 1eV
    - in SI units: $1eV = 1.602 \cdot 10^{-19}J$
    - prefixes for saving digits: 1000eV=1keV, 1000keV=1MeV, 1000MeV=1GeV
  - what's the kinetic energy of the beam in the example?
    - A = 238, ME = 0.05u, Q = 73 → M = 238.01, m = M·u
    - $E_{ion} = M \cdot u \cdot E = 238.01GeV$
    - $E_{beam} = N_{ions} \cdot E_{ion} = 10^9 \cdot 238.01GeV = 238.01 \cdot 10^{18}eV \approx 40J$
      - kinetic energy of a walking man: $E = 1/2 \cdot m \cdot v^2 = 1/2 \cdot 80kg \cdot 1(m/s)^2 = 40J$
      - if deposited in 1ml of water: heating by 10 degrees, dose 40000Gy

Relativity: Mass and Energy



- Mass and energy are equivalent

$$E = mc^2$$

- What's the energy of a mass unit?
  - speed of light (constant of nature):

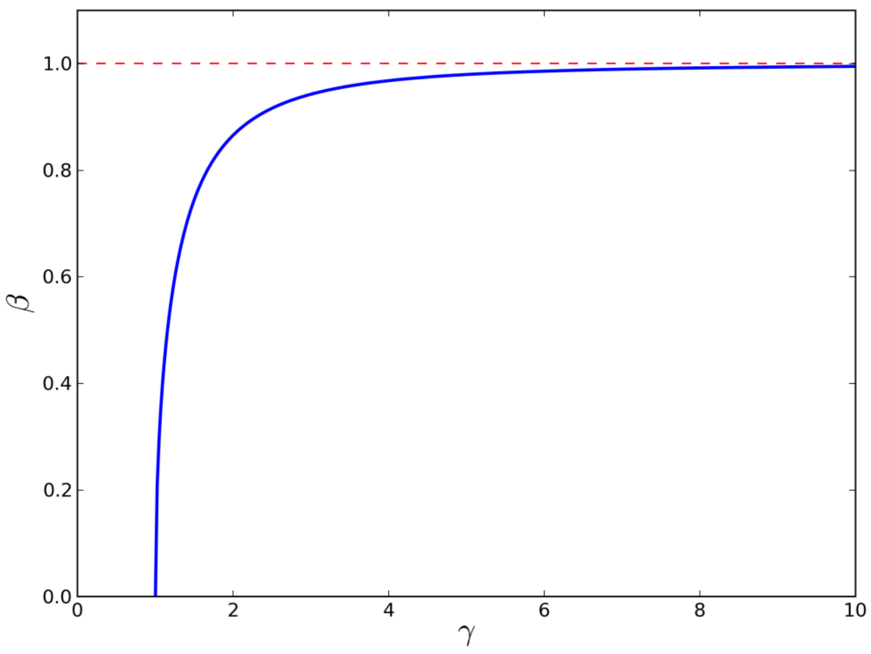
    $c = 299792458$ m/s
  - energy of 1u:
    - $u \cdot c^2 \approx 1.661 \cdot 10^{-27} kg \cdot (2.998 \cdot 10^8 m/s)^2 \approx 1.4925 \cdot 10^{-8} J$
    - $1 eV \approx 1.602 \cdot 10^{-19} J$
    - $u \cdot c^2 \approx 931.6 \cdot 10^6 eV$
    - precise value: $u \cdot c^2 = 931.494$ MeV
- Expression of masses as energies in convenient units
  - consider $^{238}U^{73+}$: $m \cdot c^2 = M \cdot u \cdot c^2 = 238.01 \cdot u \cdot c^2 = 221.7 GeV$
  - also written as: $m = 221.7 GeV/c^2$

Relativity: Energy

- Energy equivalent to mass is referred to as **rest energy**
  - will use symbol E_0 from now on: $E_0 = mc^2 = M \cdot uc^2$
  - use special symbol E_u for amu: $E_u = uc^2$
- **Kinetic energy** of ion always proportional to mass
  - write kinetic energy as: $E_{ion} = M \cdot E$
  - E is referred to as **kinetic energy per nucleon** (or per atomic mass unit)
- **Total energy** is the sum of rest and kinetic energy
  - again proportional to mass: $M \cdot E_{tot} = M \cdot (E + E_u)$
- Comparison of total energy to rest energy
  - defines the **relativistic gamma**: $\gamma = E_{tot}/E_u = 1 + E/E_u$
  - examples:
    - injection into SIS18: E = 11.4MeV/u → $\gamma = 1 + 11.4/931.5 = 1.01$
    - extraction from SIS18: E = 1GeV/u → $\gamma = 1 + 1000/931.5 = 2.07$

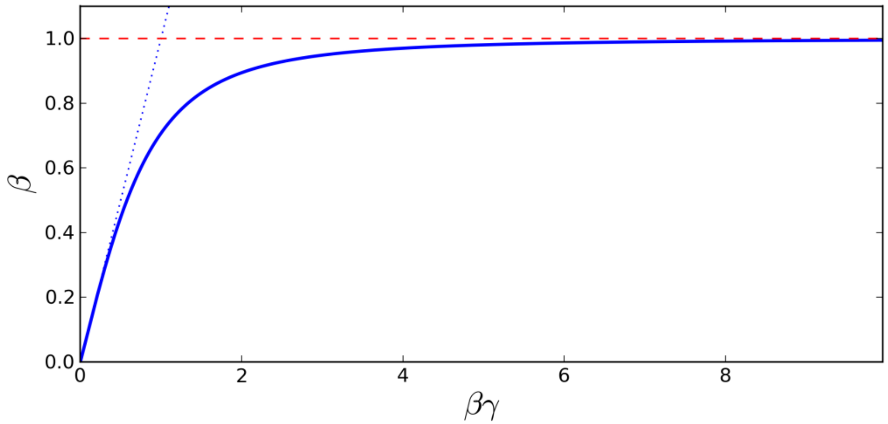
Relativity: Velocity

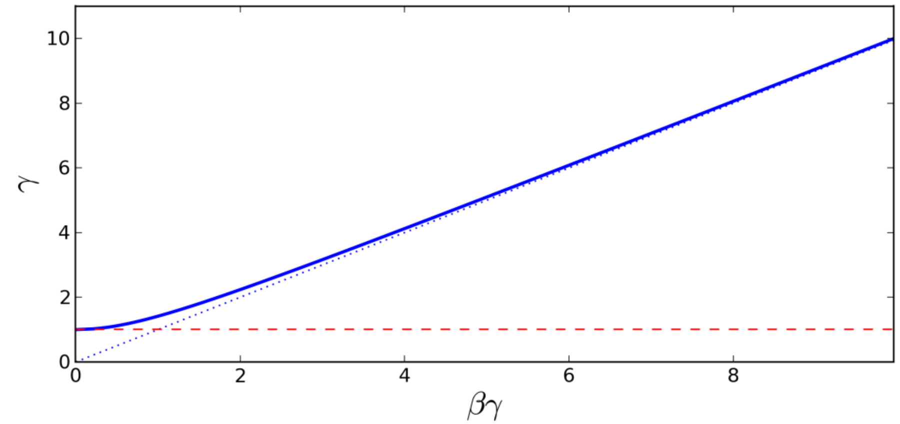
- Velocity expressed in units of the speed of light
  - defines **relativistic beta**: $v = \beta \cdot c$
- Gamma and beta are related
  - Lorentz factor: $\gamma = 1/\sqrt{(1 - \beta^2)}$
  - can be inverted: $\beta = \sqrt{(1 - 1/\gamma^2)}$
- Velocity bound by speed of light
  - Look again at $\gamma = 1 + E/E_u$
    - $E=0 \rightarrow \gamma = 1 \rightarrow \beta = 0$
    - $E \to \infty \rightarrow \gamma \to \infty \rightarrow \beta \to 1$
    - range of beta: $0 \leq \beta < 1$
  - Ions never reach the speed of light
    - but they can come quite close:
      (e.g. p@LHC: E=7TeV, $\gamma \approx 7500$, $1-\beta \approx 10^{-8}$)



Relativity: Momentum

- Momentum is an important kinematical quantity
  - conserved if no forces act on the particle
  - deflection in a magnet is proportional to momentum (we'll come to that...)
- Related to the other relativistic parameters
  - proportional to mass: $p_{ion} = M \cdot p$
  - non-linear relation to velocity: $pc = \gamma \cdot \beta \cdot E_u = \beta / \sqrt{(1 - \beta^2)} \cdot E_u$
  - non-linear relation to energy: $pc = \sqrt{(E \cdot (E + 2 \cdot E_u))}$
  - example: E = 1GeV → pc = 1692GeV or p = 1692GeV/c *here we've omitted the /u for convenience...*





Relativity: Non-Relativistic Limit

- Newton's physics recovered for small energies
  - Newton's relations for momentum:

    $p = u \cdot v$ $\qquad$ $p = \sqrt{(2 \cdot u \cdot E)}$

  - check of velocity relation:
    - let $\beta \to 0 \rightarrow pc = \beta/\sqrt{(1 - \beta^2)} \cdot E_u \to \beta \cdot E_u = c \cdot u \cdot \beta c = c \cdot u \cdot v$
  - check of energy relation:
    - let $E \to 0 \rightarrow pc = \sqrt{(E \cdot (E + 2 \cdot E_u))} \to \sqrt{(2 \cdot E_u \cdot E)} = c\sqrt{(2 \cdot u \cdot E)}$
- How far does Newton's arm reach?
  - depends on required precision
    - consider relation between momentum and velocity
    - define deviation from Newton's relation by:

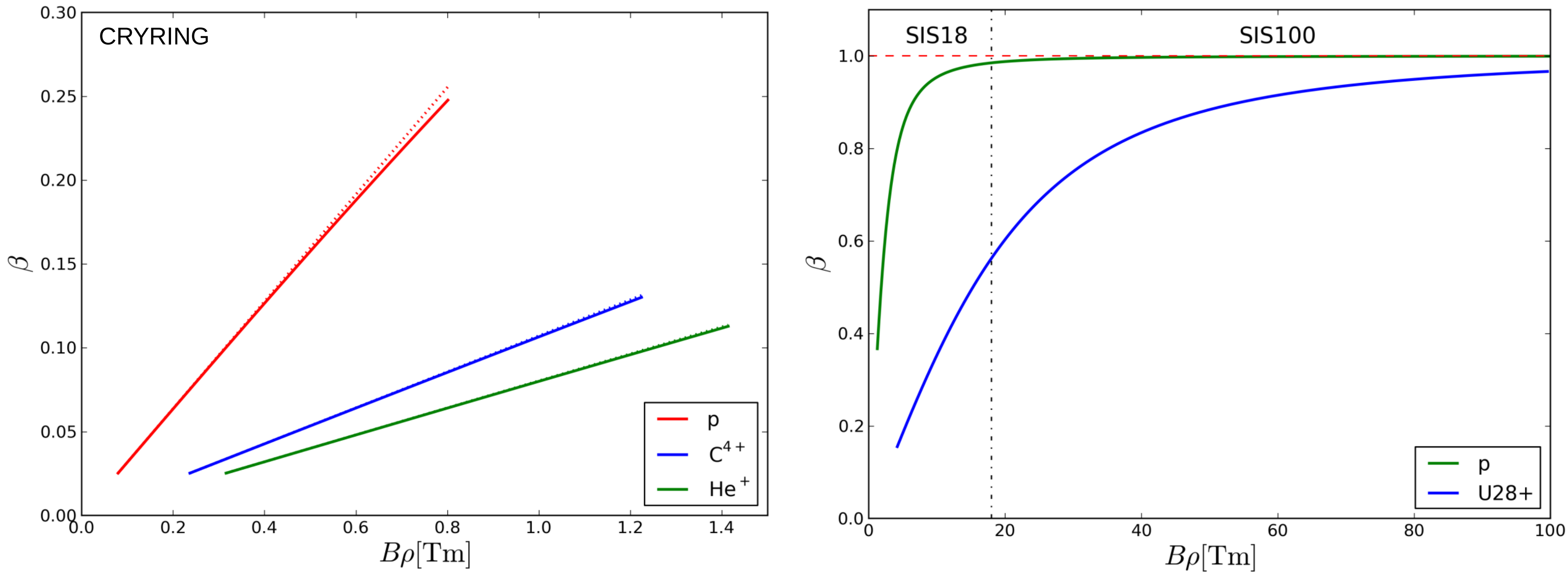
      $\beta/(\beta\gamma) = 1 - \varepsilon$ with $0 < \varepsilon < 1$

    - below this β, deviation from linearity smaller than ε

ε	γ	β	E
0.1	1.11111	0.436	103MeV/u
0.01	1.01010	0.141	9.4MeV/u
0.001	1.00100	0.045	932keV/u
0.0001	1.00010	0.014	93.2keV/u
0.00001	1.00001	0.004	9.32keV/u

last line still corresponds to 1300km/s = 1.3m/µs!

How Relativistic are GSI and FAIR?



- UNILAC (β < 0.15) and CRYRING (β < 0.25) close to non-relativistic
- SIS18 and SIS100 practically always relativistic
- SIS100 for protons at extraction pretty relativistic (γ ≈ 30)
- For comparison: LHC @ 7 TeV → γ ≈ 7500 (now this is ultra-relativistic...)

Lorentz Force and Magnetic Rigidity

- **Lorentz force** on charged particles in electromagnetic fields

$$\underline{F} = Q \cdot e\,(\underline{E} + \underline{v} \times \underline{B}) = M \cdot d\underline{p}/dt$$

  - general equation involving 3D vector quantities ($\underline{F}$, $\underline{E}$, $\underline{v}$, $\underline{B}$, $\underline{p}$)
  - electric field $\underline{E}$ will accelerate particle if aligned with $\underline{v}$
  - force by magnetic field $\underline{B}$ always perpendicular to $\underline{v}$ -> no acceleration
  - change of momentum per nucleon proportional to Q/M
- Particle in homogenous magnetic field B
  - energy remains constant
  - trajectory is circle with radius ρ satisfying: $B \cdot \rho = M/Q \cdot p$
  - fundamental relation for accelerator physics
  - quantity Bρ denoted **magnetic rigidity**
  - the larger Bρ, the harder it is to deflect the particle by a magnetic field
  - for fixed p rigidity determined by **mass-to-charge ratio** M/Q

Manetic Rigidity: Examples

Ions		SIS18 Inj.	SIS18 Ext.	SIS100 Ext.
		E=11.4MeV/u	E=1GeV/u	E=1.5GeV/u
Ion	**M/Q**	**Bρ [Tm]**	**Bρ [Tm]**	**Bρ [Tm]**
$^{40}Ar^{18+}$	2.22	1.08	12.5	16.6
$^{40}Ar^{10+}$	4.00	1.95	n.a.	30.0
$^{86}Kr^{33+}$	2.61	1.27	14.7	19.5
$^{86}Kr^{16+}$	5.38	2.62	n.a.	40.3
$^{238}U^{73+}$	3.26	1.59	18.4	24.4
$^{238}U^{28+}$	8.50	4.14	n.a.	63.7

Protons	SIS18 Inj. (UNILAC)	SIS18 Inj. (p-Linac)	SIS18 Ext.	SIS100 Ext.
	E=11.4MeV/u	E=70.0MeV/u	E=4GeV/u	E=29GeV/u
M/Q	**Bρ [Tm]**	**Bρ [Tm]**	**Bρ [Tm]**	**Bρ [Tm]**
1.007	0.49	1.26	16.3	100

Deflection in Dipoles

- Dipole with a uniform field deflects a charged particle by an angle θ
- θ depends on arc length L and magnetic field B
- L depends on θ and radius ρ

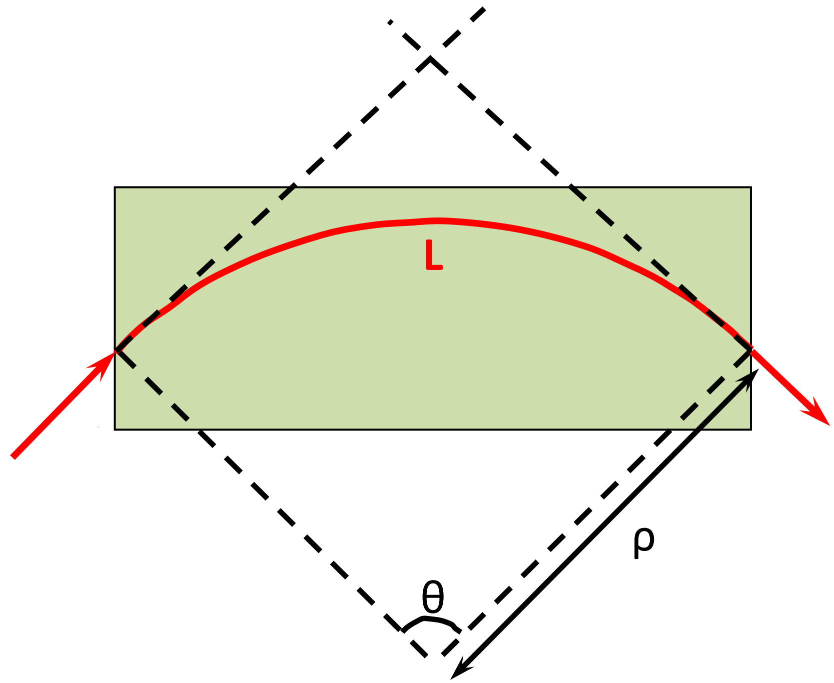
$$L = \theta \cdot \rho$$

- Particle with magnetic rigidity Bρ

$$B \cdot L = \theta \cdot B\rho$$

- Quantity B·L is referred to as **integral magnetic field**
- Deflection angle given by

$$\theta = B \cdot L / B\rho$$



- Macroscopic angles to bend transfer lines or create circular accelerators → **bending magnets**
- Microscopic angles to correct beam trajectory or closed orbit → **steering magnets**

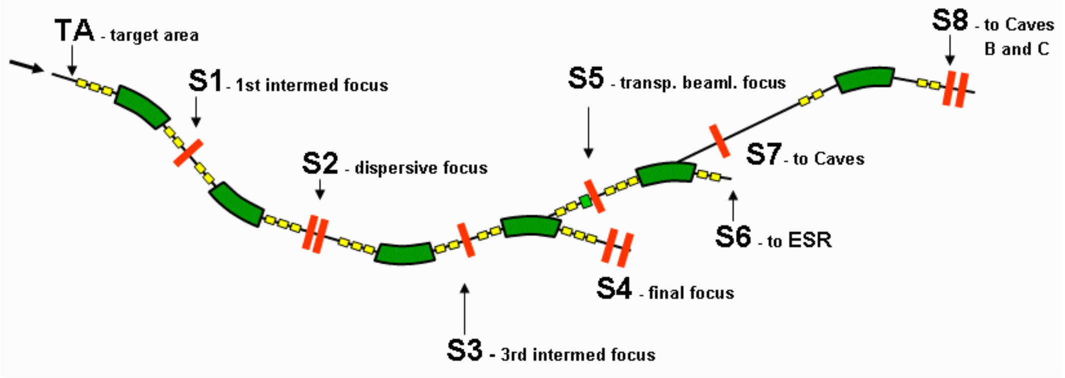
Dipoles as Separators

- Consider particles with fixed p but different M and Q in a dipole field B
  - assume B·L same for all M, Q
  - deflection angle given by

$$\theta = Q/M \cdot B \cdot L/p$$

- Charge separation
  - assume same M, but different Q
  - let only one angle θ pass and vary B, then Q proportional to 1/B
- Mass separation
  - assume same Q, but different M
  - let only one angle θ pass and vary B, then M proportional to B

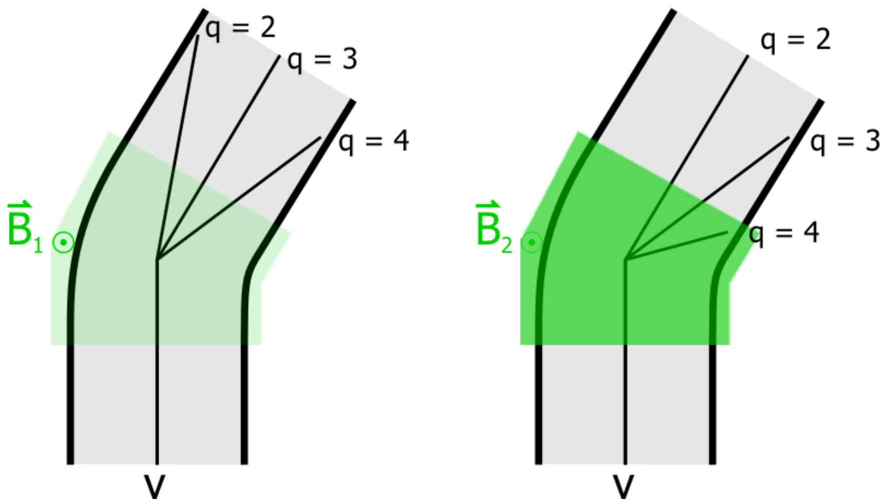
Some separator magnets are rather large...

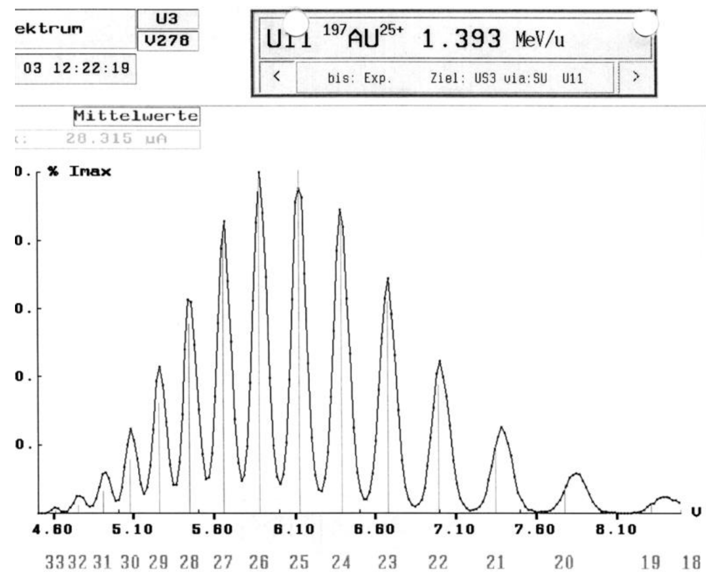


...and one can build separators with more dipoles.

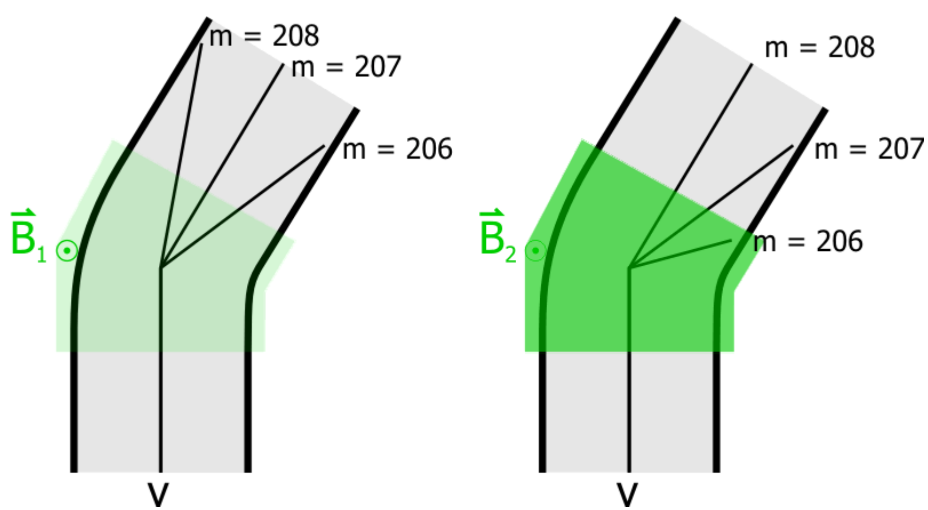
Example: Spectrometer

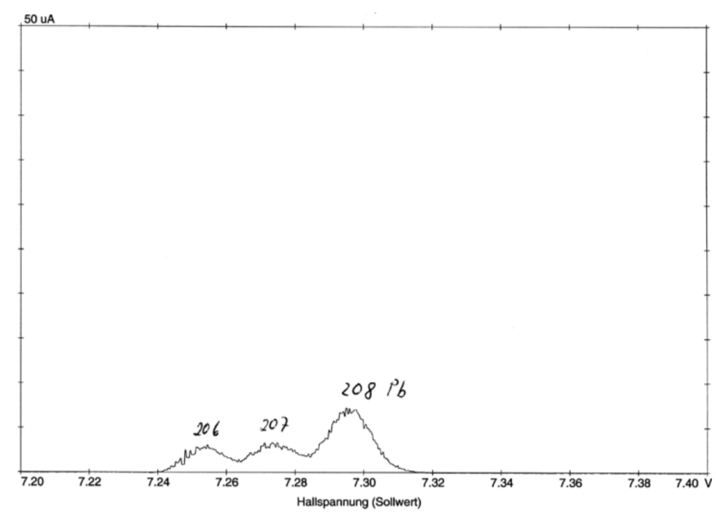
Charge separation behind gas stripper





Mass (isotope) separation





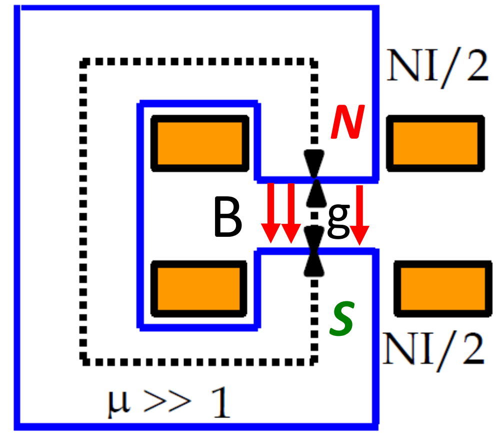
Dipole Magnets

- Homogeneous field desired
- At GSI and FAIR electromagnets with and iron core are used
- B field created in gap g between two parallel iron poles ***N*** and ***S***
- B field excited by current in coil with N windings
- B field can be calculated

$$B = \mu_0 \cdot N \cdot I / g$$

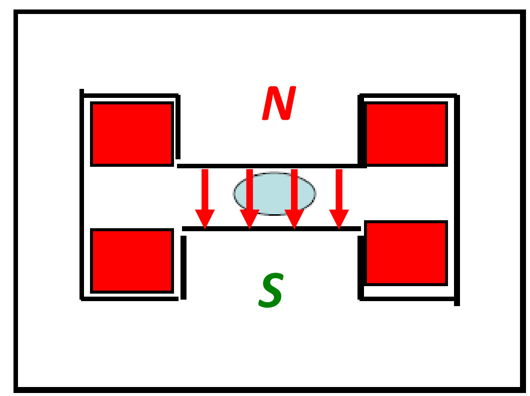
$\mu_0 = 4\pi \cdot 10^{-7}$ Tm/A (constant of nature)

- Choice of coils
  - Normal conducting (water cooled)
    - e.g. SIS18, ESR, FRS
  - Super-conducting (liquid He cooled)
    - e.g. SIS100, Super-FRS



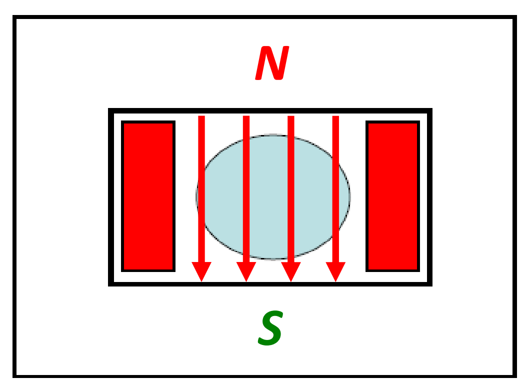
C core dipole

e.g. ESR, CRYRING



H core dipole

e.g. SIS18, SIS100



Window frame dipole

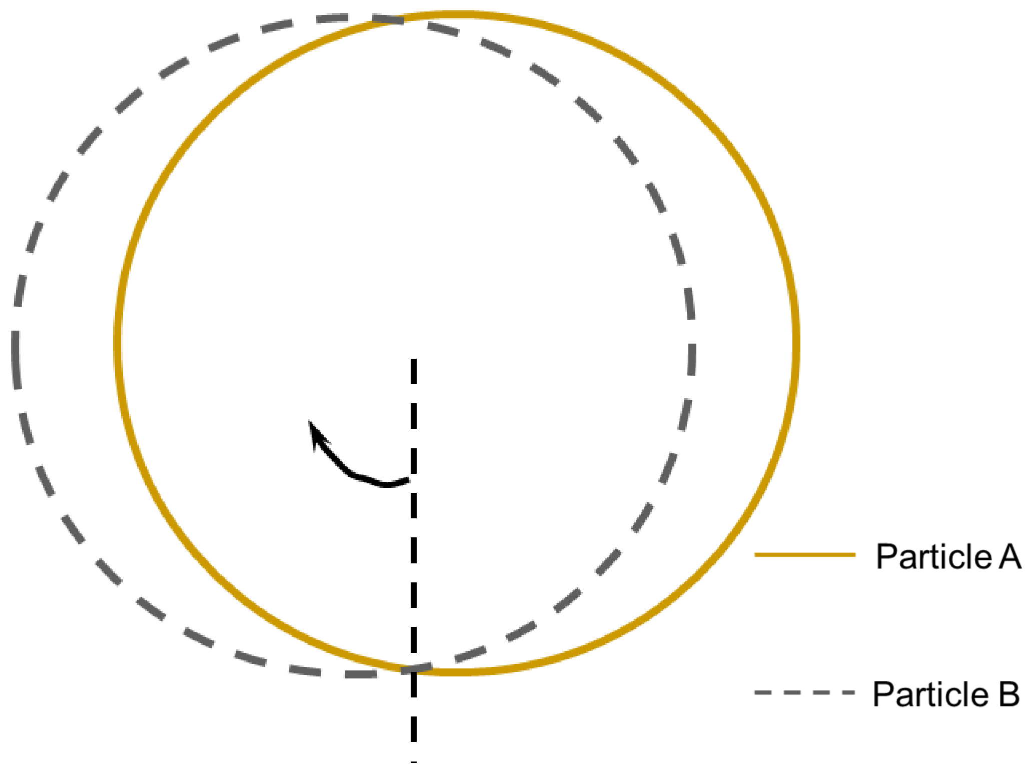
Bending Magnets @ GSI and FAIR

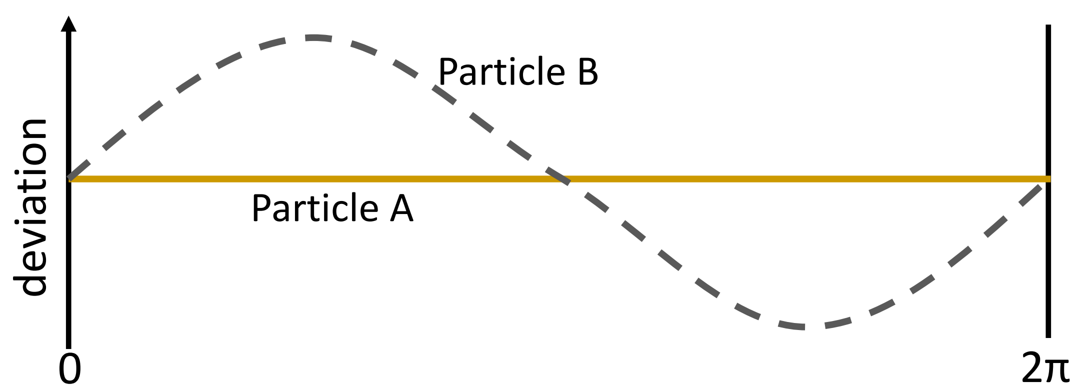
Machine	Type	N	θ [deg]	ρ [m]	L [m]	B_{max} [T]	BL_{max} [Tm]	$B\rho_{max}$ [Tm]
SIS18	H	24	15	10.4	2.7	1.8	4.9	18.7
ESR	C	6	60	6.25	6.5	1.6	10.5	10.0
CRYRING	C	12	30	1.20	0.63	1.2	0.75	1.44
SIS100	H	108	3.33	52.6	3.1	1.9	5.8	100.0
CR	H	24	15	8.13	2.1	1.6	3.4	13.0
HESR	H	44	8.18	29.4	4.2	1.7	7.1	50.0

And a large variety of bending magnets in the transfer lines and at the experiments...

Particle Motion in Dipole Magnets

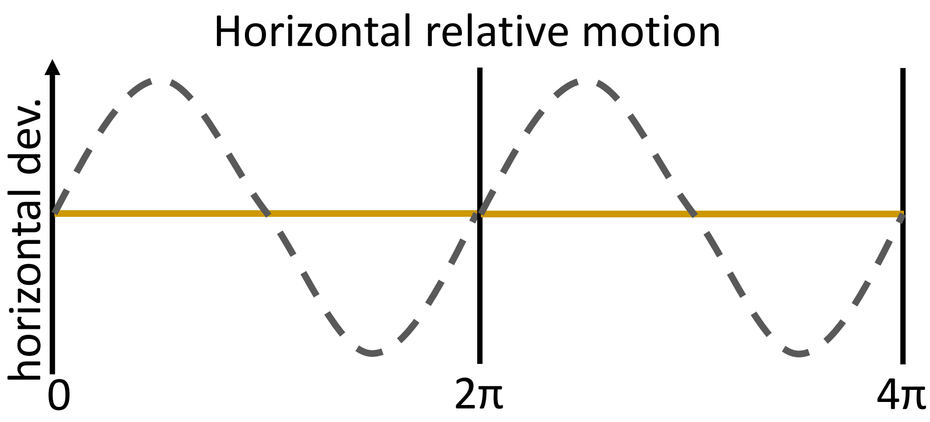
- Can we build circular accelerators from dipoles only?
- Start two particles in same field
  - same momentum, same position
  - different angle
- Use particle A's trajectory as reference and measure deviation of B's trajectory along the circle
  - particle B oscillates around particle A
  - such oscillations characterize transverse motion in accelerators
  - referred to as **betatron oscillations**
  - more about that later...
    (see talk on 'Transverse Dynamics')

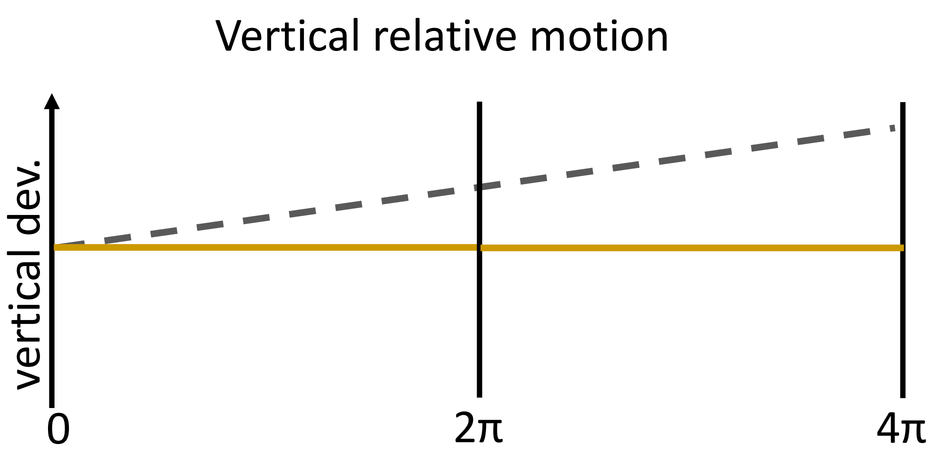




Stable or Unstable Motion

- In the previous example, horizontal trajectories close on themselves
  - could be repeated infinitely
  - motion considered to be **stable**
  - **focusing effect** of the dipole field
- What about the vertical motion?
  - particle A at zero position and angle
  - particle B with same position but small angle at start
  - the dipole field has no influence on the vertical motion of both particles
  - position of particle B grows without bounds over many turns
  - motion considered to be **unstable**
- Need focusing in the vertical direction



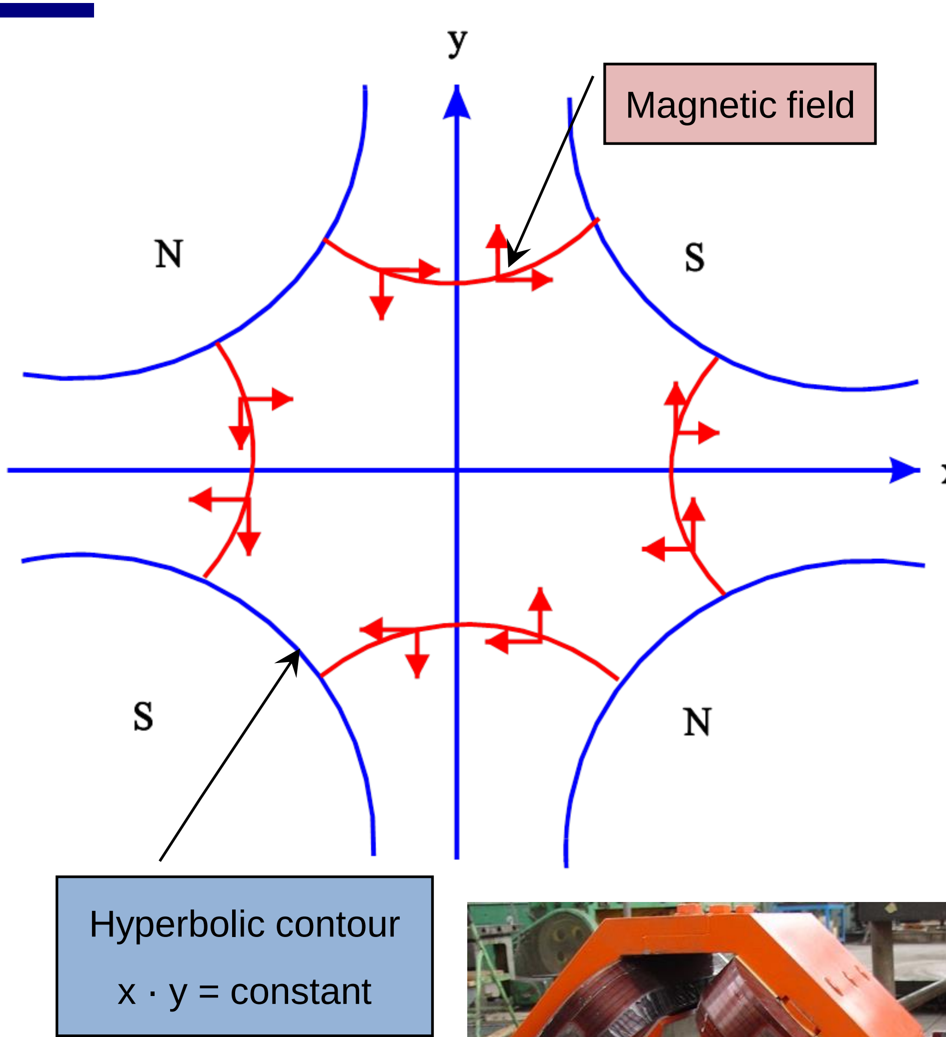


Quadrupole Magnets

- Quadrupole magnet has 4 poles symmetric about the center
- By symmetry the B field on the longitudinal axis is zero
  - particle passes straight through the center with no deflection
  - quadrupole magnet is straight
- B field depends linearly on position
  - horizontal direction: $B_y = B' \cdot x$
  - vertical direction: $B_x = -B' \cdot y$
  - B' referred to as **field gradient**
- Integral quadrupole strength

$$k \cdot L = B' \cdot L / B\rho$$

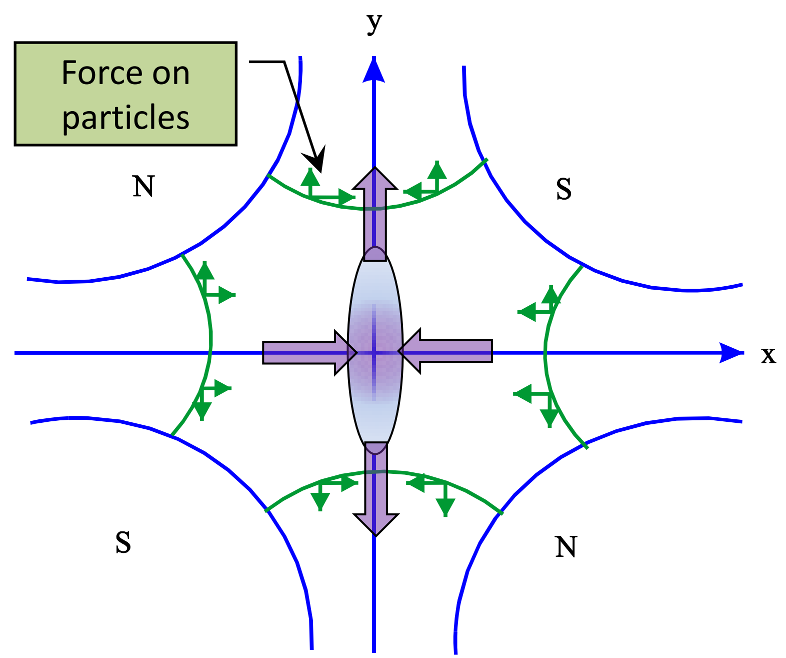
  - focusing power for particles with Bρ
  - analogous to angle of a dipole

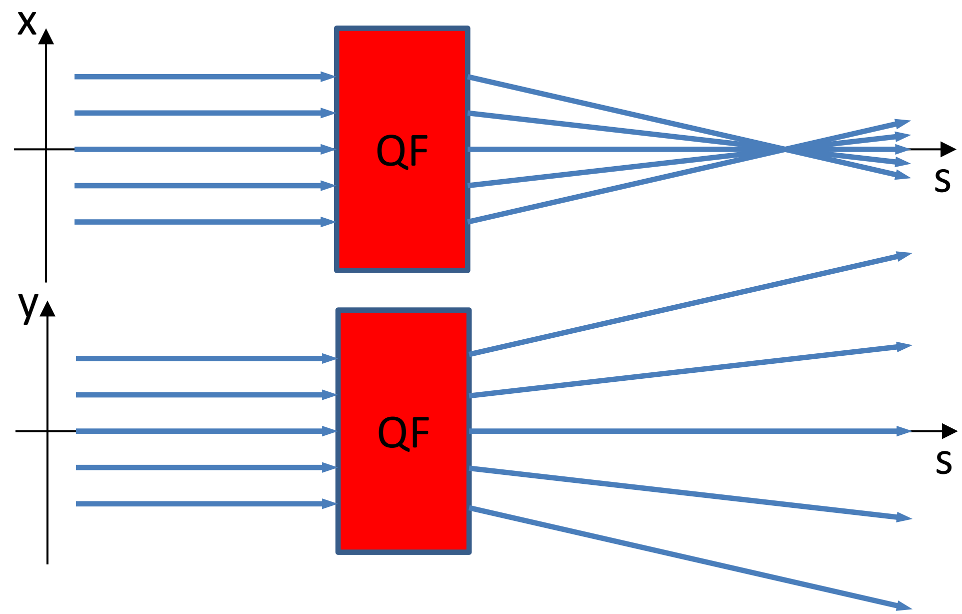


Effect of Quadrupole Magnets

- Forces on particles determined by arrangement of poles
  - arrangement N S / S N
    - focusing in horizontal plane
    - defocusing in vertical plane
    - denoted **focusing quadrupole (QF)**
  - arrangement S N / N S
    - defocusing in horizontal plane
    - focusing in vertical plane
    - denoted **defocusing quadrupole (QD)**
- Forces linear in transverse position
  - particles receive deflection proportional to their position
  - similar to lenses in optics except that lenses (de)focus in both planes

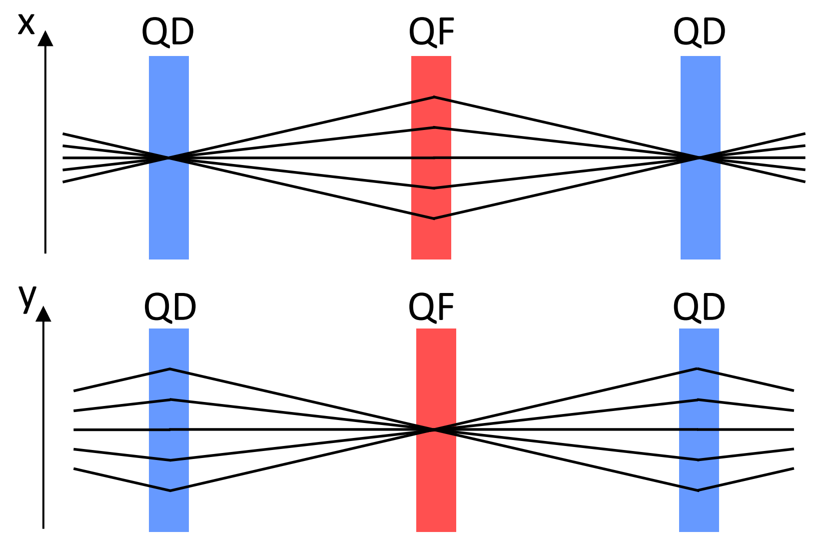
Focusing quadrupole (QF)

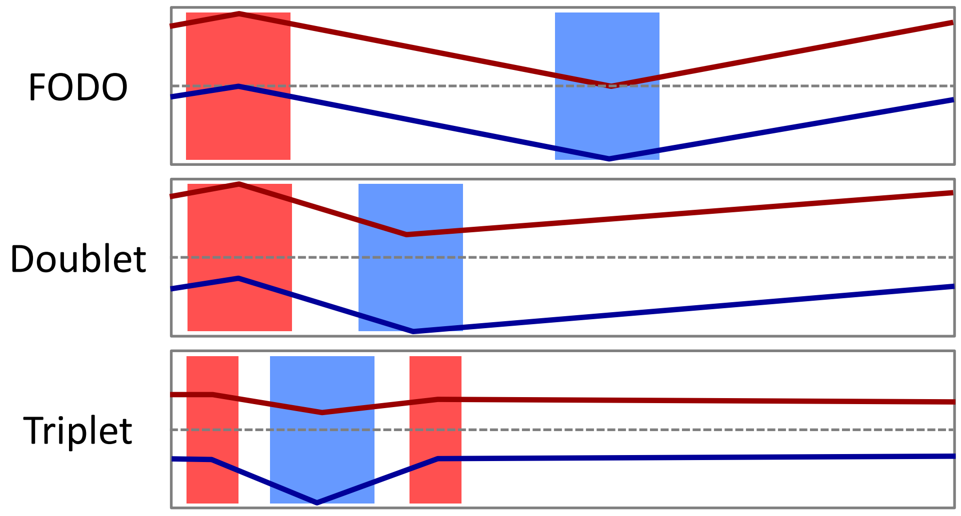




Focusing with Quadrupole Magnets

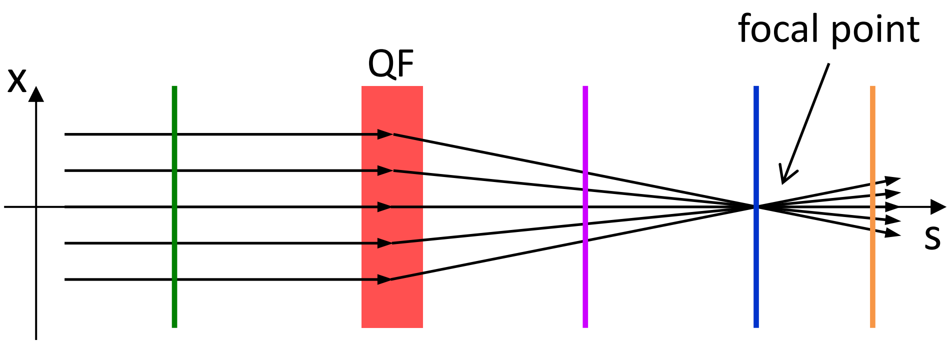
- Neither QF nor QD can focus in both x and y simultaneously, but...
- Combinations of QF and QD with overall focusing in x and y possible
- Idea:
  - put QD in place where horizontal offsets are small
  - put QF in place where vertical offsets are small
  - then effect of QD on x respectively QF on y should be small
- Different arrangements possible
  - FODO: smallest field gradients
  - Doublet: long free section
  - Triplet: symmetric beams

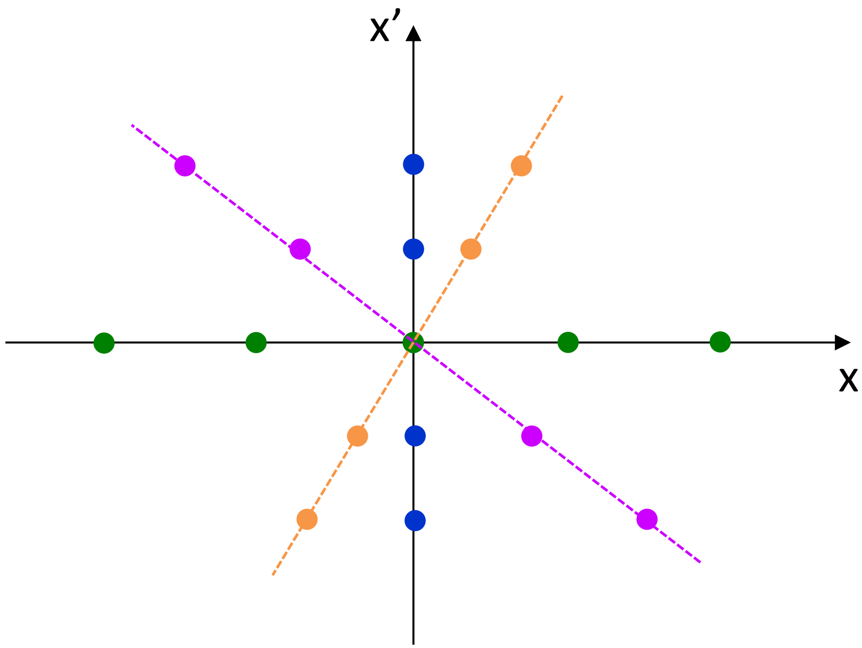




Transverse Phase Space

- Consider again focusing by QF in the horizontal plane
  - parallel input trajectories
    → particles characterized by position x
  - at focal point, all particles have x = 0
    → particles characterized by angle x'
- In general, position x and angle x' needed to fully describe particle
  - aggregated into **vector** $\begin{pmatrix} x \\ x' \end{pmatrix}$
  - can be depicted in 2D coordinate system called **phase space**
  - convergent trajectories have negative x' for positive x and vice versa
  - opposite sign for divergent trajectories
- Vertical plane completely analoguos





Quadrupole Effect Quantitatively

- Consider thin quadrupole at s = 0
- Input vector (just in front of quad)

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{in} = \begin{pmatrix} x_1 \\ 0 \end{pmatrix}$$

- Output vector (just behind quad)

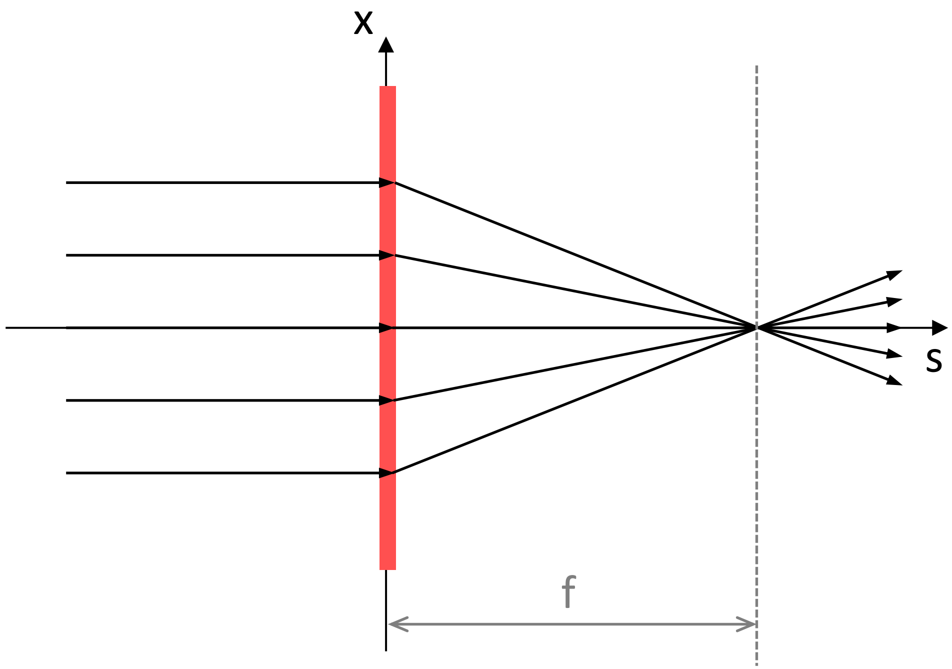
$$\begin{pmatrix} x \\ x' \end{pmatrix}_{out} = \begin{pmatrix} x_1 \\ -x_1/f \end{pmatrix}$$

  - same position x_1 (thin quad)
  - angle of trajectory: $-x_1/f$ ($x_1 \ll f$)

- Vector downstream of quad at s

$$\begin{pmatrix} x \\ x' \end{pmatrix}(s) = \begin{pmatrix} x \\ x' \end{pmatrix}_{out} + \begin{pmatrix} -x_1/f \cdot s \\ 0 \end{pmatrix}$$

  - angle fixed in **drift space**
  - position change linear in s



Matrix Formalism

- Matrices and vectors
  - mathematical structures with a very powerful calculus (linear algebra)
  - easily implemented in computer programs
  - very convenient for representing particle transport through accelerators with linear forces
  - takes some time getting used to it...

Multiplication rules for matrices and vectors

Matrix · Vector = Vector

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a \cdot x + b \cdot y \\ c \cdot x + d \cdot y \end{pmatrix}$$

Matrix · Matrix = Matrix

$$\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} \cdot \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} = \begin{pmatrix} a_1 a_2 + b_1 c_2 & a_1 b_2 + b_1 d_2 \\ c_1 a_2 + d_1 c_2 & c_1 b_2 + d_1 d_2 \end{pmatrix}$$

Multiplication not commutative...

$$M_1 \cdot M_2 \neq M_2 \cdot M_1$$

..but associative

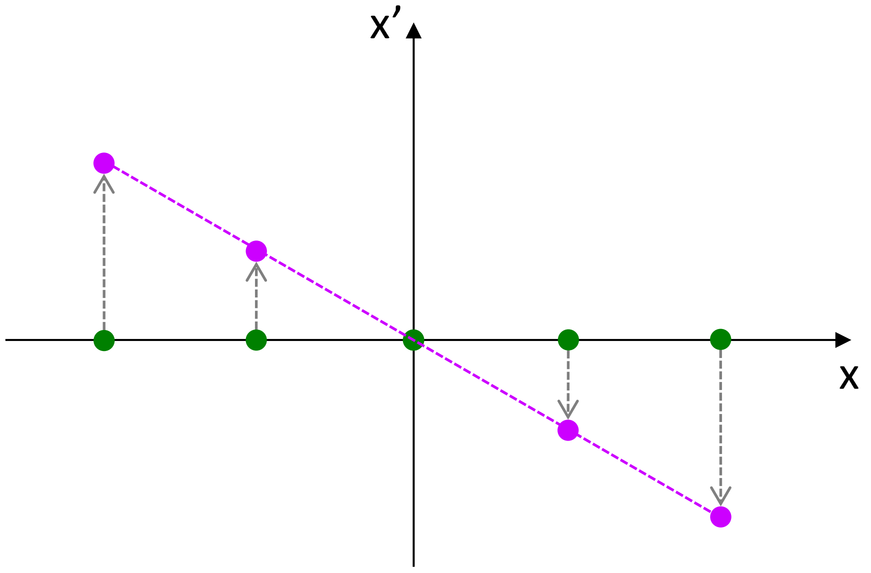
$$(M_1 \cdot M_2) \cdot M_3 = M_1 \cdot (M_2 \cdot M_3)$$

Quadrupoles and Drifts as Matrices

- Effect of thin quadrupole with focal length f described by matrix

$$\begin{pmatrix} x_2 \\ x'_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x'_1 \end{pmatrix} = \begin{pmatrix} x_1 \\ x'_1 - x_1/f \end{pmatrix}$$

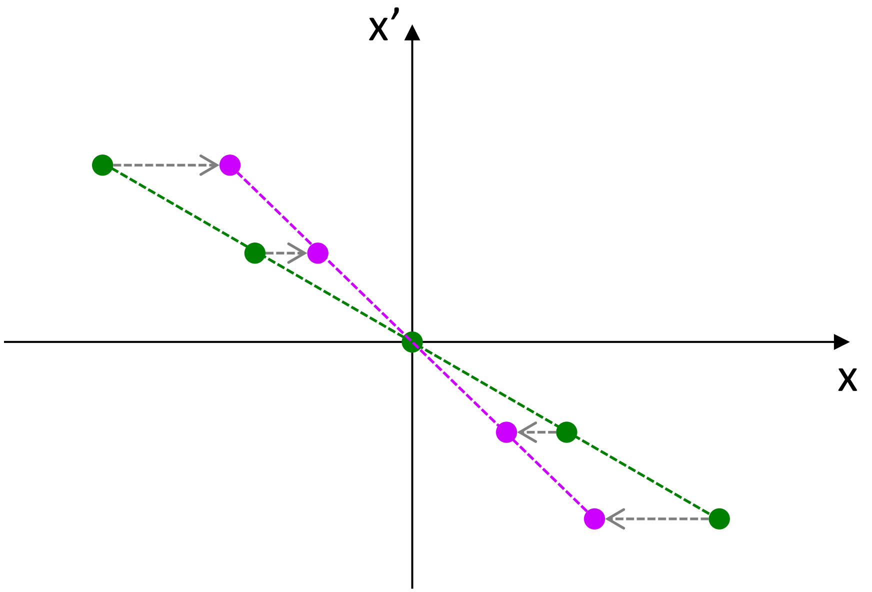
  - first row: position x is unchanged and does not depend on angle x'
  - second row: quad changes angle x' by amount proportional to position x



- Effect of drift space of length L

$$\begin{pmatrix} x_2 \\ x'_2 \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x'_1 \end{pmatrix} = \begin{pmatrix} x_1 + L \cdot x'_1 \\ x'_1 \end{pmatrix}$$

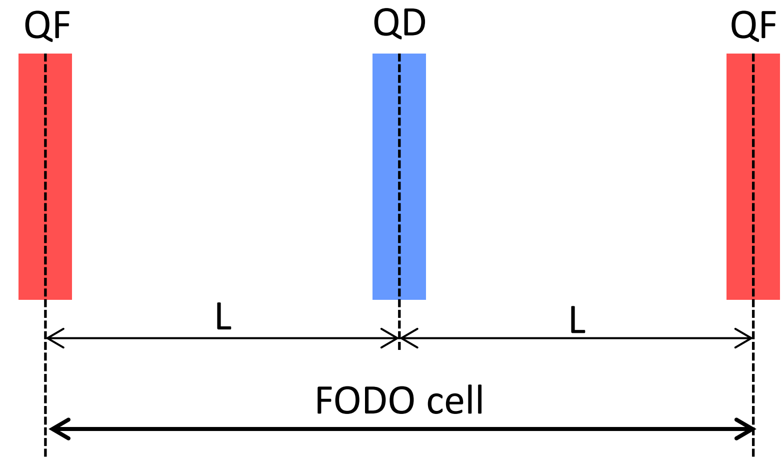
  - first row: position x changes by amount proportional to angle x'
  - second row: angle unchanged



- Can be visualized in phase space

FODO Focusing

- Alternating focusing and defocusing quadrupoles separated by drift spaces
- FODO cell can be repeated many times (periodic FODO channel)
- Dipole magnets installed between quadrupoles for curved accelerators
  - approximately like drift spaces
- Efficient focusing structure regarding necessary fields for given aperture
- Periodic solution exists
  - output beam same as input beam
  - ellipses in phase space
  - other beam shapes may lead to troubles (mismatch)

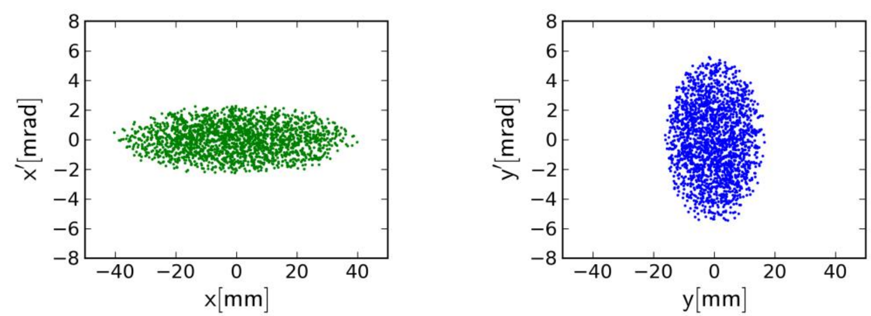


Matrix formalism allows to split quads easily...

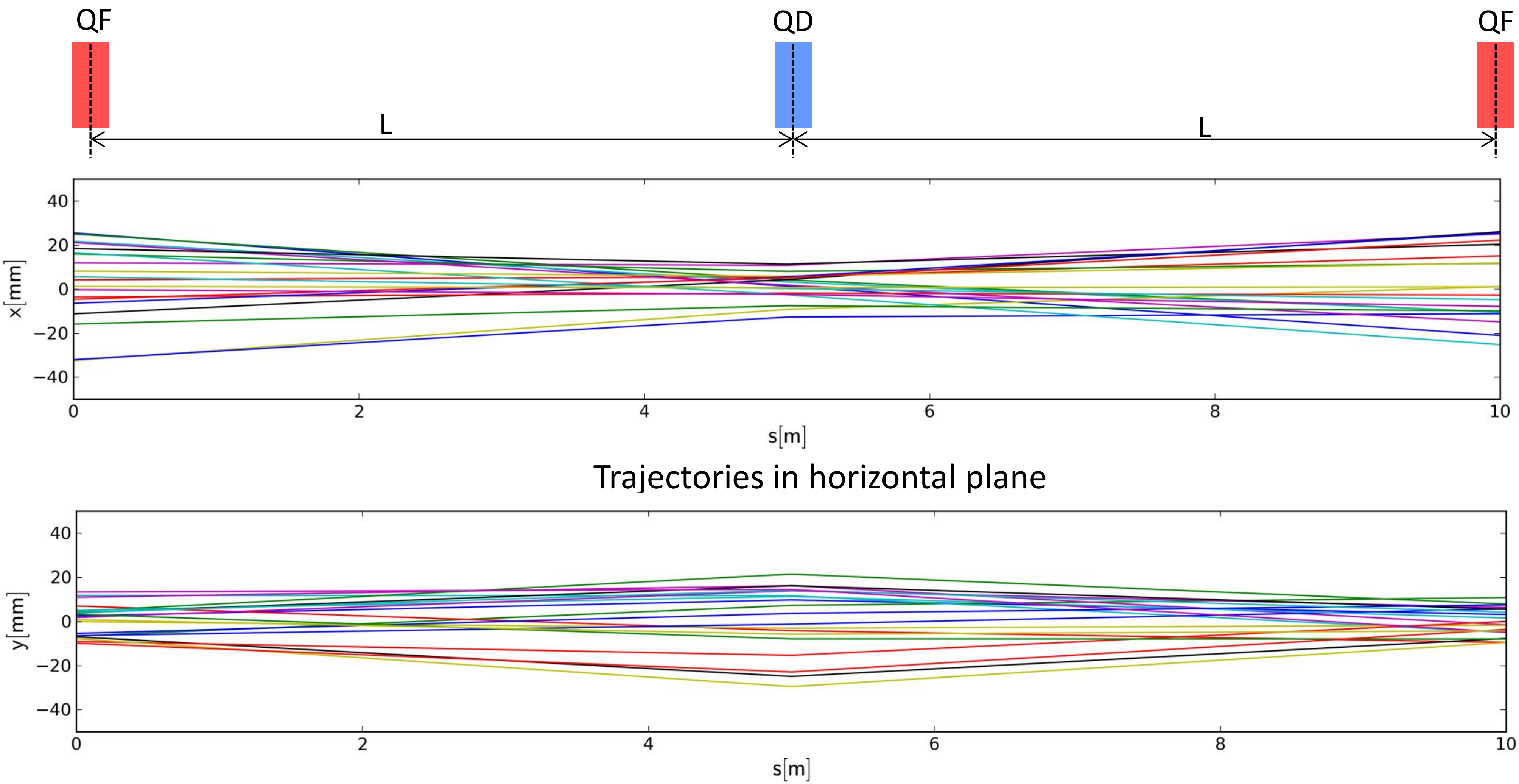
Matrix of FODO cell

$$M = M_{QF/2} \cdot M_D \cdot M_{QD} \cdot M_D \cdot M_{QF/2}$$
$$= \begin{pmatrix} 1-\frac{L^2}{2f^2} & 2L\left(1+\frac{L}{2f}\right) \\ -\frac{L^2}{2f^2}\left(1-\frac{L}{2f}\right) & 1-\frac{L^2}{2f^2} \end{pmatrix}$$

Just for the curious and brave...



FODO Tracking: Trajectories

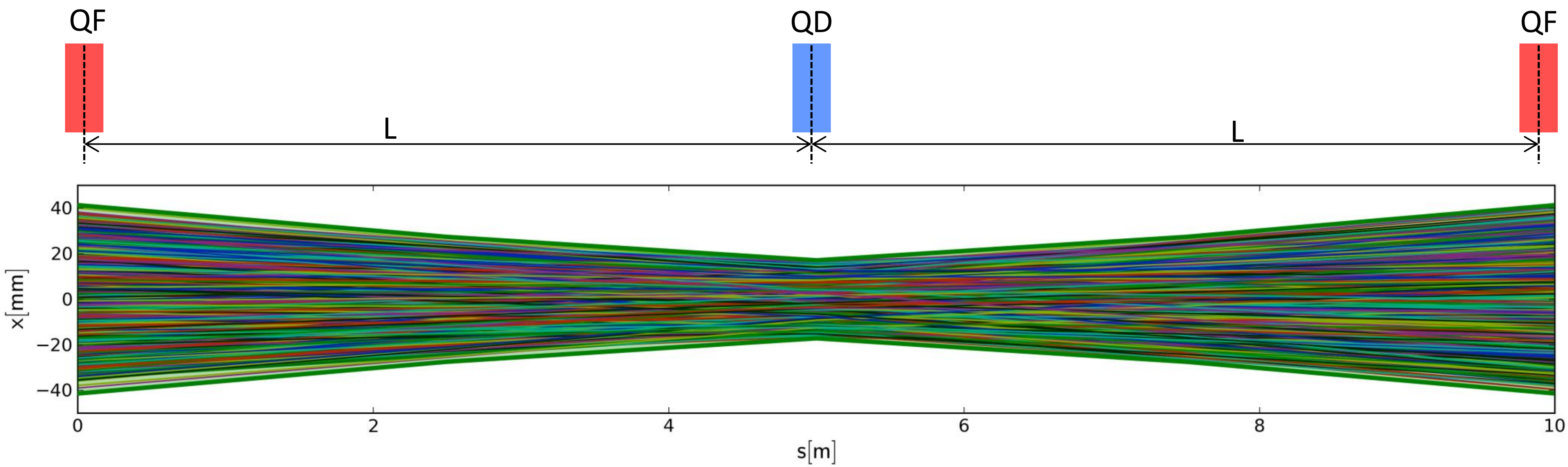


Trajectories in vertical plane

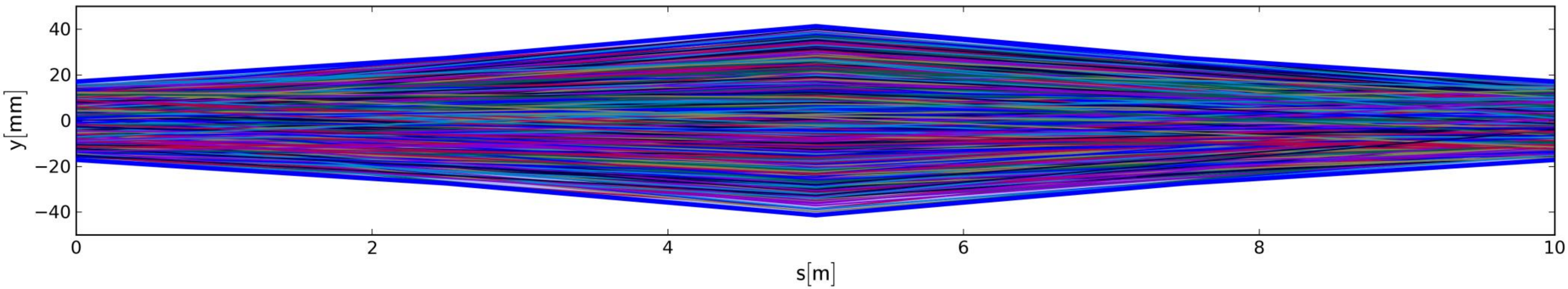
Tracks obtained by tracking 20 particles

- Particle tracks obtained by tracking random sample of particles
- General properties of FODO structure visible, but where are the limits?

FODO Tracking: Envelopes



Envelopes in horizontal plane



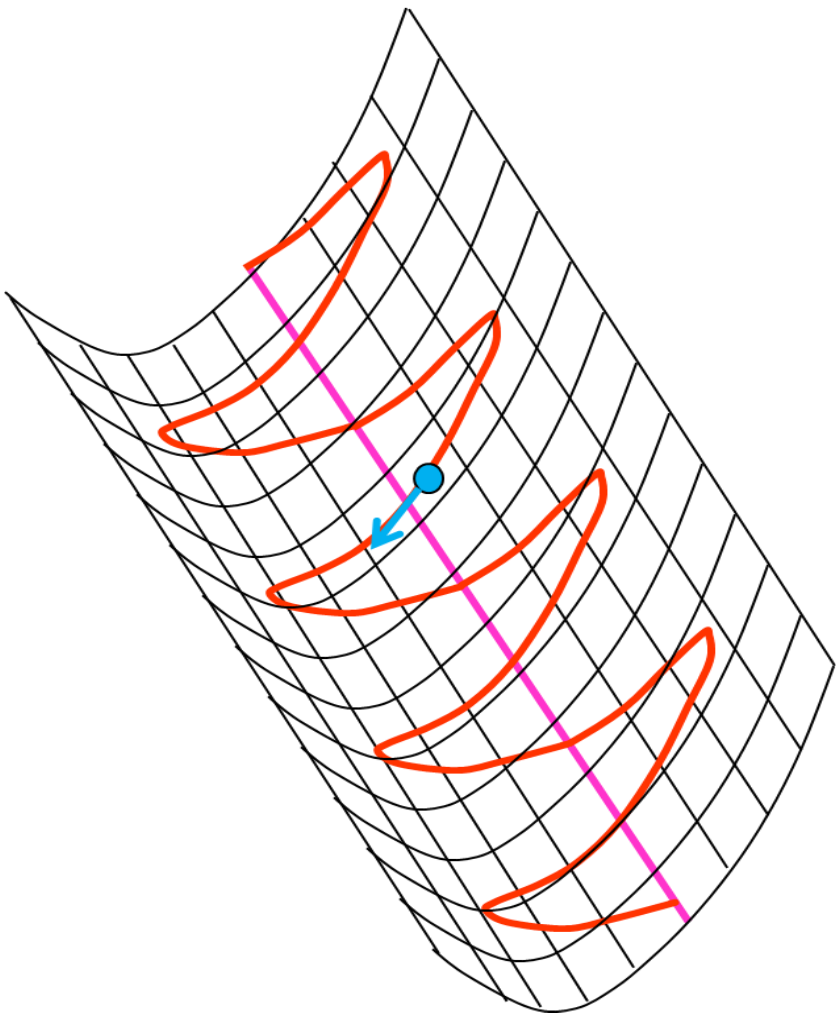
Envelopes in vertical plane

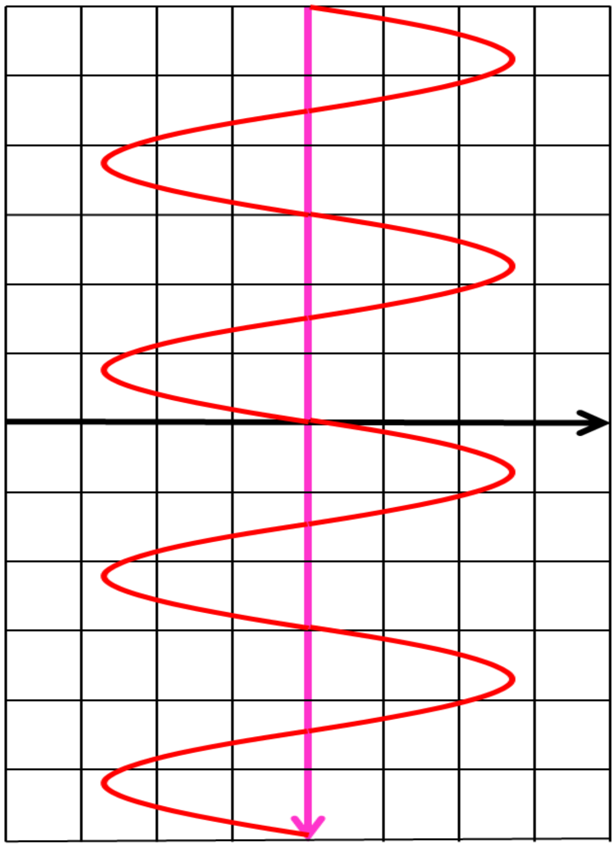
Tracks obtained by tracking 2000 particles

- Shape of beam emerges when tracking lots of particles
- Envelope can actually be calculated from L and f (→ *talk on transverse optics*)

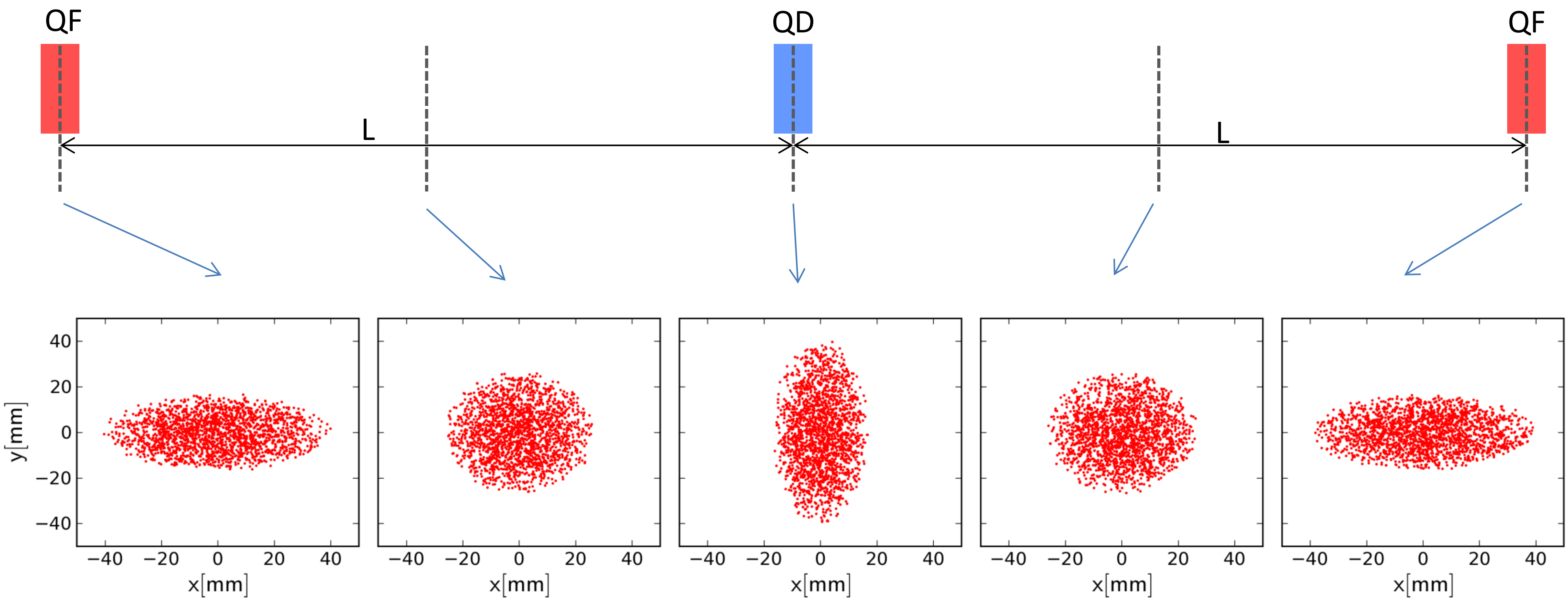
FODO Focusing: Mechanical Analogue

- Ball rolling without friction in a gutter can't escape due to force of gravity
- Particles diverging from nominal orbit focused back by quadrupoles





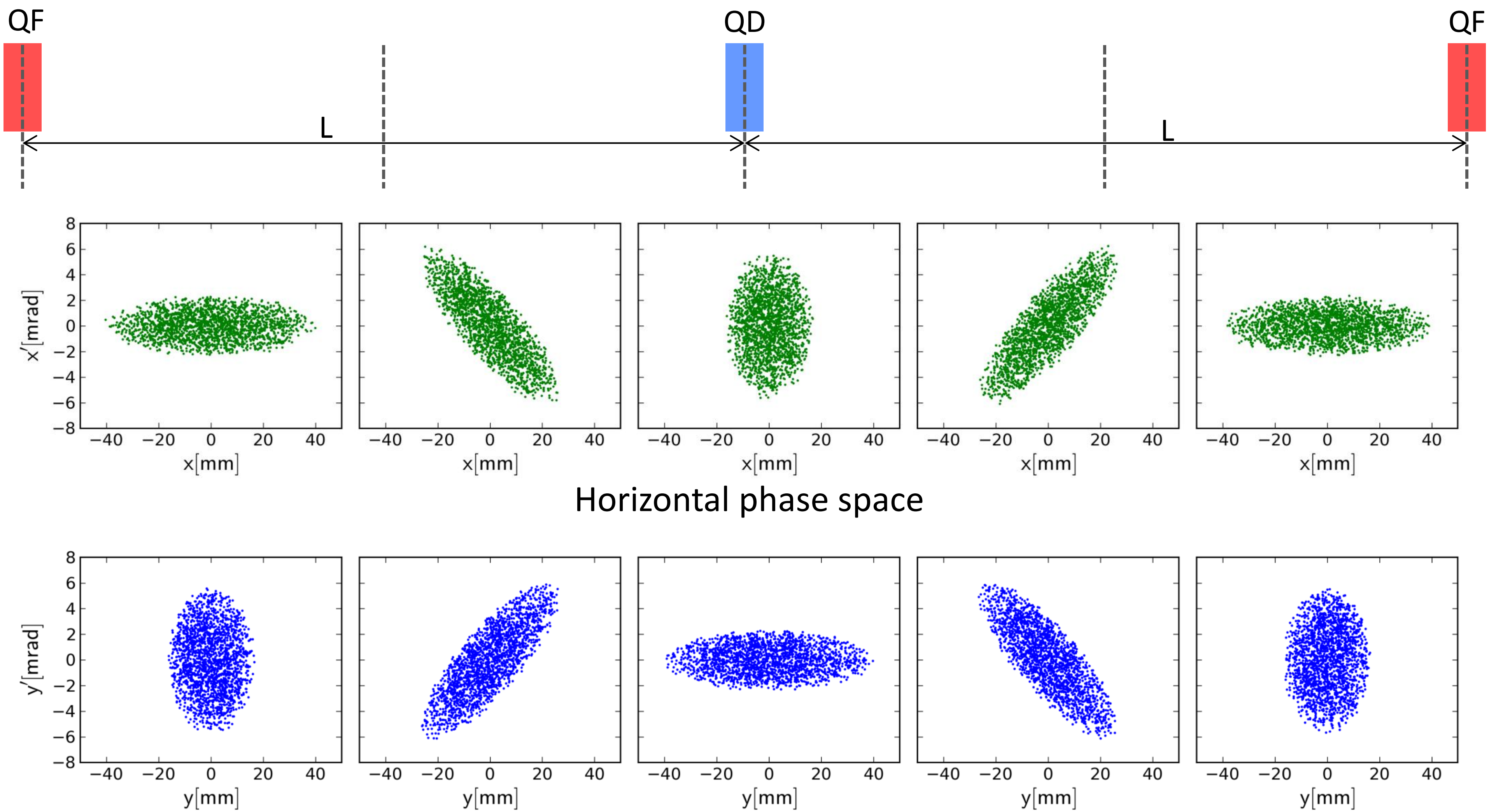
FODO Tracking: Cross section



Beam cross sections in x-y

- Cross sections reflect the change of beam size through FODO cell
- In real accelerator observable using scintillating screens
  - not possible for beam in a circular accelerator
  - typically not inside of quadrupoles, of course...

FODO Tracking: Phase Space

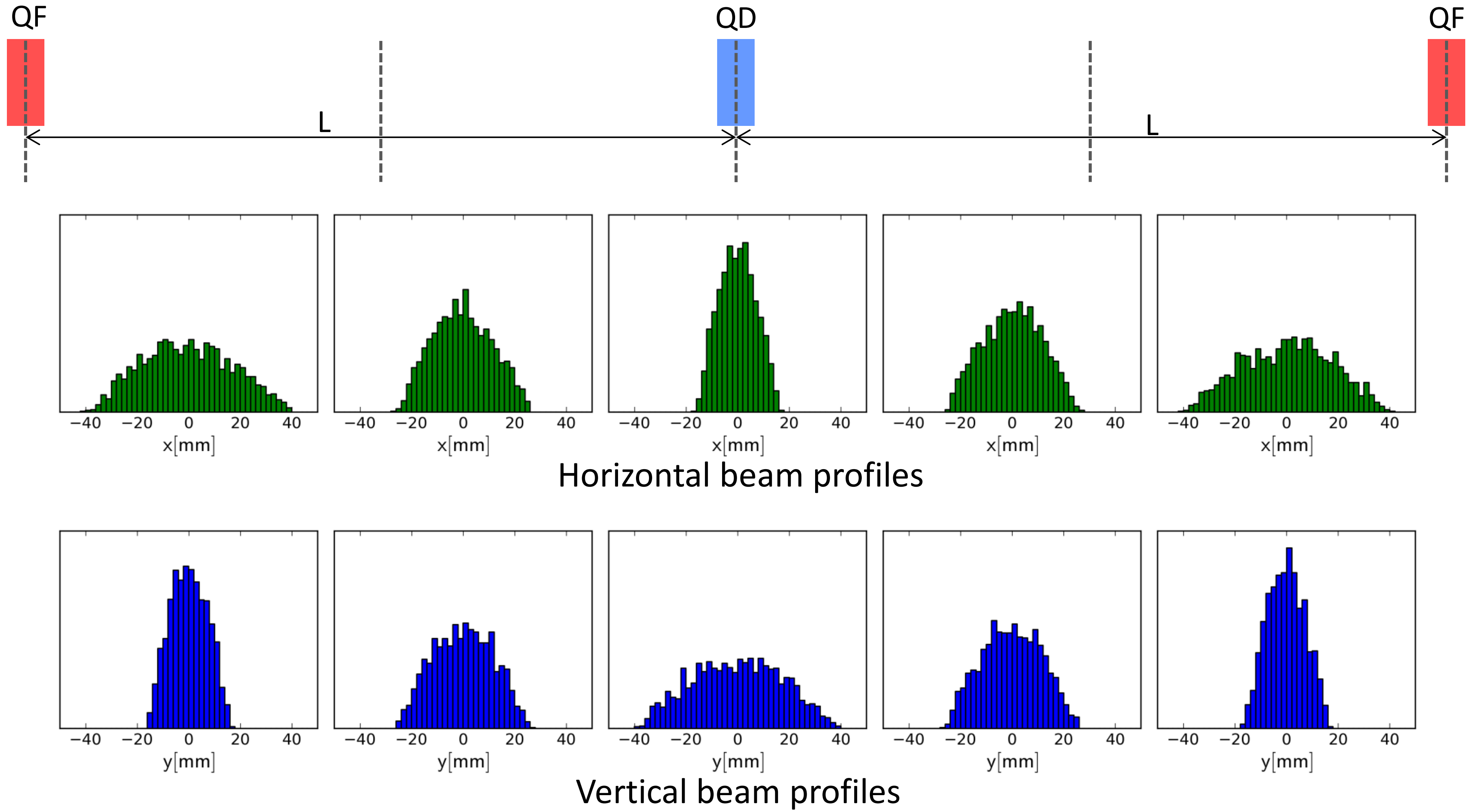


Horizontal phase space

Vertical phase space

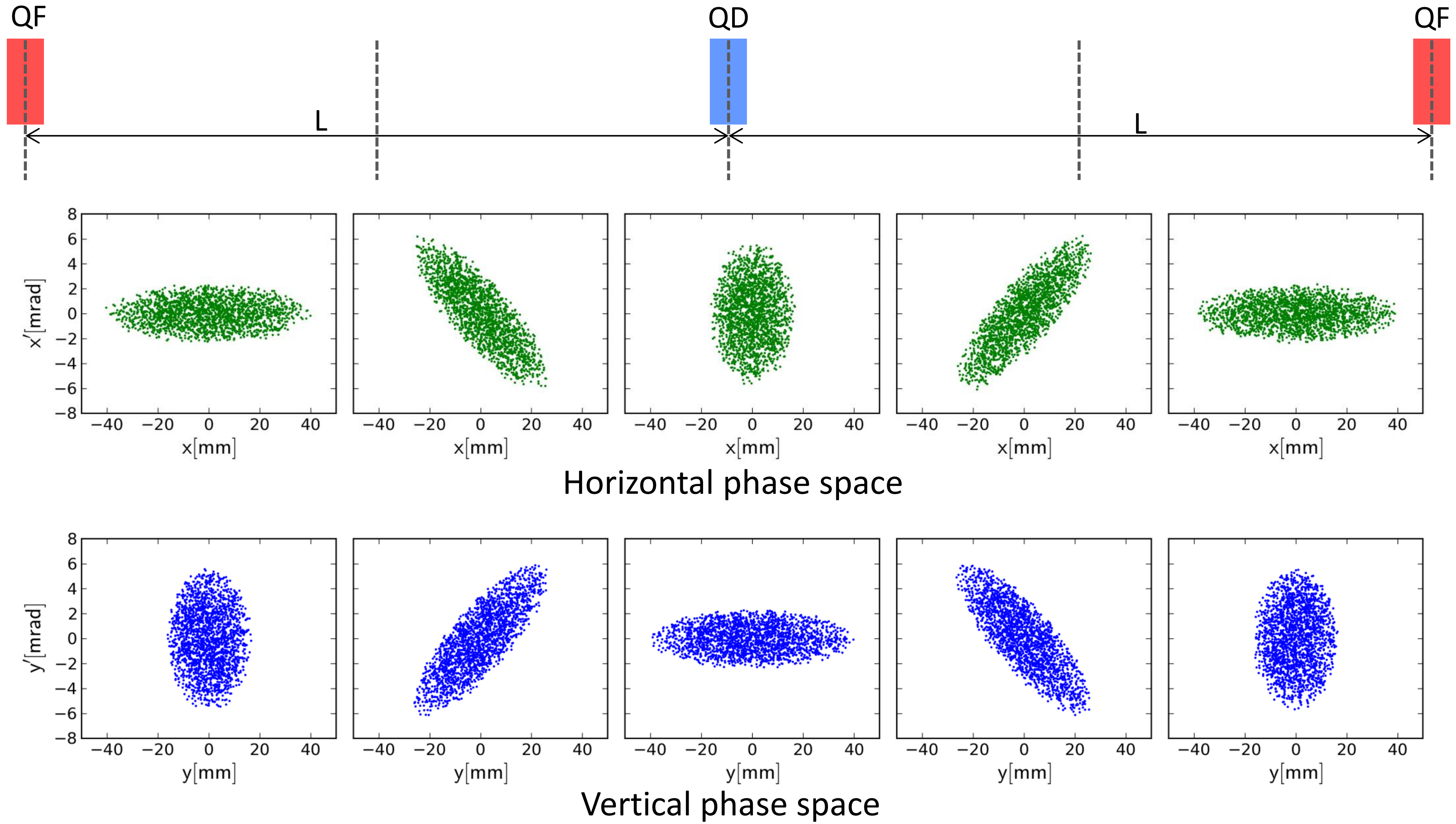
- Important property of beam, but not directly observable in a real accelerator
  - how would you measure the angle of a trajectory simultaneously with position?

FODO Tracking: Beam Profiles



- Projections of phase space onto position can be observed
  - beam profiles measured by SEM-grids, MWPCs, BIFs, IPMs
  - very important information for set-up of accelerators and beamlines

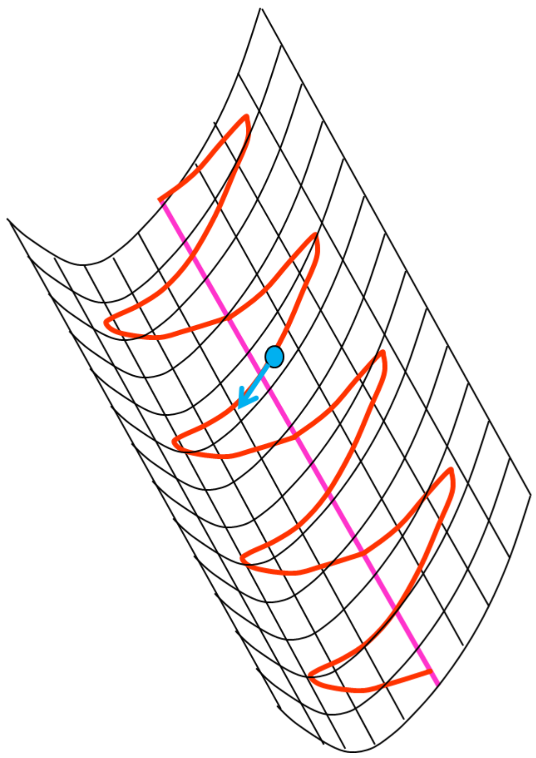
FODO Tracking: Mismatch

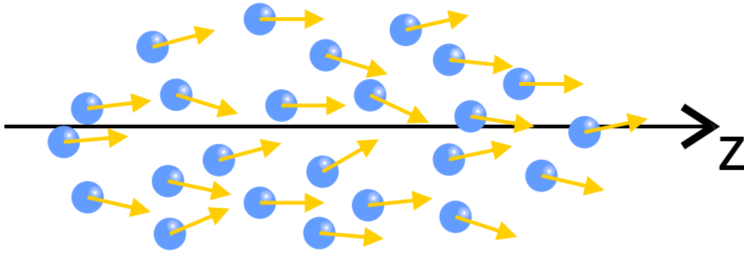


- Important property of beam, but not directly observable in a real accelerator
  – how would you measure the angle of a trajectory simultaneously with position?

Emittance

- Oscillation amplitude of single particles normally conserved
  - remember the frictionless gutter: ball will oscillate forever
  - measure of transverse energy
- Transverse energy referred to as **single particle emittance**
  - transverse energy created by random motion in the source
  - conserved (at best) until final target
  - measure of disorder of particles in the beam
  - impossible to create truly parallel beam (would be completely ordered)





Beam Size and Emittance

- Periodic solution of FODO cell characterized by ellipses
  - aspect ratio defined by quadrupoles
  - for upright ellipses, aspect ratio described by **beta functions** $\beta_{x/y}$
  - area of ellipse written as

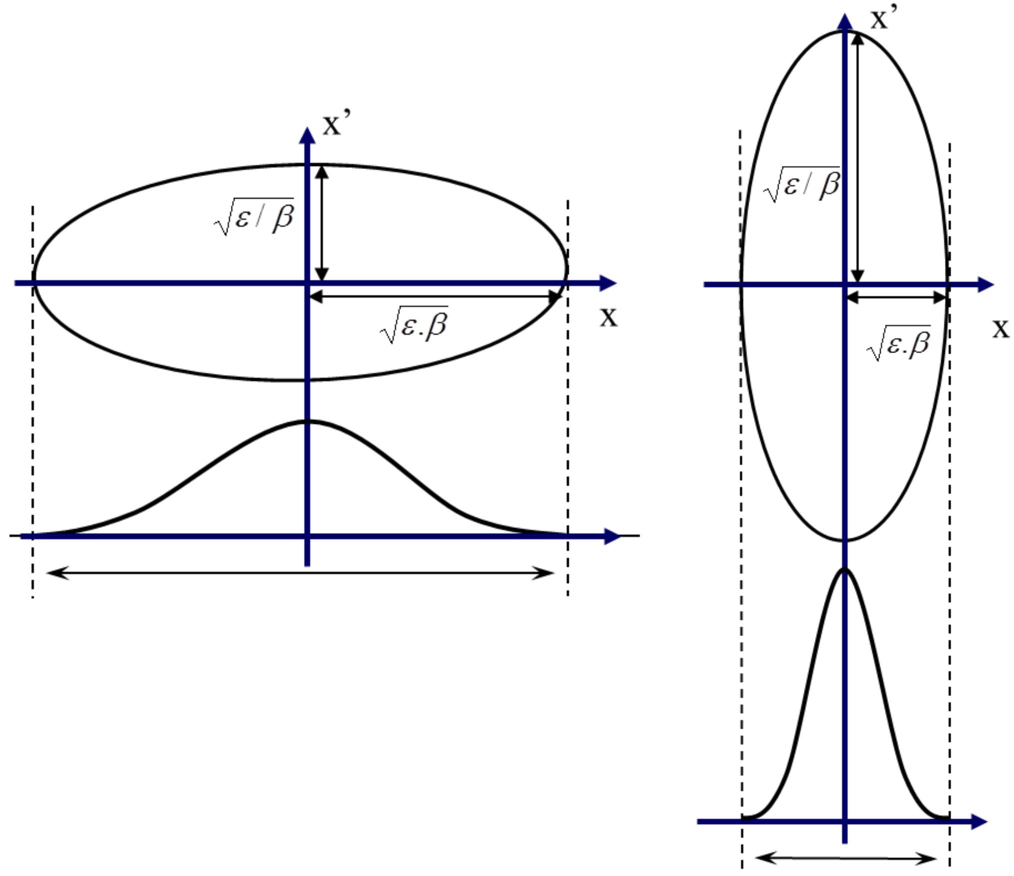
$$A_{x/y} = \pi \cdot \varepsilon_{x/y}$$

  - emittance $\varepsilon_{x/y}$ measure for beam size

$$r_{x/y} = \sqrt{\varepsilon_{x/y} \cdot \beta_{x/y}}$$

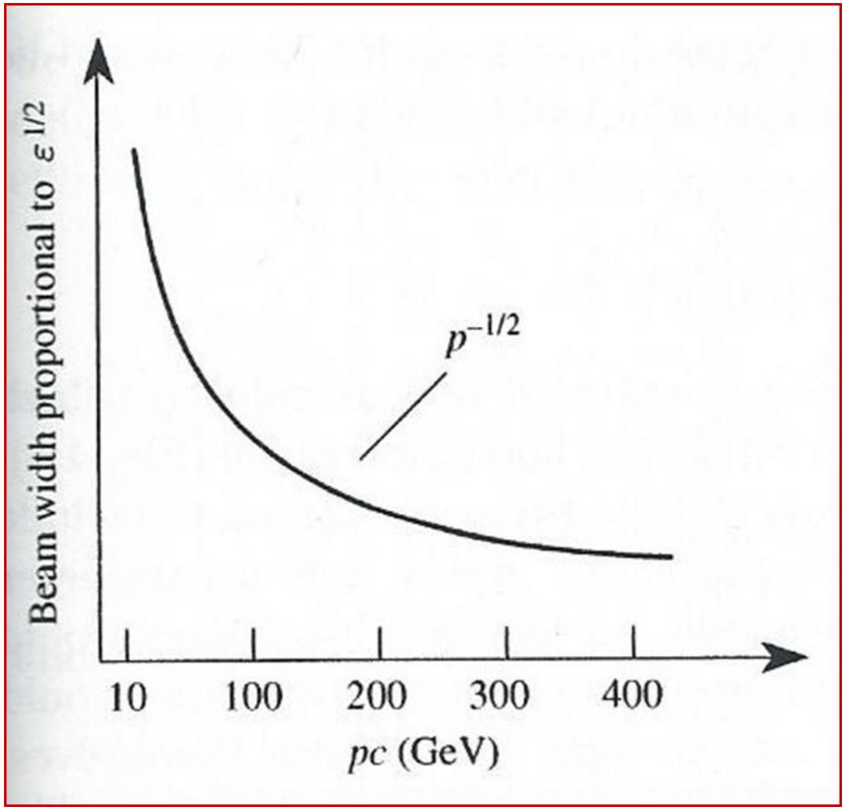
- Emittance is constant in FODO cell
  - small width implies large angle spread
  - limits the beam spot size at a target
  - impossible to create parallel beam

Skew ellipses → talk on Transverse Optics



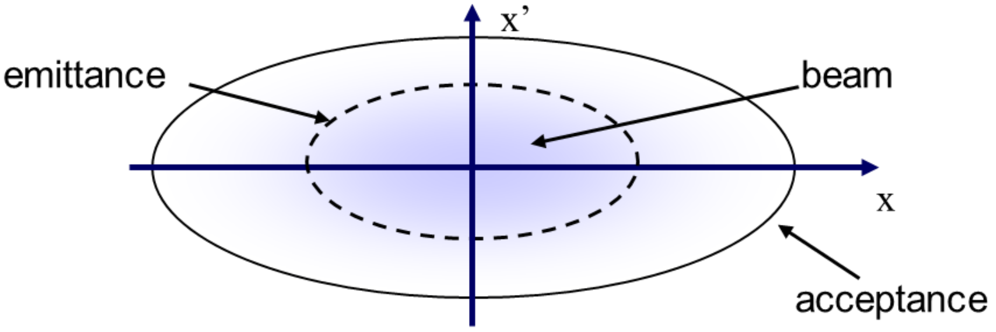
Emittance Change

- Sources of emittance growth
  - scattering in stripping foil or targets
  - ellipse mismatch
  - non-linear fields
- Ways to shrink emittance
  - acceleration shrinks emittance according to $\varepsilon \sim 1/p \sim 1/B\rho$ (**adiabatic damping**)
  - electron cooling (SIS, ESR) and stochastic cooling (ESR) available to create very small emittances
- Unnecessary emittance growth can limit performance
  - beam losses due to beam size growth
  - larger beam spot size at targets



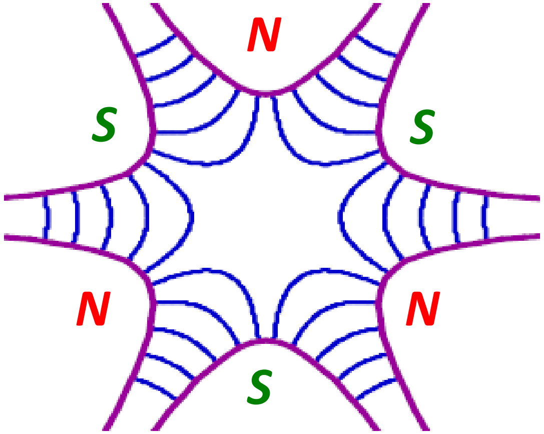
Emittance and Acceptance

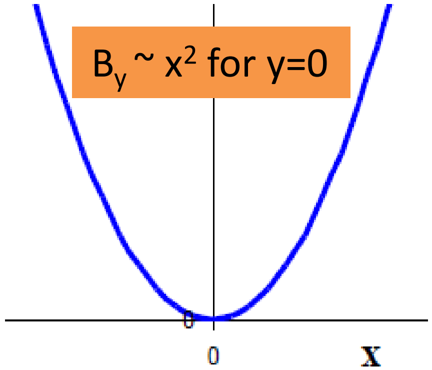
- Emittance:
  area of phase space ellipse containing a fraction of particles (e.g. 95%)
  - may grow due to mismatch, non-linear fields, beam interaction with pipe, etc.
  - does not take into account deviations of beam center from ideal orbit
- Acceptance:
  maximum area of the ellipse possible without ever losing particles (by hitting beam pipe)
  - acceptance in general smaller than physical aperture of beam pipe
  - need some margin for deviations of beam center and emittance growth



Non-Linear Magnets: Sextupoles

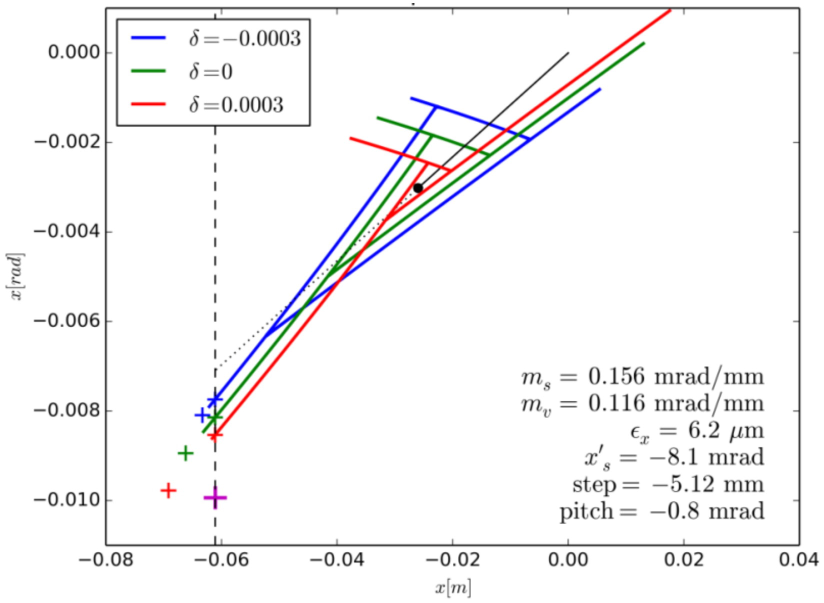
- Magnet with six poles
- Field depends quadratic on position
  - forces on particles non-linear
  - in general no analytic solutions
  - can even lead to chaotic motion



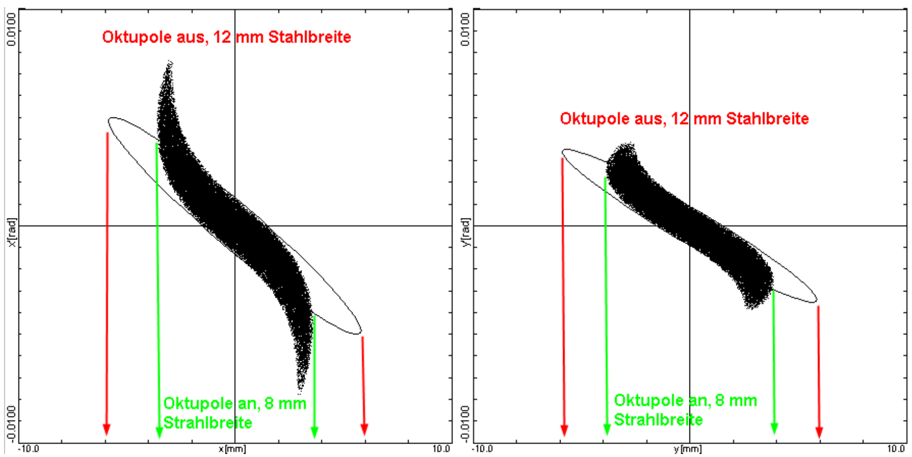


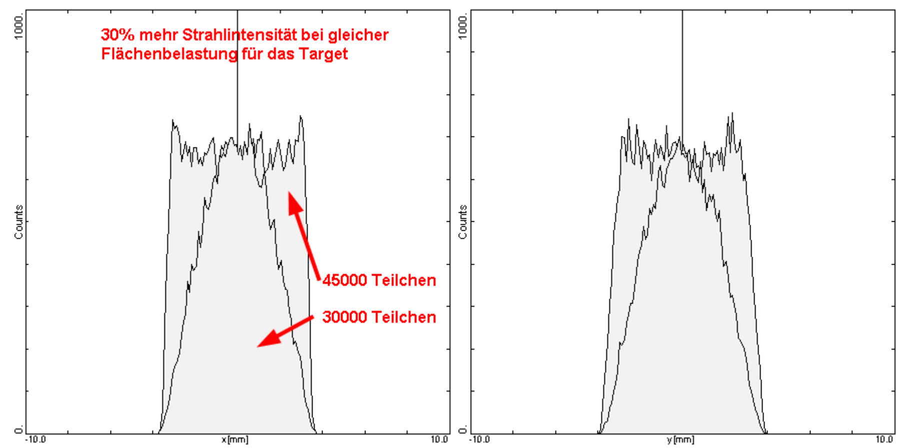
- Frequently used in accelerators
  - correction of momentum dependent effects on betatron oscillations (e.g. SIS18, ESR, FRS)
  - essential for slow extraction (SIS18)
  - compensation of unavoidable field errors in main magnets (dipoles)

Particle motion during slow extraction (SIS18)

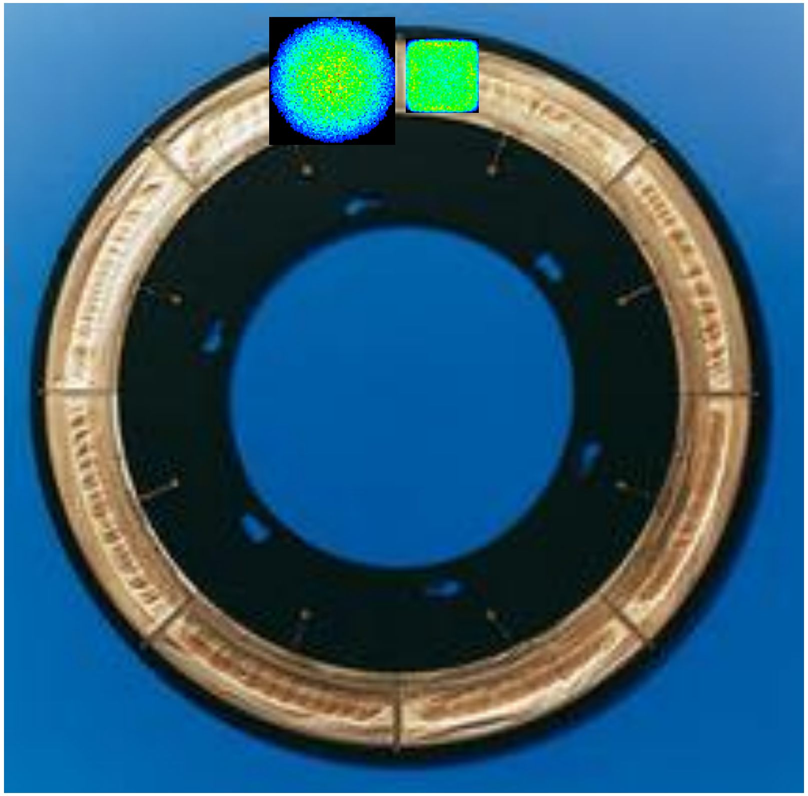


Non-Linear Magnets: Octupoles





- Octupoles for SHIP beamline
  - deformation of ellipse by non-linear field of octupoles
  - uniform distribution in cross section
  - higher beam intensity at equal maximum particle density on target



Summary

- Definition of elements and isotopes
- Special relativity and electro-dynamics
  - masses, energies and mass-energy relation
  - Relativistic particle motion and non-relativistic limit
  - Lorentz force and magnetic rigidity
- Motion of charged particles in magnetic fields
  - dipoles as bending magnets and separators
  - quadrupoles as focusing magnets
  - non-linear magnets for more sophisticated purposes
- Particle tracking and transverse phase space
  - vectors for description of particles and matrix formalism
  - FODO focusing structure
  - emittance and acceptance

Thank you for your attention!

These slides contain material I found in talks of the following colleagues:
W. Bayer and M. Maier (GSI), R. Steerenberg (CERN), K. Peach (Oxford University)

I'm very grateful for the possibility of profiting from their excellent material.