Charmonium Spectroscopy on the Lattice

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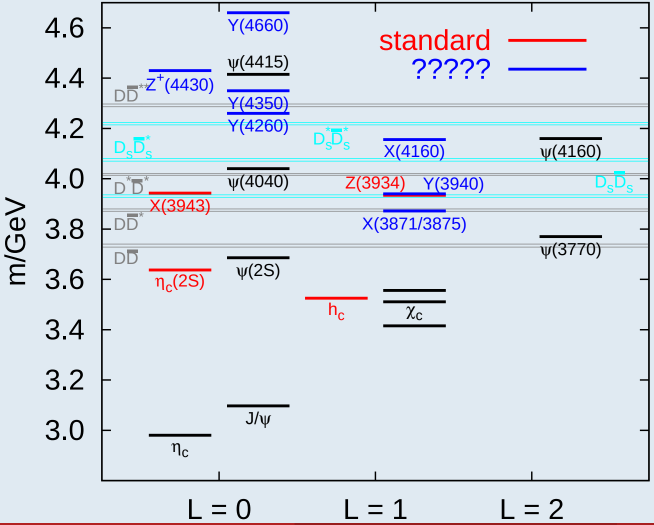
- The new Charmonia
- Lattice spectroscopy
- Fine structure
- Disconnected quark lines
- Outlook

1974 – 1977: 10 $c\bar{c}$ resonances, 1978 – 2001: 0 $c\bar{c}$'s

2002 – 2008: $\leq$ 12 new $c\bar{c}$'s found by BaBar, Belle, CLEO-c, CDF, D0



new detectors

higher luminosity

new channels:

B decays

$\gamma\gamma$

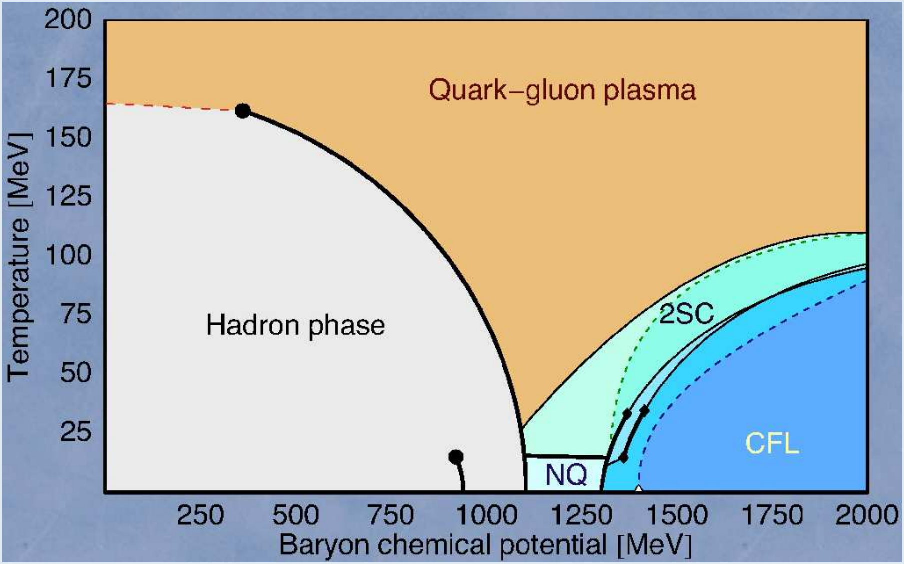
$\psi\psi$-production

gg in $p\bar{p}$ collisions.

$c\bar{q}q\bar{c}$ in $c\bar{c}$?

$cg\bar{c}$ hybrids ?

Possible QCD phase diagram: diquarks ?

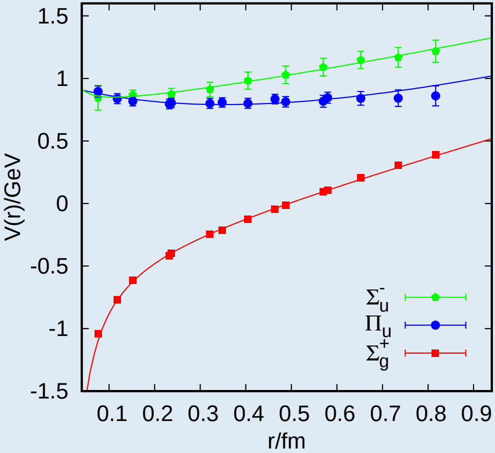


Hybrid mesons

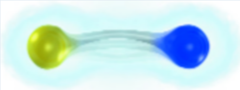
$m_c \gg \Lambda_{\rm QCD} \longrightarrow$ Adiabatic and non-relativistic approximations:

$$H\psi_{nlm} = E_{nl}\psi_{nlm} \quad , \quad H = 2m_c + \frac{p^2}{m_c} + V(r)$$

Lattice:



hybrid potential:



Input: $\mathcal{L}_{QCD} = -\frac{1}{16\pi\alpha_L} FF + \bar{q}_f(\not{D} + m_f)q_f$

$$\begin{aligned} m_N^{\text{latt}} &= m_N^{\text{phys}} &\longrightarrow\quad & a \\ m_\pi^{\text{latt}}/m_N^{\text{latt}} &= m_\pi^{\text{phys}}/m_N^{\text{phys}} &\longrightarrow\quad & m_u \approx m_d \\ &\cdots \end{aligned}$$

Output: hadron masses, matrix elements, decay constants, etc...

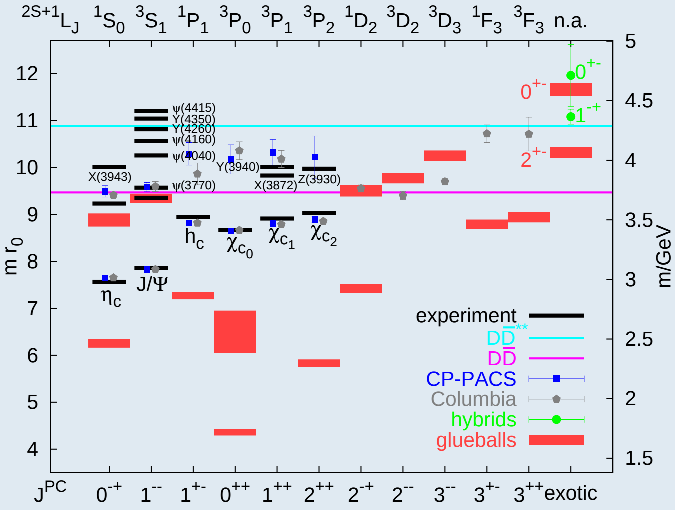
Extrapolations:

1. $a \to 0$: functional form known.
2. $L \to \infty$: harmless but often computationally expensive.
3. $m_q^{\text{latt}} \to m_q^{\text{phys}}$: chiral perturbation theory (χPT) but m_q^{latt} must be sufficiently small to start with.

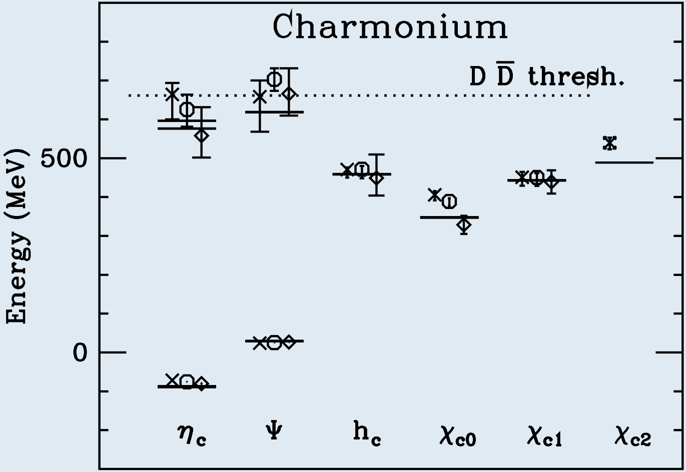
   ($m_\pi^{\text{latt}} = m_\pi^{\text{phys}}$ has only very recently been realized.)

Quenched Lattice: glueballs, charmonia and hybrids

(No "disconnected diagrams" and no sea quarks → no mixing $G, c\bar{c}, c\bar{q}q\bar{c}$, no decay !)

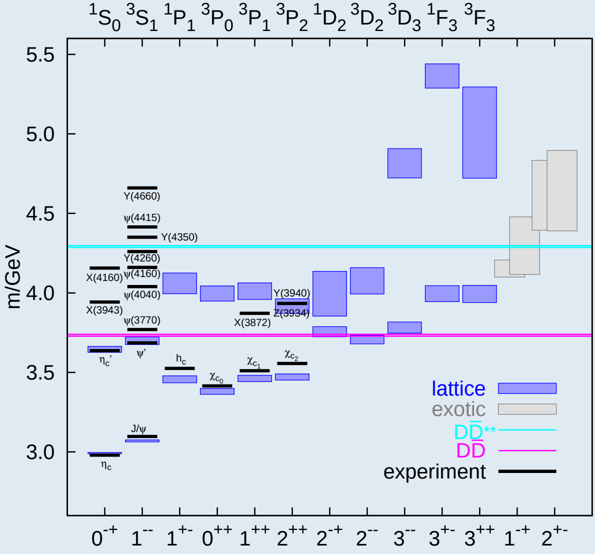


First result with sea quarks 04 FNAL+MILC ($n_f \stackrel{?}{\approx} 2+1$)



$a^{-1} \approx 1.1, 1.6, 2.3$ GeV

C Ehmann, GB ($n_f = 2$, $a^{-1} \approx 1.73$ GeV from m_N)

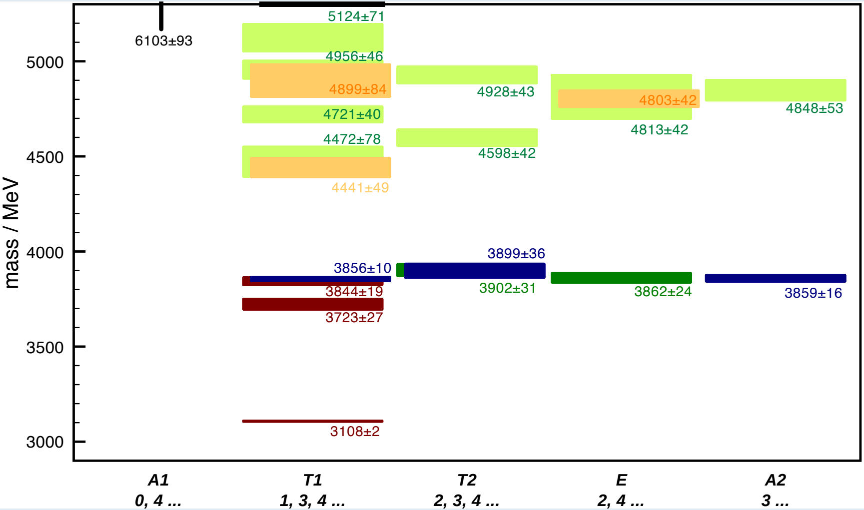


Lattice operators

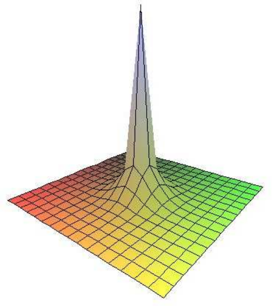
$$A_1 \rightarrow J = 0, 4, \cdots$$
$$A_2 \rightarrow J = 3, 6, \cdots$$
$$E \rightarrow J = 2, 4, \cdots$$
$$T_1 \rightarrow J = 1, 3, 4, \cdots$$
$$T_2 \rightarrow J = 2, 3, 4, \cdots$$

name	O_h repr.	J^{PC}	state	operator
a_0	A_1	0^{++}	χ_{c0}	1
π	A_1	0^{-+}	η_c	γ_5
ρ	T_1	1^{--}	J/ψ	γ_i
a_1	T_1	1^{++}	χ_{c1}	$\gamma_5\gamma_i$
b_1	T_1	1^{+-}	h_c	$\gamma_i\gamma_j$
$\pi \times \nabla$	T_1	1^{+-}	h_c	$\gamma_5\nabla_i$
$a_0 \times \nabla$	T_1	1^{--}	J/ψ	∇_i
$a_0' \times \nabla$	T_1	1^{-+}	exotic	$\gamma_4\nabla_i$
$(\rho \times \nabla)_{A_1}$	A_1	0^{++}	χ_{c0}	$\gamma_i\nabla_i$
$(\rho \times \nabla)_{T_1}$	T_1	1^{++}	χ_{c1}	$\epsilon_{ijk}\gamma_j\nabla_k$
$(\rho \times \nabla)_{T_2}$	T_2	2^{++}	χ_{c2}	$s_{ijk}\gamma_j\nabla_k$
$(a_1 \times \nabla)_{A_1}$	A_1	0^{--}	exotic	$s_{ijk}\gamma_j\nabla_k$
$(a_1 \times \nabla)_{T_2}$	T_2	2^{--}		$\gamma_5 s_{ijk}\gamma_j\nabla_k$
$(b_1 \times \nabla)_{T_1}$	T_1	1^{-+}	exotic	$\gamma_4\gamma_5\epsilon_{ijk}\gamma_j\nabla_k$
$a_0' \times D$	T_2	2^{+-}	exotic	$\gamma_4 D_i$
$(a_1 \times D)_{A_2}$	A_2	3^{++}		$\gamma_5\gamma_i D_i$
$(a_1 \times D)_{T_1}$	T_1	1^{++}	χ_{c1}	$\gamma_5 s_{ijk}\gamma_j D_k$
$(a_1 \times D)_{T_2}$	T_2	2^{++}		$\gamma_5\epsilon_{ijk}\gamma_j D_k$
$(b_1 \times D)_{A_2}$	A_2	3^{+-}		$\gamma_4\gamma_5\gamma_i D_i$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

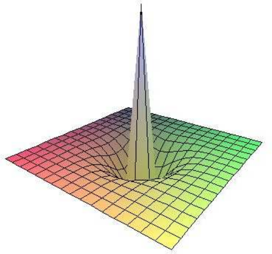
J-assignment for $PC = --$ J Dudek et al.



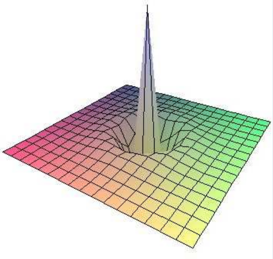
Wavefunctions after variational optimization (Coulomb gauge). CE, GB



S1

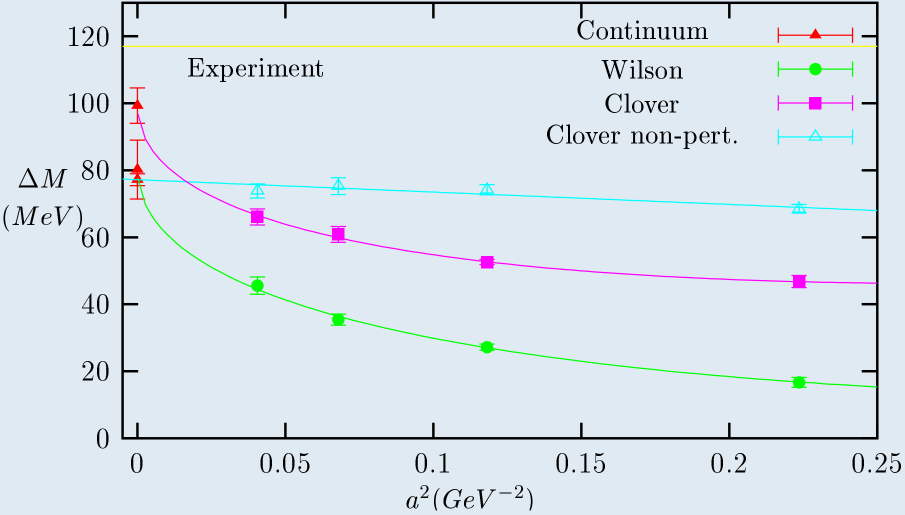


2S



3S

S Choe et al (QCD-TARO 03): fine splitting $\Delta M = m_{J/\psi} - m_{\eta_c}$ $(n_F = 0)$



NRQCD: $$\Delta M = \frac{1}{6m_c^2}\langle\psi|V_4|\psi\rangle + \cdots$$

Leading order perturbation theory: $V_4(r) = 8\pi C_F \alpha_s \delta^3(r)$.

ΔM	scale from $r_0 = 0.5$ fm	scale from $1\overline{P} - 1\overline{S}$
Columbia	72(2) MeV	83(??) MeV
CP-PACS	73(1)(4) MeV	85(4)(6) MeV
QCD-TARO	77(2)(6) MeV	89(??) MeV
χQCD	88(4) MeV	121(6) MeV
JLAB	97(6) MeV	???

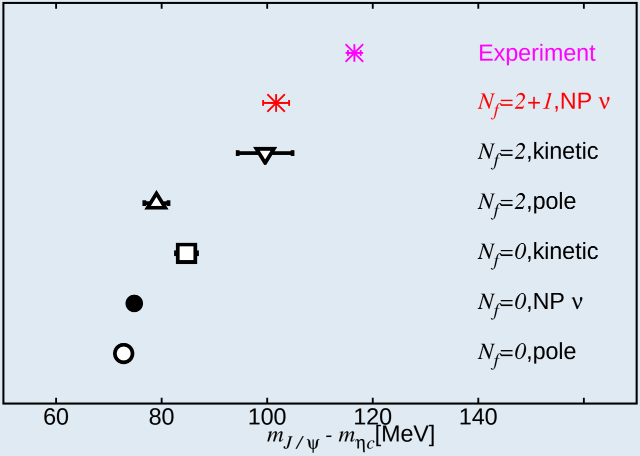
JLAB (Dudek et al): $m_c \approx 5\,\%$ too small !

χQCD (Tamhankar et al) + JLAB: only one lattice spacing a.

χQCD: $La < 0.9$ fm $\rightarrow 1\overline{P} - 1\overline{S}$ underestimated ?

Y Namekawa et al (PACS-CS, arXiv:0810.2364)

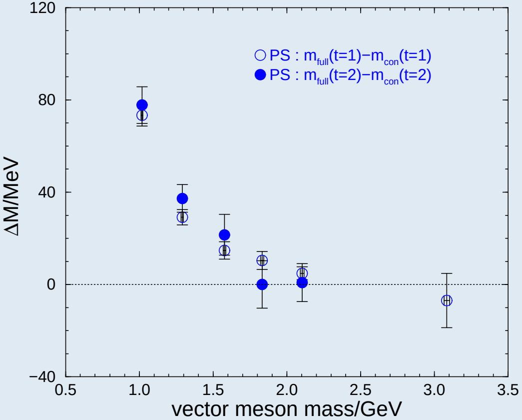
$a^{-1} \approx 2.2$ GeV from m_Ω, $310\,\mathrm{MeV} > m_\pi \geq 165\,\mathrm{MeV}$, $Na \approx 2.9$ fm.



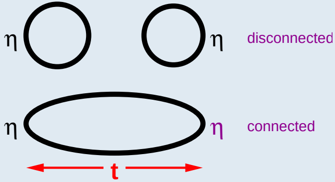
$\Delta M \to 117$ MeV as $a \to 0$?

"$I = 0$" vs. "$I = 1$" ???

Disconnected quark lines ? ($n_f = 0$: P de Forcrand et al (QCD-TARO 04))



$$\Delta M = m_{``\eta"} - m_{``\pi"}$$



Disconnected diagrams $\curvearrowright$ $m_\eta > m_\pi$ $\curvearrowright$ $m_\omega - m_\eta < m_\rho - m_\pi$

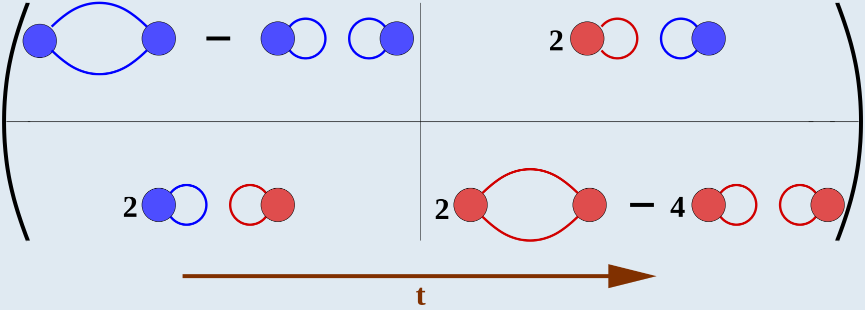
C McNeile & C Michael 04: sign change for heavy quarks ??

L Levkova & C DeTar 08: $\Delta M \approx 3 - 4$ MeV ($n_f = 0$).

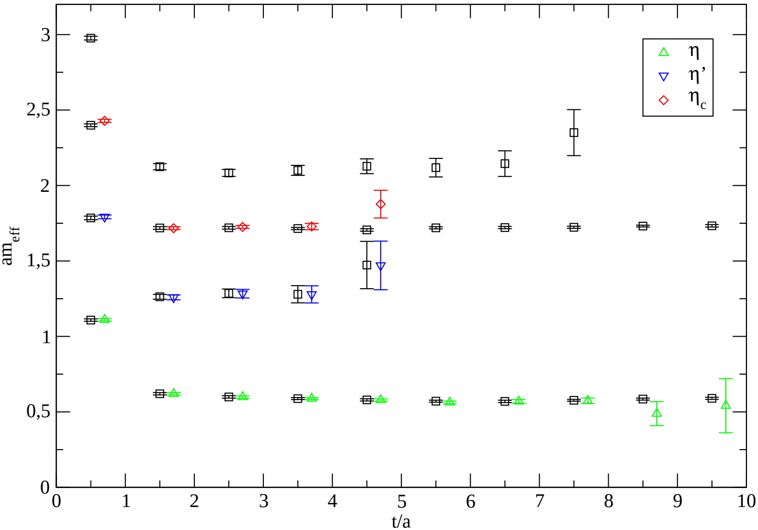
Obviously, disconnected diagrams are important, e.g.: $\psi'^{(\prime)} \leftrightarrow D\overline{D}$.

What about mixing with other $I = 0$ states?

C Ehmann, GB: $\eta_c \leftrightarrow \eta$ mixing ($n_f = 2$):

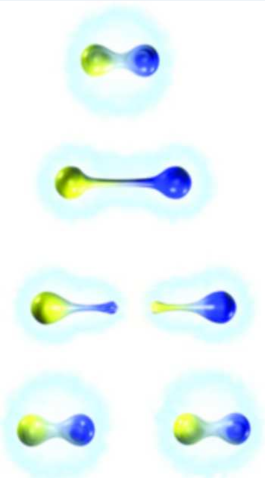


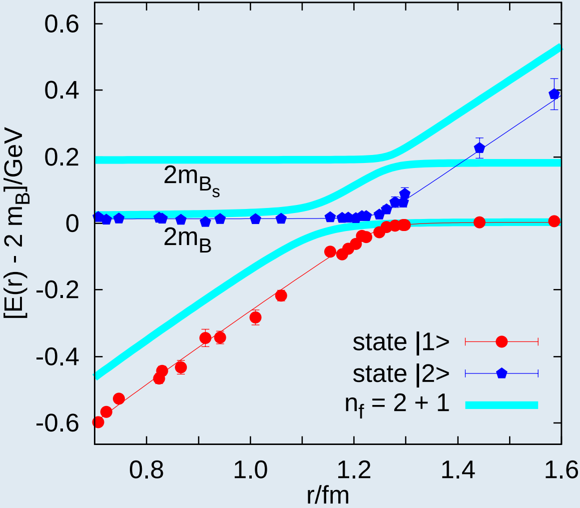
Mixing vs. no mixing:



Two state potentials

GB, H Neff, T Düssel, T Lippert, Z Prkacin, K Schilling





Coupled channel potential model for threshold effects ?

Many channels ($D\overline{D}, D^*\overline{D}, D_s\overline{D}_s, D^*\overline{D}^*, \cdots$) $\Rightarrow$ many parameters!

"Direct" calculation of the spectrum ?

We have to be able to resolve radial excitations!

(remember e.g. the very dense 1^{--} sector.)

Required: large basis of test wavefunctions including $c\bar{c}$, $c\bar{q}q\bar{c}$ and $cg\bar{c}$ operators and good statistics.

Outlook

- First $n_f = 2 + 1$ simulations near physical m_π at $a^{-1} \approx 2$ GeV.
- First precision calculations of annihilation and mixing diagrams.
- The continuum limit is important, in particular for the fine structure.
- There will be a lot of progress in sub-threshold charmonium spectroscopy in the next two years.
- Calculation of forces between pairs of static-light mesons for different S and I is on its way, to qualitatively understand 4-quark binding.
- Study of $c\bar{c} \leftrightarrow c\bar{q}q\bar{c}$, initially for $J^{PC} = 1^{--}$.