Theory of Semileptonic Charm Decays

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Outline

- Challenges in the SM
  - Hadronic form factors
  - Convergence of HQE & OPE
- Opportunities beyond the SM
  - Testing possible non-standard effects from leptonic decays
  - Precision studies of anomalous quark currents

Motivation

- Determination of V_{cs} and V_{cd} CKM matrix element moduli c.f. CKMFitter 0905.1572
  - In the SM currently best constrained through CKM unitarity, not direct measurement
  - Recently, deviations in leptonic decays measurements
- Testing ground for theoretical tools
  - Lattice QCD - need other theoretical inputs to control systematic uncertainties, connect to experiment
  - HQE & OPE - controlling power corrections in V_{ub} extraction

Exclusive semileptonic decays: Motivation

- Direct determination of V_{cs} and V_{cd} CKM moduli
  - From Unitarity
    - $|Vcd|_{UT} = 0.22508 \pm 0.00082$
    - $|Vcs|_{UT} = 0.97347 \pm 0.00019$

CKMFitter, Moriond 2009

  - From leptonic decays $D_s \to \mu(\tau)\ \nu$, $D^+ \to \mu\ \nu$
    - $|Vcd|_L = |Vcd|_{UT}\ (1.00 \pm 0.05)$
    - $|Vcs|_L = |Vcs|_{UT}\ (1.08 \pm 0.03)$

Cleo 0806.2112

Averages of Cleo & Belle measurements 0901.1216, 0901.1147, 0709.1340 taken fom Akeroyd & Mahmoudi 0902.2393

Lattice decays constants
-HPQCD 0706.1726
-European Twisted Mass 0904.0954

  - Cross-checks of Lattice, Experiment needed

Exclusive semileptonc decays D → P l ν

- Differential decay width in the SM

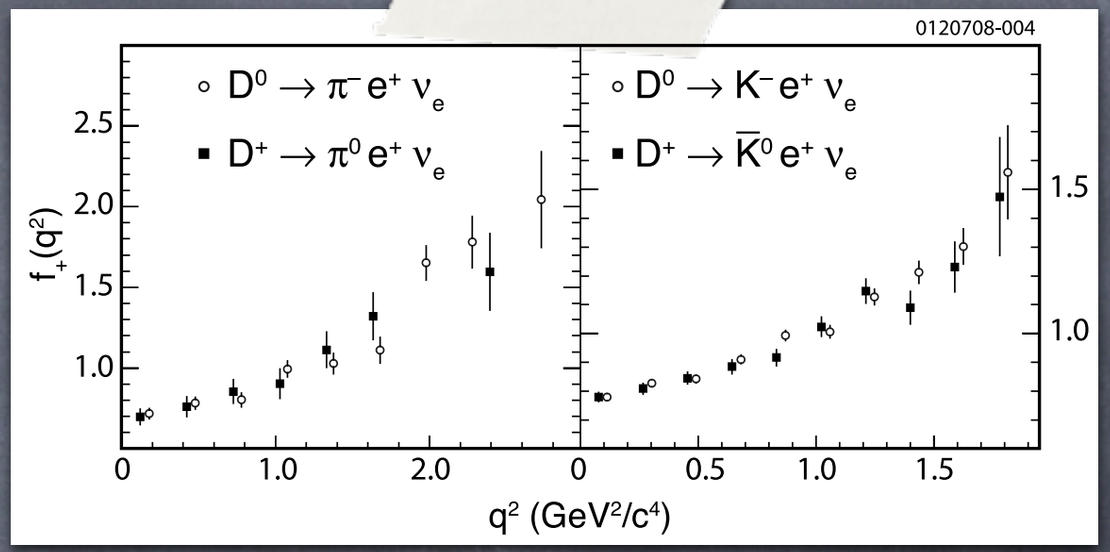
$$\frac{d\Gamma(D \to P\ell\bar{\nu}_\ell)}{dq^2\, d\cos\theta_\ell} = \frac{G_F^2|V_{cq}|^2}{32\pi^3} p^{*\,3} |f_+(q^2)|^2 \sin\theta_\ell^2$$

- Quark current matrix element and form factors (FFs)

$$\langle P(p)|\bar{q}\gamma^\mu c|D(p')\rangle = f_+(q^2)\left[(p'+p)^\mu - \frac{M_D^2 - m_P^2}{q^2} q^\mu\right] + f_0(q^2)\frac{M_D^2 - m_P^2}{q^2} q^\mu$$

- $|V_{cq}|$ can be extracted if
  - FF normalization at a single point is known (customarily at $q^2 = 0$)
  - (partial) phase space integral can be extracted
- Example: LQ Sum rules [Ball hep-ph/0608116]
  - $f^+(D\to\pi)(0) = 0.63 \pm 0.11$
  - $f^+(D\to K)(0) = 0.75 \pm 0.12$
- More precision from Lattice QCD

Cleo 0810.3878



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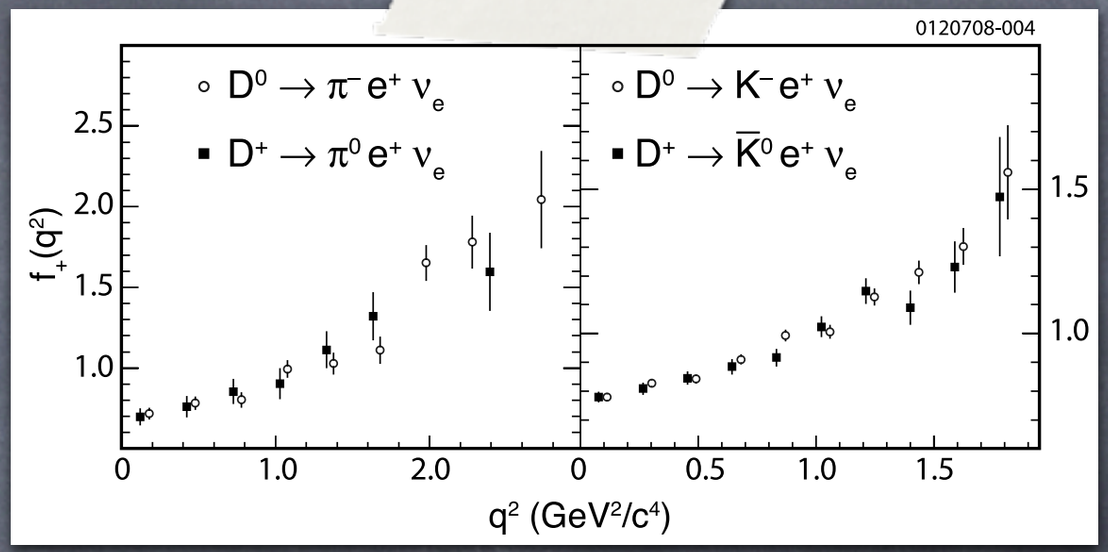
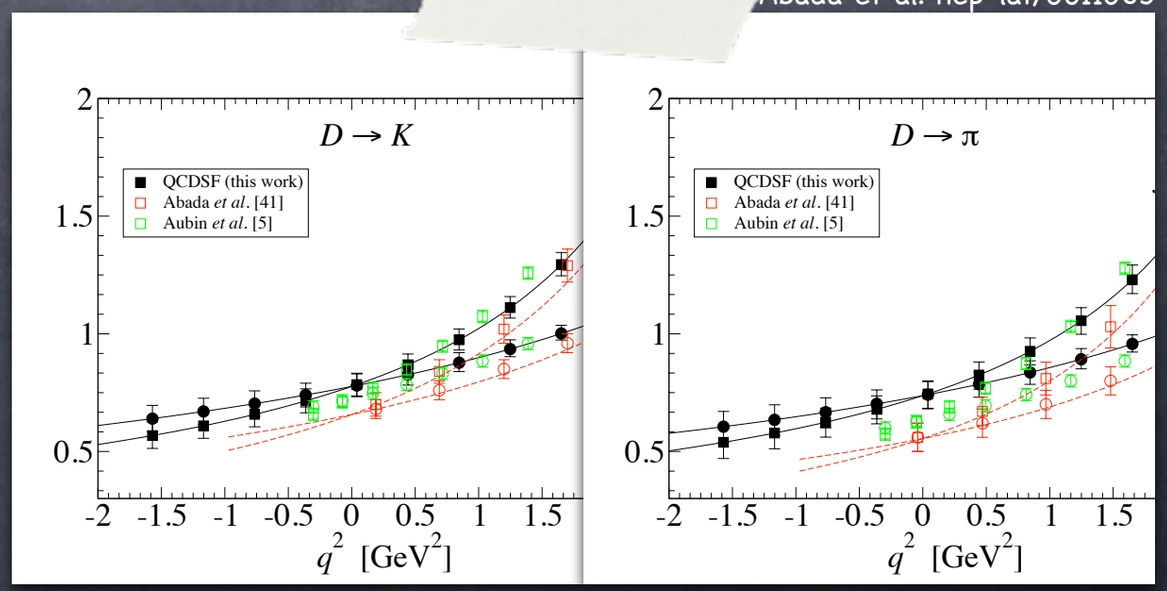
- $|V_{cq}|$ can be extracted if
  - FF normalization at a single point is known (customarily at $q^2 = 0$)
  - (partial) phase space integral can be extracted (from data)
- Problem in the past: Lattice accurate in a kinematical region of diminishing phase-space $q^2 \approx$ max
- "Extrapolation" between Lattice & Exp. needed: FF parameterizations
  - Recent lattice studies can do without in D decays

Cleo
0810.3878



QCDSF 0903.1664
Fermilab, MILC, HPQCD
hep-ph/0408306
Abada et al. hep-lat/0011065



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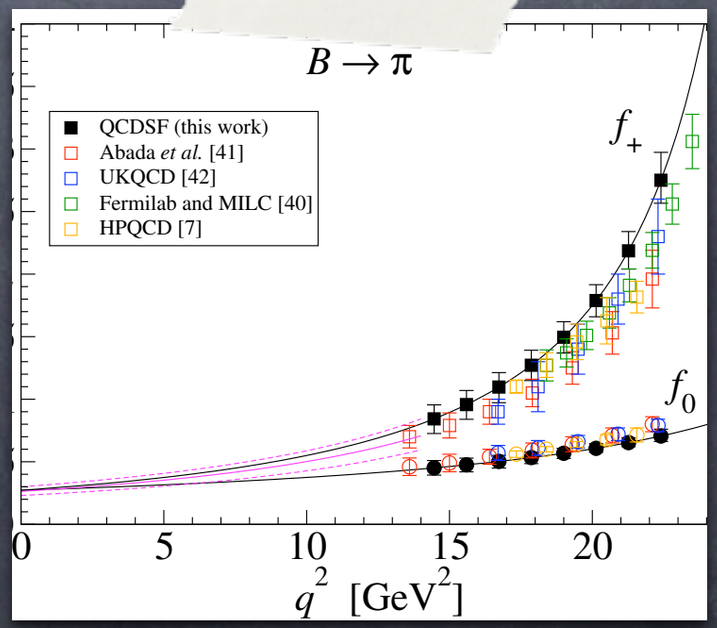
B -> π l ν

BF(total) = 1.34 10^{-4} (1 ± 0.06)
BF(q^2<16 GeV^2) = 0.93 10^{-4} (1 ± 0.07)
BF(q^2>16 GeV^2) = 0.37 10^{-4} (1 ± 0.10)

HFAG
ICHEP '08

- $|V_{cq}|$ can be extracted if
  - FF normalization at a single point is known (customarily at q^2 = 0)
  - (partial) phase space integral can be extracted (from data)
- Problem in the past: Lattice accurate in a kinematical region of diminishing phase-space q^2 = max
- "Extrapolation" between Lattice & Exp. needed: FF parameterizations
  - Recent lattice studies can do without in D decays
  - Still important in exclusive determinations of $|V_{ub}|$ from B->π - can be tested in the charm sector



QCDSF 0903.1664
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Exclusive semileptonc decays D → P l ν

- Constructing FF parametrizations:
  - Exact (analytic) shape of the FF is illusive, Lattice can calculate numerically at individual q^2 points -> how to best (extra-)interpolate
  - Expansion in functions of q^2
    - q^2 [0, $q_{max}^2 = (m_D - m_P)^2$] - Taylor expansion (K->pi)
    - $(q^2 - m_i^2)^{-1}$ - sum over t-channel resonance contributions
    - $z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$ [0,$|z_{max}|$<1], $t_+ = (m_D + m_P)^2$ - expansion around t_0

c.f.
Boyd & Savage
hep-ph/9702300

  - Dispersion relations
    - In D -> K decays, the first t-channel resonance (D_s^*) is below the physical cut at $q^2 = t_+$ (also for B->pi)
    - $$f^0(q^2) = \frac{1}{\pi}\int_{t_+}^{\infty} dt \frac{\mathrm{Im} f^0(t)}{t - q^2 - i\epsilon}$$

$$f^+(q^2) = \frac{\mathrm{Res}_{q^2 = m_{D_s^*}^2} f^+(q^2)}{m_{D_s^*}^2 - q^2} + \frac{1}{\pi}\int_{t_+}^{\infty} dt \frac{\mathrm{Im} f^+(t)}{t - q^2 - i\epsilon}$$

Related to the D_s^+ D K coupling
Descotes-Genon, Le Yaouanc 0804.0203

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c.f.
Boyd & Savage
hep-ph/9702300

  - Heavy quark scalling limits (m_c -> ∞, m_q -> 0)

Isgur & Wise
PRD42, 2388 (1990)

    - HQ, soft P: $f^+(q^2 \simeq q^2_{\max}, m_D) \sim \sqrt{m_D}\ , \quad f^0(q^2 \simeq q^2_{\max}, m_D) \sim \frac{1}{\sqrt{m_D}}\ .$
    - LQ SR, SCET: $f^{+0}(q^2 \simeq 0, m_D) \sim m_D^{-3/2}$

Charles et al. hep-ph/9812358
Beneke & Feldmann hep-ph/0008255

$$f^0 \simeq \frac{2E}{m_B} f^+$$ (Broken by $\alpha_s(m_c)$, Λ/m_c corrections)

Exclusive semileptonc decays $D \to P\, l\, \nu$

- Constructing FF parametrizations:
  - Exact (analytic) shape of the FF is illusive, Lattice can calculate numerically at individual q^2 points -> how to best (extra-)interpolate
  - Expansion in functions of q^2
    - q^2 $[0, q_{max}^2 = (m_D - m_P)^2]$ - Taylor expansion (K->pi)
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  - Single pole for $f^+(q^2)$ inconsistent with measured spectra Focus hep-ex/0410037 Cleo hep-ex/0407035

$$f^+(q^2) = \frac{f^+(0)}{(1-x)}\,, \quad x = q^2/m_{\rm pole}^2$$

Exclusive semileptonc decays D → P l ν

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  - Single pole for $f^+(q^2)$ inconsistent with measured spectra Focus hep-ex/0410037 Cleo hep-ex/0407035
  - [Becirevc & Kaidalov hep-ph/9904490] ansatz respecting LO scalling behavior

$$f^+(q^2) = \frac{f(0)}{(1-x)(1-ax)}, \quad f^0(q^2) = \frac{f(0)}{(1-bx)}$$

  - As truncated pole expansion, requires $\frac{1}{f^+(0)}\left[\frac{df^+(x)}{dx} - \frac{df^0(x)}{dx}\right]\Big|_{x\to 0} \approx 1$ or a≈b(≈1) - not valid for D->pi, D->K decays

Hill, hep-ph/0606023
Fajfer & J.F.K. hep-ph/0412140

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      c.f. Boyd & Savage hep-ph/9702300
  - Using dispersion relations

$$f_+(q^2) = \frac{1}{P(q^2)\,\phi(q^2,t_0)} \sum_{k=0}^{\infty} a_k(t_0)[z(q^2,t_0)]^k$$

    see also Bourrely, Caprini, Lellouch 0807.2722
    - $P(q^2)$ - subtracts poles below continuum threshold
    - $\phi(q^2,t_0)$ - normalization factor from perturbative OPE unitarity
    - In D->P, existing data much more constraining than unitarity bounds

      Becher & Hill hep-ph/0509090

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c.f.
Boyd & Savage
hep-ph/9702300

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see also
Bourrely, Caprini, Lellouch
0807.2722

    - In D->P, existing data much more constraining than unitarity bounds

Becher & Hill
hep-ph/0509090

      - $|V_{cd}|_{SL} = |V_{cd}|_{UT}$ (0.99 ± 0.11)
      - $|V_{cs}|_{SL} = |V_{cs}|_{UT}$ (1.05 ± 0.13)

Cleo 0810.3878
using FF normalization from
Lattice hep-ph/0408306
parametrization uncertainty?

Exclusive semileptonc decays D → P l ν

- Beyond SM, new operator contributions may contribute significantly to the decay rate
  - $$\langle P(k)|\bar{s}\sigma^{\mu\nu}c|D(p)\rangle = im_D^{-1}(p^\mu k^\nu - p^\nu k^\mu)f_2(q^2),$$
    $$\langle P(k)|\bar{s}c|D(p)\rangle = \frac{m_D^2 - m_P^2}{m_c - m_q} f_0(q^2)$$
  - May contribute differently to leptonic and semileptonic decays
    - Chiral current interactions produce similar enhancement, proportional to f^+
    - Scalar or tensor interactions need to scale with lepton mass to produce similar enhancement in tau and mu leptonic modes - negligible in semileptonic case (tau channel closed)
    - Need knowledge of one additional FF:
      - In both SCET & HQET limits $f_2 \approx 2f_+$
      - Ansatz: the relation holds throughout the kinematical region
      - Could be tested on the lattice

Isgur & Wise
PRD42, 2388 (1990)
Beneke & Feldmann
hep-ph/0008255

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    - Chiral current interactions produce similar enhancement, proportional to f^+
    - Scalar or tensor interactions need to scale with lepton mass to produce similar enhancement in tau and mu leptonic modes - negligible in semileptonic case (tau channel closed)
    - Need knowledge of one additional FF: $f_2 \approx 2f_+$
- Other observables, available in the semileptonic mode may help to discriminate among contributions

$$\mathcal{A}_\perp = \frac{\Gamma(E_{\ell\perp} > 0) - \Gamma(E_{\ell\perp} < 0)}{\Gamma(E_{\ell\perp} > 0) + \Gamma(E_{\ell\perp} < 0)} \qquad E_{\ell\perp} = \frac{p\cdot\ell}{m_D} - \frac{p\cdot q\, q\cdot\ell}{m_D q^2} = E_\ell - \frac{1}{2}(m_D - E_K)\left(1 + m_\ell^2/q^2\right)$$

  - Sensitive to interference between SM & scalar or tensor interactions
  - Very challenging experimentally?

Kronfeld 0812.2030

Exclusive semileptonc decays D → V l ν

- Polarizations of the vector meson in the final state give access to more observables
- Need to know more form factors:
  - V, $A_{0,1,2}$ in the SM (and for scalar contributions)
  - $T_{1,2,3}$ for additional tensor contributions
- Lattice can provide normalization at various kinematical points
- HQ scalling laws & relations can be used to construct useful parametrizations

Fajfer & J.F.K. hep-ph/0506051
Becirevic et al. hep-ph/0611295

  - HQ, soft V $V(q^2 \simeq q^2_{\max}, m_D) \sim \sqrt{m}_D$ $A_0(q^2 \simeq q^2_{\max}, m_D) \sim \sqrt{m}_D$
    $A_1(q^2 \simeq q^2_{\max}, m_D) \sim \frac{1}{\sqrt{m}_D}$ $A_2(q^2 \simeq q^2_{\max}, m_D) \sim \sqrt{m}_D$

Isgur & Wise PRD42, 2388 (1990)

  - SCET $V(q^2 \simeq 0, m_D) \sim m_D^{-3/2}$ $A_{0,1,2}(q^2 \simeq 0, m_D) \sim m_D^{-3/2}$

Charles et al. hep-ph/9812358
Beneke & Feldmann hep-ph/0008255

    - FF relations broken by $\alpha_s(m_c)$, Λ/m_c corrections

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  - BK analogues for H->V decays

$$V(q^2) = \frac{V(0)}{(1-x)(1-ax)} \qquad A_0(q^2) = \frac{A_0(0)}{(1-y)(1-a'y)}$$
$$A_1(q^2) = \frac{A_1(0)}{(1-bx)} \qquad A_2(q^2) = \frac{A_2(0)}{(1-bx)(1-b'x)}$$

  - $V(0)/A_1(0)/T(0) \approx 1$, $a^{(i)}, b^{(i)} \approx 1$

$$T_1(q^2) = \frac{T(0)}{(1-x)(1-ax)} \quad T_2(q^2) = \frac{T(0)}{(1-bx)} \quad T_3(q^2) = \frac{T_3(0)}{(1-bx)(1-b'x)}$$

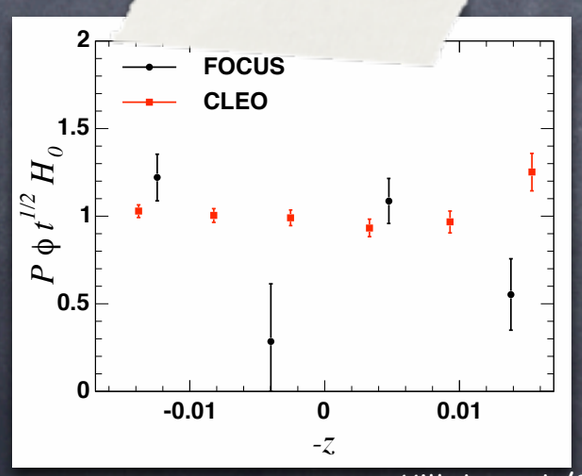
Exclusive semileptonc decays D → V l ν

- Polarizations of the vector meson in the final state give access to more observables
- Differential decay width in the SM usually written in terms of Helicity amplitudes H_{+-0}

$$\frac{d\Gamma(D \to V \ell \bar{\nu}_\ell)}{dq^2\, d\cos\theta_\ell} = \frac{G_F^2\, |V_{cq}|^2}{128\pi^3 M_D^2}\, p^*\, q^2 \left[\frac{(1-\cos\theta_\ell)^2}{2} |H_-|^2 + \frac{(1+\cos\theta_\ell)^2}{2} |H_+|^2 + \sin^2\theta_\ell |H_0|^2 \right]$$

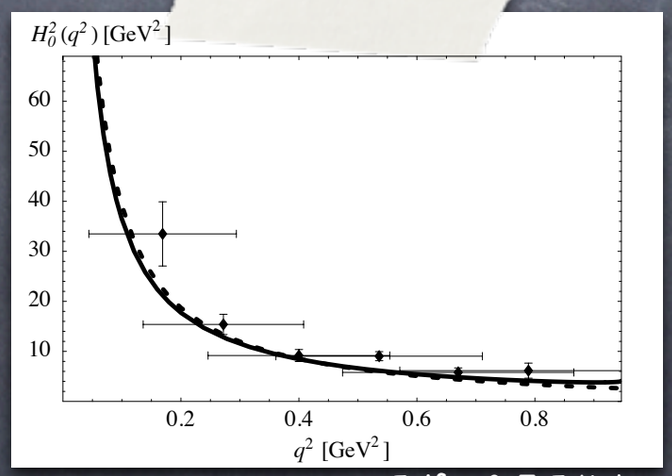
- Experimental information on H_{+-0} available, can be used to test FF parametrizations

z-expansion



Hill, hep-ph/0606023

modified pole ansatz



Fajfer & J. F. K. hep-ph/0601028

Focus hep-ex/0509027

Cleo hep-ex/0606010
0709.3247

- Access to anomalous scalar, tensor contributions

Inclusive Semileptonic Decays

- $D \to X\, l\, \nu$
- Recently determined experimentally
  - $$\begin{aligned} B(D^+ \to Xe\nu) &= (16.13 \pm 0.20 \pm 0.33)\% \\ B(D^0 \to Xe\nu) &= (6.46 \pm 0.17 \pm 0.13)\% \end{aligned}$$
  - Similar results for muons

N. E. Adam et al. [CLEO] hep-ex/0604044

M. Ablikim et al. [BES] arXiv:0804.1454

- Treating charm quark mass as heavy, one can attempt an expansion in $\alpha_s(m_c)$, Λ/m_c
  - Need to estimate local operator matrix elements (lattice, experiment)

Motivation:
Power corrections to $B \to X_u\, l\, \nu$

Lange, Neubert and Paz [hep-ph/0504071]

Andersen and Gardi [hep-ph/0509360]

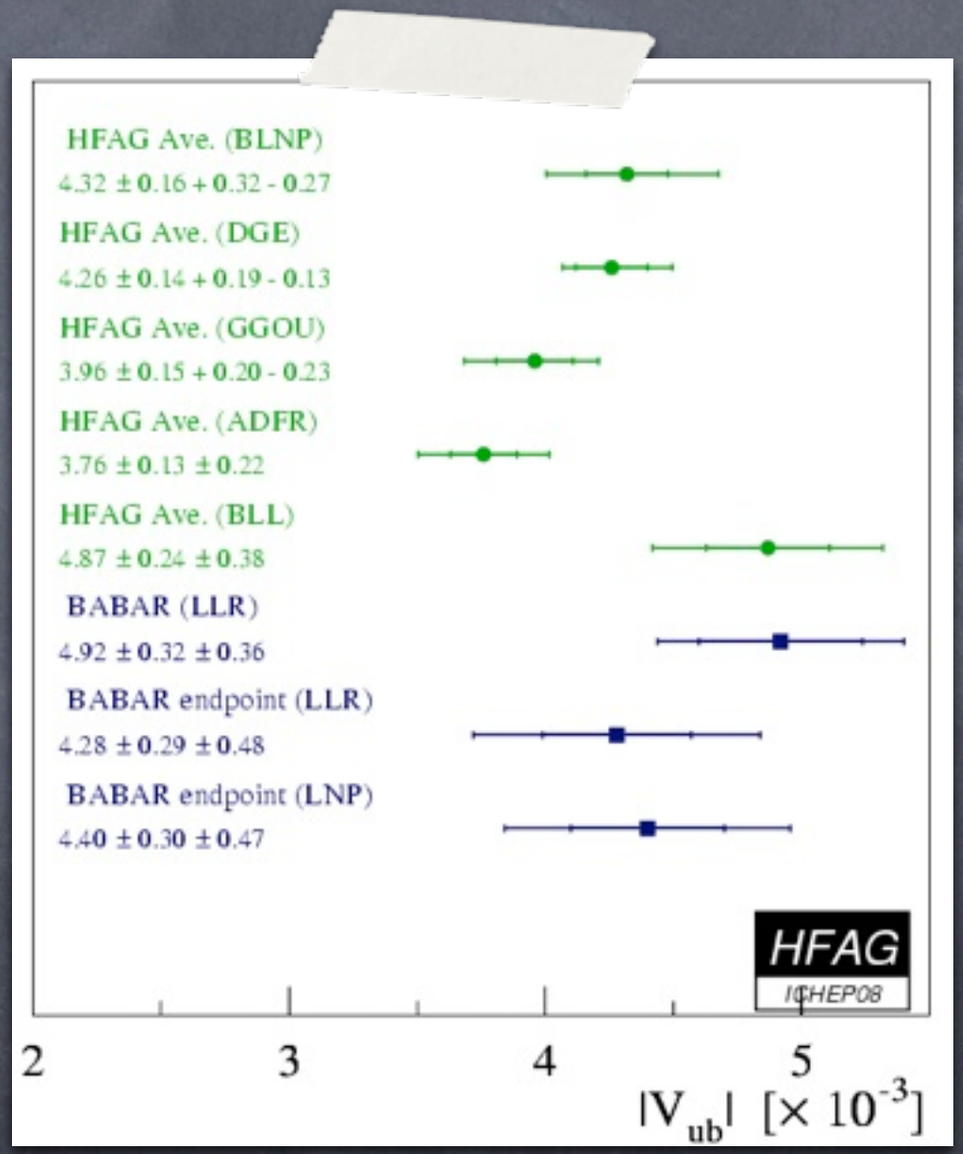
Gambino, Giordano, Ossola, Uraltsev [arXiv:0707.2493]

Aglietti, Di Lodovico, Ferrera, Ricciardi [arXiv:0711.0860]

Bauer, Ligeti and Luke [hep-ph/0107074]

- Inclusive determination of V_{ub} using OPE and HQE
- Precision around 10%
- At $1/m_b^3$ leading spectator effects due to dimension 6 four quark operators (WA contributions)
  - Cannot be extracted from inclusive b->c analysis
  - $16\pi^2$ phase space enhanced compared to LO & NLO contributions

    Not present at dim=7
    [Dassinger et al. hep-ph/0611168]



OPE and heavy quark expansion

- Optical theorem

$$\Gamma(H_{Q\bar{q}}) = \frac{1}{2m_H} \langle H_{Q\bar{q}} | \mathcal{T} | H_{Q\bar{q}} \rangle$$

$$\mathcal{T} = \mathrm{Im}\, i \int d\ x\, T\{\mathcal{H}_{eff}(x)\mathcal{H}_{eff}(0)\}$$

- Quark-hadron duality, HQE & OPE ($z=m_s^2/m_c^2$)

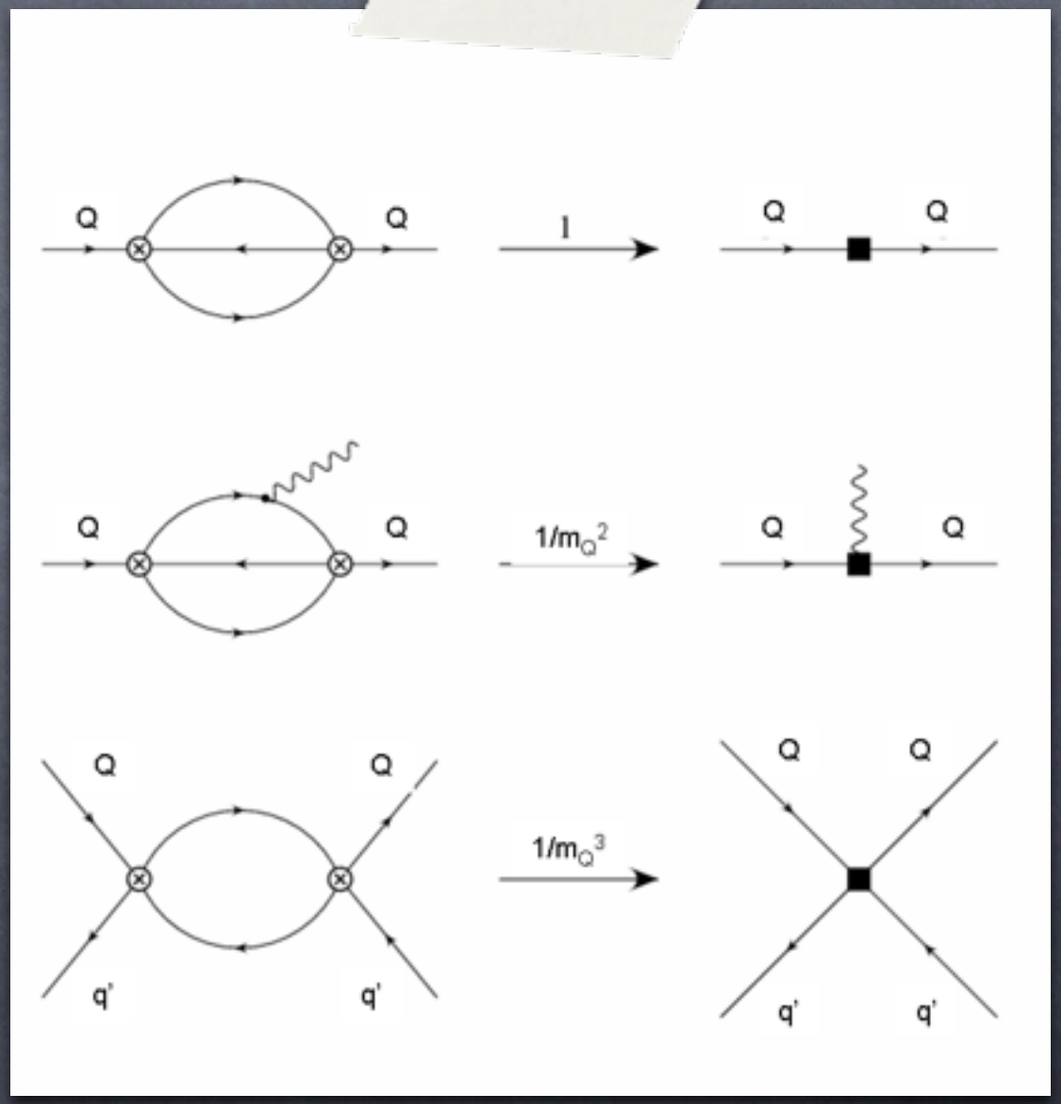
$$\Gamma(D \to X e \nu) = \frac{G_F^2 m_c^5}{192\pi^3} |V_{cs}|^2 \eta(z) \left[\frac{1}{2m_D} \left(I_0(z) \langle D|\bar{c}c|D\rangle - \frac{I_1(z)}{m_c^2} \langle D|\bar{c} g_s \sigma G c|D\rangle - \frac{16\pi^2}{2m_c^3} \langle D|\mathcal{O}_{V-A} - \mathcal{O}_{S-P}|D\rangle + \ldots \right) \right]$$

Bigi et al. [hep-ph/9207214]

Manohar and Wise, [hep-ph/9308246]

...

Additional non-enhanced power corrections



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$$\Gamma(D \to Xe\nu) = \frac{G_F^2 m_c^5}{192\pi^3}|V_{cs}|^2\eta(z) \left[\frac{1}{2m_D}\left(I_0(z)\langle D|\bar{c}c|D\rangle - \frac{I_1(z)}{m_c^2}\langle D|\bar{c}g_s\sigma Gc|D\rangle - \frac{16\pi^2}{2m_c^3}\langle D|\mathcal{O}_{V-A} - \mathcal{O}_{S-P}|D\rangle + \ldots\right)\right]$$

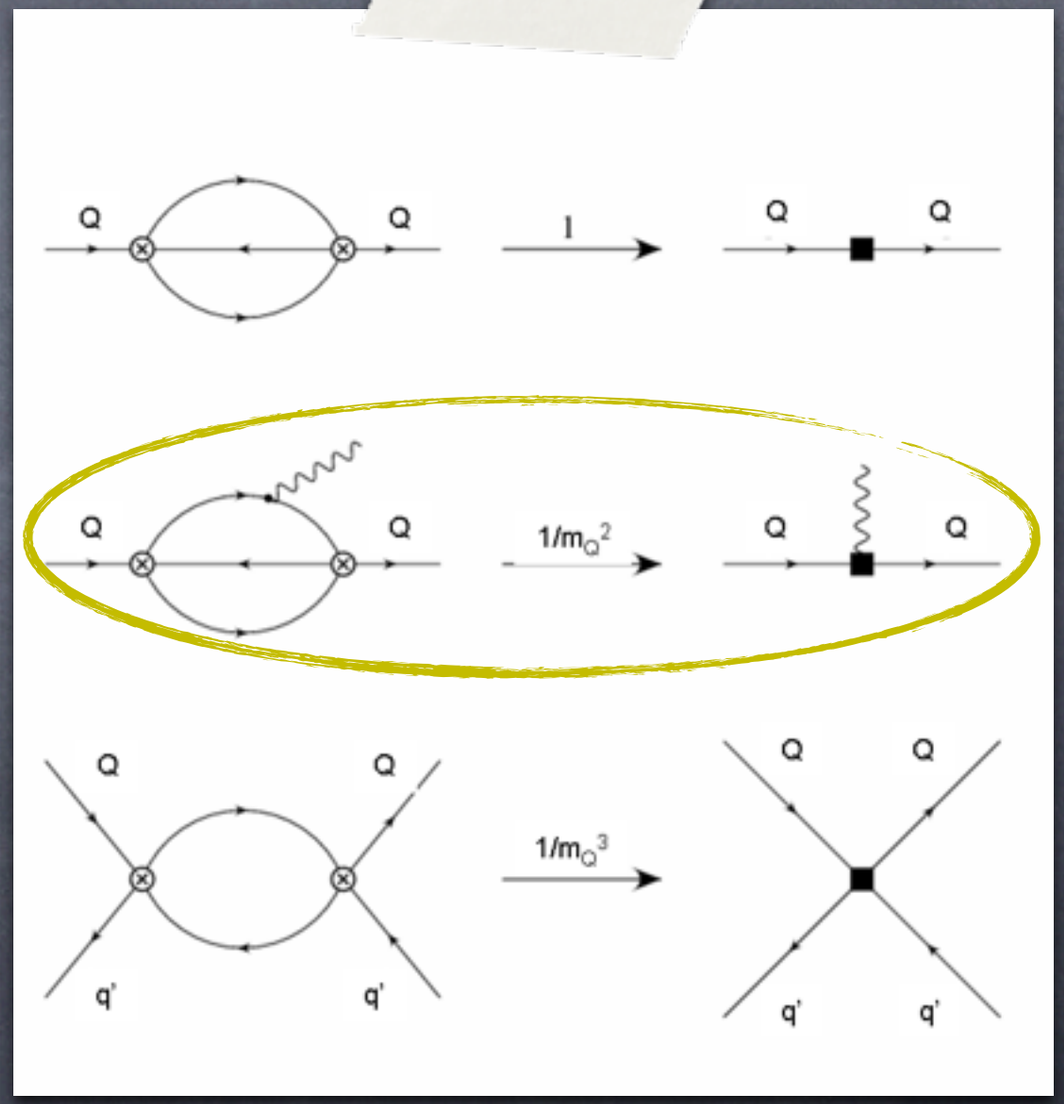
- Equations of motion

$$\bar{c}c = \bar{c}\not{p}c + \frac{1}{2m_c^2}\left(\bar{c}(iD_\perp)^2c + \bar{c}\frac{g_s}{2}\sigma.Gc\right) + \mathcal{O}(1/m_c^3)$$

- HQE parameters

$$\mu_\pi^2 = -\frac{1}{2m_D}\langle D|\bar{c}(iD_\perp)^2c|D\rangle$$

$$\mu_G^2 = \frac{1}{2m_D}\langle D|\bar{c}\frac{g_s}{2}\sigma.Bc|D\rangle$$



OPE and heavy quark expansion

- Quark-hadron duality, HQE & OPE

$$\Gamma(D \to Xe\nu) = \frac{G_F^2 m_c^5}{192\pi^3}|V_{cs}|^2\eta(z) \left[\frac{1}{2m_D}\left(I_0(z)\langle D|\bar{c}c|D\rangle - \frac{I_1(z)}{m_c^2}\langle D|\bar{c}g_s\sigma Gc|D\rangle - \frac{16\pi^2}{2m_c^3}\langle D|\mathcal{O}_{V-A} - \mathcal{O}_{S-P}|D\rangle + \ldots\right)\right]$$

- HQE parameters

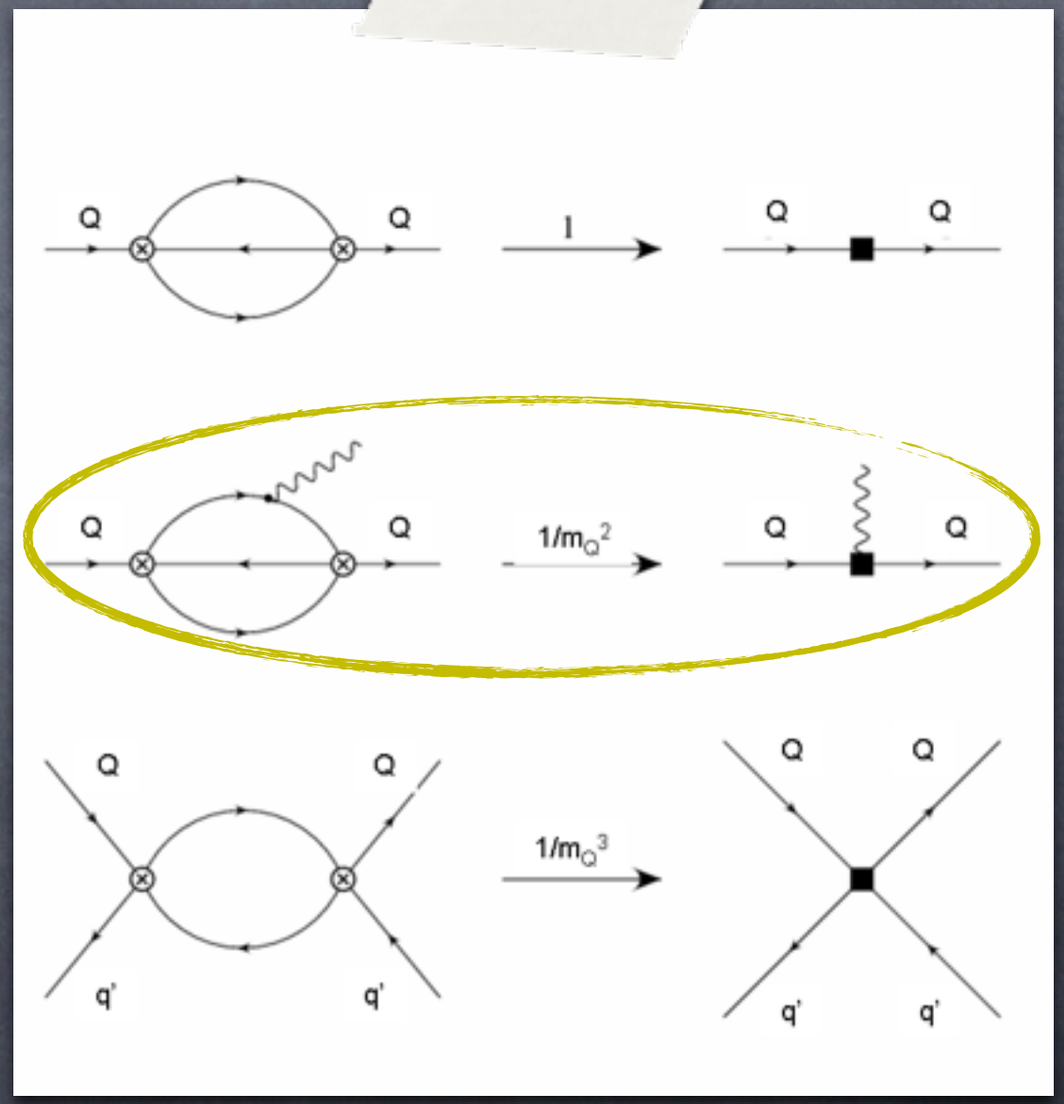
$$\frac{1}{2m_D}\langle D|\bar{c}c|D\rangle = 1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_c^2}$$

- Can be extracted experimentally

$$\mu_G^2 = (3/4)[m_{D^*}^2 - m_D^2] = 0.41\ \mathrm{GeV}^2$$

$$\mu_\pi^2 \approx 0.5(1)\ \mathrm{GeV}^2$$ From a fit to the b->c spectrum

See e.g. A Hauke [arXiv:0706.4468]



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- HQE parameters - spectator cont.

Dim=6 operators involving light flavors

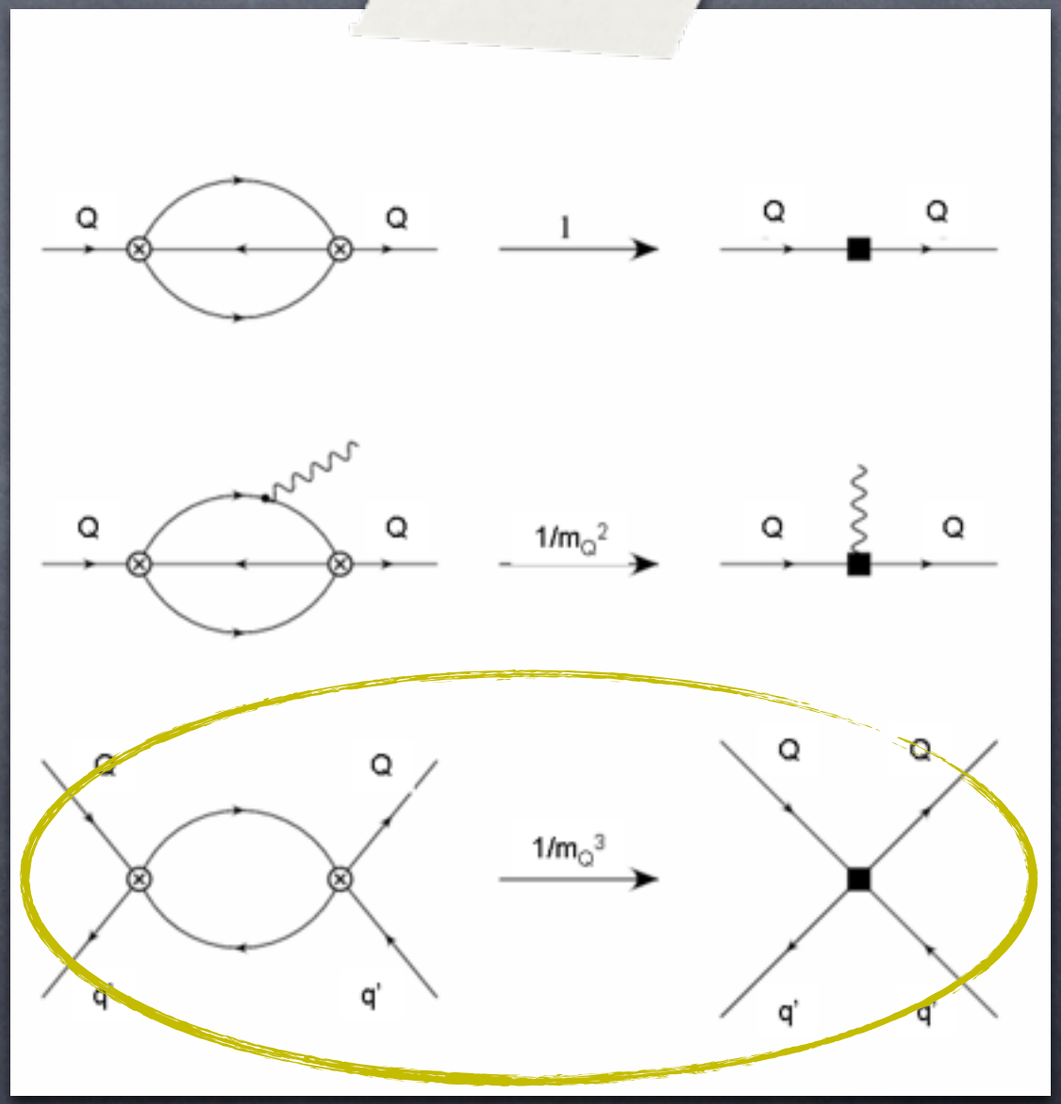
$$\langle H_{Q\bar{q}}|O^{q'}_{V-A}|H_{Q\bar{q}}\rangle \equiv \langle H_{Q\bar{q}}|\bar{Q}\gamma_\mu(1-\gamma_5)q'\,\bar{q}'\gamma^\mu(1-\gamma_5)Q|H_{Q\bar{q}}\rangle$$

$$\langle H_{Q\bar{q}}|O^{q'}_{S-P}|H_{Q\bar{q}}\rangle \equiv \langle H_{Q\bar{q}}|\bar{Q}(1-\gamma_5)q'\,\bar{q}'(1-\gamma_5)Q|H_{Q\bar{q}}\rangle$$

- Conventionally one parametrizes deviations from VSA: bag params.

$$\langle D|O_{V-A}|D\rangle = f_D^2 m_D^2 B_1\,,$$

$$\langle D|O_{S-P}|D\rangle = f_D^2 m_D^2 B_2$$



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- HQE parameters - spectator cont.

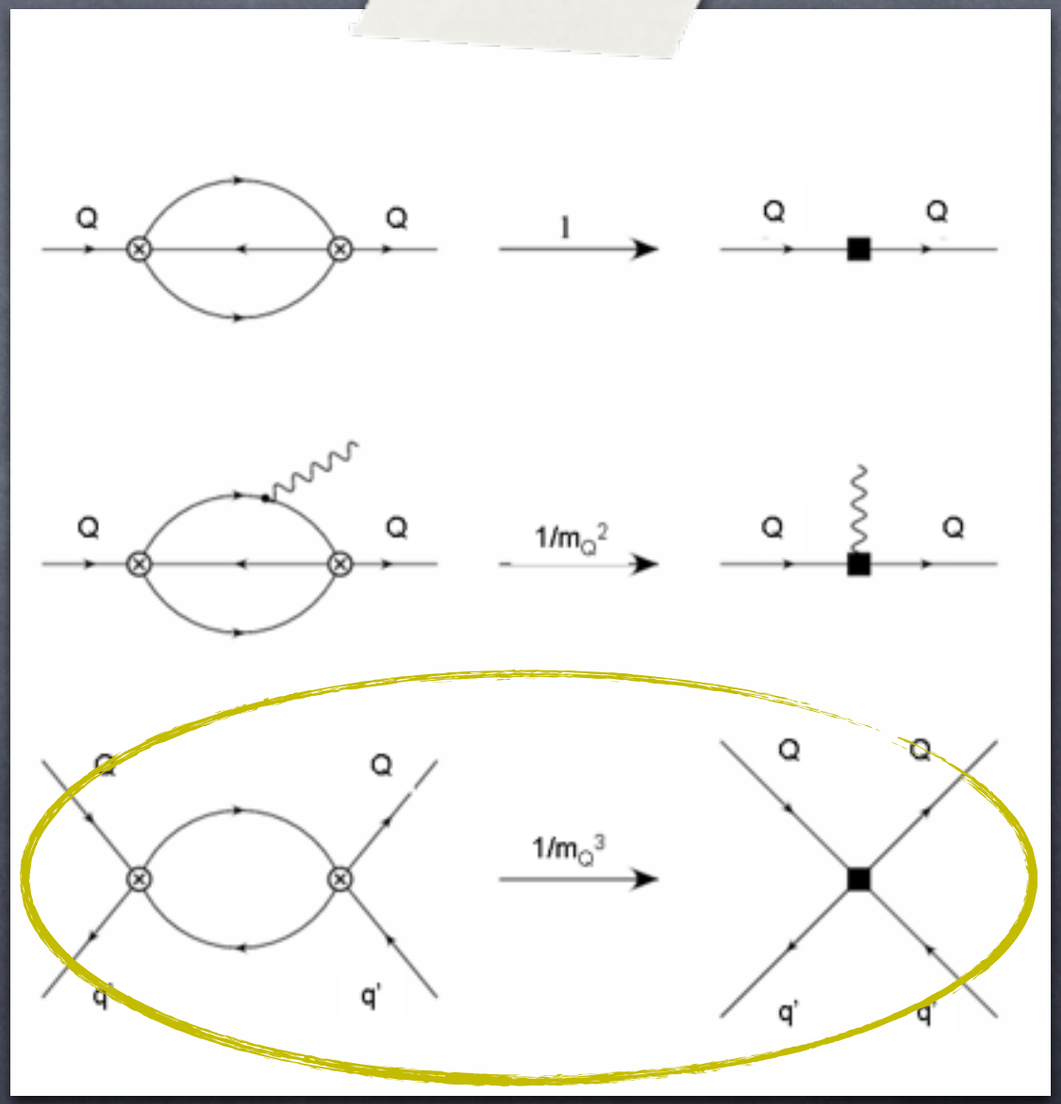
Dim=6 operators involving light flavors

$$\langle D|O_{V-A}|D\rangle = f_D^2 m_D^2 B_1\,,$$

$$\langle D|O_{S-P}|D\rangle = f_D^2 m_D^2 B_2$$

- Can mix with non-enhanced (Darwin) contributions Gambino et al. [hep-ph/0505091]
- Cannot be fully estimated on the lattice (Eye-contractions)

M. B. Voloshin, [hep-ph/0106040]



Phenomenological analysis

- Lattice, Charmonium SR, b → c spectral fits:
  - $m_c(m_c)$=1.27(2)GeV Taken from PDG'08

Allison et al. [HPQCD], [0805.2999]
Kuhn, Steinhauser and Sturm, [hep-ph/0702103].
Buchmuller & Flacher [hep-ph/0507253]

- Up to known non-enhanced $1/m_c^3$ terms saturate the experimental values to ≈70%
  - Sizable residual scale and scheme dependencies
  - Very slow perturbative & power convergence
- Assuming enhanced $1/m_c^3$ terms saturate the rate, one can obtain a guestimate of (f_D=208(4)MeV)
  - $(B_1-B_2)(2GeV) \approx 0.05$

Follana, Davies, Lepage and Shigemitsu [HPQCD], arXiv:0706.1726.

D. Becirevic, S. Fajfer, J. F. K. arXiv:0804.1750

Improving the phenomenological analysis

- Better control over OPE convergence from spectral moments analysis (as done in b->c case)?
  - Cleo published also lepton momentum spectra with a lower cut (requires more involved treatment)

hep-ex/0604044

  - More direct access to power corrections, duality violations (WA contributions dominate at the end-point)

c.f. Shifman [hep-ph/9505289] Benson et al. [hep-ph/0302262]

- Lattice QCD estimates of $B_{1,2}$
- Inclusive D_s decays would give access to valence spactator quark contributions, related to $B^+ \to X_u l \nu$, $B^0 \to X_u l \nu$ width difference via SU(3) symmetry

M. B. Voloshin, [hep-ph/0106040]

Conclusions

- Exclusive D -> P l ν decays
  - FF normalization: Lattice, SR
    - Extraction of CKM elements not yet competitive with leptonic decays
  - FF shape: test of extrapolation procedures for B decays
  - More exp. information available:
    - Testing for Scalar, RH, Tensor currents
    - Introduces (almost) no new hadronic uncertainties!
    - D -> V l ν potential
- Inclusive D -> X l ν decays
  - Test & extraction of power suppressed contributions from experiment
    - Comparison with lattice estimates
  - Convergence & validity of OPE
    - More experimental observables available than the total rate
    - Dedicated exp. & theor. analysis needed

Backup Slides

D -> X l ν Phenomenological analysis: Details

- Bad convergence of $\alpha_s(m_c) \approx 0.4$
  - Integrate out the charm at $\mu_{UV} \approx 2$GeV - Induce log corrections
- Large α_s^2 corrections in kinetic scheme for m_c at $\mu_{IR} \approx 1$GeV, large scale dependence
  - Lower $\mu_{IR} \approx 0.5$GeV - Validity of perturbative expansion?
- Kinetic scheme not applicable for strange quark mass, cannot be put to zero
  - use MSbar mass at $\mu_{UV} \approx 2$GeV - hybrid scheme
- Total rate: $\Gamma = \Gamma_0 [\, 1 - 0.4\, \alpha_s - 1\, \alpha_s^2 - 0.4_{(2)} + 0.3_{(3)} \,]$ translated OPE parameters from Gambino & Giordano [0805.0271]

  ↑ Sizable WA factorization scale dependence