Isospin breaking effects in the $X(3872)$

D. Gamermann*[1] and E. Oset[1]

[1]Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia-CSIC,, Institutos de Investigación de Paterna, Aptdo. 22085, 46071, Valencia, Spain, ,

Introduction

Since its discovery [1] the $X(3872)$ has attracted the attention of the scientific community, since it is a clear example of meson not fitting into the $q\bar{q}$ picture. This state has already been confirmed by different experiments [2, 3, 4]. It has been discovered in the decay channel $J/\psi\pi^+\pi^-$ and the $\pi^+\pi^-$ mass distribution indicates that this meson pair is coming from a ρ meson [5], which indicates isospin 1 assignment for the resonance. On the other hand, no charge partner of this meson has been found, indicating an isospin 0 state. Later on this state has been observed [6] in the decay channels $J/\psi\gamma$, which determines its C-parity as positive, and in the decay channel $J/\psi\pi^0\pi^+\pi^-$, where the three pion mass distribution indicates it comes from a ω meson, supporting an isospin 0 assignment for the state. The available data on this state has been thoroughly investigated in [7] determining that the most probable J^{PC} quantum numbers for this state are $J^P = 1^{++}$ or $J^P = 2^{-+}$.

The branching fraction ratio for the two hadronic decay channels has been measured:

$$\frac{\mathcal{B}(X \to J/\psi\pi^+\pi^-\pi^0)}{\mathcal{B}(X \to J/\psi\pi^+\pi^-)} = 1.0 \pm 0.4 \pm 0.3\,. \quad (1)$$

The fact that it is so close to 1 seems to indicate a large isospin violation in the hadronic decays of the $X(3872)$, which is not usual.

Since the mass of the $X(3872)$ is very close to the $D^0\bar{D}^{*0}$ threshold, it has been suggested by many theoretical models that the $X(3872)$ is a s-wave $D^0\bar{D}^{*0}$ molecule [8, 9, 10, 11, 12].

In this work we analyse the $X(3872)$ from the point of view that it is a dynamically generated state in coupled channels. We analyse more deeply results from a model that already described many other charmed axial states as dynamically generated ones [8]. In the model used previously we kept isospin symmetry exact. In this current study we break isospin symmetry and analyse its effects. Together with a positive C-parity state, associated with the $X(3872)$, our model also generates a negative C-parity state, for which there is so far no experimental evidence. We comment on the most probable decay channels to observe this predicted state.

Model

The model we briefly explain here is detailed elsewhere [8].

We study here the interaction of a pseudoscalar with a vector-meson that, in s-wave, has the quantum numbers of an axial: 1^+. The starting point of our model consists of the fields belonging to the 15-plet and a singlet of $SU(4)$ describing the pseudoscalar and vector-mesons. The field for the vector mesons is:

$$\mathcal{V}_\mu = \begin{pmatrix} \frac{\rho^0_\mu}{\sqrt{2}} + \frac{\omega_\mu}{\sqrt{2}} & \rho^+_\mu & K^{*+}_\mu & \bar{D}^{*0}_\mu \\ \rho^{*-}_\mu & \frac{-\rho^0_\mu}{\sqrt{2}} + \frac{\omega_\mu}{\sqrt{2}} & K^{*0}_\mu & D^{*-}_\mu \\ K^{*-}_\mu & \bar{K}^{*0}_\mu & \phi_\mu & D^{*-}_{s\mu} \\ D^{*0}_\mu & D^{*+}_\mu & D^{*+}_{s\mu} & J/\psi_\mu \end{pmatrix} \quad (2)$$

The field for the pseudoscalars we call Φ and can be found in [13]. Note that this field differ from those used in [8] because of the inclusion of ω-ϕ mixing.

For each one of these fields a current is defined:

$$J_\mu = (\partial_\mu\Phi)\Phi - \Phi\partial_\mu\Phi \quad (3)$$
$$\mathcal{J}_\mu = (\partial_\mu\mathcal{V}_\nu)\mathcal{V}^\nu - \mathcal{V}_\nu\partial_\mu\mathcal{V}^\nu\,. \quad (4)$$

The Lagrangian is constructed by coupling these currents:

$$\mathcal{L}_{PPVV} = -\frac{1}{4f^2}Tr\left(J_\mu\mathcal{J}^\mu\right). \quad (5)$$

In the way it is constructed this Lagrangian is $SU(4)$ symmetric, but we know that $SU(4)$ symmetry is badly broken in nature. The way we take this into account is by assuming vector-meson dominance and recognising that the interaction behind our Lagrangian is the exchange of a vector meson in between the two hadronic currents. If the initial and final pseudoscalars (and vector-mesons), in a given process, have different charm quantum number, it means that the vector-meson exchanged in such a process is a charmed meson, and hence a heavy one. In these cases we suppress the term in the Lagrangian containing such processes by a factor $\gamma = m_L^2/m_H^2$ where m_L is the typical value of a light vector-meson mass (800 MeV) and m_H the typical value of the heavy vector-meson mass (2050 MeV). We also suppress, in the interaction of D-mesons the amount of the interaction which is driven by a J/ψ exchange by the factor $\psi = m_L^2/m_{J/\psi}^2$. Another source of symmetry breaking will be the meson decay constant f appearing in the Lagrangian. For light mesons we use

* E-mail: daniel.gamermann@ific.uv.es

$f = f_\pi = 93$ MeV but for heavy ones $f = f_D = 165$ MeV.

So, for a given process $(P(p)V(k))_i \rightarrow (P'(p')V'(k'))_j$ we have the amplitude:

$$\mathcal{M}_{ij}(s,t,u) = -\frac{\xi_{ij}}{4f_i f_j}(s-u)\epsilon.\epsilon' \qquad (6)$$

where s and u are the usual Mandelstam variables, f_i is the pseudoscalar i meson decay constant, ϵ are the vector-meson polarization vectors and i, j refer to the initial and final channels in the coupled channel space. The coefficient matrices ξ_{ij} can be directed calculated from the Lagrangian of eq. (5) in charge basis. The ξ_{ij} coefficients can be found in [13].

The amplitude in eq. (6) is projected in s-wave and plugged into the scattering equation for the coupled channels:

$$T = V + VGT. \qquad (7)$$

In this equation G is a diagonal matrix with each one of its elements given by the loop function for each channel in the coupled channel space. For channel i with mesons of masses m_1 and m_2 G_{ii} is given by:

$$\begin{aligned} G_{ii} &= \frac{1}{16\pi^2}\Bigg(\alpha_i + Log\frac{m_1^2}{\mu^2} + \frac{m_2^2 - m_1^2 + s}{2s}Log\frac{m_2^2}{m_1^2} \\ &+ \frac{p}{\sqrt{s}}\Big(Log\frac{s - m_2^2 + m_1^2 + 2p\sqrt{s}}{-s + m_2^2 - m_1^2 + 2p\sqrt{s}} \\ &+ Log\frac{s + m_2^2 - m_1^2 + 2p\sqrt{s}}{-s - m_2^2 + m_1^2 + 2p\sqrt{s}}\Big)\Bigg) \qquad (8) \end{aligned}$$

where p is the three momentum of the two mesons in the center of mass frame. The two parameters μ and α are not independent, we fix μ=1500 MeV and change α to fit our results within reasonable values in the natural range.

The imaginary part of the loop function ensures that the T-matrix is unitary, and since this imaginary part is known, it is possible to do an analytic continuation for going from the first Riemann sheet to the second one. Possible physical states (resonances) are identified as poles in the T-matrix calculated in the second Riemann sheet for the channels which have the threshold below the resonance mass.

Couplings and decays of the $X(3872)$

By changing slightly the value of α we can move the pole position of our dynamically generated state. At first we kept isospin symmetry (charged and neutral members of the same isospin multiplet with the same mass). In this case, setting α_H=-1.34, which is equivalent to a cut-off of 830 MeV in the three momentum, we get two poles with opposite C-parity, the positive one at 3866 MeV with a width smaller than 1 MeV and the negative one at (3875-25i) MeV, which means a width around 50 MeV. The poles appear in isospin I=0. Now while increasing the value of α_H (lowering the cut-off) the poles approach the threshold (at 3876 MeV in the isospin symmetric case). The negative C-parity pole touches the threshold for α_H values bigger that -1.33 (cut-off of 820 MeV), while the positive C-parity one reaches the threshold for α_H around -1.185 (cut-off equivalent to 660 MeV). Once the pole crosses the threshold it does not appear in the second Riemann sheet, it is no longer a resonance, but becomes a virtual state. Yet a peak can be seen in the cross section of some channels, but can not be identified as a pole in the second Riemann sheet of the T-matrix.

In the isospin symmetric case there is one $D\bar{D}^*$ threshold. By using physical masses for the charged and neutral D-mesons there are two thresholds, the neutral one at 3872 MeV and the charged one at 3880 MeV. The $X(3872)$ state is a very weakly $D^0\bar{D}^{*0}$ bound state and the fact that the binding energy is much smaller than the difference between these two thresholds could reflect itself in a large isospin violation in observables.

Let us consider a simplified model with only two channels, with neutral and charged D and D^* mesons. In this model we assume the potential V to be a 2x2 matrix:

$$V = \begin{pmatrix} v & v \\ v & v \end{pmatrix}, \qquad (9)$$

with v constant, which indeed is very close to the real one in a small range of energies.

In this case the solution of the scattering equation (7) is:

$$T = \frac{V}{1 - vG_{11} - vG_{22}} \qquad (10)$$

where G_{11} and G_{22} are the loop function calculated for channels 1 and 2 respectively. If there is a pole at s=s_R we can expand T close to this pole as:

$$T_{ij} = \frac{g_i g_j}{s - s_R} \qquad (11)$$

where g_i is the coupling of the pole to the channel i. The product $g_i g_j$ is the residue at the pole and can be calculated with:

$$\begin{aligned} \lim_{s\rightarrow s_R}(s - s_R)T_{ij} &= \lim_{s\rightarrow s_R}(s - s_R) \\ &\times \frac{V_{ij}}{1 - vG_{11} - vG_{22}} \qquad (12) \end{aligned}$$

We can apply the l'Hôpital rule to this expression and we get:

$$\lim_{s\rightarrow s_R}(s - s_R)T_{ij} = \frac{V_{ij}}{-v(\frac{dG_{11}}{ds} + \frac{dG_{22}}{ds})} \qquad (13)$$

For a resonance lying right at the lowest threshold the couplings g_i will be zero, since the derivative of the loop function, in the denominator of eq. (13) is infinity at threshold. This is a general property which has its roots in basic Quantum Mechanics as shown in [14].

We also note that eq. (13) for just one channel is the method used to get couplings of bound states to their building blocks in studies of dynamically generated states following the method of the compositness condition of Weinberg [15, 16].

Let's consider the diagram in figure 1 for the decay of the X. In this figure the D mesons can be either charged or neutral. A final state with isospin I=1 involves the ρ meson, and in this case the diagrams with neutral D mesons interfere destructively with those with charged D mesons. In the situation with the ω in the final state the diagrams sum up. If the vertices have the same strength for ρ and ω production (this is the case in the framework of the hidden gauge formalism) the ratio of the amplitudes will be given by the ratio of the difference between the charged and neutral loops divided by the sum of the loops:

$$R_{\rho/\omega} = \left(\frac{G_{11} - G_{22}}{G_{11} + G_{22}}\right)^2 \tag{14}$$

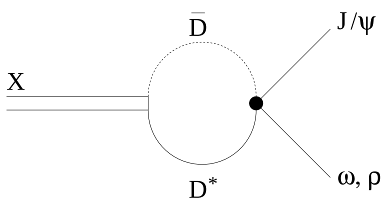

Figure 1: X decay

Actually the decays $X \to J/\psi\rho$ and $X \to J/\psi\omega$ are not allowed because of phase-space, but can occur when their mass distribution is considered and will be seen in the decays $X \to J/\psi\pi\pi$ and $X \to J/\psi\pi\pi\pi$ respectively, where the two and three pion states are the result of the decays of the ρ and ω. In order to take this phase-space into account we calculate the expression:

$$\frac{\mathcal{B}(X \to J/\psi\pi\pi)}{\mathcal{B}(X \to J/\psi\pi\pi\pi)} = \left(\frac{G_{11} - G_{22}}{G_{11} + G_{22}}\right)^2 \frac{\int_0^\infty q\mathcal{S}_\rho}{\int_0^\infty q\mathcal{S}_\omega} \times \frac{\theta\left(m_X - m_{J/\psi} - \sqrt{s}\right)\, ds}{\theta\left(m_X - m_{J/\psi} - \sqrt{s}\right)\, ds} \frac{\mathcal{B}_\rho}{\mathcal{B}_\omega} \tag{15}$$

where $\mathcal{B}_\rho$ and $\mathcal{B}_\rho$ are the branching fractions of ρ decaying into two pions ($\sim$ 100 %) and ω decaying into three pions ($\sim$ 89 %), $\theta(y)$ is the Heaviside theta function and $\mathcal{S}_V = \mathcal{S}\left(s, m_V, \Gamma_V\right)$ is the spectral function of the mesons given by:

$$\mathcal{S}\left(s, m, \Gamma\right) = -\frac{1}{\pi} Im \left(\frac{1}{s - m^2 + i\Gamma m}\right) \tag{16}$$

The isospin violation is therefore proportional to the difference between the loops for charged and neutral D-mesons which is maximum at the threshold of the $D^0\bar{D}^{*0}$. So the closer the resonance is to this threshold (the smaller the binding energy) the bigger is the isospin violation in the decay of the X. If the X is right over the threshold, the value of $R_{\rho/\omega}$, with the loops calculated with dimensional regularization for ρ and ω fixed masses, is:

$$R_{\rho/\omega} = 0.032 \tag{17}$$

This is a measure of the isospin violation in the decay of the X, which is only about 3% in spite of the fact that we have chosen the conditions to maximize it. However, even this small isospin breaking can lead to sizable values of the ratio of eq. (15) when one takes into account the mass distributions of the ρ and ω, which provide different effective phase-spaces in this two possible X decays. Thus, using eq. (15), which considers explicitly the ρ and ω mass distributions, we find the branching ratio:

$$\frac{\mathcal{B}(X \to J/\psi\pi^+\pi^-\pi^0)}{\mathcal{B}(X \to J/\psi\pi^+\pi^-)} = 1.4 \tag{18}$$

which is compatible with the value 1.0 ± 0.4 from experiment.

We show in figure 2 that the coupling of the X to $D^0\bar{D}^{*0}$goes to zero for small binding energies, and in figure 3 we show that even though the difference between the neutral and charged couplings grows for small binding energies, they are of the same order of magnitude. The wave function of the $X(3872)$ is, thus, very close to the isospin I=0 combination of $D^0\bar{D}^{*0} - c.c.$ and $D^-\bar{D}^{*+} - c.c.$ and has a sizable fraction of the $D_s^- D_s^{*+} - c.c.$ state.

From figure 3 we notice that the isospin violation in the couplings of the X to the $D\bar{D}^*$ channels is bigger for small binding energies, but it reaches a maximum of about 1.4% which is a very small value.

As we already mentioned, we find a second state with negative C-parity. Some of the channels with negative C-parity have isospin I=0 to which the resonance can decay. There are also pure isospin I=1 channels but, although the generated resonance is an isospin I=0 state, these isospin I=1 channels will couple to it since we are considering here some amount of isospin violation coming from the different masses of charged and neutral members of a same isospin multiplet.

For values of α_H similar to those used in the generation of the $X(3872)$ (α_H=-1.27), the pole with negative C-parity is in the wrong Riemann sheet, but its effects can still be seen in the cross sections of some channels. We show in figure 4 the $|T|^2$ plots of some channels.

The channels shown in figure 4 are those where there is phase-space available for the resonance to decay and to which it couples most strongly.

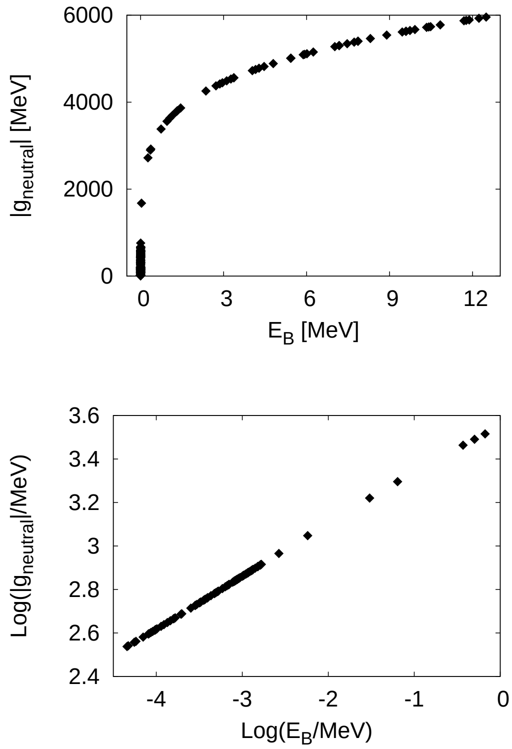

Figure 2: Coupling of the X to the $D^0\bar{D}^{*0}$ channel for different biding energies.

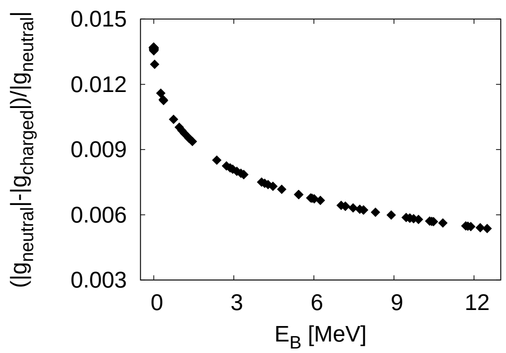

Figure 3: Difference of the coupling of the X with neutral and charged $D\bar{D}^*$ channels.

Conclusion

There is strong evidence that the $X(3872)$ state has J^{PC} quantum numbers equal to 1^{++}. This fact and the mass value of the X make it tempting to associate this state with a s-wave $D^0\bar{D}^{0*}$ molecular state.

We have investigated this possibility in order to describe the large isospin violation in the decays of the $X(3872)$. For that we make use of a phenomenological model that generates dynamically resonances in coupled channels.

Experimentally the decays of the $X(3872)$ into J/ψ with two and three pions have been measured to be of the same order of magnitude, suggesting a huge isospin violation. The couplings of the X to charged and neutral D mesons are very similar, with at most 1.4% of isospin violation, in our model. Considering the decay of X to $J/\psi\rho$ or $J/\psi\omega$ pairs as going through $D\bar{D}^*$ loops the $J/\psi\rho$ production should be suppressed in relation to the $J/\psi\omega$ by a factor around 30. This factor 30 is compensated in the decays of the X with two and three pions in the final state once one considers also the mass distribution for the ρ meson to decay into two pions and the ω meson to decay into three pions. Since the ρ meson has a much bigger width than the ω, although the decay of the X to the ρ meson is suppressed, one still observes a sizable branching fraction for the X to decay into $J/\psi\pi\pi$.

We predict also a negative C-parity state in a framework which describes many low lying axial states and also most of the already observed axial charmed resonances.Our model shows that the channels to which this resonance couples mostly are $\eta\phi$, $\eta\omega$, $\eta\prime\omega$ and $\eta_c\omega$. We also made predictions for observables where the negative C-parity state should in principle be seen and we hope these results stimulate experimental efforts in this direction.

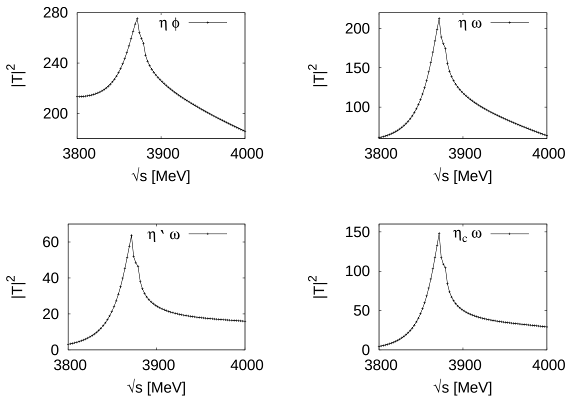

Figure 4: The $|T|^2$ plot for some of the negative C-parity channels.

References

[1] S. K. Choi *et al.* [Belle Collaboration], Phys. Rev. Lett. **91**, 262001 (2003) [arXiv:hep-ex/0309032].

[2] D. E. Acosta *et al.* [CDF II Collaboration], Phys. Rev. Lett. **93**, 072001 (2004) [arXiv:hep-ex/0312021].

[3] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. Lett. **93**, 162002 (2004) [arXiv:hep-ex/0405004].

[4] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **71**, 071103 (2005) [arXiv:hep-ex/0406022].

[5] A. Abulencia *et al.* [CDF Collaboration], Phys. Rev. Lett. **96**, 102002 (2006) [arXiv:hep-ex/0512074].

[6] K. Abe *et al.*, arXiv:hep-ex/0505037.

[7] A. Abulencia *et al.* [CDF Collaboration], Phys. Rev. Lett. **98**, 132002 (2007) [arXiv:hep-ex/0612053].

[8] D. Gamermann and E. Oset, Eur. Phys. J. A **33**, 119 (2007) [arXiv:0704.2314 [hep-ph]].

[9] Y. R. Liu, X. Liu, W. Z. Deng and S. L. Zhu, Eur. Phys. J. C **56**, 63 (2008) [arXiv:0801.3540 [hep-ph]].

[10] Y. b. Dong, A. Faessler, T. Gutsche and V. E. Lyubovitskij, Phys. Rev. D **77**, 094013 (2008) [arXiv:0802.3610 [hep-ph]].

[11] E. S. Swanson, Phys. Lett. B **588**, 189 (2004) [arXiv:hep-ph/0311229].

[12] M. B. Voloshin, Phys. Lett. B **604**, 69 (2004) [arXiv:hep-ph/0408321].

[13] D. Gamermann and E. Oset, Phys. Rev. D **80**, 014003 (2009) [arXiv:0905.0402 [hep-ph]].

[14] H. Toki, C. Garcia-Recio and J. Nieves, Phys. Rev. D **77**, 034001 (2008) [arXiv:0711.3536 [hep-ph]].

[15] V. Baru, J. Haidenbauer, C. Hanhart, Yu. Kalashnikova and A. E. Kudryavtsev, Phys. Lett. B **586** (2004) 53 [arXiv:hep-ph/0308129].

[16] S. Weinberg, Phys. Rev. **130**, 776 (1963).