Quarkonium production: a theoretical status

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Introduction

Heavy-quarkonium bound states provide a very rich ground to to probe QCD, the $SU(3)$ gauge theory of the strong interactions. Even if the confinement effects are not tractable from first principles, the mass of the heavy quarks introduces a high energy scale at which the strong interactions can be treated perturbatively, by virtue of the asymptotic freedom behavior of QCD. Many properties of heavy quarkonium spectra, including the striking narrow width of the J/ψ, can be explained within this framework.

The use of perturbative QCD in the prediction of heavy-quarkonium production or decay rates is based on a factorization principle, that isolates the low-energy non-perturbative effects into a set of universal parameters in the expression of the rates. One of the first attempts at such a factorization procedure is the Colour-Singlet Model (CSM) [1, 2, 3], in which the perturbative heavy quarks are assumed to be created on-shell, with the same quantum numbers as the physical quarkonium state. The total rate is expressed as the product of a process-dependent factor associated with the creation or the annihilation of the perturbative heavy quarks, and a universal factor that takes into account the transition between the heavy quarks and the physical quarkonium. Usually, this last factor is approximated by the value of a Schrödinger wave function at the origin, or its derivatives.

The CSM gives a simple prescription for the computation of the rates and is very predictive, but it has a certain number of drawbacks. For S-wave quarkonium decays, calculation beyond the static approximation revealed a very large relativistic correction at order v^2 [4], with v the velocity of the heavy quark in the quarkonium rest frame. As the CSM does not provide a correct prescription to compute the relativistic corrections in general, higher correction terms are a priori not under control. The situation is even worst for the P-waves, where the presence of infrared divergences appearing at higher orders in α_s explicitly breaks the validity of the factorization procedure adopted by the CSM.

In the search for a more rigorous factorization procedure, the theory of Non-Relativistic QCD was introduced in 1994 [5]. In this theory, perturbative amplitudes are systematically expanded in powers of v. In this approach, transitions in which the perturbative heavy-quark pair is created in a different quantum state than the physical quarkonium are also included, as such transitions arise naturally in the v expansion. For each transition $Q\bar{Q}(n) \rightarrow \mathcal{Q}$, a non-perturbative parameter enters in the expression of the decay/production rate. In NRQCD, these parameters, also called long-distance matrix elements (shortened as LDME), have a precise definition in terms of vacuum expectation of quantum field operators. They encode the low energy effects associated to the evolution of the heavy-quark pair into a quarkonium state. The infrared divergences that arise in the computation of the perturbative factors are consistently reabsorbed into these long-distance matrix elements through a matching procedure. As a result, one can properly handle the computation of quarkonium rates beyond the leading order in α_s. In particular, the QCD corrections to P-wave decay/production rates are then tractable [6, 7].

Despite its theoretical appeal and the undeniable successes, not all the predictions of the NRQCD factorization approach have been firmly established, and the experimental evidence of the universality of the LDME's still requires further phenomenological investigation. Among the questions still open stands the relevance of the colour-octet contributions in the J/ψ production: they seem to play a dominant role at the Tevatron and in $\gamma\gamma$ collisions, but they look marginal at e^+e^- at low energy, in photoproduction at HERA [8] and in fixed-target experiments [9]. Moreover, NRQCD predicts a sizeable transverse polarisation for J/ψ 's at high-p_T at the Tevatron, in contrast to the latest data, which now clearly indicate that J/ψ's are not transversely polarized [10].

During the past few years, several efforts have been provided to try to solve these puzzles in quarkonium production. In this review, I first mention the recent advances in establishing the proof of factorization in quarkonium production. Next I compare the NRQCD predcitions to the recent data collected at B factories. I move on to J/ψ photoproduction at HERA. I then discuss the production of quarkonium states in hadron collisions. I give my conclusion in the last section.

Proof of the factorization

As we have recalled in the previous section, the formalism of NRQCD allows to factorize the non-perturbative effects inherent to the quarkonium states into a finite number of parameters, expressed as matrix elements of 4-fermion operators. The uncancelled infrared pôles that arise beyond the leading order in α_s must be reabsorbed into these NRQCD long-distance matrix elements. A well known example of application is the production of a P-wave quarkonium state from the fragmentation of a gluon.

Whether such a procedure can be applied at all order in α_s and in v is not obvious. In particular, it is not clear that all the uncanceled pôles in the computation of the short-distance coefficients can always be matched by the NRQCD matrix elements. The investigation of this issue is of capital importance, as it is the fundamental ground for the use of perturbative QCD in the computation of heavy quarkonium cross sections. Several efforts have recently been made to analyze the validity of the NRQCD factorization approach at higher order in α_s. We briefly review some of these new results.

Nayak et al. [11, 12] have investigated the production of quarkonium states at large P_T from gluon fragmentation. At NNLO accuracy in α_s, they found uncanceled pôles that do not appear in the perturbative calculation of the long-distance matrix elements. They showed that, if the colour-octet matrix elements are modified by the introduction of Wilson lines that make them gauge invariant, the factorization is restored at NNLO. This restoration makes use of the fact that the IR pôle does not depend on the direction of the Wilson line. It is therefore unclear whether the factorization holds at higher order in α_S.

In more recent papers [13, 14], Nayak *et al* have analyzed associated heavy quarkonium production, i.e. the production of a quarkonium state plus a heavy-quark pair of the same flavor. They showed that the standard NRQCD factorization procedure does not apply in phase-space regions in which one of the passive quarks is co-moving with the active heavy-quark pair with almost zero relative momentum. In such a configuration, colour exchange between the active and passive heavy quarks leads to an infrared pôle that cannot be matched by any of the standard NRQCD long distance matrix elements.

Recently Bodwin *et al.* [15] have outlined the proof of the factorization theorems for exclusive two-body charmonium production in B-meson decay and in e^+e^- annihilation, both of these processes being particularly relevant from the phenomenological point of view. Considering all orders in the strong coupling constant, they find that factorized expressions hold up to corrections of order m_c/m_b in B-meson decay, and up to corrections of order m_c^2/s in e^+e^- annihilation, where m_c, m_b and $\sqrt{s}$ are the charm-quark mass, the bottom quark mass and the e^+e^- center-of-mass energy, respectively. For the specific process of $e^+e^- \to J/\psi + \chi_{c0}$, the NLO cross section has been computed explicitly in [29]. The factorization has been found to hold exactly at this order, provided that the relative velocity inside the S-wave quarkonium state is set to zero.

Before ending this section, we mention that beside the predictions of quarkonium rates in the collinear factorization scheme, various works have focused on the production of quarkonium states within the k_t factorization scheme, both in electron-proton and in hadron-hadron collisions. In the later case, though, Collins and Qiu [16] showed that in general the k_T-factorisation theorem does not hold in production of high-P_T particles in hadron-collision processes.

Charmonium production at B factories

Quarkonium production in electron-positron annihilation provides a rich ground to test the factorization principles of NRQCD. Both exclusive and inclusive measurements have been reported at B-factories [17, 18, 19], and have been compared intensively with theoretical predictions for several years.

Exclusive double charmonium production

One of the most striking result reported by the Belle and BABAR collaboration is the cross section for exclusive $J/\psi + \eta_c$ production [18, 19]. The measurement appeared to be one order of magnitude larger than the leading order NRQCD prediction [20, 21], a situation which has been referred to as the largest discrepancy in the standard model [22]. The full NLO correction, computed in [23, 24], was shown to be large, pushing the predicted cross section closer to the measurement. As the v^2 correction [20] proved to be large, it was then suggested that a non-relativistic expansion was not appropriate to describe this process, and an alternative approach in term of a light cone wave function was proposed to reproduce the measured cross section [25]. A following work then showed that part of the enhancement in the light-cone approach actually corresponds to correction of relative-order α_s in the NRQCD formalism [26]. Moreover, a resummation of a class of relativistic corrections to S-wave states production [27] demonstrates that the NRQCD expansion is actually well behaved. Applying this resummation and taking into account the fragmentation contribution to the pure QED amplitude, the inclusion of the effects of the running of α and the inclusion of the contribution that arises from the interference between the relativistic corrections and the corrections of next-to-leading order in α_s, a refined analysis of the theoretical uncertainties showed that the discrepancy was resolved [28]. More recently, the factorization theorem on which this NRQCD result is based was proved to hold at all orders in α_s up to corrections of order m_c^2/s [15]. The Belle collaboration also reported a measurement of double production of S-wave + P-wave charmonium states. The corresponding NRQCD prediction at NLO accuracy [29] in α_s was shown to reach the lower bound of experiment.

Inclusive J/ψ production

Another chalenge was addressed to the quarkonium production community with the measurement of the inclusive J/ψ cross section [17, 30, 34]. In this case as well, the leading-order NRQCD prediction [31, 32, 33] turns out to underestimate the experimental yield. Moreover, the Belle collaboration reported that of most the reconstructed J/ψ are produced in association with charm mesons: the most recent measurement [34] for the ratio

$$R_{cc} = \frac{\sigma[e^+e^- \to J/\psi + c\bar{c} + X]}{\sigma[e^+e^- \to J/\psi + X]} = 0.63 \pm 0.23 \quad (1)$$

is in disagreement with the NRQCD leading-order prediction $R_{cc} = 0.1 - 0.3$. The computation of the QCD correction for both processes $e^+e^- \to J/\psi + c\bar{c} + X$ and $e^+e^- \to J/\psi + gg + X$ have been completed [35, 36] recently, and found to bring the ratio R_{cc} in better agreement with the experimental result. In the meantime, standard NRQCD factorization has been shown to break down for the associated production at NNLO in α_s, due to colour transfer between co-moving active and passive charm quarks [13, 14]. This effect is expected to catalyze the production rate when one of the charm quark is produced in the same direction as the J/ψ. It could in principle be quantified experimentally by measuring the angular separation bewteen the J/ψ and the charm meson.

J/ψ production at Hera

Charmonium production in ep collisions at HERA is dominated by photon-gluon fusion: a photon emitted from the incoming electron or positron interacts with a gluon from the proton to produce a $c\bar{c}$ pair that evolves into a charmonium state. The bulk of the production is initiated by a photon with a low virtuality Q^2, in which case we talk about photoproduction. The quasi-real photon can then interacts directly with the charm quark (direct processes), or it can interact via its hadronic component (resolved processes). The latter mechanism contributes mainly to the production of low-energy charmonium states, and is therefore relevant at low z —the fraction of charmonium energy taken from the photon in the proton rest frame—.

The ZEUS and H1 collaborations have published several measurements for the photoproduction of J/ψ and $\psi(2S)$ [37, 38]. Samples of J/ψ events resulting from their selection cuts are dominated by inelastic production in which the J/ψ's do not originate from the decay of a heavier resonance. However these measurements are still subject to sub-dominant diffractive background, as well as feed-down contributions from the production of $\psi(2S)$ and χ_c mesons and from the production of b flavoured hadrons. These feed-down contributions —of which relative importance depends on the kinematic region— are usually neglected in the theoretical prediction.

The measurements of J/ψ cross sections and polarisation parameters reported by the ZEUS and H1 collaborations have been intensively compared to NRQCD prediction at leading order in α_s. In these predictions the standard truncation in v is used, in which the independent matrix elements are $\langle\mathcal{O}_1^{J/\psi}\left({}^3S_1\right)\rangle$, $\langle\mathcal{O}_8^{J/\psi}\left({}^3S_8\right)\rangle$, $\langle\mathcal{O}_8^{J/\psi}\left({}^1S_0\right)\rangle$ and $\langle\mathcal{O}_8^{J/\psi}\left({}^3P_0\right)\rangle$. Prediction in the Colour-Singlet Model [3] amounts to set all these matrix elements to zero except the first one. Usually the values of the colour-octet matrix elements are extracted from the Tevatron data, in which case the resulting predictions for J/ψ production at HERA and their comparison to the data offers the opportunity to assess the universality of the long distance matrix elements.

Differential rates

The comparison between the predictions of the production rates at leading order and the data is summarized in the QWG review [22]. On the theoretical side, large uncertainties arise from the sensitivity to the input parameters (the mass of the charm quark, the factorization and renormalization scales as well as the values of the colour-octet matrix elements fit to the Tevatron data). One also expect large corrections due to the omission of higher-order-in-α_s terms. Because of these theoretical uncertainties, the rôle of each transition in charmonium photoproduction is still unclear. Fixed-order tree-level prediction of the colour-octet contribution —based on the central value of the long-distance matrix elements extracted from the Tevatron data— gives rise to a large peak in the z distribution near the end-point region $z \sim 1$. This problem, first interpreted as a failure of the universality of the long-distance matrix elements, was then been attributed to the break down of the NRQCD expansion near the kinematic end-point $z \sim 1$. In this region large perturbative and non-perturbative corrections need to be resumed to all orders in α_s and v. The resummation in the colour-octet channel has been carried out, leading to a significant broadening of the peak at large z [39].

J/ψ photoproduction at HERA has also been studied in the k_t factorization scheme [40, 41]. In this framework, it has been argued that the Colour-Singlet Model alone can explain the data collected at HERA. Prediction for the colour-singlet yield at leading order in α_s in k_t factorization reproduces reasonably well the measured shapes of events, although additional uncertainties arise from the choice of the unintegrated PDF set.

The pioneer calculation of the NLO correction to the direct J/ψ colour-singlet cross-section in the collinear factorization scheme [43] also suggested that the colour-singlet transition is the main mechanism under work in J/ψ photoproduction. In this case, the radiative correction is seen to affect not only the normalisation of the predicted distributions of events but also the shape in transverse momentum (P_T), pushing the colour-singlet yield in better agreement with the data. The large impact of the radiative correction at large transverse momentum is easily understood as a consequence of the strong kinematic suppression that applies to the colour-singlet production mechanism at leading order in α_s.

Although the NLO correction is seen to bring the colour-singlet prediction closer to the data at large P_T, theoretical uncertainties are too large to exclude other components. In the region of low transverse momentum —i.e. the dominant region for J/ψ production at HERA— the sensitivity to the factorization and renormalization scales is dramatically large and hence complicates the identification of the dominant production mechanism from the data collected at HERA. In more recent works [44, 45, 46], it has been emphasized that a natural choice of the factorization and renormalization scales $\mu \sim 2m_c$ leads to predictions for differential cross sections in P_T^2 or in z that underestimate the H1 and ZEUS measurements.

The impact of the radiative corrections in direct J/ψ photoproduction via colour-octet transitions is currently being analysed [46]. The preliminary results indicate that the NRQCD prediction of the cross section at NLO accuracy is in better agreement if one includes the colour-octet transitions with the corresponding long-distance matrix elements extracted from the Tevatron data. In this scheme, α_s correction enhances the total cross section by a factor ~ 2. The NLO prediction suffers from large uncertainties, partially related to the values of the colour-octet long-distance matrix elements fit to the Tevatron data.

Polarisation observables

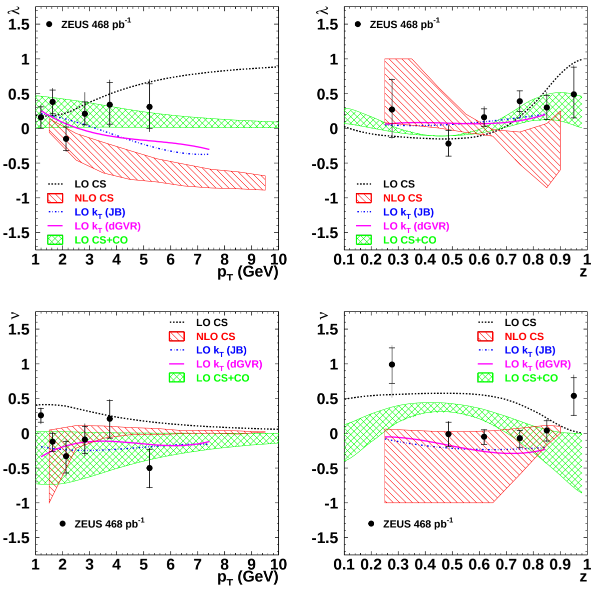

Figure 1: The helicity parameters λ and ν in the target frame as a function of p_T and z. The ZEUS measurement is compared to the colour-singlet prediction at leading order (identified by the label LO CS), the colour-singlet prediction at next-to-leading order (identified by the label NLO CS), the predictions in the k_t factorization approach with two different sets of unintegrated PDF (identified by the labels LO k_t (JB) and LO k_t (dGRV)), and the NRQCD prediction at leading order (identified by the label LO CS+CO). From ref. [42].

Beside these studies on J/ψ production rates in photoproduction, recent analyses have also focused on the polarisation of the J/ψ events in photoproduction as this observable can help in identifying the transitions at work. Experimentally, the polarisation is extracted from the angular distribution of the leptons originating from the J/ψ. In the J/ψ rest frame, the general distribution takes the form

$$\frac{d\Gamma(J/\psi \to l^+l^-)}{d\Omega} \propto 1 + \lambda \cos^2\theta + \mu \sin 2\theta \cos\phi + \frac{\nu}{2}\sin^2\theta \cos 2\phi \qquad (2)$$

where θ and ϕ refer to the polar and azimuthal angles of the l^+ three-momentum with respect to a coordinate system that is defined in the J/ψ rest frame. While the H1 collaboration is currently carrying a new experimental analysis on J/ψ photoproduction, the ZEUS collaboration has already published a new measurement of the parameters λ and ν in the target frame, using an integrated luminosity of 468 pb^{-1} [42]. Figure 1 shows the comparison of this measurement with several theoretical predictions. The curves identified by LO k_T are the predictions in the k_t factorization scheme with two different sets of unintegrated PDF, from Ref. [47]. The band that is identified by CS NLO is the recent prediction for the colour-singlet contribution at next-to-leading order in α_s, computed in Ref. [44, 45]. The comparison with the prediction for the colour-singlet contribution at leading order [48] shows the large impact of the α_s correction on the polarisation parameters. The uncertainty band associated to the CS NLO prediction results from the large sensitivity to the factorization and renormalization scales. In this scheme, the λ parameter is predicted to decrease with P_T. This trend is also observed for the leading-order prediction in the k_t formalism —which effectively accounts for some topologies occurring at higher-order-in-α_s in the collinear factorization scheme— but is substantially different from the measured pattern reported by the ZEUS collaboration. In contrast, the radiative correction puts the colour-singlet prediction for the ν parameter as a function of P_T in better agreement with the ZEUS data. The colour-singlet NLO prediction of the polarisation parameters as a function of z is marked by a very large scale sensitivity, pointing out that the prediction at this fixed order in α_s is not really reliable in the very low P_T region.

Discussion

In spite of these recent advances, it is still unclear which mechanisms are at work in J/ψ photoproduction at HERA. On the one hand, the new computations of QCD corrections to both differential cross sections and polarisation parameters in the collinear factorization scheme point out a sizable contribution of the colour-octet transitions. On the other hand, recent analyses in the k_t factorization scheme show a reasonable agreement of the CSM with the data. In both cases, theoretical uncertainties remain substantial. The ongoing experimental analysis carried by the H1 collaboration might shed lights on the production mechanisms in photoproduction.

Quarkonium hadroproduction

More then ten years ago, the CDF Collaboration of the direct production of J/ψ and ψ' at $\sqrt{s} = 1.8$ TeV [49, 50] brought to light a striking puzzle. They indeed found much larger rates than the leading-order prediction of the QCD-based approach of the Colour-Singlet Model (CSM) [51].

With the advent of the more rigorous and general framework of Non-Relativistic QCD, it has been realised that the CSM as well as other models like the Colour-Evaporation Model (CEM) [52] correspond to specific assumptions of the NRQCD long-distance matrix elements [53], and as

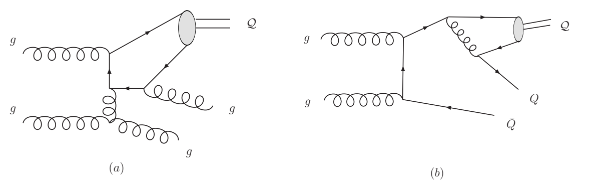

Figure 2: Typical Feynman diagrams for the production of a vector quarkonium state at NLO in α_s :(a) J/ψ or Υ hadroproduction + two gluons, (b) J/ψ or Υ production plus a heavy-quark pair of the same flavor.

such may provide only a crude approximation of the production rates. For recent reviews, the reader is guided to [54, 22, 8].

Despite the theoretical advances, a clear picture of the meachnisms at work for quarkonium hadroproduction that would explain in a consistent way experimental studies of both cross-section and polarisation measurements for charmonia at the Tevatron [10, 55] along with the cross sections measured by PHENIX and STAR at RHIC [56, 57] is still awaited for. For instance, given the P_T differential cross sections measured by the CDF collaboration, it seems that colour-octet transitions are needed to fill the gap between the colour-singlet yield and the data [58, 59, 60, 8] collected at the Tevatron. But once we use the colour-octet LDME's fitted to reproduce the cross section, the NRQCD prediction for the polarisation [61, 62, 63, 64, 65] disagrees with the CDF measurement [10]. The most natural interpretation of such a disagreement is that the charmonium system is too light for relativistic effects to be neglected and that the quark-velocity expansion (v) of NRQCD [5] may not be truncated to the lowest orders for the rather "light" $c\bar{c}$ system. In this context, one expects a better agreement between theory and the available experimental data on production in pp of the significantly heavier Υ. In this case, relativistic corrections are expected to be less important.

Differential rates

During the past few years, the computation of the production rate of a vector quarkoniun state in hadron collisions has been completed for the colour-singlet [66, 67, 68] and the S-wave colour-octet [70]. channels.

In the colour-singlet channel, QCD correction raises significantly the cross section for J/ψ hadroproduction at large transverse momentum, as expected. Indeed, at α_s^4, new channels become available to produce a high-P_T J/ψ at a lower kinematic price. At high transverse momentum, the NLO parton-level processes $gg \to J/\psi + gg$ and $gg \to J/\psi + c\bar{c}$ (see Fig. 2) bring a $\frac{1}{P_T^6}$ and a $\frac{1}{P_T^4}$ components, respectively, whereas the born-level contribution is suppressed ($1/P_T^8$ scaling). This also explains why the associated production $J/\psi + c\bar{c}$ dominates the colour-singlet NLO yield at large P_T. Although NLO contributions reduce the gap between the colour-singlet prediction and the

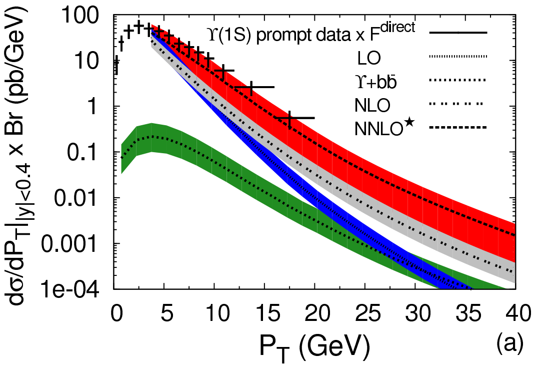

Figure 3: Comparison between differential cross sections at NLO and NNLO* accuracy as function as function of the $\Upsilon(1S)$ transverse momentum P_T at the Tevatron ($\sqrt{s} = 1.96$ TeV) and the data [74]. Taken from [69].

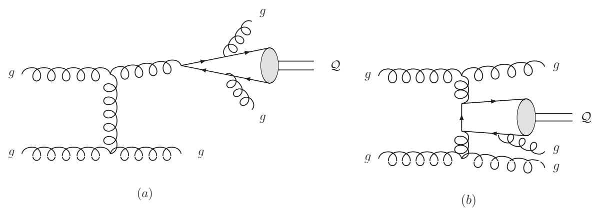

Figure 4: Typical Feynman diagrams for J/ψ or Υ hadroproduction + three jets: (a) colour-singlet gluon fragmentation channel (b) high-energy-enhanced contribution.

CDF data, there is still a discrepancy of more than than an order of magnitude. The situation is similar in the case of Υ production. The impact of the NLO corrections to the CS channels reduces the discrepancy between the CS yield and the data collected by the CDF collaboration, as illustrated in Fig. 3. However the predicted curve is still dropping to fast at large P_T, pointing out that another production mechanism is at work in this phase-space region.

At order α_S^5 new kinematic configurations appear among the CS channels, including the gluon fragmentation (Fig. 4, a) and the double t-channel gluon exchange (Fig. 4, b) topologies. It is interesting to note that the contribution of the last topology has been estimated in the k_t factorization scheme. Working in the collinear factorization scheme, the authors of [69] have estimated these 3-jet channels. This contribution, when added to the NLO yield, may fill the remaining gap between the CS yield at NLO and the data as illustrated by the band labeled NNLO* in Fig. 3, although large theretical uncertainties remains. In the case of J/ψ and $\psi(2S)$, the upper value of the NNLO* evaluation – that includes the real α_s^5 channels– is compatible with CDF results at low P_T as well as with the STAR results[57] at $\sqrt{s} = 200$ GeV up to $P_T = 8$ GeV. At larger P_T, a gap is opening between the CS yield and the data.

QCD corrections have a milder impact in the S-wave colour-octet channels. As computed in [70], they enhance the S-wave spin-singlet channel by 14% in the region $P_T >$

5 GeV, and the S-wave spin-triplet channel by 24 % in the same region. Beside the effect on the normalization, is it interesting to note that the shape of the transverse momentum distribution is insensitive to the radiative correction in the case of the spin-triplet channel, which is dominated by fragmentation contribution already at the Born-level. In the case of the spin-singlet channel, the k-factor (i.e. the ratio of the NLO to the LO contributions) is increasing with P_T. Again, these scaling properties result directly from the kinematics of the dominant channels opening at α_s^3 and α_s^4 order. QCD corrections to the P-wave colour-octet channels have not been computed yet.

Parallel to these new results, s-channel $c\bar{c}$-cut contributions to the J/ψ hadroproduction were investigated in [71, 72] in the framework of a phenomenological model. A first evaluation of the latter incorporating constraints for the low- and large-P_T (the scaling limit) region supported rates significantly larger than the usual CSM cut and allowed for a fit of the data. However, by evaluating the leading-order contribution of the s-channel $c\bar{c}$-cut amplitude in the framework of NRQCD, Artoisenet and Braaten showed [73] that the previous estimate was unrealistically large and that the s-channel $c\bar{c}$-cut could only account for a negligible fraction of the J/ψ production rate measured by the CDF collaboration.

Polarisation observables

As mentioned earlier in the case of photoproduction, the polarisation of a vector quarkonium state can be deduced from the angular distribution of the leptons originating from the quarkonium state. In the rest frame of the quarkonium, the distribution in the polar angle θ is proportional to

$$1 + \alpha cos^2\theta \tag{3}$$

On the experimental side, measuring the angular dependence is important since the reconstruction efficiency strongly depends on it. On the theory side, the quantity α can be written as a ratio of polarised cross sections, in which most of the theoretical uncertainties from quantities such as the scale (μ_F, μ_R) choices and the heavy-quark mass, partially or totally cancel. In the case of CO mediated production of e.g. $\psi(2S)$, the polarisation depends mostly on the ratio of CO LDME, not on their magnitude.

So far, nearly all the theoretical studies for any process involving polarisation predictions were done using the quarkonium momentum as the quantisation axis, this is referred to an analysis in the helicity frame (HF). The analyses by CDF[55, 74, 10], D∅ [75] and PHENIX[76] were also done in the HF, while some analysis at fixed target experiments were done in the Collins-Soper frame (CSF) [77, 78]. Recently, the Hera-B collaboration carried out the analysis in the latter two frames and in the Gottfried Jackson frame (the quantisation axis is then the beam) [79]. Faccioli *et al.* [80] have recently made a global analysis and have shown that various results obtained in the CSF and HF are compatible when the experiemental rapidity range is accounted for using a simple parametrisation. In any case, more analyses, like the HERA-B one, in different frames would be very informative. Recently, Braaten *et al.* [81] proposed a specific choice of the quantisation axis maximising the significance of the measurement of the J/ψ polarisation under the assumption that the colour-octet channels give the dominant contribution.

According to CDF polarisation measurements for ψ [55, 10], the prompt yield is increasingly longitudinal when P_T grows. The polarisation of the $\Upsilon(1S)$ was also measured by the CDF collaboration [74] within sizable experimental errors. They don't see any substantial polarisation of the Υ's, in contradiction with the measurement performed by the D∅ collaboration [75]. The PHENIX data (for P_T between 0 to 5 GeV) indicate a polarisation compatible with zero for prompt J/ψ with a trend for longitudinal yield when P_T grows.

Except for the $\psi(2S)$ and $\Upsilon(3S)$, these measurements are subject to a contamination due to feed-down from excited states. These feed-down fractions of events are not removed from the prompt measurements. In the case of colour-octet channels, the effect of the feed-down have been taken into account in the theoretical predictions at tree-level. Recently the prediction of the polarisation of the direct component has been derived at NLO accuracy: according to the result in [70] the polarisation from CO transitions appears not to be affected by radiative correction. The polarisation from CO is then still predicted to be massively transverse at large P_T, in fragrant contradiction with the available experimental measurements.

The direct Colour-Singlet prediction at NLO and NNLO* [68, 69] point at an increasingly longitudinal yield in both cases of $\psi(nS)$ and $\Upsilon(nS)$ direct production. This trend is found similar in the various k_T factorisation CS evaluations [82, 40, 83, 84] and in in the gluon tower approach [85]. In all cases, the trend from the data is less marked than from the latter theoretical evaluations. Further analyses that take into account the contribution of the feed-down are needed for a more meaningful comparison with the data.

Conclusion

In this review, I have presented the recent theoretical advances in quarkonium production. At B-factories, the evaluation of higher-order corrections in α_s and in v for several processes has remarkably reduced the previous apparent discrepancies between the NRQCD predictions and the data. To what concerns quarkonium production in electron-proton or hadron-hadron collisions, the situation is much less satisfying, in spite of the large efforts that have been provided to improve the accuracy of the predictions.

In the future, measurements of quarkonium production rates at the Large Hadron Collider are expected to shed light on the mechanisms at work in quarkonium hadroproduction. Indeed, the possibility of exploring a new region of very high transverse momentum will provide a valuable information to identify the dominant transitions that govern the production of quarkonium states in hadron collisions. Moreover, the expected large production rates at the LHC open the door to new possible more exclusive studies –i.e. where other particles in the quarkonium events are reconstructed– giving access to new observables that can improve our understanding of the production mechanisms.

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