Neutron star equations of state with optical potential constraint

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NAVI PHYSICS DAYS

26. -27. FEBRUARY 2015

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Content

1) Introduction

- combine available observations to deduce the underlying equation of state (EOS) of dense matter
- observed pulsars with two solar masses - challenge for theoretical description
- EoS has to be sufficiently stiff to support such a high mass of compact stars

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- observed pulsars with two solar masses - challenge for theoretical description
- EoS has to be sufficiently stiff to support such a high mass of compact stars

- models with nucleonic and leptonic degrees of freedom:
  - able to produce high masses if effective interaction between the nucleons becoms strongly repulsive at high barion densities

- additional hadronic particle species at densities 2 or 3 times saturation density
  - softening of the EoS and reduced maximum mass below observed values

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- other then observations we have also constraints from experiment

- the energy dependence of optical potential is experimentally extracted
  - **standard RMF** can not reproduce it
  - **extensions of RMF:** introducing new derivative couplings between nucleons and mesons to reproduce the energy dependence of optical potential

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- the energy dependence of optical potential is experimentally extracted
  - **standard RMF** can not reproduce it
  - **extensions of RMF:** introducing new derivative couplings between nucleons and mesons to reproduce the energy dependence of optical potential

- we will study the effect of the optical potential constraint on the mass-radius relation of neutron star

2) Lagrangian density and field equestions

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$$\mathcal{L}_{\text{int}} = \Gamma_\sigma \sigma \overline{\Psi}\Psi - \Gamma_\omega \omega_\mu \overline{\Psi}\gamma^\mu \Psi - \Gamma_\rho \boldsymbol{\rho}_\mu \overline{\Psi}\boldsymbol{\tau}\gamma^\mu \Psi$$

- coupling constants
- meson fields
- scalar and vector density

Standard RMF

minimal coupling of the nucleons to the meson fields

Interaction in DD-NLD model

- Derivative coupling model*: $\Psi, \overline{\Psi}$ is replaced by $\mathcal{D}_m\Psi, \overline{\mathcal{D}_m\Psi}$
- Operator functions:

$$\mathcal{D}_m(x) = \sum_{n=0}^{\infty} \frac{d_n^{(m)}}{n!} x^n$$

Numerical coefficents

Argument x:
$$x = v^\beta i\partial_\beta - sm$$

- $$\overrightarrow{\mathcal{D}}_m = \sum_{k=0}^{\infty} C_k^{(m)} (v^\beta i \overrightarrow{\partial}_\beta)^k$$
- $$\overleftarrow{\mathcal{D}}_m = \sum_{k=0}^{\infty} C_k^{(m)} (-v^\beta i \overleftarrow{\partial}_\beta)^k$$

$$C_k^{(m)} = \sum_{n=0}^{k} \frac{d_n^{(m)}}{n!} \binom{n}{k} (-sm)^{n-k}$$

(* T. Gaitanos, M.M. Kuskulov, Nucl.Phys. A899) (2013))

$$
\begin{aligned}
\mathcal{L}_{\text{int}}^{(NLD)} &= \frac{1}{2}\Gamma_\sigma \boldsymbol{\sigma} \left(\overline{\Psi} \overleftarrow{\mathcal{D}}_\sigma \Psi + \overline{\Psi} \overrightarrow{\mathcal{D}}_\sigma \Psi \right) \\
&- \frac{1}{2}\Gamma_\omega \boldsymbol{\omega}_\mu \left(\overline{\Psi} \overleftarrow{\mathcal{D}}_\omega \gamma^\mu \Psi + \overline{\Psi} \gamma^\mu \overrightarrow{\mathcal{D}}_\omega \Psi \right) \\
&- \frac{1}{2}\Gamma_\rho \boldsymbol{\rho}_\mu \left(\overline{\Psi} \overleftarrow{\mathcal{D}}_\rho \gamma^\mu \boldsymbol{\tau} \Psi + \overline{\Psi} \boldsymbol{\tau} \gamma^\mu \overrightarrow{\mathcal{D}}_\rho \Psi \right)
\end{aligned}
$$

$$\mathcal{L}_{\text{int}}^{(NLD)} = \frac{1}{2}\Gamma_\sigma \sigma \left(\overline{\Psi} \overleftarrow{\mathcal{D}}_\sigma \Psi + \overline{\Psi} \overrightarrow{\mathcal{D}}_\sigma \Psi \right) - \frac{1}{2}\Gamma_\omega \omega_\mu \left(\overline{\Psi} \overleftarrow{\mathcal{D}}_\omega \gamma^\mu \Psi + \overline{\Psi} \gamma^\mu \overrightarrow{\mathcal{D}}_\omega \Psi \right) - \frac{1}{2}\Gamma_\rho \boldsymbol{\rho}_\mu \left(\overline{\Psi} \overleftarrow{\mathcal{D}}_\rho \gamma^\mu \boldsymbol{\tau} \Psi + \overline{\Psi} \boldsymbol{\tau} \gamma^\mu \overrightarrow{\mathcal{D}}_\rho \Psi \right)$$

- meson - nucleon couplings:

  depend on the vector density $n_v = \sqrt{J_v^\mu J_{v\mu}}$, $J_v^\mu = \sum_{i=p,n} \langle \overline{\Psi}_i \gamma^\mu \Psi_i \rangle$

$$\mathcal{L}_{\text{int}}^{(DD-NLD)} = \frac{1}{2}\Gamma_\sigma(n_v) \sigma \left(\overline{\Psi} \overleftarrow{\mathcal{D}}_\sigma \Psi + \overline{\Psi} \overrightarrow{\mathcal{D}}_\sigma \Psi \right) - \frac{1}{2}\Gamma_\omega(n_v) \omega_\mu \left(\overline{\Psi} \overleftarrow{\mathcal{D}}_\omega \gamma^\mu \Psi + \overline{\Psi} \gamma^\mu \overrightarrow{\mathcal{D}}_\omega \Psi \right) - \frac{1}{2}\Gamma_\rho(n_v) \boldsymbol{\rho}_\mu \left(\overline{\Psi} \overleftarrow{\mathcal{D}}_\rho \gamma^\mu \boldsymbol{\tau} \Psi + \overline{\Psi} \boldsymbol{\tau} \gamma^\mu \overrightarrow{\mathcal{D}}_\rho \Psi \right)$$

Field equations

$$\frac{\partial \mathcal{L}}{\partial \varphi_r} + \sum_{i=1}^{\infty} (-)^i \partial_{\alpha_1,\ldots,\alpha_i} \frac{\partial \mathcal{L}}{\partial(\partial_{\alpha_1,\ldots,\alpha_i}\varphi_r)} = 0$$

GENERALIZED EULER - LAGRANGE EQUATION

- NUCLEONS: higher order derivatives of nucleon fields
- Dirac equation:

$$\left[\gamma_\mu \left(i\partial^\mu - \Sigma^\mu\right) - (m - \Sigma)\right] \Psi = 0$$

Vector self-energy

Scalar self-energy

- Scalar self-energy: $\Sigma = \Gamma_\sigma \sigma \overrightarrow{D}_\sigma$
- Vector self-energy: $\Sigma^\mu = \Gamma_\omega \omega^\mu \overrightarrow{D}_\omega + \Gamma_\rho \boldsymbol{\tau} \cdot \boldsymbol{\rho}^\mu \overrightarrow{D}_\rho + \Sigma_R^\mu$

'REARRANGEMENT' CONTRIBUTION

$$\Sigma_R^\mu = \frac{j^\mu}{n_v}\Big[\Gamma'_\omega \omega^\nu \frac{1}{2}\left(\overline{\Psi}\overleftarrow{D}_\omega \gamma_\nu \Psi + \overline{\Psi}\gamma_\nu \overrightarrow{D}_\omega \Psi\right) + \Gamma'_\rho \boldsymbol{\rho}^\nu \frac{1}{2}\left(\overline{\Psi}\overleftarrow{D}_\rho \gamma_\nu \boldsymbol{\tau}\Psi + \overline{\Psi}\gamma_\nu \boldsymbol{\tau}\overrightarrow{D}_\rho \Psi\right) - \Gamma'_\sigma \sigma \frac{1}{2}\left(\overline{\Psi}\overleftarrow{D}_\sigma \Psi + \overline{\Psi}\overrightarrow{D}_\sigma \Psi\right)\Big]$$

Derivatives of coupling functions:

$$\Gamma'_i = \frac{d\Gamma_i}{dn_v}$$

Field equations

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STANDARD EULER - LAGRANGE EQUATION

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Field equations

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STANDARD EULER - LAGRANGE EQUATION

- MESONS:

$$\begin{aligned}
\partial_\mu \partial^\mu \sigma + m_\sigma^2 \sigma &= \frac{1}{2}\Gamma_\sigma \left(\overline{\Psi} \overleftarrow{\mathcal{D}}_\sigma \Psi + \overline{\Psi} \overrightarrow{\mathcal{D}}_\sigma \Psi \right) \\
\partial_\mu F^{(\omega)\mu\nu} + m_\omega^2 \omega^\nu &= \frac{1}{2}\Gamma_\omega \left(\overline{\Psi} \overleftarrow{\mathcal{D}}_\omega \gamma^\nu \Psi + \overline{\Psi} \gamma^\nu \overrightarrow{\mathcal{D}}_\omega \Psi \right) \\
\partial_\mu \boldsymbol{F}^{(\rho)\mu\nu} + m_\rho^2 \boldsymbol{\rho}^\nu &= \frac{1}{2}\Gamma_\rho \left(\overline{\Psi} \overleftarrow{\mathcal{D}}_\rho \gamma^\nu \boldsymbol{\tau} \Psi + \overline{\Psi} \boldsymbol{\tau} \gamma^\nu \overrightarrow{\mathcal{D}}_\rho \Psi \right)
\end{aligned}$$

- Conserved baryon current in DD-NLD:

$$J^{\mu}=\sum_{i=p,n}\langle\overline{\Psi}_i N^{\mu}\Psi_i\rangle$$

$$N^{\mu}=\gamma^{\mu}+\Gamma_{\sigma}\sigma\left(\partial_p^{\mu}\mathcal{D}_{\sigma}\right)-\Gamma_{\omega}\omega_{\alpha}\gamma^{\alpha}\left(\partial_p^{\mu}\mathcal{D}_{\omega}\right)-\Gamma_{\rho}\boldsymbol{\rho}_{\alpha}\gamma^{\alpha}\boldsymbol{\tau}\left(\partial_p^{\mu}\mathcal{D}_{\rho}\right)$$

- The energy - momentum tensor:

$$T^{\mu\nu}=\sum_{i=p,n}\langle\overline{\Psi}_i N^{\mu}p^{\nu}\Psi_i\rangle-g^{\mu\nu}\langle\mathcal{L}\rangle$$

Energy density : $\epsilon=T^{00}$

Pressure: $P=\sum_{i=1}^{3}T^{ii}/3$

3) DD-NLD model for nuclear matter

- Stationary nuclear matter:
  - simplification of equations (homogenous system, meson fields treated as classical)
  - positive energy solution of Dirac equation

    - plane waves (for p and n): $\Psi_i = u_i \, exp(-ip_i^\mu x_\mu)$

    - normalized: $\overline{\Psi}_i N^0 \Psi_i = \overline{u_i} N^0 u_i = 1$

- $$\overrightarrow{\mathcal{D}}_m = \sum_{k=0}^{\infty} C_k^{(m)} (v^\beta i \overrightarrow{\partial}_\beta)^k \quad \rightarrow \quad p_i^\mu = (E_i, \vec{p}_i)$$

  - simple D_m function depending on energy and momentum of the nucleon

3) DD-NLD model for nuclear matter

- Meson fields:

$$\sigma = \frac{\Gamma_\sigma}{m_\sigma^2} n_\sigma = \frac{\Gamma_\sigma}{m_\sigma^2} \sum_{i=p,n} \langle \overline{\Psi}_i D_\sigma \Psi_i \rangle$$

$$\omega = \frac{\Gamma_\omega}{m_\omega^2} n_\omega = \frac{\Gamma_\omega}{m_\omega^2} \sum_{i=p,n} \langle \overline{\Psi}_i \gamma^0 D_\omega \Psi_i \rangle$$

$$\rho = \frac{\Gamma_\rho}{m_\rho^2} n_\rho = \frac{\Gamma_\rho}{m_\rho^2} \sum_{i=p,n} \langle \overline{\Psi}_i \gamma^0 \tau_3 D_\rho \Psi_i \rangle$$

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- Self-energies:

$$S_i = \Gamma_\sigma \sigma \mathcal{D}_\sigma$$
$$V_i = \Gamma_\omega \omega \mathcal{D}_\omega + \Gamma_\rho \tau_{3,i} \rho \mathcal{D}_\rho + \Sigma_R^0$$

- 'Rearrangement' contribution:

$$\Sigma_R^0 = \Gamma'_\omega \omega n_\omega + \Gamma'_\rho \rho n_\rho - \Gamma'_\sigma \sigma n_\sigma$$

- Dispersion relation:

$$E_i = \sqrt{p^2 + (m_i - S_i)^2} + V_i$$

4) Parametrization of DD-NLD model

- Form of D functions:
  - D1 - constant $D = 1$, usual RMF with DD coupling
  - D2 - a Lorentzian form: $D = 1/(1+x^2)$
  - D3 - an exponential dependence: $D = exp(-x)$

$$x = \frac{(E_i - m_i)}{\Lambda}$$

regulates strength of energy dependence

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$$x = \frac{(E_i - m_i)}{\Lambda}$$

Λ regulates strength of energy dependence

- Meson - nucleon couplings:
  - $\sigma, \omega: \quad \Gamma_m(n_v) = \Gamma_m(n_{ref}) a_m \frac{1 + b_m (x + d_m)^2}{1 + c_m (x + d_m)^2}$
  - $\rho: \quad \Gamma_\rho(n_v) = \Gamma_\rho(n_{ref}) \exp(-a_\rho (x - 1))$

$$x = \frac{n_v}{n_{ref}}$$

- reduction of number of independent parameters:

$$f_\sigma(1) = f_\omega(1) = 1 \quad \text{and} \quad f''_\sigma(0) = f''_\omega(0) = 0$$

- only two independent coeff. for each isoscalar meson
- saturation properties:

$$n_{sat} = 0.15 \ fm^{-3}$$
$$B = 16 \ MeV$$
$$K = 240 \ MeV$$
$$J = 32 \ MeV$$
$$L = 60 \ MeV$$
$$m_{eff} = 0.5625 \, m_{nuc}$$
$$f'_\omega(1)/f_\omega(1) = -0.15$$
$$f''_\omega(1)/f'_\omega(1) = -1.0$$

⇒

$$n_{ref}$$
$$\Gamma_m(n_{ref})$$
$$a_m$$
$$b_m$$
$$c_m$$
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fixed ratios, values close to those of DD2* - fitted to the properties of nuclei and predicts mass of 2.4 solar masses for neutron star

⇒

$$n_{ref}$$
$$\Gamma_m(n_{ref})$$
$$a_m$$
$$b_m$$
$$c_m$$
$$d_m$$

(* S.Typel, G.Ropke, T.Klahn, D.Blaschke, H.Wolter, Phys.Rev. C81 (2010))

5) Results

- Introduction of nonlinear derivative couplings?
  - to improve energy dependence of optical potential

OPTICAL POTENTIAL

$$U_{opt}(E) = \frac{E}{m_{nuc}}V - S + \frac{S^2 - V^2}{2m_{nuc}}$$

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OPTICAL POTENTIAL

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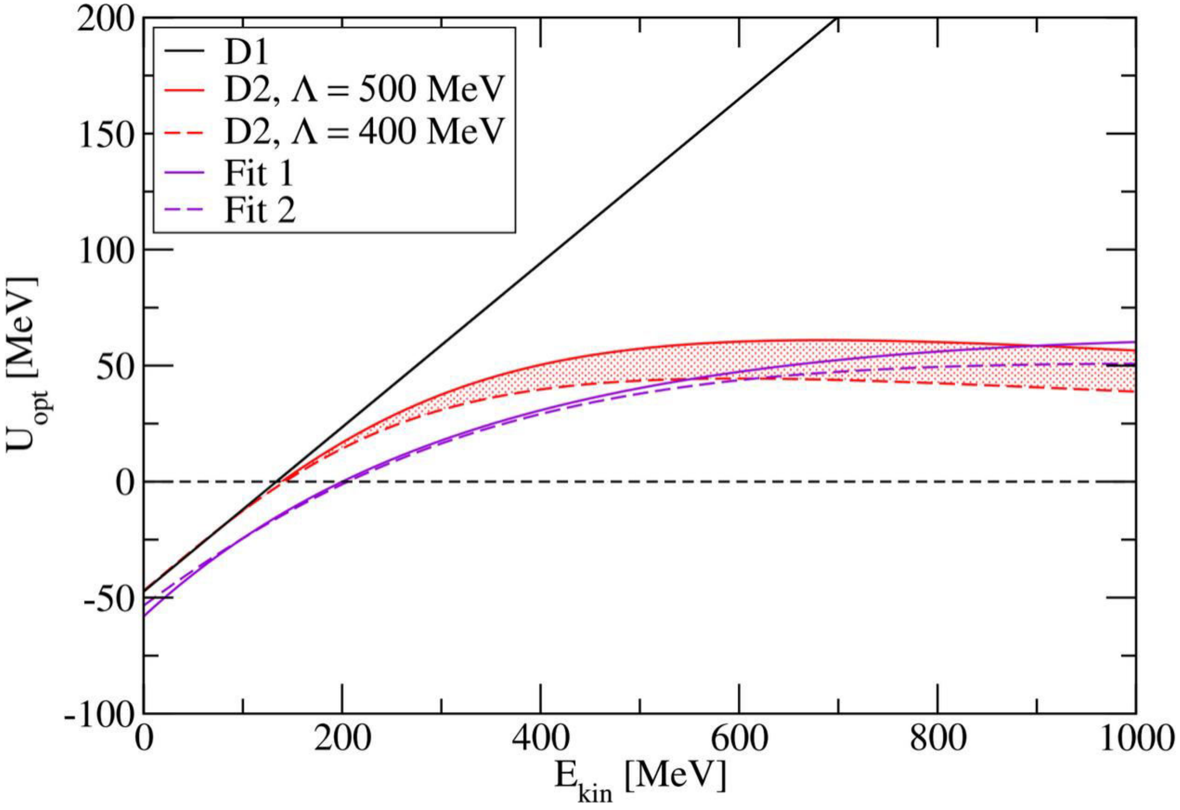
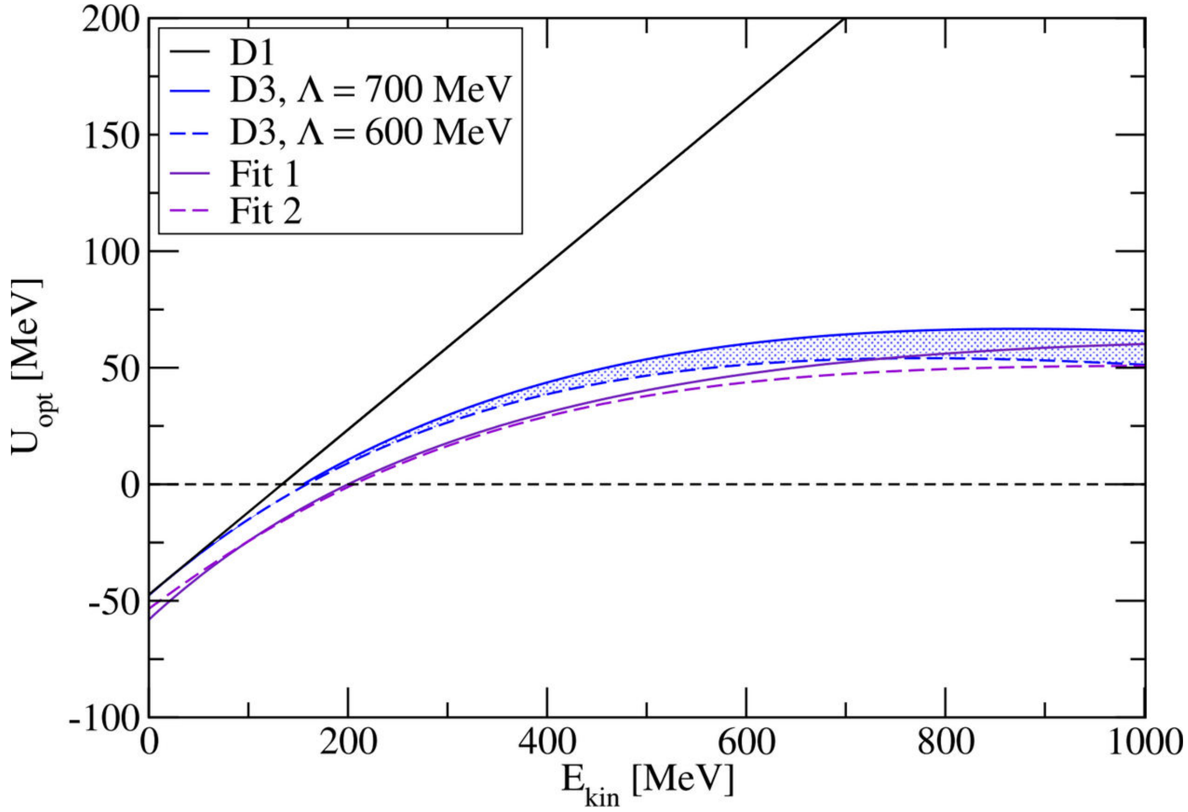
- From p scattering on nuclei of different A, optical potential obtained in symmetric nuclear matter at saturation density as a function of kinetic energy

5a) Results - nuclear matter





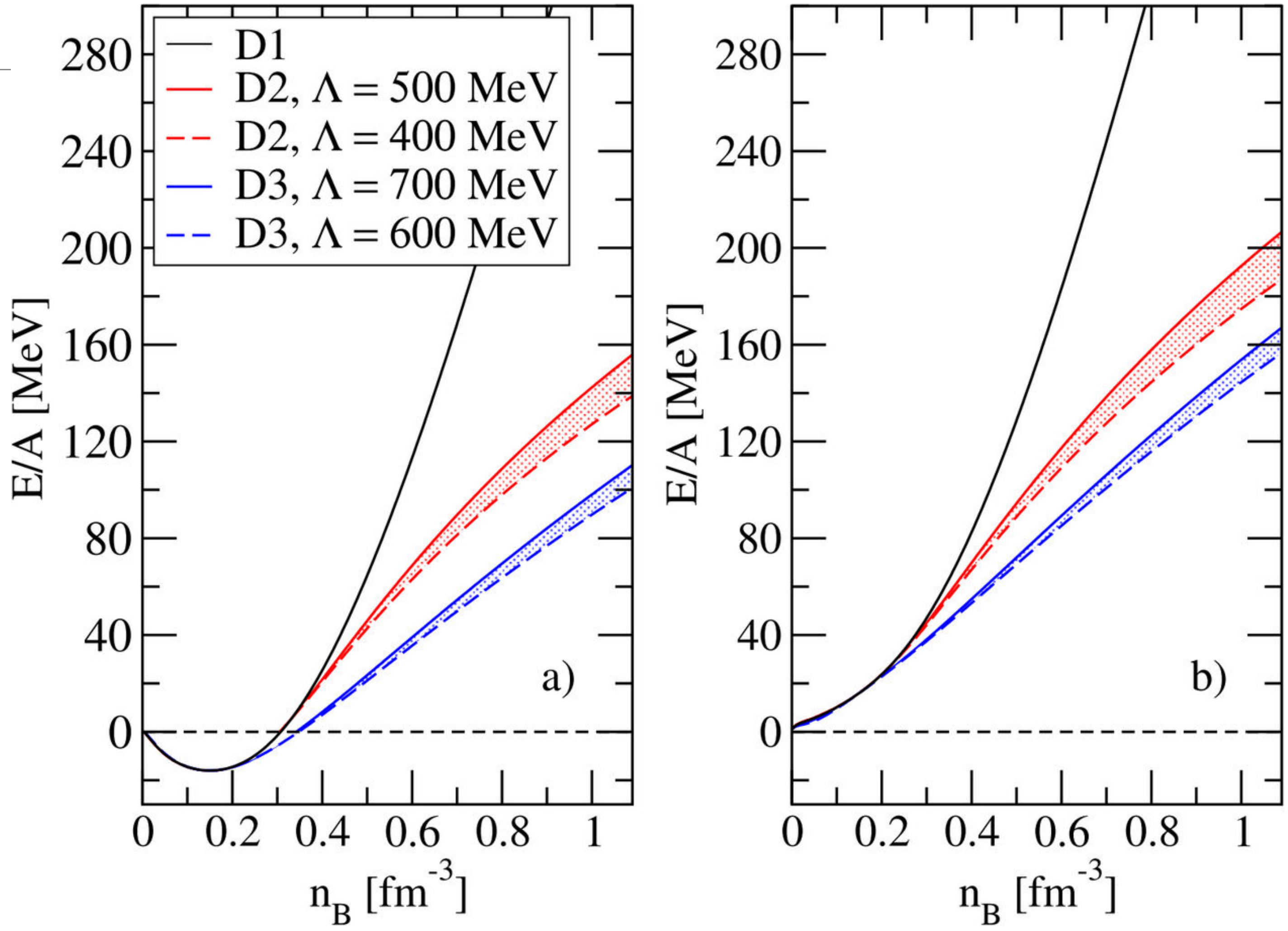
- Fit1 and Fit2: fits from Dirac phenomenology *

*(S.Hama, B. Clark, E.Cooper, H.Sherif,R.Mercer, Phys.Rev. C41 (1990) 2737-2755)

5a) Results - nuclear matter

- Reduction of the optical potential at high kinetic energies reflected also in the EoS

- softening of the EoS



5b) Results - neutron stars

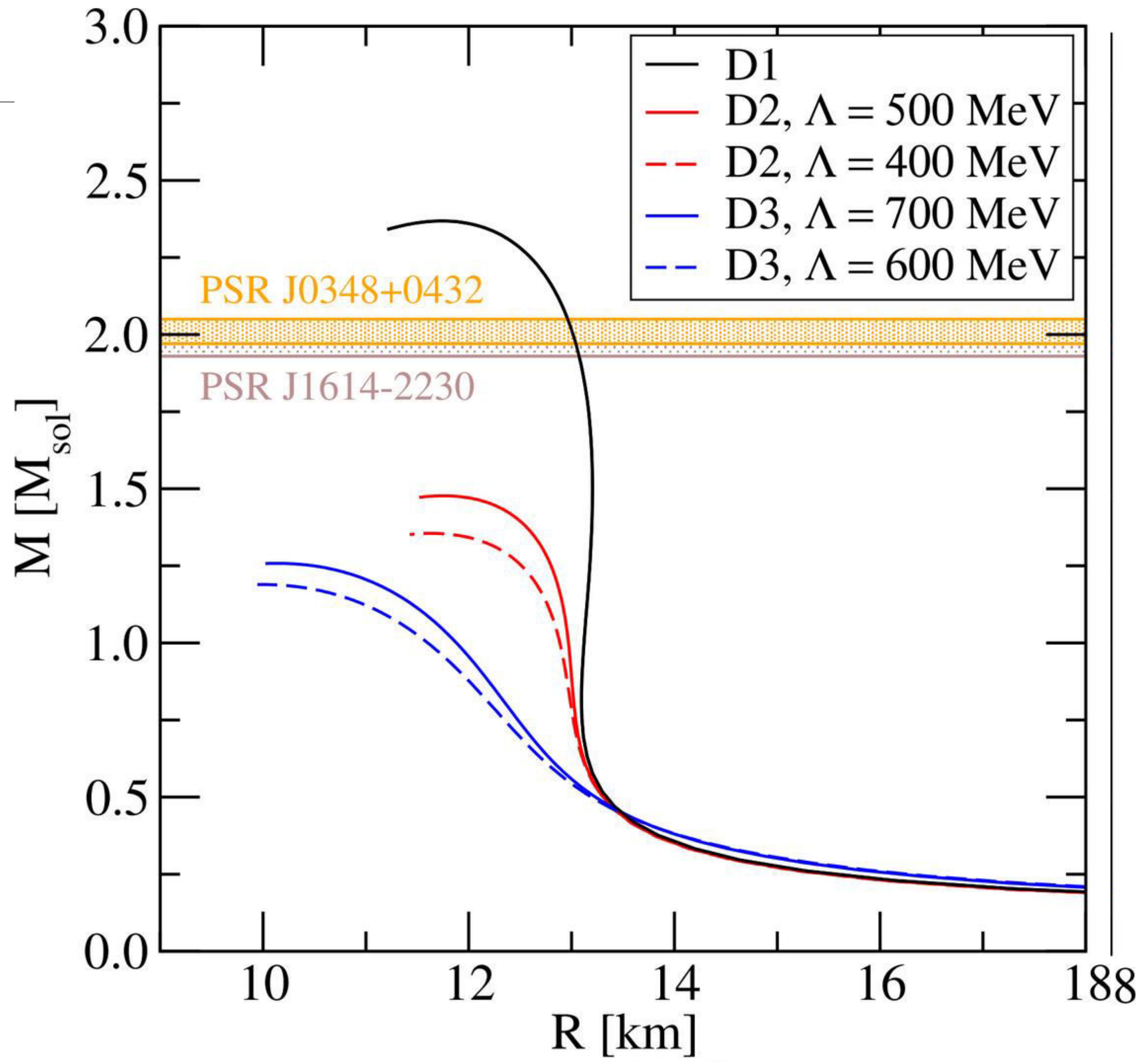
- properties of Neutron stars with DD-NLD model?

- we need EoS of stellar matter:
  - adding contribution of electrons → $\mathcal{L}_e = \overline{\Psi}_e(i\gamma^\mu\partial_\mu - m_e)\Psi_e$
  - charge neutrality (fixing lepton density) → $n_e = n_p$
  - β-equilibrium (fixing p-n asymmetry) → $\mu_n = \mu_p + \mu_e$

- by solving Tolman-Oppenheimer-Volkoff equations we get mass-radius relation for NS $\frac{dp}{dr} = -\frac{G}{c^4}\frac{(mc^2 + 4\pi pr^3)(\epsilon + P)}{r(r - 2Gm/c^2)}, \quad \frac{dm}{dr} = 4\pi\frac{\epsilon}{c^2}r^2$

5b) Results - neutron stars

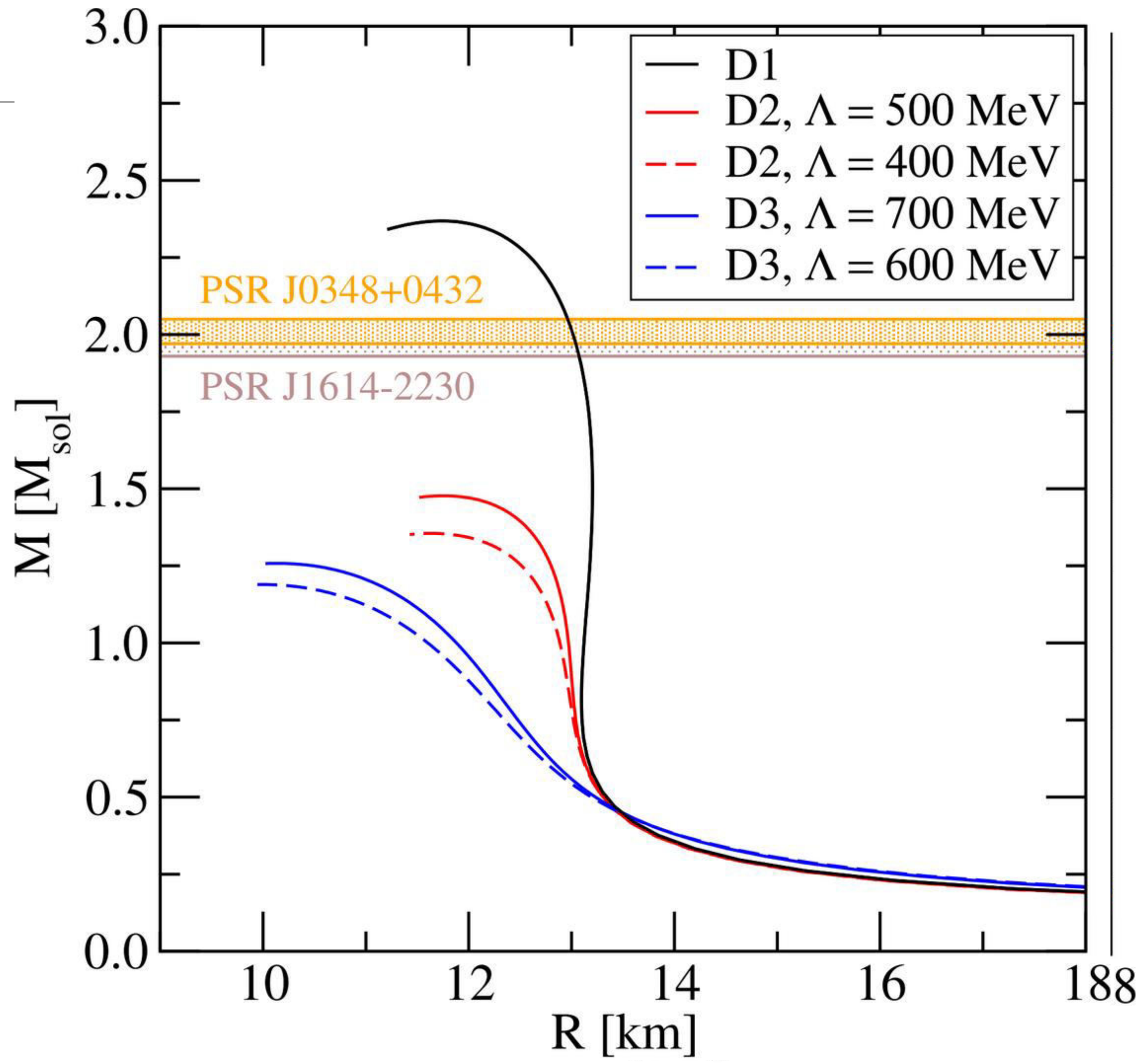
- DD-NLD model - homogeneous matter
- at low densities - standard Baym-Pethick-Sutherland (BPS) crust EoS



5b) Results - neutron stars

- DD-NLD model - homogeneous matter
- at low densities - standard Baym-Pethick-Sutherland (BPS) crust EoS

- D1:
  - no energy dependence
  - easily describes maximum NS mass
- D2 & D3:
  - consistent with optical potential constraint
  - serious reduction of maximum NS mass



6) Conclusion

- current version of DD-NLD:
  - energy dependence of self-energies: softening of the EoS at high densities
  - independent of appearance of additional degrees of freedom
  - with this parametrization - not possible to obtain 2 Solar masses for NS
  - only few energy dependence tested - freedom (find suitable parametrization consistent with optical potential and NS mass constraint)

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- optical potential has to be taken seriously into account in the development of realistic phenomenological models for dense matter

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- models with self-energies depending explicitly on energy/momentum
  - application to HIC simulations using relativistic transport approaches
  - constrain the parameters of present model at supra saturation densities

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- Future perspectives?
- application of DD-NLD approach to description of nuclei
- better control of the parameters

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  - application to HIC simulations using relativistic transport approaches
  - constrain the parameters of present model at supra saturation densities

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- application of DD-NLD approach to description of nuclei
- better control of the parameters

Thank You !