Microscopic description of nuclear structure and reactions with astrophysical applications

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Overview

Realistic Effective Nucleon-Nucleon interaction:

Unitary Correlation Operator Method

Many-Body Approach:

Fermionic Molecular Dynamics

Applications:

^{3}He(α,γ)^{7}Be Radiative Capture Reaction

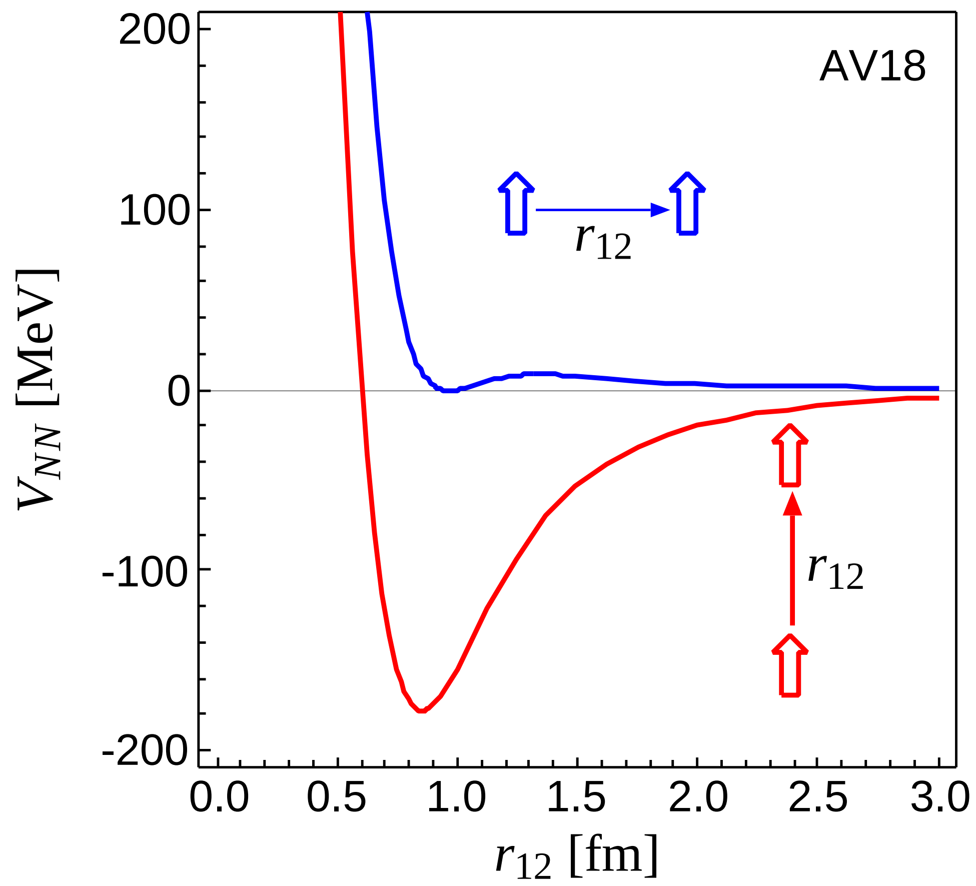
^{12}C in the Microscopic Cluster Model

^{12}C in Fermionic Molecular Dynamics

Nuclear Force

Argonne V18 (T=0)

spins aligned parallel or perpendicular to the relative distance vector



- strong repulsive core: nucleons can not get closer than ≈ 0.5 fm

➽ **central correlations**

- strong dependence on the orientation of the spins due to the tensor force

➽ **tensor correlations**

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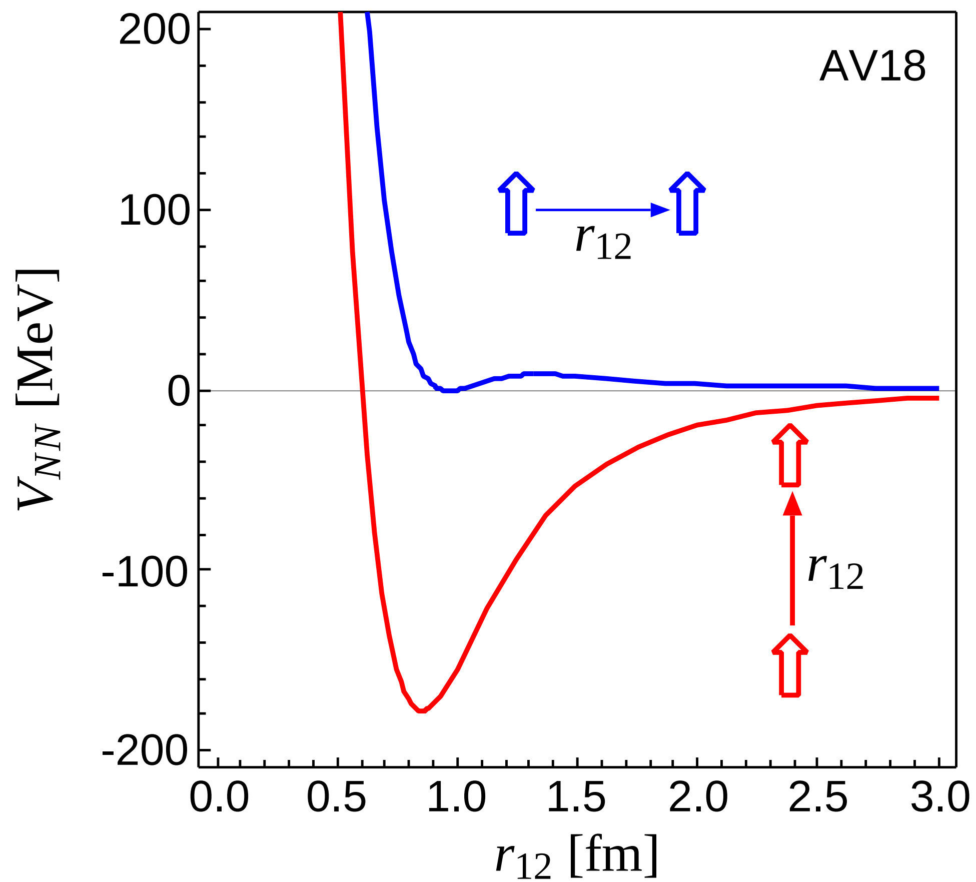

- strong repulsive core: nucleons can not get closer than ≈ 0.5 fm

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➽ **tensor correlations**

the nuclear force will induce **strong short-range correlations** in the nuclear wave function

Unitary Correlation Operator Method

Correlation Operator

- induce short-range (two-body) central and tensor correlations into the many-body state

$$\underset{\sim}{C} = \underset{\sim}{C}_\Omega \underset{\sim}{C}_r = \exp\Big[-i\sum_{i<j} \underset{\sim}{g}_{\Omega,ij}\Big] \exp\Big[-i\sum_{i<j} \underset{\sim}{g}_{r,ij}\Big] \quad , \quad \underset{\sim}{C}^\dagger \underset{\sim}{C} = \underset{\sim}{1}$$

- correlation operator should conserve the symmetries of the Hamiltonian and should be of finite-range

Correlated Operators

- correlated operators will have contributions in higher cluster orders

$$\underset{\sim}{C}^\dagger \underset{\sim}{O} \underset{\sim}{C} = \underset{\sim}{\hat{O}}^{[1]} + \underset{\sim}{\hat{O}}^{[2]} + \underset{\sim}{\hat{O}}^{[3]} + \ldots$$

- two-body approximation: correlation range should be small compared to mean particle distance

Correlated Interaction

$$\underset{\sim}{C}^\dagger (\underset{\sim}{T} + \underset{\sim}{V}) \underset{\sim}{C} = \underset{\sim}{T} + \underset{\sim}{V}_{\mathrm{UCOM}} + \underset{\sim}{V}^{[3]}_{\mathrm{UCOM}} + \ldots$$

- correlated interaction **phase shift equivalent** to bare interaction by construction

Intrinsic Basis States

Fermionic

Slater determinant

$$\left| Q \right\rangle = \underset{\sim}{\mathcal{A}}\Big(\left| q_1 \right\rangle \otimes \cdots \otimes \left| q_A \right\rangle \Big)$$

- antisymmetrized A-body state

Feldmeier, Schnack, Rev. Mod. Phys. **72** (2000) 655

Neff, Feldmeier, Nucl. Phys. **A738** (2004) 357

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Molecular

single-particle states

$$\left\langle \mathbf{x} \middle| q \right\rangle = \sum_i c_i \exp\left\{ -\frac{(\mathbf{x} - \mathbf{b}_i)^2}{2a_i} \right\} \otimes \left| \chi^{\uparrow}_i, \chi^{\downarrow}_i \right\rangle \otimes \left| \xi \right\rangle$$

- Gaussian wave-packets in phase-space (complex parameter $\mathbf{b}_i$ encodes mean position and mean momentum), spin is free, isospin is fixed
- width a_i is an independent variational parameter for each wave packet
- use one or two wave packets for each single particle state

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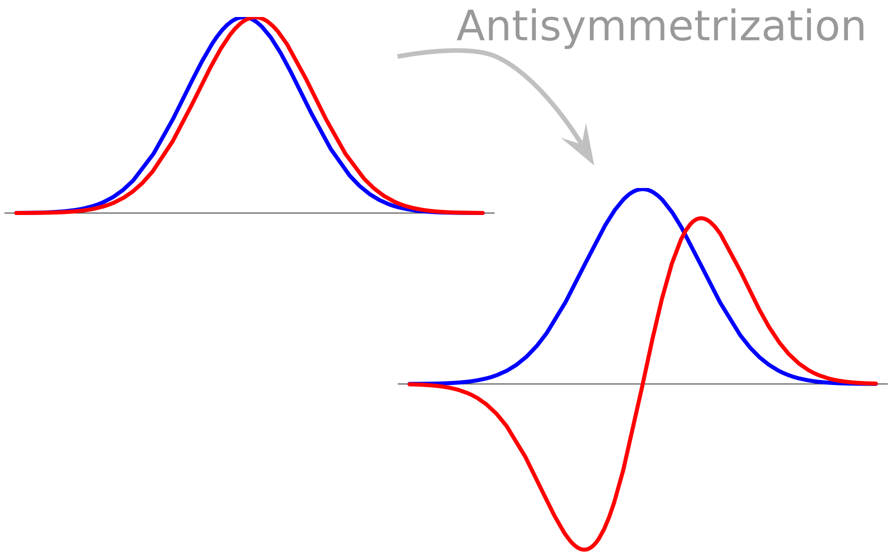

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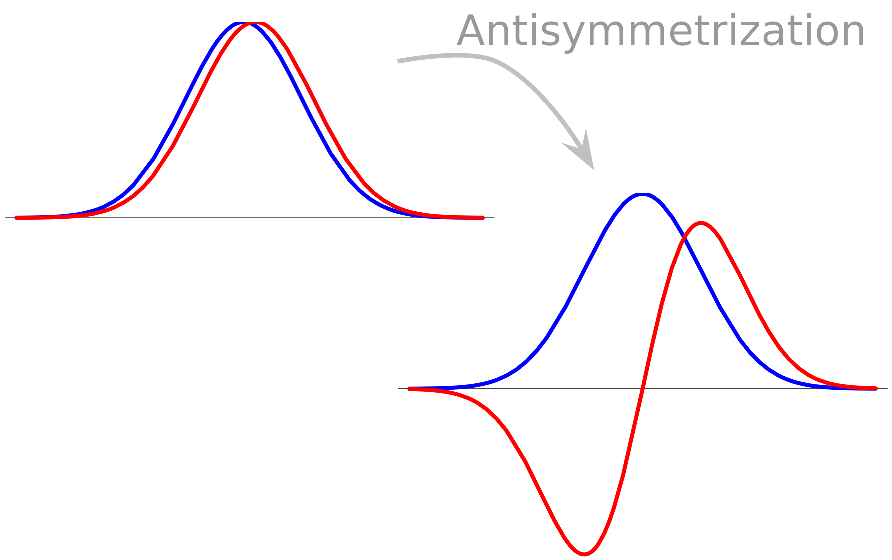

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FMD basis contains
HO shell model and
microscopic cluster model
as limiting cases

Feldmeier, Schnack, Rev. Mod. Phys. **72** (2000) 655

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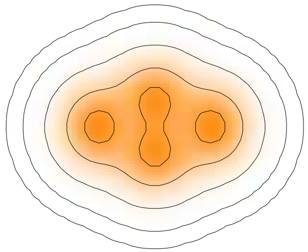
Symmetries and Projection

Breaking of symmetries



- Slater determinants $\left| Q \right\rangle$ may break symmetries of the Hamiltonian with respect to parity, rotations and translations

Projection

- Restore symmetries by projection

$$\underset{\sim}{P}^{\pi} = \frac{1}{2}(1+\pi\underset{\sim}{\Pi}), \qquad \underset{\sim}{P}^{J}_{MK} = \frac{2J+1}{8\pi^2}\int d^3\Omega\, {D^{J}_{MK}}^{\star}(\Omega)\underset{\sim}{R}(\Omega), \qquad \underset{\sim}{P}^{\mathbf{P}} = \frac{1}{(2\pi)^3}\int d^3X\, \exp\{-i(\underset{\sim}{\mathbf{P}}-\mathbf{P})\cdot\mathbf{X}\}$$

Multiconfiguration Mixing

- **diagonalize** Hamiltonian in a set of projected intrinsic states $\left\{ \left| Q^{(a)} \right\rangle, a=1,\ldots,N \right\}$

$$\left| \Psi; J^{\pi} M \alpha \right\rangle = \sum_{Ka} \underset{\sim}{P}^{\pi} \underset{\sim}{P}^{J}_{MK} \underset{\sim}{P}^{\mathbf{P}=0} \left| Q^{(a)} \right\rangle c^{\alpha}_{Ka}$$

$$\sum_{K'b} \underbrace{\left\langle Q^{(a)} \right| \underset{\sim}{H} \underset{\sim}{P}^{\pi} \underset{\sim}{P}^{J}_{KK'} \underset{\sim}{P}^{\mathbf{P}=0} \left| Q^{(b)} \right\rangle}_{\text{Hamiltonian kernel}} c^{\alpha}_{K'b} = E^{J^{\pi}\alpha} \sum_{K'b} \underbrace{\left\langle Q^{(a)} \right| \underset{\sim}{P}^{\pi} \underset{\sim}{P}^{J}_{KK'} \underset{\sim}{P}^{\mathbf{P}=0} \left| Q^{(b)} \right\rangle}_{\text{norm kernel}} c^{\alpha}_{K'b}$$

^{3}He(α,γ)^{7}Be radiative capture

one of the key reactions in the solar pp-chains

Effective Nucleon-Nucleon interaction:

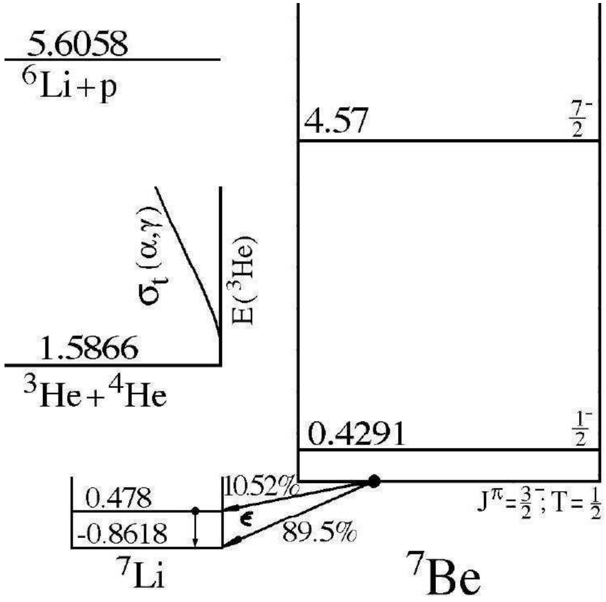
AV18-UCOM(SRG)

$\alpha = 0.20\ \mathrm{fm}^4$ – $\lambda \approx 1.5\ \mathrm{fm}^{-1}$

Many-Body Approach:

Fermionic Molecular Dynamics

- **Internal region:**
  VAP configurations with radius constraint
- **External region:**
  Brink-type cluster configurations
- **Matching to Coulomb solutions:**
  Microscopic *R*-matrix method



Neff, Phys. Rev. Lett. 106, 042502 (2011)

FMD model space

Frozen configurations

- antisymmetrized wave function built with ^{4}He and ^{3}He FMD clusters up to channel radius a=12 fm

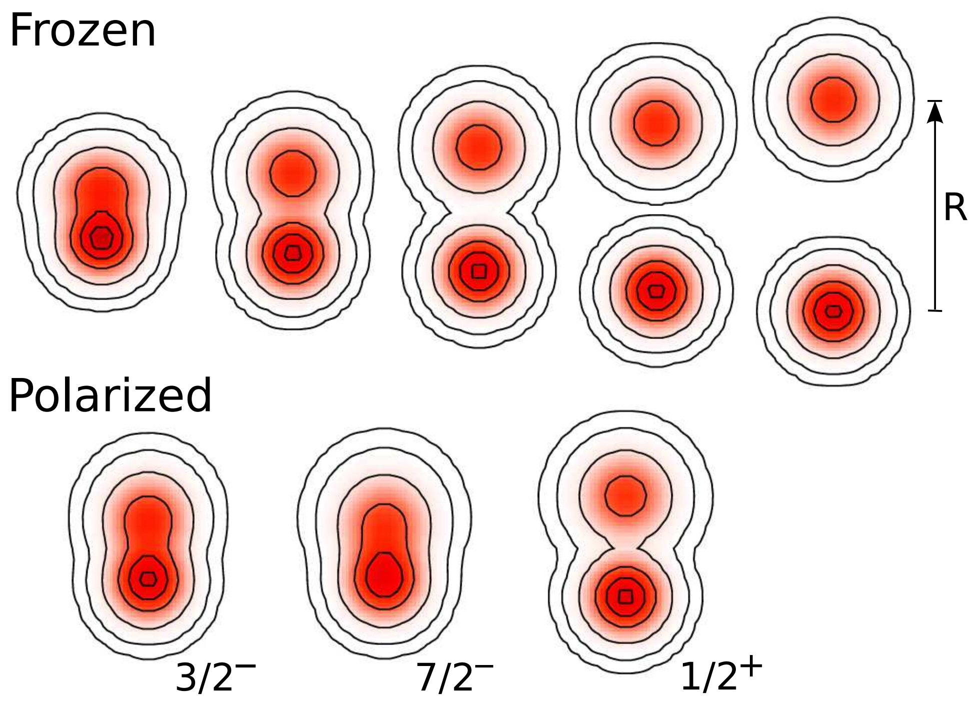
Polarized configurations

- FMD wave functions obtained by **Variation after Projection** on $1/2^-$, $3/2^-$, $5/2^-$, $7/2^-$ and $1/2^+$, $3/2^+$ and $5/2^+$ combined with radius constraint in the interaction region

Boundary conditions

- Match relative motion of clusters at channel radius to Whittaker/Coulomb functions with the **microscopic *R*-matrix** method of the Brussels group

  D. Baye, P.-H. Heenen, P. Descouvemont



p-wave Bound and Scattering States

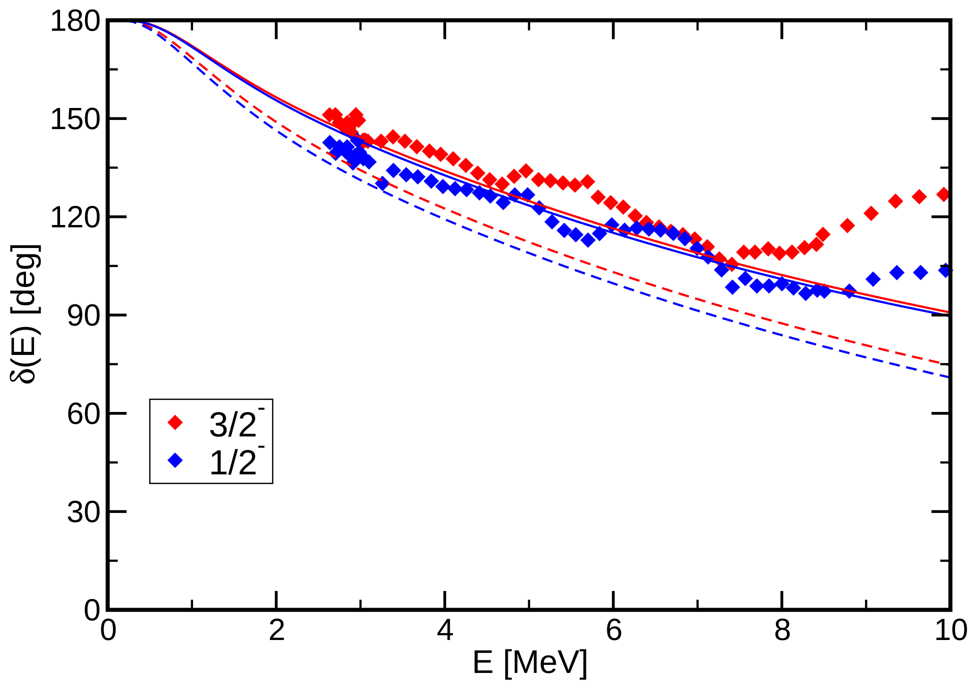
Bound states

		Experiment	FMD
^{7}Be	$E_{3/2-}$	-1.59 MeV	-1.49 MeV
	$E_{1/2-}$	-1.15 MeV	-1.31 MeV
	r_{ch}	2.647(17) fm	2.67 fm
	Q	–	-6.83 e fm^2
^{7}Li	$E_{3/2-}$	-2.467 MeV	-2.39 MeV
	$E_{1/2-}$	-1.989 MeV	-2.17 MeV
	r_{ch}	2.444(43) fm	2.46 fm
	Q	-4.00(3) e fm^2	-3.91 e fm^2

- centroid of bound state energies well described if polarized configurations included
- tail of wave functions tested by charge radii and quadrupole moments

Phase shift analysis:

Spiger and Tombrello, PR **163**, 964 (1967)

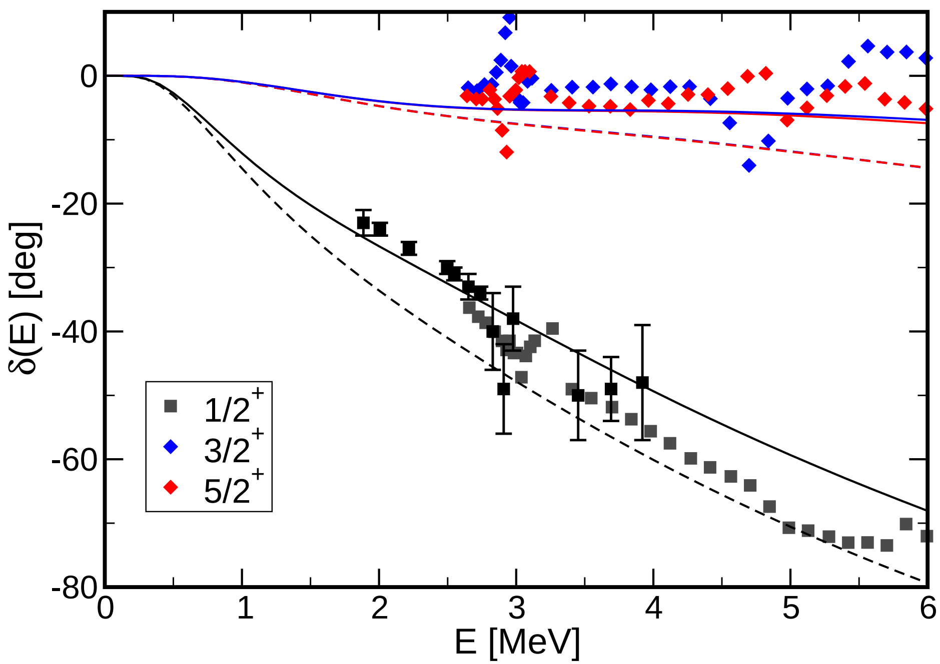


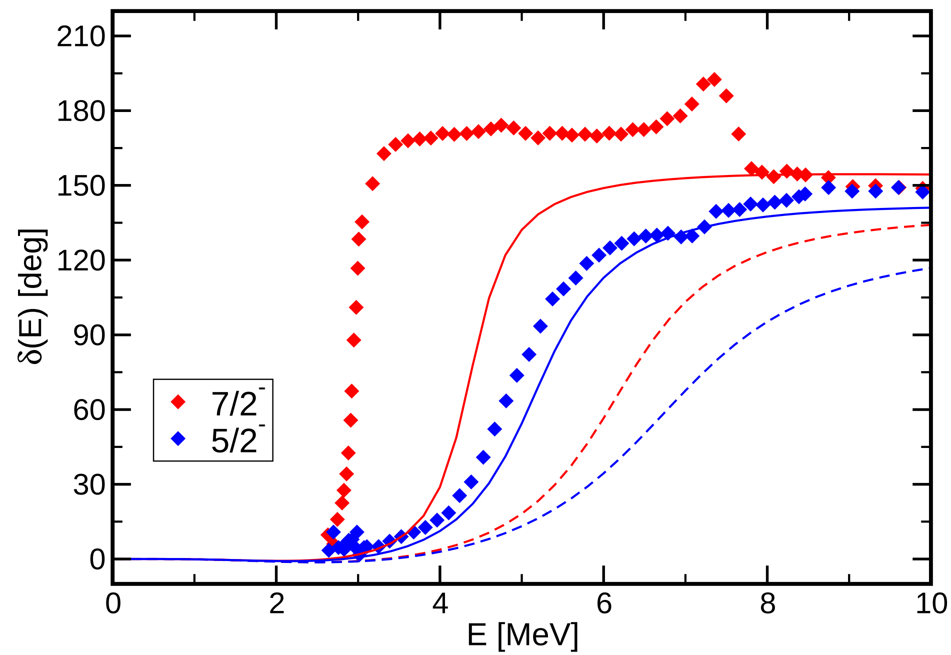
dashed lines – frozen configurations only

solid lines – polarized configurations in interaction region included

- Scattering phase shifts well described, polarization effects important

s-, *d*- and *f*-wave Scattering States

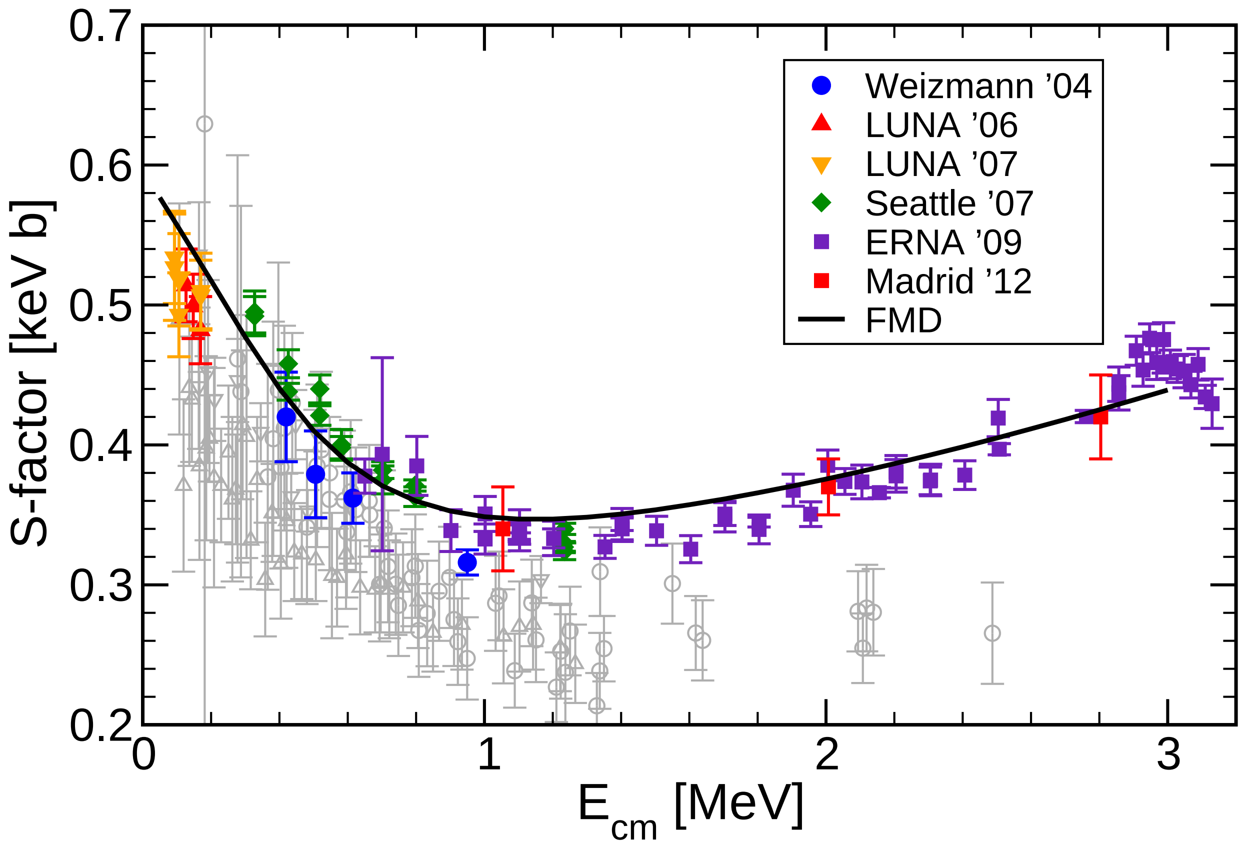




dashed lines – frozen configurations only – solid lines – FMD configurations in interaction region included

- polarization effects important
- *s*- and *d*-wave scattering phase shifts well described
- *f*-wave splittings too small, additional spin-orbit strength from three-body forces expected

S-Factor



S-factor:

$$S(E) = \sigma(E) E \exp\{2\pi\eta\}$$

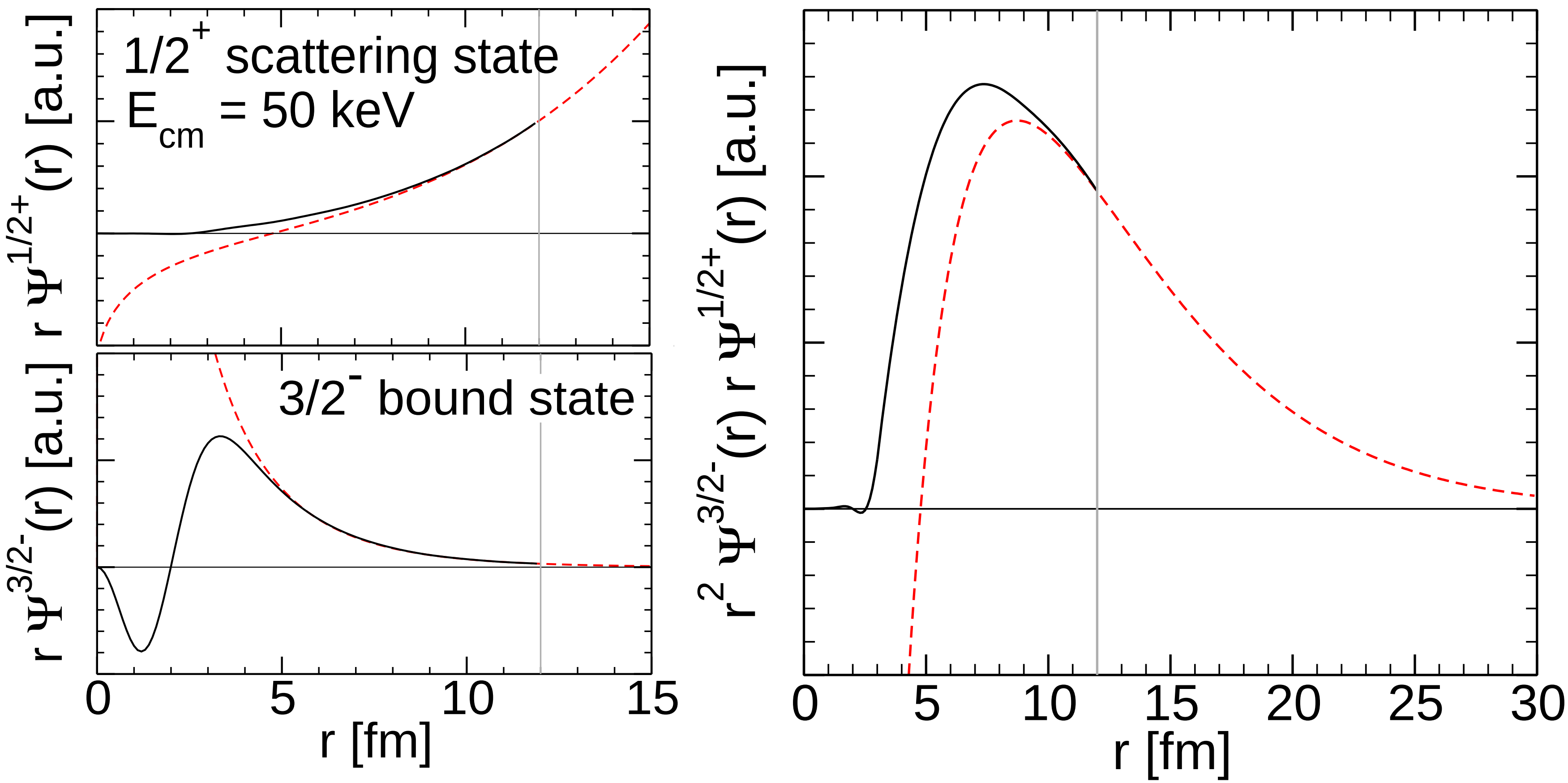
$$\eta = \frac{\mu Z_1 Z_2 e^2}{k}$$

Nara Singh *et al.*, PRL **93**, 262503 (2004)
Bemmerer *et al.*, PRL **97**, 122502 (2006)
Confortola *et al.*, PRC **75**, 065803 (2007)
Brown *et al.*, PRC **76**, 055801 (2007)
Di Leva *et al.*, PRL **102**, 232502 (2009)

- dipole transitions from $1/2^+$, $3/2^+$, $5/2^+$ scattering states into $3/2^-$, $1/2^-$ bound states
- FMD is the only model that describes well the energy dependence and normalization of new high quality data
- fully microscopic calculation, bound and scattering states are described consistently

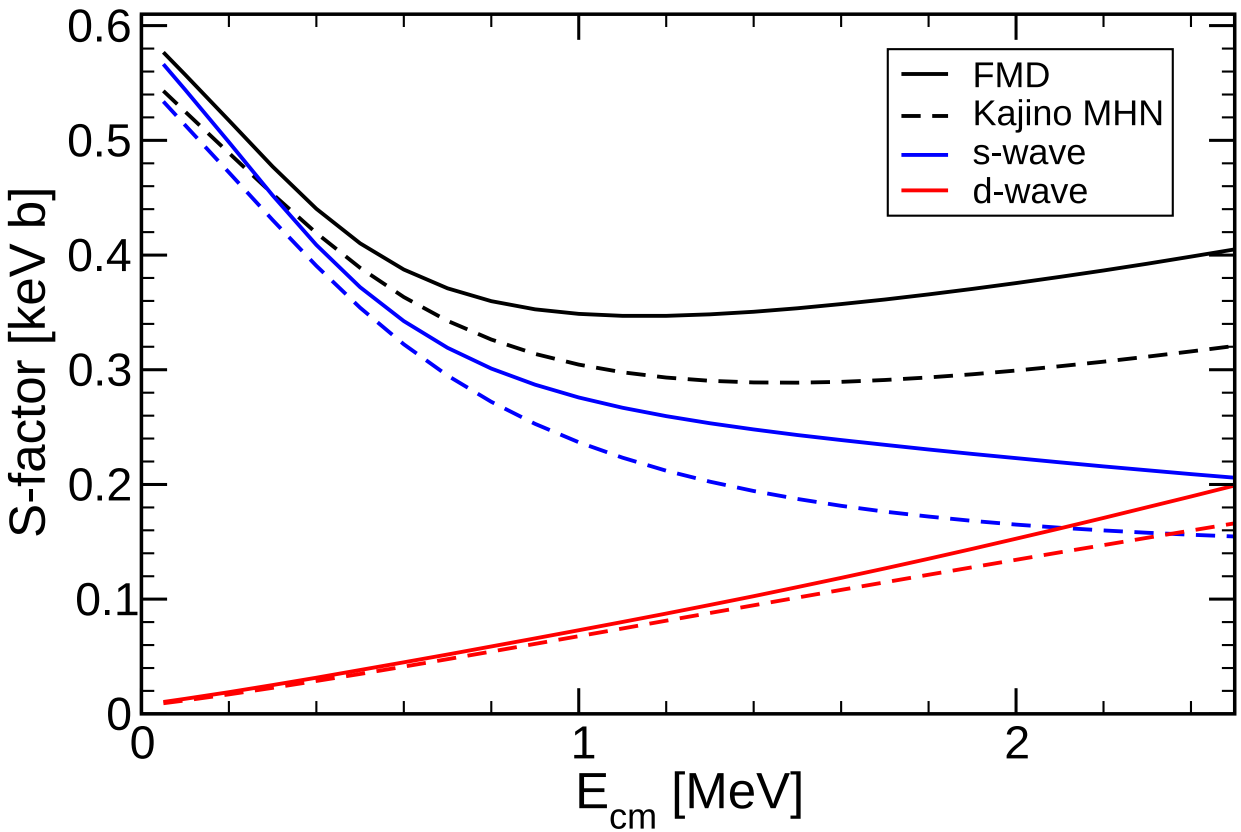
$^3He(\alpha,\gamma)^7Be$

Overlap Functions and Dipole Matrixelements



- Overlap functions from projection on RGM-cluster states
- Coulomb and Whittaker functions matched at channel radius a=12 fm
- Dipole matrix elements calculated from overlap functions reproduce full calculation within 2%
- cross section depends significantly on internal part of wave function, description as an “external” capture is too simplified

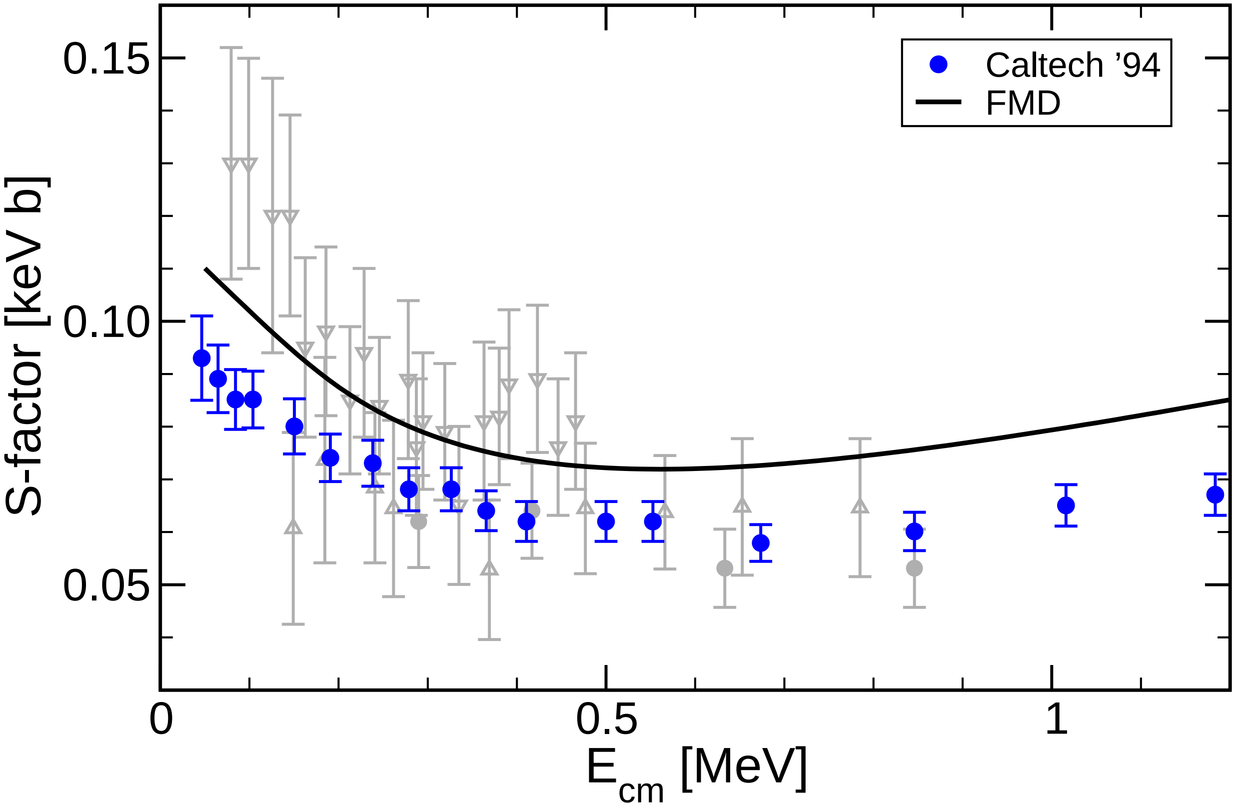
Energy dependence of the S-Factor



- low-energy *S*-factor dominated by *s*-wave capture
- at 2.5 MeV equal contributions of *s*- and *d*-wave capture

➽ **investigate angular distributions:**
NAVI collaboration with Tamás Szücs and Daniel Bemmerer

S-Factor



S-factor:

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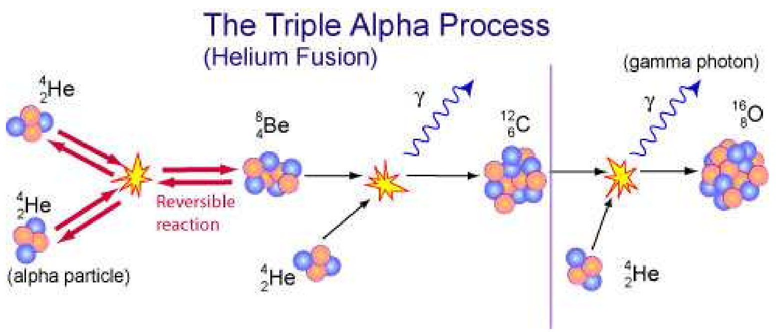
$$\eta = \frac{\mu Z_1 Z_2 e^2}{k}$$

Brune *et al.*, PRC **50**, 2205 (1994)

- isospin mirror reaction of $^{3}He(\alpha,\gamma)^{7}Be$
- ^{7}Li bound state properties and phase shifts well described

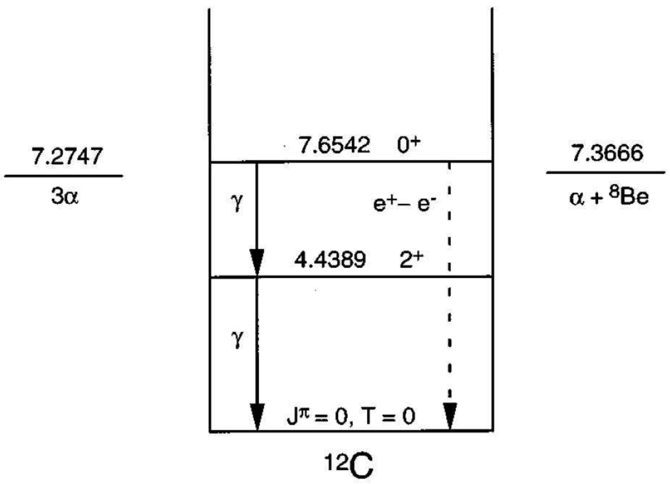
➼ FMD calculation describes energy dependence of Brune *et al.* data but cross section is larger by about 15%

Cluster States in ^{12}C



Structure

- **Is the Hoyle state a pure α-cluster state ?**
- **Second 2^+ state**
  Zimmermann *et al.*, Phys. Rev. Lett. 110, 152502 (2013)
- **Second 4^+ state**
  Freer *et al.*, Phys. Rev. C 83, 034314 (2011)
- **Other states in the continuum**
  Fynbo *et al.*, . . .

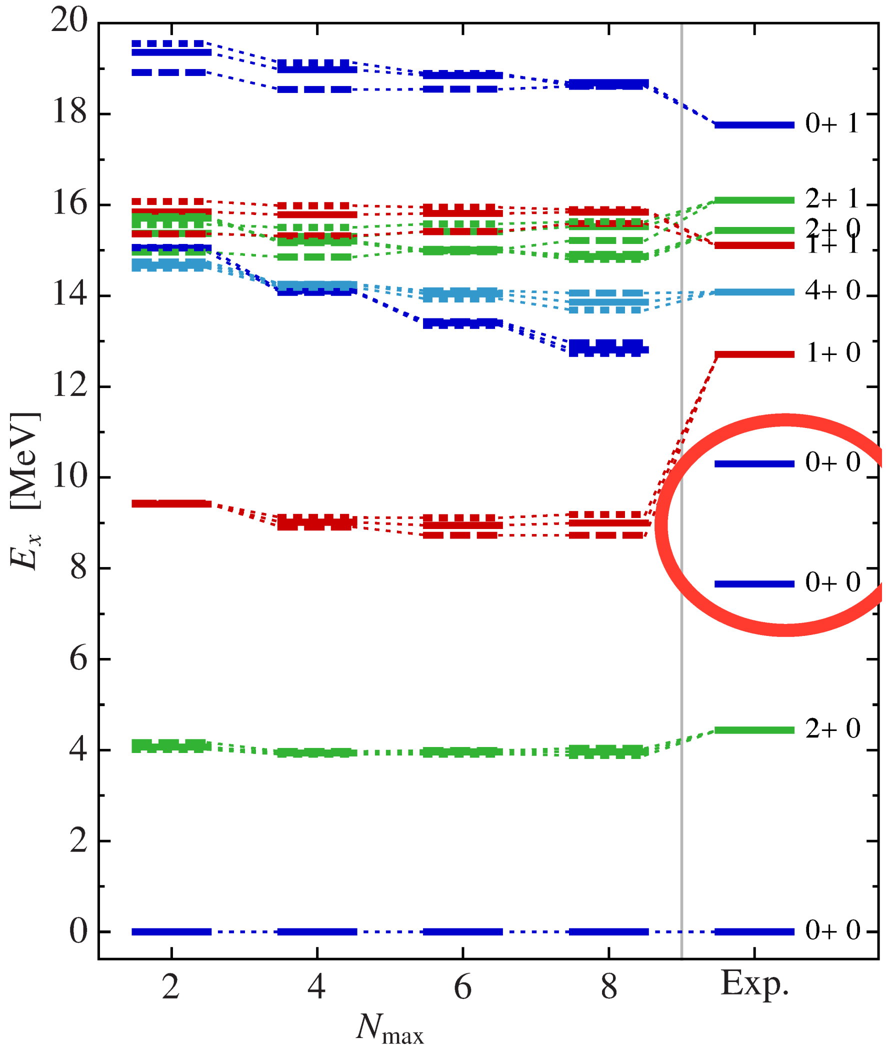


➽ **Include continuum in the calculation!**

➽ **Compare microscopic α-cluster model and FMD**

Neff, Feldmeier, arXiv:1409.3726

Cluster States in the NCSM ?

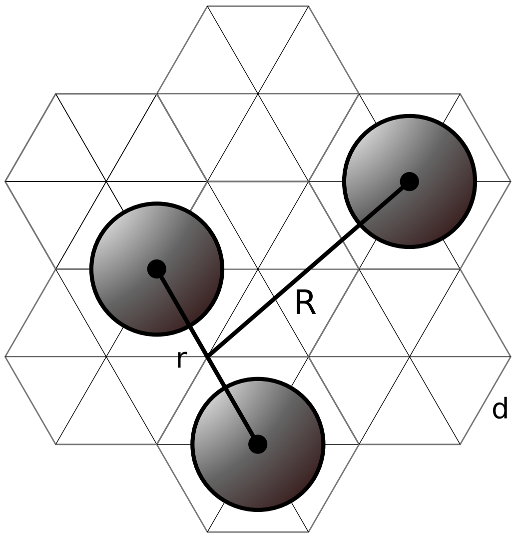


State of the art calculation with chiral NN+NNN forces

Hoyle state and other cluster states missing !

Maris, Vary, Calci, Langhammer, Binder, Roth, Phys. Rev. C **90**, 014314 (2014)

Model space in internal region



$$\rho^2 = \frac{1}{2}\mathbf{r}^2 + \frac{2}{3}\mathbf{R}^2$$

Hyperradius

Kamimura, Nuc. Phys. **A351** (1981) 456

Funaki et al., Phys. Rev. C **67** (2003) 051306(R)

Model Space

- include all possible configurations on triangular grid ($d = 1.4\,\mathrm{fm}$) up to a certain hyperradius ρ
- no restriction on relative angular momenta

Basis States

- Intrinsic states are projected on parity and angular momentum

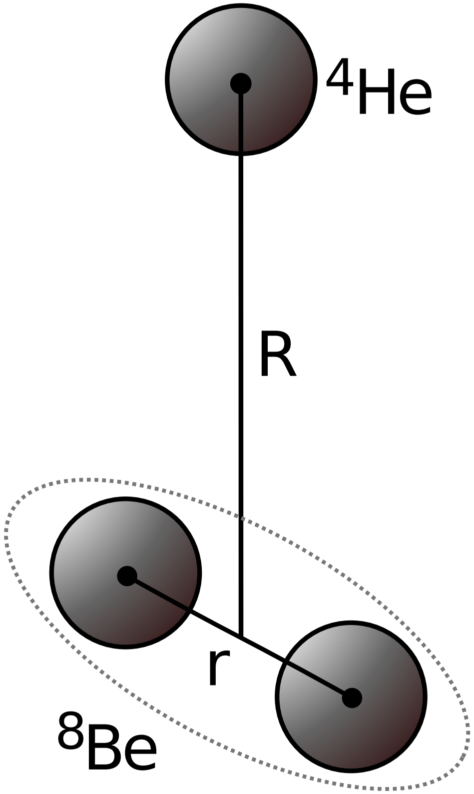
$$\left| \Psi^{3\alpha}_{JMK\pi}(\mathbf{R}_1, \mathbf{R}_2, \mathbf{R}_3) \right\rangle = \underset{\sim}{P}^{\pi} \underset{\sim}{P}^{J}_{MK} \underset{\sim}{\mathcal{A}} \left\{ \left| \Psi^{^4\mathrm{He}}(\mathbf{R}_1) \right\rangle \otimes \left| \Psi^{^4\mathrm{He}}(\mathbf{R}_2) \right\rangle \otimes \left| \Psi^{^4\mathrm{He}}(\mathbf{R}_3) \right\rangle \right\}$$

Volkov Interaction

- simple central interaction
- parameters adjusted to give reasonable α binding energy and radius, $\alpha - \alpha$ scattering data, adjusted to reproduce ^{12}C ground state energy

✘ only reasonable for ^{4}He, ^{8}Be and ^{12}C nuclei

Model space in external region



Model Space

- ^{8}Be-^{4}He cluster configurations with generator coordinate R
- ^{8}Be ground state (0^+_1) and pseudo states (2^+_1, 0^+_2, 2^+_2, 4^+_1) obtained by diagonalizing α-α configurations up to $r = 10\,\mathrm{fm}$

Basis States

- ^{12}C basis states are obtained by **double projection**:

Project first ^{8}Be

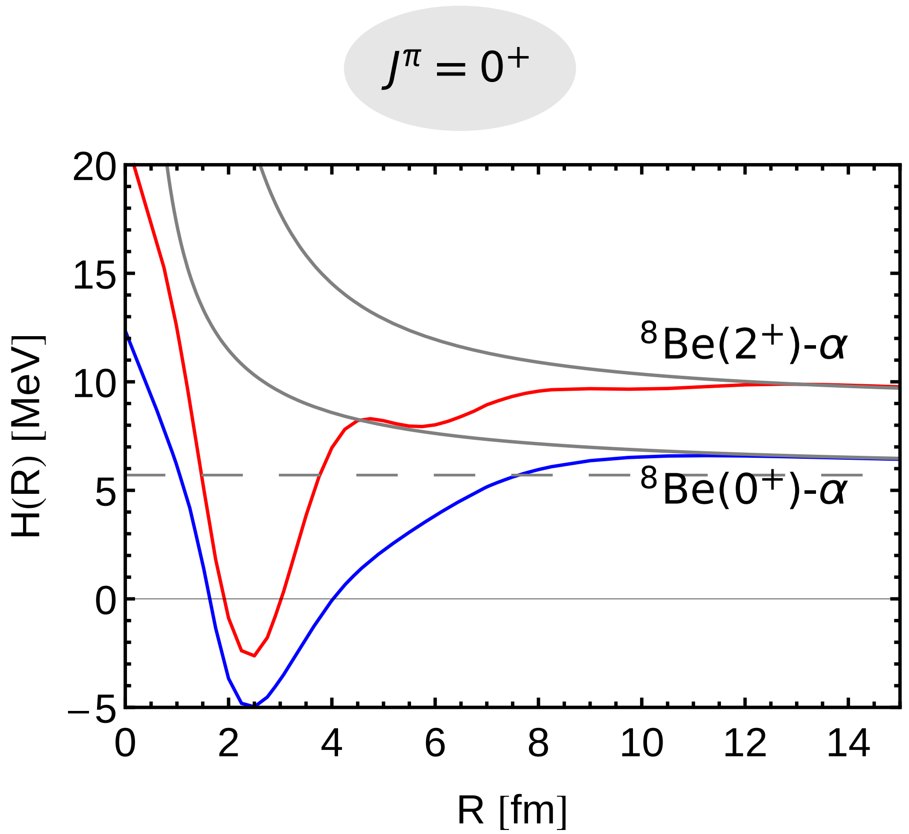
$$\left| \Psi^{^8\mathrm{Be}}_{IK} \right\rangle = \sum_i \underset{\sim}{P}^{I}_{K0} \underset{\sim}{\mathcal{A}} \left\{ \left| \Psi^{^4\mathrm{He}}(-\tfrac{r_i}{2}\mathbf{e}_z \right\rangle \otimes \left| \Psi^{^4\mathrm{He}}(+\tfrac{r_i}{2}\mathbf{e}_z \right\rangle \right\} c^I_i$$

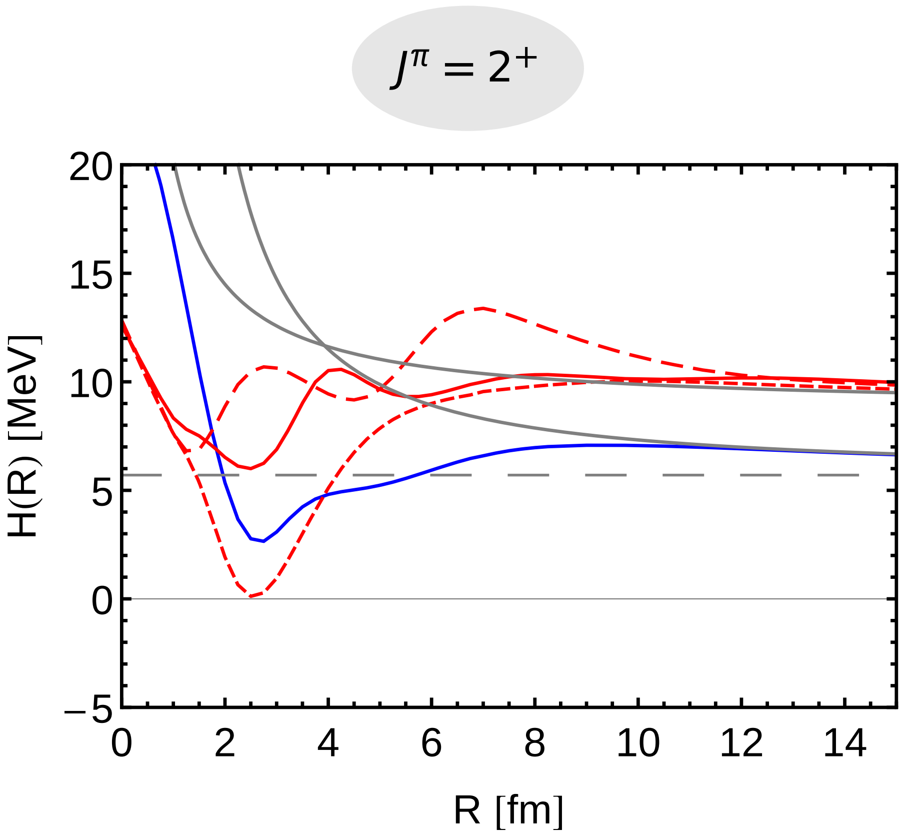
then the combined wave function

$$\left| \Psi^{^8\mathrm{Be},^4\mathrm{He}}_{IK;JM\pi}(R_j) \right\rangle = \underset{\sim}{P}^{\pi} \underset{\sim}{P}^{J}_{MK} \underset{\sim}{\mathcal{A}} \left\{ \left| \Psi^{^8\mathrm{Be}}_{IK}(-\tfrac{R_j}{3}\mathbf{e}_z) \right\rangle \otimes \left| \Psi^{^4\mathrm{He}}(+\tfrac{2R_j}{3}\mathbf{e}_z \right\rangle \right\}$$

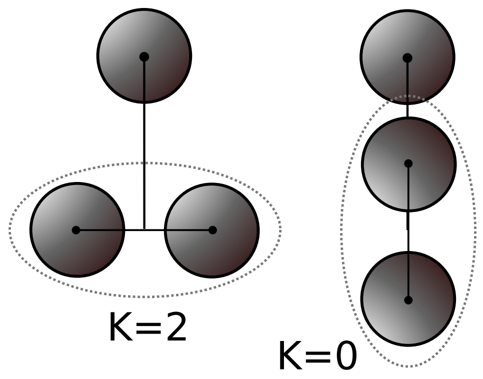
- will allow to match to Coulomb asymptotics

^{8}Be-α Energy Surfaces





- energy surfaces contain localization energy for relative motion of ^{8}Be and α
- 2^+ energy surface depends strongly on orientation of ^{8}Be 2^+ state: $K = 2$ most attractive



Bound state approximation – Convergence ?

	$\rho < 6\,\mathrm{fm}$	$\rho < 6\,\mathrm{fm}$, $R < 9\,\mathrm{fm}$	$\rho < 6\,\mathrm{fm}$, $R < 12\,\mathrm{fm}$	$\rho < 6\,\mathrm{fm}$, $R < 15\,\mathrm{fm}$	Experiment	
$E(0^+_1)$	-89.63	-89.64	-89.64	-89.64	-92.16	
$E^*(2^+_1)$	2.53	2.54	2.54	2.54	4.44	
$E^*(0^+_2)$, $\Gamma_\alpha(0^+_2)$	8.53	7.82	7.78	7.76	7.65, $(8.5 \pm 1.0)10^{-6}$	
$E^*(2^+_2)$, $\Gamma_\alpha(2^+_2)$	10.11	9.18	9.08	8.93	10.13(5), $2.08^{+0.33}_{-0.26}$	[3]
$r_{\text{charge}}(0^+_1)$	2.53	2.53	2.53	2.53	2.47(2)	
$r(0^+_1)$	2.39	2.39	2.39	2.39	–	
$r(0^+_2)$	3.21	3.68	3.78	3.89	–	
$B(E2, 2^+_1 \to 0^+_1)$	9.03	9.12	9.08	9.08	7.6(4)	
$M(E0, 0^+_1 \to 0^+_2)$	7.20	6.55	6.40	6.27	5.47(9)	[2]
$B(E2, 2^+_2 \to 0^+_1)$	3.65	2.48	2.09	1.33	$1.57^{+0.14}_{-0.11}$	[3]

- properties of resonances (Hoyle state and second 2^+ state) can not be determined in bound state approximation in an unambigouos way

[1] Ajzenberg-Selove, Nuc. Phys. **A506**, 1 (1990)
[2] Chernykh et al., Phys. Rev. Lett. **105**, 022501 (2010)
[3] Zimmermann et al., Phys. Rev. Lett. **110**, 152502 (2013); H. Weller, *private communication*

Matching to Coulomb asymptotics

Model Space

- Internal region: 3-α configurations on a grid
- External region: ^{8}Be$(0^+, 2^+, 4^+)$-α configurations
- Asymptotically: only Coulomb interaction between ^{8}Be and ^{4}He clusters

GCM basis state expressed in RGM basis

- Microscopic GCM wave functions are functions of single-particle coordinates: internal wave functions of cluster, the relative motion of the clusters and the total center-of-mass motion are entangled
- Write GCM basis state in external region with RGM basis states

$$\left| \Psi^{^8\mathrm{Be},^4\mathrm{He}}_{IK;JM\pi}(R_j) \right\rangle = \sum_L \left\langle \begin{matrix} I & L \\ K & 0 \end{matrix} \middle| \begin{matrix} J \\ K \end{matrix} \right\rangle \int dr r^2 \, \Gamma_L(R_j; r) \left| \Phi^{^8\mathrm{Be},^4\mathrm{He}}_{(IL)JM\pi}(r) \right\rangle \otimes \left| \Phi^{\mathrm{cm}} \right\rangle$$

with $(\pi = (-1)^L)$

$$\left\langle \boldsymbol{\rho}, \xi_a, \xi_b \middle| \Phi^{^8\mathrm{Be},^4\mathrm{He}}_{(IL)JM\pi}(r) \right\rangle = \sum_{M_I, M_L} \left\langle \begin{matrix} I & L \\ M_I & M_L \end{matrix} \middle| \begin{matrix} J \\ M \end{matrix} \right\rangle \underset{\sim}{\mathcal{A}} \left\{ \frac{\delta(\rho - r)}{r^2} \, \Phi^{^8\mathrm{Be}}_{IM_I}(\xi_a) \, \Phi^{^4\mathrm{He}}(\xi_b) \, Y_{LM_L}(\hat{\rho}) \right\}$$

- asymptotically RGM states have good channel spin I and orbital angular momentum L

Matching to Coulomb asymptotics

RGM norm kernel and Overlap functions

- RGM norm kernel reflects effects of antisymmetrization, channel $c = (IL)J$

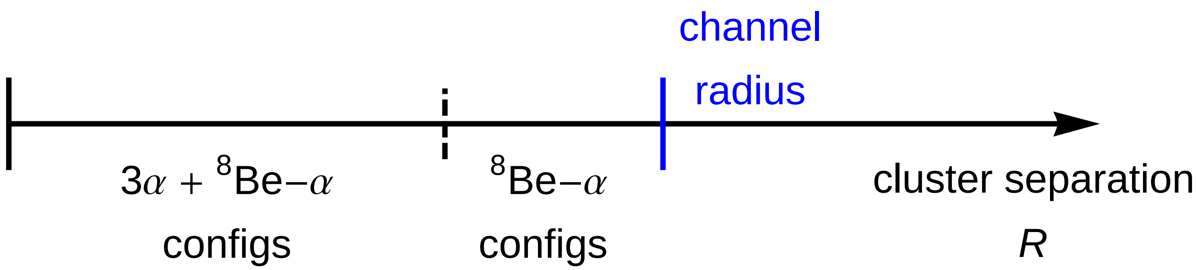
$$N_{c,c'}(r,r') = \langle\, \Phi_c(r) \,|\, \Phi_{c'}(r') \,\rangle \xrightarrow{r,r'\to\infty} \delta_{cc'} \frac{\delta(r-r')}{rr'}$$

- Overlap functions can be interpreted as wave functions for point-like clusters

$$\psi_c(r) = \int dr' r'^2\, N^{-1/2}_{c,c'}(r,r') \langle\, \Phi_{c'}(r') \,|\, \Psi \,\rangle$$

Matching to the asymptotic solution

- Use using multichannel microscopic R-matrix approach
  Descouvemont, Baye, Phys. Rept. 73, 036301 (2010)
- Check that results are independent from channel radius: used $a = 16.5$ fm here



Matching to Coulomb asymptotics

Bound states

- Whittaker functions

$$\psi_c(r) = A_c \frac{1}{r} W_{-\eta_c, L_c+1/2}(2\kappa_c r), \qquad \kappa_c = \sqrt{-2\mu(E-E_c)}$$

Resonances

- purely outgoing Coulomb, k complex

$$\psi_c(r) = A_c \frac{1}{r} O_{L_c}(\eta_c, k_c r), \qquad k_c = \sqrt{2\mu(E-E_c)}$$

Scattering states

- in- and outgoing Coulomb (incoming channel c_0)

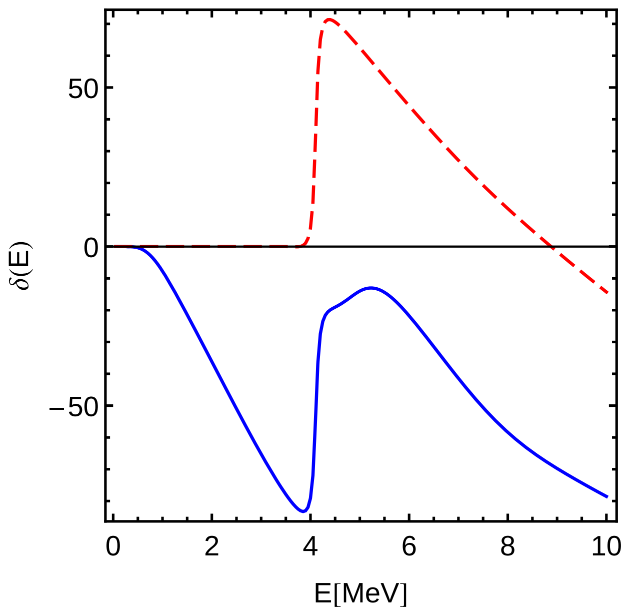
$$\psi_c(r) = \frac{1}{r} \left\{ \delta_{L_c,L_0} I_{L_c}(\eta_c, k_c r) - S_{c,c_0} O_{L_c}(\eta_c, k_c r) \right\}, \qquad k_c = \sqrt{2\mu(E-E_c)}$$

- Diagonal phase shifts and inelasticity parameters: $S_{cc} = \eta_c \exp\{2i\delta_c\}$
- Eigenphases: $S = U^{-1} D U, D_{\alpha\alpha} = \exp\{2i\delta_\alpha\}$

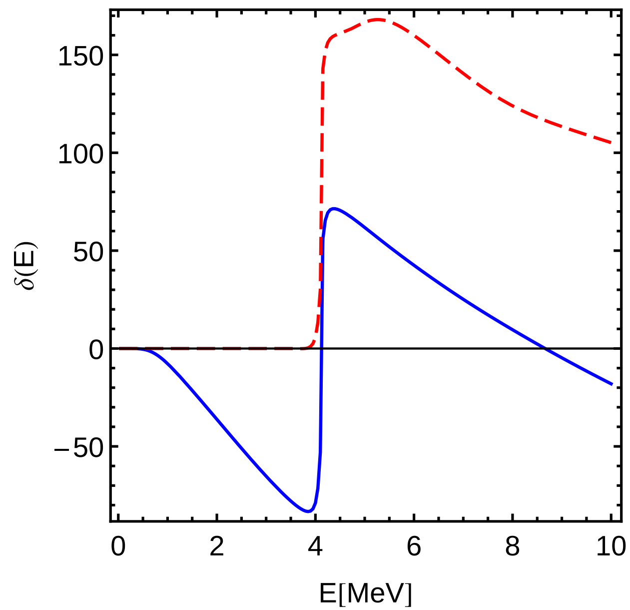
Cluster Model: ^{8}Be(0_1^+,2_1^+)-α Continuum

0^+ Phase shifts

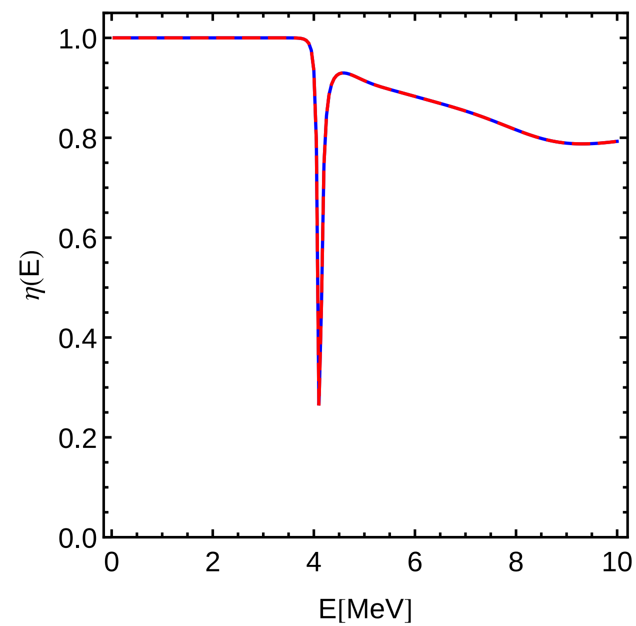
Eigenphaseshifts



Phaseshifts



Inelasticities



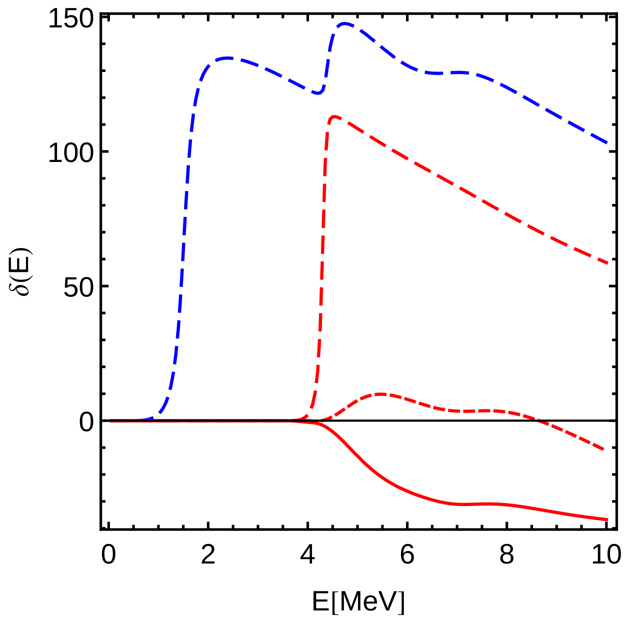
Gamow states

	E [MeV]	Γ_α [MeV]	
0_2^+	0.29	$1.78 \cdot 10^{-5}$	
0_3^+	4.11	0.12	
0_4^+	4.76	1.57	(?)

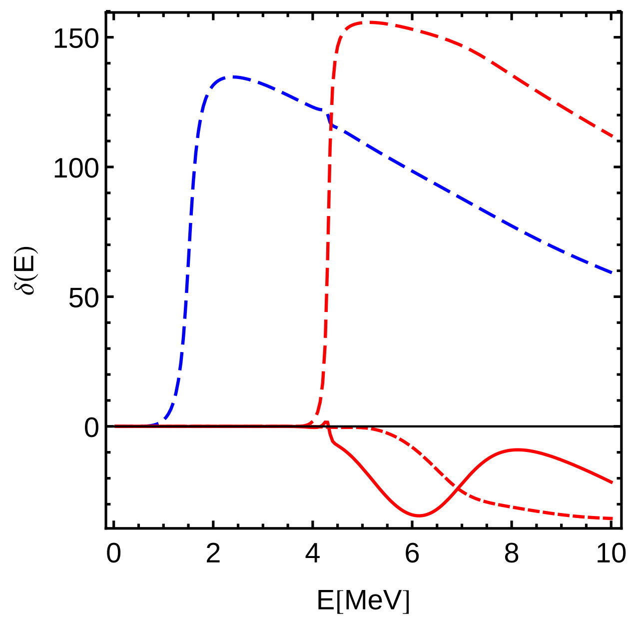
- non-resonant background
- strong coupling between ^{8}Be(0^+) and ^{8}Be(2^+) channel at 4.1 MeV
- Hoyle state missed when scanning the phase shifts
- stability of broad resonance with respect to channel radius ?

Cluster Model: ^{8}Be(0^+_1,2^+_1)-α Continuum
2^+ Phase shifts

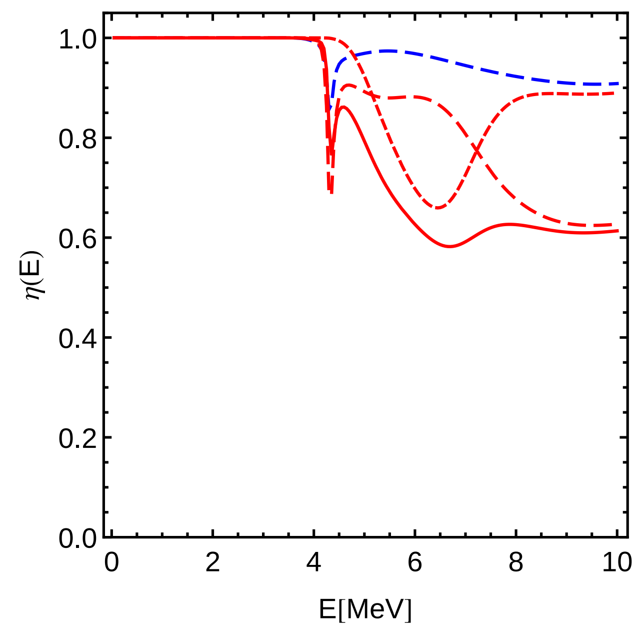
Eigenphaseshifts



Phaseshifts



Inelasticities



Gamow states

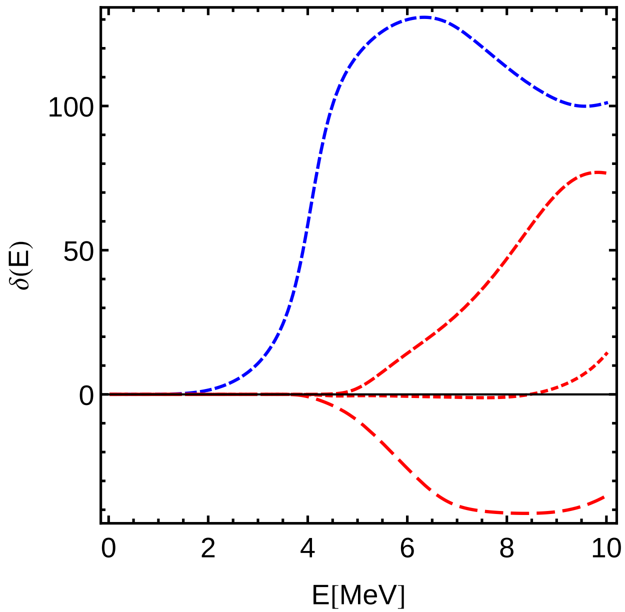
	E [MeV]	Γ_α [MeV]
2^+_2	1.51	0.32
2^+_3	4.31	0.14
. . .		

- non-resonant background
- $L = 2$ ^{8}Be(0^+) and ^{8}Be(2^+) resonances

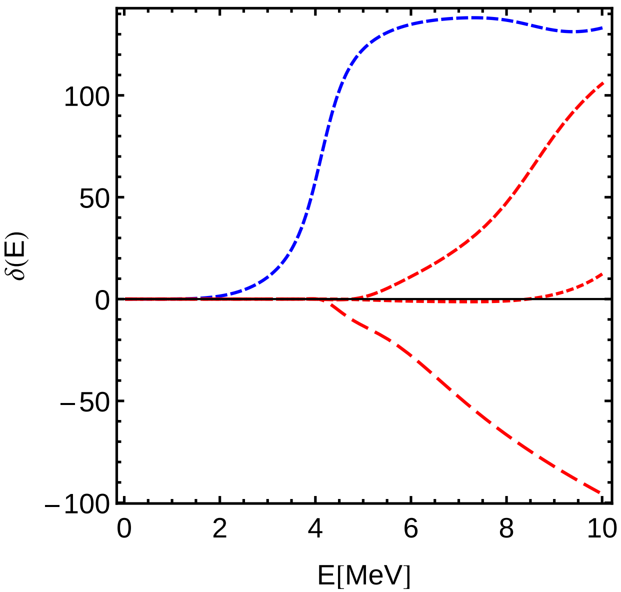
Cluster Model: ^{8}Be(0^+_1,2^+_1)-α Continuum

4^+ Phase shifts

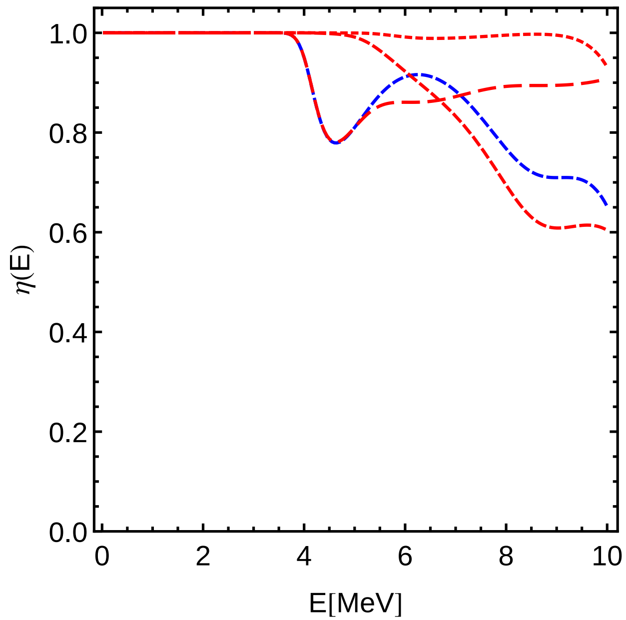
Eigenphaseshifts



Phaseshifts



Inelasticities



Gamow states

	E [MeV]	Γ_α [MeV]
4^+_1	1.17	$8.07{\cdot}10^{-6}$
4^+_2	4.06	0.98
...		

- 4_1 state (ground state band) very narrow, missed when scanning phase shifts
- 4^+_2 state mostly ^{8}Be(0+) but some mixing

Including Continuum

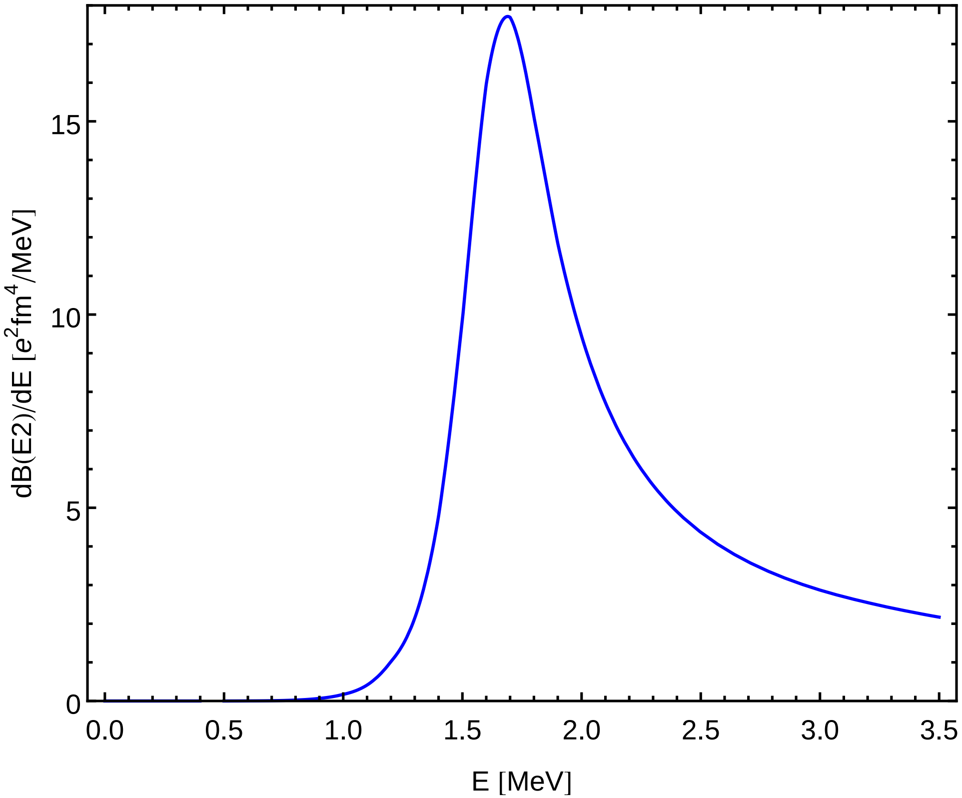
	$\rho < 6\,\text{fm}$ $R < 9\,\text{fm}$	$\rho < 6\,\text{fm}$ $R < 12\,\text{fm}$	$\rho < 6\,\text{fm}$ $R < 15\,\text{fm}$	$\rho < 6\,\text{fm}$ Continuum	Experiment
$E(0_1^+)$	-89.64	-89.64	-89.64	-89.64	-92.16
$E^*(2_1^+)$	2.54	2.54	2.54	2.54	4.44
$E^*(0_2^+)$, $\Gamma_\alpha(0_2^+)$	7.82	7.78	7.76	7.76, $3.04 \cdot 10^{-3}$	7.65, $(8.5 \pm 1.0) \cdot 10^{-6}$
$E^*(2_2^+)$, $\Gamma_\alpha(2_2^+)$	9.18	9.08	8.93	8.98, 0.46	10.13(5), $2.08^{+0.33}_{-0.26}$
$r_{\text{charge}}(0_1^+)$	2.53	2.53	2.53	2.53	2.47(2)
$r(0_1^+)$	2.39	2.39	2.39	2.39	–
$r(0_2^+)$	3.68	3.78	3.89	4.08 + 0.07i	–
$B(E2, 2_1^+ \to 0_1^+)$	9.12	9.08	9.08	9.08	7.6(4)
$M(E0, 0_1^+ \to 0_2^+)$	6.55	6.40	6.27	6.15 + 0.01i	5.47(9)
$B(E2, 2_2^+ \to 0_1^+)$	2.48	2.09	1.33	2.14 + 1.45i	$1.57^{+0.14}_{-0.11}$

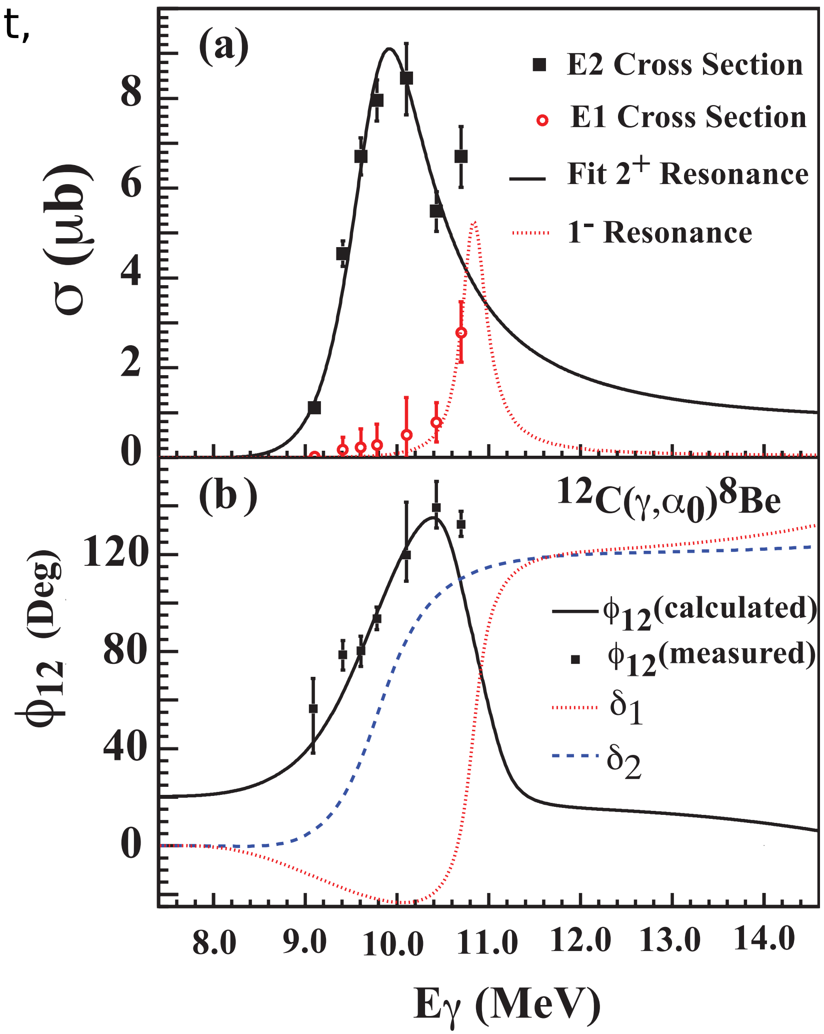
- Resonances are calculated as Gamow states
- Matrix elements including resonances are regulated according to Berggren and Gyarmati
- Imaginary part provides information about uncertainty of matrix elements

Berggren, Nucl. Phys. **A109**, 265 (1968)
Gyarmati, Krisztinkovics, Vertse, Phys. Lett. **B41**, 475 (1972)
Berggren, Phys. Lett. **B373**, 1 (1996)

Strength distributions

- Use real continuum (scattering states)
- Might be the better way to compare to experiment, especially for broad and overlapping resonances (background contributions)





Zimmermann et al.,
Phys. Rev. Lett. **110**, 152502 (2013)

✘ $E1$ transition isospin-forbidden in cluster model !

Work in Progress:
FMD calculations with ^{8}Be-α continuum

UCOM interaction

- **AV18 UCOM(SRG) (α=0.20 fm^4, λ=1.5 fm^{-1})**
- **Increase strength of spin-orbit force by a factor of two to partially account for omitted three-body forces**

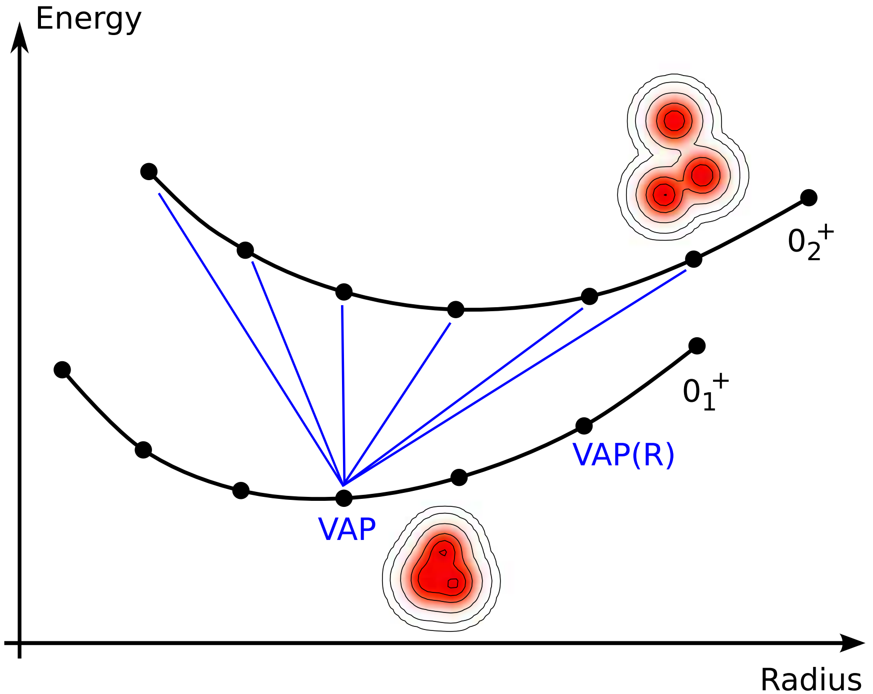
^{8}Be-α Continuum

- **To get a reasonable description of ^{8}Be it is essential to include polarized configurations**

➽ **Calculate strength distributions**

➽ **Investigate non-cluster states: non-natural parity states, $T = 1$ states, M1 transitions, ^{12}B and ^{12}N β-decay into ^{12}C, . . .**

Model space in internal region



Model Space

- no assumption of α-clustering
- complete basis not feasible, find the "most important" basis states
- determine wave packet parameters by variation

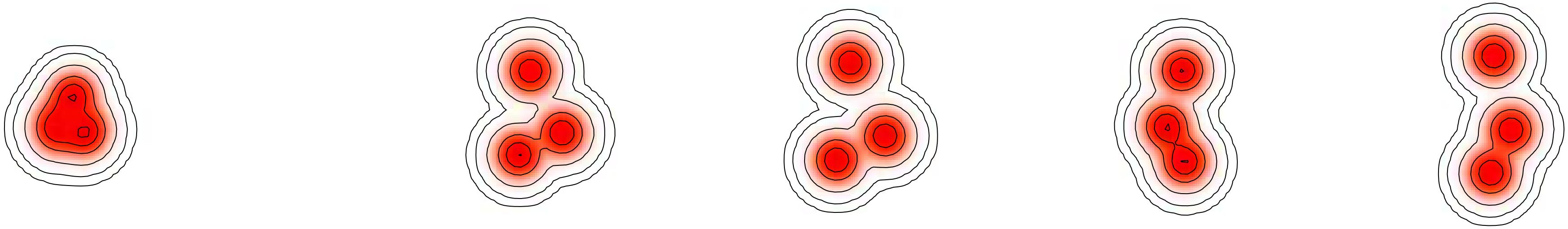
VAP, VAP with constraints, Multiconfiguration-VAP

For each angular momentum $(0^+, 1^+, 2^+, \ldots)$

- **VAP**: vary energy of projected Slater determinant $\underset{\sim}{P}^{\pi}\underset{\sim}{P}^{J}_{MK}\left| Q(q_i) \right\rangle$ with respect to all parameters q_i
- **VAP(R)**: create additional basis states by variation with a constraint on the radius of the intrinsic state
- **MC-VAP**: keep VAP state fix and vary the parameters of a second Slater determinant to minimize the energy of the second eigenstate in a multiconfiguration mixing calculation
- **MC-VAP(R)**: create additional basis states by adding a constraint on the radius of the second intrinsic state

Important Configurations

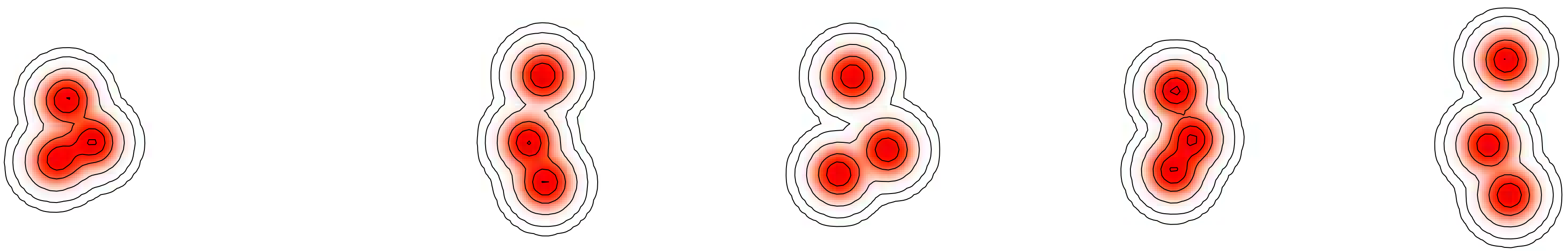
- Calculate the overlap with FMD basis states to find the most important contributions to the eigenstates



$\left|\langle \cdot \,|\, 0_1^+ \rangle\right| = 0.94$

$\left|\langle \cdot \,|\, 2_1^+ \rangle\right| = 0.93$

$\left|\langle \cdot \,|\, 0_2^+ \rangle\right| = 0.64$ $\left|\langle \cdot \,|\, 0_2^+ \rangle\right| = 0.58$ $\left|\langle \cdot \,|\, 0_2^+ \rangle\right| = 0.57$ $\left|\langle \cdot \,|\, 0_2^+ \rangle\right| = 0.45$

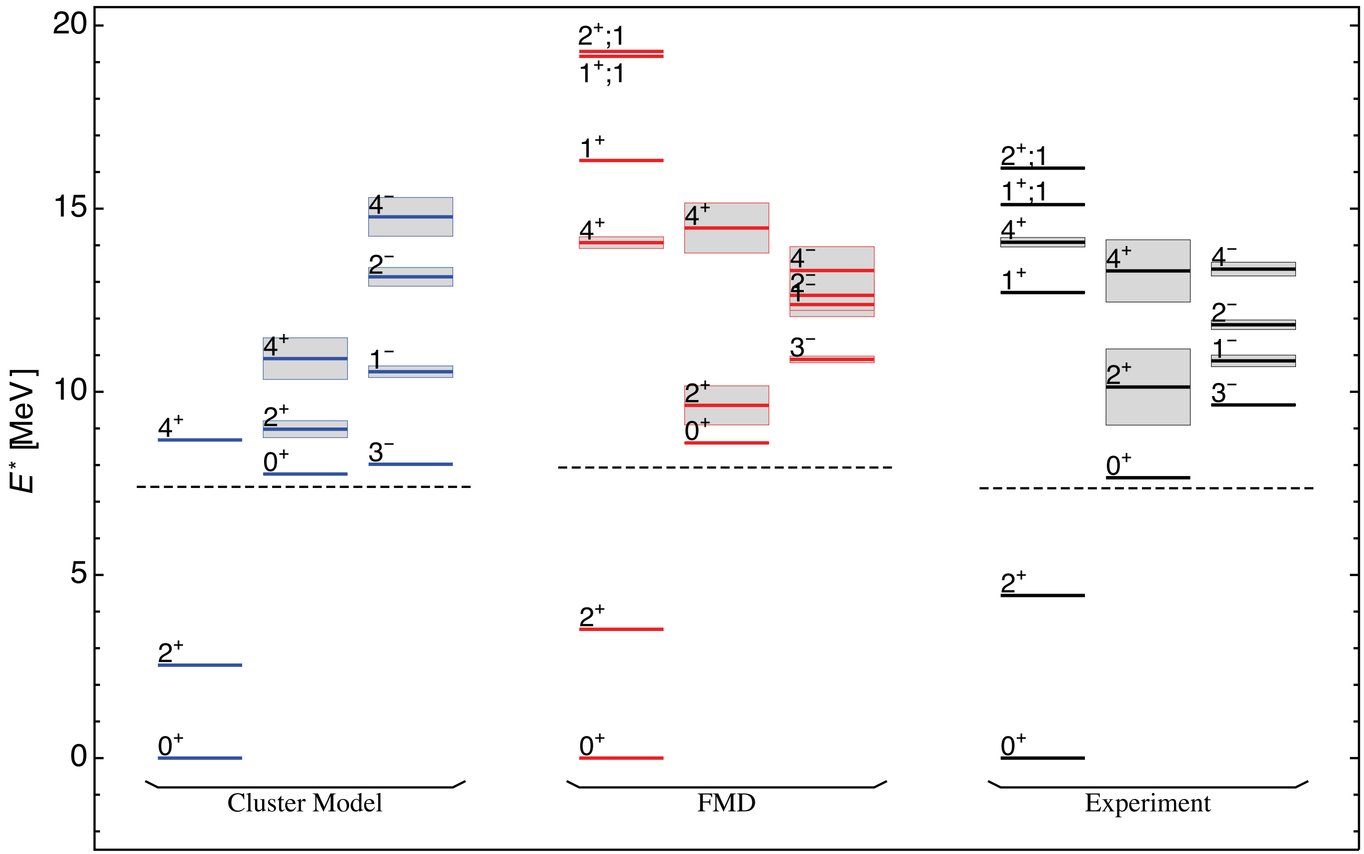


$\left|\langle \cdot \,|\, 3_1^- \rangle\right| = 0.91$ $\left|\langle \cdot \,|\, 2_2^+ \rangle\right| = 0.50$ $\left|\langle \cdot \,|\, 2_2^+ \rangle\right| = 0.49$ $\left|\langle \cdot \,|\, 2_2^+ \rangle\right| = 0.44$ $\left|\langle \cdot \,|\, 2_2^+ \rangle\right| = 0.41$

FMD basis states are not orthogonal!

0_2^+ and 2_2^+ states have no rigid intrinsic structure

Spectrum



- FMD provides a reasonable description of the ground state band, the cluster states related to the Hoyle state and the negative parity states
- Spin-flip states (1^+ $T = 0, 1$ and 2^+ $T = 1$) appear to be reasonably well described although they are somewhat too high in energy

Summary

Unitary Correlation Operator Method

- Explicit description of short-range central and tensor correlations

Fermionic Molecular Dynamics

- Gaussian wave-packet basis contains HO shell model and Brink-type cluster states

^{3}He(α,γ)^{7}Be Radiative Capture

- Bound states, scattering states, transitions from the continuum

Microscopic cluster model for ^{12}C

- Model space with 3 α and ^{8}Be-α configurations
- Matching with Coulomb continuum, resonances and scattering states
- Hoyle state band build on ^{8}Be(gs)-α

FMD calculations for ^{12}C

- VAP and Multiconfig-VAP in internal region, ^{8}Be-α in external region

➽ Investigate EM and GT transitions to the continuum

➽ ^{8}Be-α vs real three-body asymptotics ?