Modelling Light exotic nuclei in Fermionic Molecular Dynamics

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Definition: Exotic nuclei

Nuclei which are either sufficiently proton-neutron-rich to lie far out of the "valley of stability" on the Segré plot.

Aim:

To model the structure of light exotic nuclei in order to calculate stellar nucleosynthetic reaction rates and to help elucidate how changing the proton:neutron (p:n) ratio affects nucleon distribution and nuclear structure.

Why exotic nuclei?

- Affect reaction rates in stellar nucleosynthesis.
- A "skew" p:n ratio will bring in different correlations and therefore new structure phenomena.

Structure phenomena of exotic nuclei:

- Haloes (a nucleon with more than 50% probability to be outside the core is a halo nucleon).
- Deformation.
- Cluster structures.

Exotic nuclei and Astrophysics applications:

Light exotic nuclei are important for nuclear astrophysics, mainly because they are intermediates in stellar nucleosynthesis processes. Their structure (in S-factors) is thus important input into calculations of nucleosynthetic reaction rates. Modelling the structure of light exotics is also important for testing interactions for asymmetric nuclear matter (which in turn may be used to obtain neutron-star equations of state).

Modelling exotic nuclear structure:

Challenges which affect modelling exotic nuclei: extended nucleon distributions and cluster configurations all require a very large model-space. Clustering behaviour is often observed alongside "shell-model" type behaviour, so a model should treat both on an equal footing. Asymptotic behaviour (the "tail" of the wavefunction) should also be modelled correctly to model extended distributions accurately (it should drop off exponentially).

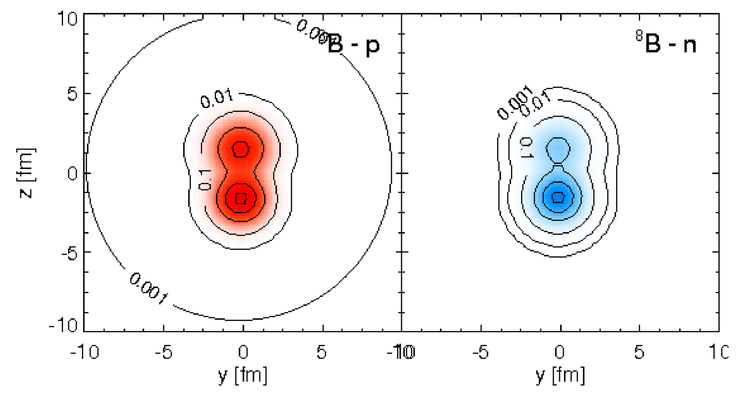

Figure: A plot of proton and neutron densities for an FMD state in ^{8}B.

Fermionic Molecular Dynamics (FMD)

This is a microscopic, time-dependent model of nuclear matter, using nucleon degrees of freedom. The many-body states are Slater determinants
$|Q(t)\rangle = \hat{\mathcal{A}}\{|q_1\rangle \otimes |q_2\rangle ... \otimes |q_A\rangle\}$,
and the single-particle states are Gaussians in phase space,
i.e. $\langle x|q\rangle = exp\{(\overline{x} - \overline{b})^2/(2a)\}$
Many-body states are created by minimising the expectation value of the Hamiltonian, subject to one or more constraints, and projecting out eigenstates of good angular momentum *i.e.*.

$$\min_{\{q_\nu\}} \frac{\langle Q|\hat{H} - \hat{T}_{cm}|Q\rangle}{\langle Q|Q\rangle},$$

where q_ν are the properties (*e.g.* width, mean position) of the single-particle states. The projection may be performed either before the minimisation (Variation after Projection (VAP)) or afterwards (Projection after Variation (PAV)). The former is more computationally-expensive, but more accurate in that the state obtained is truly the minimum-energy state.
The projection operator for angular momentum is given by:

$$\hat{P}^J_{MK} = \frac{2J+1}{8\pi^2} \int d\Omega \, {D^J_{MK}}^*(\Omega) \hat{R}(\Omega)$$

In addition to these basis states, clustering may be included explicitly in basis states by making "frozen" states $\mathcal{A}\{|r, l\rangle \otimes |Q_{c1}\rangle \otimes |Q_{c2}\rangle\}$ in which the clustering into clusters $|Q_{c1}\rangle$ and $|Q_{c2}\rangle$ is included explicitly.
The Hamiltonian is diagonalised in a Hilbert space of these Slater determinants and the eigenstates are the allowed states of the nucleus.
The interaction used is a Unitary Correlated Operator Method (UCOM) transformed AV18 Hamiltonian, which reproduces the phase shifts of light nuclei and is "soft" enough for use in a tractable model-space.
A range of observables may then be extracted from these many-body states (*e.g.* radii, quadrupole moments, transition strength). This is for comparing to experiment.
Notable successes of the model include modelling the second 0^+ state of ^{12}C (the famous Hoyle state), and modelling halo structures in the helium isotopes ^{8}He and ^{8}He.

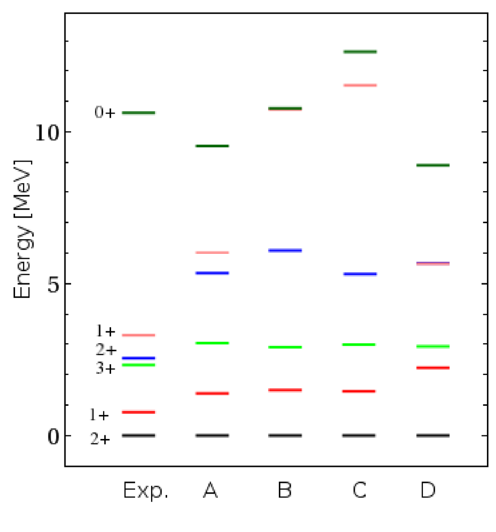

Figure: *A level-scheme for ^{8}B using various FMD Hilbert spaces, compared to the experimental scheme (Exp.).*

Why FMD for exotic nuclei?

- Flexible basis: Shell-model type states and cluster states can be accessed.
- Gaussian single-particle wavefunctions have correct asymptotics.
- Cluster states and extended distributions are accessed without requiring an excessively-large model-space.

Figure: *The study of light exotic nuclei plays an important role in nuclear astrophysics.*

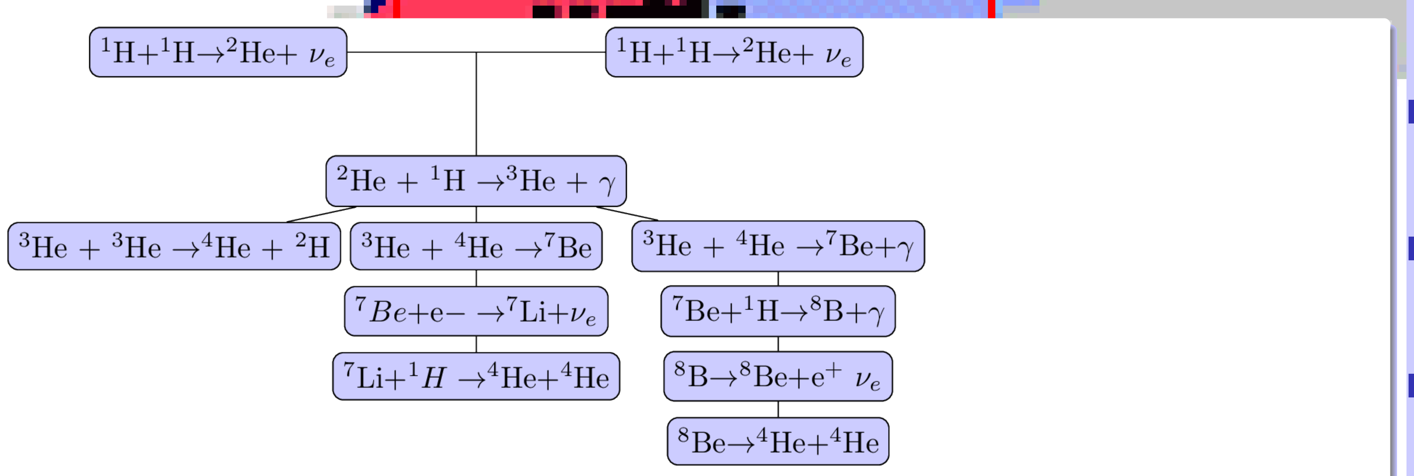

Figure: The reactions of the pp chain of stellar nucleosynthesis, indicating the importance of the light exotic ^{8}B in stellar nucleosynthesis.

Explicit cluster states

- FMD nuclear states may be divided into "internal" and "external" configurations.
- The "internal" states are the Slater determinants $|Q\rangle$.
- The "external" states are joined Slater determinants of clusters.
- These together describe the full range of nuclear structures.
- Adding clustering in explicitly is necessary to model states where the structures are very wide apart or unusually arranged (*e.g.* in the Hoyle state).

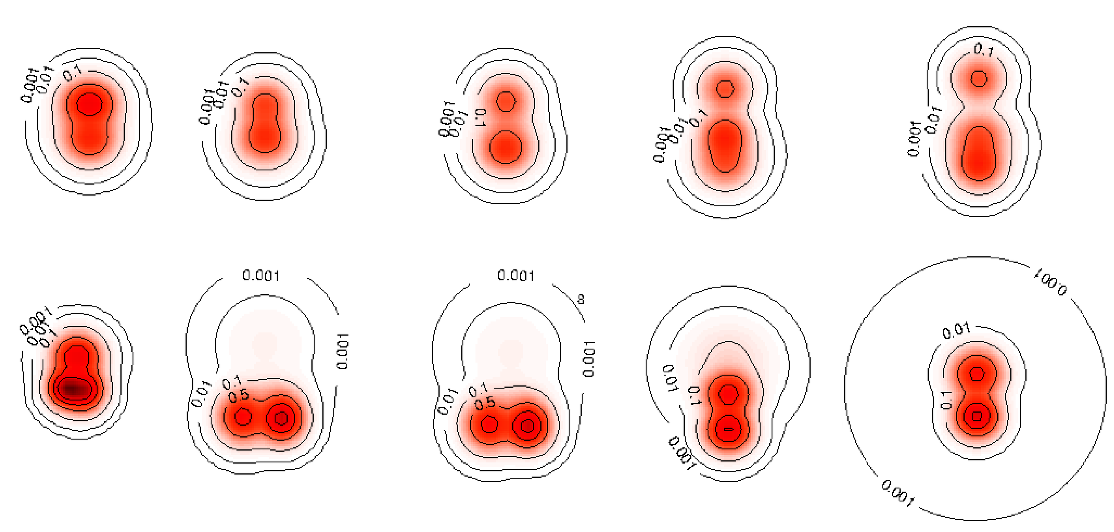

Figure: Density plots of proton density for some FMD states of ^{8}B with constraints on radii.