Hydrodynamic attractors and transport in small systems

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Thanks to Sandra Brandstetter, Lars Heyen, Giuliano Giacalone, and Ilya Selyuzhenkov for helpful input.

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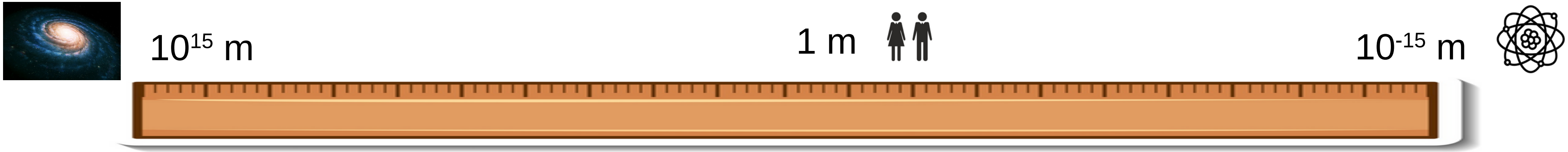
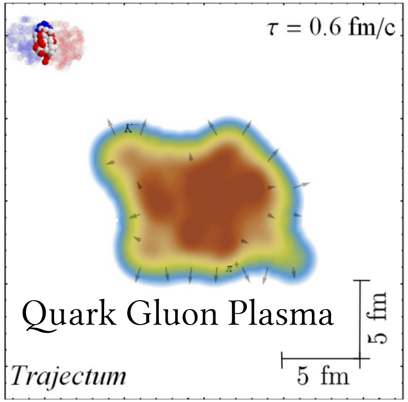
Hydrodynamics - universal effective theory

- Separation of scales (time, distance) between constituents and fields
- (Near) local equilibrium $\longrightarrow$ equation of state, transport coefficients





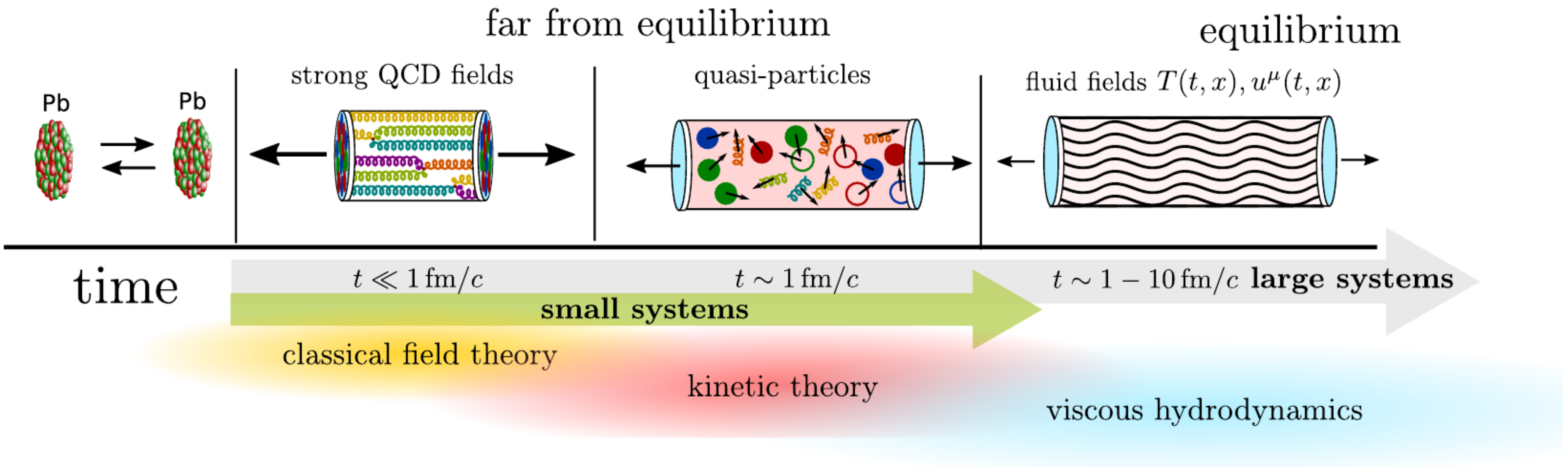
Observation of collectivity in heavy ion collisions challenges hydro paradigm!

See talk by Ilya Selyuzhenkov

QCD thermalisation in heavy-ion collisions

High-energy (weak coupling $\alpha_s \ll 1$) limit

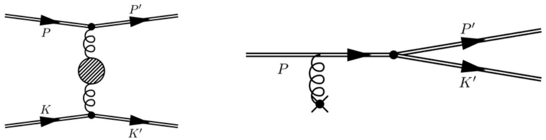
Berges, Heller, AM, Venugopalan RMP (2021)



QCD kinetic theory of quarks and gluons

Arnold, Moore, Yaffe JHEP (2003)

$$\partial_t f(t,\mathbf{x},\mathbf{p}) + \frac{\mathbf{p}}{|p|}\cdot\nabla_{\mathbf{x}} f(t,\mathbf{x},\mathbf{p}) = -\mathcal{C}_{2\leftrightarrow2}[f] - \mathcal{C}_{1\leftrightarrow2}[f]$$



Kurkela, Zhu PRL (2015), Keegan, Kurkela, AM, Teaney JHEP (2016), Kurkela, AM, Paquet, Schlichting and Teaney PRL (2018)

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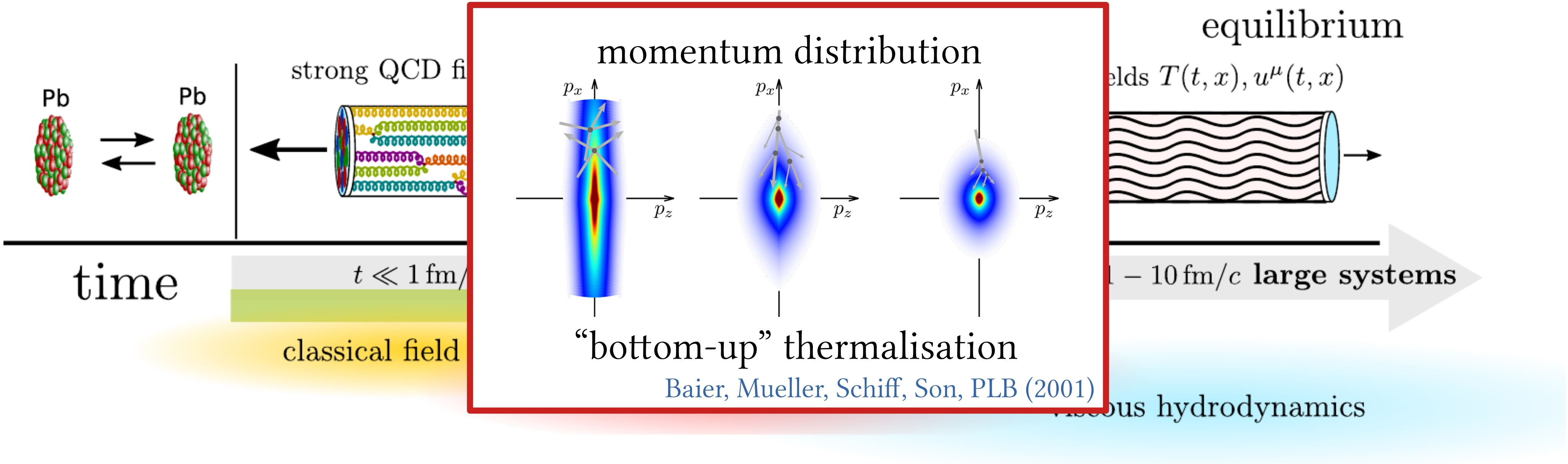

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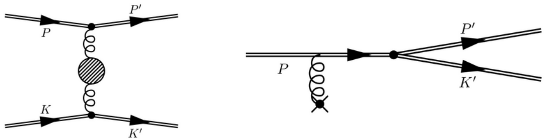

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Hydrodynamic modeling of QGP expansion

Equations of motion

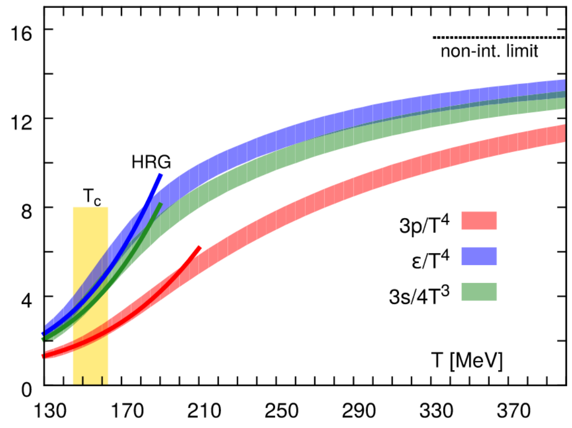
$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = e u^\mu u^\nu + (P + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

Transport coefficients: shear and bulk viscosity

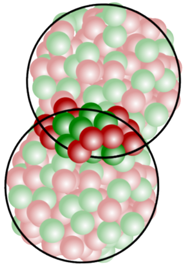
$$\pi^{\mu\nu} \sim \eta \partial^{(\mu} u^{\nu)} \qquad \Pi \sim \zeta \partial_\mu u^\mu$$

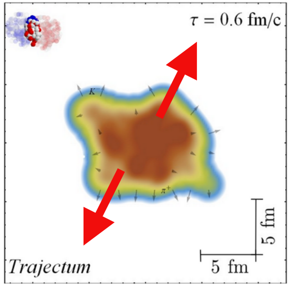
Equation of state (lattice QCD)

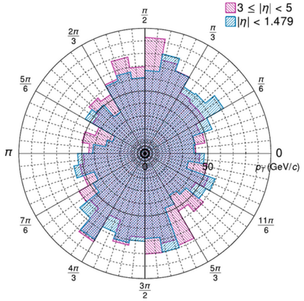


HotQCD PRD (2014)

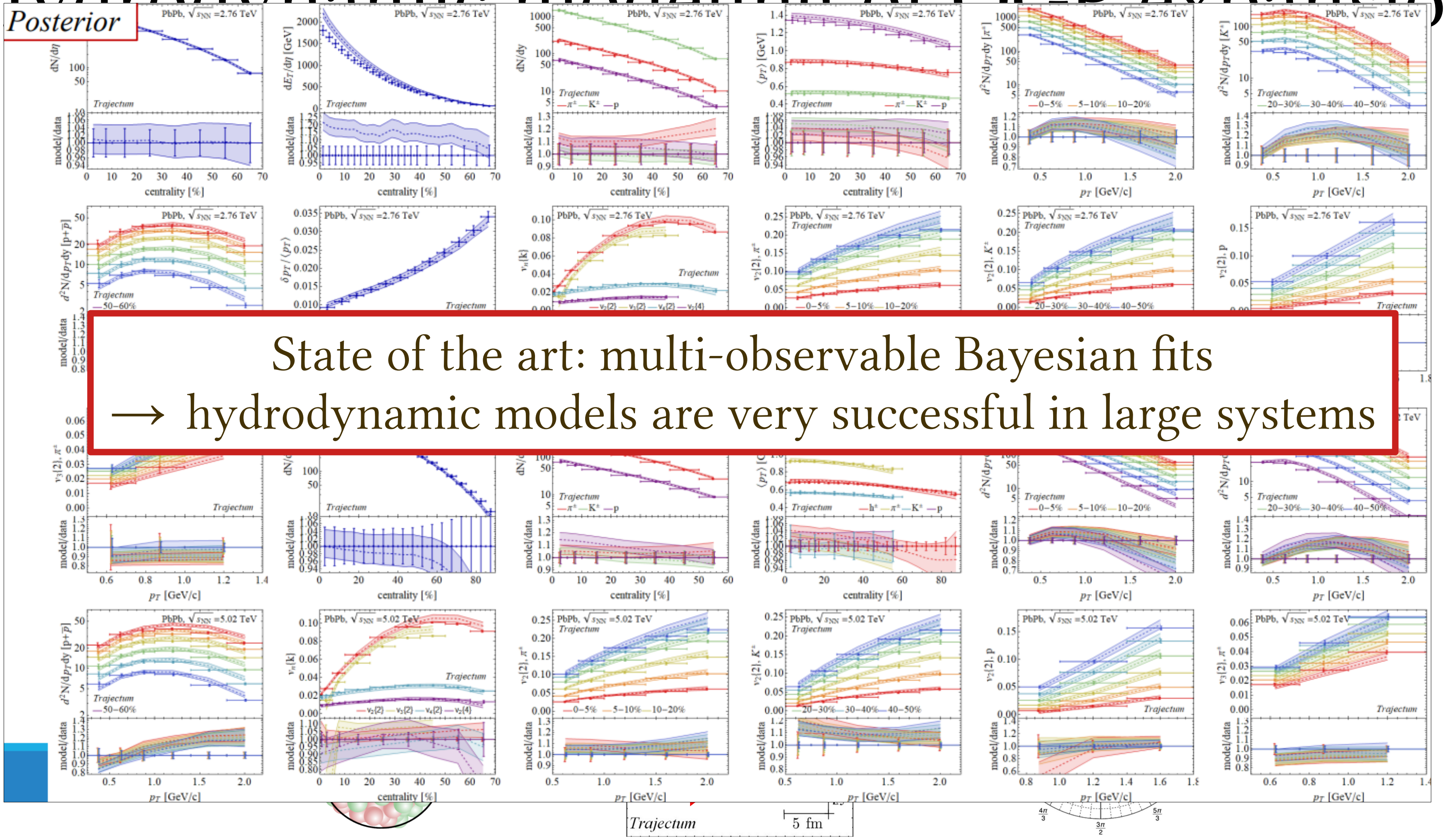
spatial anisotropy ⟶ hydro expansion ⟶ momentum anisotropy







Hydrodynamic modeling of QGP expansion



State of the art: multi-observable Bayesian fits
→ hydrodynamic models are very successful in large systems

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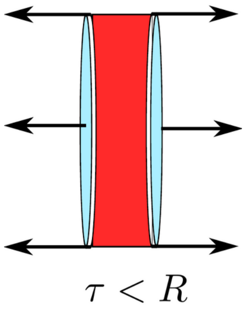
Hydrodynamic attractor



Heller, Spaliński PRL (2015)

Early times in large systems $\longrightarrow$ 1D expansion

$$\frac{\tau \partial_\tau e}{e} = -1 - \frac{T^{zz}}{e} = -\frac{4}{3} + \frac{16}{9}\frac{\eta}{(e+P)\tau} + \ldots$$

viscous correction

$$\tau_R \sim \frac{\eta}{e+P} \qquad \partial_\mu u^\mu = \tau^{-1}$$

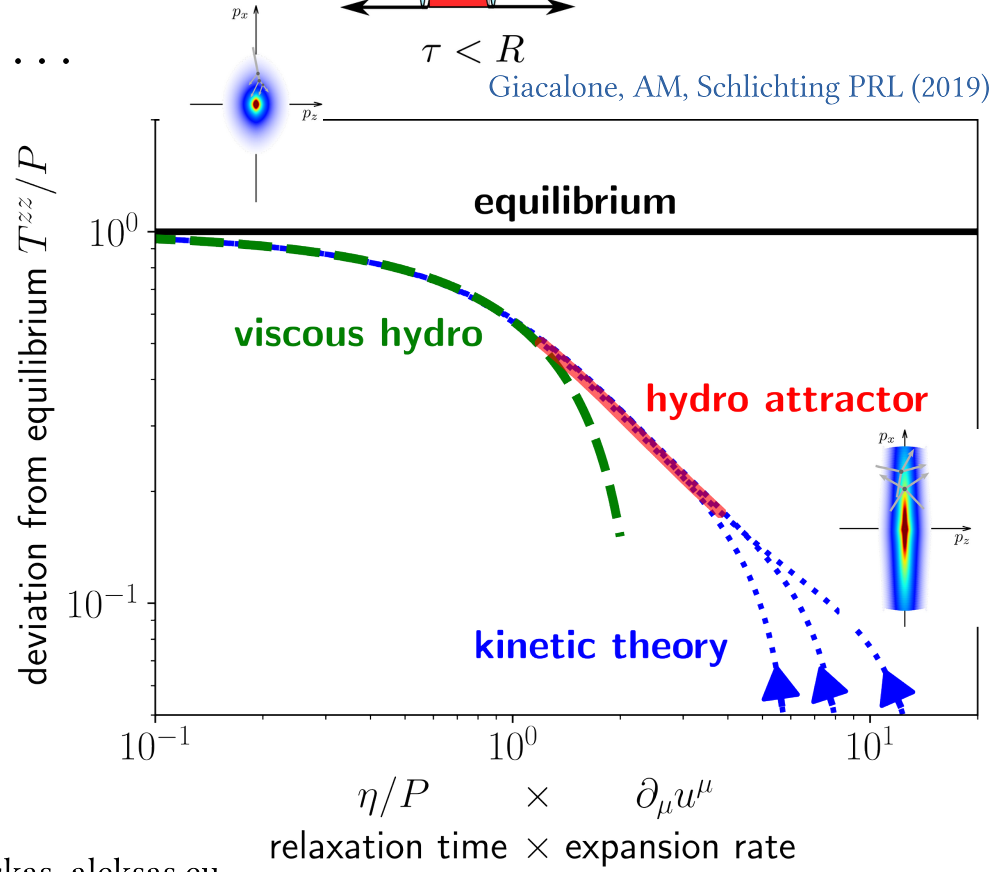
Standard hydro condition: $\tau_R \partial_\mu u^\mu \ll 1$

Microscopic simulations show rapid loss of details $\quad \tau_R \partial_\mu u^\mu \sim 1$

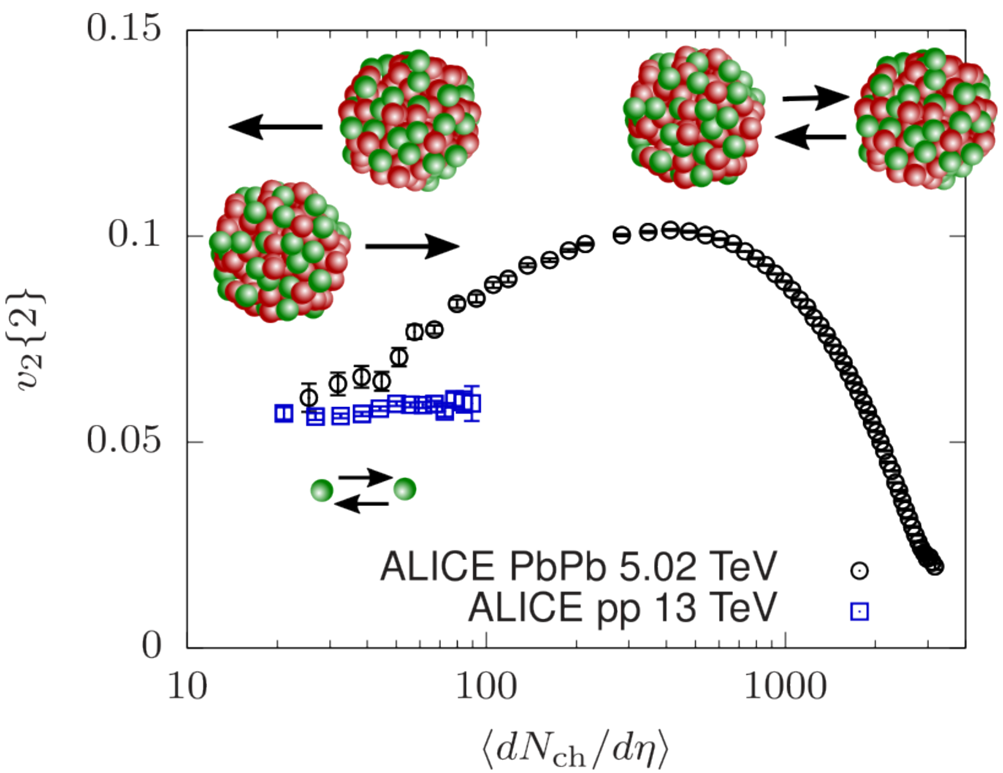
$\longrightarrow$ hydrodynamic attractor

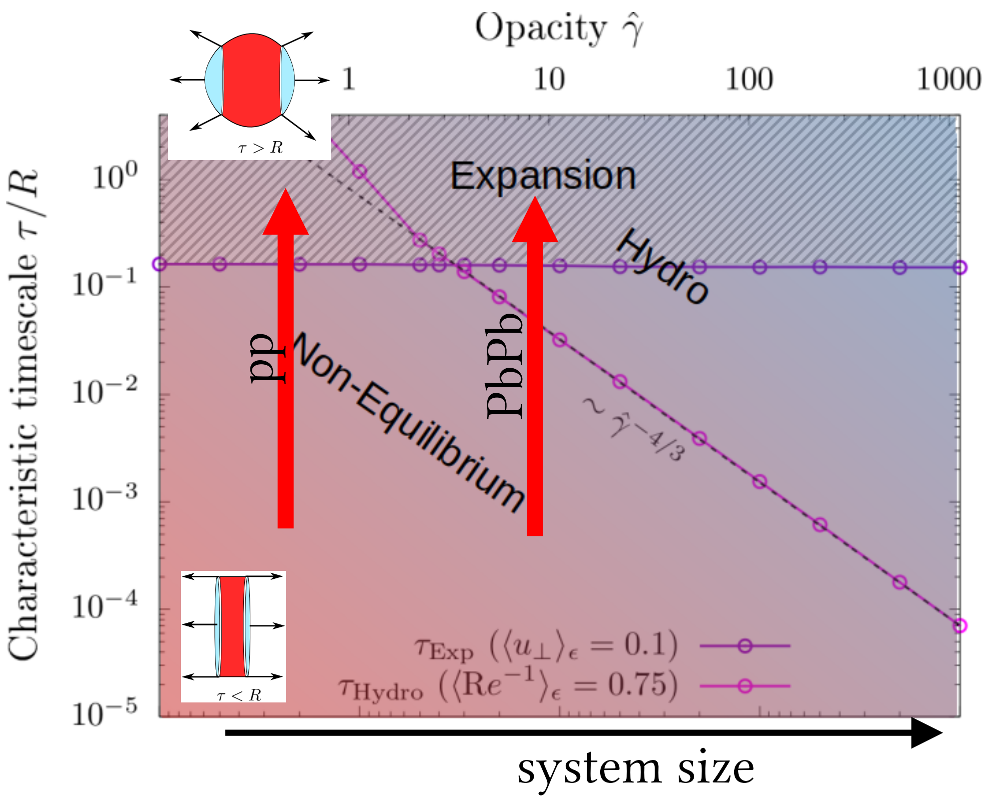
$\longrightarrow$ justification for hydro in large collision systems $\quad \tau > 1\,\mathrm{fm}/c$

Giacalone, AM, Schlichting PRL (2019)



Collectivity in small collision systems





Ambrus,Schlichting, Werthmann PRL (2023)

Small systems live too short to reach hydro phase.
What is the origin of collectivity in small systems?

Collective phenomena with few ultracold atoms

See talk by Sandra Brandstetter

Theory proposal: Flörchinger, Giacalone, Heyen, Tharwat, PRC 105 (2022) 4, 044908
Experimental realisation: Brandstetter, Lunt, Heintze, Giacalone, Heyen, Gałka, Subramanian, Holten, Preiss, Floerchinger, Jochim arXiv:2308.09699

CRC 1225: Isolated quantum systems and universality in extreme conditions
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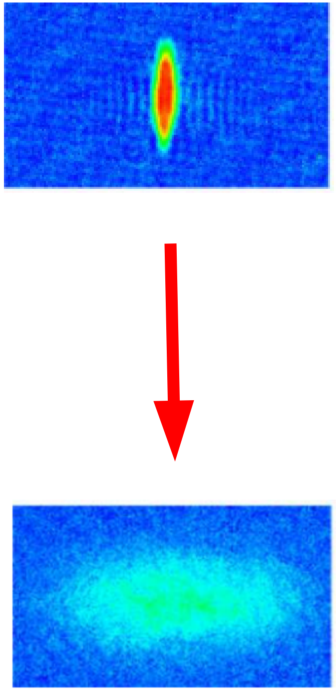
Geometry inversion - signature of fluid flow

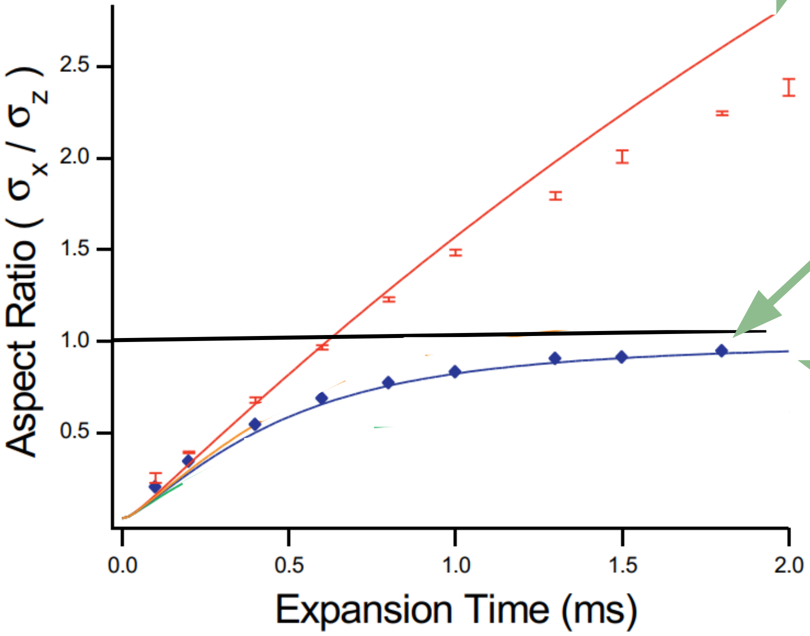
Cold atom experiments with 10^5 Lithium atoms



100 µs

2000 µs

Single measurement



ideal hydrodynamic prediction

strongly-interacting expansion

non-interacting expansion

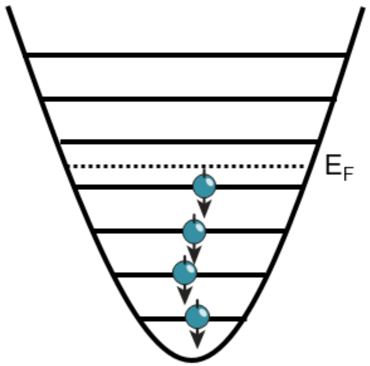
free streaming prediction

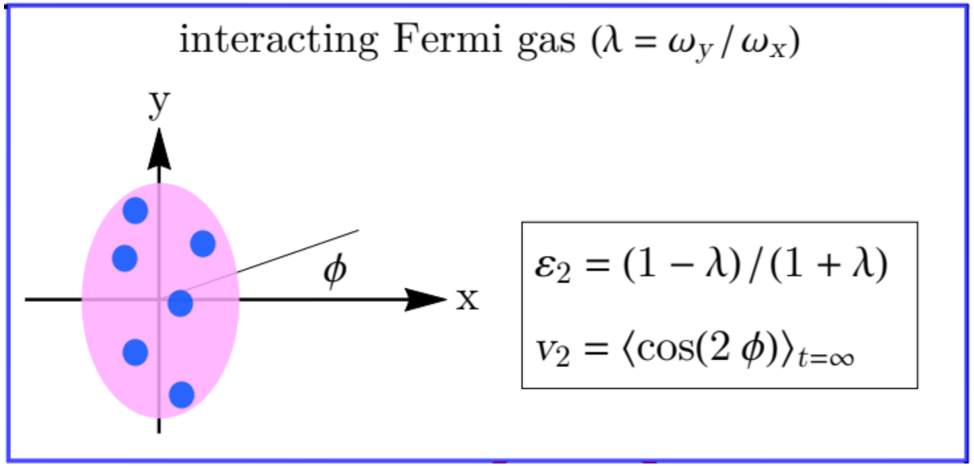
O'Hara et al., Science (2002)
Menotti, Pedri, Stringari, PRL (2002)

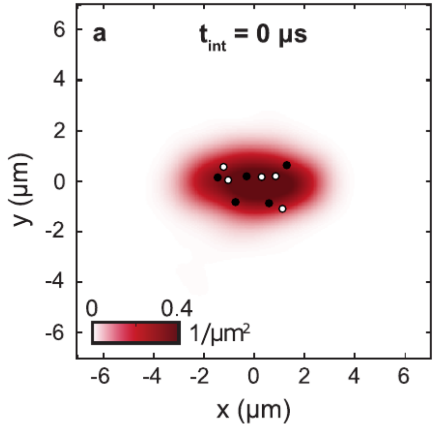
Testing collectivity in few-fermion systems

See talk by Sandra Brandstetter

- 5+5 fermion quantum wavefunction
  → repeated measurements for expectation values
- Initial (trap) geometry known (unlike in HI)
  → Measure distribution of atoms in coordinate and momentum space after expansion







Intrinsic anisotropy of wave-function

Flörchinger, Giacalone, Heyen, Tharwat, PRC (2022)

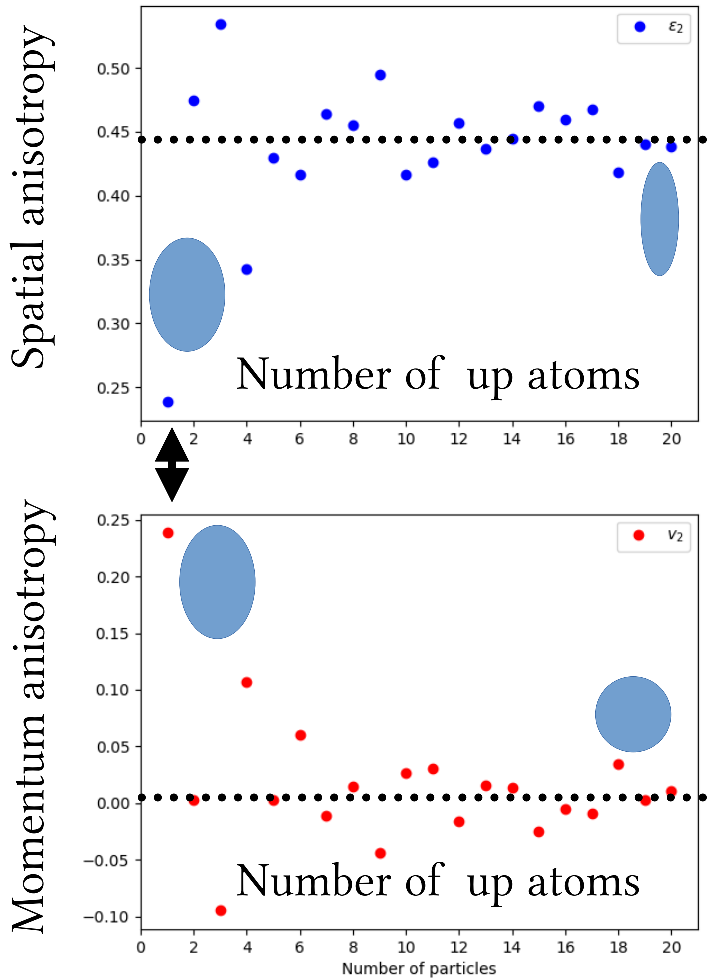
Non-interacting N fermion wavefunction

$$\psi_N(x,y) \to \mathrm{FT} \to \psi_N(p_x,p_y)$$

$\longrightarrow$ compute spatial (ε_2) momentum (v_2) anisotropy as a function of N

$N=1$: $\varepsilon_2 \approx v_2$ (Heisenberg uncertainty)
$N \gg 1$: $\varepsilon_2 = \mathrm{const}, v_2 \approx 0$ (round Fermi surface)

$\longrightarrow$ compare interacting vs non-interacting expansions



Intrinsic anisotropy of wave-function

Flörchinger, Giacalon

Non-interac

$\psi_N(x,y)$

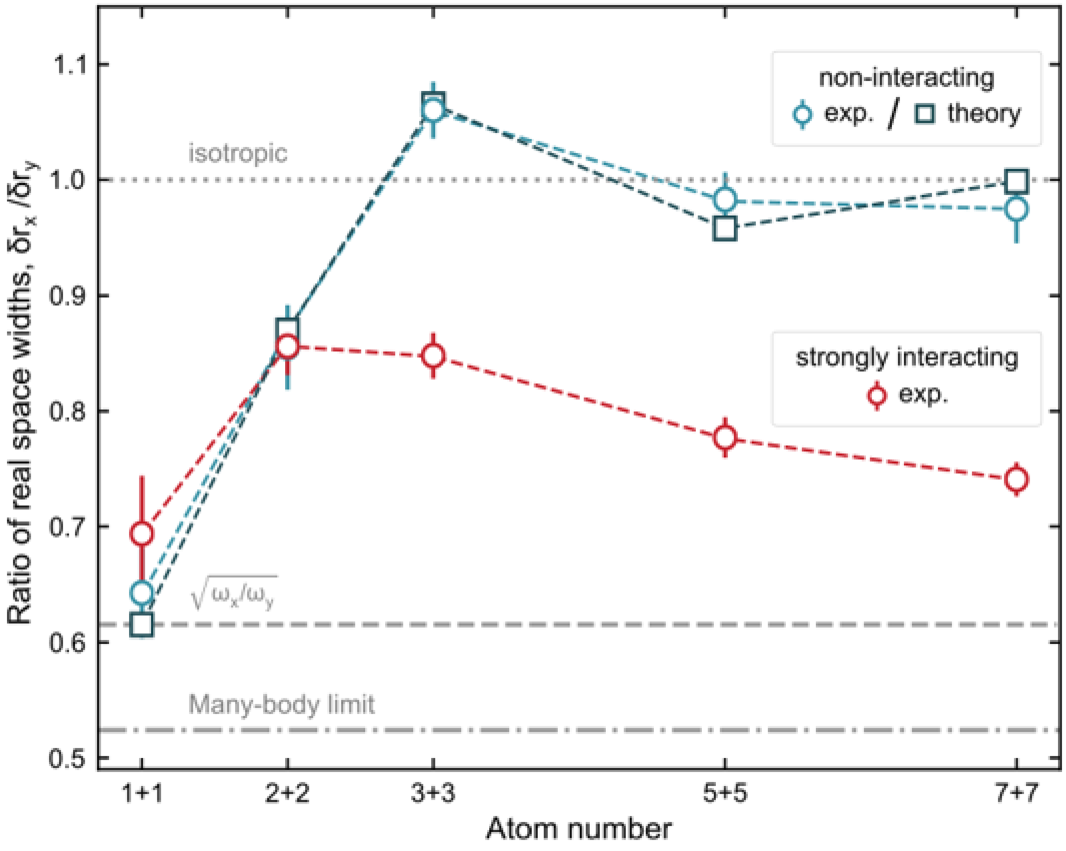
$\longrightarrow$ compute

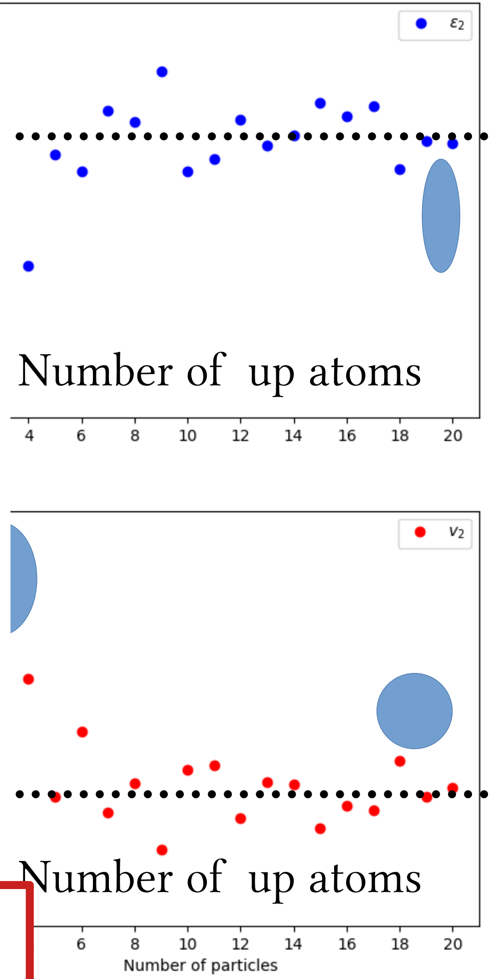
anisotropy

$N\!=\!1:\ \varepsilon_2 \approx$

$N\!\gg\!1:\ \varepsilon_2 =$ c

$\longrightarrow$ compare i





Interaction driven anisotropy for $N>3$!

Hydrodynamic modeling of CA expansion

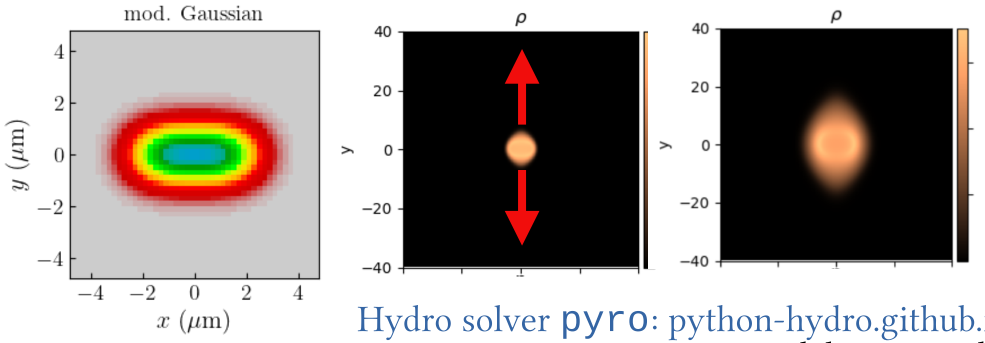
Equations of motion

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = 0\,,$$

$$\rho(\partial_t + \mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla P$$

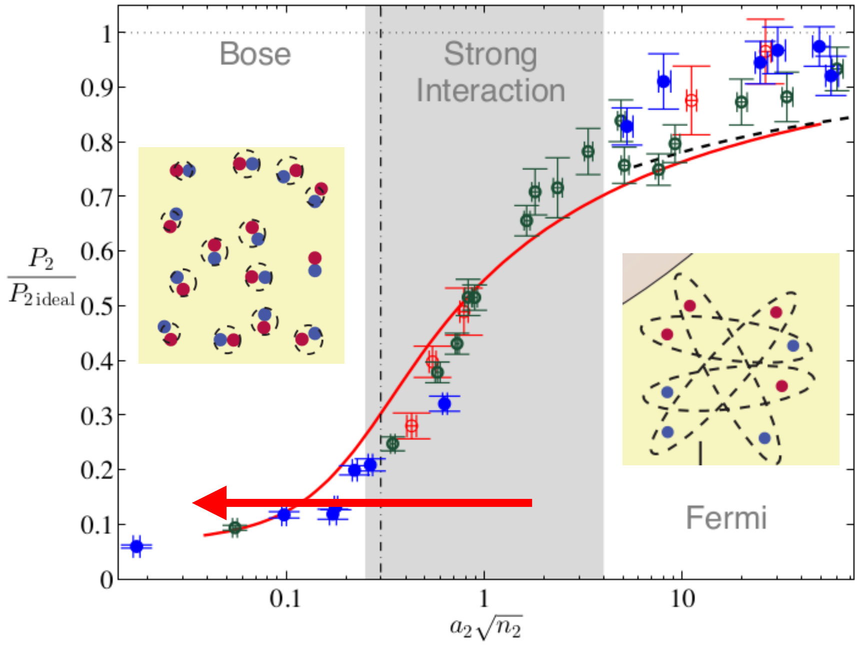
Transport coefficients: ideal/superfluid motion

average geometry ⟶ expansion ⟶ inversion



Hydro solver pyro: python-hydro.github.io/pyro2

Equation of state (many-body)



$$P_{\mathrm{ideal}} = \frac{\pi\hbar^2}{2m_{\mathrm{Li}}} n^2$$

Makhalov, Martiyanov, and Turlapov, PRL (2014)

Hydrodynamic modeling of CA expansion

Equatio

$\partial_t \rho + \nabla$

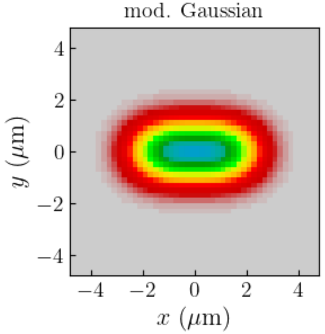
$\rho(\partial_t + \mathbf{v}$

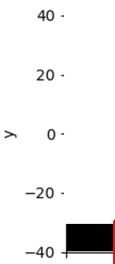
Transp

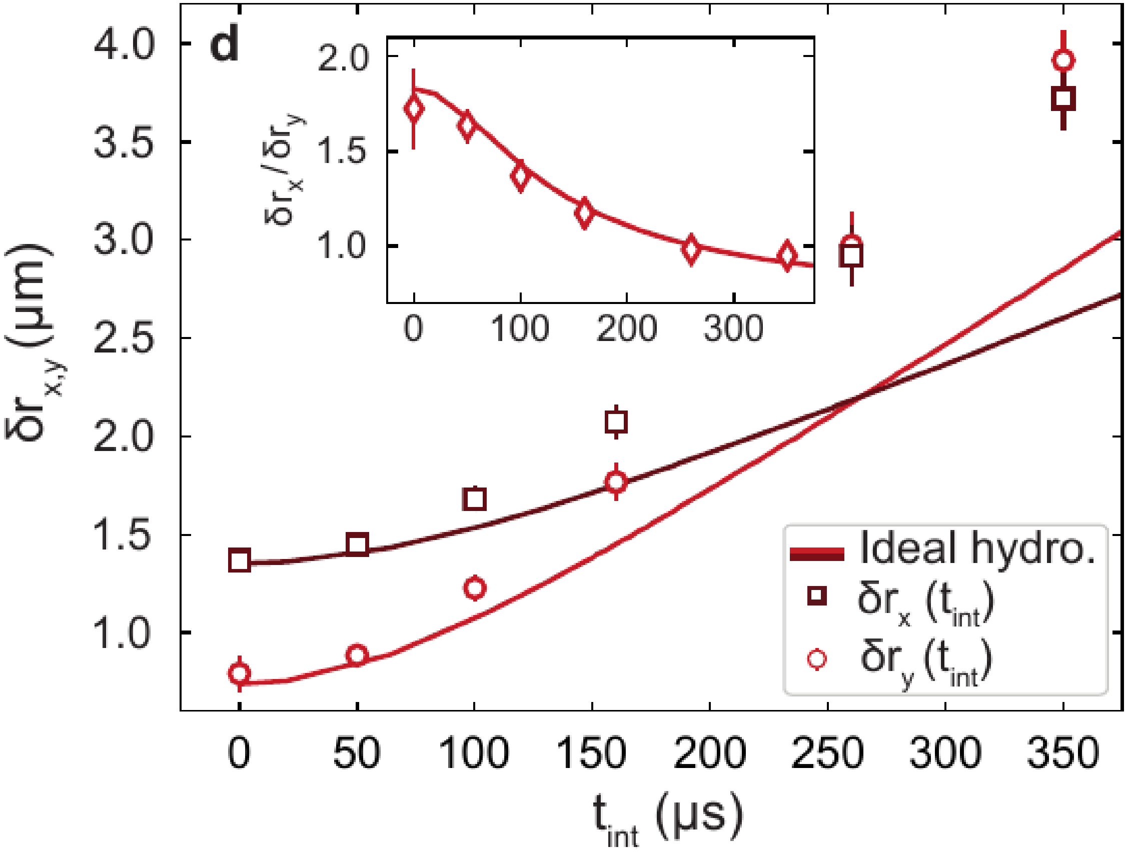
ideal/s

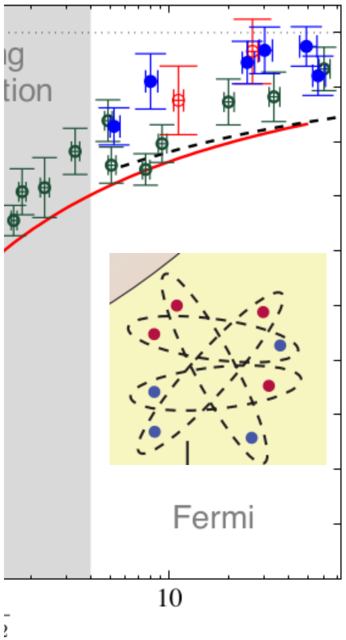
(many-body)

average geometry









$\frac{\pi\hbar^2}{\ }n^2$

Makhalov, Martiyanov, and Turlapov, PRL (2014)

Hydro

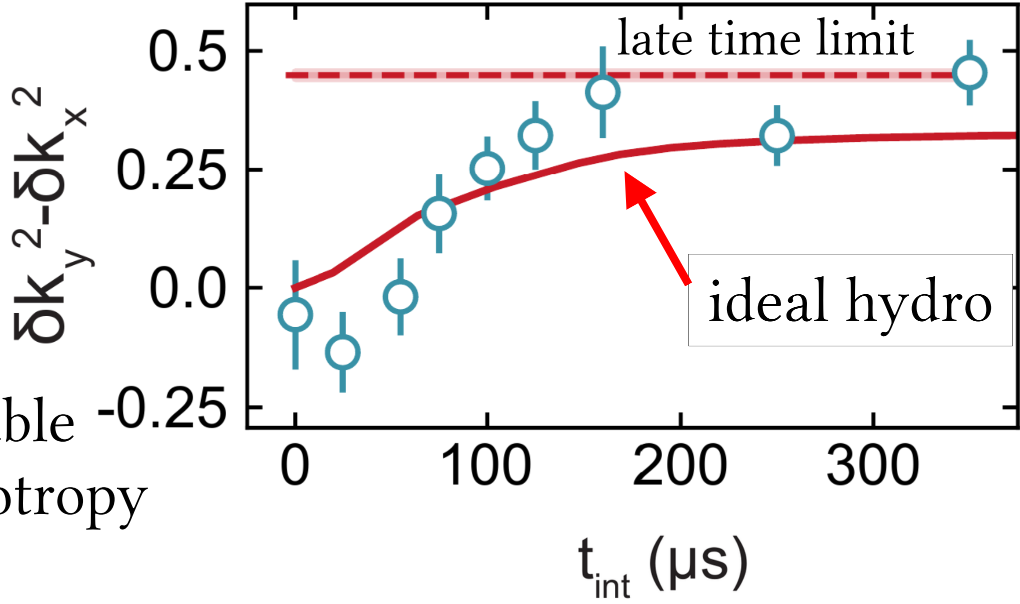
Good description of spatial aspect ratio vs time

Build-up of momentum anisotropy

Momentum anisotropy → anisotropy in energy-momentum tensor

$$T^{ij}(\vec{r}) = \int d^2p \left\{ \frac{p_j p_k}{m} f(t, \vec{r}, \vec{p}) \right\}$$

$$\langle k_x^2 \rangle - \langle k_y^2 \rangle \propto \int_{\vec{r}} [T^{xx}(\vec{r}) - T^{yy}(\vec{r})]$$



Ideal (equilibrium) hydro gives a reasonable description of averaged momentum anisotropy

What is local momentum anisotropy?
→ Opportunity to study non-equilibrium!

Hydrodynamic attractors in cold atoms?

Can we reach local non-equilibrium?

- Need finite relaxation time
  $\to$ higher temperatures

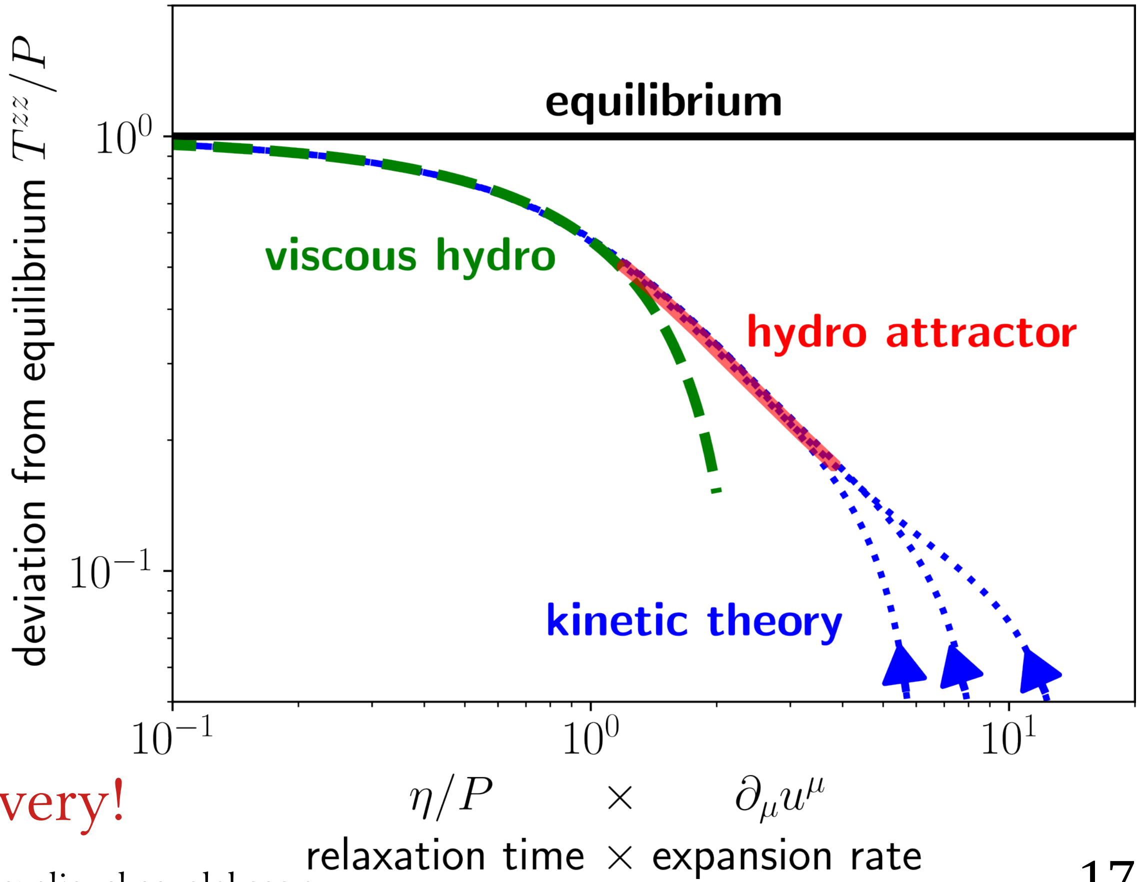
$$\tau_R \sim \frac{\eta}{P}$$

- Large expansion rates
  $\to$ trap frequency modulation

$$\Omega = \Omega_0 + A\cos(\omega t)$$

$$\partial_\mu u^\mu \propto A\cos(\omega t)$$

Opportunity for experimental discovery!



Conclusions and outlook

- Collective behaviour is a widespread phenomena in small collision systems
- Surprising effectiveness of hydro descriptions out-of-equilibrium
  → hydrodynamic attractors
- Cold atom experiments offer unrivaled toolbox to study collectivity
- Origins of collectivity in few-body systems is a pressing interdisciplinary issue
  → Call a Rapid Reaction Task Force!

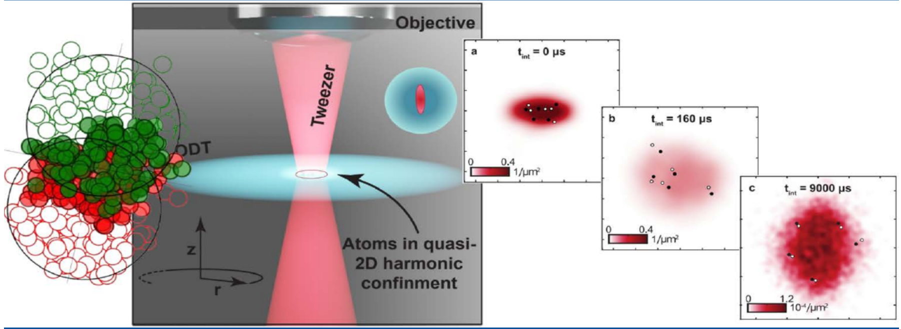
ExtreMe Matter Institute EMMI

EMMI Rapid Reaction Task Force

Deciphering Many-Body Dynamics in Mesoscopic Quantum Gases

March 18-21, 2024

Heidelberg University, Germany



Starts next week!

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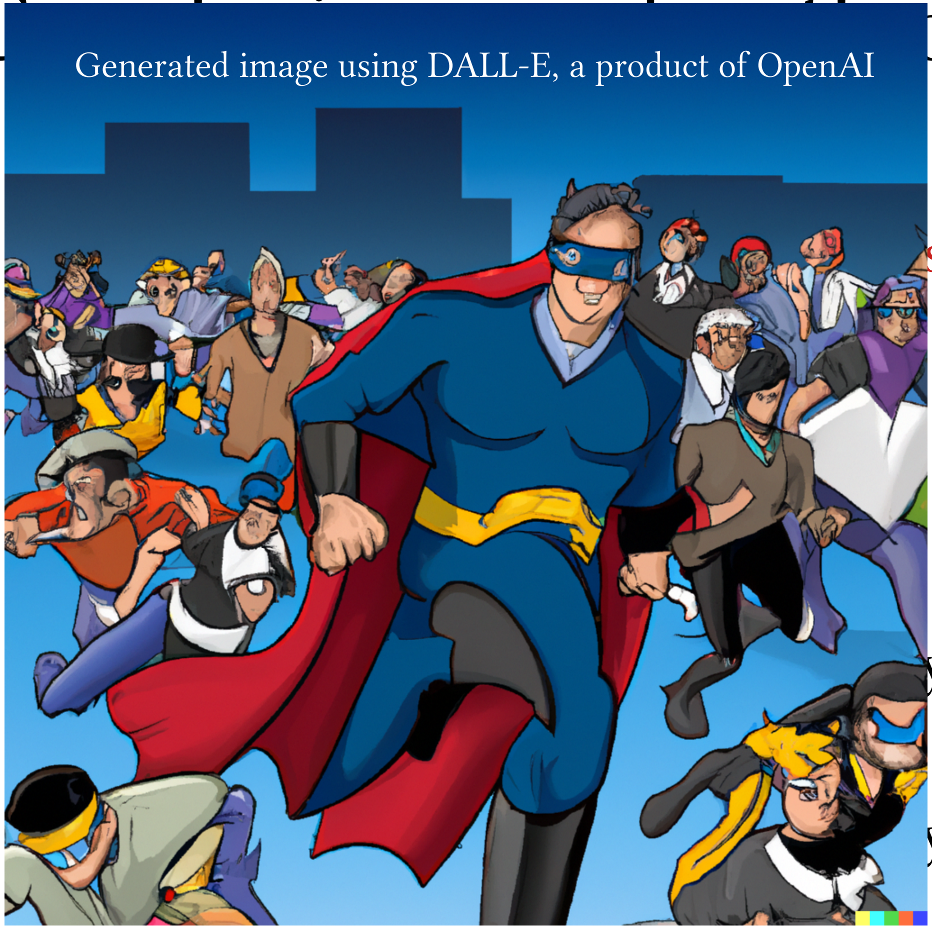

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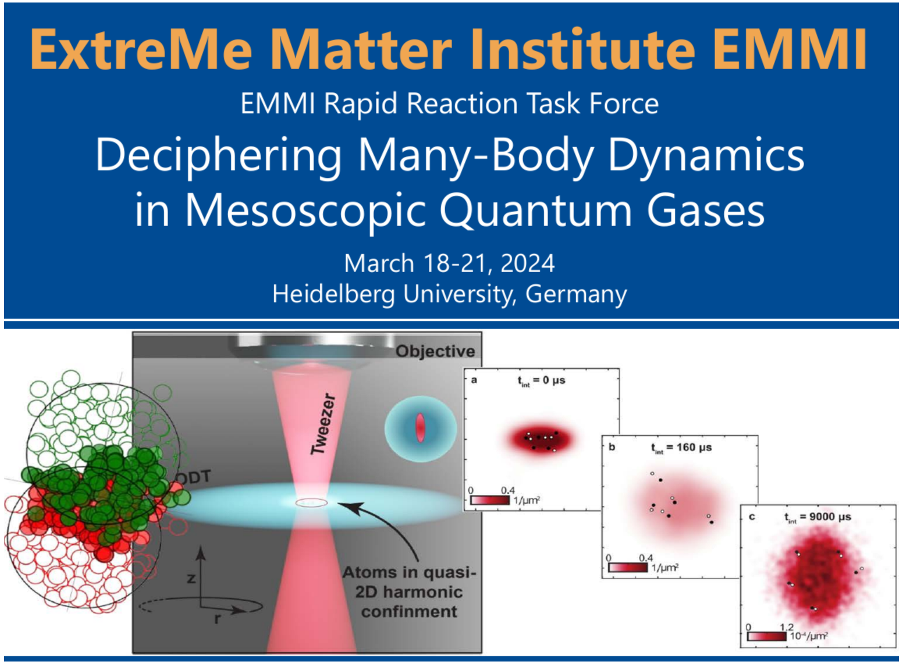

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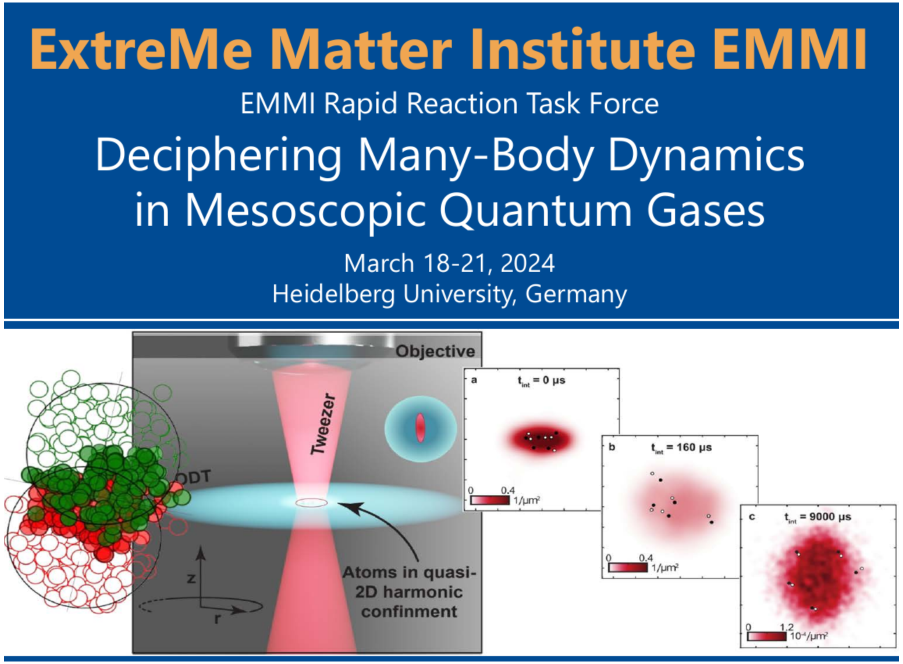

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