

# Theory of strong-interaction matter

Gergely Endrődi

University of Bielefeld



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in collaboration with:

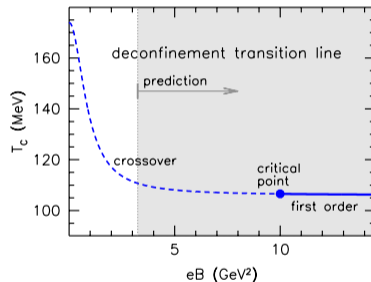
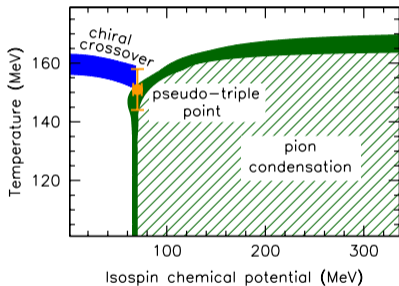
Bielefeld: Bastian Brandt, Eduardo Garnacho, Javier Hernández, Gergely Markó,  
Laurin Pannullo, Leon Sandbote, Dean Valois

Frankfurt, Darmstadt: [crc-tr211.org](http://crc-tr211.org)

my colleagues from Budapest, Regensburg, Wuppertal, Graz, Nicosia, Mumbai,  
Seattle, Sao Paulo, Tuscaloosa

# Appetizer

fundamental phase diagrams of QCD  
with possible phenomenological implications



 Brandt, Endrödi, Schmalzbauer '18

 Brandt, Endrödi '19

 Endrödi '15

 D'Elia, Maio, Sanfilippo, Stanzione '21

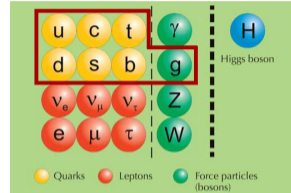
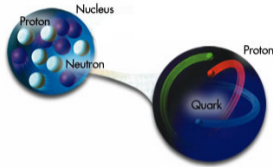
# Outline

- ▶ introduction: strongly interacting matter in
  - ▶ strong electromagnetic fields
  - ▶ nonzero isospin density
- ▶ lattice simulation techniques
- ▶ phase diagrams: current status
- ▶ application: cosmic trajectory
- ▶ further electromagnetic effects:  
    inhomogeneities, topology and chirality
- ▶ summary

# Introduction

# Strong interactions

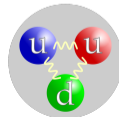
- ▶ explain 99.9% of visible matter in the Universe



- ▶ elementary particles: quarks and gluons
- ▶ elementary fields:  $\psi(x)$  and  $A_\mu(x)$  enter the QCD Lagrangian

$$\mathcal{L}_{\text{QCD}} = \frac{1}{4} \text{Tr} F_{\mu\nu}(g_s, A)^2 + \bar{\psi}[\gamma_\mu(\partial_\mu + ig_s A_\mu) + m]\psi$$

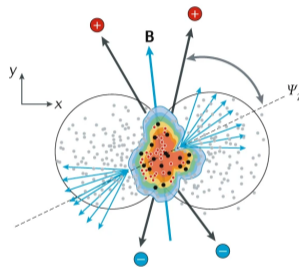
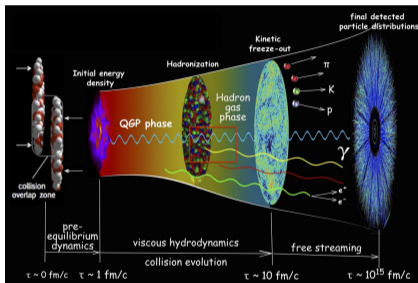
- ▶  $g_s = \mathcal{O}(1) \rightsquigarrow$  confinement  
 $m_u, m_d \approx 3 - 5 \text{ MeV}, \quad m_p = 938 \text{ MeV}$



- ▶ asymptotic freedom at high energy scales  $\rightsquigarrow$  deconfinement

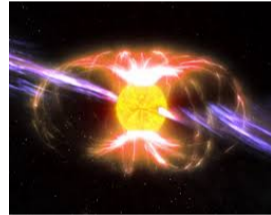
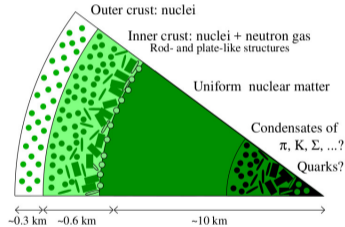
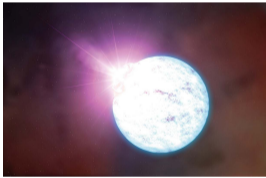
# Quarks and gluons in extreme conditions

- ▶ heavy ion collisions  $T \lesssim 10^{12} \text{ }^\circ\text{C} = 200 \text{ MeV}$ ,  $n \lesssim 0.12 \text{ fm}^{-3}$   
 $B \lesssim 10^{19} \text{ G} = 0.3 \text{ GeV}^2/e$



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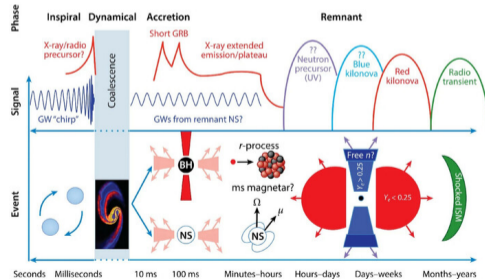
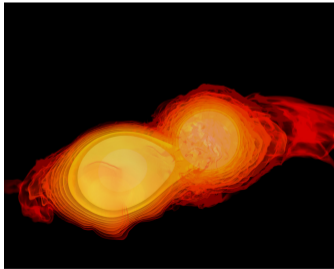


 Lattimer, Nature Astronomy 2019



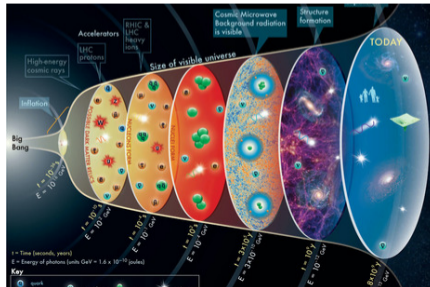
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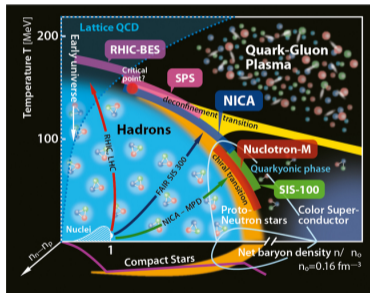
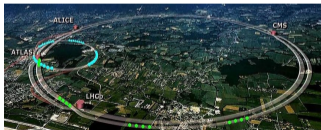


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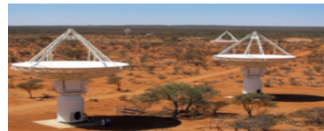
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- ▶ early universe, QCD epoch  $T \lesssim 200 \text{ MeV}$   
standard scenario:  $n \approx 0$



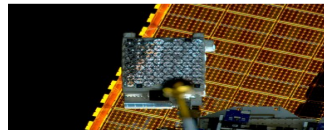
# Major experimental and observational campaigns



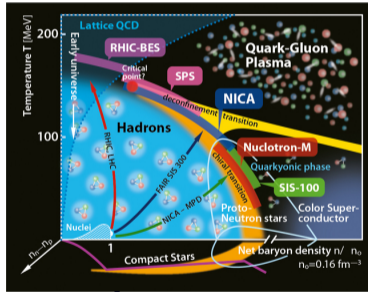
Observational astronomy



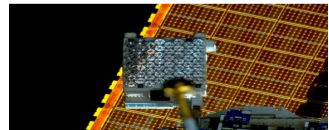
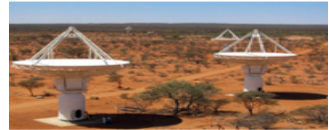
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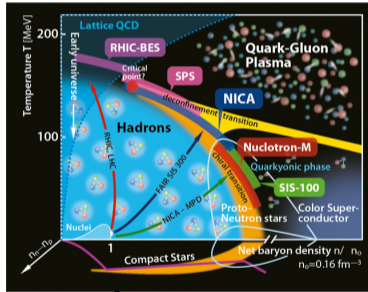
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**Observational astronomy**



**Heavy ion collisions**



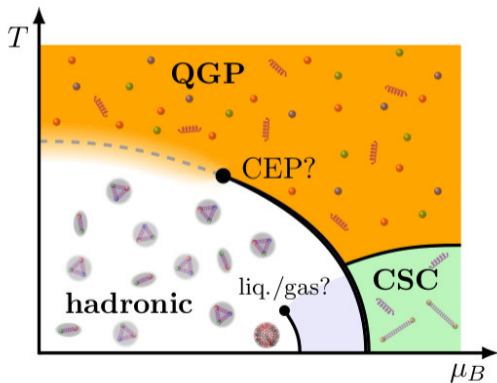
## QCD phase diagram(s)

# Phase diagram

- ▶ control parameters:  $T, n \leftrightarrow \mu, B$        $\mu_{\{u,d,s\}} / \mu_{\{B,Q,S\}} / \mu_{\{B,I,S\}}$

# Phase diagram

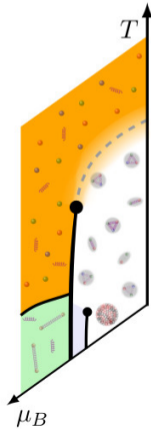
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- ▶ well-known famous phase diagram





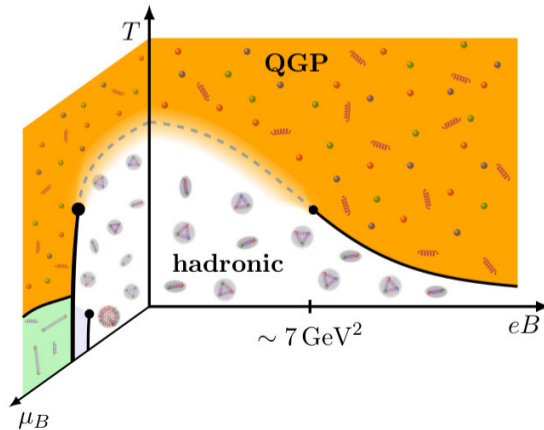
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- ▶ well-known famous phase diagram
- ▶ well-known, less famous phase diagram:



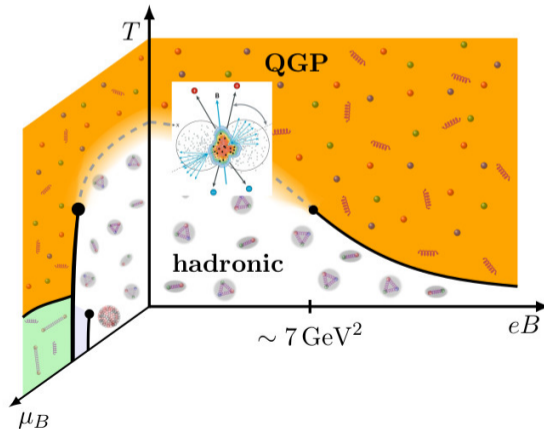
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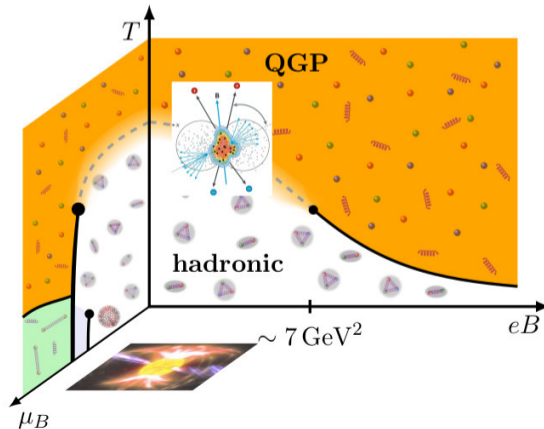
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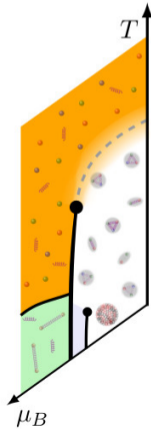
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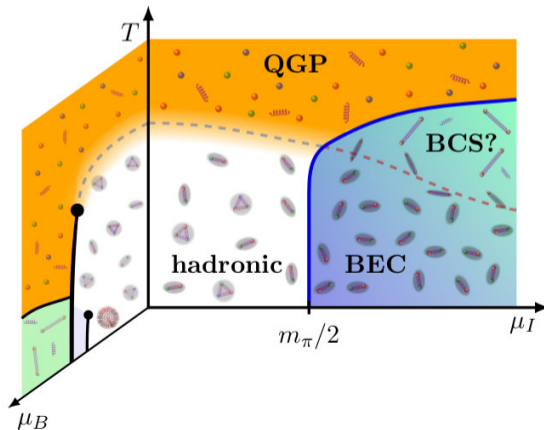
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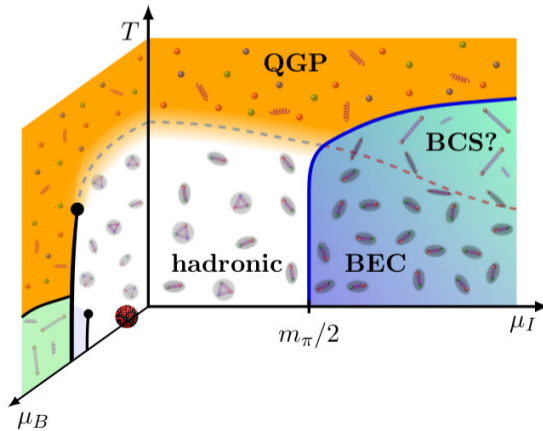
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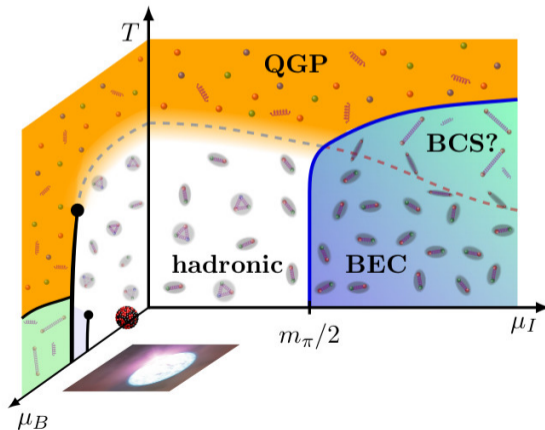
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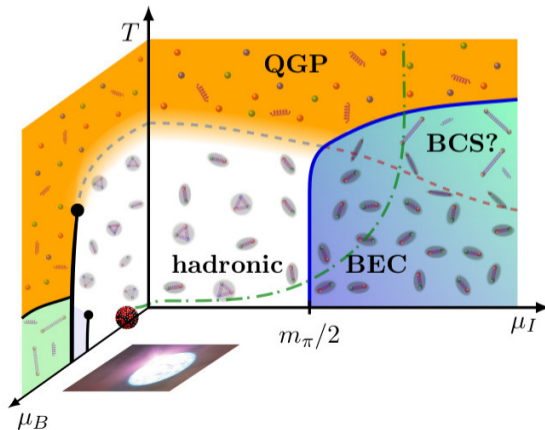
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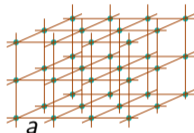
## Lattice QCD simulations

# Lattice simulations

- ▶ path integral [Feynman '48](#)

$$\mathcal{Z} = \int \mathcal{D}A_\mu \mathcal{D}\bar{\psi} \mathcal{D}\psi \exp\left(-\int d^4x \mathcal{L}_{\text{QCD}}(x)\right)$$

- ▶ discretize QCD action on space-time lattice [Wilson '74](#)



continuum limit  $a \rightarrow 0$  in a fixed physical volume:  $N \rightarrow \infty$

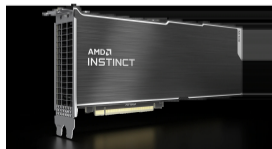
- ▶ dimensionality of lattice path integral:  $10^9-10^{10} \rightsquigarrow$  computationally very demanding



[SuperMUC-NG](#)



[nvidia.com](#)



[amd.com](#)



[Bielefeld GPU cluster](#)

# Monte Carlo simulations

- ▶ Euclidean QCD path integral over gauge field  $\mathcal{A}$

$$\mathcal{Z} = \int \mathcal{D}\mathcal{A} e^{-S_g[\mathcal{A}]} \det[\not{D}[\mathcal{A}] + m]$$

- ▶ Monte-Carlo simulations need:  $\det[\not{D} + m] \in \mathbb{R}^+$   
for that one needs  $\Gamma$  so that

$$\Gamma \not{D} \Gamma^\dagger = \not{D}^\dagger, \quad \Gamma^\dagger \Gamma = 1$$

$$\det[\not{D} + m] = \det[\Gamma^\dagger \Gamma (\not{D} + m)] = \det[\Gamma (\not{D} + m) \Gamma^\dagger] = \det[\not{D}^\dagger + m] = \det[\not{D} + m]^*$$

- ▶ usually positivity can also be shown
- ▶ such a  $\Gamma$  exists:  $B, \mu_I, i\mu_B, iE$  ✓
- ▶ no  $\Gamma$  exists: complex action problem  $\mu_B, E$  ⚡

**Phase diagram: nonzero isospin**

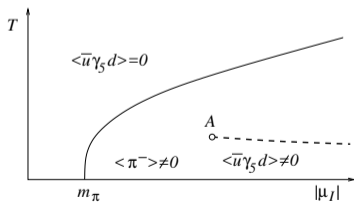
# Pion condensation

- ▶ isospin chemical potential:  $\mu_u = \mu_l, \mu_d = -\mu_l, \mu_s = 0$ 
  - ▶ QCD at low energies  $\approx$  pions  
chiral perturbation theory
  - ▶ charged pion chemical potential:  $\mu_\pi = 2\mu_l$



at zero temperature  $\mu_\pi < m_\pi$  vacuum state  
 $\mu_\pi \geq m_\pi$  Bose-Einstein condensation

 Son, Stephanov '00



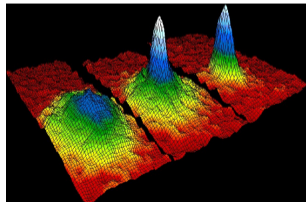
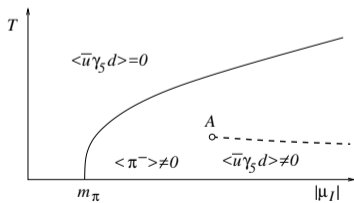
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- ▶ trapped Rb atoms at low temperature ✍ Anderson et al '95 JILA-NIST/University of Colorado

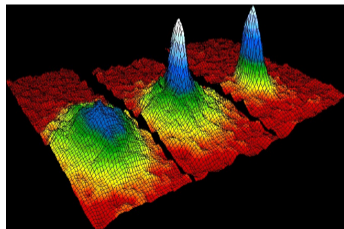
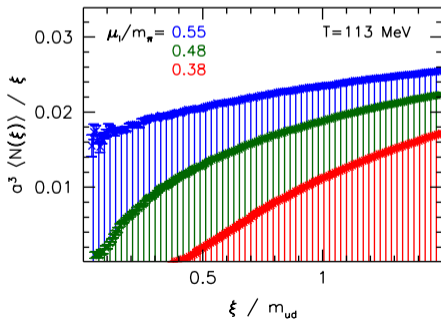
# Condensation of states

- ▶ singular values of the Dirac operator  $\not{D}$  Kanazawa, Wettig, Yamamoto '11

$$(\not{D} + m)^\dagger (\not{D} + m)\chi = \xi^2 \chi$$

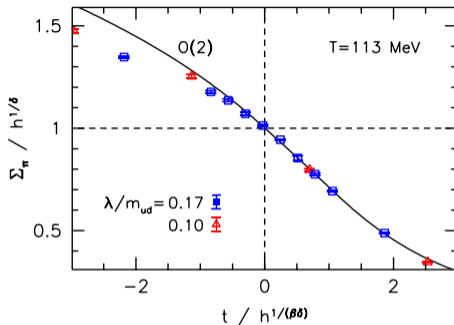
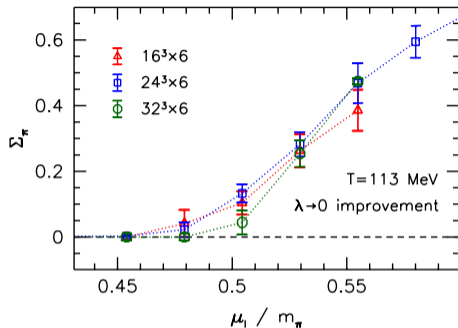
- ▶ pion condensate via Banks-Casher-type relation  $\not{D}$  Brandt, Endrödi, Schmalzbauer '17

$$\Sigma_\pi \propto \langle \rho(\xi = 0) \rangle$$



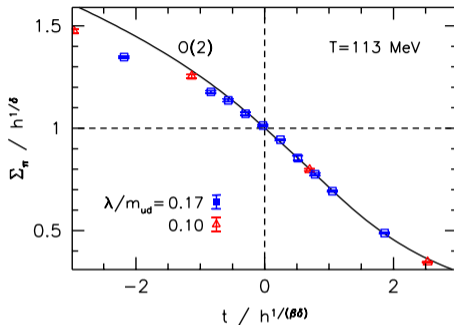
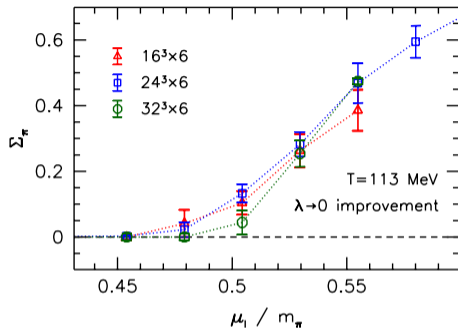


# Order of the transition



- ▶ volume scaling of order parameter shows typical sharpening
- ▶ collapse according to  $O(2)$  critical exponents [Ejiri et al '09](#)

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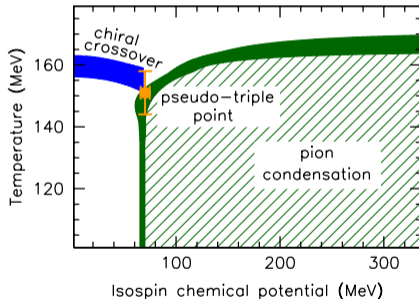
- ▶ volume scaling of order parameter shows typical sharpening
- ▶ collapse according to  $O(2)$  critical exponents [Ejiri et al '09](#)
- ▶ indications for a second order phase transition at  $\mu_I = m_\pi/2$ , in the  $O(2)$  universality class

# Phase diagram

- ▶ phases in the  $T - \mu_I$  phase diagram: hadronic (confined), quark-gluon plasma (deconfined), pion condensation (confined)

✍ Brandt, Endrődi, Schmalzbauer '17

✍ Brandt, Endrődi '19

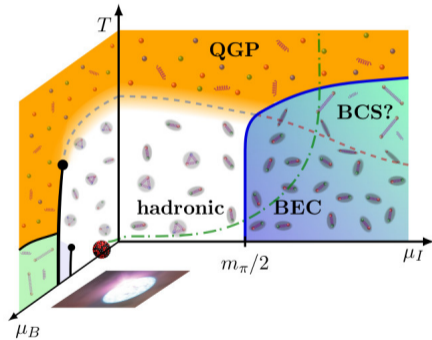
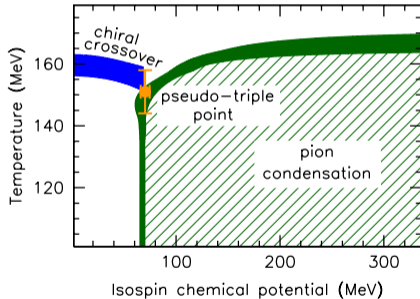


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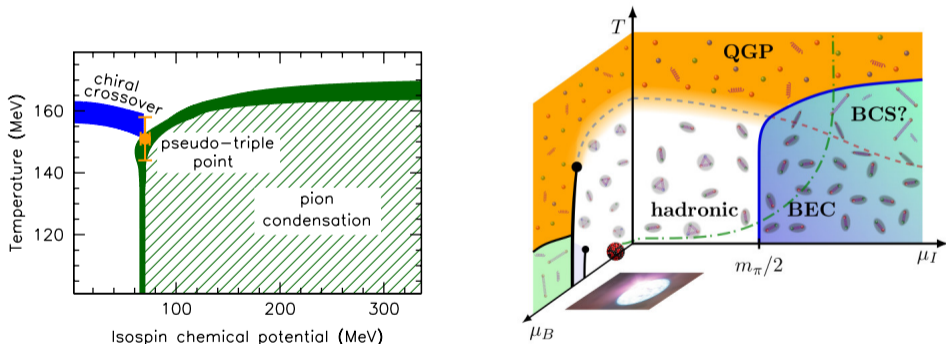
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🔗 Brandt, Endrődi, Schmalzbauer '17   🔗 Brandt, Endrődi '19



- ▶ comparison to effective models,  $\chi$ PT, Q2CD, ...

🔗 Adhikari et al. '18   🔗 Zhokhov et al. '19   🔗 Adhikari et al. '20   🔗 Boz et al. '20

🔗 Astrakhantsev et al. '20   🔗 Andersen et al. '23

**Equation of state: nonzero isospin**

# Equation of state

- ▶ equilibrium description of matter

$$\epsilon(p)$$

relevant for:

- ▶ neutron star physics (TOV equations)
  - ▶ cosmology, evolution of early Universe (Friedmann equation)
  - ▶ heavy-ion collision phenomenology (charge fluctuations)
- ▶ thermodynamic relations

$$p = \frac{T}{V} \log \mathcal{Z}, \quad s = \frac{\partial p}{\partial T}, \quad n_I = \frac{\partial p}{\partial \mu_I}, \quad \epsilon = -p + Ts + \mu_I n_I$$

$$l = \epsilon - 3p, \quad c_s^2 = \left. \frac{\partial p}{\partial \epsilon} \right|_{s/n_I}$$

## Equation of state on the lattice: $T \approx 0$

- ▶ integral method to calculate differences

$$n_I = \frac{T}{V} \frac{\partial \log \mathcal{Z}}{\partial \mu_I}, \quad p(T, \mu_I) - p(T, 0) = \int_0^{\mu_I} d\mu'_I n_I(\mu'_I)$$



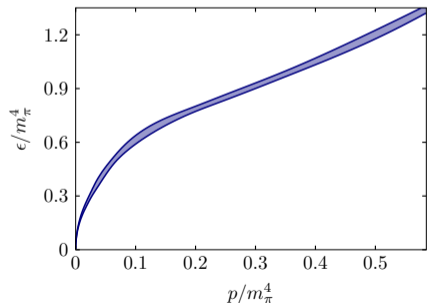
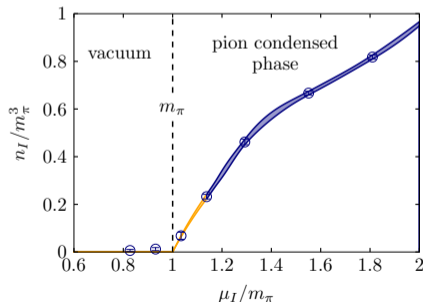
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- ▶ pressure and energy density

✍ Brandt, Endrődi, Fraga, Hippert, Schaffner-Bielich, Schmalzbauer '18



# Equation of state on the lattice: $T \approx 0$

- ▶ integral method to calculate differences

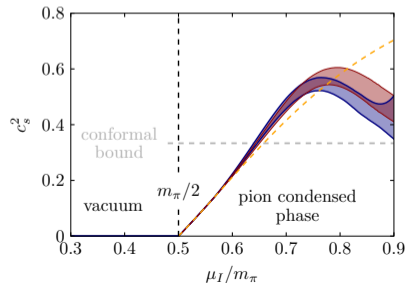
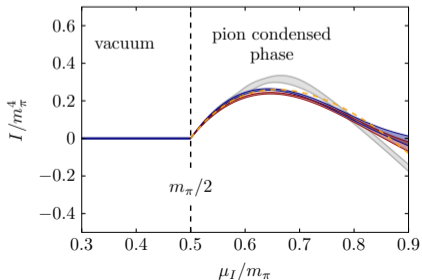
$$n_l = \frac{T}{V} \frac{\partial \log \mathcal{Z}}{\partial \mu_l}, \quad p(T, \mu_l) - p(T, 0) = \int_0^{\mu_l} d\mu'_l n_l(\mu'_l)$$

- ▶ pressure and energy density

✍ Brandt, Endrődi, Fraga, Hippert, Schaffner-Bielich, Schmalzbauer '18

- ▶ interaction measure and speed of sound

✍ Brandt, Cuteri, Endrődi '22



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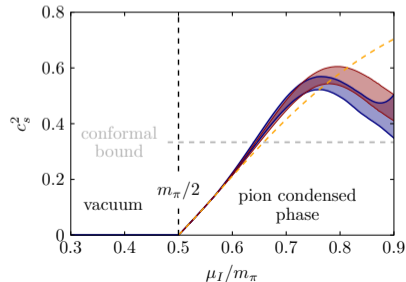
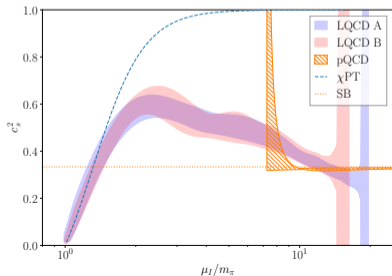
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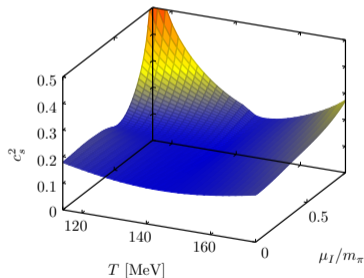
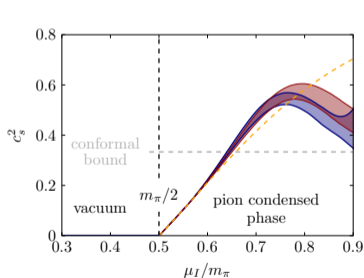
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✍ Brandt, Cuteri, Endrődi '22   ✍ Abbott et al. '23



# Equation of state on the lattice: $T > 0$

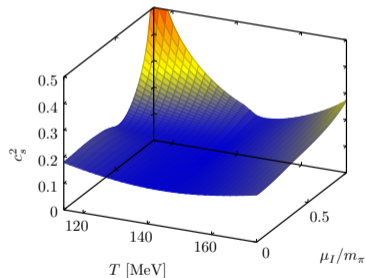
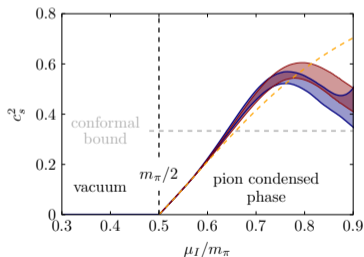
- ▶ nonzero temperature results ✍ Brandt, Cuteri, Endrődi '22  
✍ Vovchenko, Brandt, Cuteri, Endrődi, Hajkarim, Schaffner-Bielich '20
- ▶ interaction measure negative at high  $\mu_I$ , low  $T$
- ▶ speed of sound **above  $1/\sqrt{3}$**  at high  $\mu_I$  and intermediate  $T$



- ▶ EoS can get very stiff inside pion condensation phase

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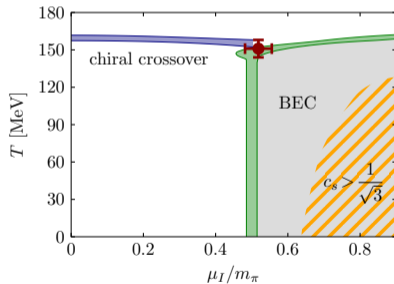
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- ▶ comparison:  $\chi$ PT, models [Adhikari et al. '21](#) [Avancini et al. '19](#)

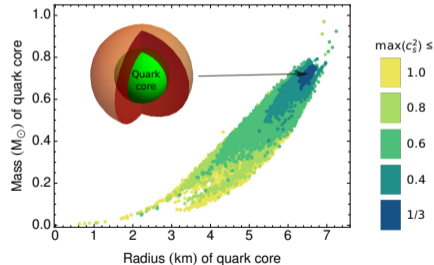
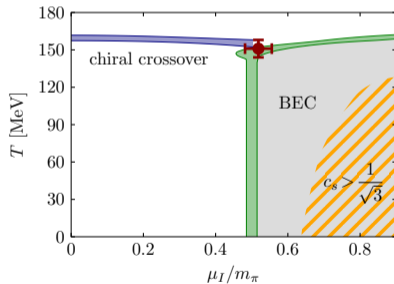
# Speed of sound

- ▶ 'supersonic' region of pion condensate
- ▶ first time that  $c_s > 1/\sqrt{3}$  found in a first-principles lattice QCD calculation



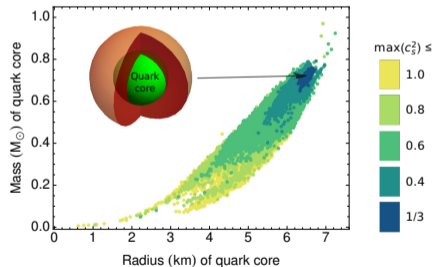
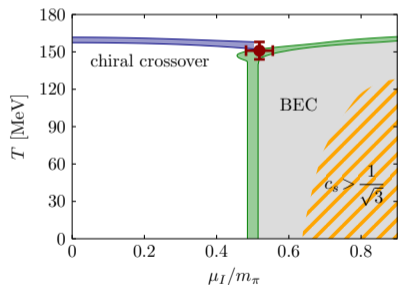
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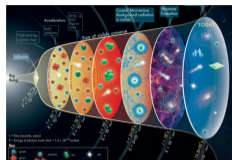


- ▶  $c_s$  at  $\mu_B > 0$  from FRG and  $\chi$ EFT [Braun, Schallmo '22](#) [Leonhardt et al. '20](#)



## **Cosmological implications**

# Cosmic trajectories



- ▶ early Universe

- ▶ conservation equations for isentropic expansion

$$\frac{n_B}{s} = b, \quad \frac{n_Q}{s} = 0, \quad \frac{n_{L_\alpha}}{s} = l_\alpha \quad (\alpha \in \{e, \mu, \tau\})$$

- ▶ parameters:  $T$ ,  $\mu_B$ ,  $\mu_Q$ ,  $\mu_{L_\alpha}$

- ▶ experimental constraints [Planck collaboration '15](#) [Oldengott, Schwarz '17](#)

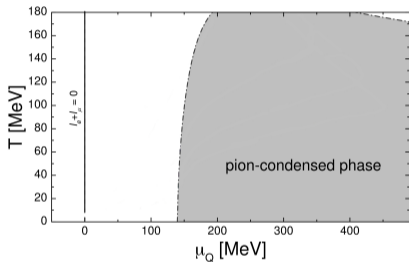
$$b = (8.60 \pm 0.06) \cdot 10^{-11}, \quad |l_e + l_\mu + l_\tau| < 0.012$$

(the individual  $l_\alpha$  may have opposite signs)

- ▶  $n_Q = 0$  with  $l_e > 0$  allows equilibrium of  $e^-$ ,  $\nu_e$ ,  $\pi^+$  [Abuki, Brauner, Warringa '09](#)

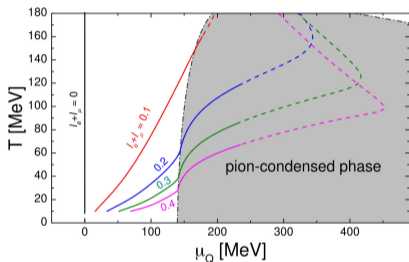
# Cosmic trajectories

- ▶ cosmic trajectory  $T(\mu_Q)$  is solved for
- ▶ standard scenario ( $l_\alpha = 0$ ):  $\mu_Q = 0$  for all  $T$



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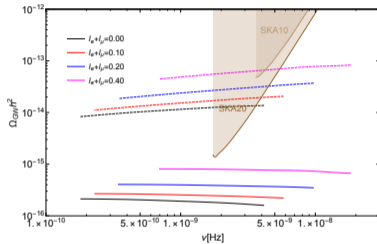
- ▶ cosmic trajectory enters BEC phase for lepton asymmetries allowed by observations [Vovchenko, Brandt, Cuteri, Endrödi, Hajkarim, Schaffner-Bielich '20](#)
- ▶ condition for pion condensation to occur:

$$|l_e + l_\mu + l_\tau| < 0.012$$

$$|l_e + l_\mu| \gtrsim 0.1$$

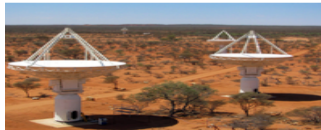
# Signatures of the condensed phase

- ▶ relic density of primordial gravitational waves is enhanced with respect to amplitude at  $I_e + I_\mu = 0$



✍ Vovchenko, Brandt, Cuteri, Endrődi, Hajkarim, Schaffner-Bielich '20

- ▶ to be detected experimentally (SKA)

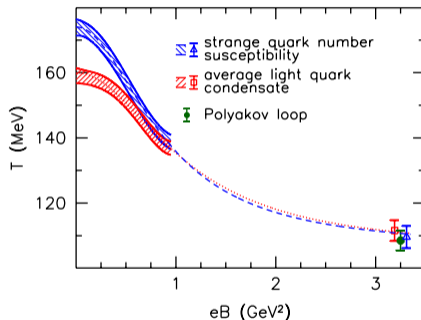


## Phase diagram: magnetic fields

# Magnetic phase diagram

## ► QCD crossover temperature in the phase diagram

✍ Bali, Bruckmann, Endrődi, Fodor, Katz et al. '11 ✍ '12 ✍ Endrődi '15



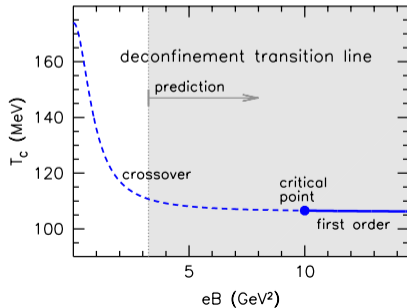
## ► $T_c$ is reduced by $B$

contrary to almost all effective theories and low-energy models of QCD

✍ Andersen, Naylor, Tranberg '14

# Magnetic phase diagram and critical point

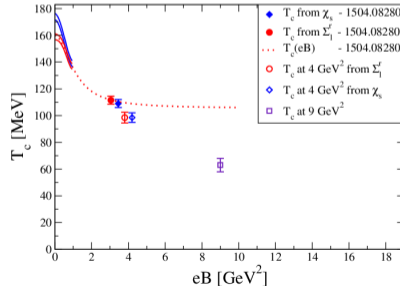
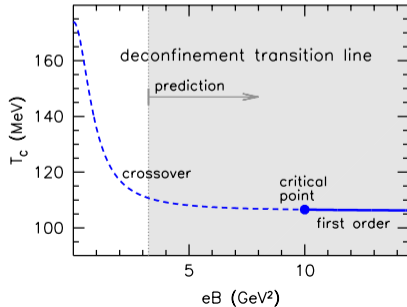
- ▶ effective theory of QCD at  $B \rightarrow \infty$ : first-order deconfinement transition  
⇒ **critical point!** *✍ Miransky, Shovkovy '02*
- ▶ location of critical point based on extrapolation from  $0 < eB \lesssim 3 \text{ GeV}^2$   
⇒  $eB_c \approx 10(2) \text{ GeV}^2$  *✍ Endrődi '15*





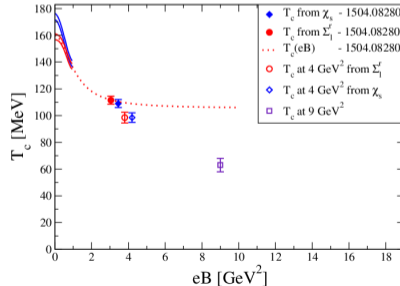
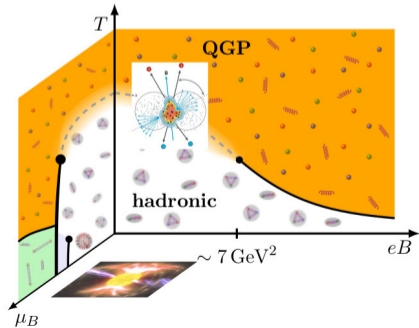
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- ▶ simulating up to  $eB \approx 9 \text{ GeV}^2$   
⇒  $4 \text{ GeV}^2 < eB_c < 9 \text{ GeV}^2$  ✍ D'Elia, Maio, Sanfilippo, Stanzione '21



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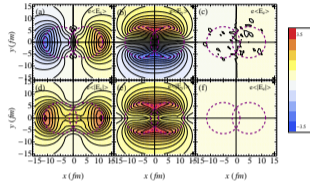
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**Beyond constant magnetic fields:  
inhomogeneities**

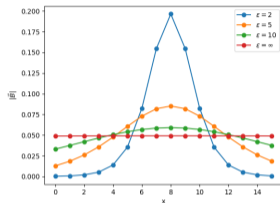
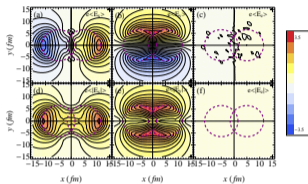
# Inhomogeneous magnetic fields

- ▶ off-central heavy-ion collisions: inhomogeneous magnetic fields [Deng et al. '12](#)



# Inhomogeneous magnetic fields

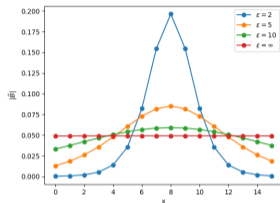
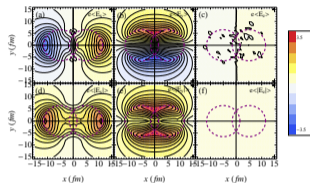
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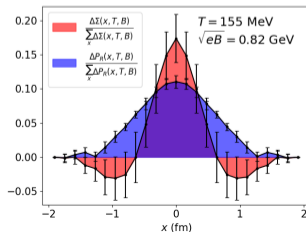
- ▶ consider profile  $B(x) = B \cosh^{-2}(x/\epsilon)$  [Dunne '04](#)

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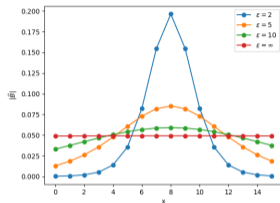
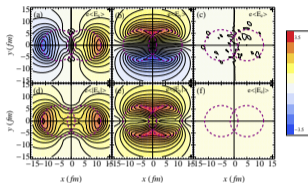


- ▶ consider profile  $B(x) = B \cosh^{-2}(x/\epsilon)$  [Dunne '04](#)
- ▶ impact: **condensate**, **Polyakov loop** [Brandt, Cuteri, Endrődi, Markó, Sandbote, Valois '23](#)

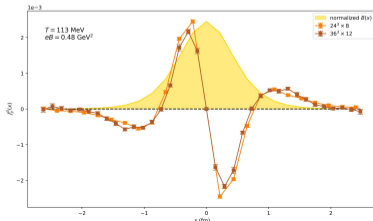


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- ▶ impact: electric current [Valois et al. upcoming](#) [D. Valois Tue 16:00 HK21.2](#)



**Magnetic fields and chiral imbalance:  
anomalous transport**



# Anomalous transport

- ▶ usual transport:  
vector current due to electric field

$$\langle \vec{J} \rangle = \sigma \cdot \vec{E}$$

- ▶ chiral magnetic effect (CME): *✍ Fukushima, Kharzeev, Warringa '08*  
vector current due to chirality and magnetic field

$$\langle \vec{J} \rangle = \sigma_{\text{CME}} \cdot \vec{B}$$

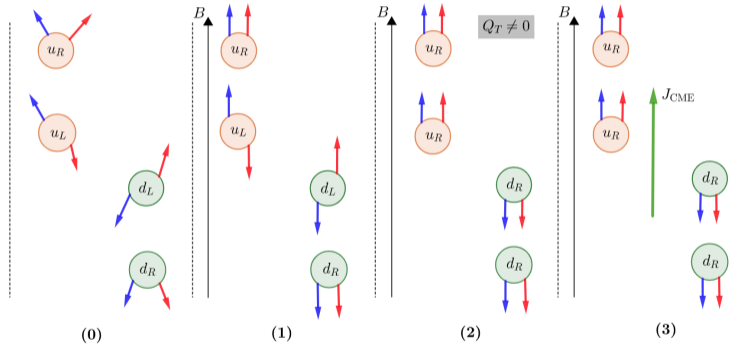
- ▶ chiral separation effect (CSE):  
axial current due to baryon number and magnetic field

$$\langle \vec{J}_5 \rangle = \sigma_{\text{CSE}} \cdot \vec{B}$$

- ▶ probe CP-odd domains in heavy-ion collisions *✍ Kharzeev, Liao, Voloshin, Wang '16*
- ▶ experimental searches for CME and related observables *✍ STAR collaboration '21*

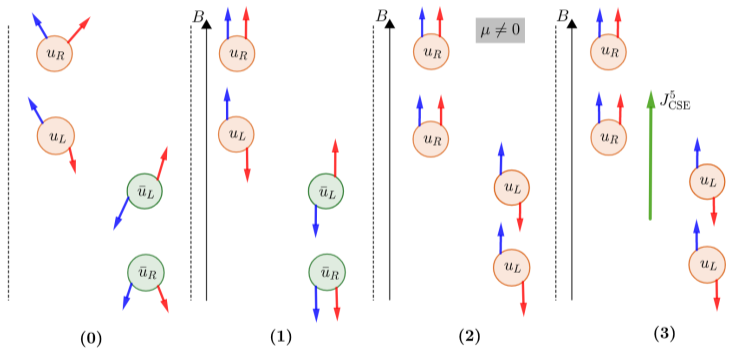
# General (handwaving) argument

## ► spin, momentum CME



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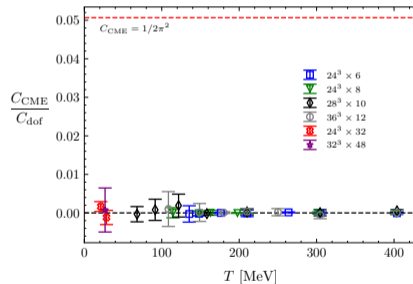
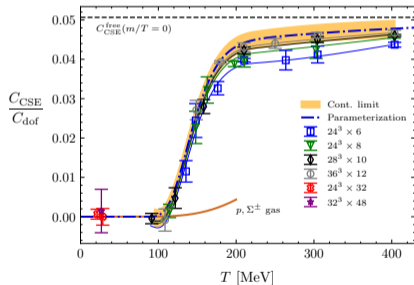


# CME and CSE from lattice QCD

- ▶ first determination of **in-equilibrium** CME/CSE coefficients with continuum extrapolated lattice simulations

CSE: ✍ Brandt, Endrődi, Garnacho, Markó, '23

CME: ✍ Brandt, Endrődi, Garnacho, Markó, upcoming



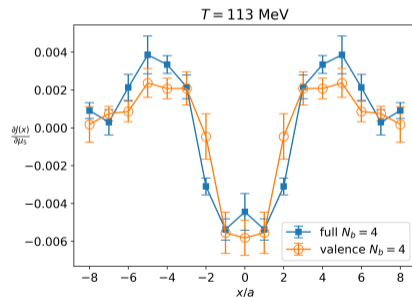
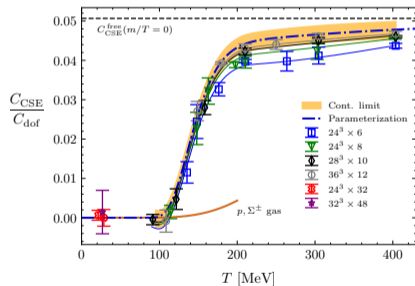
- ▶ clarifying contradictory results in the literature 🔍 E. Garnacho Tue 15:45 HK21.1

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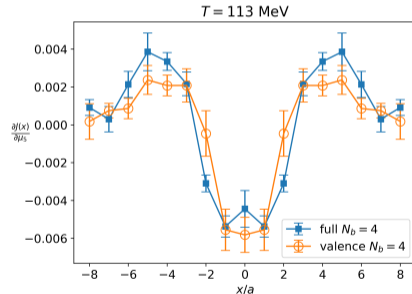
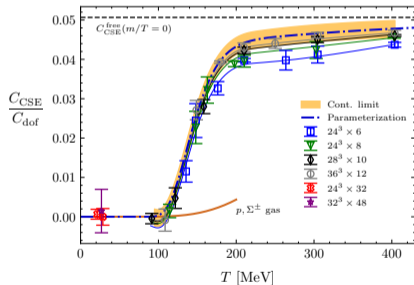
- ▶ clarifying contradictory results in the literature [E. Garnacho Tue 15:45 HK21.1](#)
- ▶ non-trivial response in inhomogeneous field  $B(x)$  [D. Valois Tue 16:00 HK21.2](#)

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- ▶ non-trivial response in inhomogeneous field  $B(x)$  [D. Valois Tue 16:00 HK21.2](#)
- ▶ this is not the **out-of-equilibrium** effect

**Magnetic and electric fields:  
axion-photon coupling**

# Axions as dark matter

- ▶ extend Standard Model with new field: axion  $a$

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{2} \partial_\mu a \partial_\mu a + a \underbrace{\text{Tr } G_{\mu\nu} \tilde{G}_{\mu\nu}}_{Q_{\text{top}}} + a g_{a\gamma\gamma} \underbrace{F_{\mu\nu} \tilde{F}_{\mu\nu}}_{\text{E}\cdot\text{B}}$$

- ▶ provides solution to 'strong CP problem'

✍ Peccei, Quinn '77   ✍ Weinberg '78   ✍ Wilczek '78

- ▶ is a possible dark matter candidate

- ▶ extensive experimental campaign:

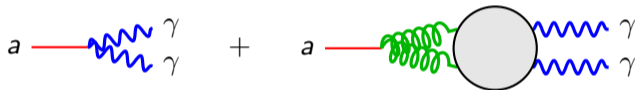
haloscopes and helioscopes   ✍ CAST   ✍ ADMX   ✍ XENON1T





# Axion-photon coupling

- ▶ most relevant parameter for experimental detection
- ▶ direct coupling (model-dependent)  
plus  
indirect coupling through quark/gluon loops



- ▶ chiral perturbation theory predicts two terms of similar magnitude and opposite sign *di Cortona et al. '16*
- ▶ QCD contribution, for slowly varying  $a$  fields

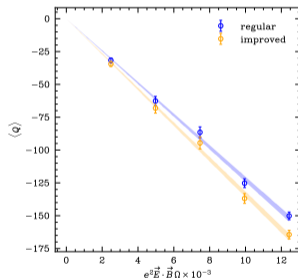
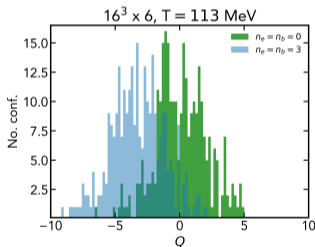
$$g_{a\gamma\gamma}^{\text{QCD}} = \frac{\partial^2 \log \mathcal{Z}}{\partial a \partial(\mathbf{E} \cdot \mathbf{B})} = \frac{\partial \langle Q_{\text{top}} \rangle}{\partial(i\mathbf{E} \cdot \mathbf{B})}$$



# Axion-photon coupling on the lattice

- ▶ QCD contribution

$$g_{a\gamma\gamma}^{\text{QCD}} = \frac{\partial \langle Q_{\text{top}} \rangle}{\partial (i\mathbf{E} \cdot \mathbf{B})}$$

- ▶ shift in mean topology by parallel magnetic *and* imaginary electric fields



- ▶ first results for  $g_{a\gamma\gamma}^{\text{QCD}}$   Brandt, Cuteri, Endrődi, Hernández, Markó '22
- ▶ challenging to approach continuum limit  J.Hernández Wed 17:45 HK48.2

## Summary

# Summary

- ▶  $T - \mu_I$  phase diagram and pion condensation
- ▶ cosmic trajectory may enter pion condensed phase
- ▶  $T - B$  phase diagram and the critical point
- ▶ in-equilibrium anomalous transport phenomena from lattice QCD

