Strong Interactions under Extreme Conditions: a Review

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Outline

- Motivation to study QCD under extreme conditions
- What we know already and how we know it
- What we do and don't know: Equation of State
- What we do and don't know: Dynamics
- Areas of active research

Reminder: QCD and what we know well

QCD is theory to describe protons+neutrons and other *hadrons* in terms of their constituents, *quarks and gluons*.

We know *degrees of freedom* and the *Lagrangian*

$$\mathcal{L} = \frac{1}{2g_s^2}\,\mathrm{Tr}\,G_{\mu\nu}G^{\mu\nu} + \sum_{i=udscbt} \overline{\psi}_i(\not{D} + m_i)\psi_i$$

We know the (scale-dependent) parameters g_s^2 and m_{udscbt} accurately

We know the *bound-state spectrum* $(\pi, K, \eta, \rho, p, n, \Delta, \ldots)$ from both experiment and theory

We know the *high-energy behavior* (jets and their properties) from a mix of experiment and theory

What we don't know quite as well

Few-body scattering at intermediate energies
(Experiments: pp, $pD \to pn$, $p\pi$, pK, but not $\pi\pi$, $KK, \ldots$)
large coupling, difficult for first-principles tools like lattice QCD

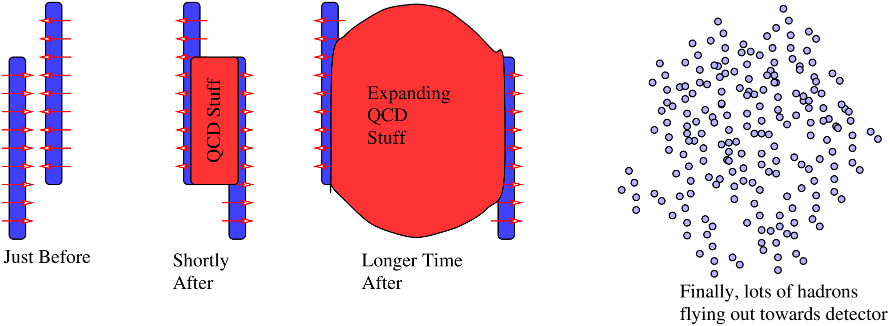
Many-body physics and collective dynamics

- Equilibrium: Equation of state $P(T, \mu_B, \mu_I, \mu_S)$
- Nonequilibrium: dynamics of large assemblages of QCD matter with lots of energy
  - Heavy ion collisions
  - Neutron star mergers
  - Early Universe

I will concentrate on these questions

Probing QCD collective dynamics in Laboratory

Collide (largest available) nuclei together at (largest available) high energy

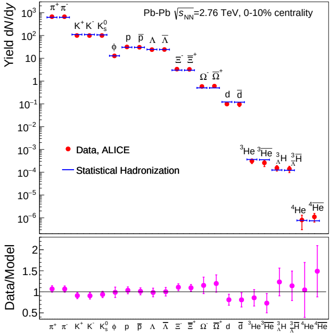


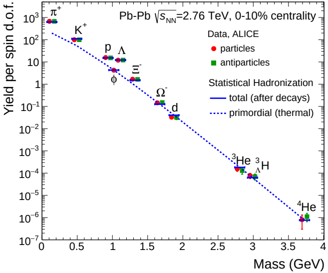
Some complex dynamics lead to final-state expanding ball of hadrons.

System gets close to equilibrium!

If system in equilibrium at temp T, chem potential μ, volume V

- Thermal statistical distribution $1/(\exp((-\mu_i + \sqrt{p^2 + m_i^2})/T) \mp 1)$
- Include unstable particles, but decay them (feeddown)
- Vary T, μ_B, V to fit particle yield data





Andronic

Braun-Munzinger

Redlich

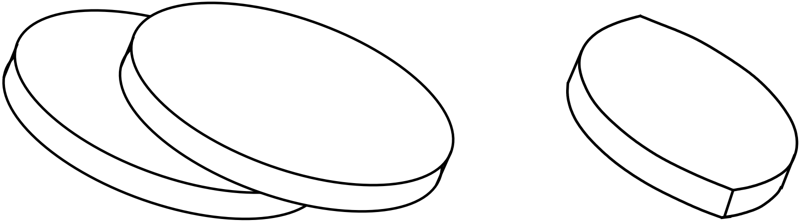
Stachel

arXiv:2101.05747

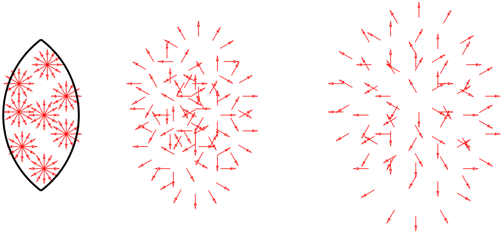
Works very well. (For pp or e^+e^- it does not.)

Initial geometry

Nuclear collisions generally not perfectly central leading to

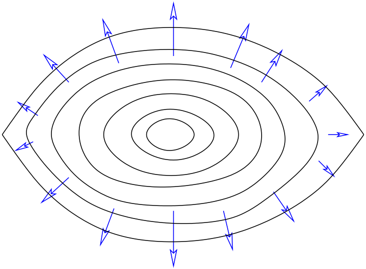


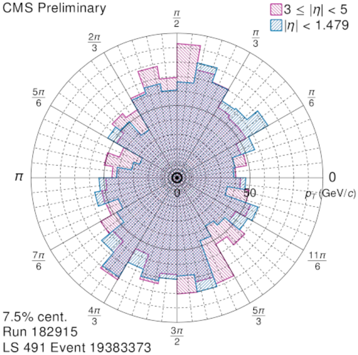
Without re-interactions: final state isotropic.



Re-interaction and final-state flow

Strong reinsteraction+fluid behavior:
Pressure contours imply system will “blow out” to the sides.
Coincides with appearance of real events (CMS):

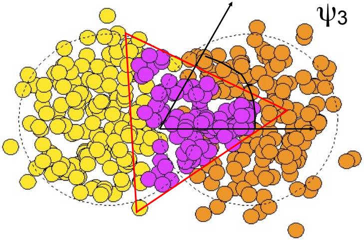


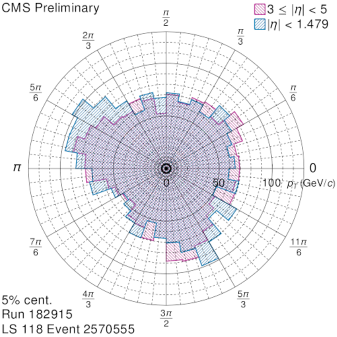


Fluctuations and Triangular Flow

Geometry of collision:
Elongated (elliptical) region
Random fluctuations:
triangular/square/pentagon....

Alver+Roland arxiv.org:1003.0194



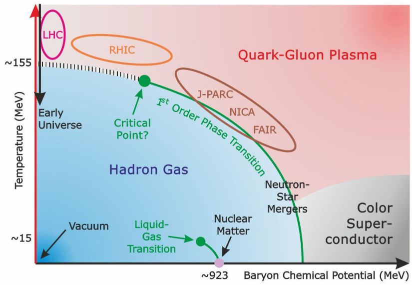


Some events clearly display this pattern.
Fluctuations in geometry are important.
system must be near-equilibrium to turn the geometry into flow in this way.

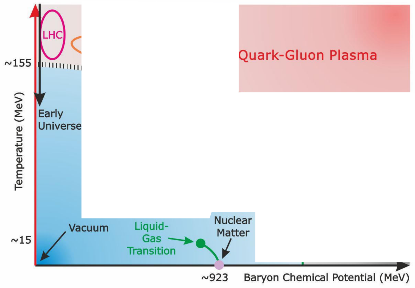
Makes sense to ask about thermodynamics

Thermodynamics

Thermodynamics means Phase Diagram + Equation of State.
So what do we expect – and what do we know for sure?



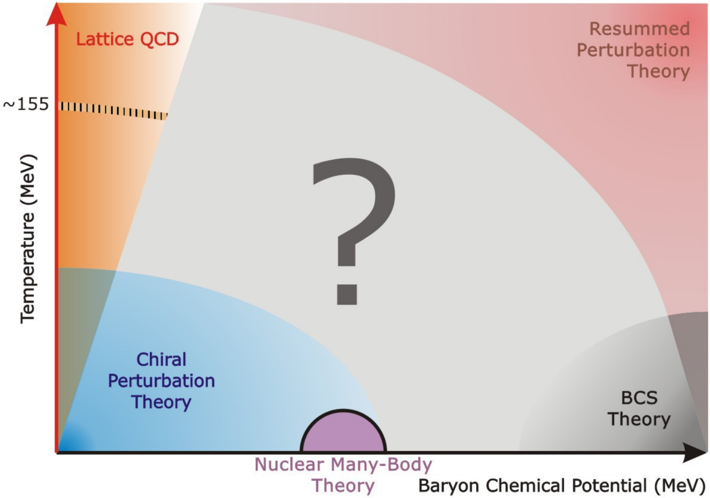
What we expect



What we know for sure

First-Principles Theory Tools

What first-principles tools do we have, and where do they work?



What makes it so hard?

Lagrangian from second slide

$$\mathcal{L} = \frac{1}{2g_s^2} \operatorname{Tr} G_{\mu\nu} G^{\mu\nu} + \sum_{i=udscbt} \overline{\psi}_i (\not{D} + m_i) \psi_i$$

Use this in the *Path Integral*

$$Z = e^{-iHt} = \int \mathcal{D}(G_\mu, \overline{\psi}, \psi) \; \exp\left(i \int d^4x \; \mathcal{L} \right)$$

Problems: Integral involves phases, phase cancellations
Stationary phases works *if* g^2 small.
But g^2 is scale-dependent, large at long distances
Perturbation theory expands about stationary phases –
works if all relevant scales are short distance (large T and/or μ)

Euclidean path integral

For thermodynamics, sufficient to explore

$$Z_{\text{therm}} = e^{-H/T} = \int \mathcal{D}(G_\mu, \overline{\psi}, \psi) \; \exp\left(- \int_0^{1/T} dt \int d^3x \, \mathcal{H} \right)$$

Now integral converges without phases (if $\mu_B = 0$ anyways).
Thermodynamic information all still there
But information about *dynamics* is now all *indirect.*

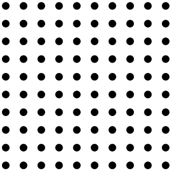
But it's one integration variable

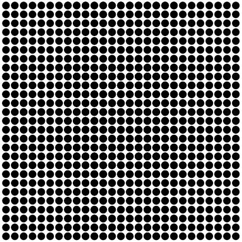
- per direction G_μ (4)
- per color component G_μ^a (4×8)
- per point in spacetime $G_\mu^a(x)$ $(4 \times 8 \times \infty^4)$

Lattice Gauge Theory

Replace continuous spacetime with finite-volume lattice

$$\text{Answer} = \text{Correct} + \mathcal{O}(a^2) + \mathcal{O}(\exp(-m_\pi L))$$



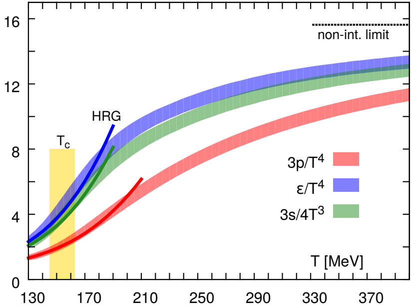


Lattice implementation preserves *gauge symmetry exactly.*
Fermions are trickier, but OK in $a \to 0$ limit.
Modern simulations: $\sim 1\%$ statistical + systematic errors for spectrum, thermodynamics $(P, \varepsilon, s, \chi_B, \ldots)$ etc.
Dynamics (Scattering, transport, etc) **much** harder....

Old-days of quenched approx. or unphysically heavy quarks are over.

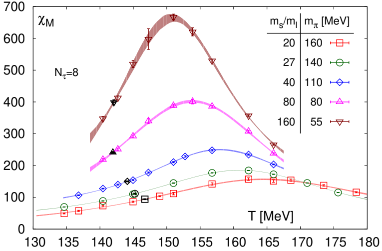
Some Nice Lattice Results

Smooth crossover from hadronic to quark-gluon behavior.
Would be sharper if up, down quarks were lighter (right)



Equation of State

HotQCD arxiv:1407.6387



Chiral susceptibility

HotQCD arxiv:1903.04801

Where the lattice struggles

Suppose we want finite density? Need chemical potentials μ.
Lattice techniques depend on importance-sampling

$$Z(\mu_q) = \int \mathcal{D}G_\mu \exp^{-\int d^4x\, \mathcal{H}_{\mathrm{bos.}}} \prod_{q=udsc} \mathrm{Det}\left(\not{D} + m_q + \mu_q \gamma^0 \right)$$

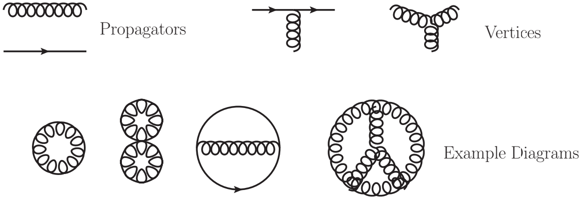
which requires $\mathcal{H}_{\mathrm{bos}}$ *and* Determinant to be real
Determinant generally complex for $\mu_i \neq 0$.

Perturb in small μ_q: results up to 6'th order.
Sufficient for a "slice" of μ values but not $\mu \gg T$.

Alternative techniques not yet quantitatively reliable at physical m_q.

Diagram-based techniques

Perturbation theory:
assume propagation
dominates interaction
do Loop Expansion

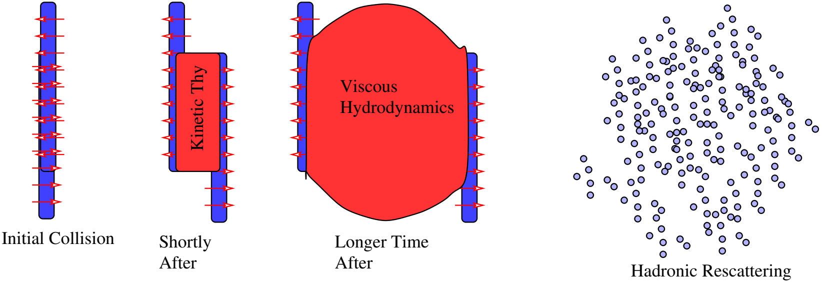


Convergence very poor without resummation techniques.
With resummations, work for $T > 350$ MeV.
"Low" T, n_B $(T < 100\,\mathrm{MeV}, \mu \lesssim m_p)$: works in terms of (π, n, p)

Functional Renormalization Group: resummation technique which also allows bound states, condensation, collective dynamics ..
Also challenging to apply at $\mu_q \gg T$.

What about Dynamics?

Total time scale $\sim 20\,\mathrm{fm}$ long compared to inverse-energies $p_\perp^{-1} \sim 0.4\,\mathrm{fm}$



Series of “epochs” which may need different physics descriptions

- Initial: statistical Glauber, pQCD, Saturation/Colored Glass
- Early dynamics: kinetic theory
- Intermediate times/densities: relativistic viscous Hydro
- Freezeout and hadronic interactions

Relativistic Hydrodynamics

Local thermal equilibrium: *conserved charge densities* $(\varepsilon, \vec{\pi}) = T^{\mu 0}, n_B, \rho_e, n_s$ determine everything.

Stress tensor $T^{\mu\nu} = \varepsilon u^\mu u^\nu + P\eta^{\mu\nu}\,,$ Currents $j^\mu_{B,e,s} = n_{B,e,s} u^\mu$

Their evolution determined by conservation equations

$$\partial_\mu T^{\mu\nu} = 0 \qquad \partial_\mu j^\mu_{B,e,s} = 0$$

Together with equation of state $P = P(\varepsilon, n_{B,e,s})$, equations close. Near equilibrium, derivative corrections

$$T^{\mu\nu} = T^{\mu\nu}_{\mathrm{eq}} - \eta\left(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\nabla\cdot u\right) - \zeta\Delta^{\mu\nu}\nabla\cdot u$$

Nice introduction: Teaney arXiv:0905.2433; Schäfer and Teaney arXiv:0904.3107

Viscous Relativistic Hydro

Agnostic about Degrees of Freedom. Works for

- weakly-coupled quarks + gluons
- strongly-coupled “stuff” with no particle description
- dense gas of hadrons

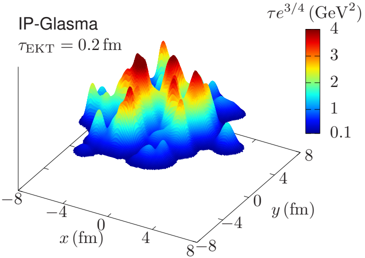
provided that mean free path $\ll$ system age.
Large systems: this works. (Gold/Lead at RHIC/LHC)
Small systems: we’ll see how/where it breaks down!

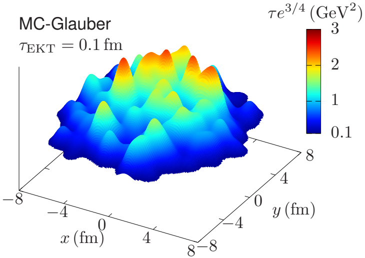
- Input required: Stress tensor from initial collision
- Output: energy+momentum density $\Rightarrow$ temperature+flow velocity
- Output: local final anisotropy

Convert into hadrons and feed into final-state scattering model

Initial Conditions

If Hydrodynamics applies at intermediate times,
All we need is the initial **Stress Tensor** $T^{\mu\nu}(\mathrm{early})$.



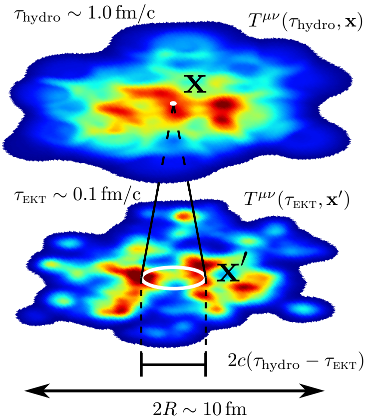


Different models on market, eg, IPGlasma (left), TRENTo (right)
Shown: 2D slice of initial energy density, to be fed into hydro

See eg Kurkela *et al* arXiv:1805.00961

Which model is “right” (or “best”)? Let the data decide.

Early-time dynamics



Early dynamics propagate $T^{\mu\nu}$ a short distance,
smoothing initial features...

Kurkela *et al* arxiv:1805.01604

Including this dynamics weakens dependence on “thermalization time” which may be ~ 1.5 Fermi/c.

Late-time Hadronic Interactions

Old approach: end Hydrodynamics abruptly at “freezeout time”
Switch from perfectly-strong scattering to free streaming.

Modern approach: end Hydrodynamics when system is hadronic.
Switch to hadronic description but keep tracking hadronic scattering
At time of switch, hadronic description approximately hydrodynamic
Insensitive to exact time+details of switch

Interplay between stages

Each phase has unknowns. Roughly

- Initial/early: how much energy density, how clumpy?
- Hydrodynamics: Transport coefficients η, ζ, σ as functions of (T, n_B)
- Transition to hadrons: Transition temperature. How $T^{\mu\nu} - T^{\mu\nu}_{\text{equil.}}$ shared between high/low momentum particles
- Final rescattering: details of hadronic interactions

Accommodated with *fitting parameters* which are varied to reproduce data

Old approach: experts on one aspect would vary fitting parameters in *their* stage of the process, holding other stages fixed.
Problem: changing, say, η/s changes best-fit initial energy density.
Choices in other stages influence best-fit in practitioner's stage.

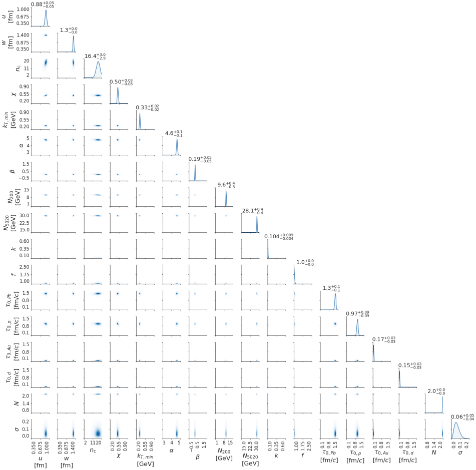
Modern Bayesian approach

Collaboration with expertise in all phases JETSCAPE, or `arXiv:2306.08665`

Code which interfaces output of one phase to input of the next

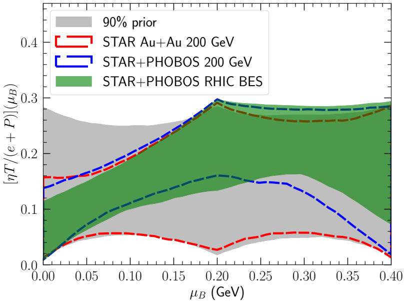
Fit *all* data while varying *all* parameters in grand-fit Bayesian analysis

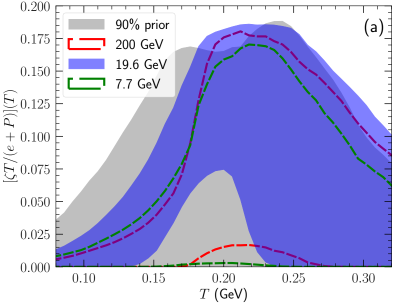
Parameter	**Symbol**	**Range**
Form width [fm]	u	0.35 – 1.0
Nucleon width [fm]	w	0.35 – 1.0
Constituent number	n_c	2.0 – 20.0
Structure	χ	0.2 – 0.9
Transverse mom. scale	$k_{\mathrm{T,min}}$	0.2 – 0.9
Shape parameter	α	3.0 – 5.0
Shape parameter	β	-0.5 – 1.5
Fireball norm. [GeV] ($\sqrt{s_{\mathrm{NN}}} = 200$ GeV)	N_{200}	1.0 – 15.0
Fireball norm. [GeV] ($\sqrt{s_{\mathrm{NN}}} = 5.02$ TeV)	N_{5020}	15.0 – 30.0
Fluctuation	k	0.1 – 0.6
Flatness	f	1.0 – 2.5
Hydrodyn. time [fm/c] (Pb–Pb 5.02 TeV)	$\tau_{0,\mathrm{Pb}}$	0.1 – 1.5
Hydrodyn. time [fm/c] (p–Pb 5.02 TeV)	$\tau_{0,p}$	0.1 – 1.5
Hydrodyn. time [fm/c] (Au–Au 200 GeV)	$\tau_{0,\mathrm{Au}}$	0.1 – 1.5
Hydrodyn. time [fm/c] (d–Au 200 GeV)	$\tau_{0,d}$	0.1 – 1.5
Overall scale	N_{scale}	0.8 – 2.0



Shear and bulk viscosity

Old goal: find η/s of Quark-Gluon Plasma.
New goal: find T, μ_B dependence of $\eta/s, \zeta/s$





Shen Schenke Zhao arXiv:2310.10787

Hard probes

Initial collision also produces

- Heavy quarks
- High-energy quarks and gluons $\Rightarrow$ jets

Initial production (mostly) under control based on pp ...

Propagation through medium modifies energy distribution
Opportunity to learn medium properties / predict modification

Coupling large enough that many interactions nonperturbative
Makes quantitative first-principles predictions challenging

Well characterized

First-principles calculations already do a good job on

- Equation of state at $\mu_B/T < \pi$ from the lattice
- Hydrodynamic description of high-energy heavy-ion systems
- Extraction of η/s etc from data using Bayesian framework
- First-principles dynamics at extreme (unachievable) temperatures

Active areas of investigation

We are making progress but there is much work to be done

- Initial conditions
- Early-time dynamics
- Hydrodynamics – or not? for small systems
- Hadronic reinteractions, late-stage evolution
- Hard probes – modification of heavy quarks and high-energy jets
- Equation of state at $T \sim 100$ MeV, $\mu_B \sim 1$ GeV
- Transport from the lattice
- Learning Equation of State from neutron stars $+$ NS mergers

Conclusions

- Heavy ion collisions give a window into many-body Quantum Chromodynamics
- Thermodynamics at small μ_B: first principles lattice methods. Mature field, results well under control
- Thermodynamics at larger μ: theoretically much harder. Phase diagram partly conjecture
- Dynamics: Heavy Ion Collisions progress in stages:
  Initial conditions, early evolution, near-equilibrium/hydro, final-state hadrons
- Substantial progress in last 20+ years, with more to come!