

3-BODY PROBLEM FROM PHENOMENOLOGY AND LATTICE QCD

MAXIM MAI

University of Bonn | The George Washington University



NSF PHY-2012289



DOE DE-SC0016582

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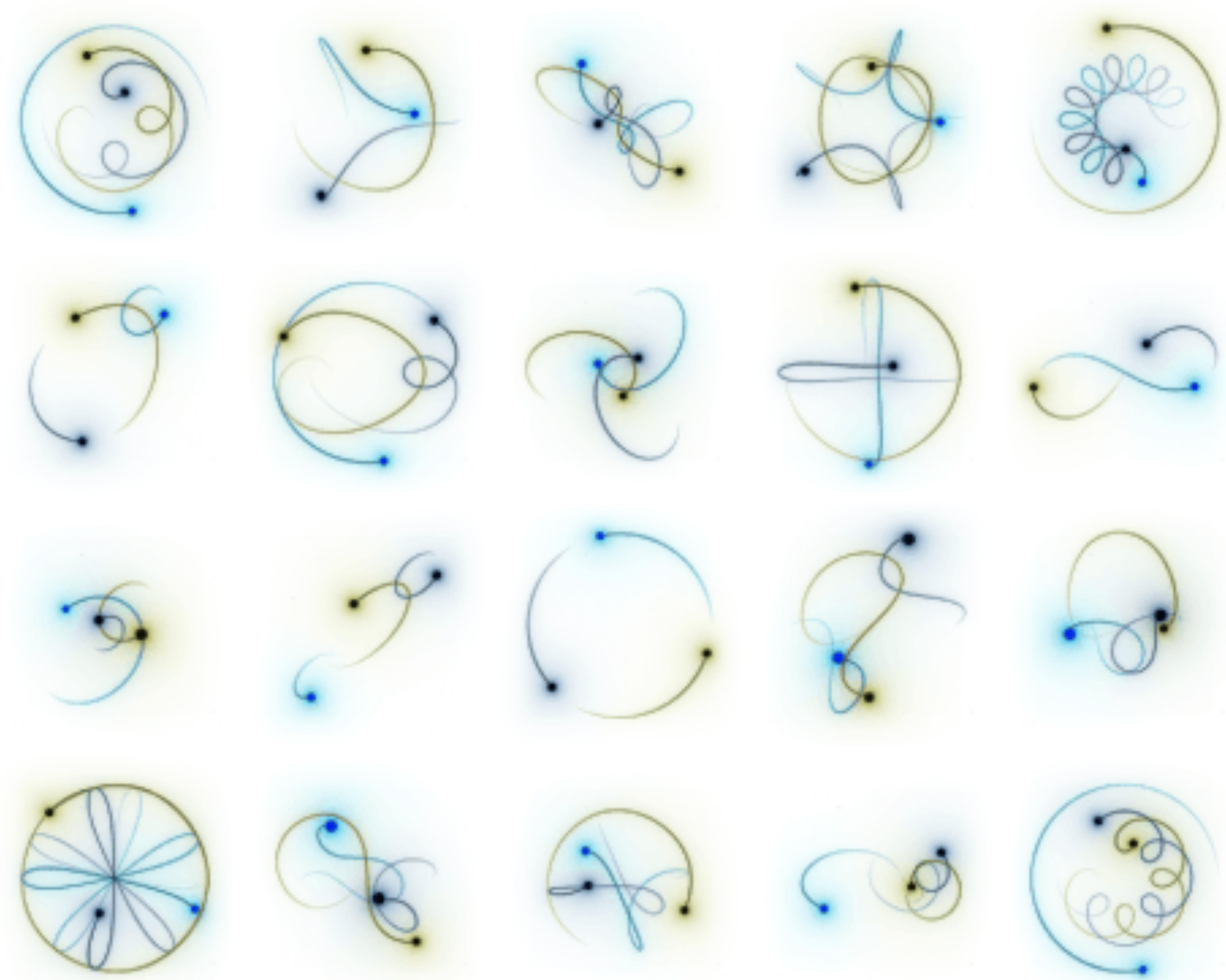
DFG CRC 110



THREE-BODY PROBLEM

Gravitational three-body problem

- goal: space-time trajectories
- challenges:
 - no closed solutions
 - in general non-repeating (few exceptions^[1])
- birth of mathematical chaos^[2]



[1] Šuvakov/Dmitrašinović PhysRevLett.110.114301 (2013)

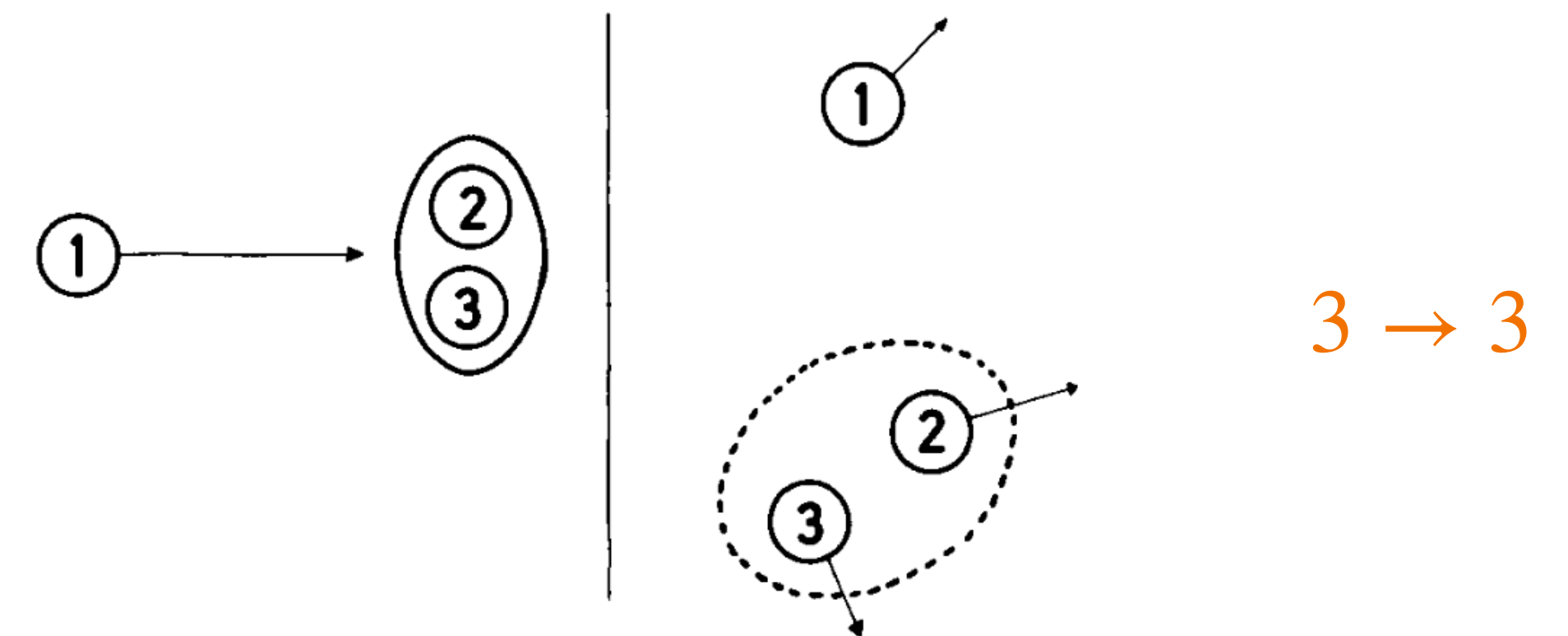
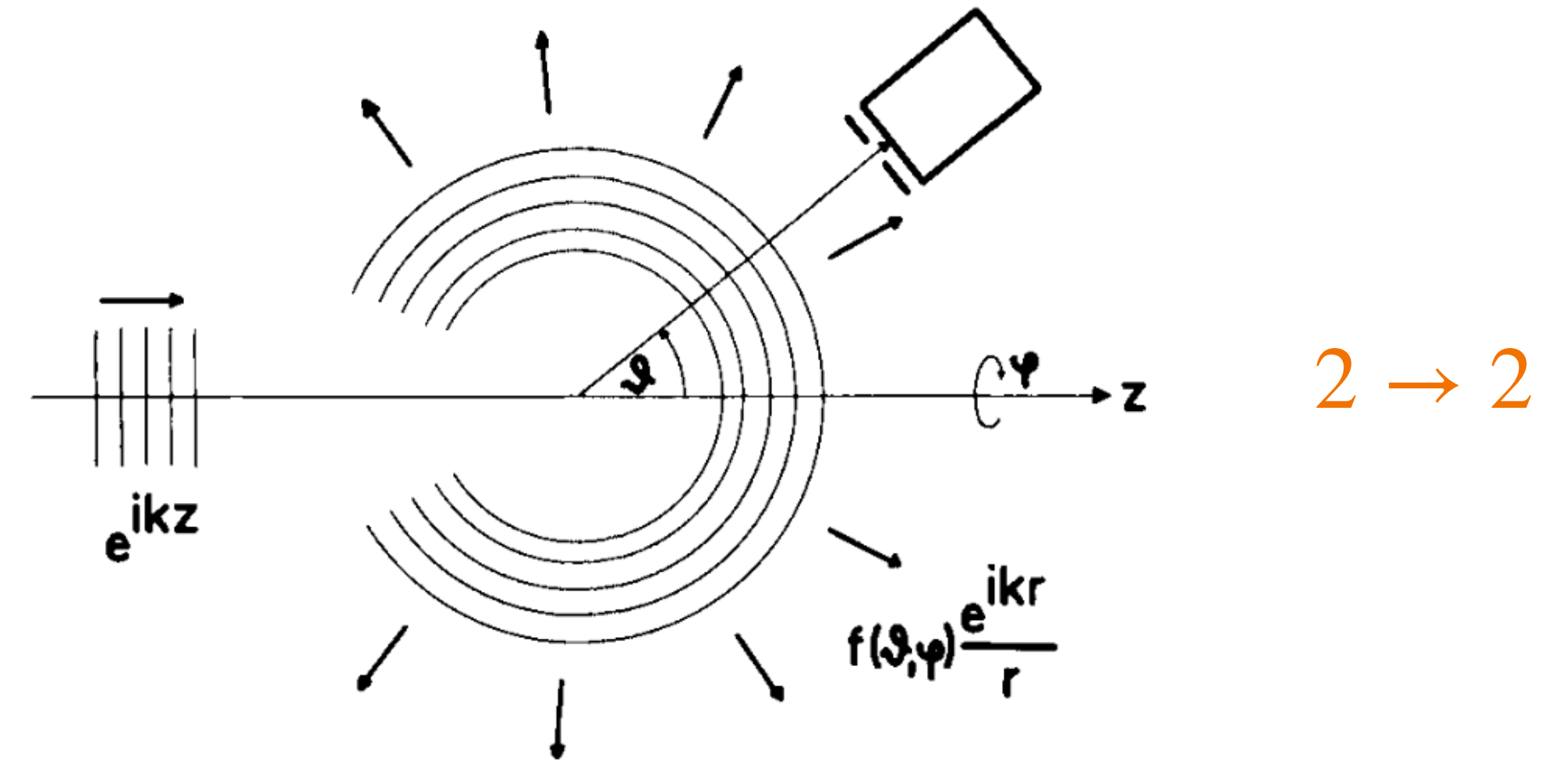
[2] Poincaré

[FIG] Adapted from Canagrisa, CC BY-SA 4.0 via Wikimedia Commons

THREE-BODY PROBLEM

Quantum mechanical three-body problem

- goal: rigorous scattering theory
- challenges:
 - continuum of two-body scattering states^[1]
 - 8 kinematic degrees of freedom

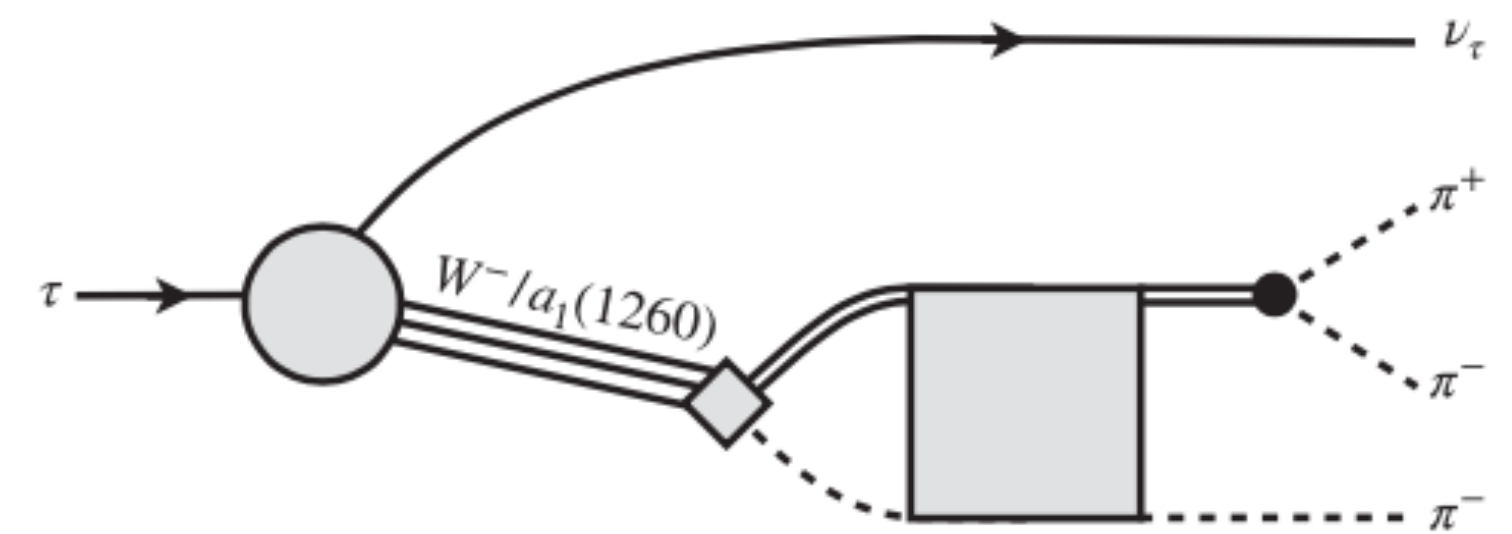
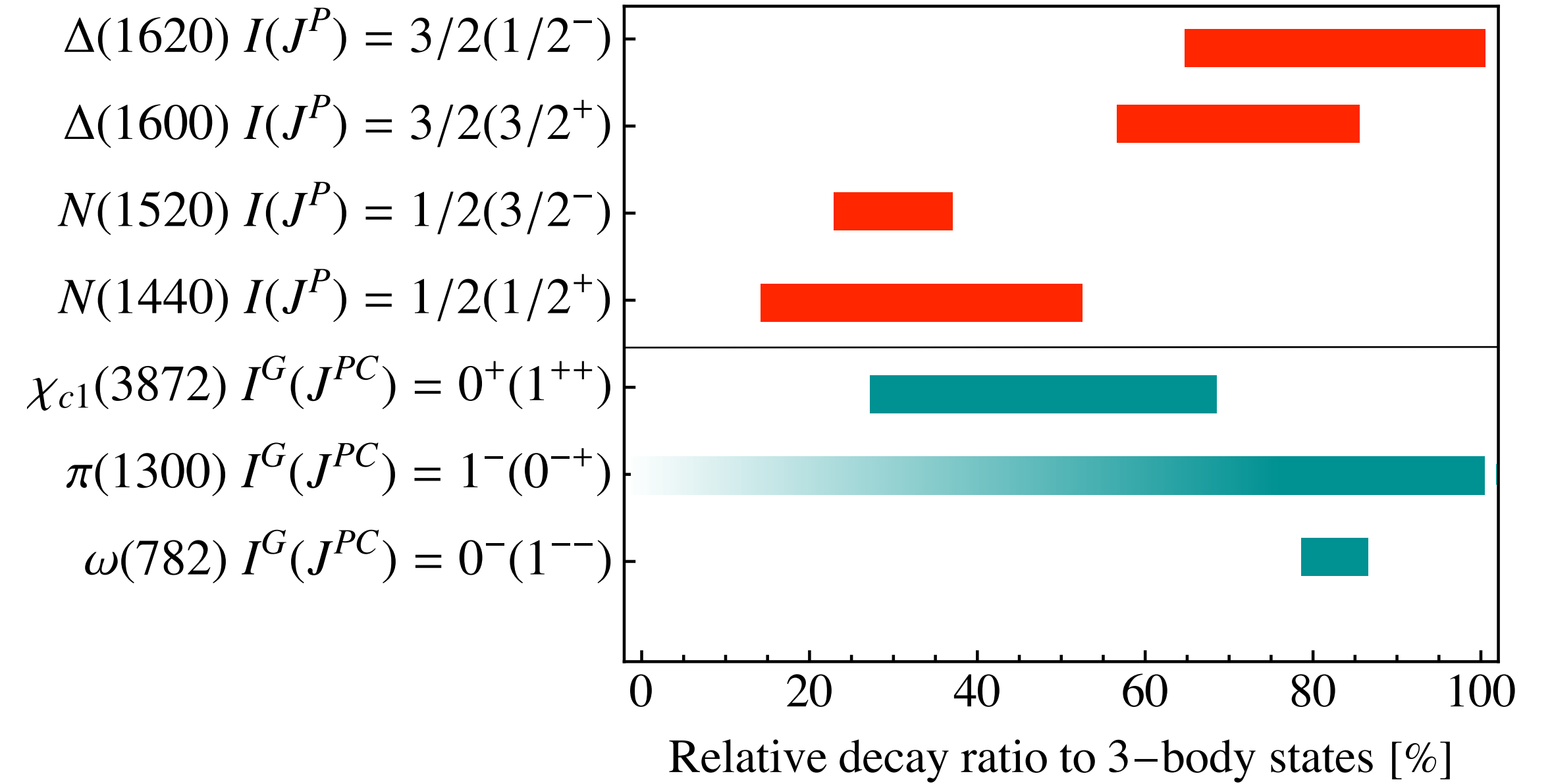


[1] Faddeev,
[FIG] Schmid/Ziegelmann Pergamon Press 1974

HADRON SPECTRUM

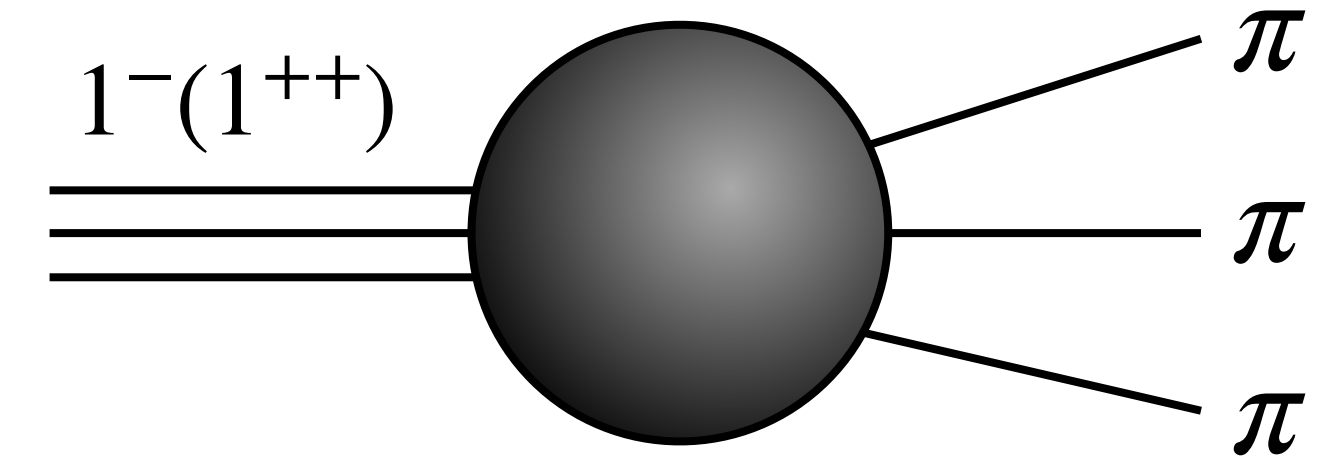
Impact

- Many known states have large 3-body content
 - Roper $N(1440)$
 - $X(3872)$
 - $a_1(1260)$, $a_1(1420)$?
- Beyond Standard Model searches (τ -EDM/...)
- Exotic states of matter^[1]



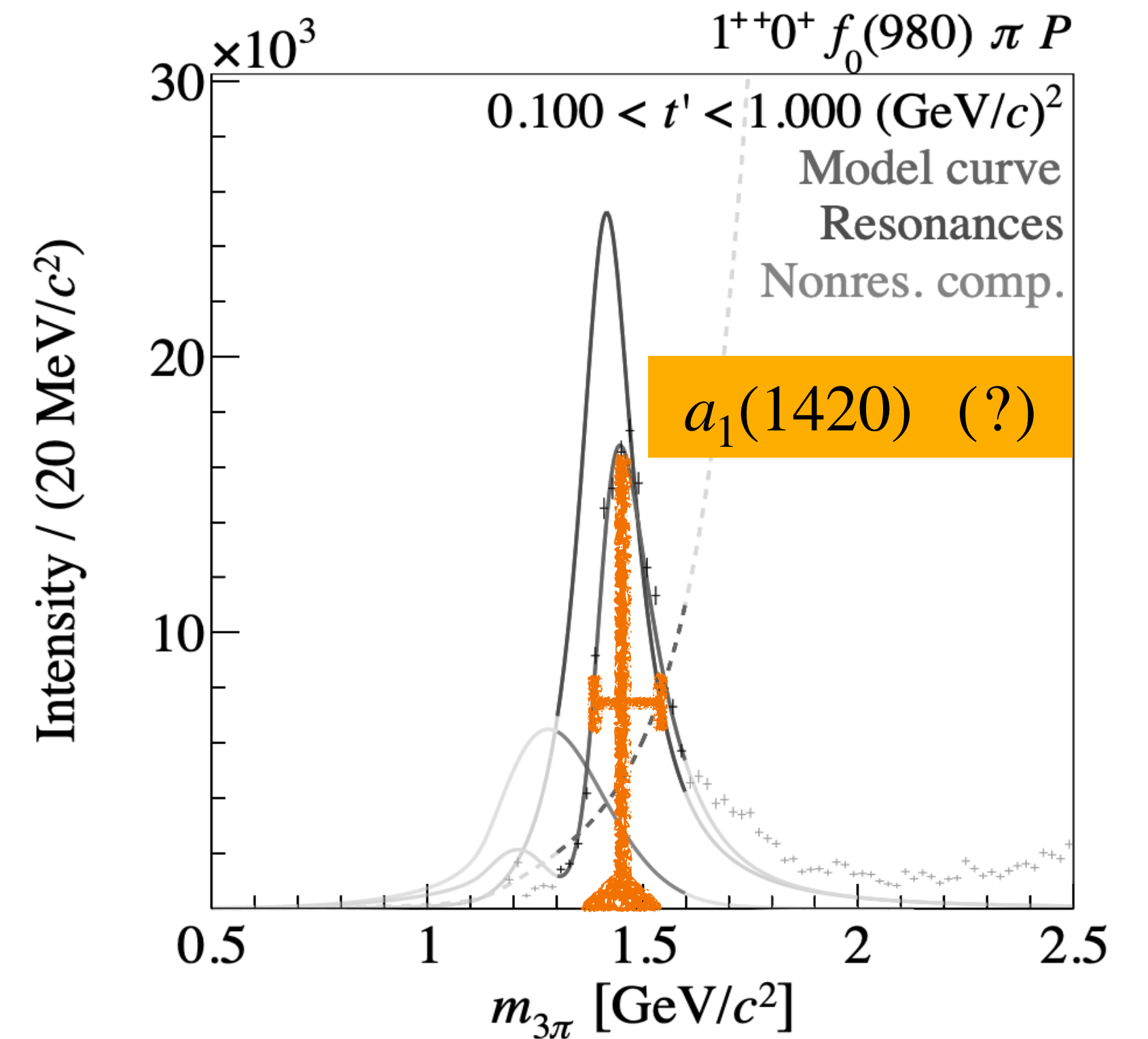
[1] Experimental programs: GlueX@JLAB; COMPASS@CERN;
 [FIG] Data from Workman et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2022, 083C01 (2022)

RESONANCE PARAMETER



Experimental input

- many high-precision experiments^[2] → line-shapes resonances ↔ increased interaction rates
- mod reaction-type
- mod kinematic singularities^[3]

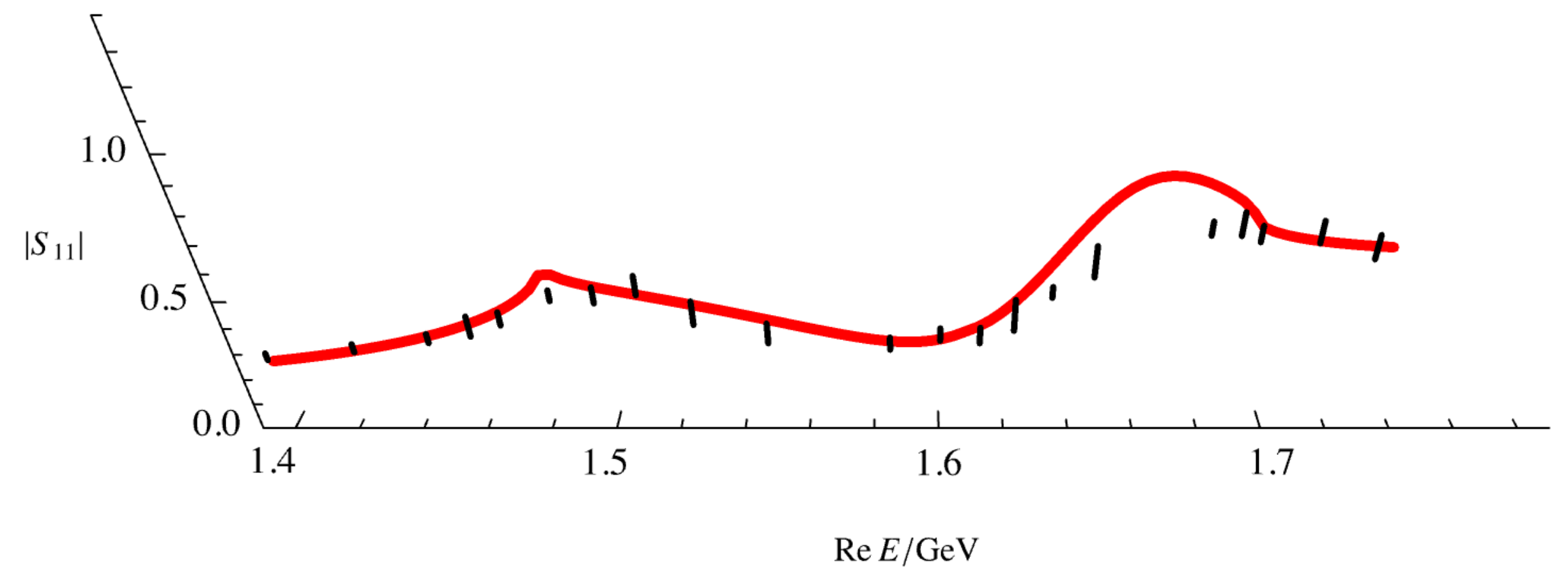


[2] CLAS12, GlueX, ...
[3,FIG] [COMPASS] Phys.Rev.Lett. 115 (2015) 8. Review: Ketzer/Grube/Ryabchikov Prog.Part.Nucl.Phys. 113 (2020) 103755

RESONANCE PARAMETER

Universal resonance parameter

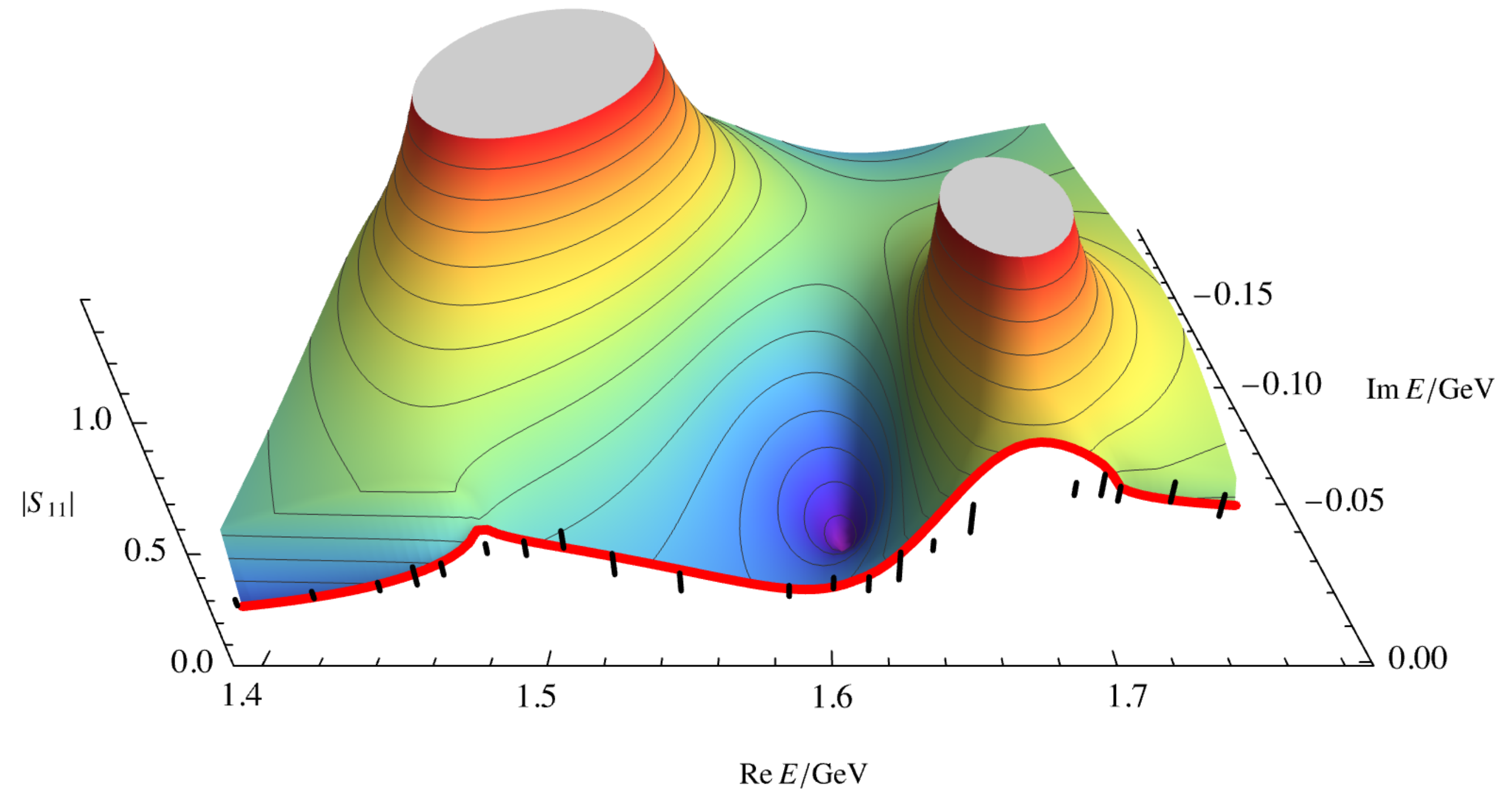
- S-matrix theory: *transition amplitude*
 - Unitarity/Analyticity/Crossing symmetry
 - Poles on unphysical Riemann Sheets
- Boundary ($E \in \mathbb{R}$):
 - Experiment
 - Lattice QCD
 - CHPT



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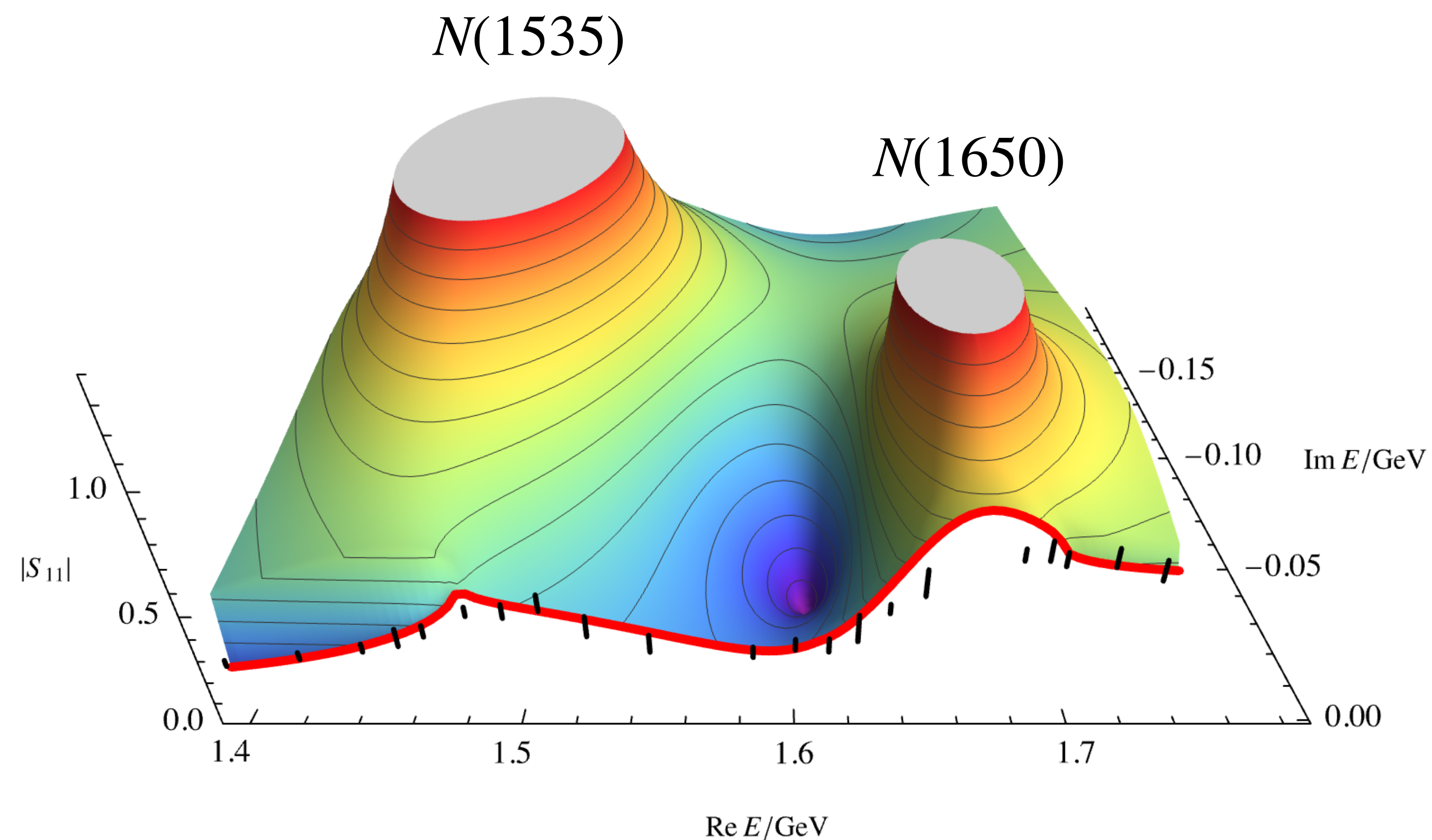
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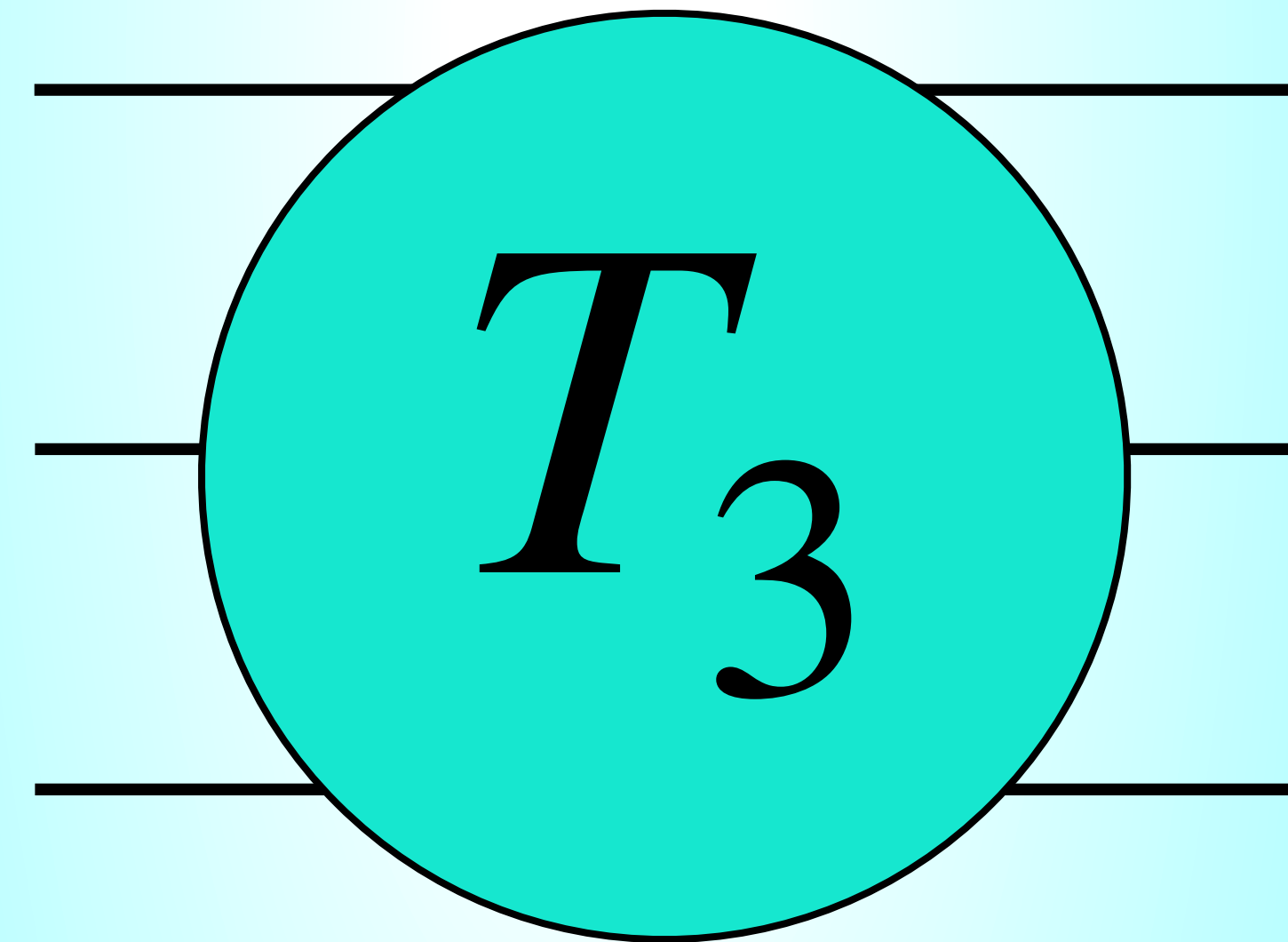
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Tridge (Midland, MI/USA)

**TRANSITION
AMPLITUDE**



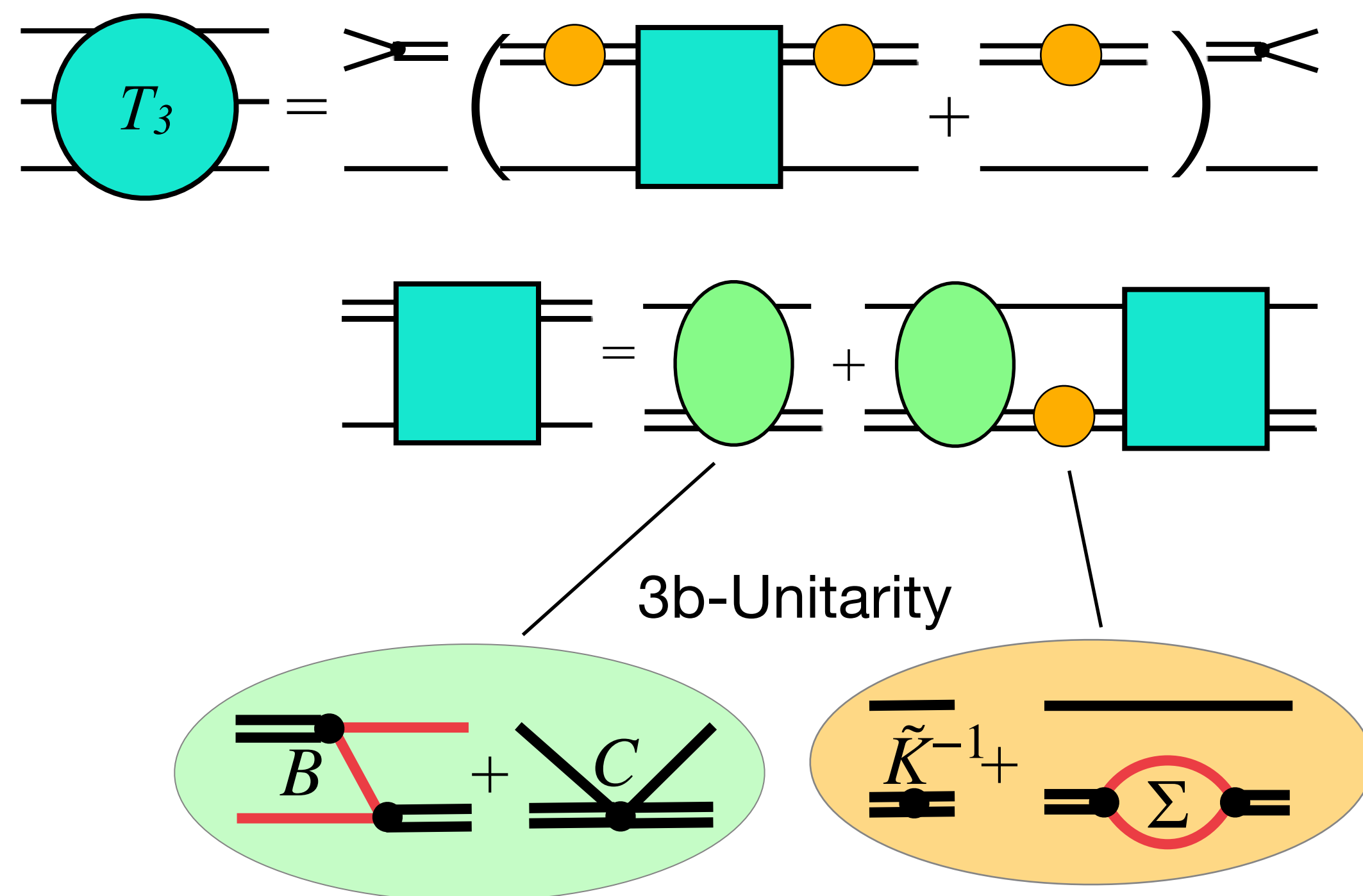
TRANSITION AMPLITUDE

“Infinite Volume Unitarity” – IVU formalism^[1]

- Express 3-body through a 2+1 system^[2]
 - Unitarity(on-shell configurations): $B, \Sigma \in \mathbb{C}$
 - Dynamics(input): $C, \tilde{K} \in \mathbb{R}$

IVU

$$T^c = B + C + \int \frac{d^3\ell}{(2\pi)^3} \frac{(B + C)}{2E_\ell} \frac{1}{\tilde{K}_n^{-1} - \Sigma_n} T^c$$



[1] MM/Hu/Döring/Pilloni/Szczepaniak Eur.Phys.J.A 53 (2017)

[2] Related approaches: Hansen/Sharpe(2014)...; Wunderlich et al. JHEP 08 (2019); Jackura et al. Eur.Phys.J.C 79 (2019);

HILBERT'S HOTEL

“Infinite Volume Unitarity” – IVU formalism

- Analytic structure of the one-particle exchange
 - Left-hand cuts^[1] – $T_{cc}(3875)$ etc..
 - Landau singularities
 - Triangles^[2] + Boxes + Boxes+...^[3]



<https://www.ias.edu/ideas/2016/pires-hilbert-hotel>

[1] Du et al. Phys.Rev.Lett. 131 (2023) 13; Hansen et al. 2401.06609 [hep-lat]

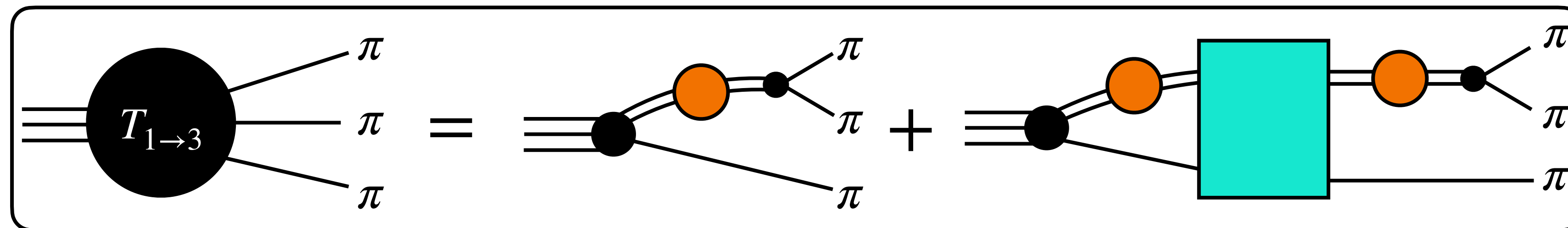
[2] Korpa/Lutz/Guo/Heo Phys.Rev.D 107 (2023) 3; Isken et al. 2309.09695; ... Ketzner/Mikhashenko/Aceti/Dai/Oset/Bayar/Guo...

[3] Sakthivasan/MM in preparation

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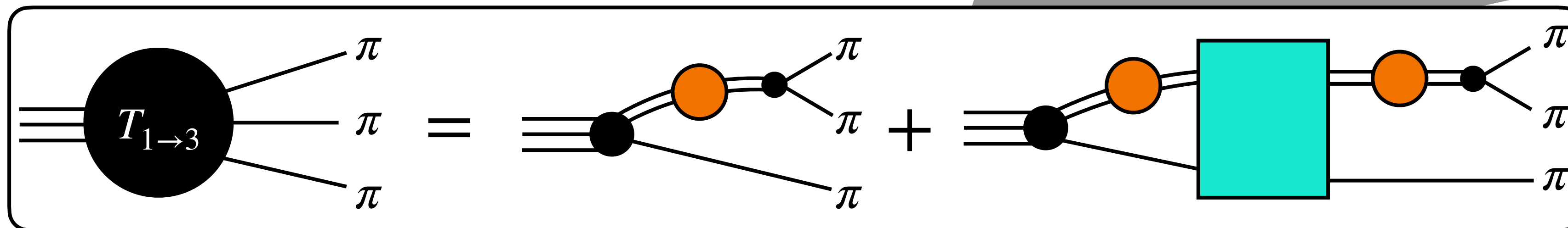
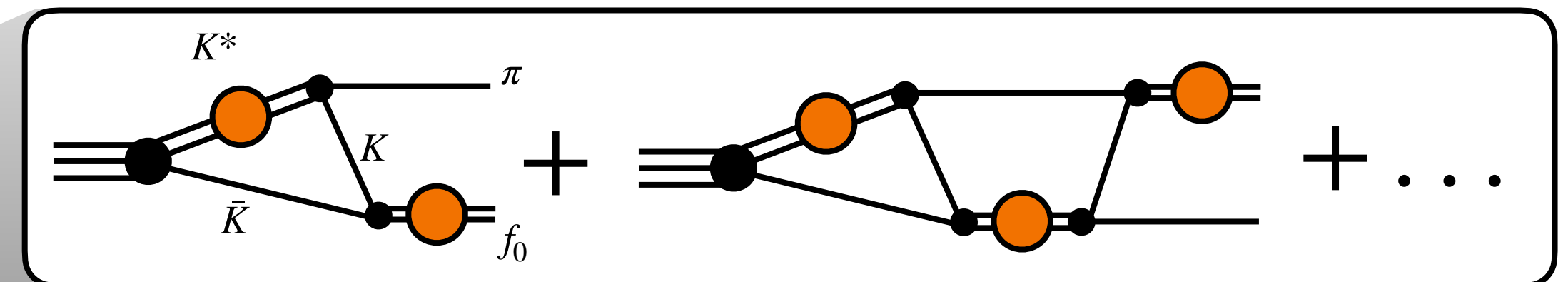
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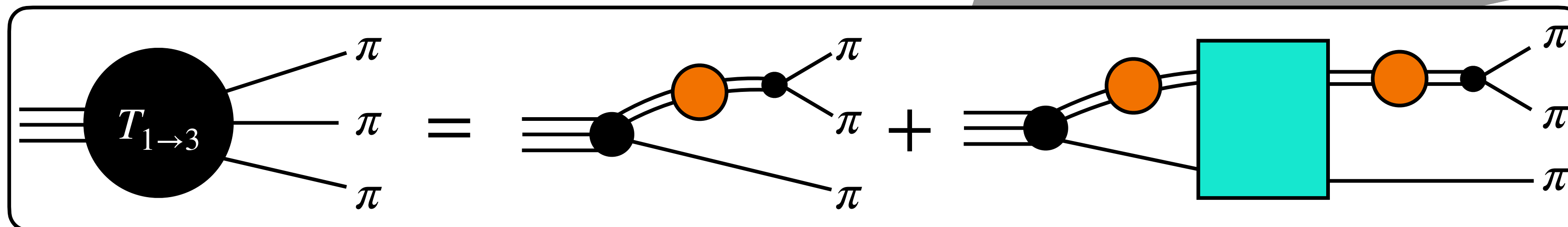
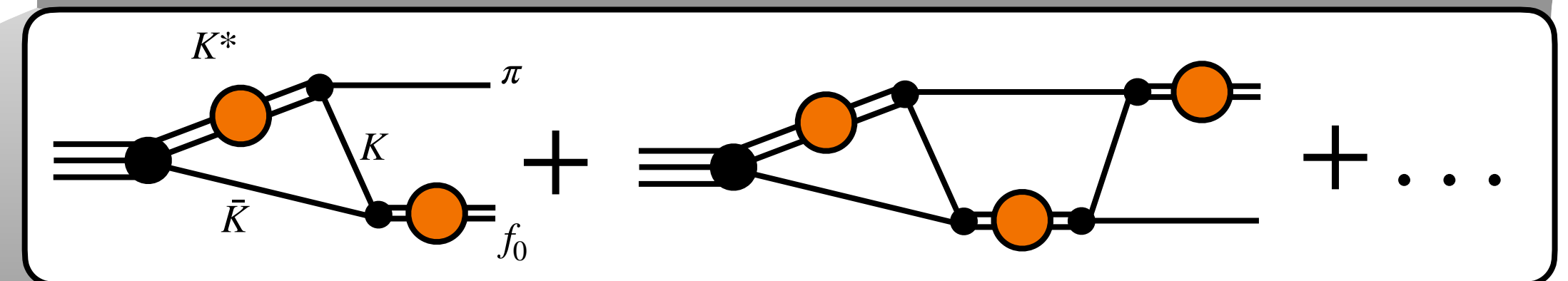
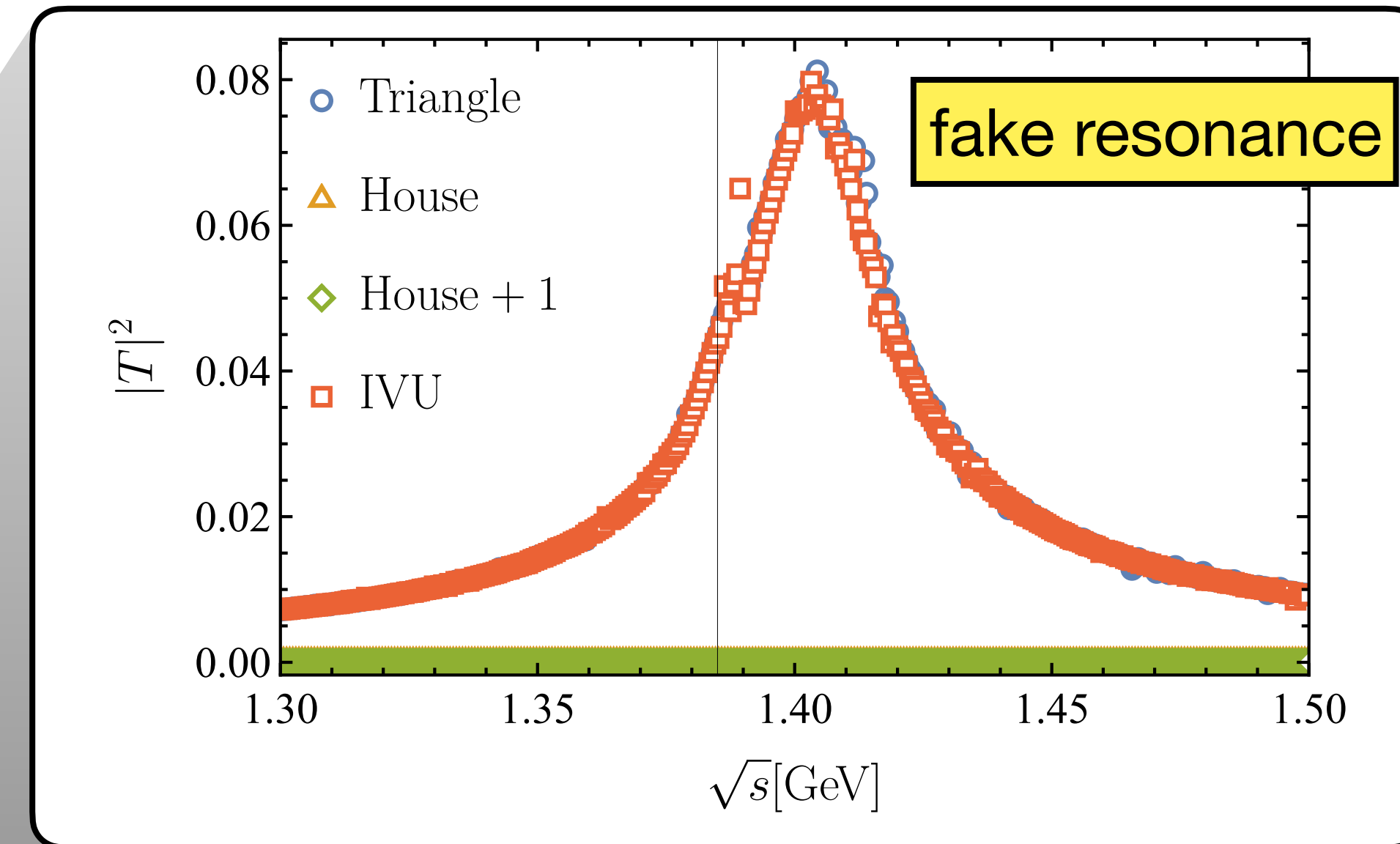
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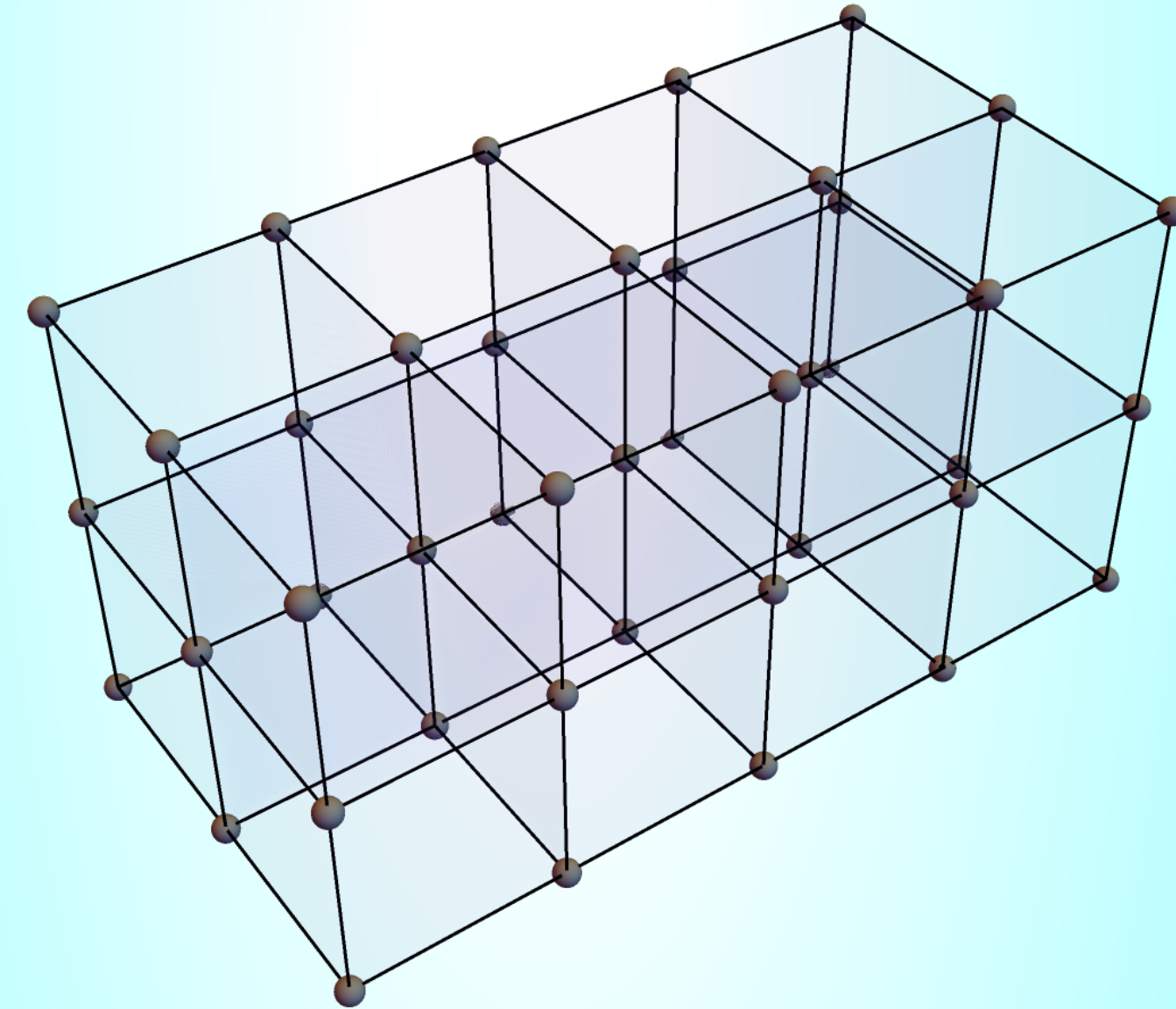
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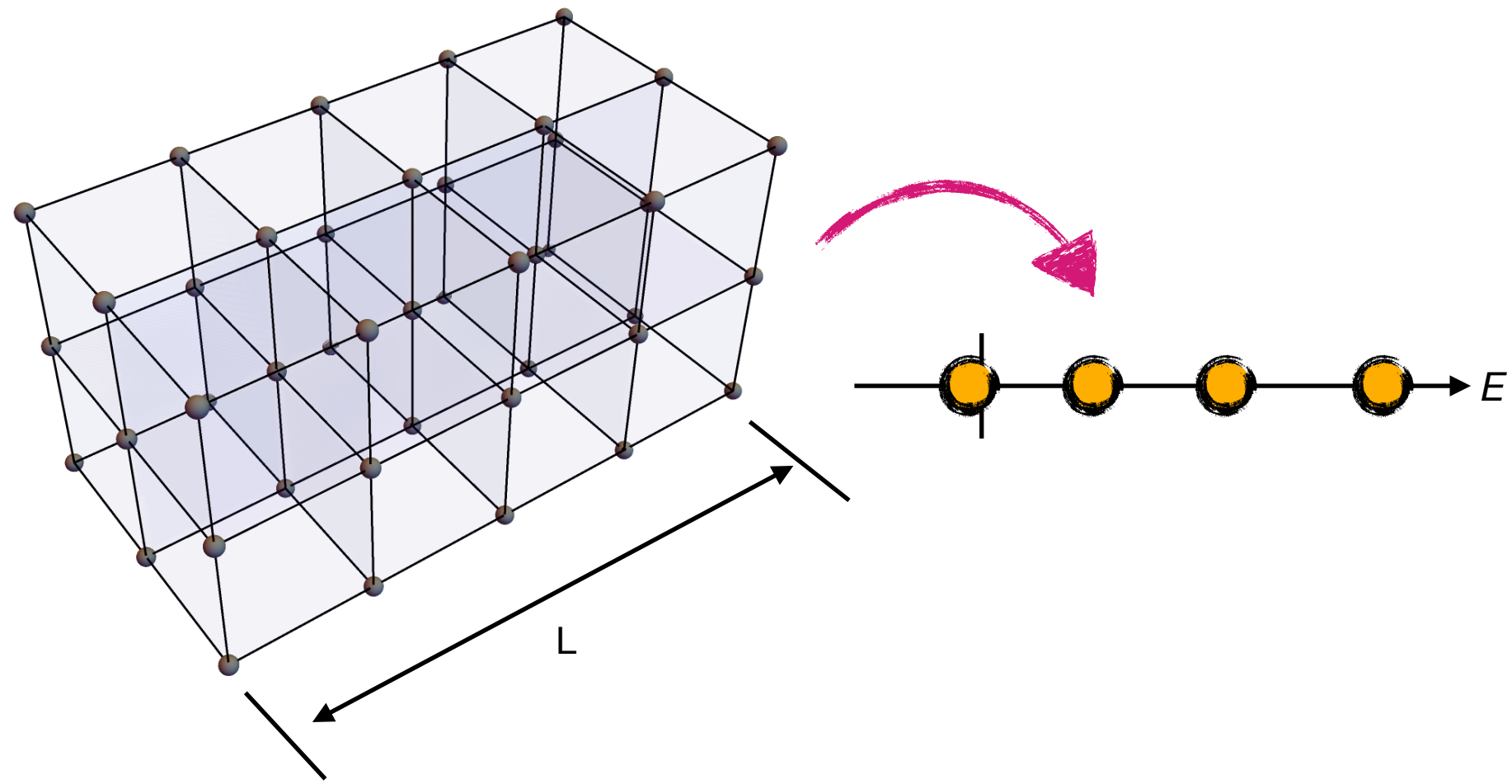
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APPLICATIONS TO LATTICE QCD

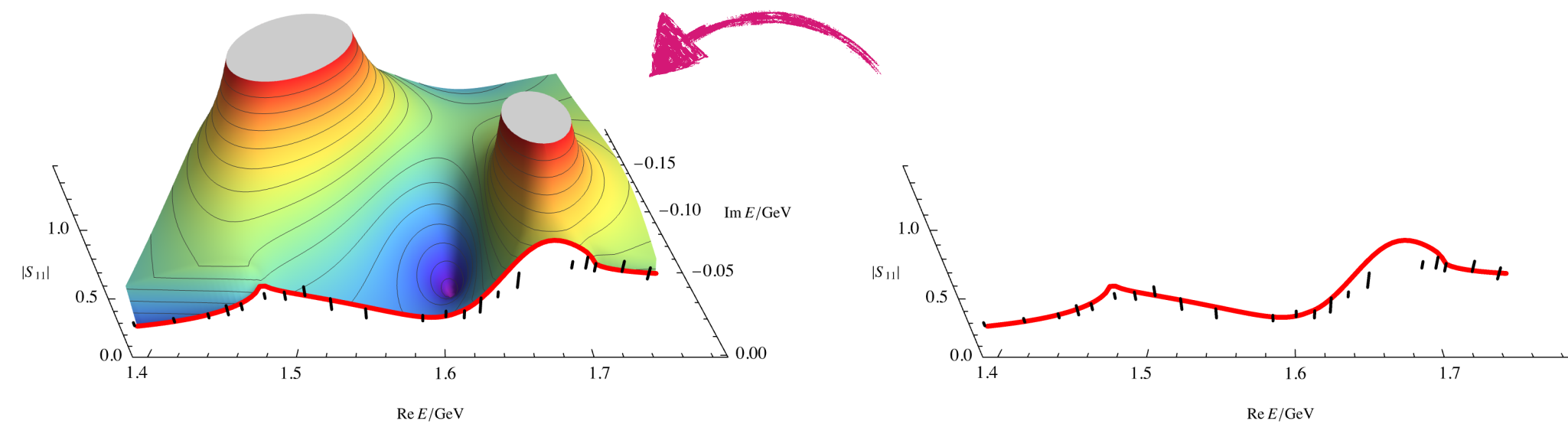


FINITE-VOLUME SPECTRUM

Lattice QCD: numerical access to QCD Green's functions:
Euclidean space-time / unphysical pion mass / **finite-volume**



S-matrix, phenomenology, experiment...

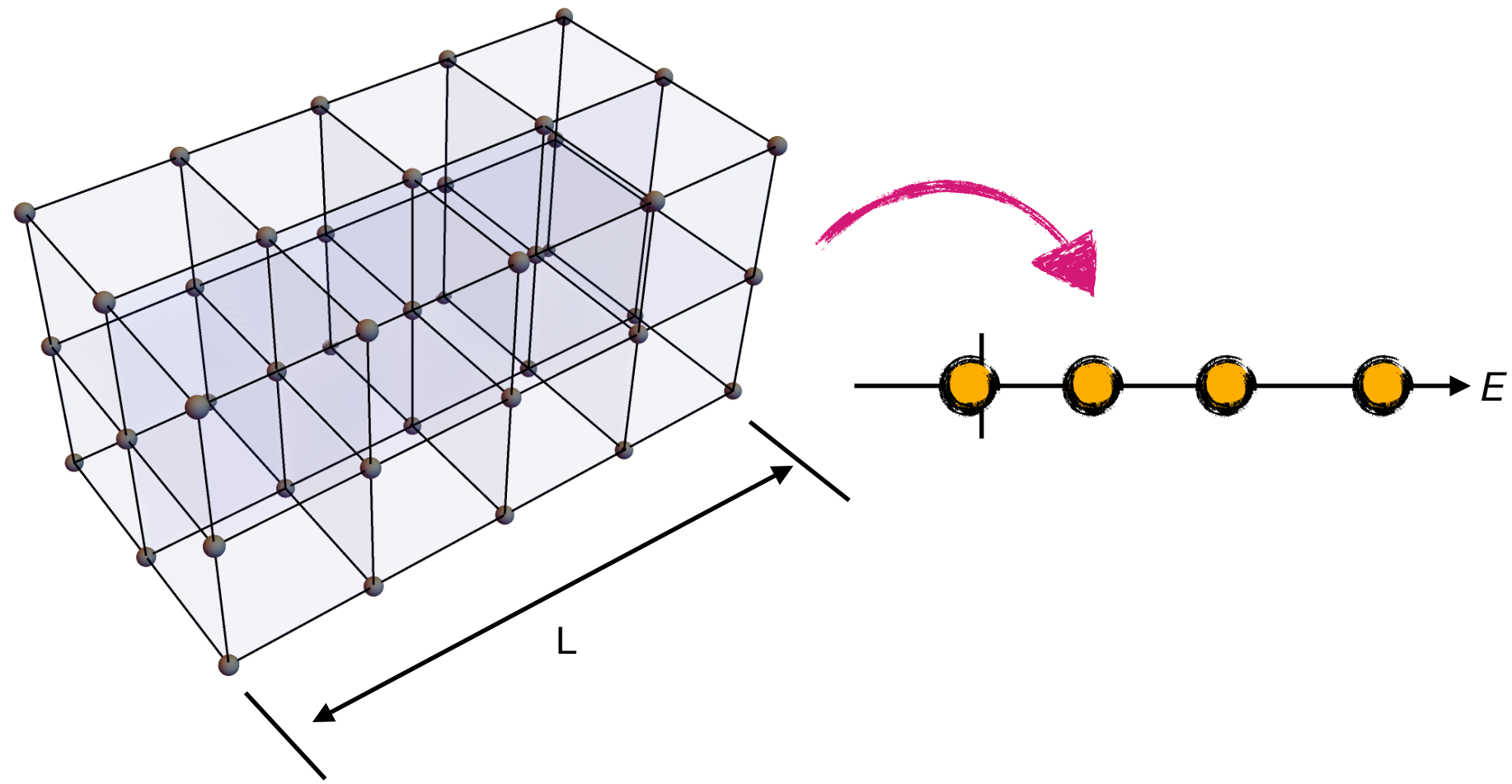


[1] Lüscher, Gottlieb, Rummukainen, Feng, Li, Döring, Briceño, Meißner, Rusetsky, Hansen, MM, Blanton, ...

Reviews: Briceño/Dudek/Young (2017) Rev.Mod.Phys. 90 (2018) 2 Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019); MM/Doring/Rusetsky Eur.Phys.J.ST 230 (2021);

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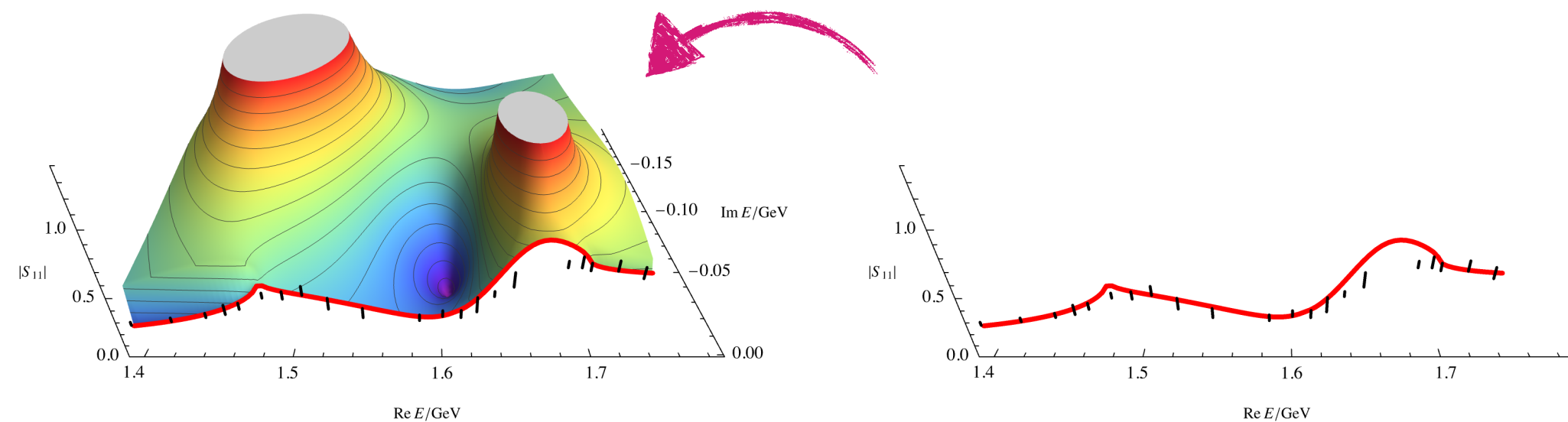
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S-matrix, phenomenology, experiment...

“Finite Volume Unitarity” – FVU formalism

- On-shell particles “feel” the box-size
- Three-body **quantization condition**

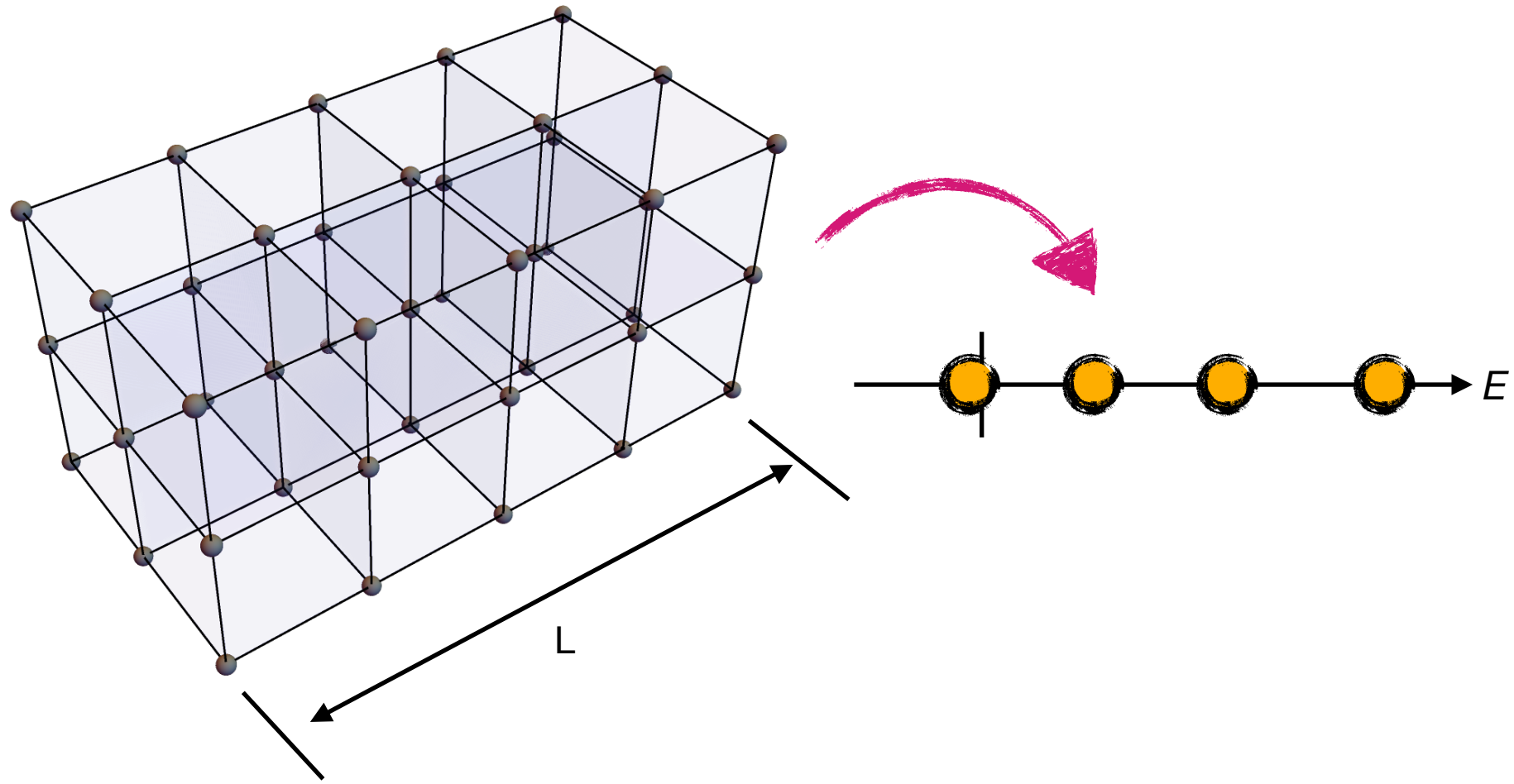


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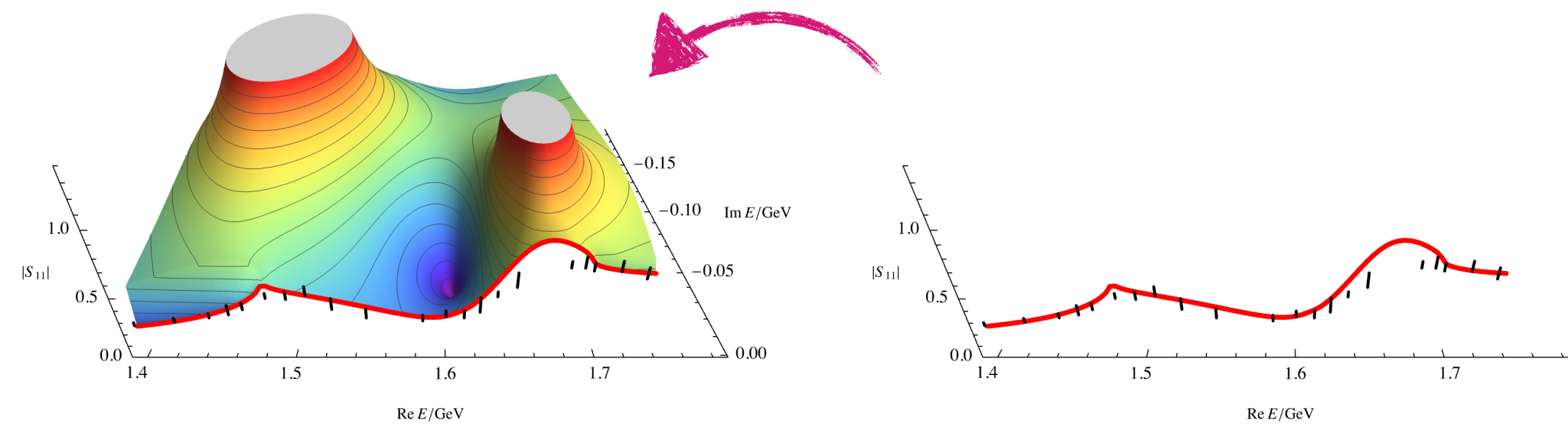
FVU

$$\det \left[2L^3 E_p \left(\tilde{K}_2^{-1} - \Sigma_2^L \right) - B - C \right]^{T_{1g}}$$

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IVU

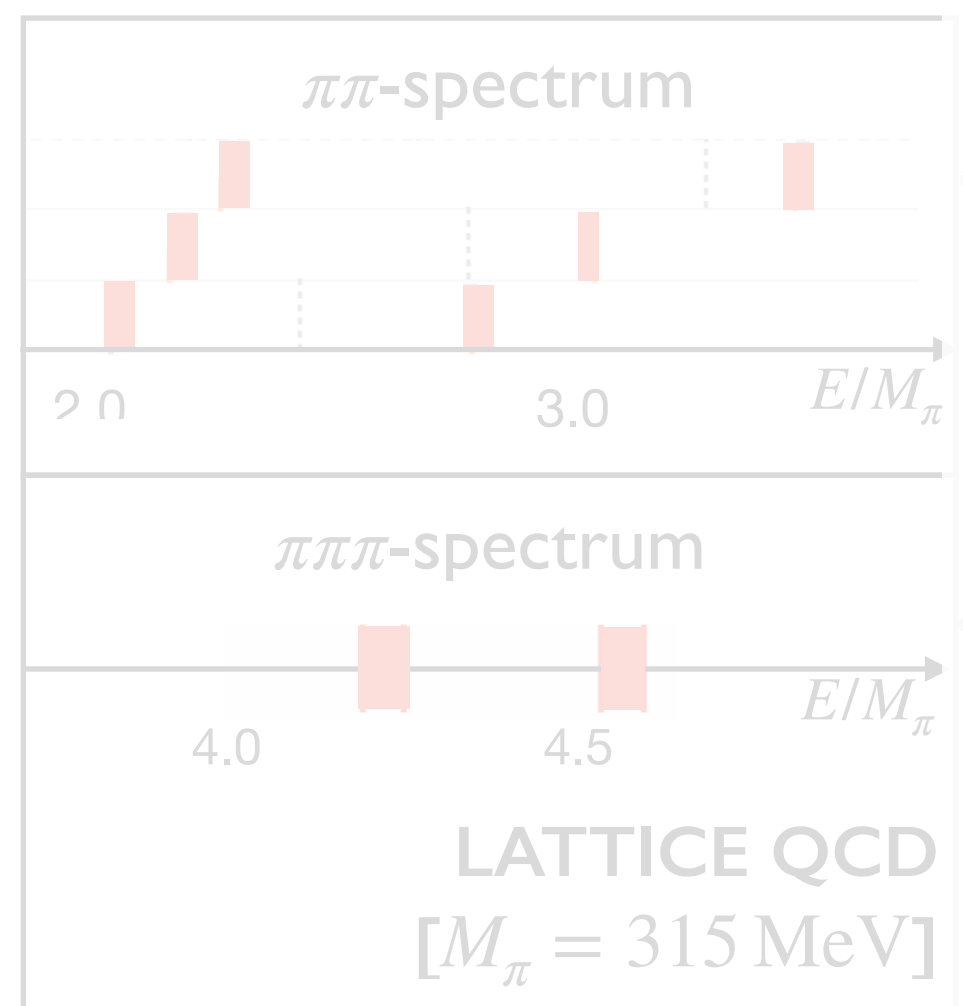
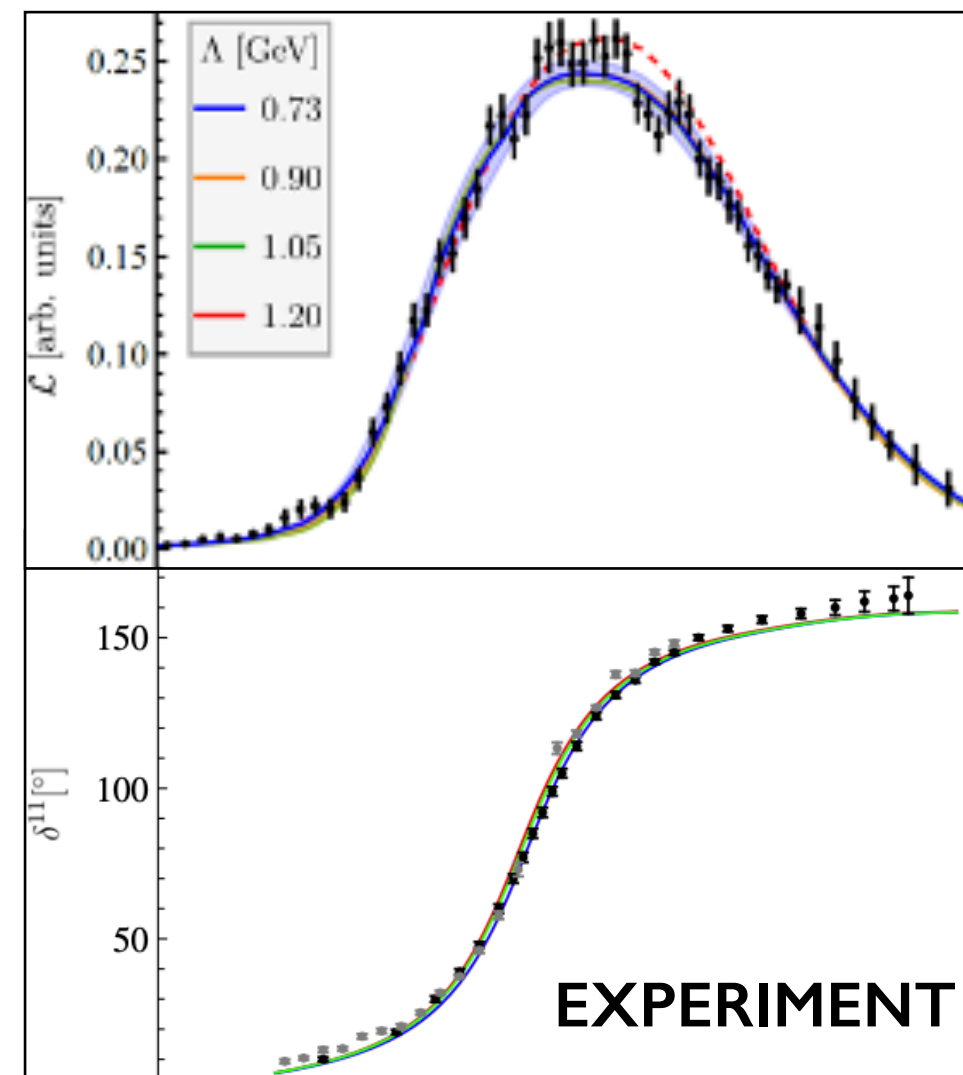
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BLUEPRINT — $a_1(1260)$

INPUT[1]



TRANSITION AMPLITUDES

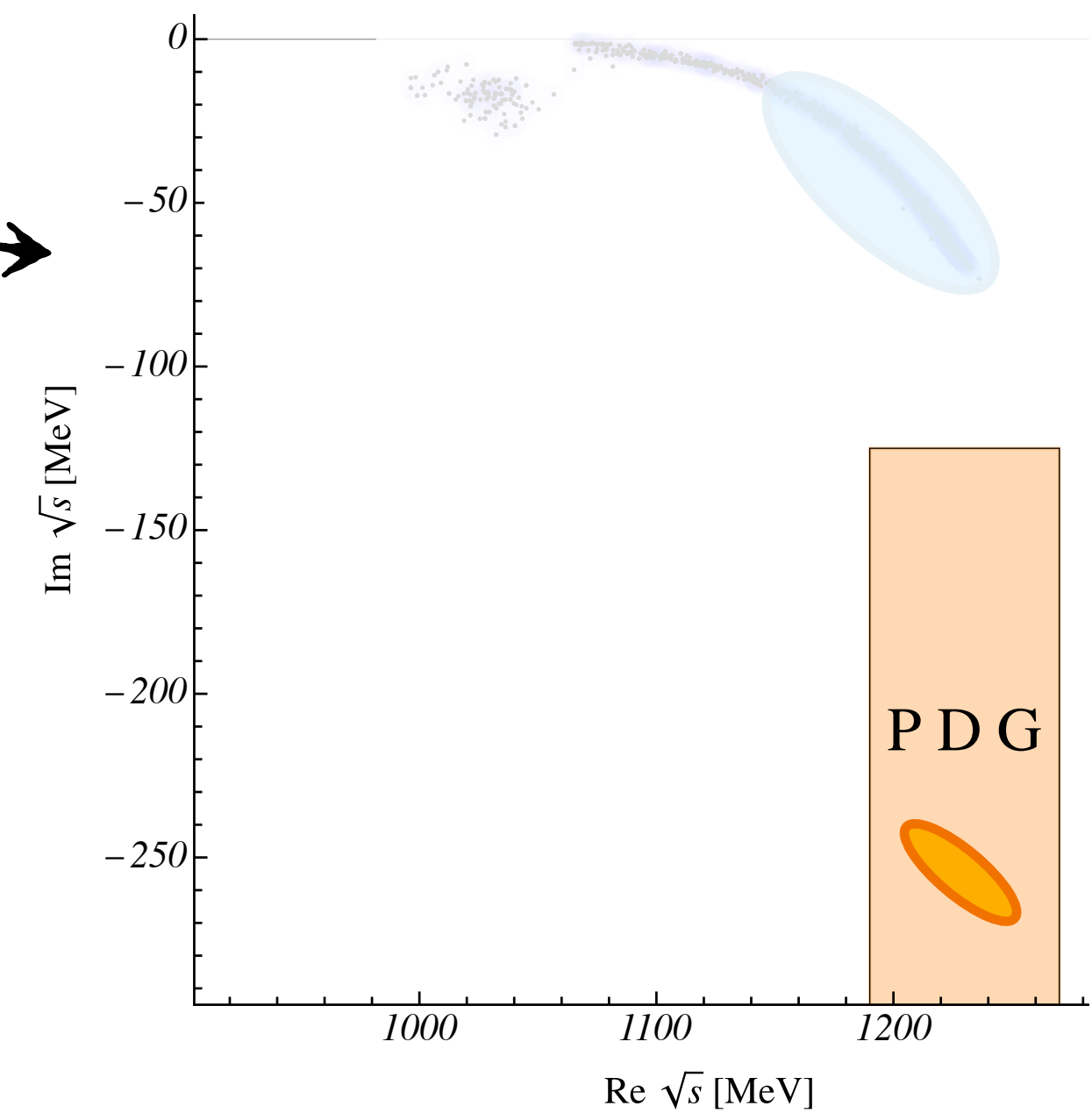
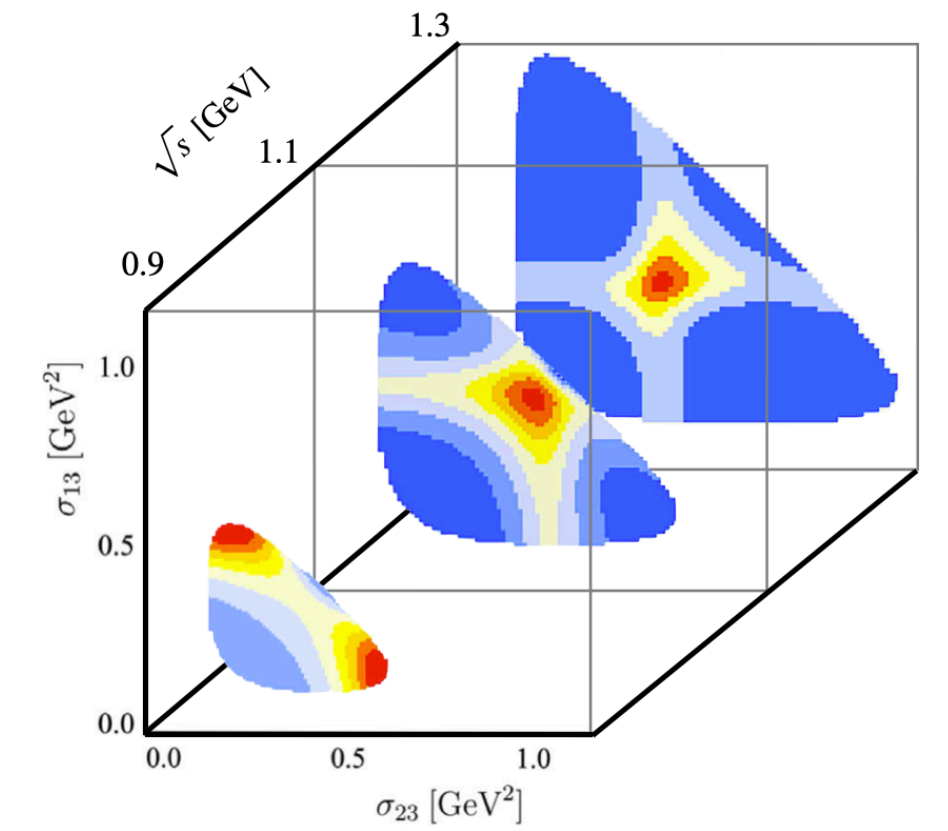
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FVU

$$\det \left[2L^3 E_p (\tilde{K}_2^{-1} - \Sigma_2^L) - B - C \right] T_{1g}$$

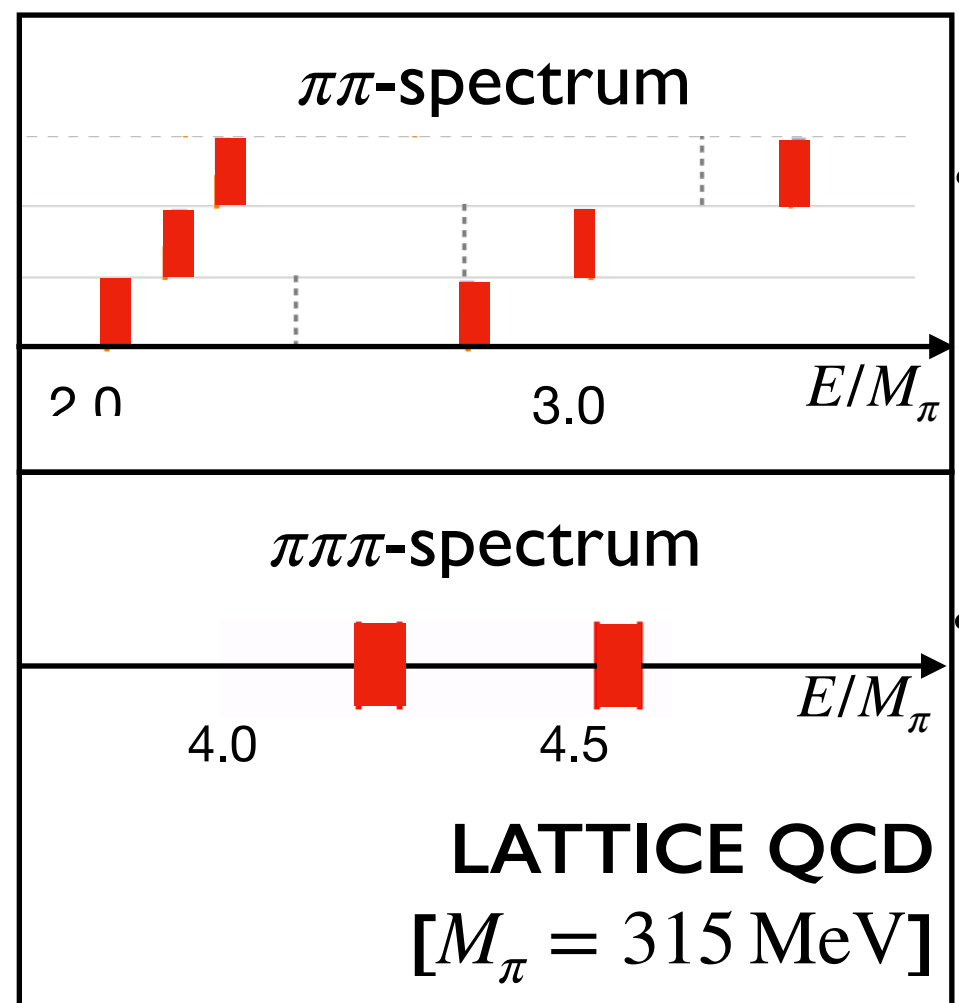
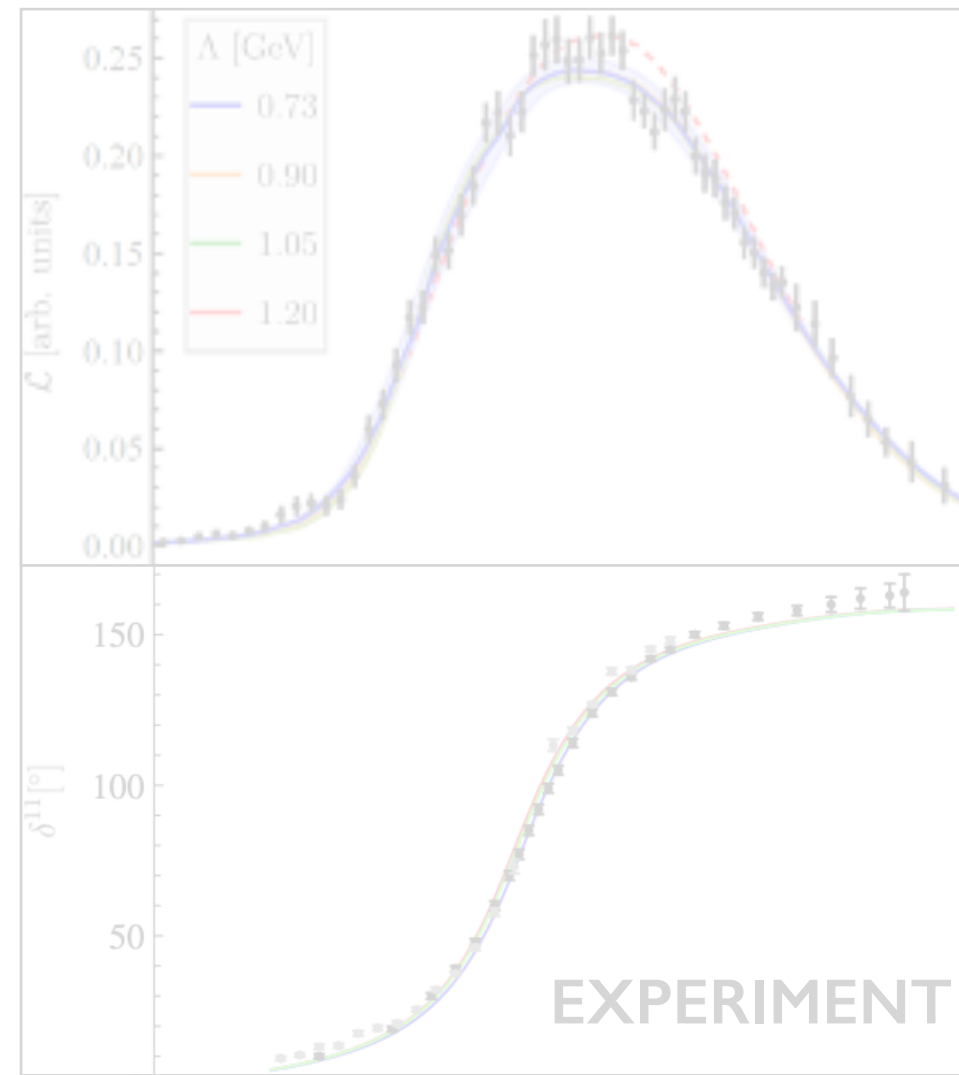
OUTPUT[2]



[1] Schael [ALEPH] Phys.Rept. 421 (2005); Nucl.Phys.B 79; Phys.Rev.D 7; [GWQCD] PRD94(2016) PRD98 (2018) PRD 100(2019)
 [2] Sadasivan/MM/Döring/Alexandru/Culver/Lee Phys.Rev.D 101 (2020); MM/Culver/Sadasivan/Brett/Döring/Alexandru/Lee [GWQCD] PRL 127 (2021)
 other phenomenological determinations: JPAC/....

BLUEPRINT — $a_1(1260)$

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TRANSITION AMPLITUDES

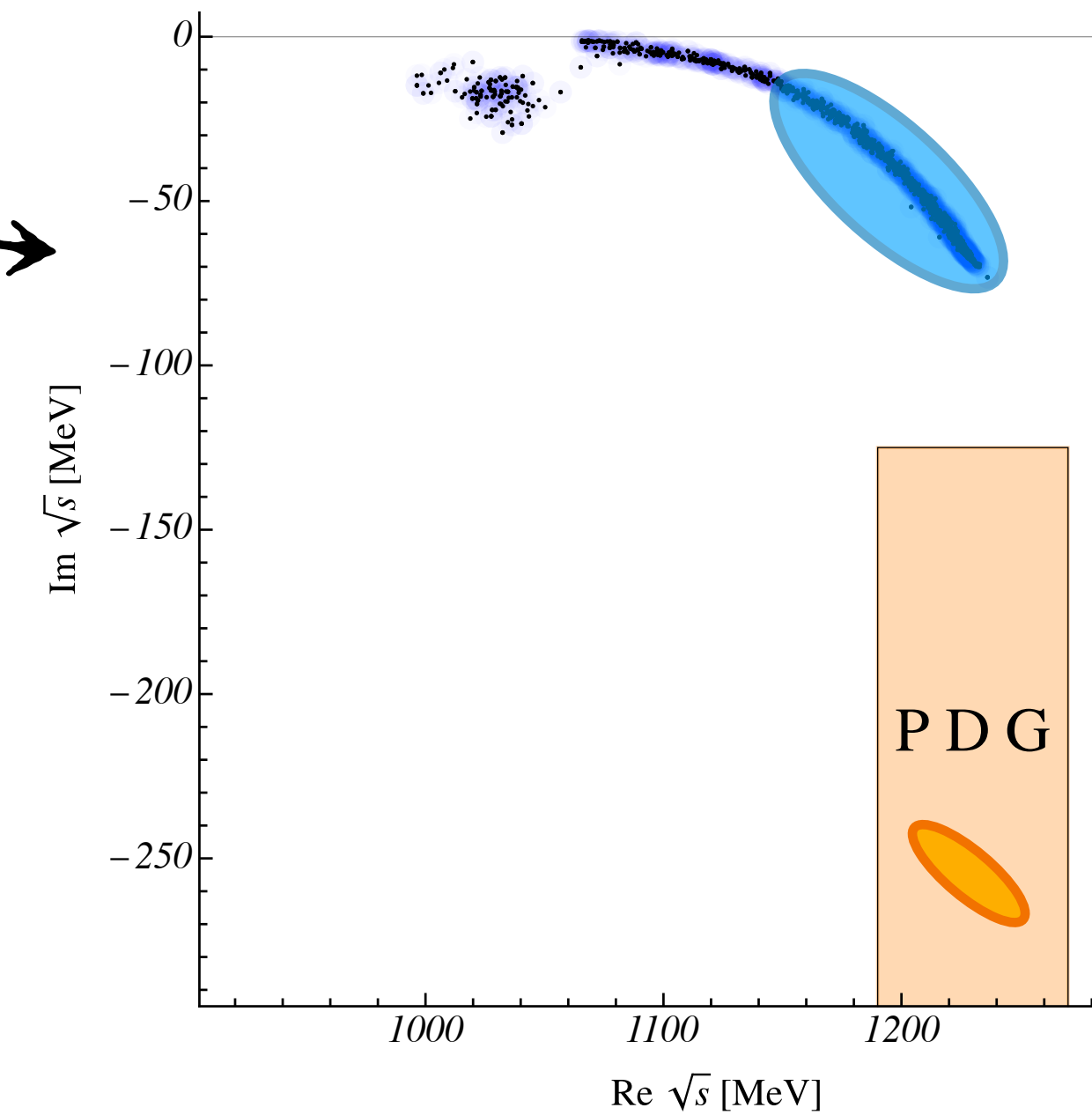
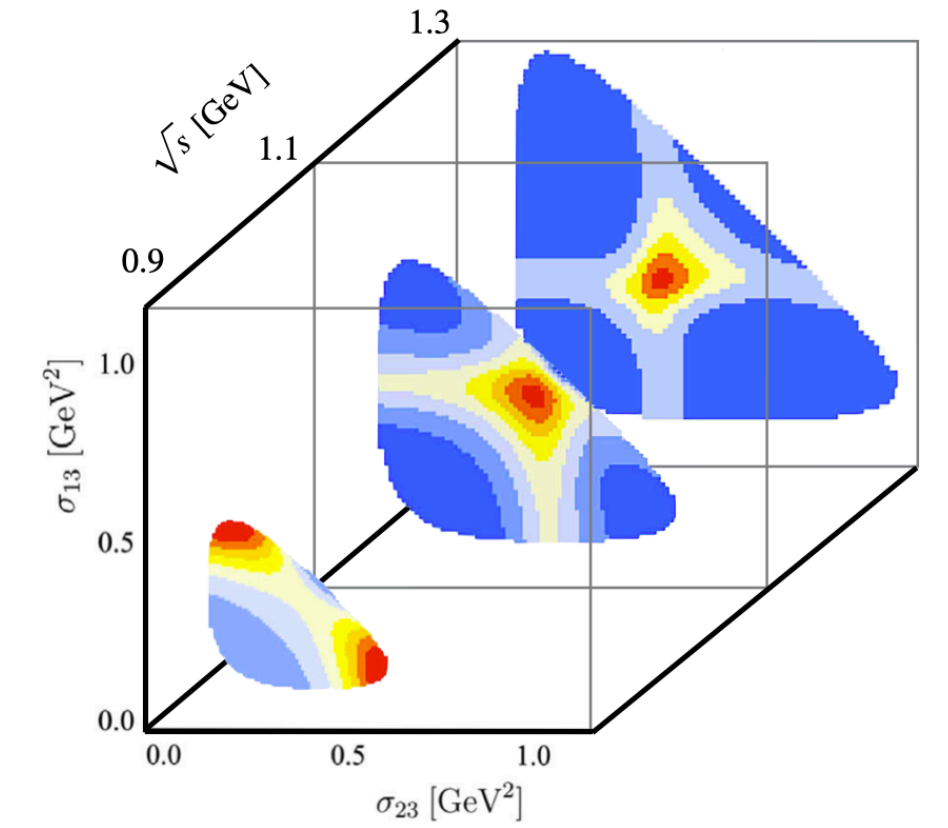
IVU

$$T^c = B + C + \int \frac{d^3\ell}{(2\pi)^3} \frac{(B+C)}{2E_\ell} \frac{1}{\tilde{K}_n^{-1} - \Sigma_n} T^c$$

FVU

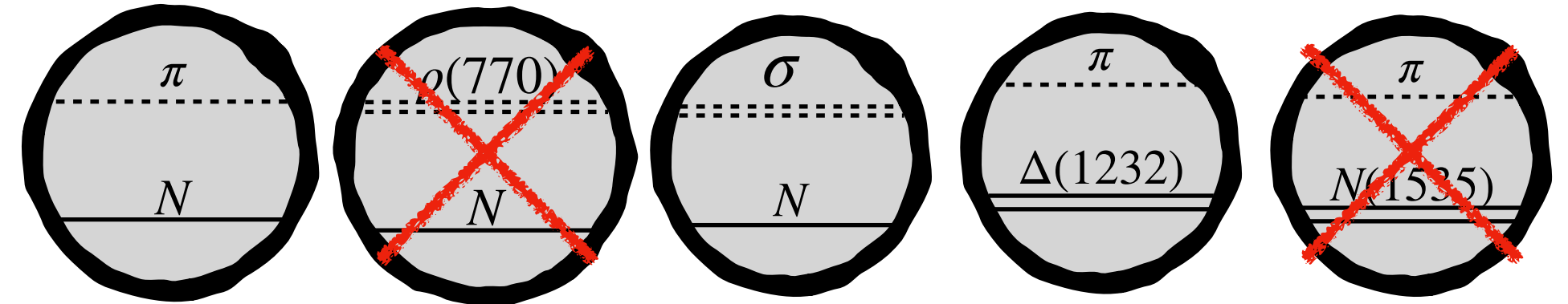
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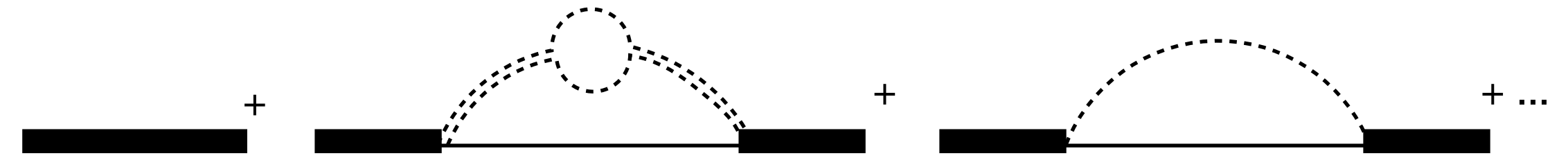
ROPER $N(1440)$ – FINITE VOLUME



~~... and more in SU(3)~~

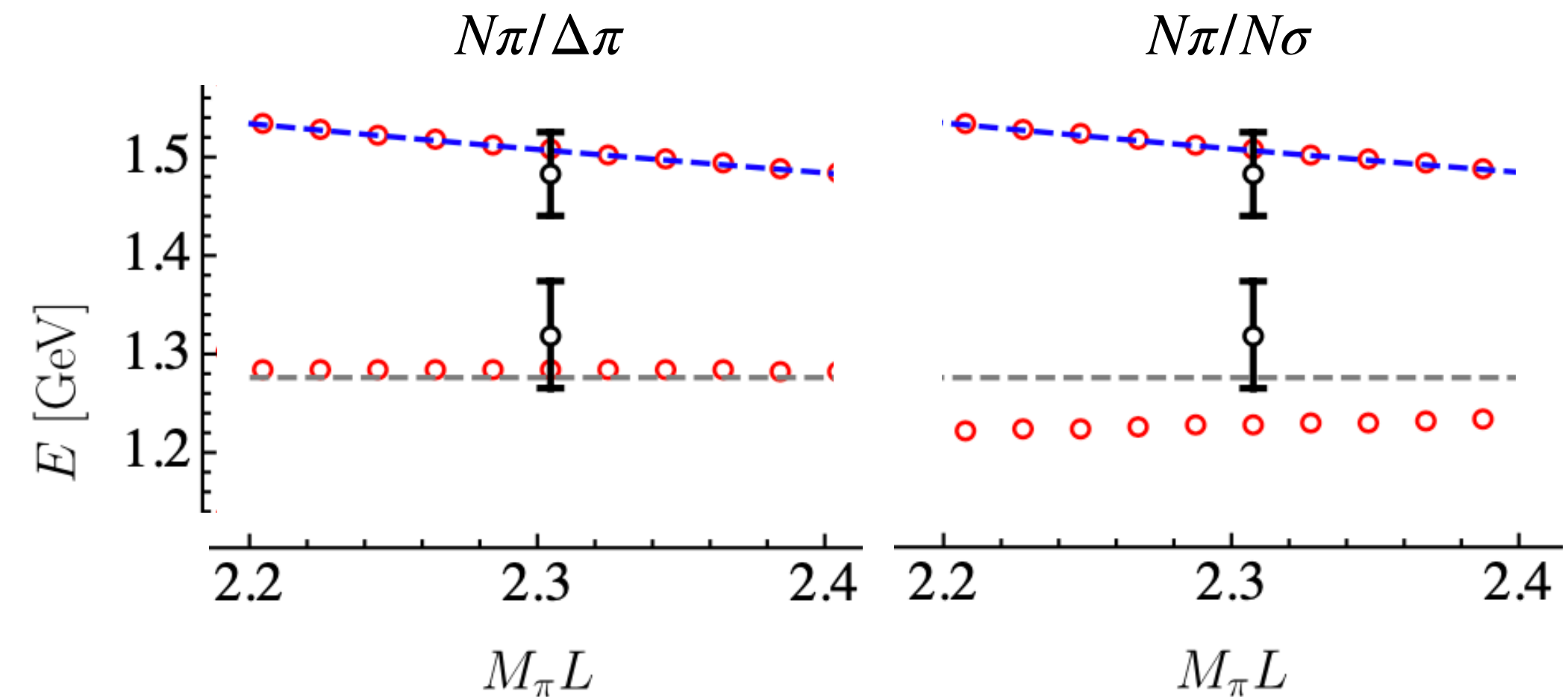
Simplified pilot study^[1]

- self-energy formalism via particle-dimer Lagrangian
- ⚠ no particle-exchange diagrams



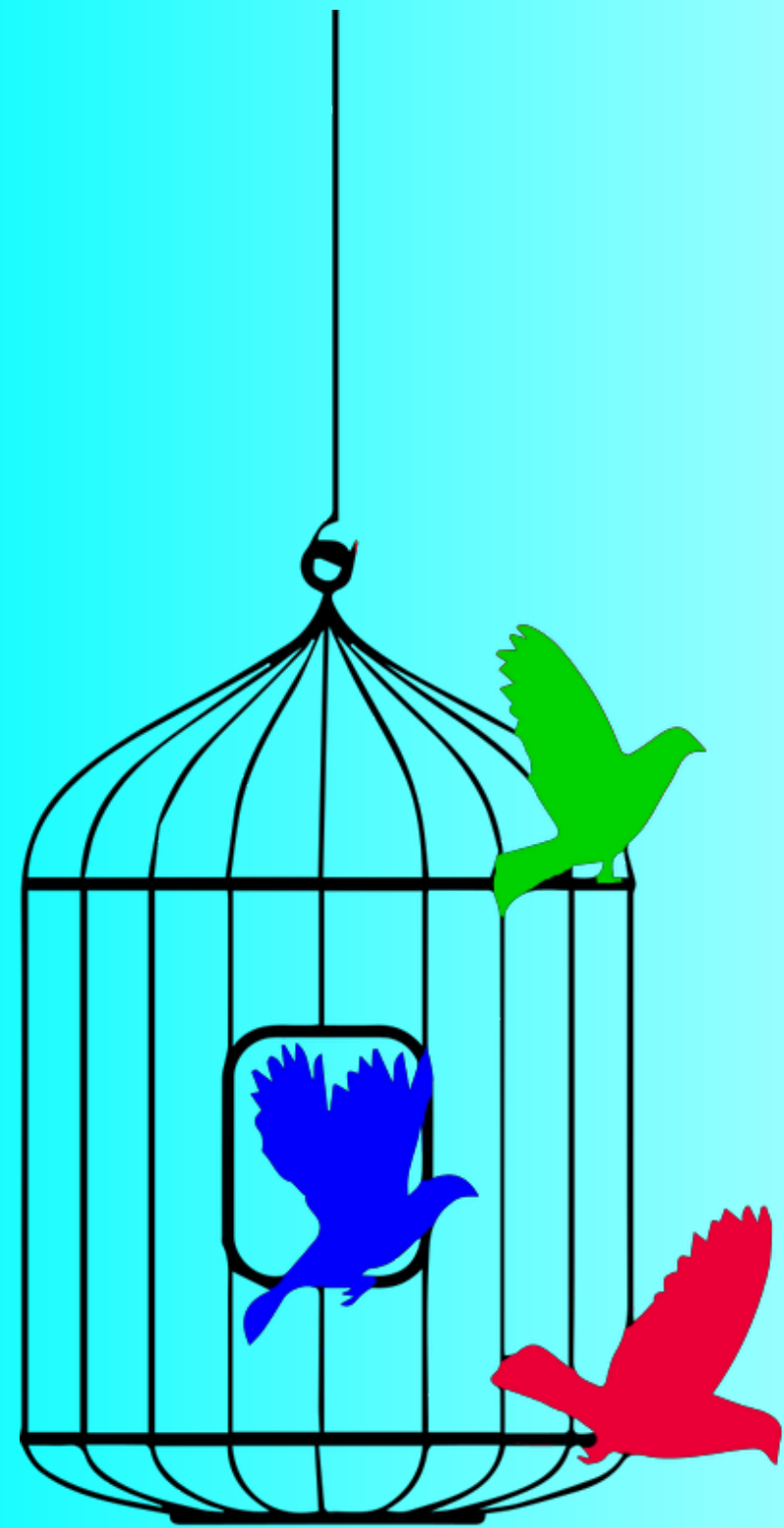
Predict finite-volume spectrum for fixed parameters

- energy shifts very small (opposing effects of $N\sigma$ and $\Delta\pi$ channels)
- phenomenological input necessary



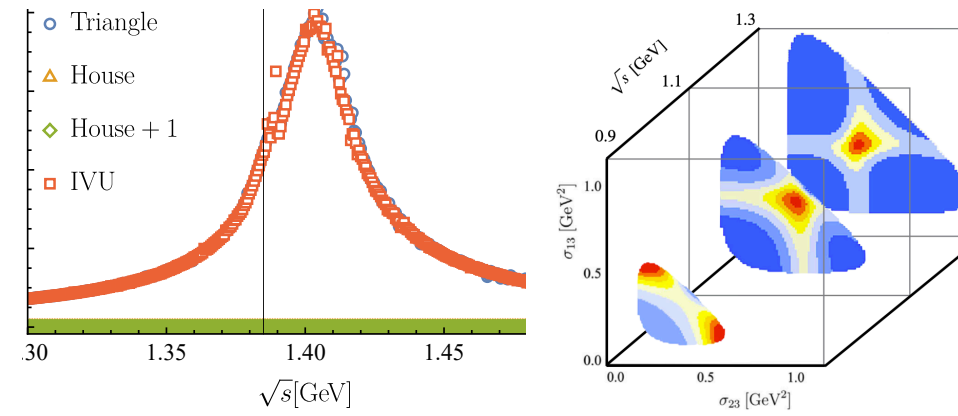
[1] Severt, MM, Ulf-G.Meißner *JHEP* 04 (2023) 100
 [2] Lattice values (black dots) Lang et al. *Phys.Rev.D* 95 (2017) 1

SUMMARY



IVU

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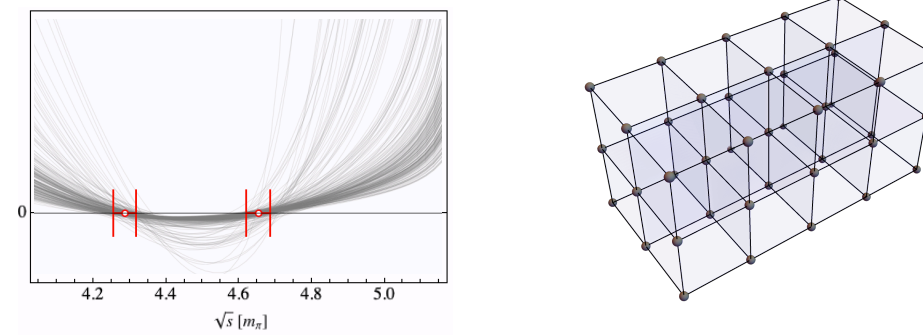


Infinite volume three-body formalism

- Unitarity induced analytic structure
- universal resonance parameter
- singularity structure/Landau singularities

FVU

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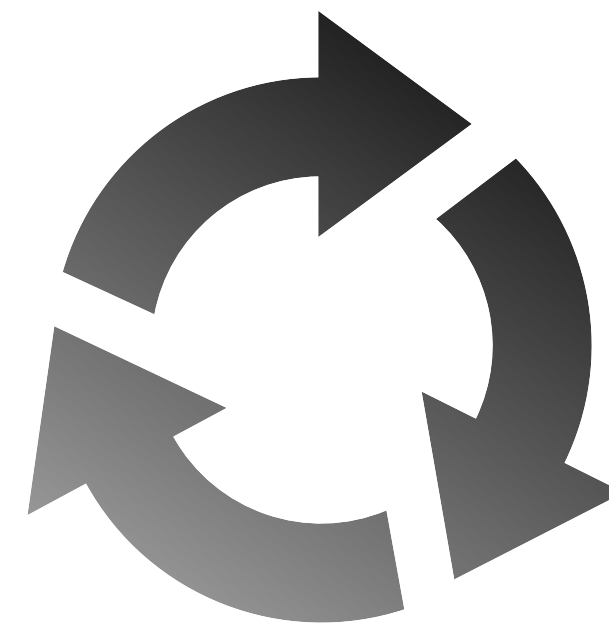


Finite-volume three-body formalism FVU

- 3b quantization condition
- several applications
- first chiral trajectories of 3b-resonances

OUTLOOK

- $\pi\pi N$ content of Roper-resonance
 - ... connections to DCC global studies
- $\pi\pi\Lambda$ and strangeness resonances (?)
- $\bar{K}d$ scattering
- ...



THANK YOU

