Charmonium, glueballs and their mixing from lattice QCD

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Physics opportunities with proton beams at SIS100

FOR5269: Future methods for studying confined gluons in QCD

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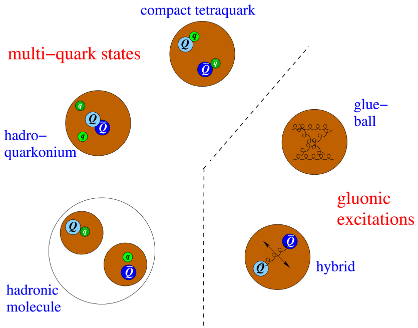
Collaboration between **physics** and **applied math** at BUW, DESY Zeuthen and Trinity College Dublin.
Main goals:

- Glueballs in dynamical QCD.
- Disconnected contributions in charmonium.
- String breaking in hybrid potentials.
- New schemes for molecular dynamics.
- Distillation + Multigrid framework.
- ...

https://confluence.desy.de/display/for5269

Beyond the quark model (XYZ, exotic candidates)

Some observed states are not compatible with a $\bar{q}q$ composition. Alternatives (See N. Brambilla *et al.*, 1907.07583):



We need to understand the dynamics of **glueballs**, **hybrids**, etc... to better understand these states.

Lattice QCD

Simulate QCD via Monte-Carlo methods in a Euclidean space-time lattice.

- Discretization introduces lattice spacing a.
- Quarks ψ live in lattice sites, gluons U live in links between sites.
- Lattice Dirac operator $\mathbf{D}$ is a **large** but sparse matrix $(10^7 \times 10^7)$
- Action $S[\bar{\psi}, \psi, U]$ recovers correct $a \to 0$ limit.

Measure expected values of observables $\mathcal{O}$:

- $\langle \mathcal{O} \rangle$ gives physical information, e.g energies.
- Sample gluon configurations distributed as $\propto e^{-S}$.
- Statistical errors $\propto \dfrac{1}{\sqrt{\#\text{ of measurements}}}$ + other effects

Hadron spectroscopy in lattice QCD

Nature: What is the mass of a $J^{PC} = 0^{-+}$ $\bar{c}c$ state, e.g η_c?

- SO(3) reduces to cubic group $\mathbb{O}$: $(0^{\pm\pm}, 1^{\pm\pm}, 2^{\pm\pm}, ...) \to (A_1^{\pm\pm}, A_2^{\pm\pm}, E^{\pm\pm}, T_1^{\pm\pm}, T_2^{\pm\pm})$.
- Flavor-singlet channel is blind to quark content.

Lattice: What is the mass of a specific A_1^{-+} state?

1. Define $\mathcal{O}[\bar{\psi}, \psi, U]$ with fixed quantum numbers, e.g meson, glueball, baryon, etc...
2. Calculate two-point temporal correlation function

$$\langle \mathcal{O}(t)\bar{\mathcal{O}}(0) \rangle = \frac{1}{Z} \int d\psi d\bar{\psi} dU \mathcal{O}(t)\bar{\mathcal{O}}(0) e^{-S}$$

$$\approx \frac{1}{N} \sum_i {\color{red}(...)} \to \text{Monte Carlo for} \int dU$$

$$= \sum_n |\langle n| \hat{O}^\dagger |\Omega\rangle|^2 e^{-E_n t} \overset{t\to\infty}{\approx} |\langle 0| \hat{O}^\dagger |\Omega\rangle|^2 e^{-E_0 t}$$

Hadron spectroscopy in lattice QCD

Even better, create correlation matrix between different operators with equal quantum numbers

$$C_{ij}(t) = \left\langle \mathcal{O}_i(t)\bar{\mathcal{O}}_j(0) \right\rangle$$

and solve a generalized eigenvalue problem (GEVP)

$$C(t)w_n(t, t_G) = \rho_n(t, t_G)C(t_G)w_n(t, t_G)$$

to get

$$\rho_n(t, t_G) \overset{t\to\infty}{\approx} c_n e^{-E_n t} \to \text{Energies of states}$$

$$\tilde{\mathcal{O}}_n = \sum_k w_n^{(k)}(t_1, t_G)\mathcal{O}_k \to \text{Operator closest to } |n\rangle$$

See Lüscher and Wolff (1999), Blossier et al. (2009).

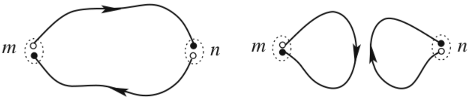
Charmonium on the lattice

Choose operator

$$\mathcal{O}(t) = \bar{c}(t)\Gamma c(t), \;\; \Gamma = \{\gamma_5, \gamma_i, \gamma_5\gamma_i, \nabla_i, ...\}$$

Build correlation function:

$$\begin{aligned} C(t) = &- \left\langle \mathrm{Tr}\left(\Gamma D^{-1}[t,0]\Gamma D^{-1}[0,t]\right)\right\rangle_{\text{gauge}} \quad \text{Connected} \\ &+ \left\langle \mathrm{Tr}\left(\Gamma D^{-1}[t,t]\right) \mathrm{Tr}\left(\Gamma D^{-1}[0,0]\right)\right\rangle_{\text{gauge}} \quad \text{Disconnected} \end{aligned}$$



- Inversions D^{-1} are the main computational cost.
- Disconnected contribution is the most expensive and noisy, being often **neglected** (OZI suppression).

Charmonium on the lattice

Variety of Γ:

- Several Γ for different J^{PC} can be studied on the lattice, e.g exotic 1^{-+}.
- Gluonic excitations via $\mathbb{B}_i = \epsilon_{ijk}\nabla_j\nabla_k$, e.g 0^{++} can be $\bar{c}\mathbb{I}c$ or $\bar{c}\gamma_4\gamma_5\gamma_i\mathbb{B}_i c$ (Hybrid operator).

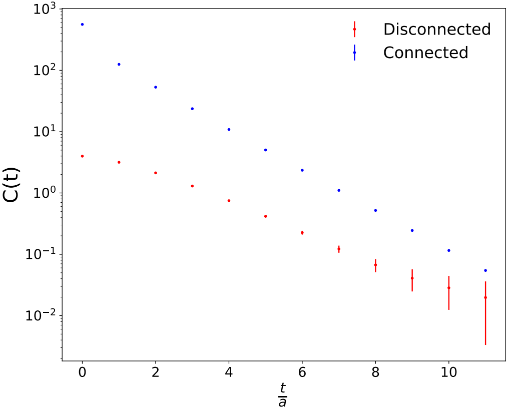
Improved Distillation:

- Restrict quark fields to subspace of smooth, gauge-covariant fields at each time t: range $(V[t])$.
- Optimize the restriction for each Γ and energy level.
- Perambulators: $\tau[t_1, t_2] = V[t_1]^\dagger \mathbf{D}^{-1} V[t_2] \rightarrow$ Most **expensive** calculation!

See Phys. Rev. D 106, 034501 (2022), Phys. Rev. D 80, 054506 (2009).

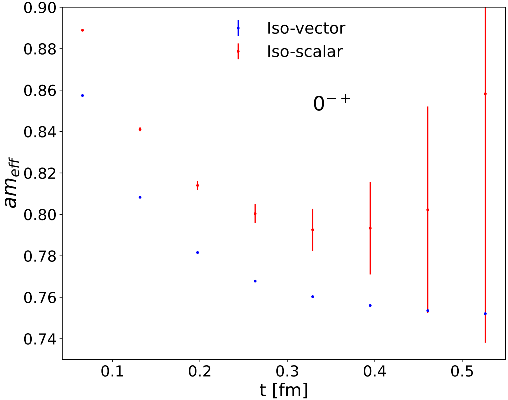
Charmonium on the lattice

$N_f = 2$, $24^3 \times 48$ lattice, Wilson quarks at half the physical charm quark mass, 0^{-+} with $\bar{c}\gamma_5 c$.



Disconnected contribution: **small** magnitude, **large** error.

Charmonium on the lattice



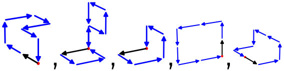
- Effective mass: $am_{eff}(t) = \ln\left(\frac{C(t)}{C(t+a)}\right)$
- Small window of opportunity for flavor-singlet.

Glueballs on the lattice

Bound states of only gluons arising from their self-interaction. Experimental detection is difficult due to decays and mixing with mesons. → ”Experimentally (...) their status remains unclear and controversial” [F. Brünner and A. Rebhan, (2015)]

On the lattice:

- Correlations are heavily affected by **signal-to-noise problem**, needing **large** statistics.
- Mixing with mesons makes **identification difficult**.

Popular operators:  → 3D Wilson loops.

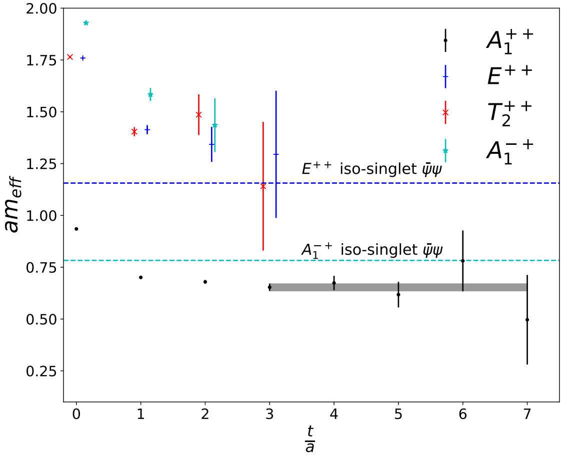
Quenched lattice QCD [C. Morningstar and M. Peardon, (1999)]:

$0^{++} : 1730 \pm 80$ MeV $\rightarrow f_0(1710)$?

$2^{++} : 2400 \pm 120$ MeV, $0^{-+} : 2590 \pm 130$ MeV

Glueballs are **unstable** in full dynamical QCD!

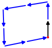
Glueballs on the lattice



- Exponential signal-to-noise problem.
- Gluonic operators see the lightest 0^{++} state clearly.

Charmonium - Glueball Mixing

What is a pure meson/glueball state with dynamical quarks?

- Lattice gluonic $\hat{G} =$  and mesonic $\bar{q}q$ operators can be in the same symmetry channel.
- $\langle \eta_c | \hat{G} | \Omega \rangle \neq 0$: Needs further disentanglement!

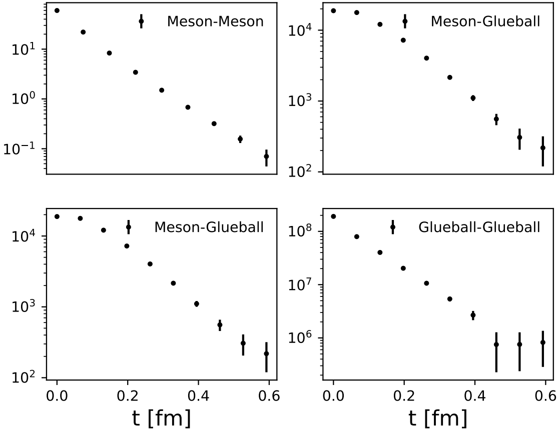
A first (limited) approach:

- Which $|n\rangle$ is more dominant in the state we create?

$$(\bar{q}q)\,|\Omega\rangle = \sum_n \langle n|\,(\bar{q}q)^\dagger\,|\Omega\rangle\,|n\rangle\,,\ \ \hat{G}^\dagger\,|\Omega\rangle = \sum_n \langle n|\,\hat{G}^\dagger\,|\Omega\rangle\,|n\rangle$$

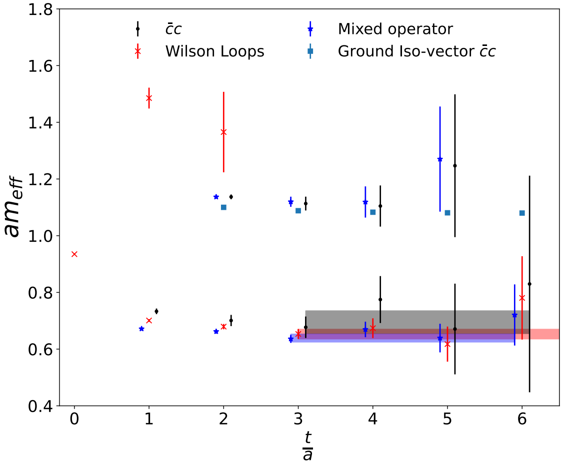
If $\dfrac{|\,\langle n|\,(\bar{q}q)\,|\Omega\rangle\,|}{|\,\langle n+1|\,(\bar{q}q)\,|\Omega\rangle\,|} > 1$, $\dfrac{|\,\langle n|\,\hat{G}\,|\Omega\rangle\,|}{|\,\langle n+1|\,\hat{G}\,|\Omega\rangle\,|} < 1$
then $|n\rangle$ mostly **mesonic**, $|n+1\rangle$ mostly **gluonic**.

Charmonium - Glueball Mixing



Non-zero off-diagonals: There is mixing between operators.

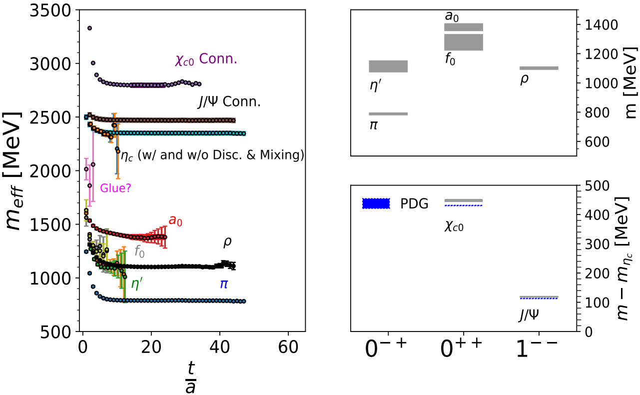
Charmonium - Glueball Mixing



- $\frac{\langle 0|\bar{c}c|\Omega\rangle}{\langle \mathbf{1}|\bar{c}c|\Omega\rangle} = -0.569(48)$, $\frac{\langle \mathbf{0}|G|\Omega\rangle}{\langle 1|G|\Omega\rangle} = -5.6(2.1)$
- $|0\rangle$ mainly gluonic, $|1\rangle$ mainly mesonic.

Towards a physical setup

$N_f = 3 + 1$ at SU(3) flavor-symmetric point + 800 MeV pions



2 (heavy) pion threshold for scalar glueball ≈ 1.6 GeV.
Need 2-particle operators! (See 2312.16740 for other details.)

Conclusions and Outlook

- State-of-the-art methods are needed, particularly for flavor-singlet states: Suitable operators, improved distillation, GEVP, etc... **Disconnected** contributions!
- Glueball spectroscopy is a **difficult** hunt: SNR problem, noisy operators, cost of statistics. **Disconnected**-like correlations.
- Characterizing a glueball state is not easy due to **hadronic decays**.

Work in progress within FOR5269:

- Better operators for $\bar{c}c$, glueballs, **multi-particle** states, **static-light** mesons, **static** potentials, ...
- Better methods to tackle SNR problem, e.g multi-level updates for quenched QCD in 2312.11372.
- Better methods to tackle computational bottlenecks: Solve $Dx = b$ with very particular b.

Thank you for your attention!