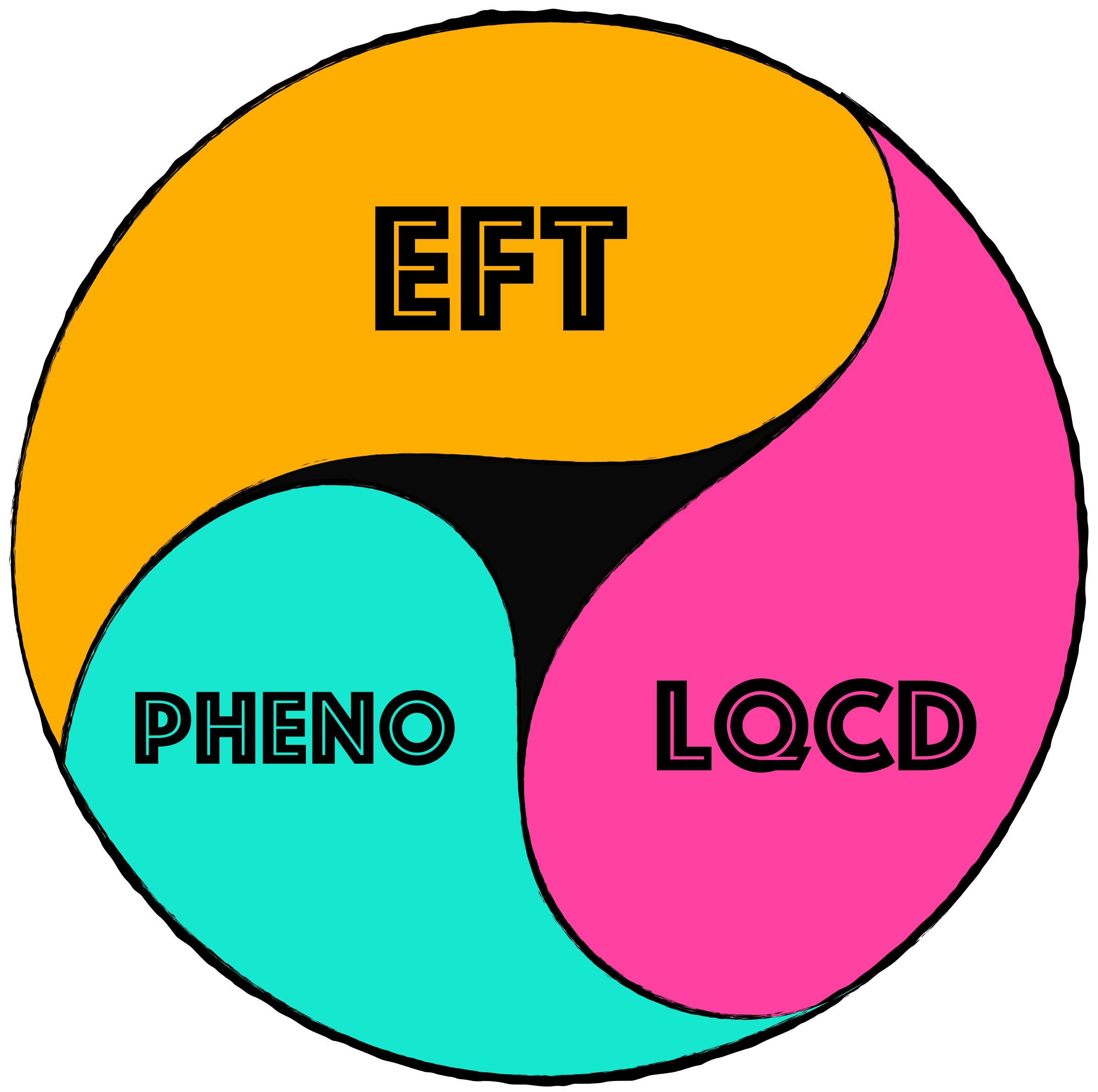
MULTI-HADRON SYSTEMS FROM EFT/LQCD/PHENOMENOLOGY



MAXIM MAI

University of Bonn / The George Washington University

NSF PHY-2012289

DOE DE-SC0016582

DOE DE-SC0016583

DFG CRC 110

Physics opportunities with proton beams at SIS100
Wuppertal (2024)

HADRON SPECTRUM

Mostly excited states[1]

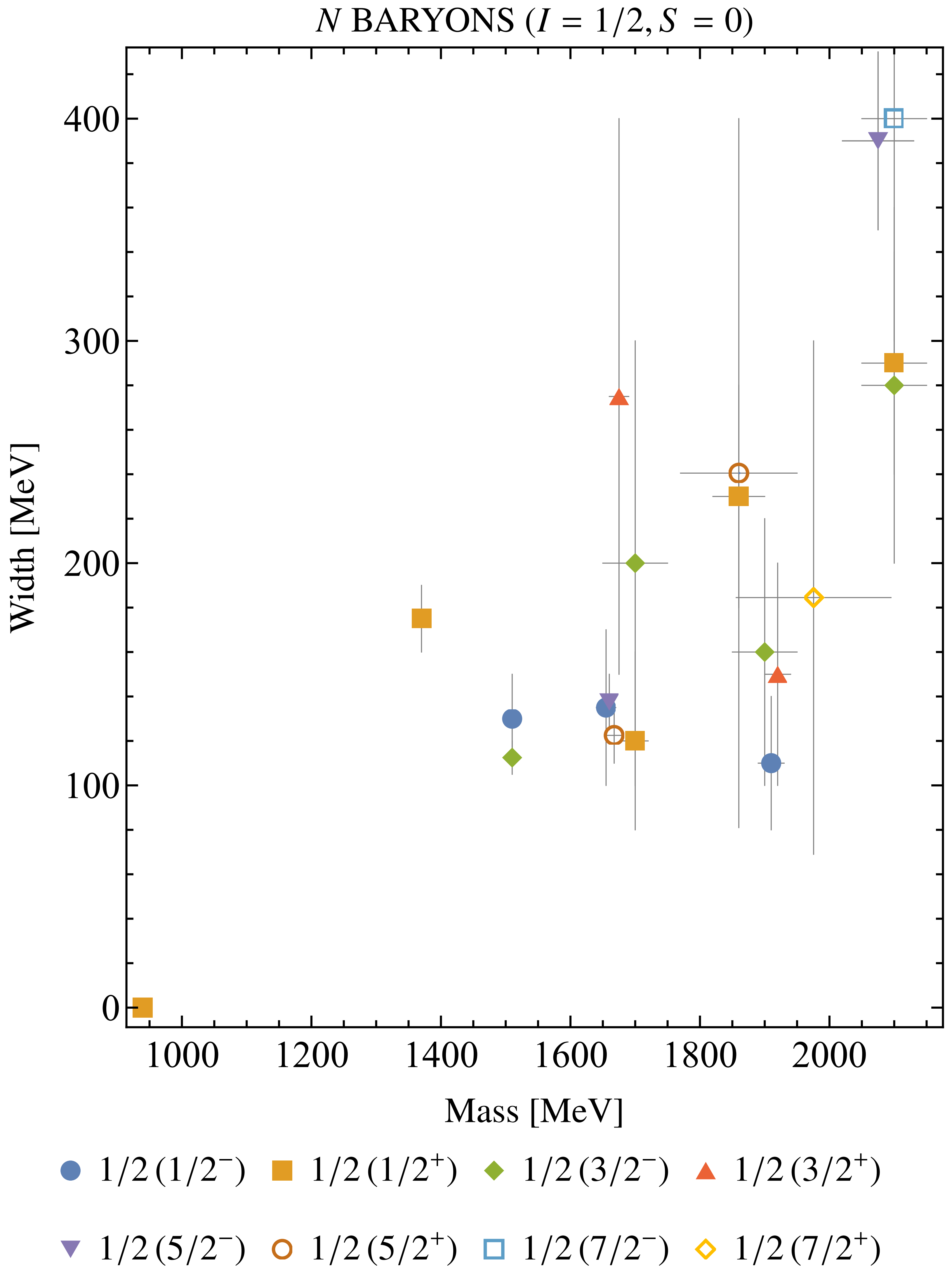
≈100 mesons & ≈50 baryons (****)

Key questions

"what is the pattern of these states?"

"how are they formed?"

- *Quark models*
- *Functional methods[2]*
- *Dynamical coupled-channel models*
- *Chiral EFT[3]*
- *Lattice QCD*



[1] Particle Data Group (Workman et al.) see talk M. Bleicher
Talks: [2] C. Fischer; G. Eichmann [3] E. Epelbaum

UNIVERSAL PARAMETERS

- pole positions on unphysical Riemann Sheets
- central quantity: ***transition amplitudes***[1]
  - S-matrix theory
    - Unitarity, Analyticity, Crossing symmetry, …
  - Constrains
    - Experiment
    - CHPT
    - Lattice QCD

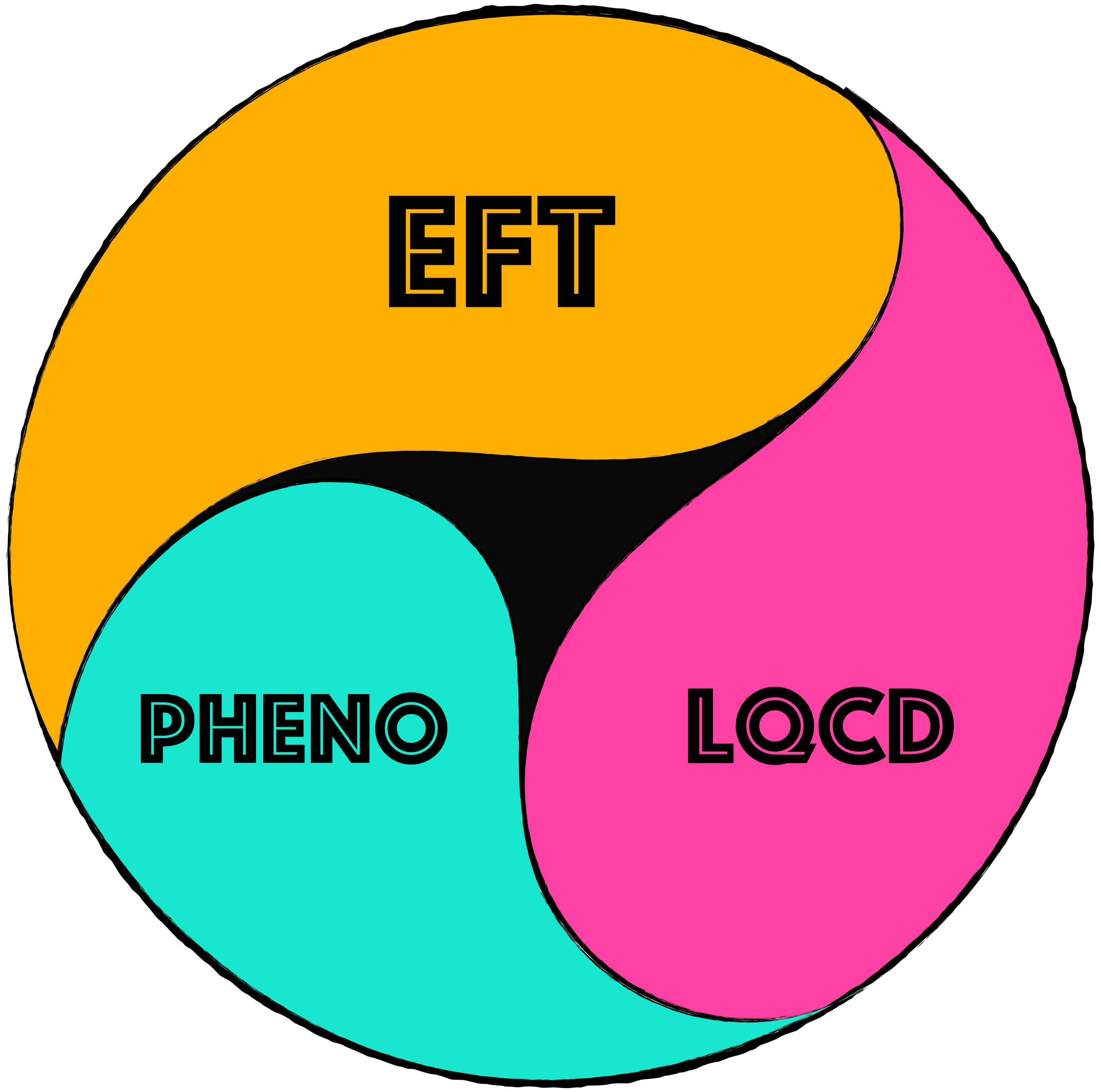
Tridge (Midland, MI/USA)

[1] **Review**: MM/Meißner/Urbach ``Towards a theory of hadron resonances'' Phys. Rept. 1001 (2023)

EXAMPLE I

MESON-BARYON RESONANCES

CHPT/UNITARITY/PHENOMENOLOGY



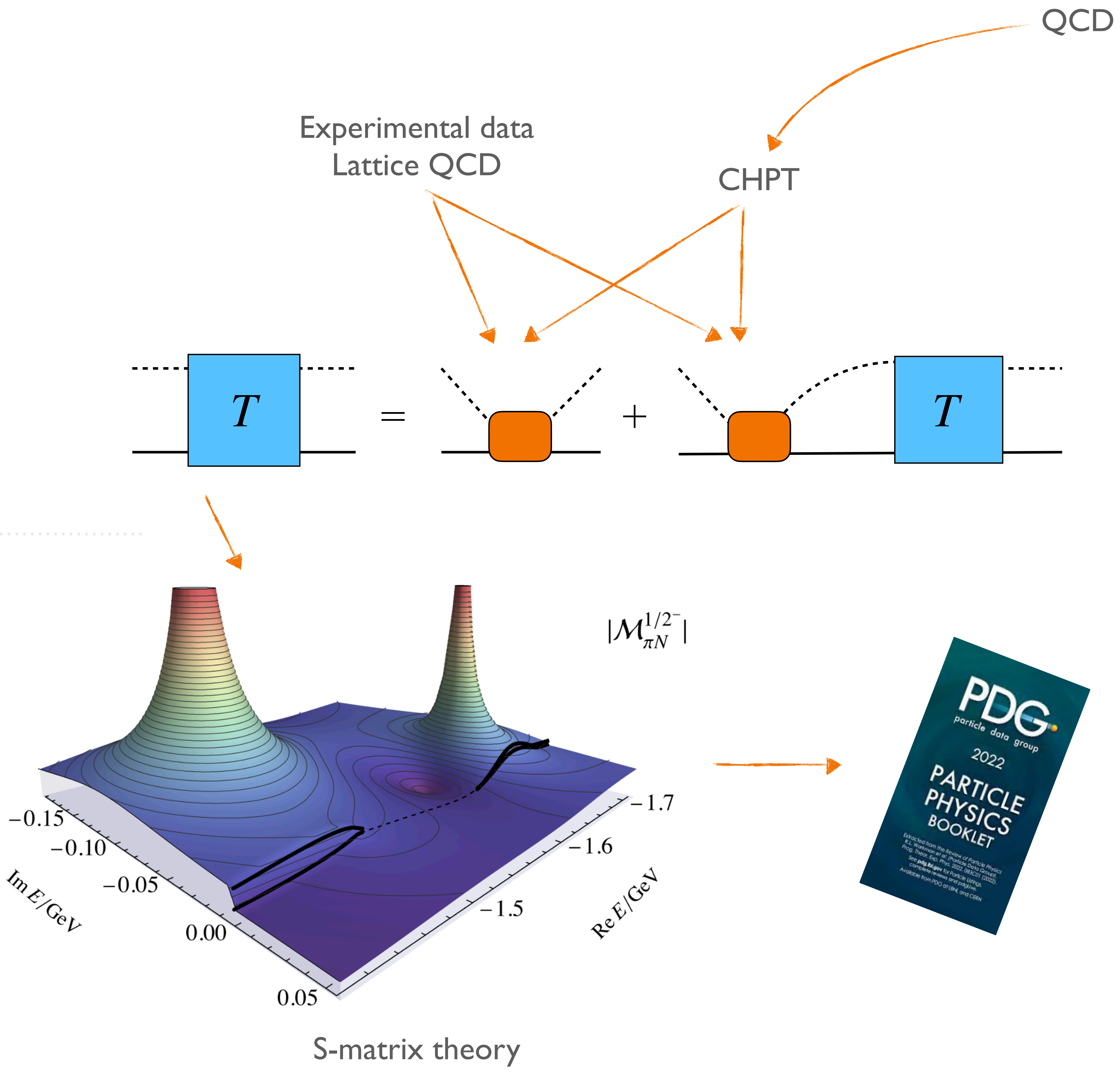
- J.-X. Lu, L.-S. Geng, MM, M.Döring [Phys.Rev.Lett. 130 (2023) 7]
- F-K Guo, Y. Kamyia, MM, Ulf-G. Meißner [Phys.Lett.B 846 (2023) 138264]
- D. Sadasivan et al. Front.Phys. 11 (2023) 1139236

TRANSITION AMPLITUDE

Chiral unitary approach[1]

- Chiral Perturbation Theory (#QCD#EFT) dictates the form of the interaction at low energies
- **Unitary amplitude from the Bethe-Salpeter equation**
  - Fit free parameters to experimental data / LQCD
  - Record complex pole positions
  - Many states can be explained[2]

$$N^*(1535), N^*(1650), \Lambda(1380), \Lambda(1405), \ldots$$



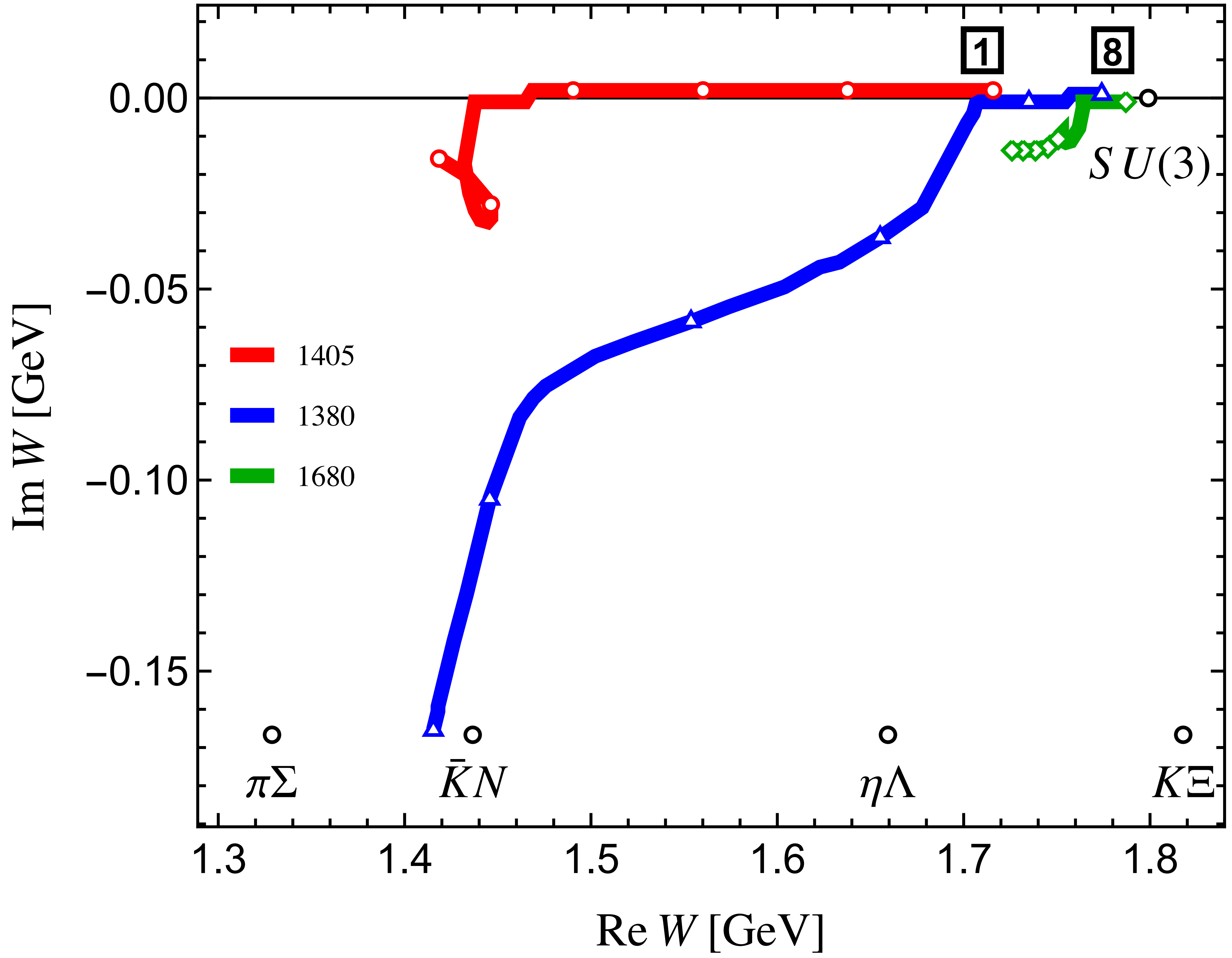
[1] Weise/Kaiser/Meißner/Lutz/Oset/Oller/Ramos/Hyodo/Borasoy…
[2] Kaiser/Siegel/Weise Phys.Lett.B 362 (1995) Lutz/Soyeur Nucl.Phys.A 773 (2006); MM et al. Phys.Lett.B 697 (2011); …

UNPHYSICAL QUARK MASSES

CHPT encodes quark mass dependence

- SU(3) limit provides a simpler resonance structure
  - 1 singlet + 2 octet poles
  - LO/NLO "tracks" differ[2]

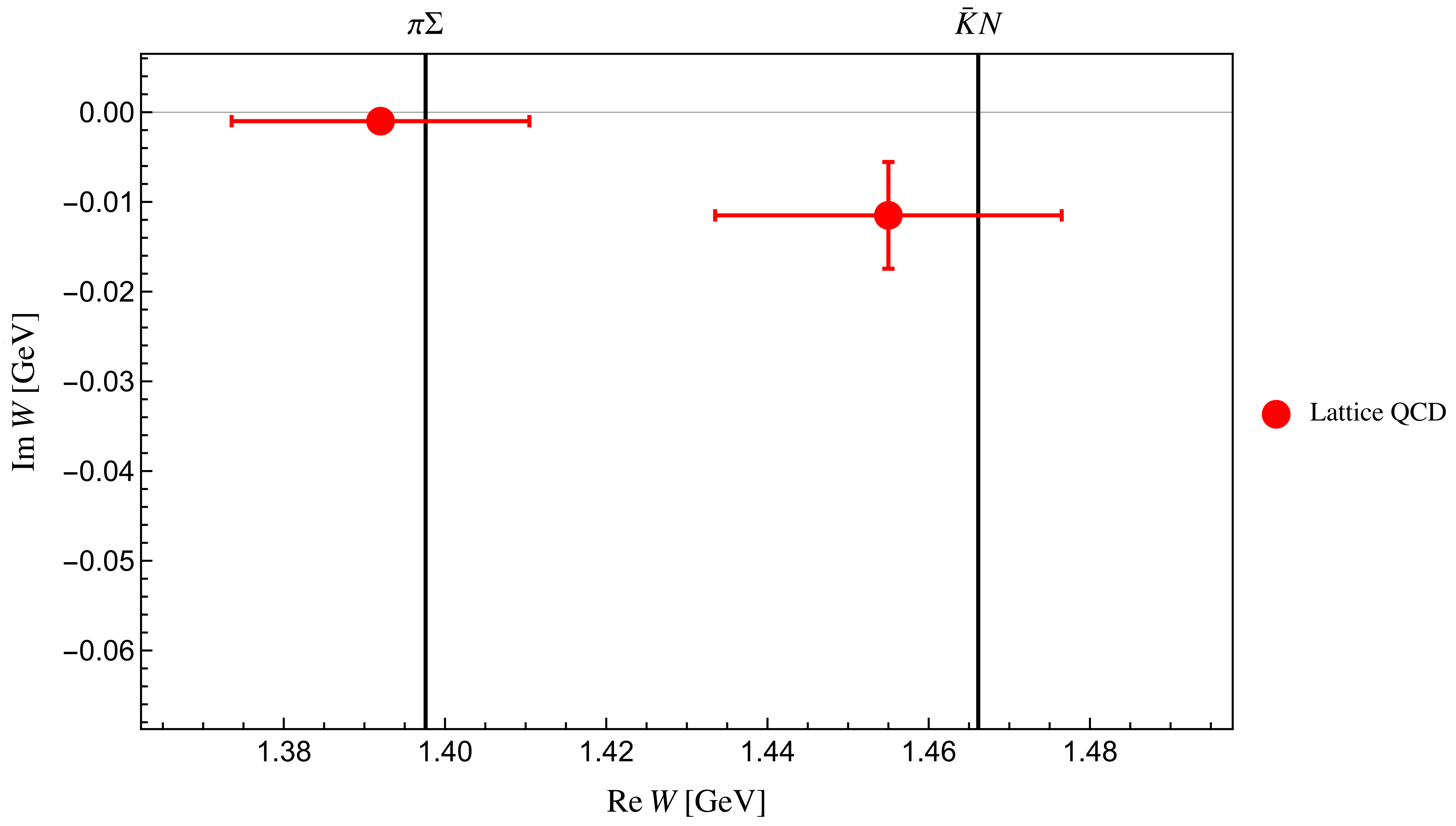
... Lattice QCD (?)



[1] Jido et al. Nucl.Phys.A 725 (2003); Bruns/Cieply, SU(3) Nucl. Phys. A 1019 (2022);
[2] Guo/Kamyia/MM/Meißner Phys.Lett.B 846 (2023)

CHPT encodes quark mass dependence

- Available Lattice spectrum — BaSc setup[3]
  - $M_\pi = 200\,\mathrm{MeV}$
  - compare to prediction of UHPT



[1] Jido et al. Nucl.Phys.A 725 (2003); Bruns/Cieply, SU(3) Nucl. Phys. A 1019 (2022);
[3] [BaSc] Bulava et al. 2307.10413; 2307.13471

UNPHYSICAL QUARK MASSES

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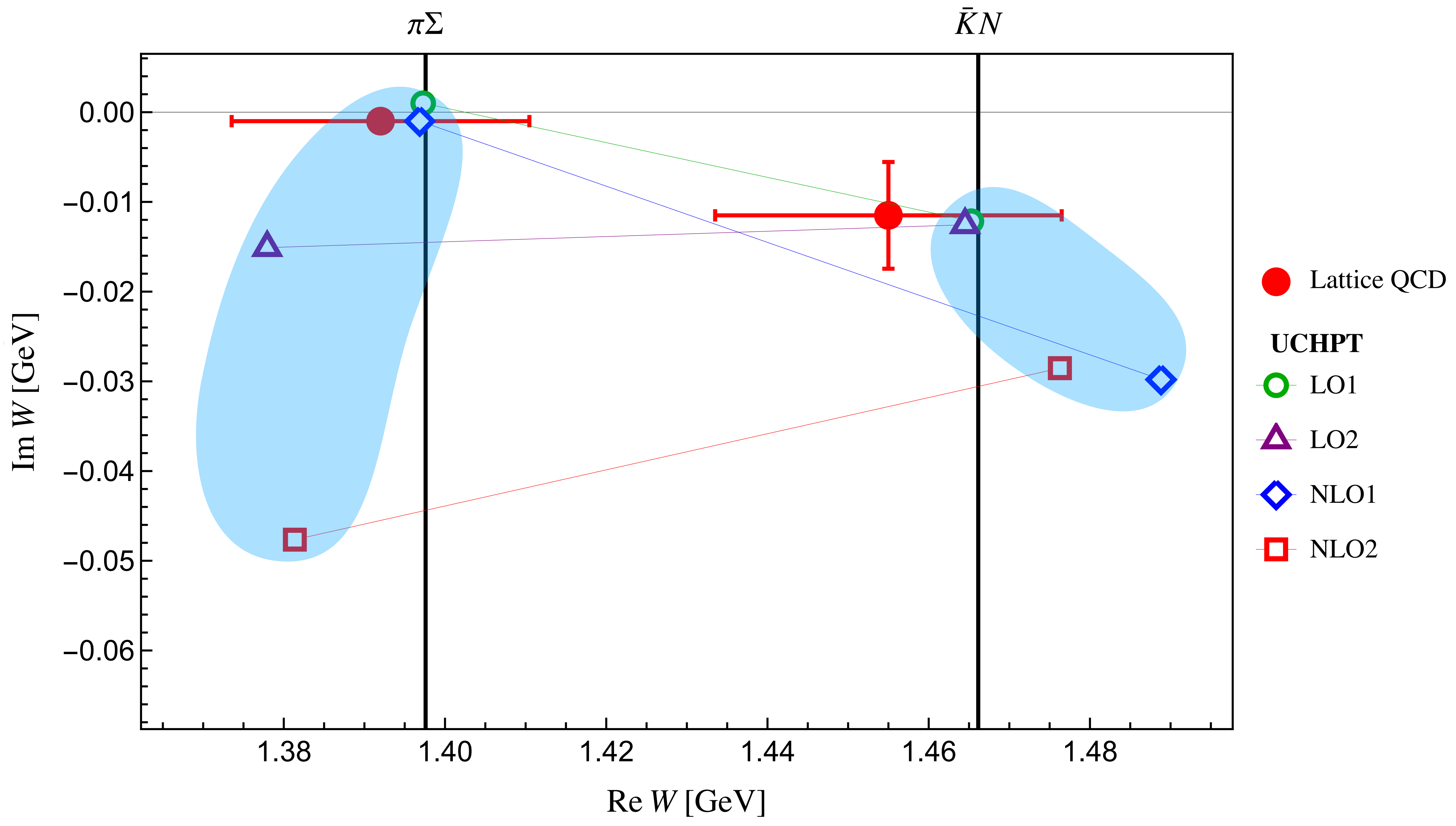
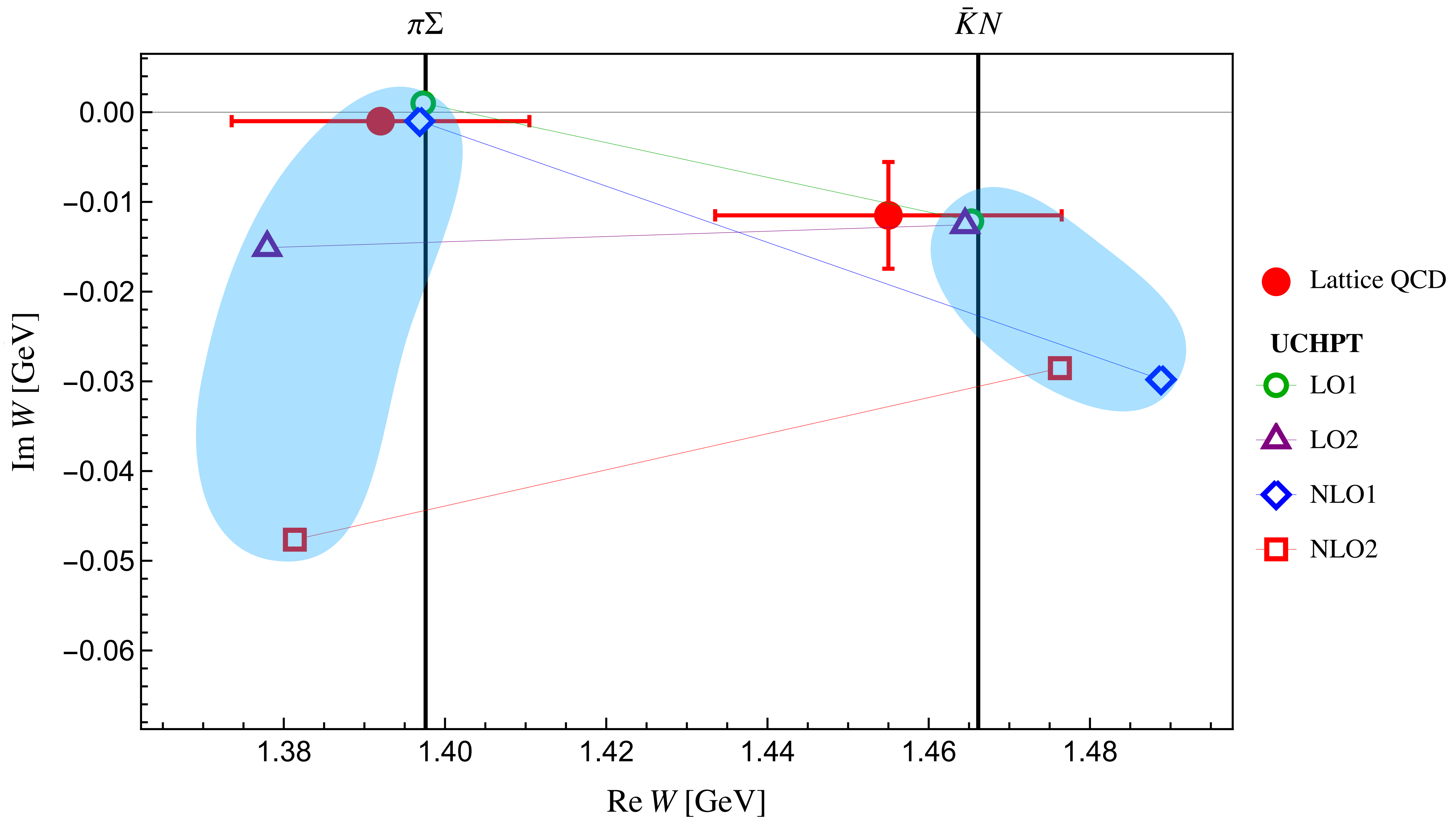


[1] Jido et al. Nucl.Phys.A 725 (2003); Bruns/Cieply, SU(3) Nucl. Phys. A 1019 (2022);
[3] [BaSc] Bulava et al. 2307.10413; 2307.13471

UNPHYSICAL QUARK MASSES

CHPT encodes quark mass dependence

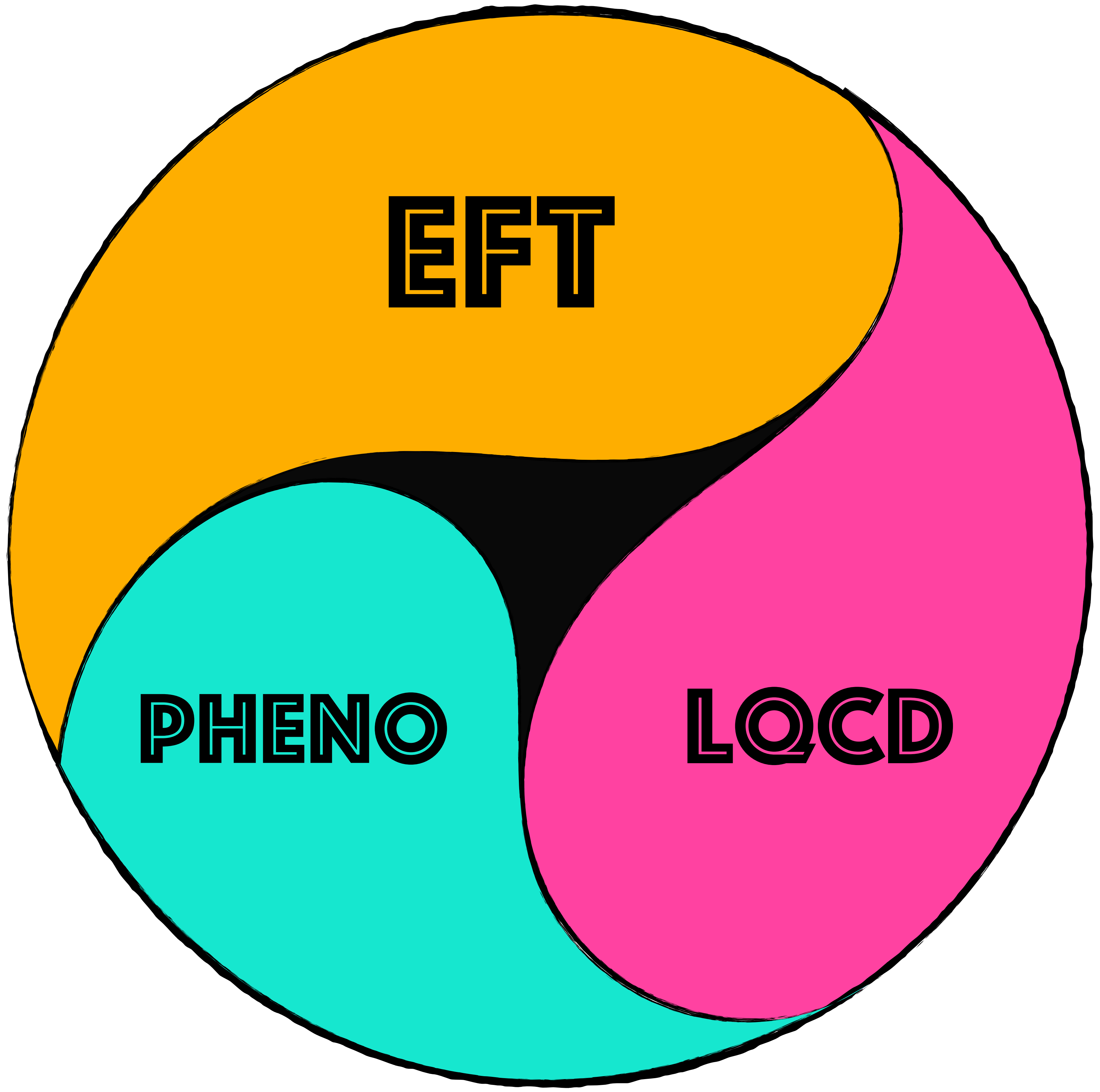
- Available Lattice spectrum — BaSc setup[3]
  - $M_\pi = 200\,\mathrm{MeV}$
  - compare to prediction of UHPT
- In progress:
  1. LQCD + UCHPT (+ EXPERIMENT?)…
  2. **Hyperons:** unified study of $S = -1, -2 - 3 (\& \, 0, 1, \ldots ?)$



[1] Jido et al. Nucl.Phys.A 725 (2003); Bruns/Cieply, SU(3) Nucl. Phys. A 1019 (2022);
[3] [BaSc] Bulava et al. 2307.10413; 2307.13471

EXAMPLE 2

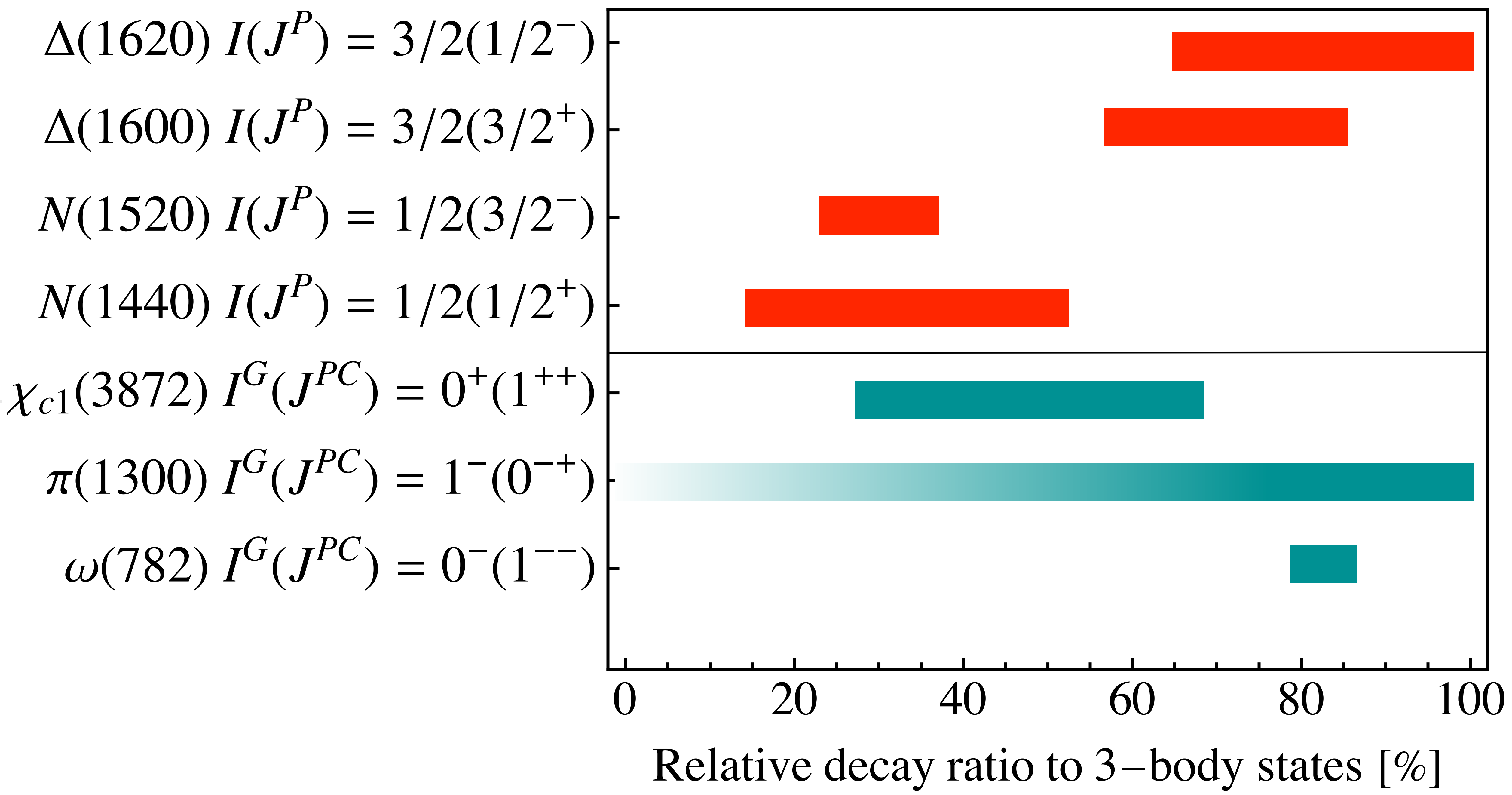
THREE-HADRON STATES



- D. Sadasivan et al. Phys.Rev.D 105 (2022) 5
- MM et al. Phys.Rev.Lett. 127 (2021) 22
- S. Severt, MM, Ulf-G.Meißner JHEP 04 (2023) 100
- [JBW] MM et al. Eur.Phys.J.A 59 (2023) 12

HADRONIC 3-BODY PROBLEM

- Many known states have large 3-body content
  - Roper $N(1440)$
  - $X(3872)$
  - $a_1(1260)\ldots$
- Beyond Standard Model searches (τ-EDM/…)
- Exotic states of matter[1]



[1] Experimental programs: GlueX@JLAB; COMPASS@CERN;
[2] Figure data Workman et al. (Particle Data Group), Prog.Theor. Exp. Phys. 2022, 083C01 (2022)

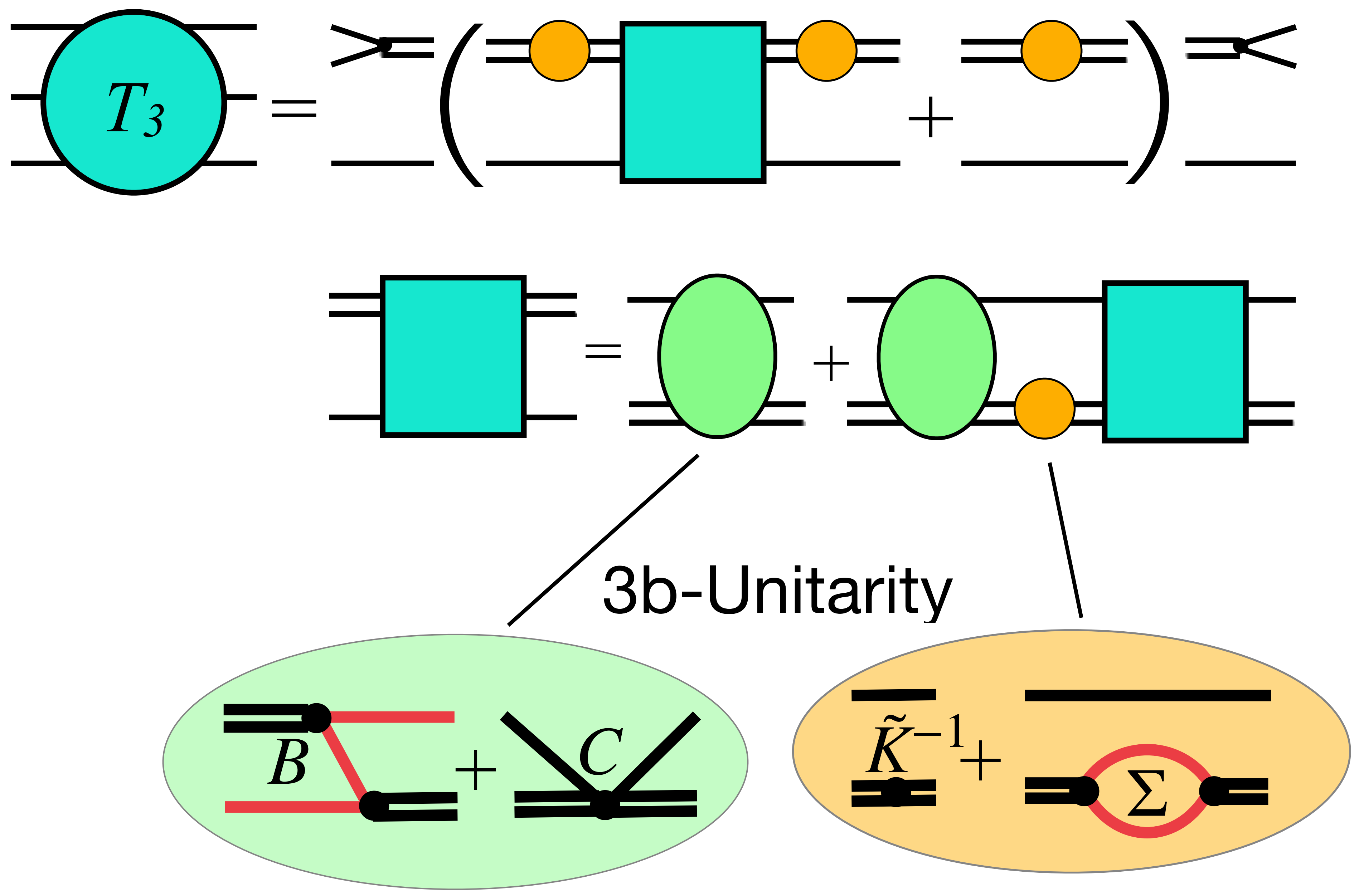
TRANSITION AMPLITUDE

"Infinite Volume Unitarity" — IVU formalism

- Three-body scattering amplitude[1,2]
- Express 3-body through 2+1 system
- Input: C and K
- **On-shell configurations** fixed through Unitarity

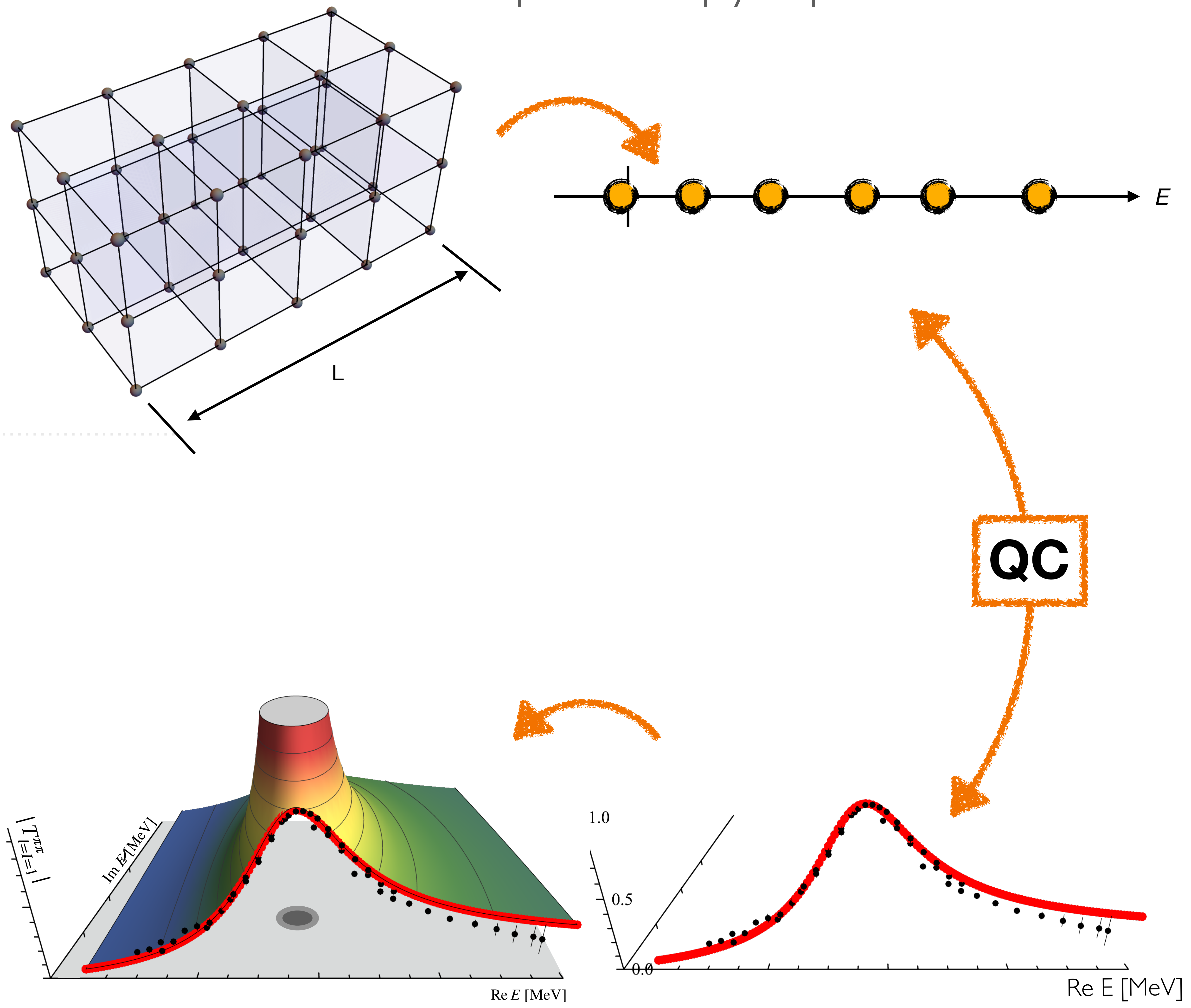
IVU

$$T^c = B + C + \int \frac{d^3\ell}{(2\pi)^3} \frac{(B+C)}{2E_l} \frac{1}{\tilde{K}_n^{-1} - \Sigma_n} T^c$$



[1] MM/Hu/Döring/Pilloni/Szczepaniak Eur.Phys.J.A 53 (2017)
[2] Related approaches: Hansen/Sharpe(2014)….;Wunderlich et al. JHEP 08 (2019); Jackura et al. Eur.Phys.J.C 79 (2019);

Lattice QCD: numerical access to QCD Green's fct.
Euclidean space-time / unphysical pion mass / **finite-volume**



"Finite Volume Unitarity" — FVU formalism

- On-shell particles "feel" the box-size
- Three-body quantization condition

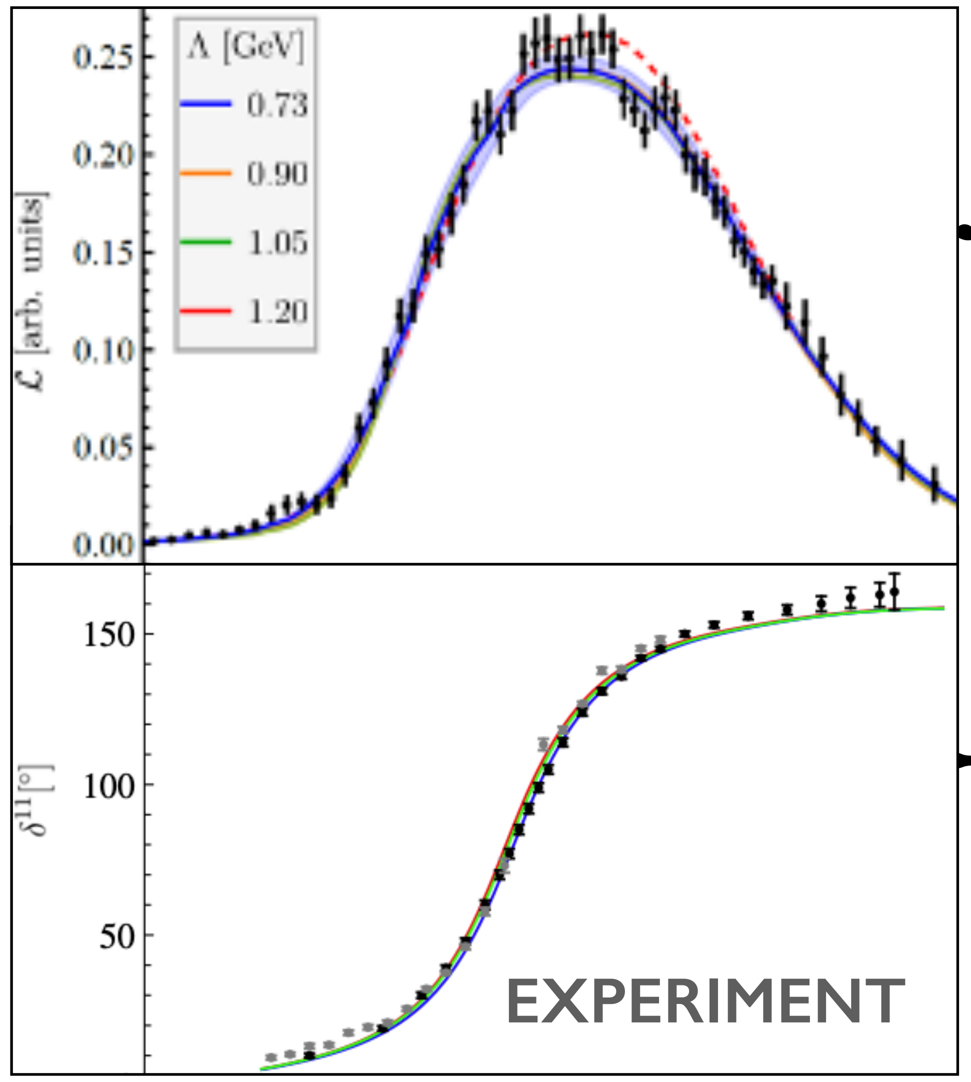
FVU

$$\det\left[\,2L^3E_{\mathbf{p}}\left(\tilde{K}_2^{-1}-\Sigma_2^L\right)-B-C\,\right]^{T_{1g}}$$

[1] Lüscher, Gottlieb, Rummukainen, Feng, Li, Döring, Briceño, Meißner, Rusetsky, Hansen, MM, Blanton, …
Reviews: Briceno/Dudek/Young (2017) Rev.Mod.Phys. 90 (2018) 2 Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019); MM/Doring/Rusetsky Eur.Phys.J.ST 230 (2021);

BLUEPRINT — $a_1(1260)$

INPUT[1]



TRANSITION AMPLITUDES

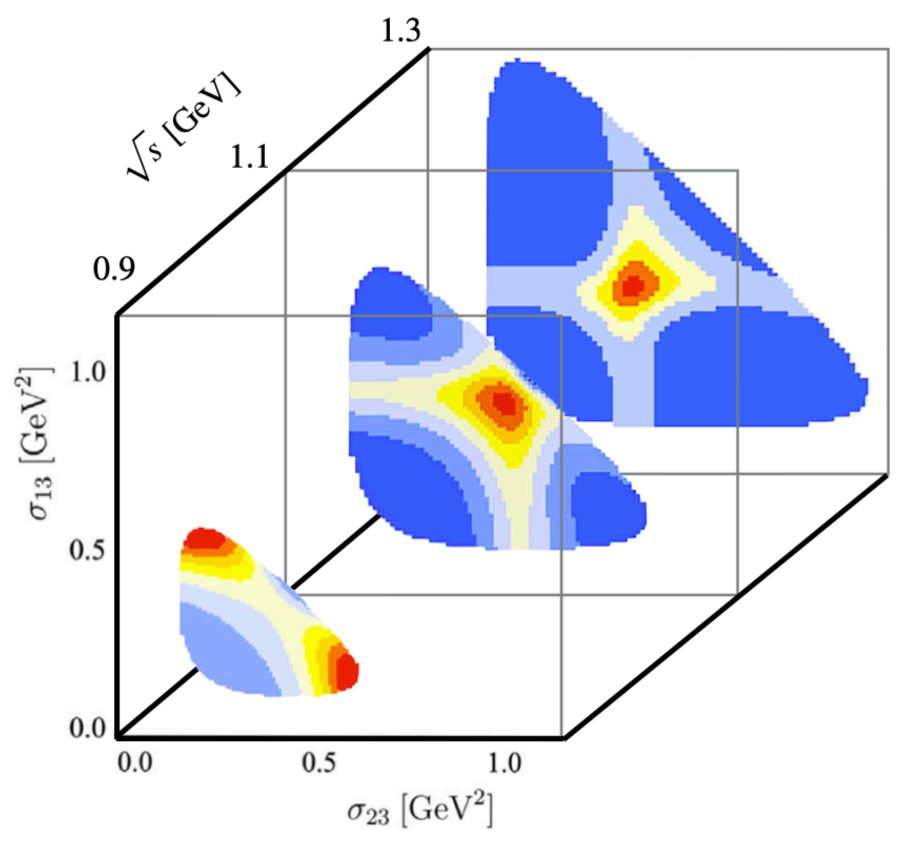
IVU

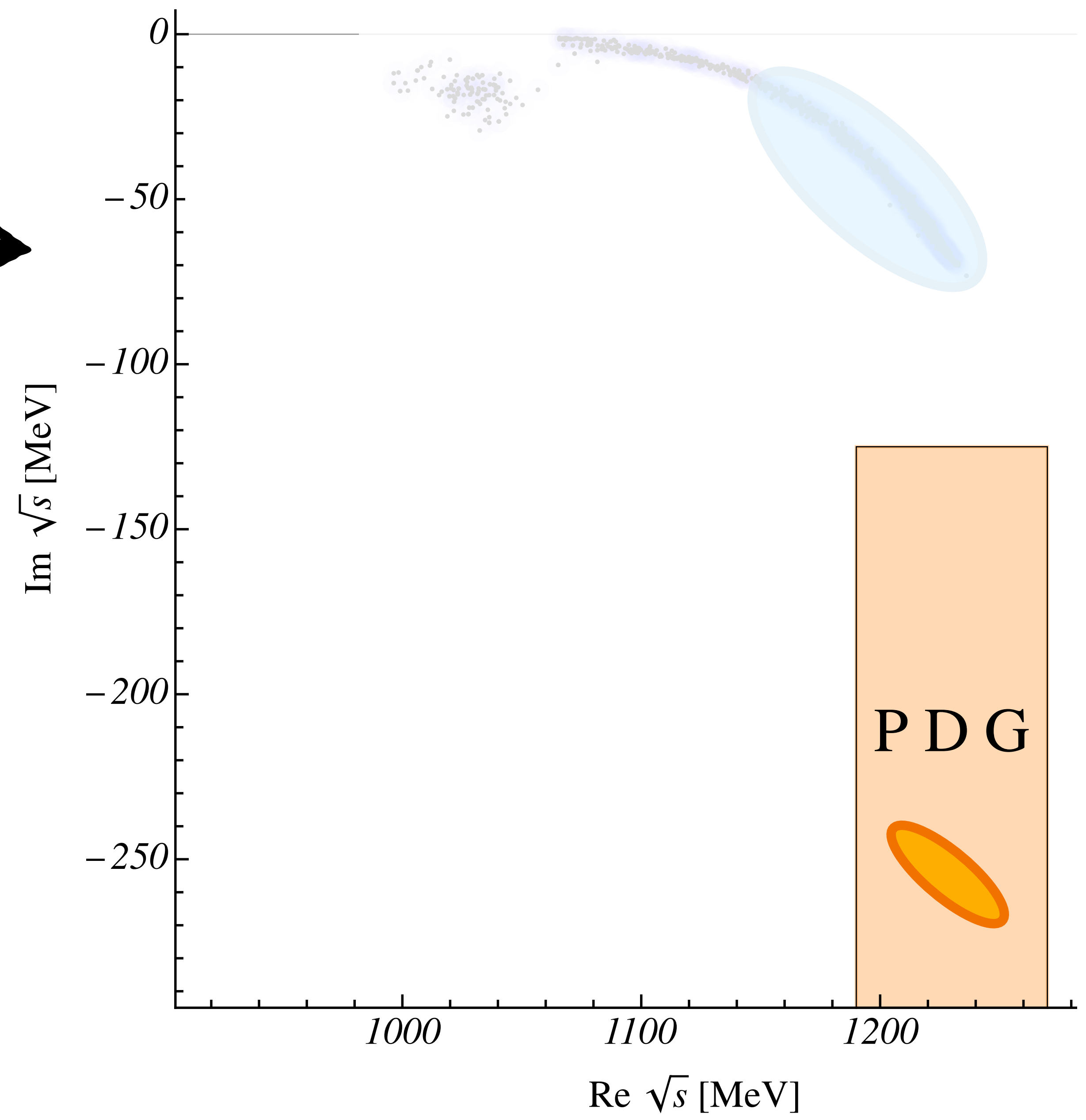
$$T^c = B + C + \int \frac{d^3\ell}{(2\pi)^3}\,\frac{(B+C)}{2E_l}\,\frac{1}{\tilde{K}_n^{-1}-\Sigma_n}\,T^c$$

FVU

$$\det\left[\,2L^3E_{\mathbf{p}}\left(\tilde{K}_2^{-1}-\Sigma_2^L\right)-B-C\,\right]^{T_{1g}}$$

OUTPUT[2]

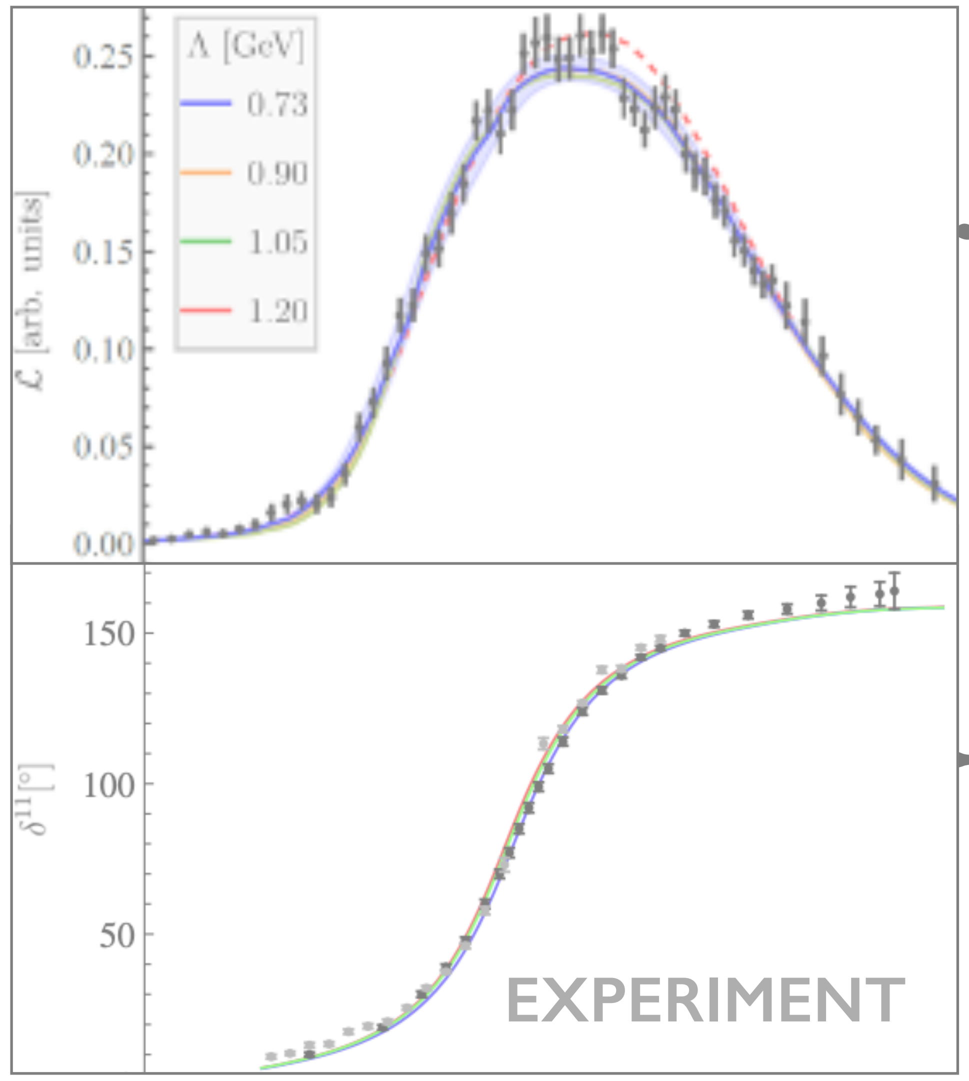


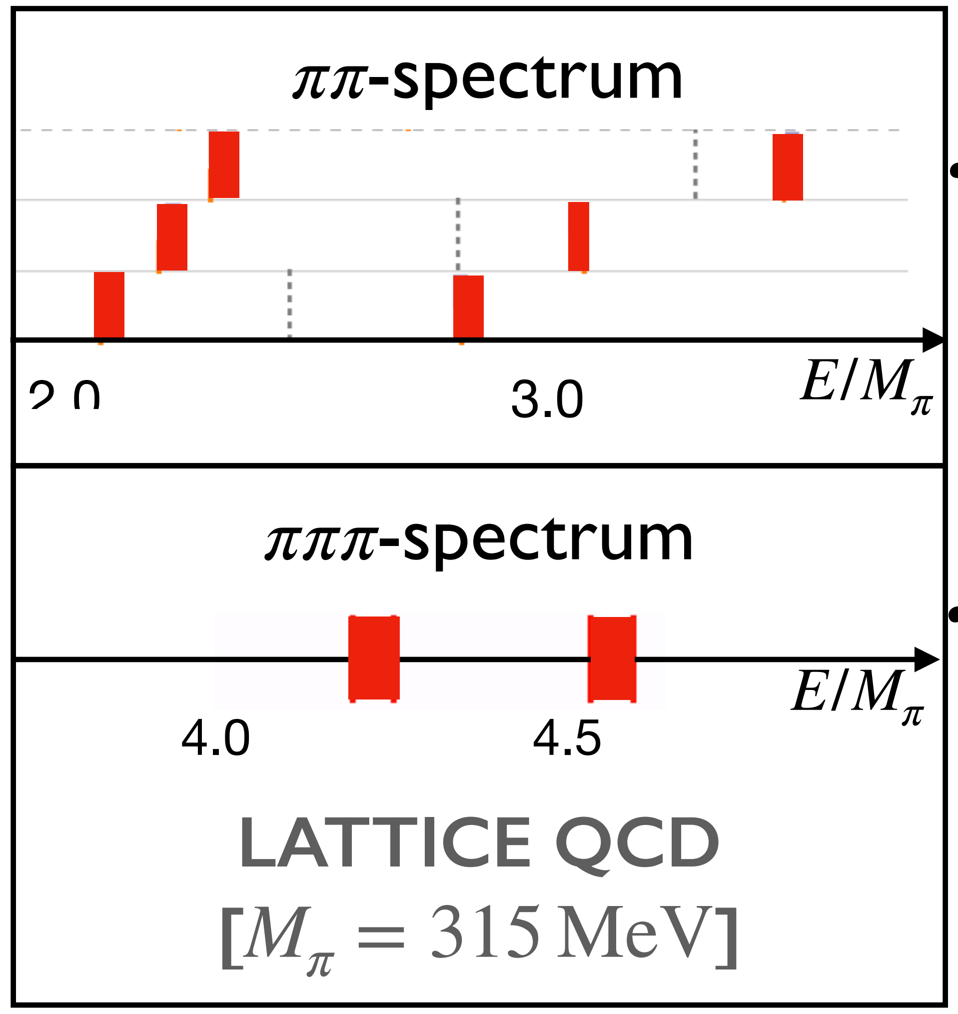


[1] Schael [ALEPH] Phys.Rept. 421 (2005); Nucl.Phys.B 79; Phys.Rev.D 7; [GWQCD] PRD94(2016) PRD98 (2018) PRD 100(2019)
[2] Sadasivan/MM/Döring/Alexandru/Culver/Lee Phys.Rev.D 101 (2020); MM/Culver/Sadasivan/Brett/Döring/Alexandru/Lee [GWQCD] PRL 127 (2021)
other phenomenological determinations: JPAC/….

BLUEPRINT — $a_1(1260)$

INPUT[1]





TRANSITION AMPLITUDES

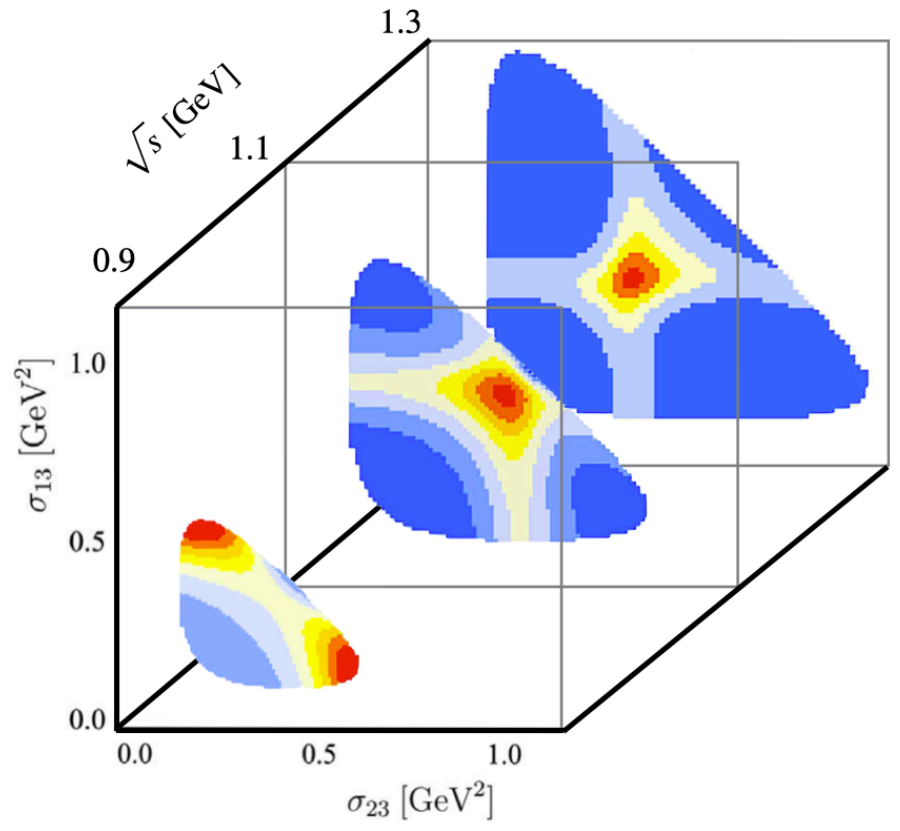
IVU

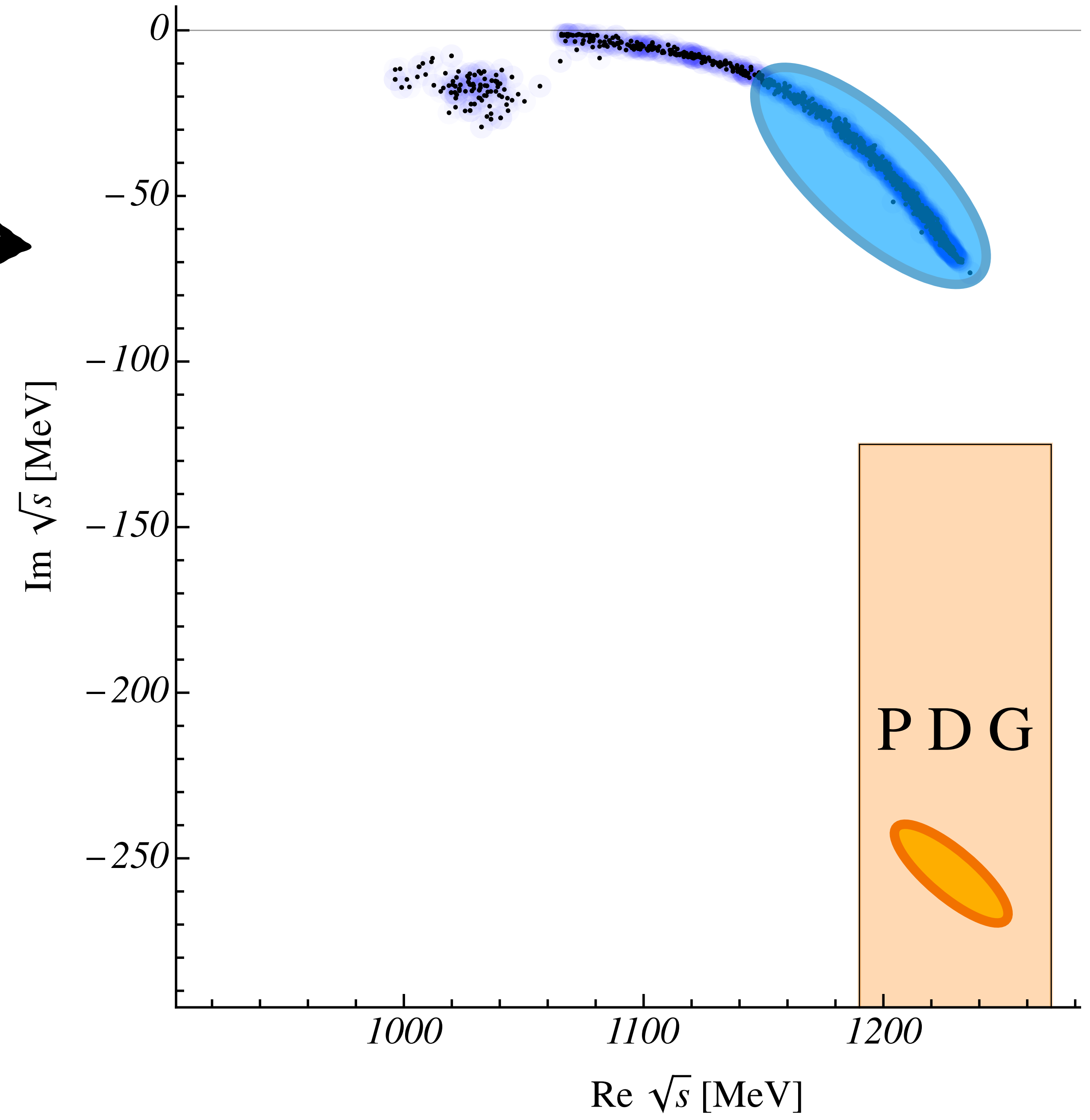
$$T^c = B + C + \int \frac{d^3\ell}{(2\pi)^3} \frac{(B+C)}{2E_l} \frac{1}{\tilde{K}_n^{-1} - \Sigma_n} T^c$$

FVU

$$\det\left[\, 2L^3 E_{\mathbf{p}} \left(\tilde{K}_2^{-1} - \Sigma_2^L\right) - B - C \,\right]^{T_{1g}}$$

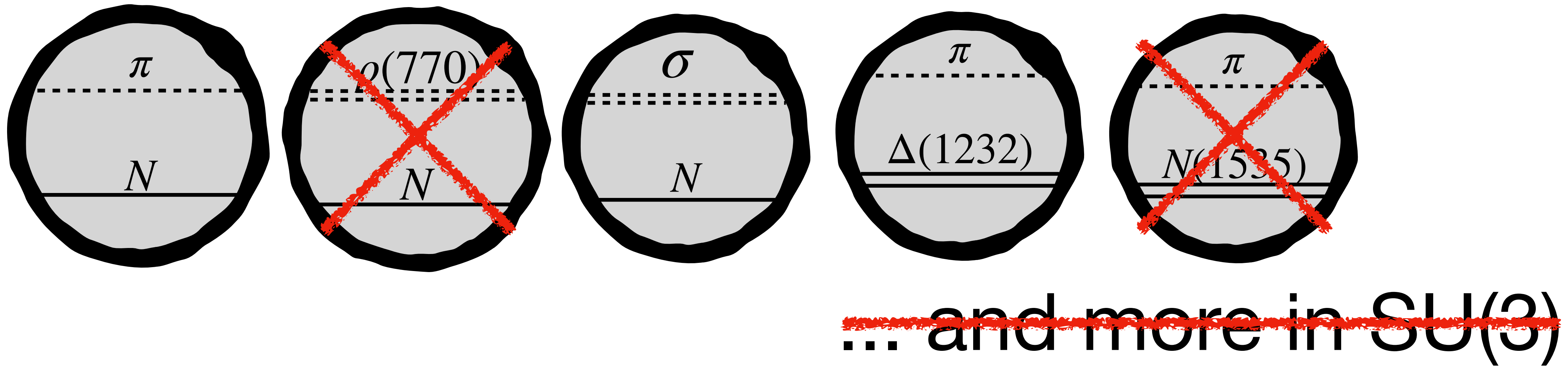
OUTPUT[2]





[1] Schael [ALEPH] Phys.Rept. 421 (2005); Nucl.Phys.B 79; Phys.Rev.D 7; [GWQCD] PRD94(2016) PRD98 (2018) PRD 100(2019)
[2] Sadasivan/MM/Döring/Alexandru/Culver/Lee Phys.Rev.D 101 (2020); MM/Culver/Sadasivan/Brett/Döring/Alexandru/Lee [GWQCD] PRL 127 (2021)

ROPER $N^*(1440)$



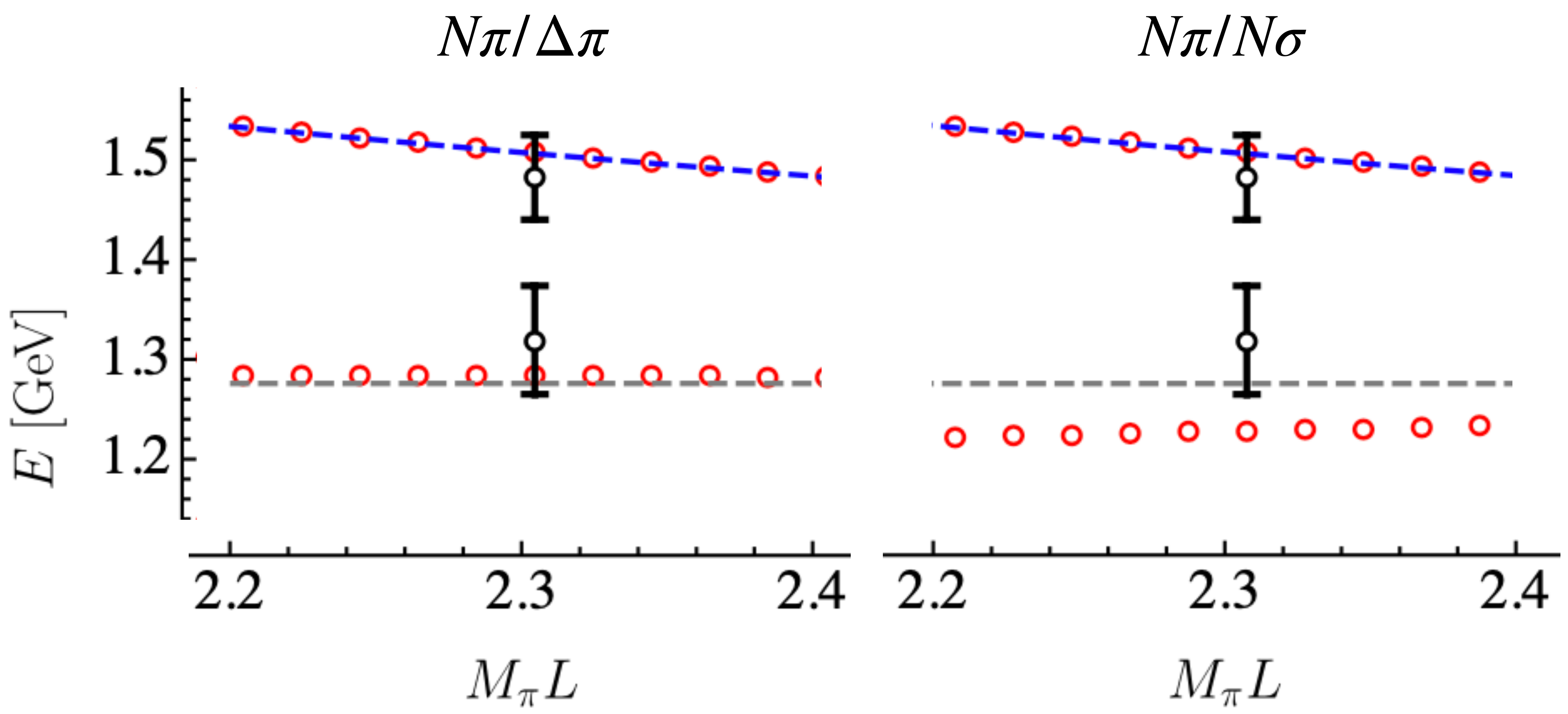
Simplified pilot study[1]

- self-energy formalism from a particle-dimer Lagrangian

⚠ no particle-exchange diagrams

Predict finite-volume spectrum for fixed parameters

- energy shifts very small
- opposing effects of $N\boldsymbol{\sigma}$ and $\boldsymbol{\Delta\pi}$ channels



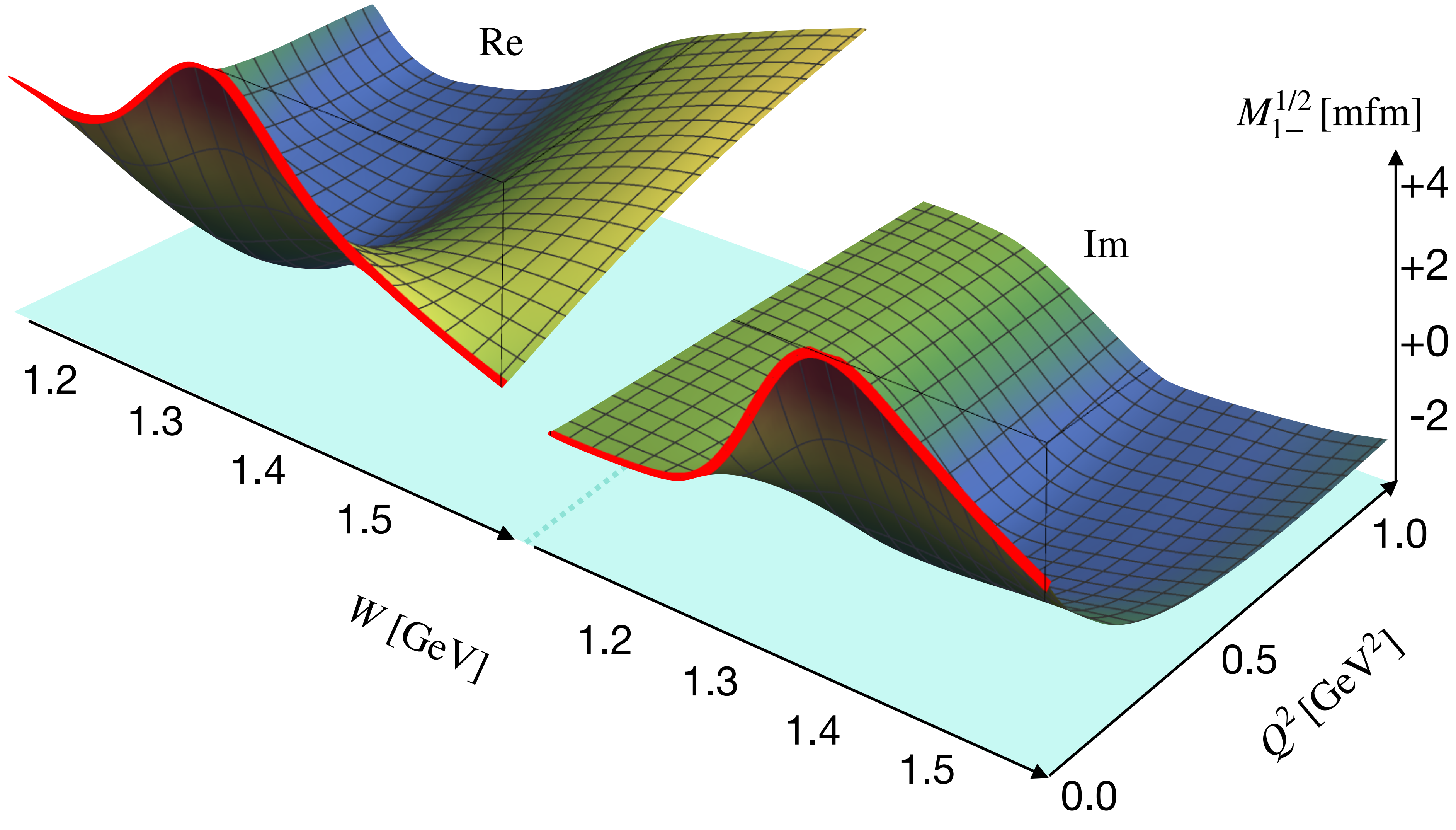
⚠

[1] S. Severt, MM, Ulf-G.Meißner *JHEP* 04 (2023) 100
[2] Lattice values (black dots) Lang et al. Phys.Rev.D 95 (2017) 1

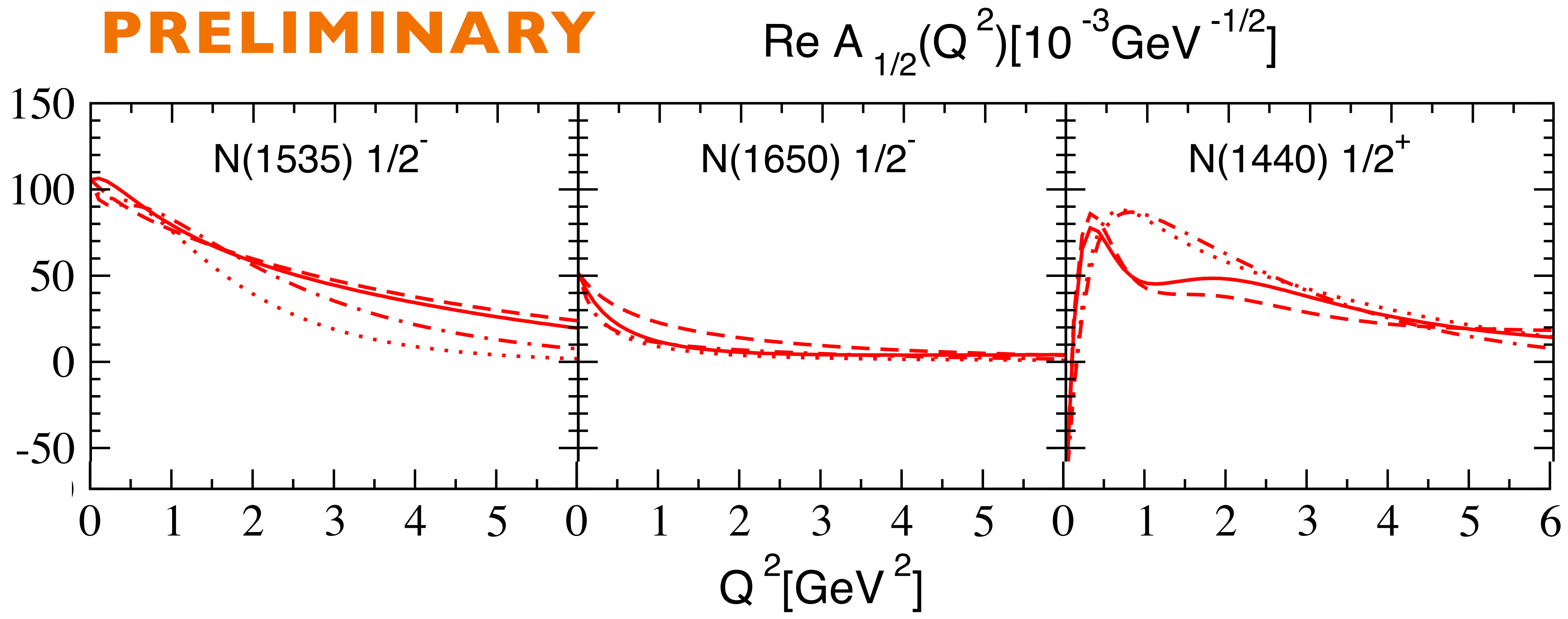
ROPER $N^*(1440)$

Global analysis (bird's view)

- many experimental data & ongoing experiments
  - @MAMI@ELSA@JLAB,…
  - $\gamma p \to \pi N$, $\pi\pi N$, $K\Lambda$, ..
- Jülich-Bonn-Washington[1,2] DCC jbw.phys.gwu.edu/
  - Roper has very unusual Q^2 dependence: $\pi\pi N$ effect
  - In progress: **transition form factors**[3]



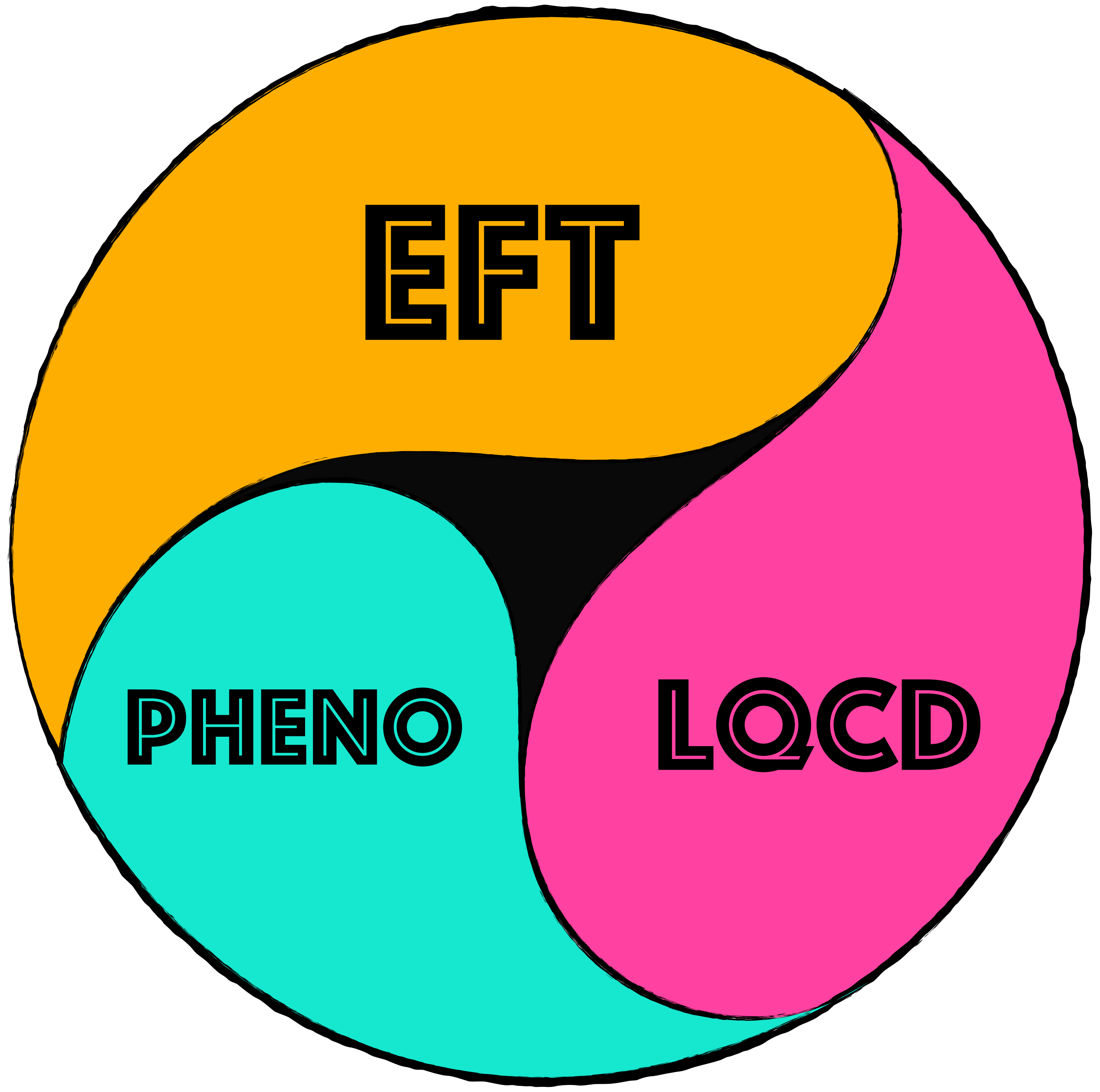
PRELIMINARY



[1] [JBW] MM et al. Phys.Rev.C 103 (2021) 6; Phys.Rev.C 106 (2022) 015201; Eur.Phys.J.A 59 (2023) 12
[2] related approaches MAID/SAID/Gent/ANLOsaka
[3] Talks: Fischer, Leupold, Pena

SUMMARY

Synergetic approaches to universal parameters



- Chiral unitary models & LQCD
  - deep insights into strangeness resonances

  Outlook: unified scenarios across strangeness sectors [Hyperons]

- 3-body methodology has matured

→ Bridge from lattice QCD to phenomenology of complex states

Outlook: Roper resonance

… helicity amplitudes

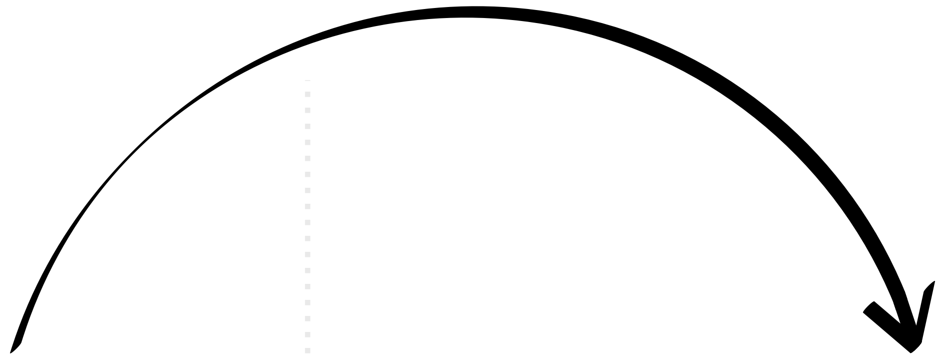
… 3-body dynamics

THANK YOU

TRANSITION AMPLITUDE

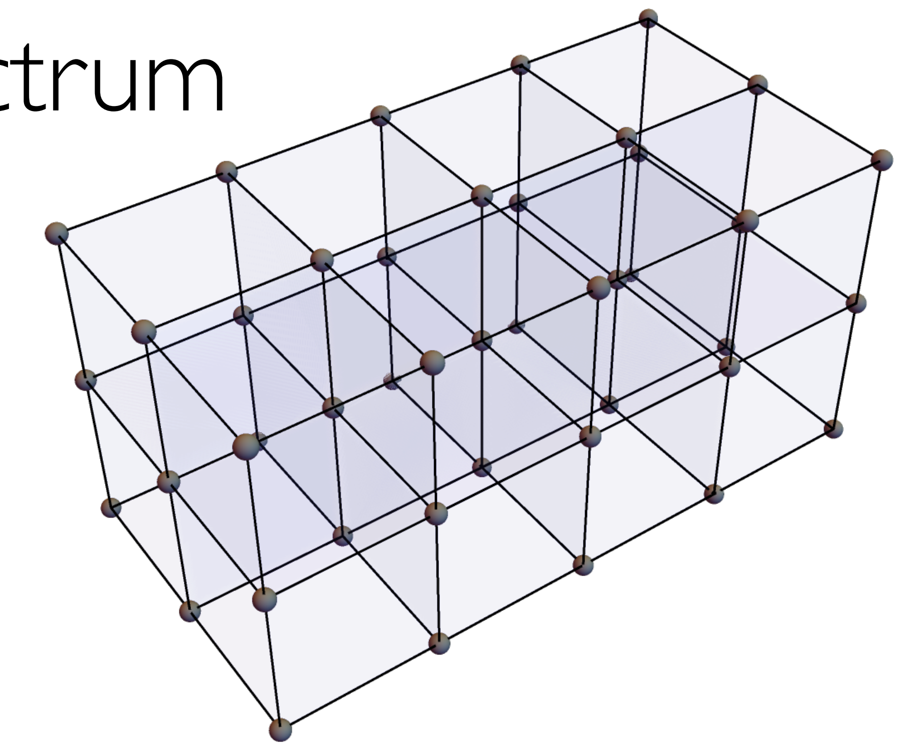
"Infinite Volume Unitarity" — IVU formalism

- Three-body scattering amplitude[1,2]
- Express 3-body through 2+1 system
- Input: C and K
- **On-shell configurations are fixed by Unitarity**



"Finite Volume Unitarity" — FVU formalism

- Three-body quantization condition
- Finite volume lattice QCD spectrum

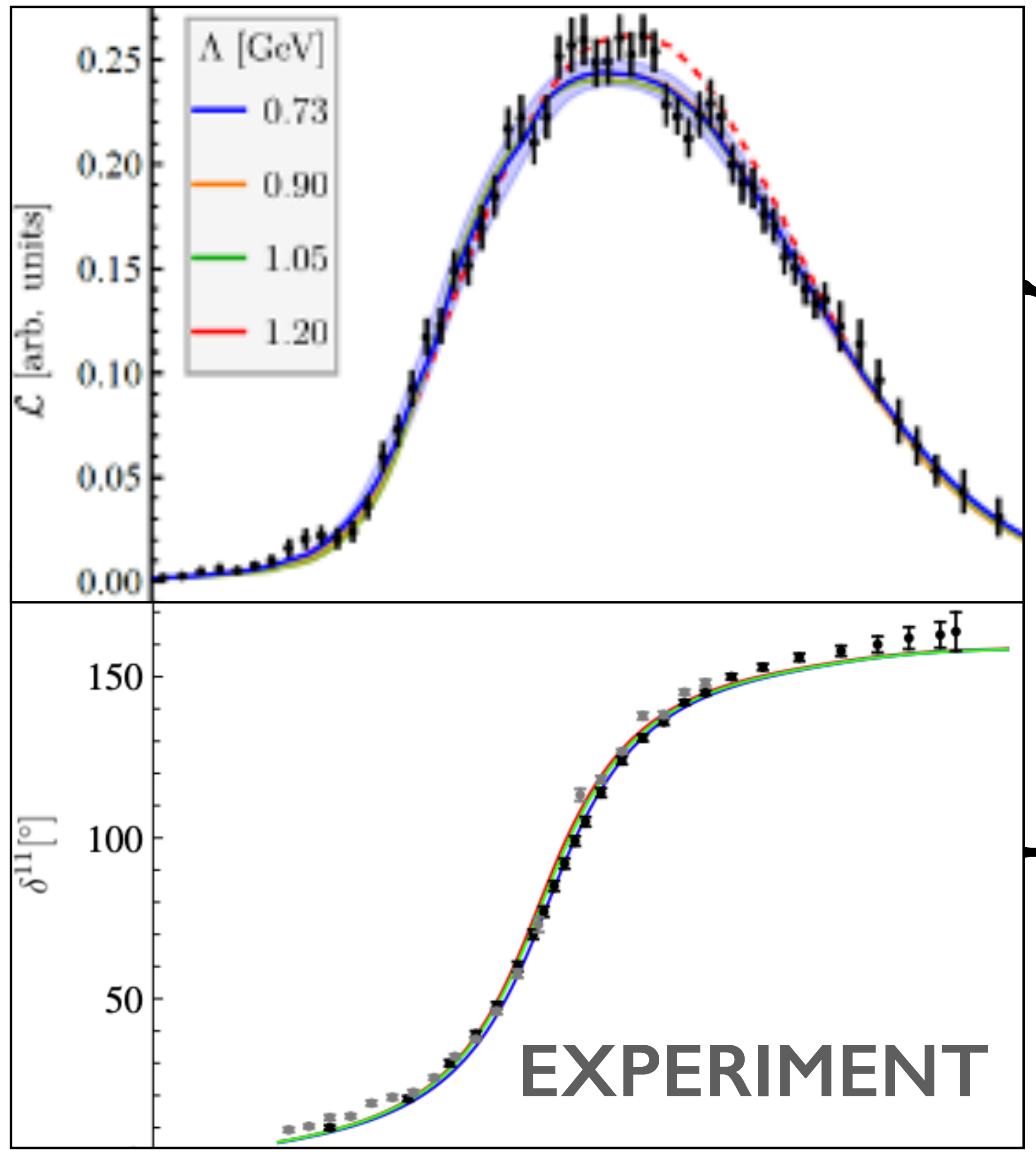


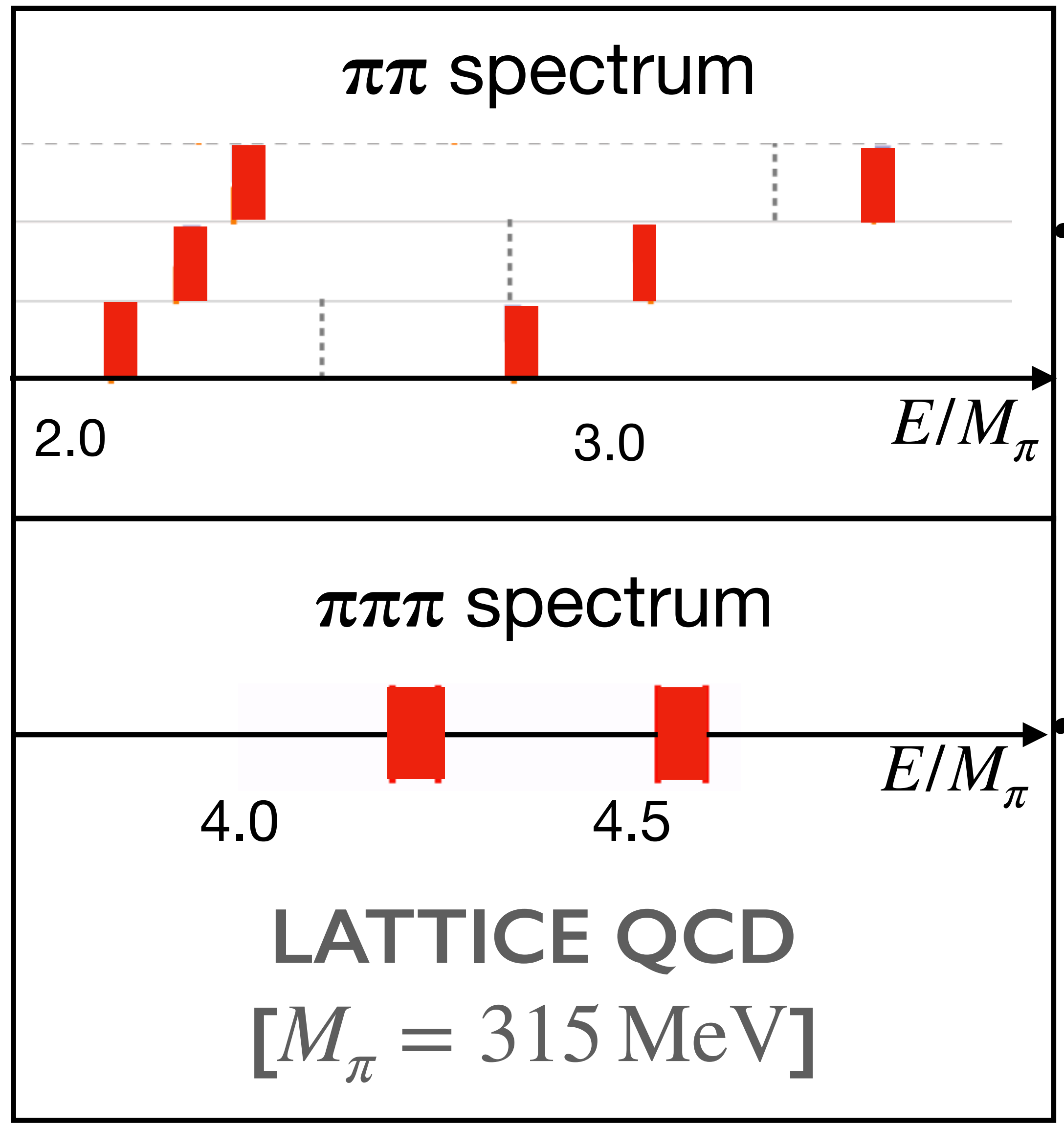
[1] MM et al. Eur.Phys.J.A 53 (2017);
[2] MM/Doring *Eur.Phys.J.A* 53 (2017) 12, Phys.Rev.Lett. 122 (2019) 6
Related approaches: Hansen/Sharpe *Phys.Rev.D* 90 (2014) 11; Wunderlich et al. JHEP 08 (2019); Jackura et al. Eur.Phys.J.C 79 (2019); Meng/Epelbaum *JHEP* 10 (2021) 051; …

INPUT[1]

TRANSITION AMPLITUDES

OUTPUT[2]



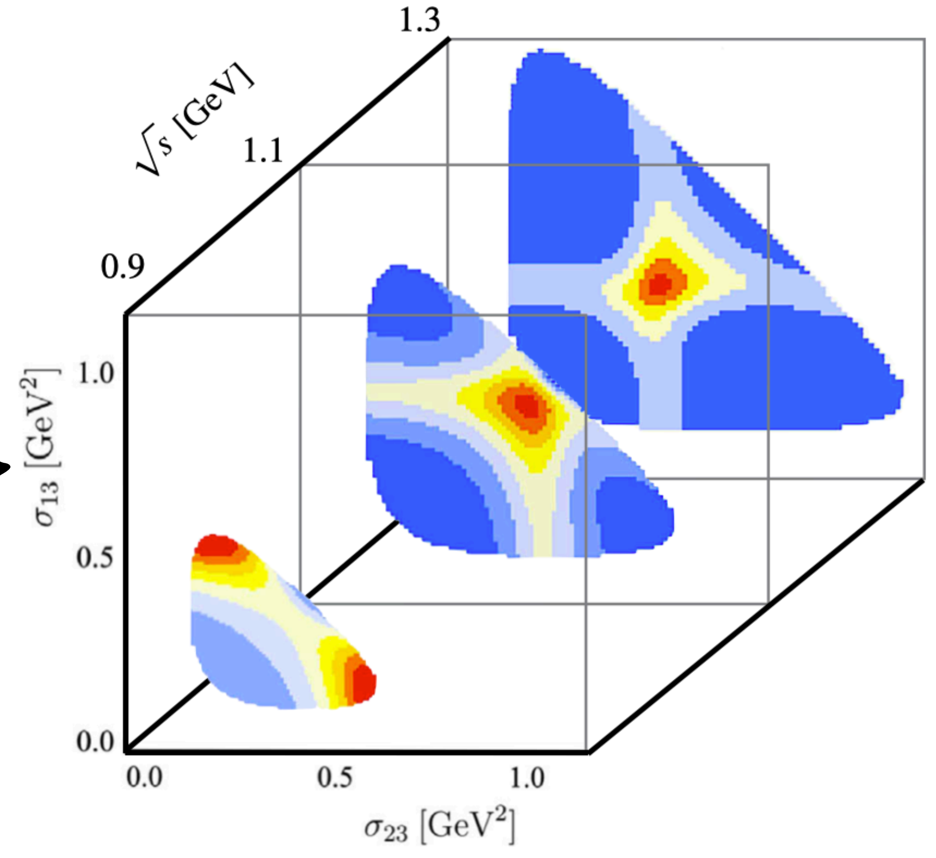


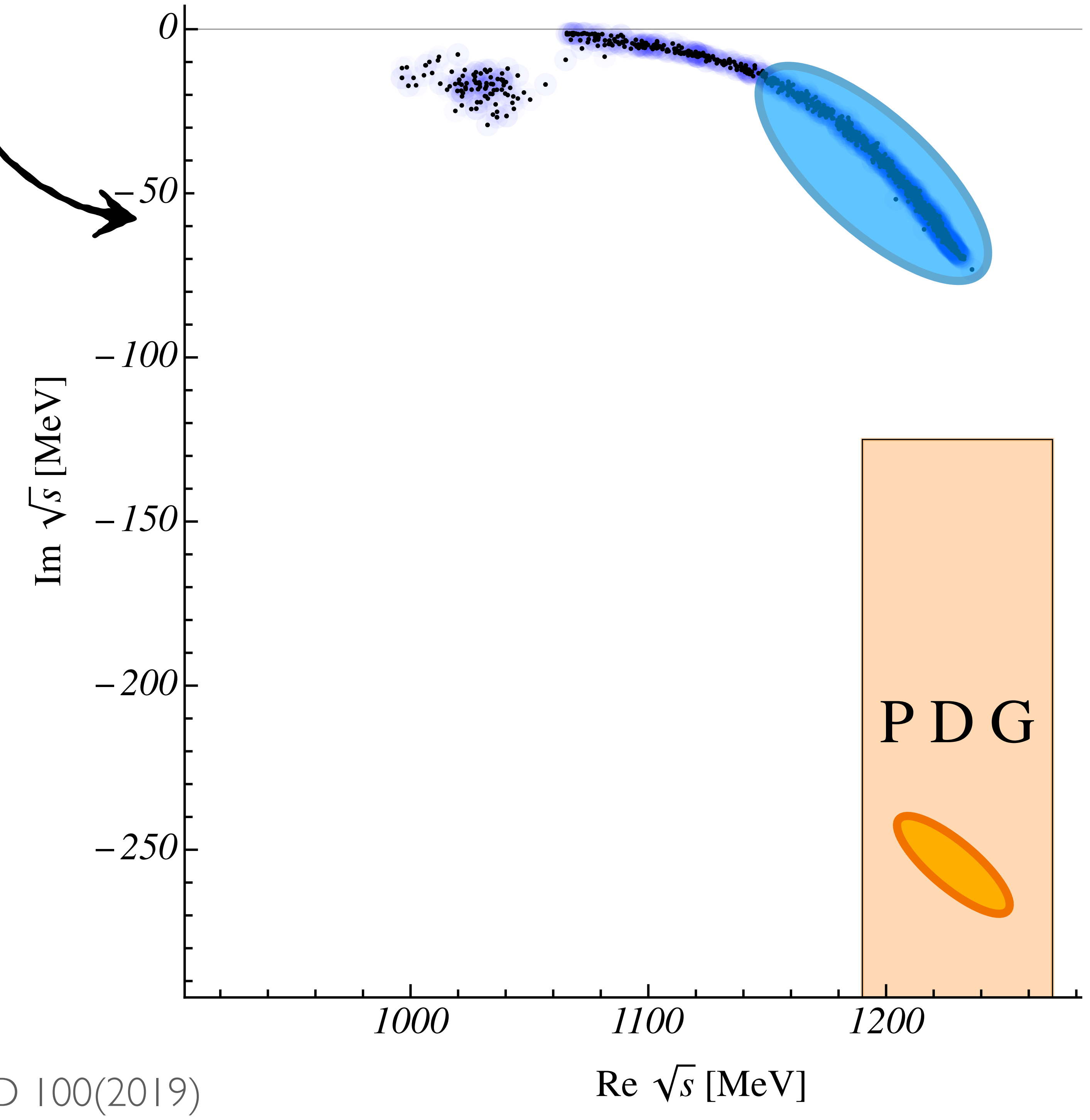
IVU

$$T^c = B + C + \int \frac{d^3\ell}{(2\pi)^3} \frac{(B+C)}{2E_l} \frac{1}{\tilde{K}_n^{-1} - \Sigma_n} T^c$$

FVU

$$\det\left[\,2L^3 E_{\mathbf{p}}\left(\tilde{K}_2^{-1} - \Sigma_2^L\right) - B - C\,\right]^{T_{1g}}$$

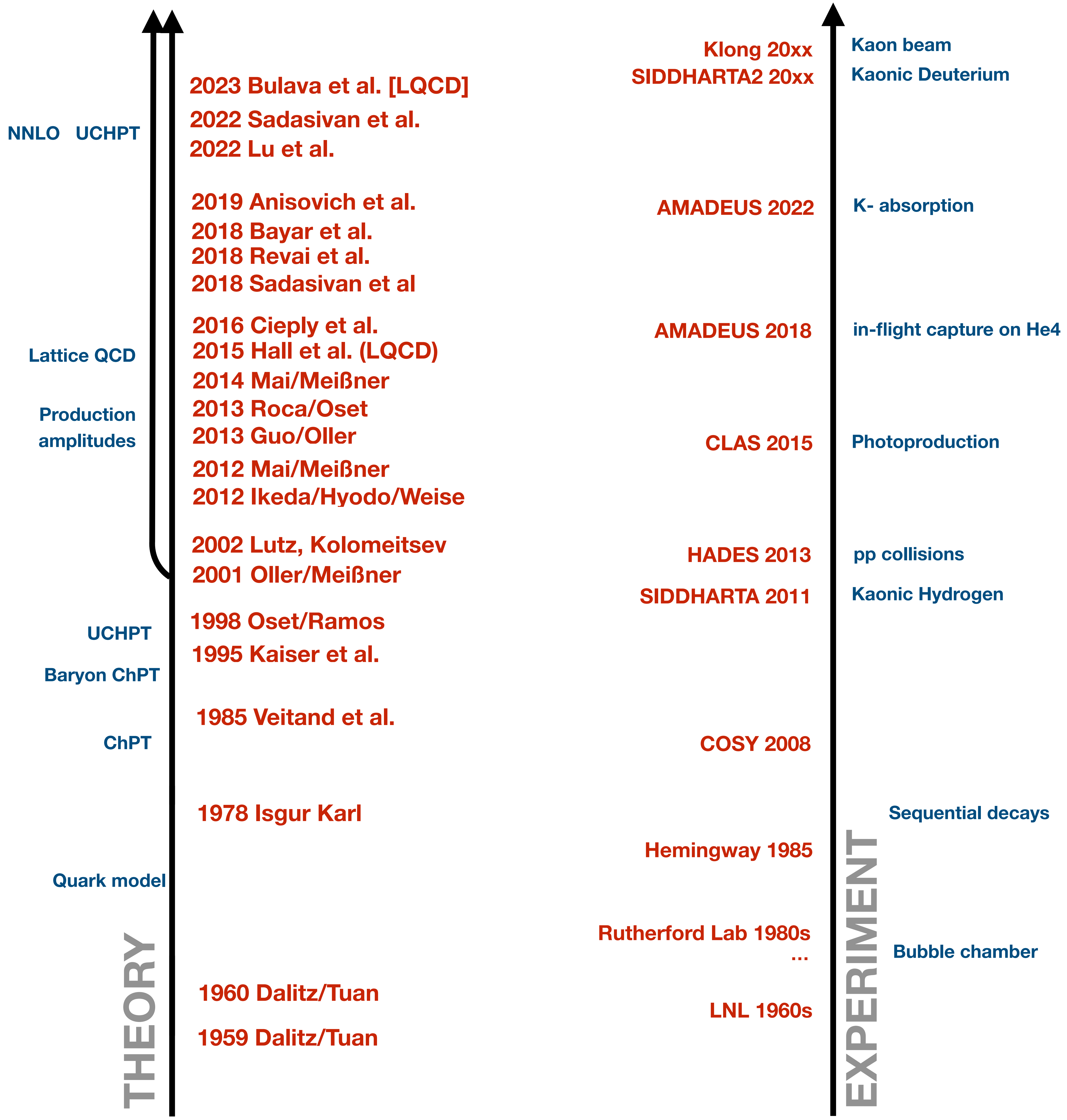




[1] Schael [ALEPH] Phys.Rept. 421 (2005); Nucl.Phys.B 79; Phys.Rev.D 7; [GWQCD] PRD94(2016) PRD98 (2018) PRD 100(2019)
[2] Sadasivan/MM/Döring/Alexandru/Culver/Lee Phys.Rev.D 101 (2020); MM/Culver/Sadasivan/Brett/Döring/Alexandru/Lee [GWQCD] PRL 127 (2022)

THE ENIGMA OF THE Λ(1405)

- Long history of experimental and theoretical efforts[1]
- Second state predicted from UCHPT **Λ**(1380)
    - no direct experimental verification
    - confirmed by many critical tests

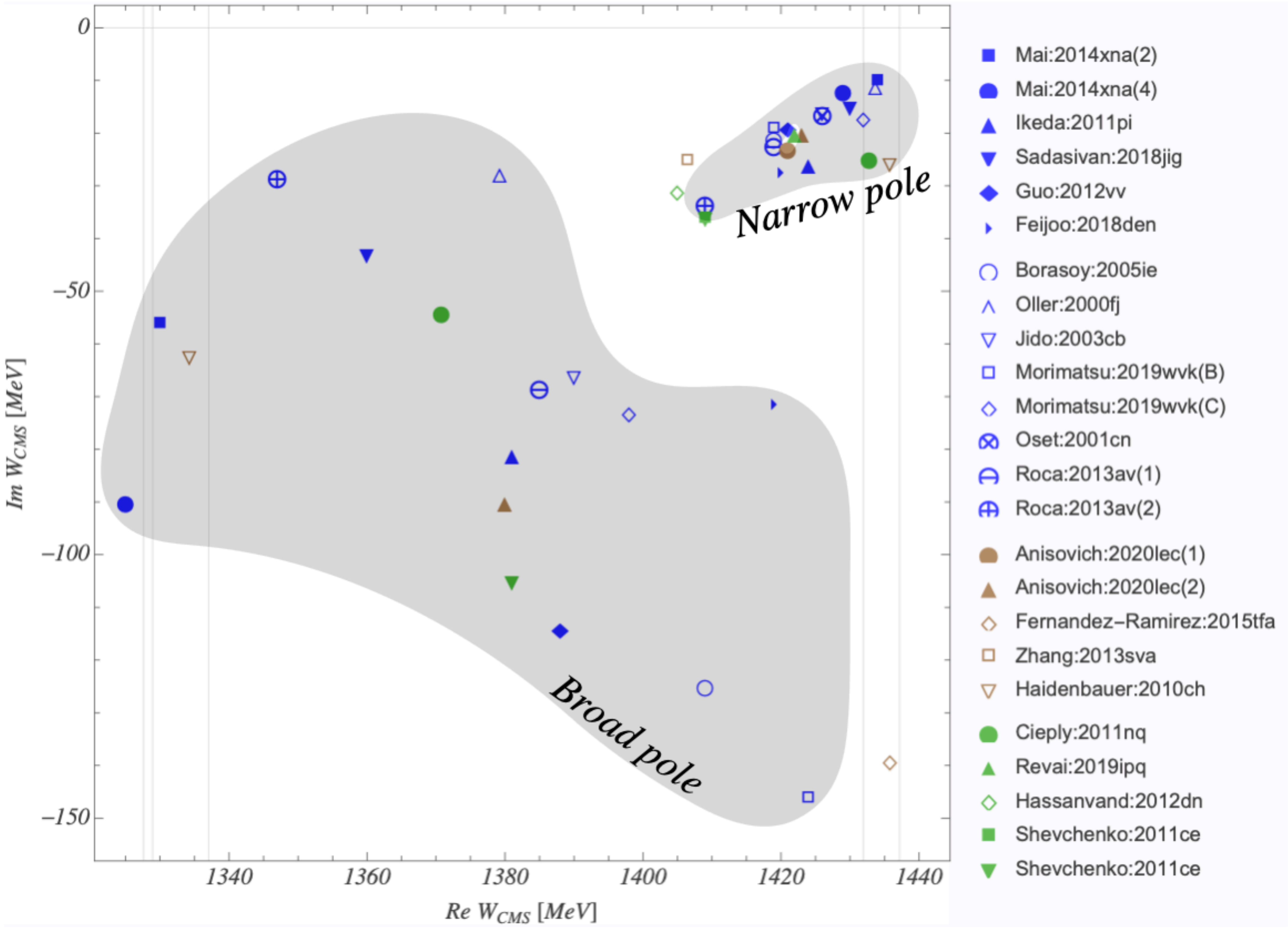


[1] MM EPJST 230 (2021) "Review of the Λ(1405) A curious case of a strangeness resonance";

THE ENIGMA OF THE Λ(1405)

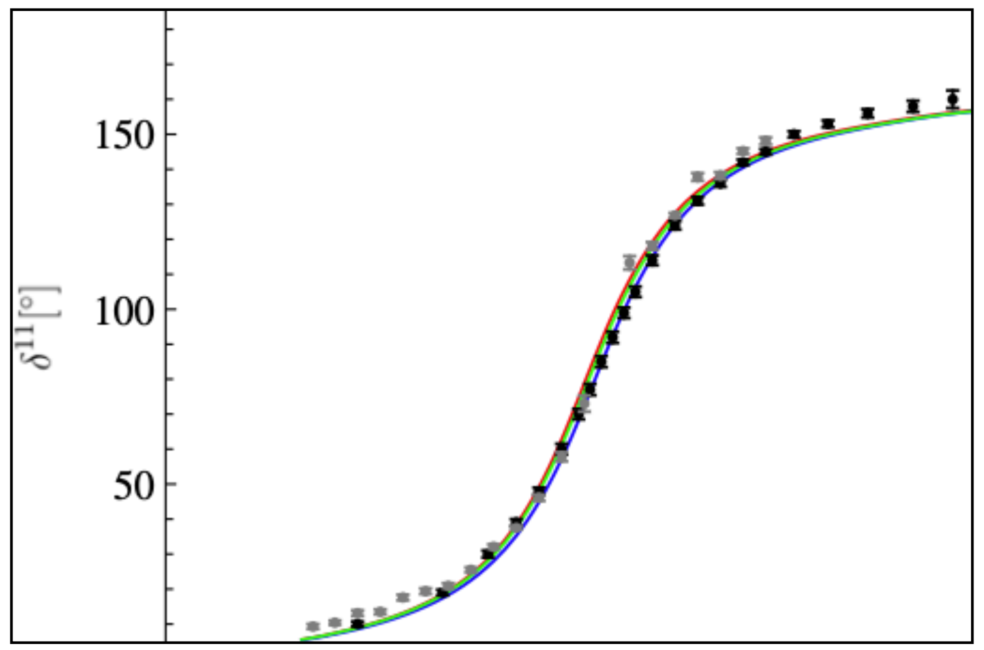
...

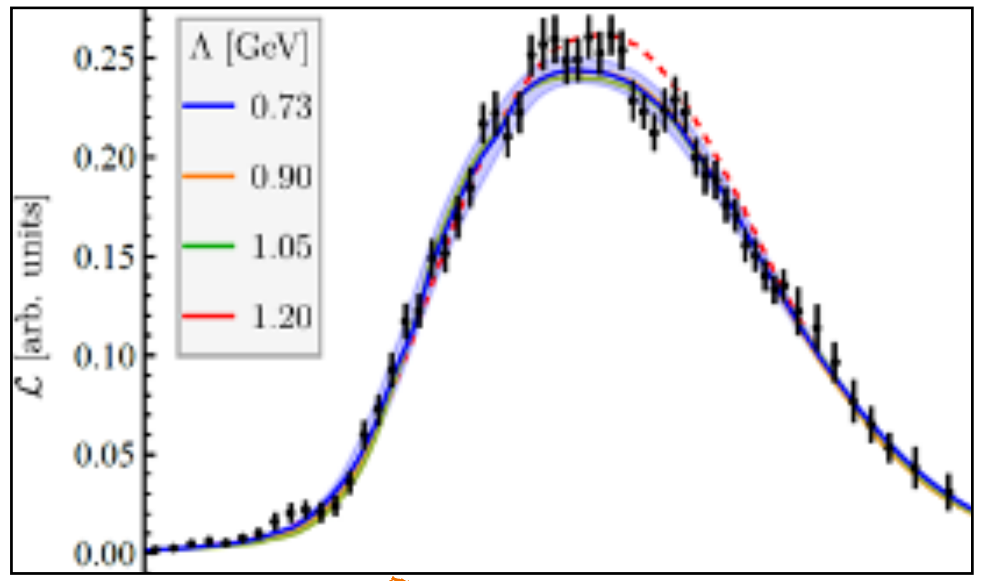
- Theory frontier: NNLO UCHPT determination[1]
- Consistently two poles, but the second pole is less well known
  - second pole below KbarN threshold
  - line-shape only through $\gamma p \to K\pi\Sigma$ [2]

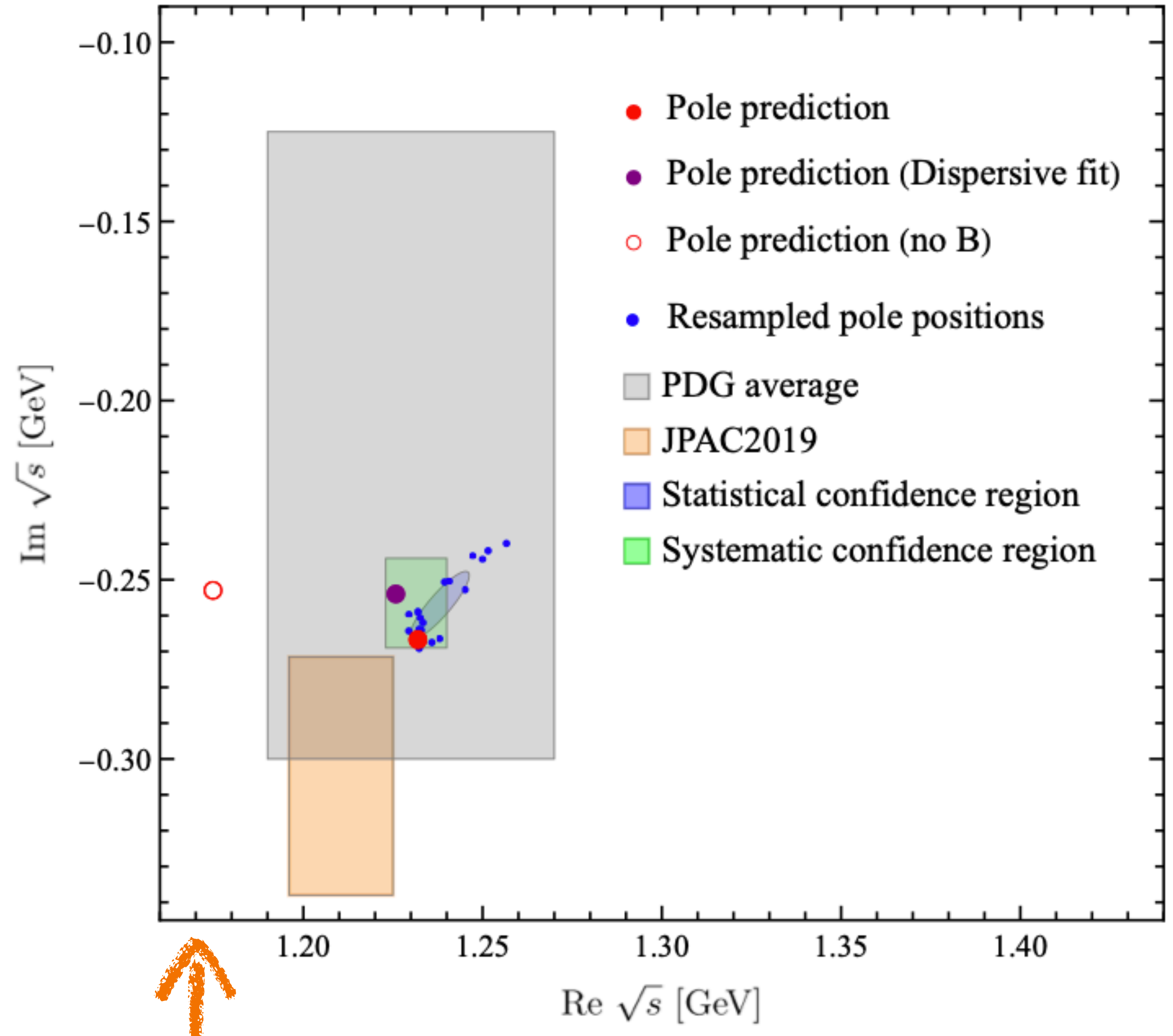


[1] Lu/Geng/Döring/MM Phys.Rev.Lett. 130 (2023)
[2] [CLAS] Moriya et al (2013)

APPLICATION: a₁(1260)

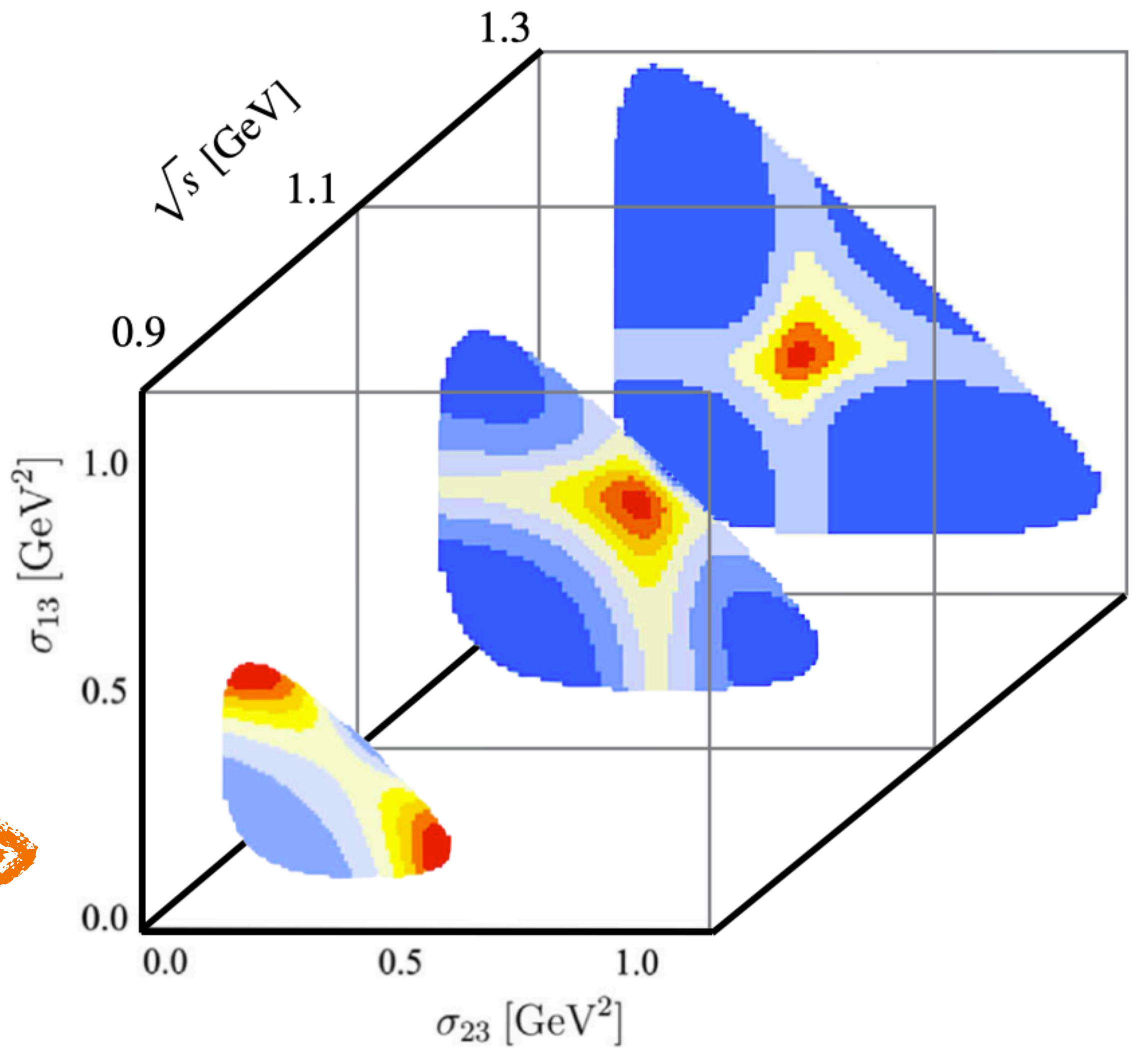






- $\pi\rho$ dynamics dominates the $1^-(1^{++})$ system
- Integral equation solved
  - Helicity formalism
  - complex momentum mapping
- $\pi\rho/\pi\sigma/\pi(\pi\pi)_2$ extended…

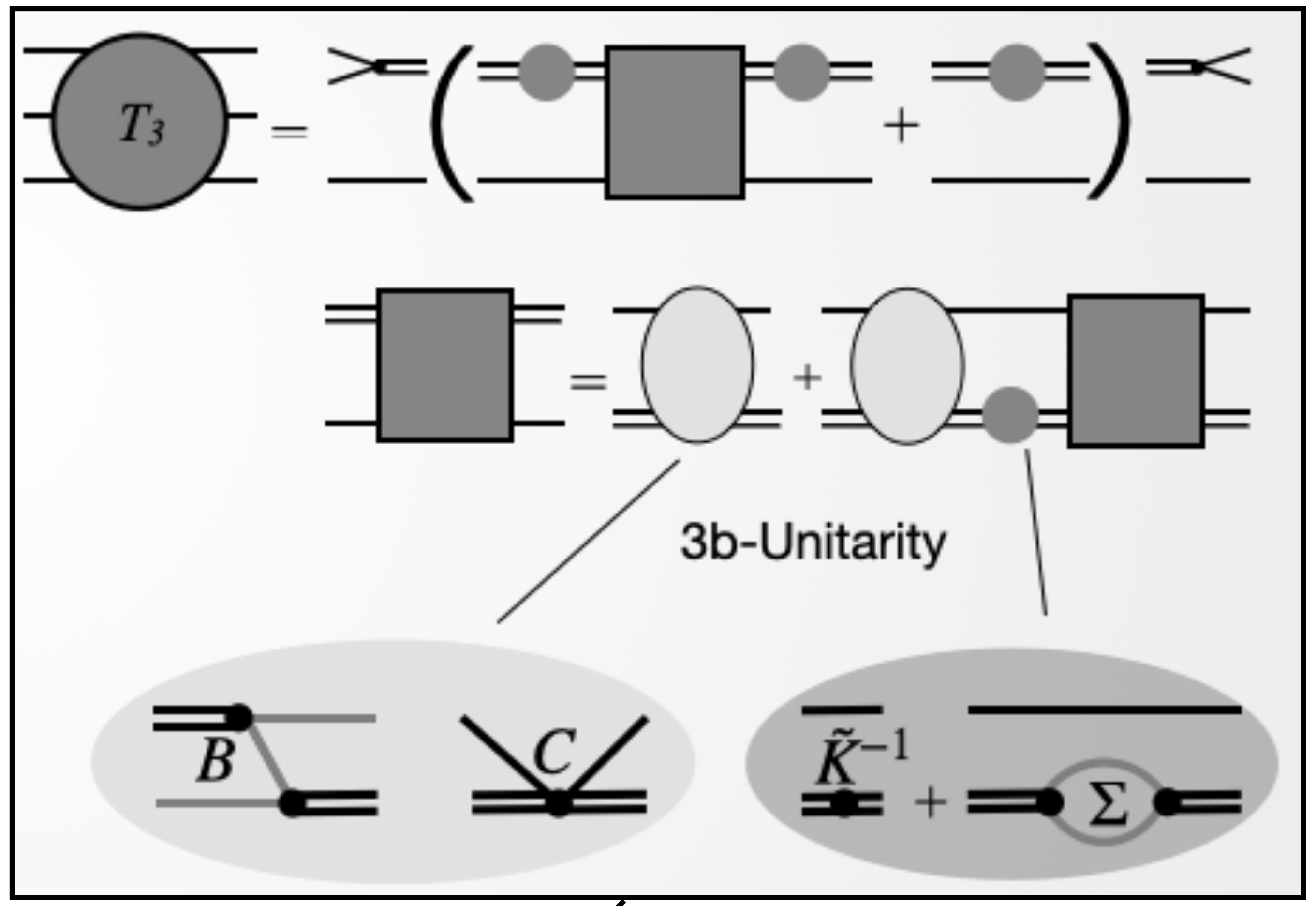
$$T^c = B + C + \int \frac{d^3\ell}{(2\pi)^3} \frac{(B+C)}{2E_l} \frac{1}{\tilde{K}_n^{-1} - \Sigma_n} T^c$$



Data: Schael [ALEPH] Phys.Rept. 421 (2005); Nucl.Phys.B 79; Phys.Rev.D 7;
Pole extraction: Sadasivan/MM/Döring/Alexandru/Culver/Lee Phys.Rev.D 101 (2020)

3-BODY QUANTIZATION CONDITION (FVU)

reminder



- **Finite-volume unitarity (FVU[1])**
  - heavily simplified:
    - on-shell particle-configurations: **Δ**E~mL
    - off-shell particle-configurations: **Δ**E~e^{-mL}
  - *Unitary 3-body amplitude separates these effects*
  - unknown volume independent quantities (K, C)

$$0 = \det\left[2L^3E\left(\tilde{K}_n^{-1} - \Sigma\right) - B - C\right]_{\mathbf{p}'\mathbf{p}}$$

[1] MM/Döring Phys.Rev.Lett. 122 (2019) 6
Reviews: Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019); MM/Doring/Rusetsky Eur.Phys.J.ST 230 (2021);

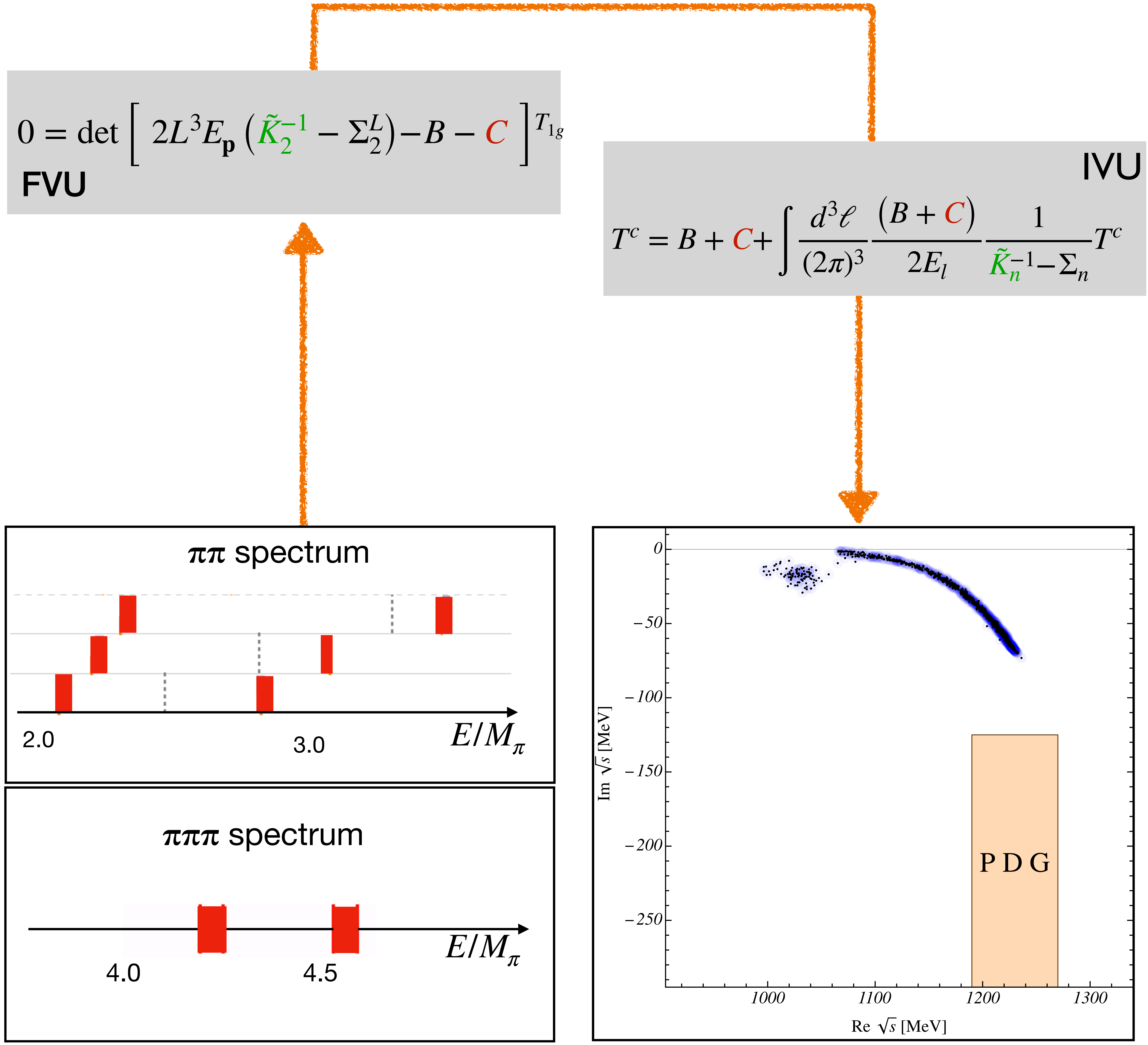
APPLICATION: a₁(1260)

Input:

- 2- and 3-body lattice results with multi-hadron operators[1]
- Unphysical pion mass

Determine infinite-volume quantities

- Pole position of the *a₁(1260)*[2,3]
- Chiral trajectory



[1] [GWQCD] PRD94(2016) PRD98 (2018) PRD 100(2019)
[2] MM/Culver/Sadasivan/Brett/Döring/Alexandru/Lee [GWQCD] PRL 127 (2022)

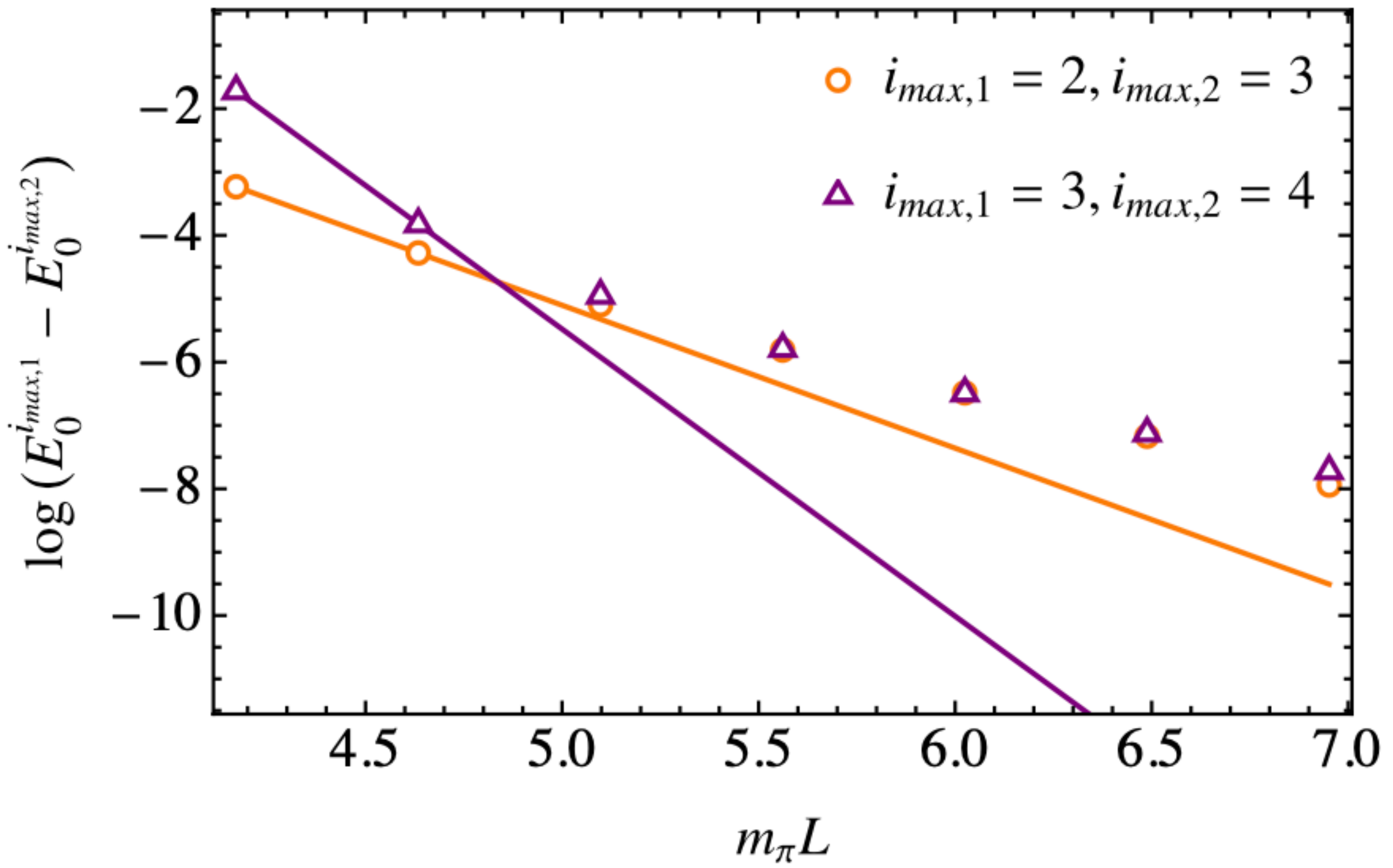
CUTOFF DEPENDENCE[1]

Consider fixed C, K then increase <u>hard</u> cutoff

- 3-body amplitude = genuine integral equation
  - spectator can carry arbitrary momentum away
  - cutoff required (form factors, hard cutoff,…)

$$0 = \det\left[2L^3E\left(\tilde{K}_n^{-1} - \Sigma\right) - B - C\right]_{\mathbf{p}'\mathbf{p}}$$

$$B(\sqrt{s}) = \frac{1}{\sqrt{s} - \sqrt{s_{\rm on}} + i\epsilon}$$



- energy eigenvalues change slower than $\Delta E \sim e^{-mL}$
- one-particle exchange falls off not rapidly enough

[1] paper in preparation

CUTOFF DEPENDENCE[1]

Consider fixed C, K then increase hard cutoff

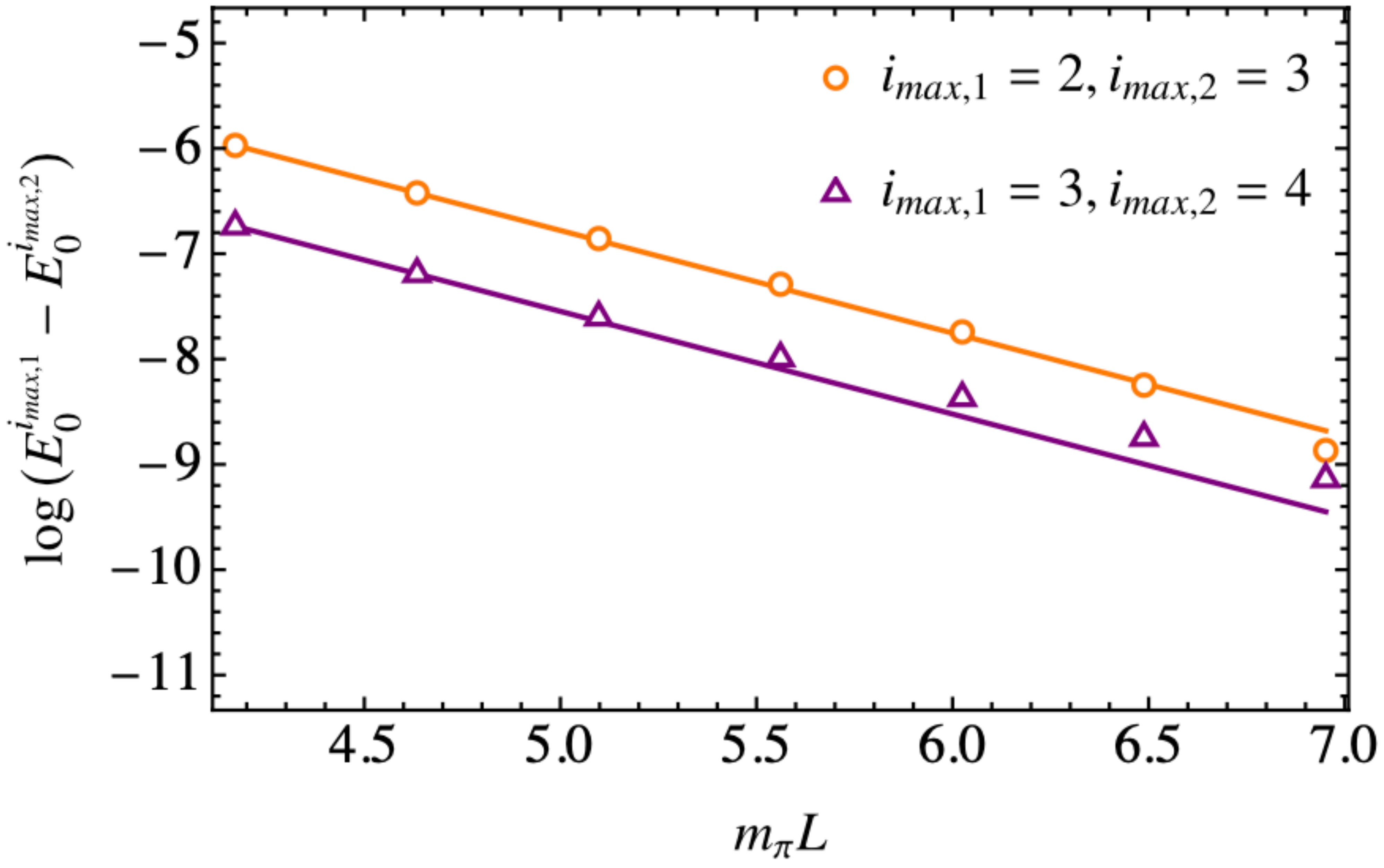
… over-subtract OPE

$$\overline{B}(\sqrt{s}) = B(0) + B'(0)\sqrt{s} + \frac{s}{s_{\text{on}}}\frac{N}{2E_{p+p'}}\frac{1}{\sqrt{s}-\sqrt{s_{\text{on}}}+i\epsilon}$$

- 3-body amplitude = genuine integral equation
  - spectator can carry arbitrary momentum away
  - cutoff required (form factors, hard cutoff,…)

$$0 = \det\left[2L^3E\left(\tilde{K}_n^{-1} - \Sigma\right) - B - C\right]_{\mathbf{p}'\mathbf{p}}$$

$$B(\sqrt{s}) = \frac{1}{\sqrt{s}-\sqrt{s_{\text{on}}}+i\epsilon}$$



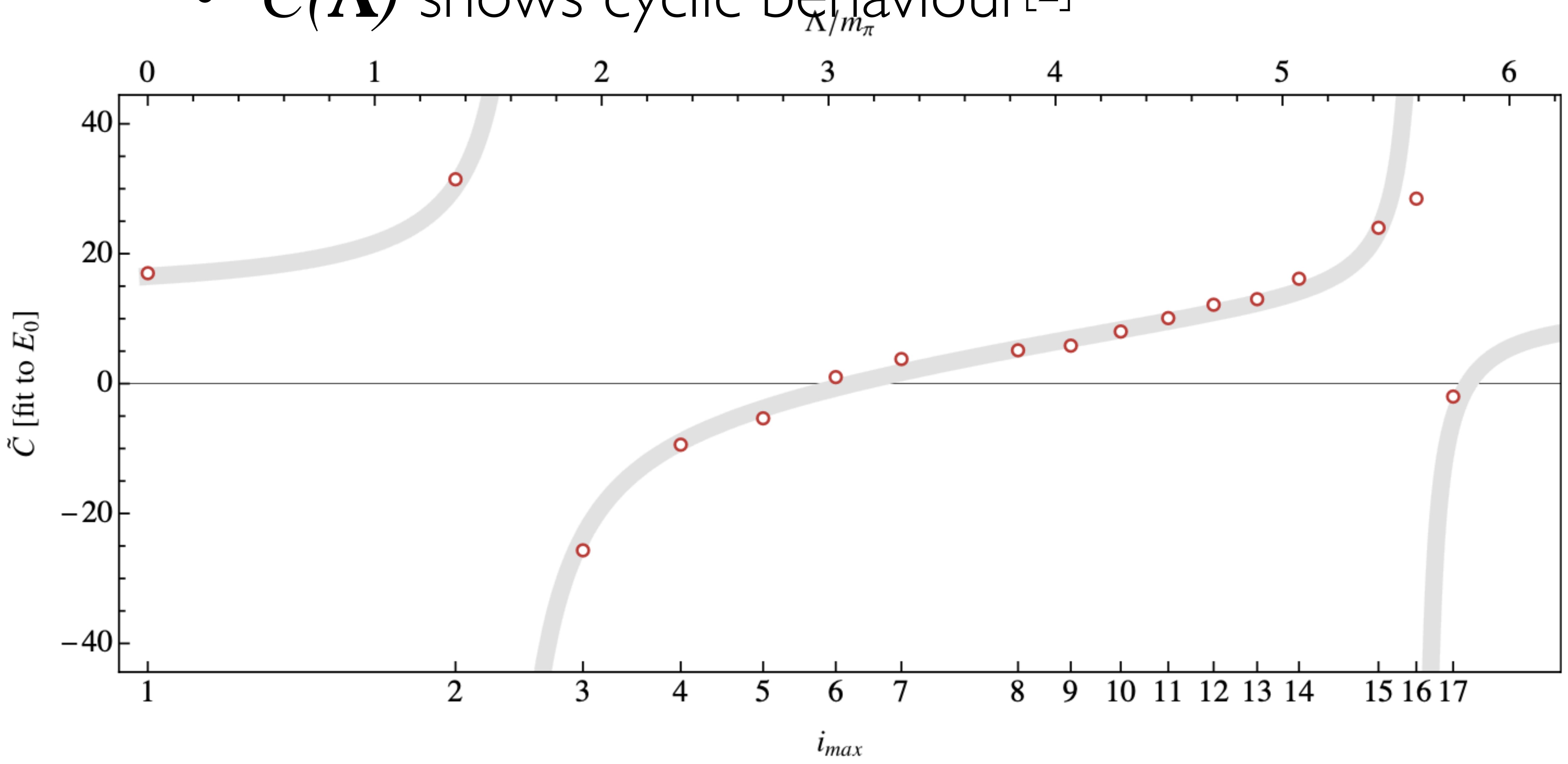
- energy eigenvalues change as $\Delta E \sim e^{-mL}$

[1] paper in preparation

CUTOFF DEPENDENCE[1]

Consider fixed ground-state finite-volume level (E_0)

- change cutoff & refit C
- $\boldsymbol{\pi\rho/\pi(\pi\pi)_2}$ repulsiv system
- $C(\boldsymbol{\Lambda})$ shows cyclic behaviour[2]



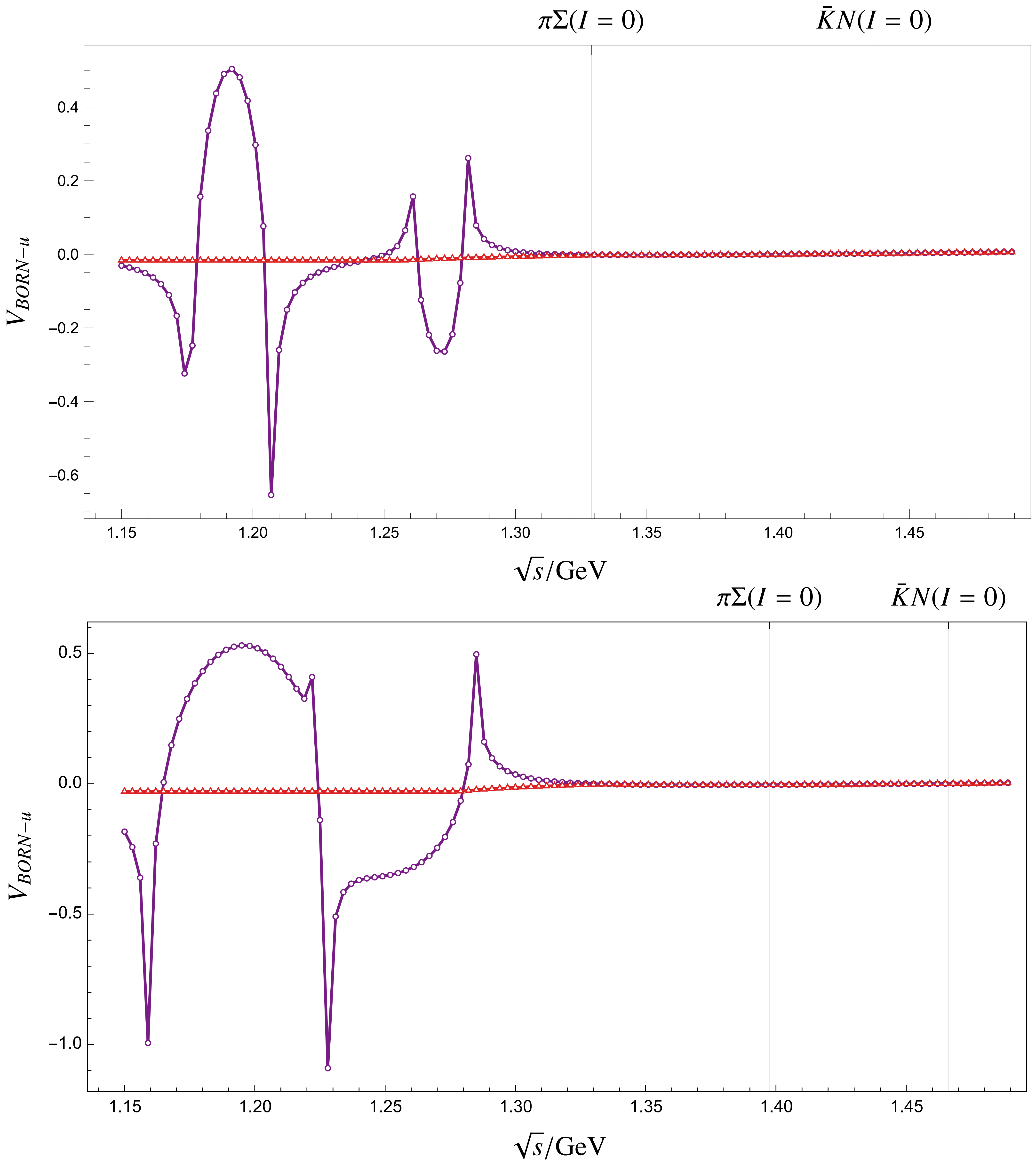
- 3-body amplitude = genuine integral equation
  - spectator can carry arbitrary momentum away
  - cutoff required (form factors, hard cutoff,…)

$$0 = \det\left[2L^3E\left(\tilde{K}_n^{-1} - \Sigma\right) - B - C\right]_{\mathbf{p}'\mathbf{p}}$$

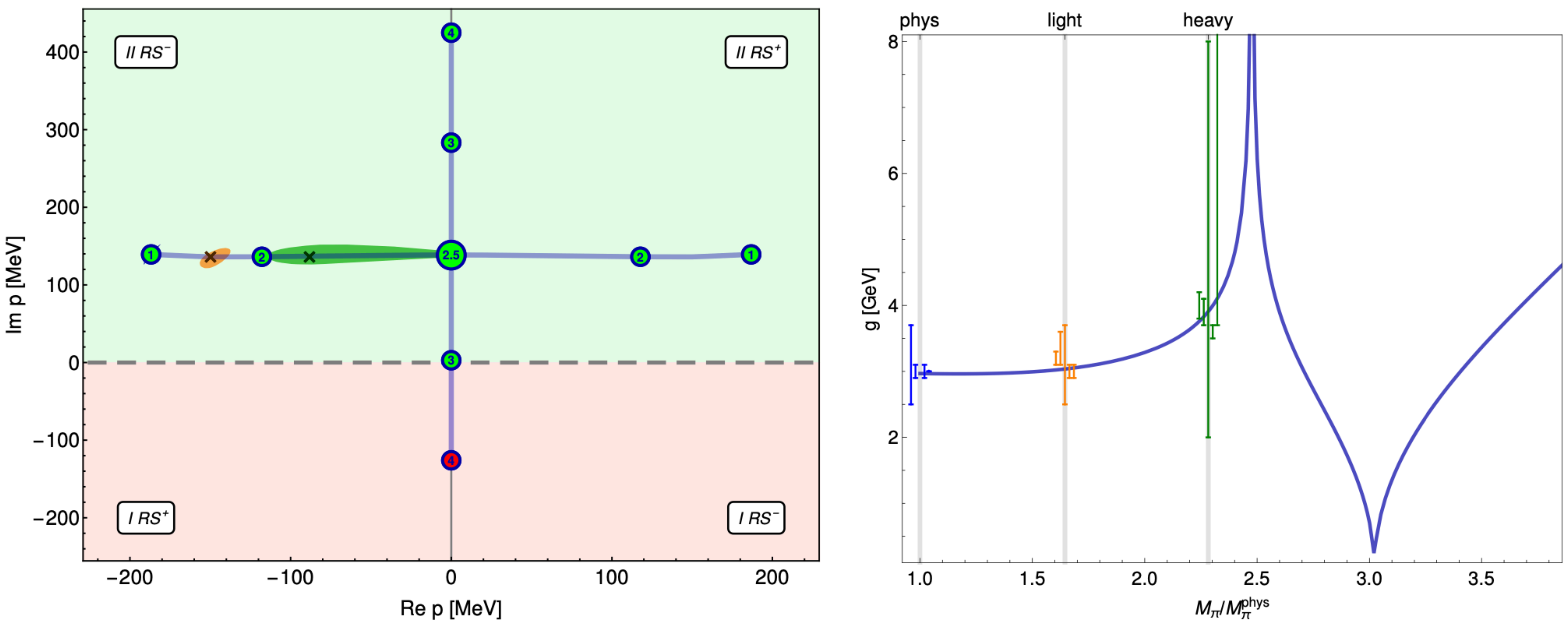
[1] paper in preparation
[2] Bedaque/Hammer/van Kolck, Phys. Rev. Lett. 82 (1999) 463; Bedaque/Hammer/van Kolck, Nucl.Phys. A 646 (1999) 444

U-CHANNEL IN THE Λ(1405)

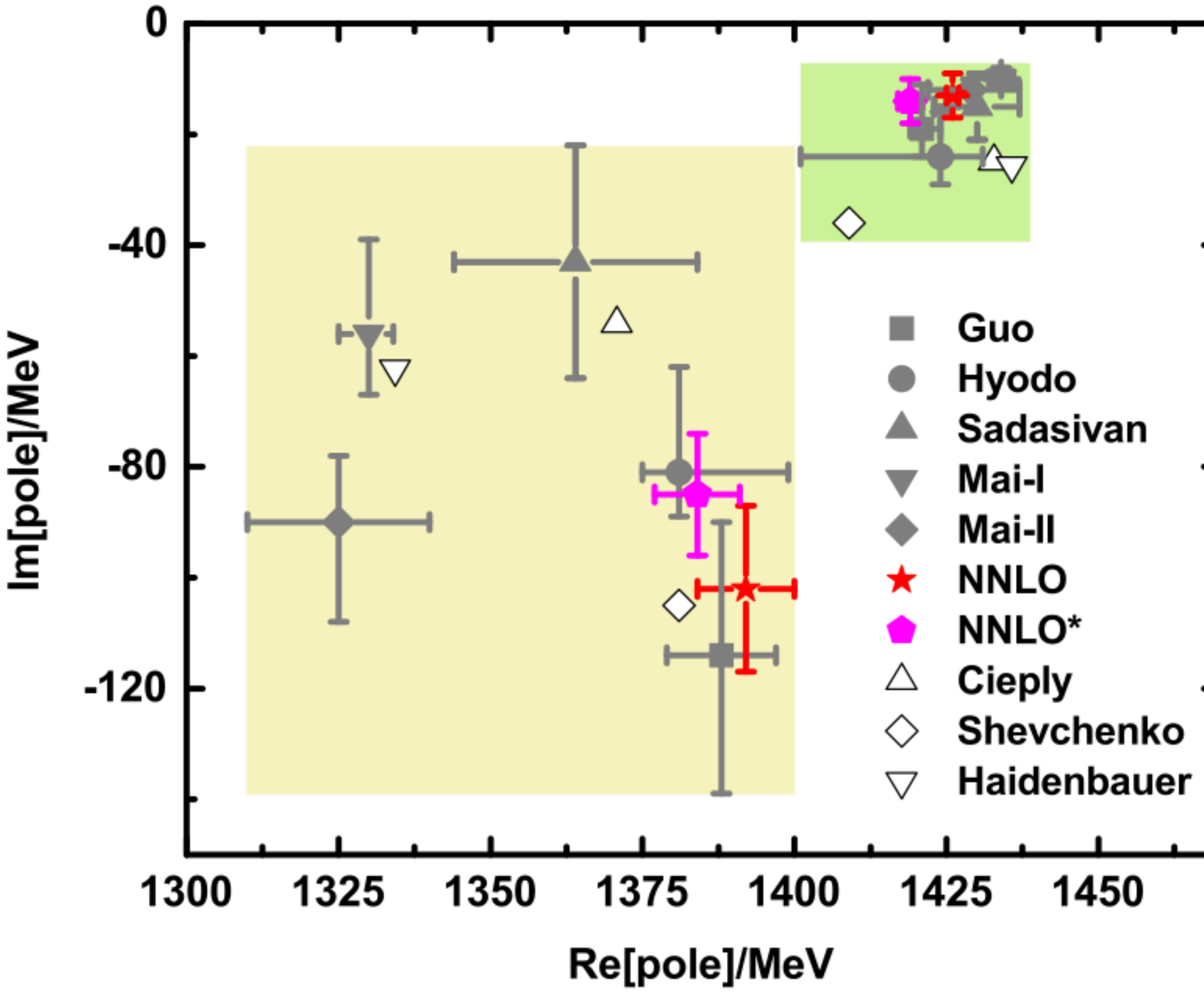
- New insights[1] from LQCD [next talk]
  - confirming two-pole scenario
- Chiral extrapolations (through UCHPT)[2]
  - u-channel baryon exchange may complicate the picture (3-body)
  - sub-leading effect



[1] [BaSc] Bulava et al. 2307.10413; 2307.13471
[2] Guo/Kamyia/MM/Meißner Phys.Lett.B 846 (2023)



II RS⁻
II RS⁺
I RS⁺
I RS⁻
Im p [MeV]
Re p [MeV]
phys
light
heavy
g [GeV]
$M_\pi/M_\pi^{\rm phys}$



0
-40
-80
-120
Im[pole]/MeV
1300
1325
1350
1375
1400
1425
1450
Re[pole]/MeV
Guo
Hyodo
Sadasivan
Mai-I
Mai-II
NNLO
NNLO*
Cieply
Shevchenko
Haidenbauer

$$\{1, 8_s, 8_a, 10, \overline{10}, 27\}$$

$$\begin{pmatrix} |\pi\Sigma\rangle \\ |\bar{K}N\rangle \\ |\eta\Lambda\rangle \\ |K\Xi\rangle \end{pmatrix} = \frac{1}{\sqrt{40}} \begin{pmatrix} \sqrt{15} & -\sqrt{24} & 0 & -1 \\ -\sqrt{10} & -2 & \sqrt{20} & -\sqrt{6} \\ -\sqrt{5} & -\sqrt{8} & 0 & 3\sqrt{3} \\ \sqrt{10} & 2 & 2\sqrt{5} & \sqrt{6} \end{pmatrix} \begin{pmatrix} |1\rangle \\ |8\rangle \\ |8'\rangle \\ |27\rangle \end{pmatrix},$$

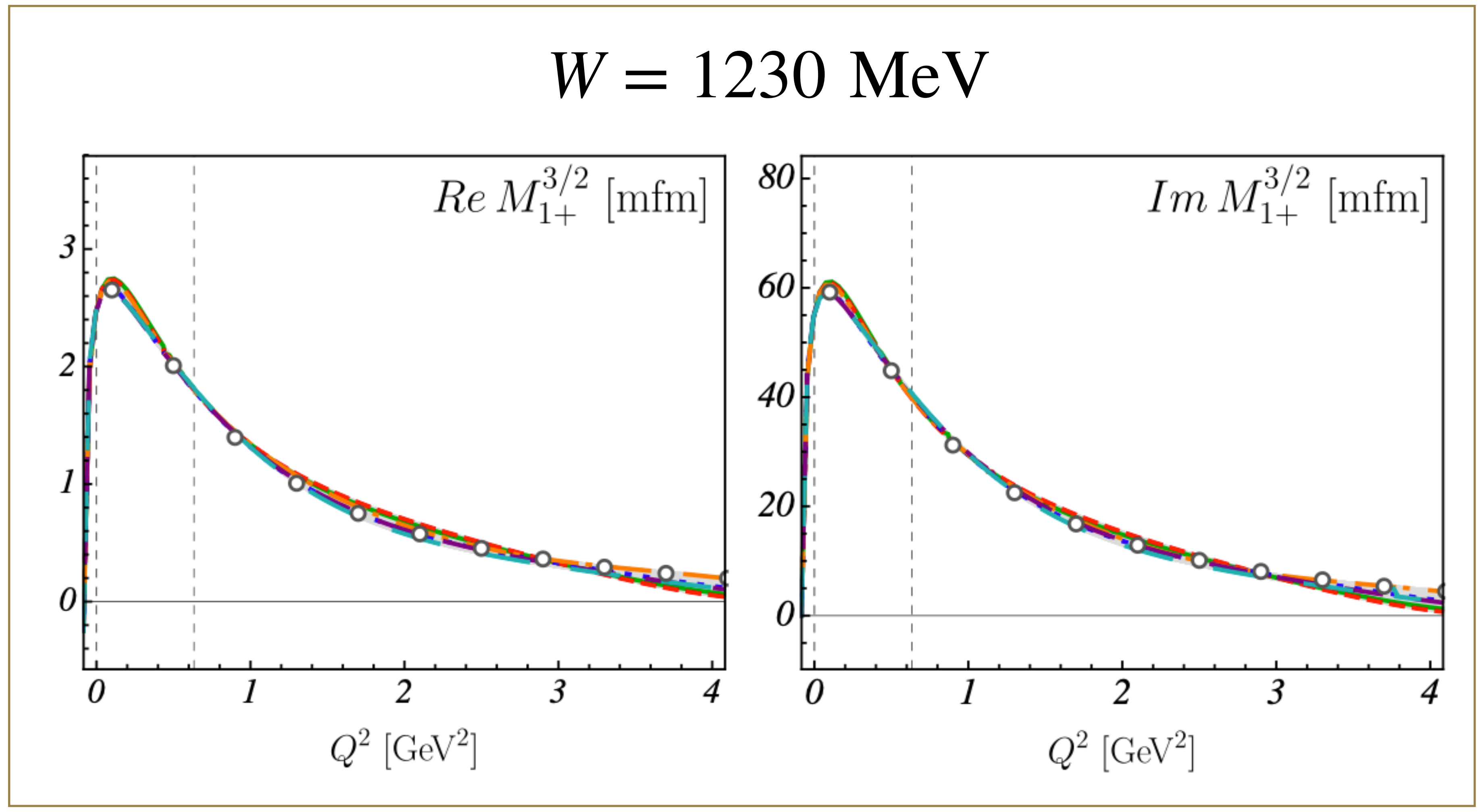
$$C_{\alpha\beta} = \begin{pmatrix} 6 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix} \quad \text{for} \quad \alpha, \beta \in \{1, 8, 8', 27\}.$$

$$C^{\mathrm{NLO1}}_{\alpha\beta} = \begin{pmatrix} \frac{4}{3}(3b_0 + 7b_D)m_q & 0 & 0 & 0 \\ 0 & \frac{2}{3}(6b_0 + b_D)m_q & -\sqrt{20}b_F m_q & 0 \\ 0 & -\sqrt{20}b_F m_q & 2(2b_0 + 3b_D)m_q & 0 \\ 0 & 0 & 0 & 4(b_0 + b_D)m_q \end{pmatrix},$$

$$C^{\mathrm{NLO2}}_{\alpha\beta} = \begin{pmatrix} -3d_2 + \frac{9}{2}d_3 + d_4 & 0 & 0 & 0 \\ 0 & \frac{1}{2}(-3d_2 + d_3 + 2d_4) & -\frac{\sqrt{5}}{2}d_1 & 0 \\ 0 & -\frac{\sqrt{5}}{2}d_1 & \frac{1}{2}(9d_2 - d_3 + 2d_4) & 0 \\ 0 & 0 & 0 & \frac{1}{2}(2d_2 + d_3 + 2d_4) \end{pmatrix}.$$

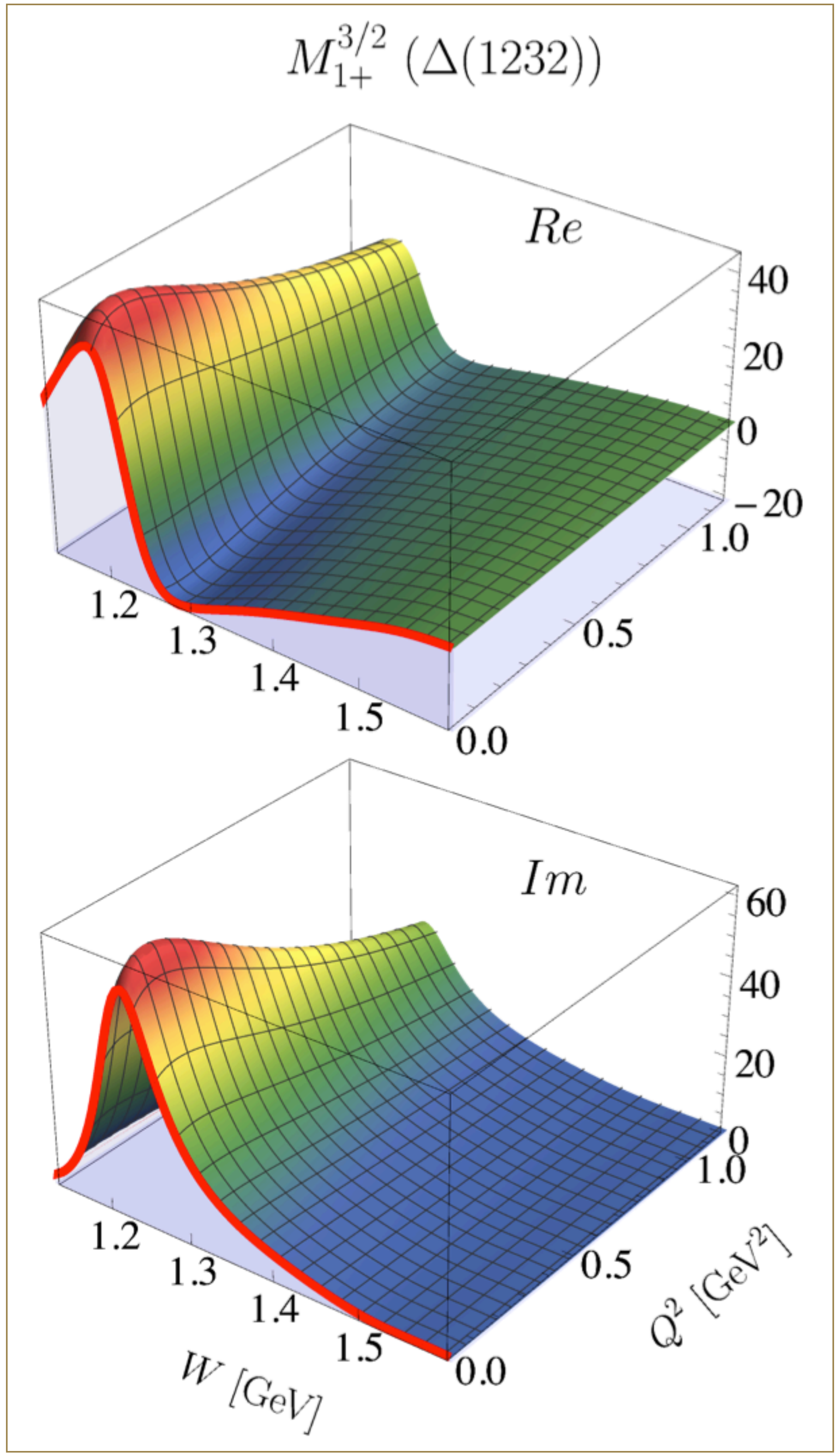
NLO breaks accidental octet symmetry

RESULTS



Delta(1232):

- Large multipoles well determined
- simple Q^2 dependence



[JBW] MM et al. *Phys.Rev.C* 103 (2021) 6; Phys.Rev.C 106 (2022) 015201

HADRONIC 3-BODY PROBLEM: IMPACT

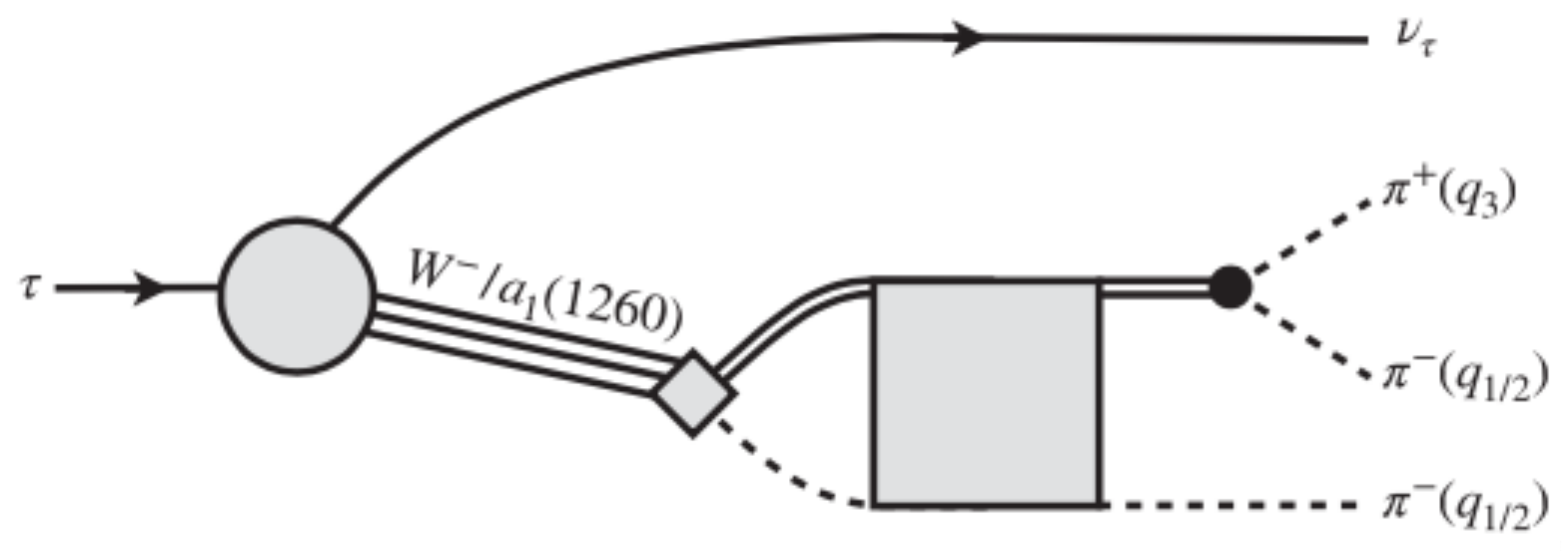
Intricate kinematics/dynamics

- 8 variables
- 2-body sub-channel dynamics

Hadron spectroscopy riddles

- Roper(1440) → **$\pi\pi$N** [first FV evaluations[1]]
- X(3872) → **DbarDπ**
- a_1(1260) → **$\pi\pi\pi$**
- ...

- Beyond Standard Model: τ-EDM



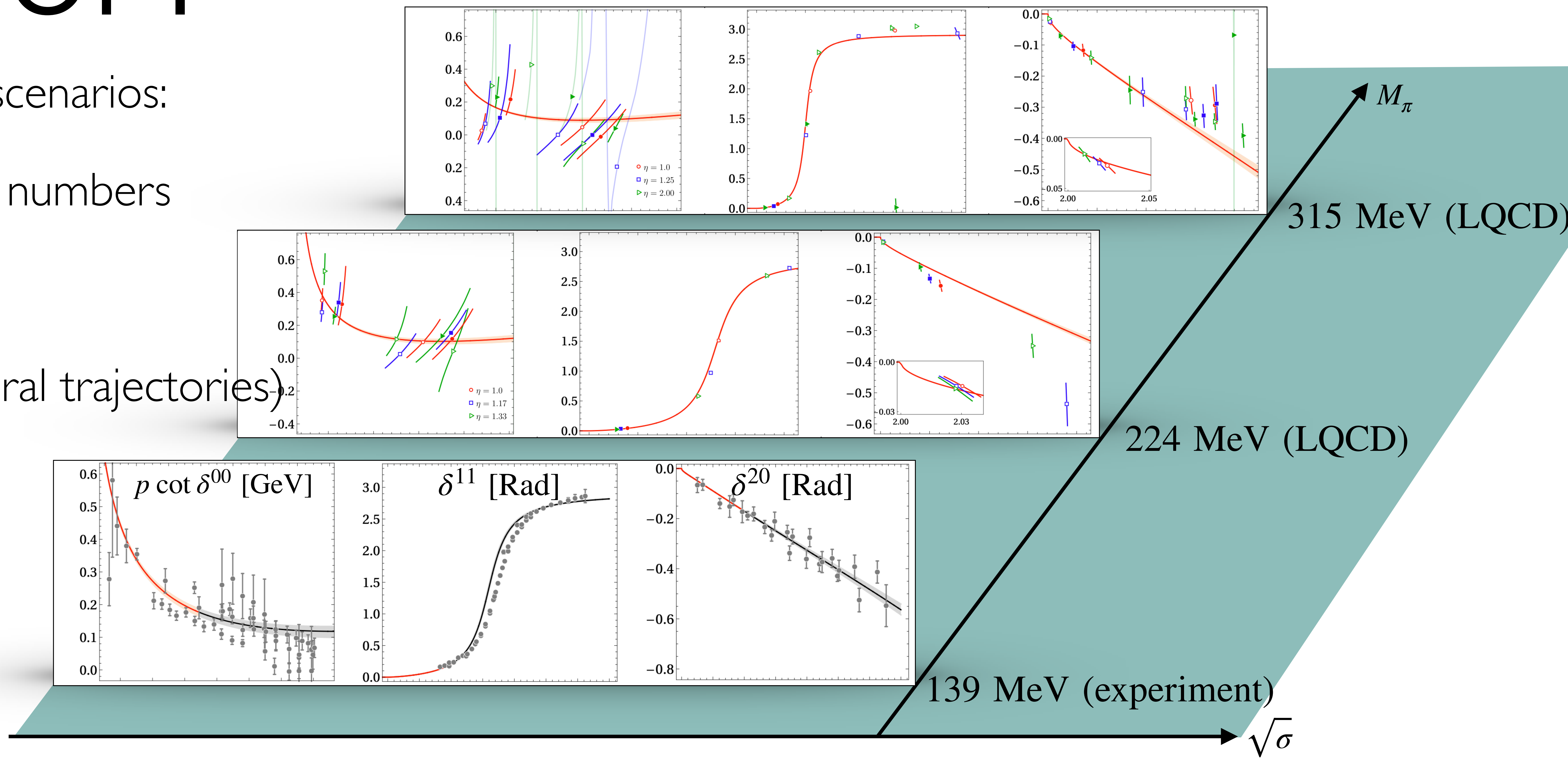
- Precision physics: rare hadronic W-decays[2]
- Exotic states of matter[3]

1) Severt/MM/Meißner JHEP04(2023) >>> PHD talk on Friday
2) Sirunyan et al. [CMS@CERN] PRL122
3) Experimental programs: GlueX@JLAB; COMPASS@CERN;

LATTICE HADRON SPECTROSCOPY

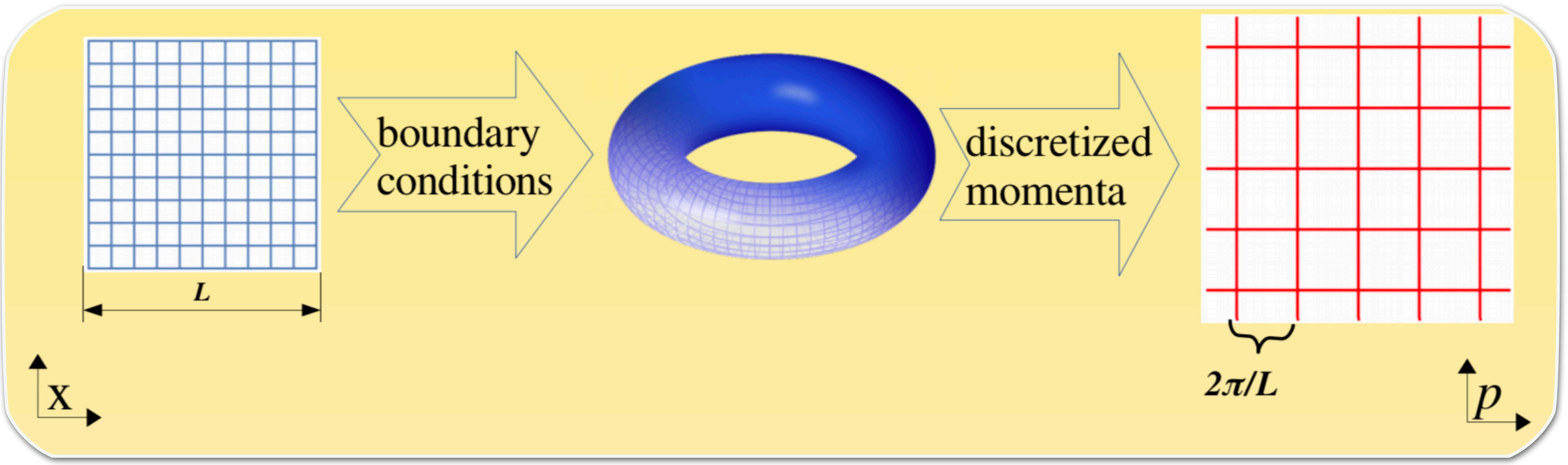
- Experimentally inaccessible scenarios:
  - Unconventional quantum numbers
  - Three-body scattering
  - Unphysical pion mass (chiral trajectories)

…



MM/Culver/Brett/Alexandru/Döring/Lee *Phys.Rev.D* 100 (2019)
Review: MM/Döring/Rusetsky EPJ ST (2021)

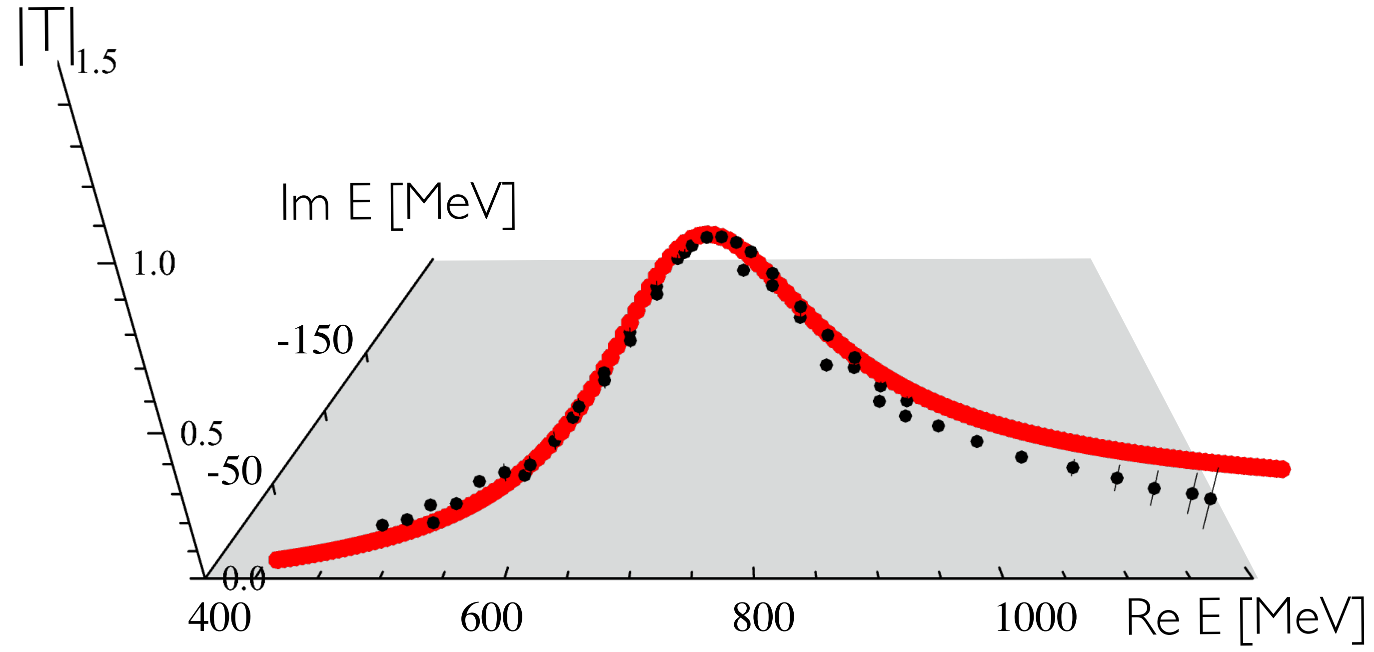
HADRONS IN A BOX



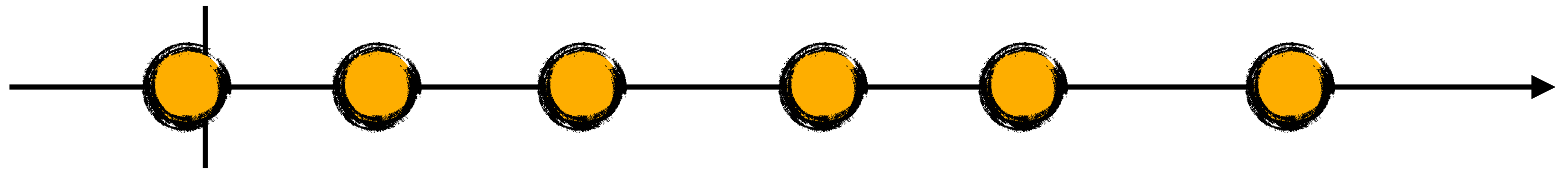
🤗 Heavily simplified:

on-shell particle-configurations: $\boldsymbol{\Delta}E\sim mL$

off-shell particle-configurations: $\boldsymbol{\Delta}E\sim e^{-mL}$

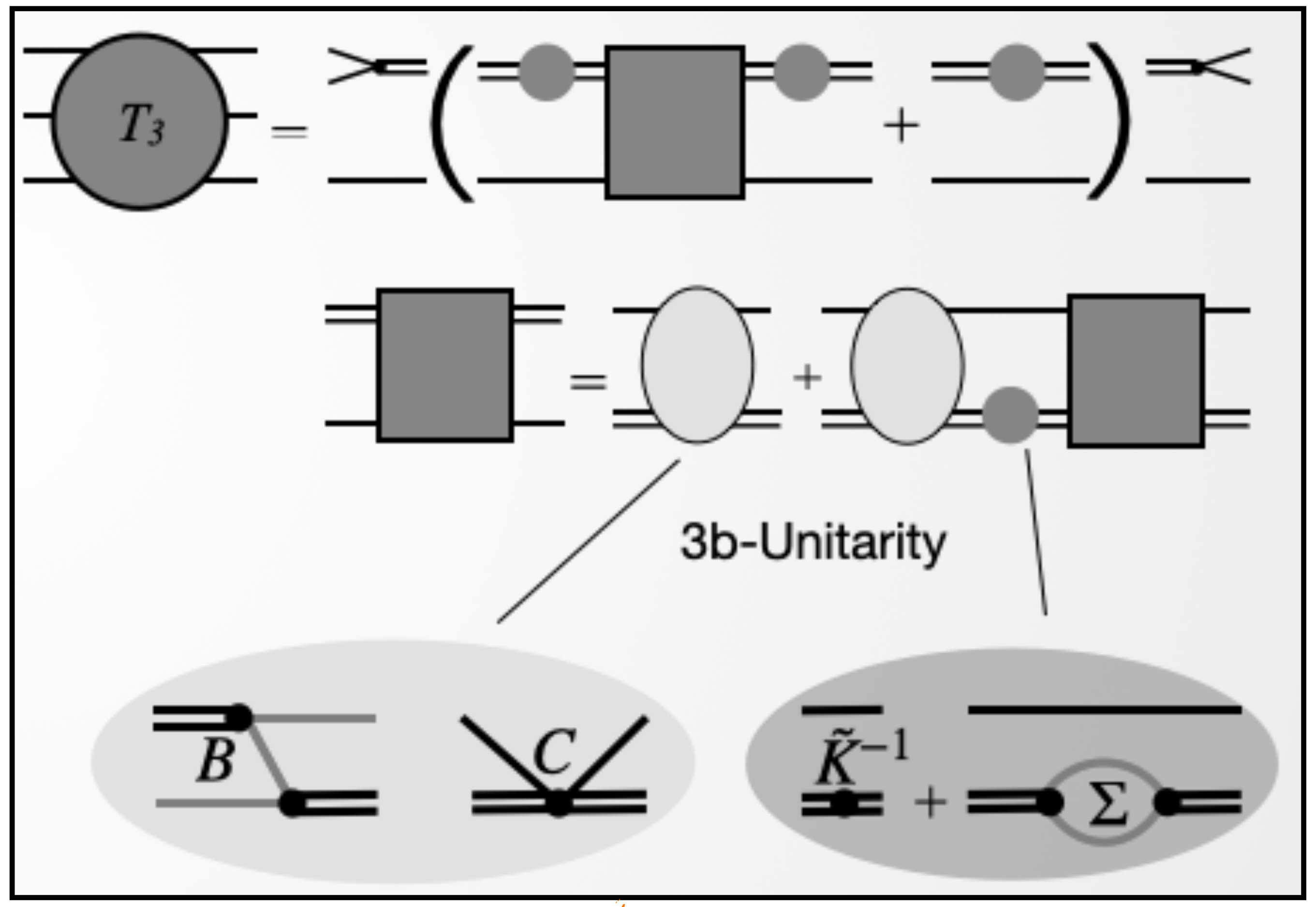


🤩 *A unitary "T-matrix" accounts for all O(mL) effects!*



1) Lüscher, Gottlieb, Rummukainen, Feng, Li, Döring, Briceño, Meißner, Rusetsky, Hansen, MM, Blanton, ...
2) Reviews: Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019); MM/Döring/Rusetsky Eur.Phys.J.ST 230 (2021);

3-BODY QUANTISATION CONDITION



Reminder

Finite-volume unitarity (FVU)[1,2]

- separates volume dependent terms
- volume independent terms connect infinite/finite-volume spectra

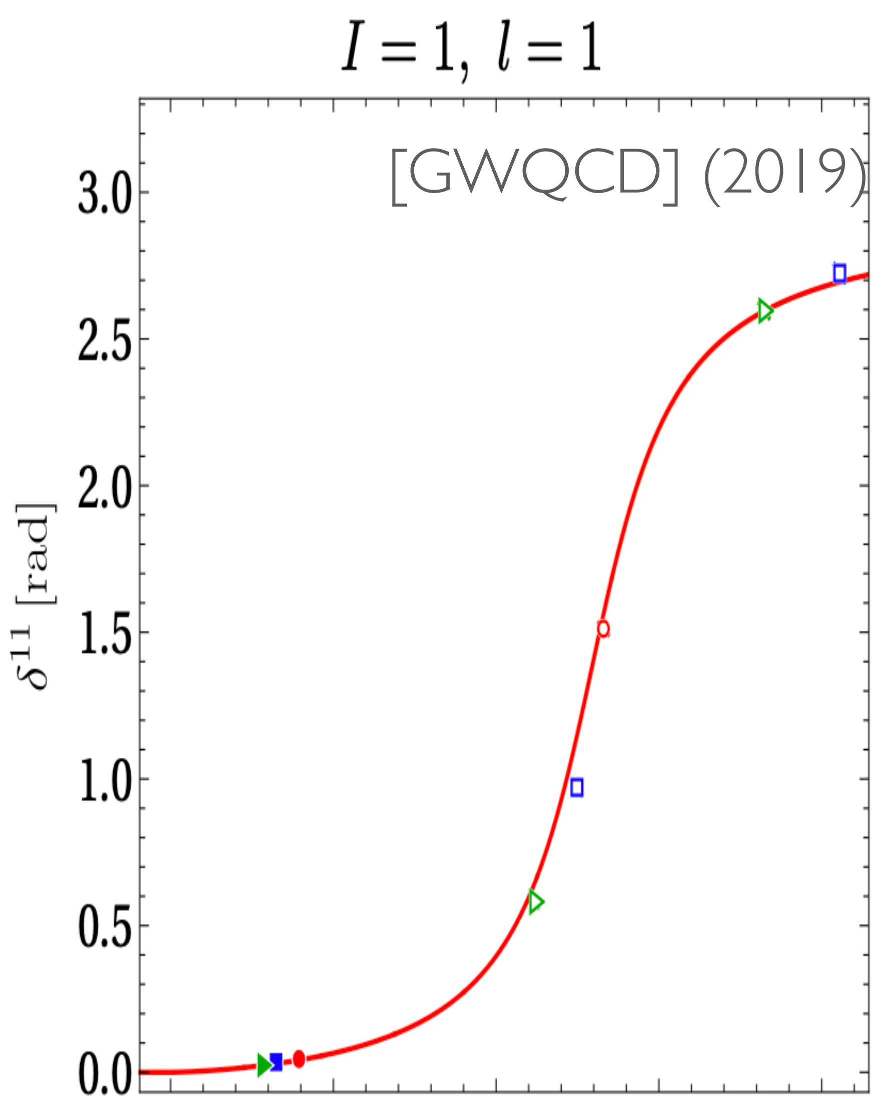
$$0 = \det\left[2L^3E\left(\tilde{K}_n^{-1} - \Sigma\right) - B - C\right]_{\mathbf{p}'\mathbf{p}}$$

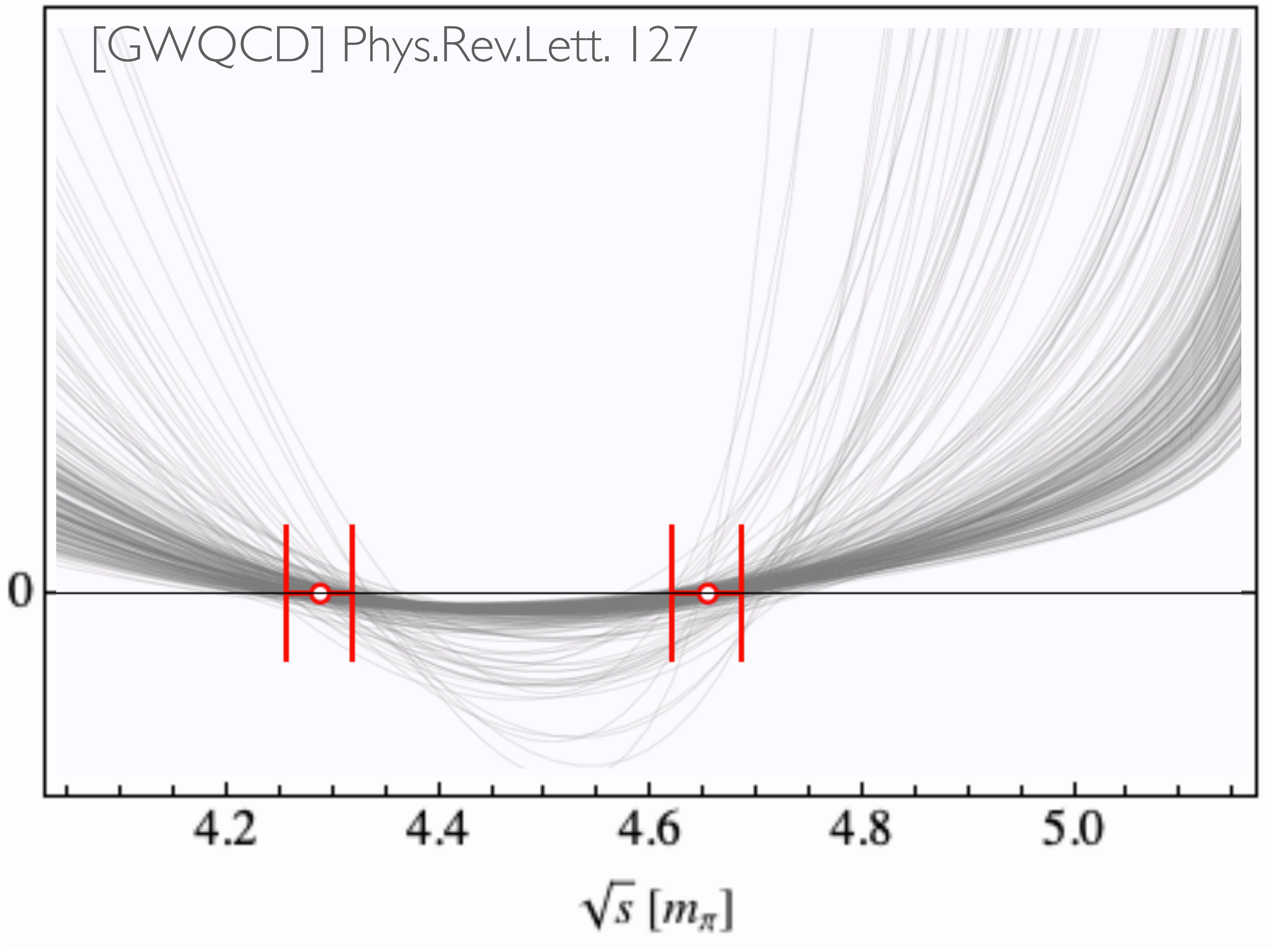
1) Lüscher, Gottlieb, Rummukainen, Feng, Li, Döring, Briceño, Meißner, Rusetsky, Hansen, MM, Blanton, ...
2) Reviews: Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019); MM/Döring/Rusetsky Eur.Phys.J.ST 230 (2021);

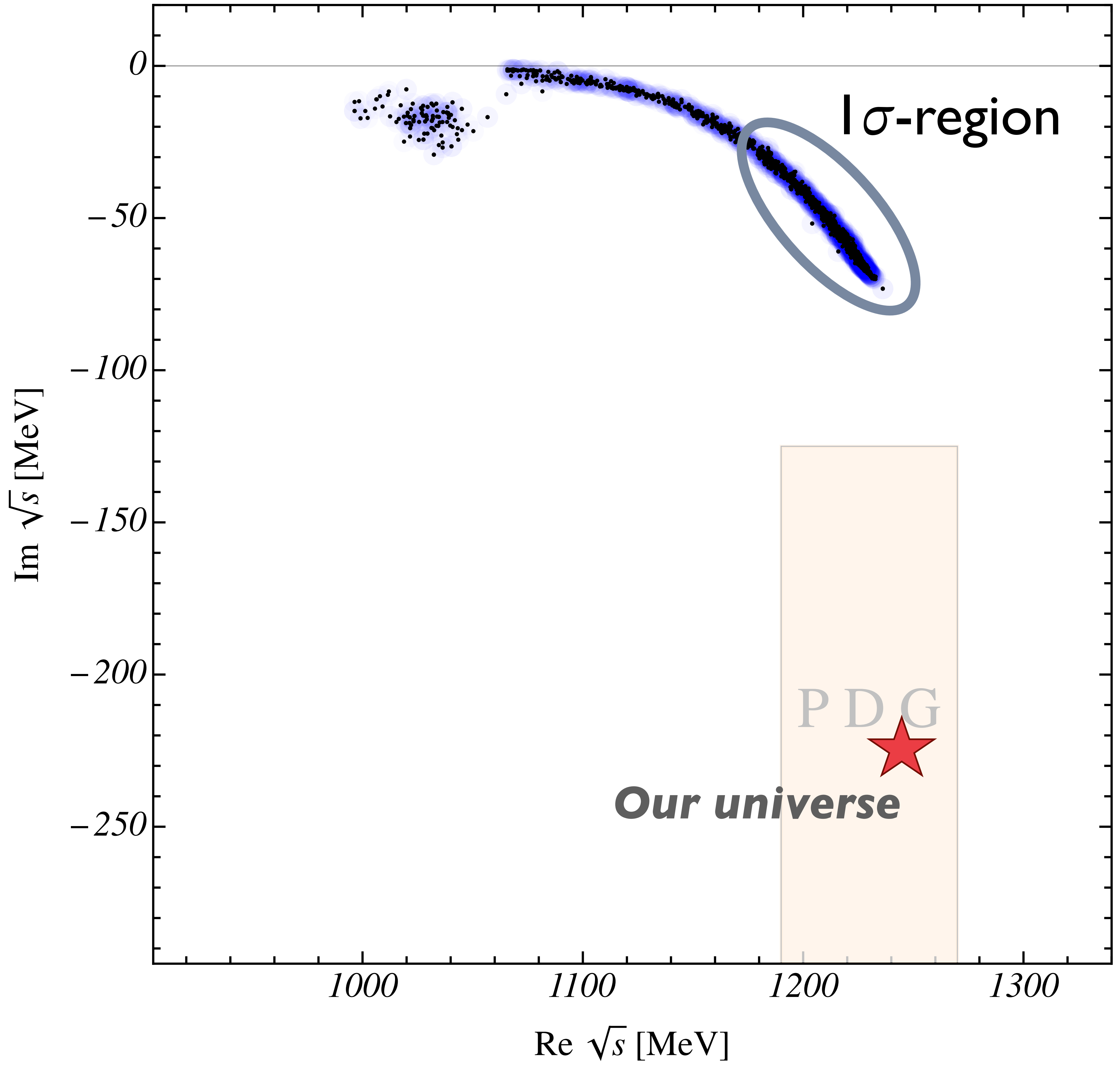
"Heavier Universe"

$$0 = \det\left[2L^3E\left(\tilde{K}_n^{-1} - \Sigma\right) - B - C\right]_{\mathbf{p}'\mathbf{p}}$$

$$T^c = B + C + \int \frac{d^3\ell}{(2\pi)^3}\frac{(B+C)}{2E_l}\frac{1}{\tilde{K}_n^{-1} - \Sigma_n}T^c$$







1) MM/Culver/Döring/Alexandru/Lee/Brett/Sadasivan [GWQCD] Phys.Rev.Lett. 127

HADRONS IN A BOX

Finite-volume spectrum is real and discrete!

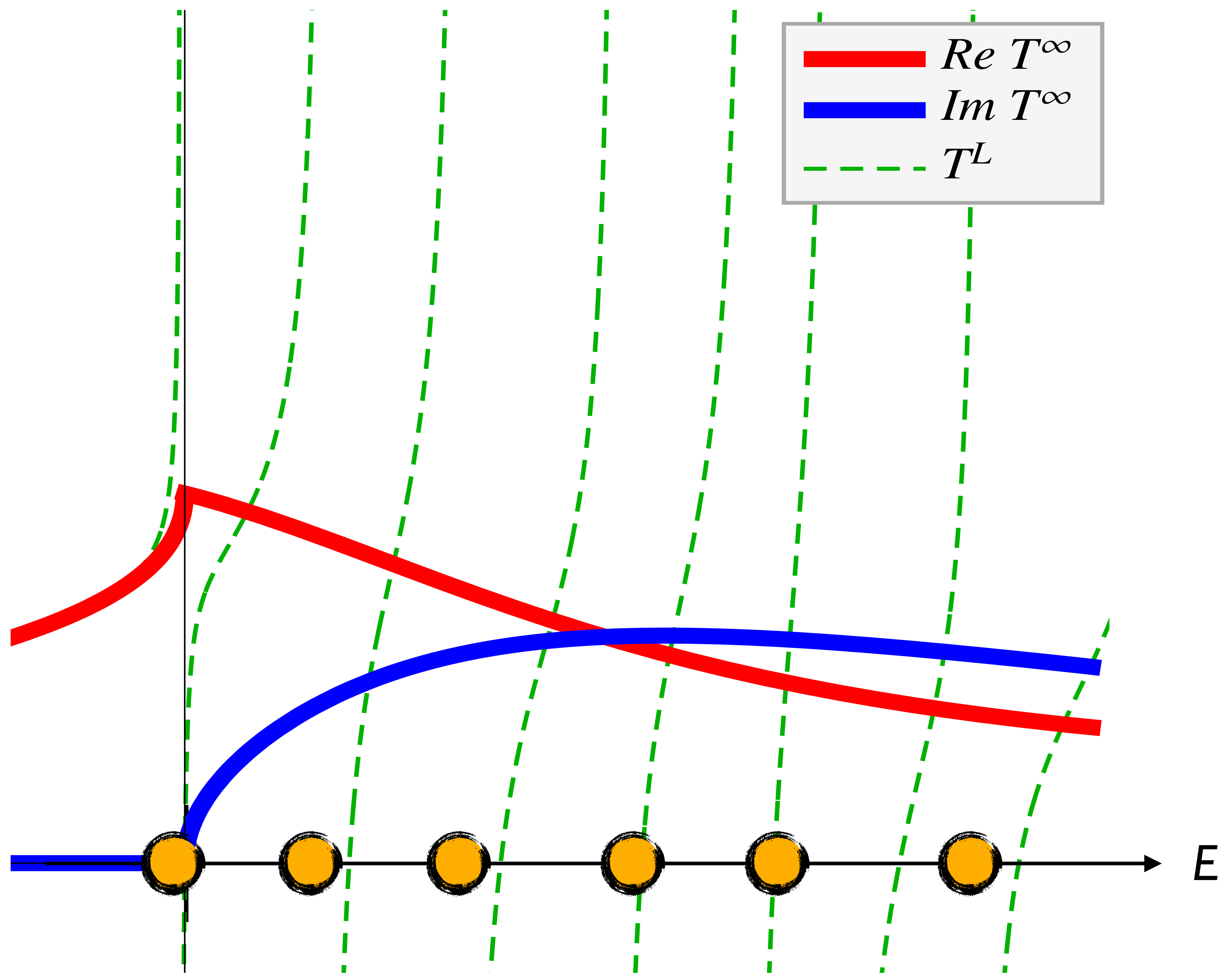
... requires mapping: Quantization condition[1,2]

🤗 Heavily simplified:

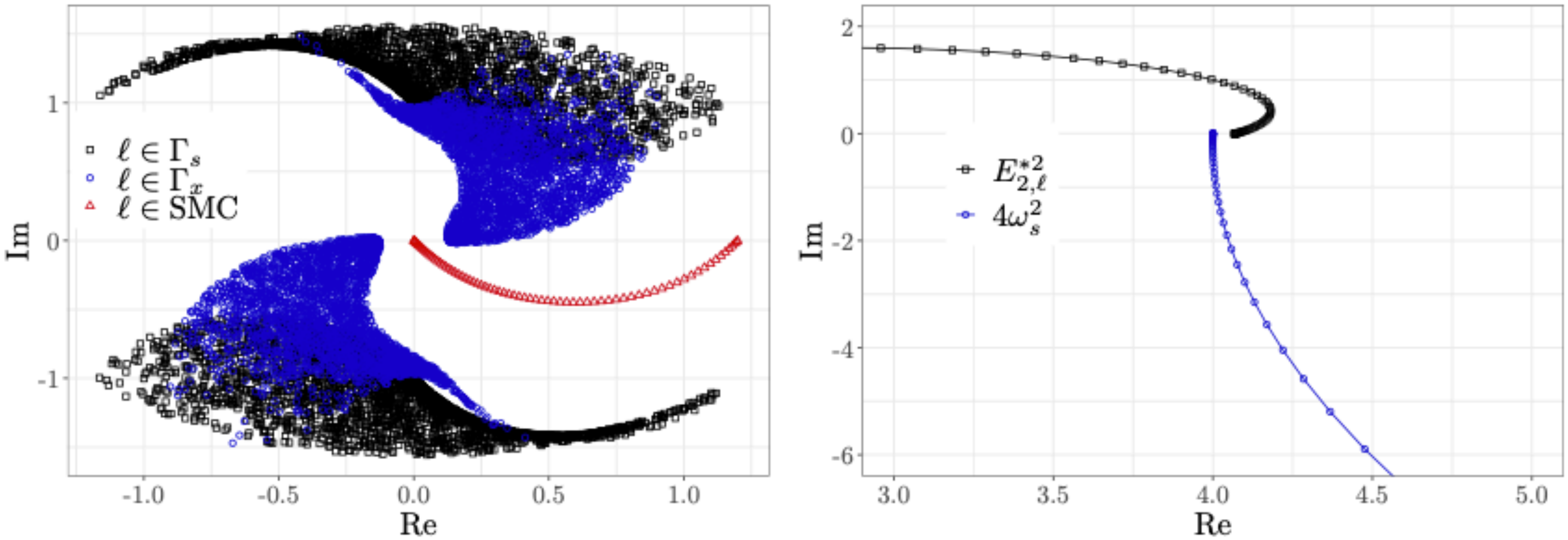
on-shell particle-configurations: $\boldsymbol{\Delta}E\sim mL$

off-shell particle-configurations: $\boldsymbol{\Delta}E\sim e^{-mL}$

🤩 *A unitary "T-matrix" accounts for all O(mL) effects!*



1) Lüscher, Gottlieb, Rummukainen, Feng, Li, Döring, Briceño, Meißner, Rusetsky, Hansen, MM, Blanton, ...
2) Reviews: Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019); MM/Döring/Rusetsky Eur.Phys.J.ST 230 (2021);



Im
Re
$\ell \in \Gamma_s$
$\ell \in \Gamma_x$
$\ell \in$ SMC
$E^{*2}_{2,\ell}$
$4\omega^2_s$

Current frontier: 3-body dynamics from LQCD

- ➥ 3-body Quantization Conditions[1]
  - ➥ RFT / FVU / NREFT
    - ➥ many perturbatively interacting systems are studied[2]

$$0 = \det\left(L^3\left(\tilde{F}/3 - \tilde{F}(\tilde{K}_2^{-1} + \tilde{F} + \tilde{G})^{-1}\tilde{F}\right)^{-1} + K_{\mathrm{df},3}\right)$$ RFT

$$0 = \det\left(B_0 + C_0 - E_L\left(K^{-1}/(32\pi) + \Sigma_L\right)\right)$$ FVU

3-body force

2-body interaction

one-particle exchange

2-body self-energy

1) Rusetsky, Bedaque, Grießhammer, Sharpe, Meißner, Döring, Hansen, Davoudi, Guo....
Reviews:
Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019);
MM/Döring/Rusetsky Eur.Phys.J.ST 230 (2021);
2) MM/Döring PRL122(2019); Blanton et al. PRL 124 (2020); Hansen et al. PRL 126 (2021); ....

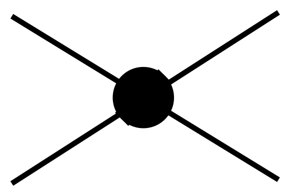
AVOIDED LEVEL CROSS

g		a	m_1	c_0	c_1	m'_1	c'_0	c'_1	$\chi^2_{\rm dof}$
5	FVU	−0.1512(9)	3.0229(1)	−0.0188(35)	–	–	–	–	2.9
	RFT	−0.1522(12)	–	–	–	3.0232(2)	31.6(8.4)	–	2.5
	FVU	−0.1569(12)	3.0233(2)	−0.0297(57)	2.29(38)	–	–	–	1.5
	RFT	−0.1571(10)	–	–	–	3.0237(2)	37.6(9.0)	2789(540)	1.5
10	FVU	−0.1521(11)	3.0205(2)	−0.0475(66)	–	–	–	–	1.7
	RFT	−0.1531(13)	–	–	–	3.0212(3)	80(14)	–	1.6
	FVU	−0.1549(16)	3.0205(2)	−0.0595(99)	0.93(41)	–	–	–	1.5
	RFT	−0.1563(27)	–	–	–	3.0213(3)	97(16)	1773(980)	1.4
20	FVU	−0.1444(11)	3.0184(2)	−0.1136(77)	–	–	–	–	1.6
	RFT	−0.1450(17)	–	–	–	3.0199(2)	178(17)	–	1.6
	FVU	−0.1464(14)	3.0183(2)	−0.1363(148)	0.84(39)	–	–	–	1.3
	RFT	−0.1484(16)	–	–	–	3.0200(2)	210(23)	2227(600)	1.2

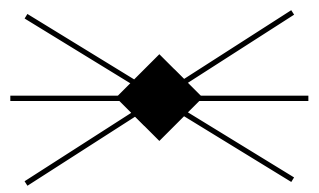
Variate g($\varphi_1 \rightarrow \varphi_0\varphi_0\varphi_0$) coupling:

- avoided level crossing becomes wider
- RFT and FVU

$$q^* \cot\delta = \frac{1}{aM_0}$$



$$C = \frac{c_0}{E_3^3 - m_1^2} + c_1$$



... same fit quality

... observables determined consistently

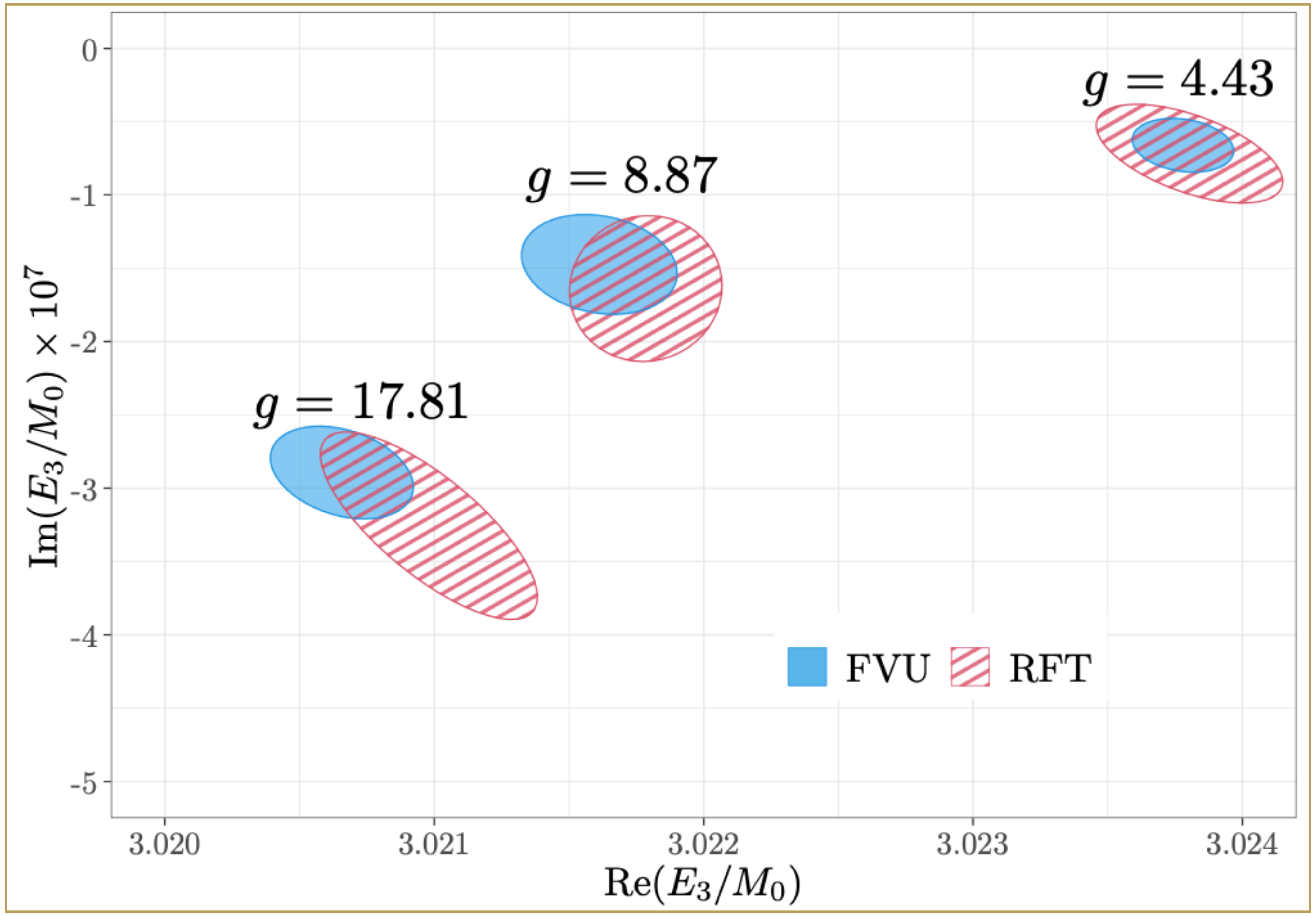
Pole positions

- FVU: complex energy-plane analysis[1]

  -- resonance width grows ~ ***g²***

  -- avoided level crossing gap >> width

- Similarly from RFT with Breit-Wigner like approximation



1) Sadasivan/MM/.. *Phys.Rev.D* 101 (2020)