Femtoscopy studies with HADES

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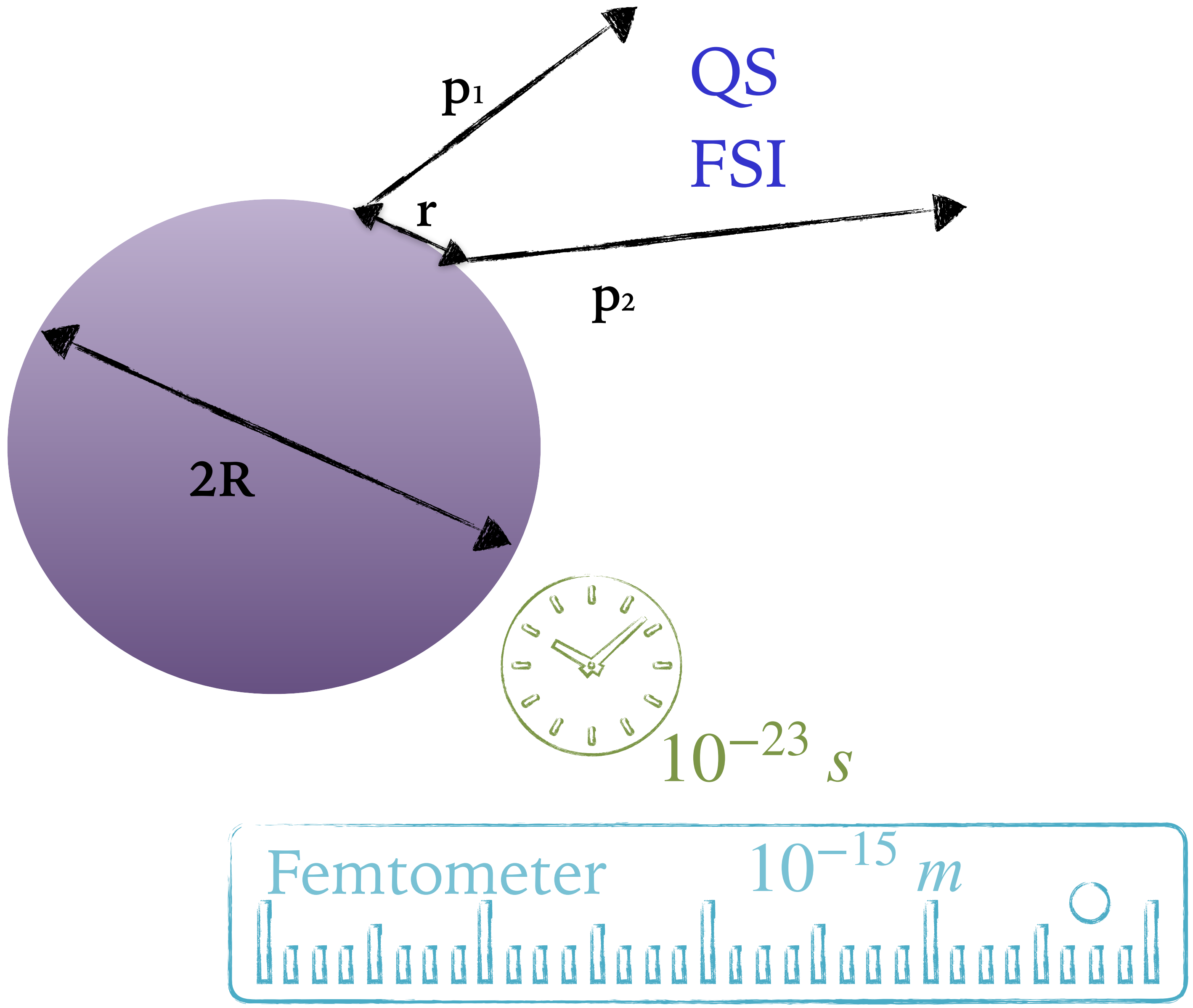
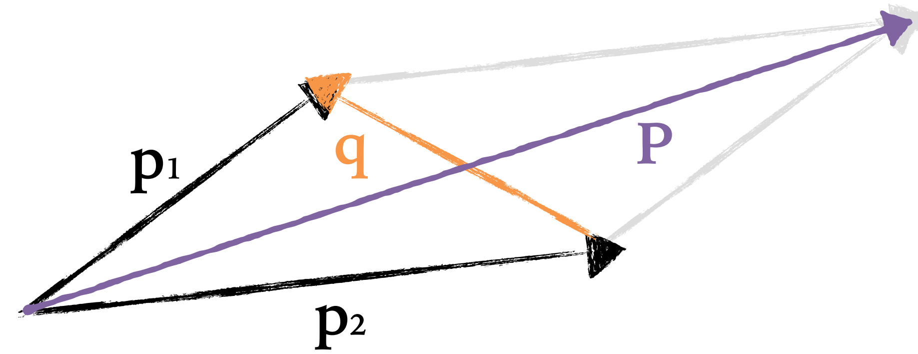
Femtoscopy

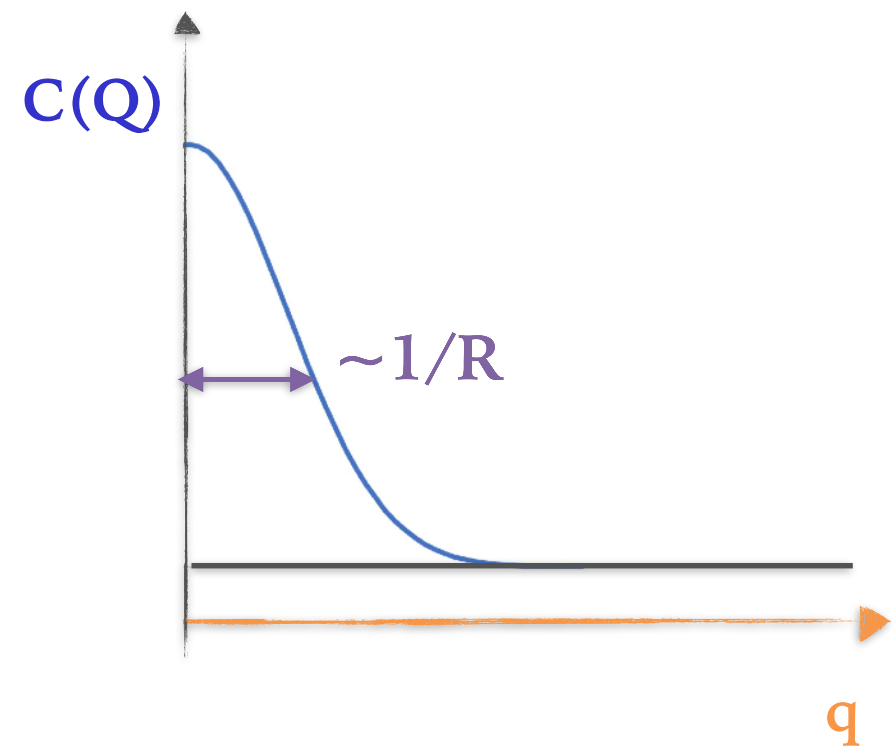
NS, NSM

FSI studies: NY, K_0^S, clusters, …

Physics opportunity with proton beams at SIS100, Wuppertal, February 6-9, 2024





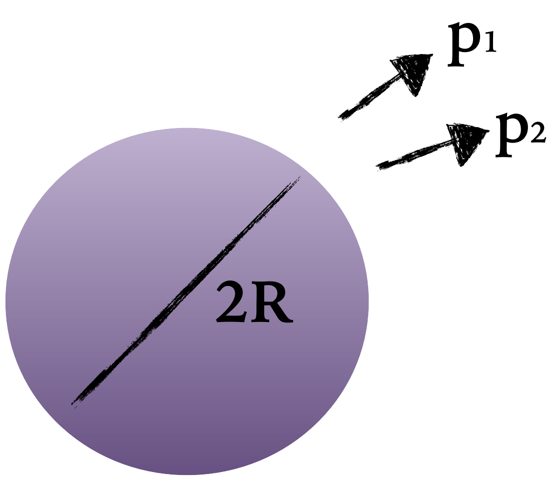


Femtoscopy

… the method to probe **geometric** and **dynamic** properties of the source (emission region, range of correlations-interactions, phase-space cloud, …)

Classic femtoscopy

Femtoscopy (originating from HBT):
the method to probe **geometric** and **dynamic** properties of the source



Space-time properties ($10^{-15}m$, $10^{-23}s$) determined thanks to two-particle correlations:

Quantum Statistics (Fermi-Dirac, Bose-Einstein);

Final State Interactions (Coulomb, strong)

determined | assumed | measured

$$C(k^*, r^*) = \int S(r^*) |\Psi(k^*, r^*)|^2 d^3 r^* = \frac{Sgnl(k^*)}{Bckg(k^*)}$$

k^* - momentum of the first particle in PRF

r^* - Separation between emission points

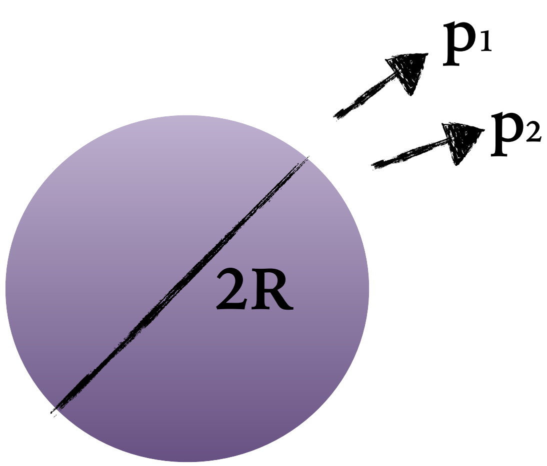
$S(r^*)$ - source function

$\Psi(k^*, r^*)$ - two-particle wave function (includes e.g. FSI interactions)

$\frac{Sgnl(k^*)}{Bckg(k^*)}$ - correlation function

Gateway to study interactions

If we assume we know the source function, measured correlations are used to determine interactions in the final state.



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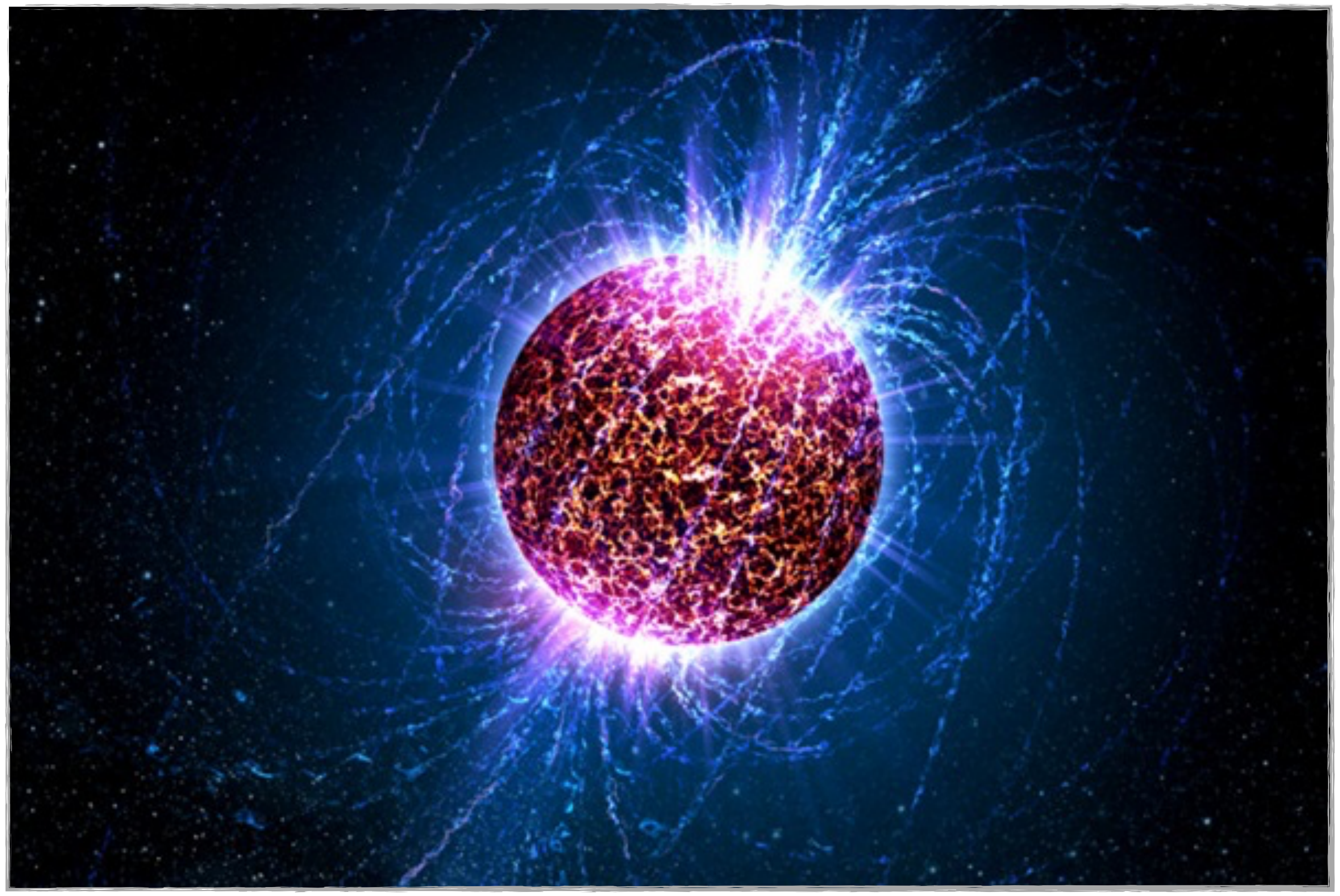
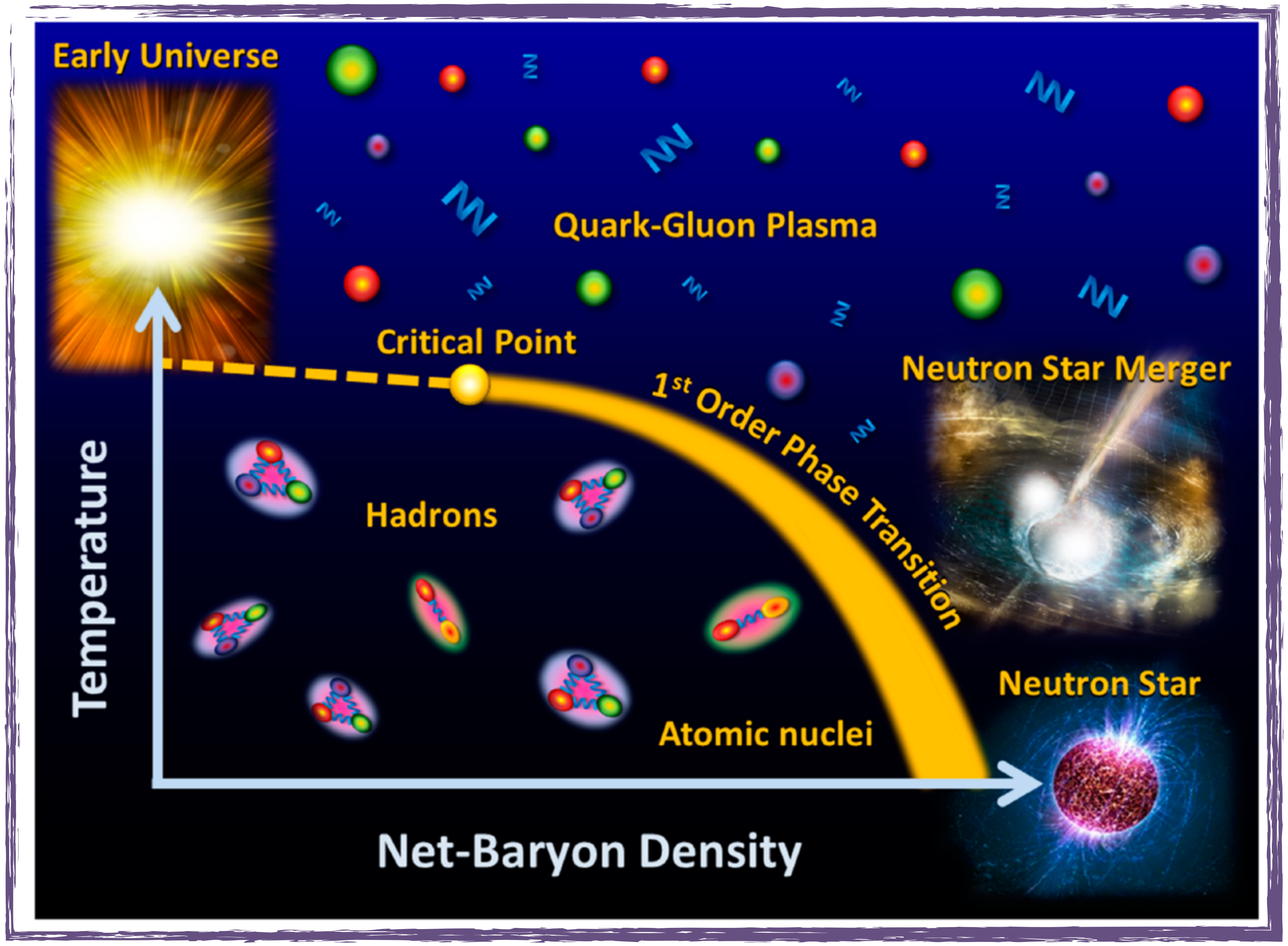


Neutron stars
Neutron star mergers



https://www.researchgate.net

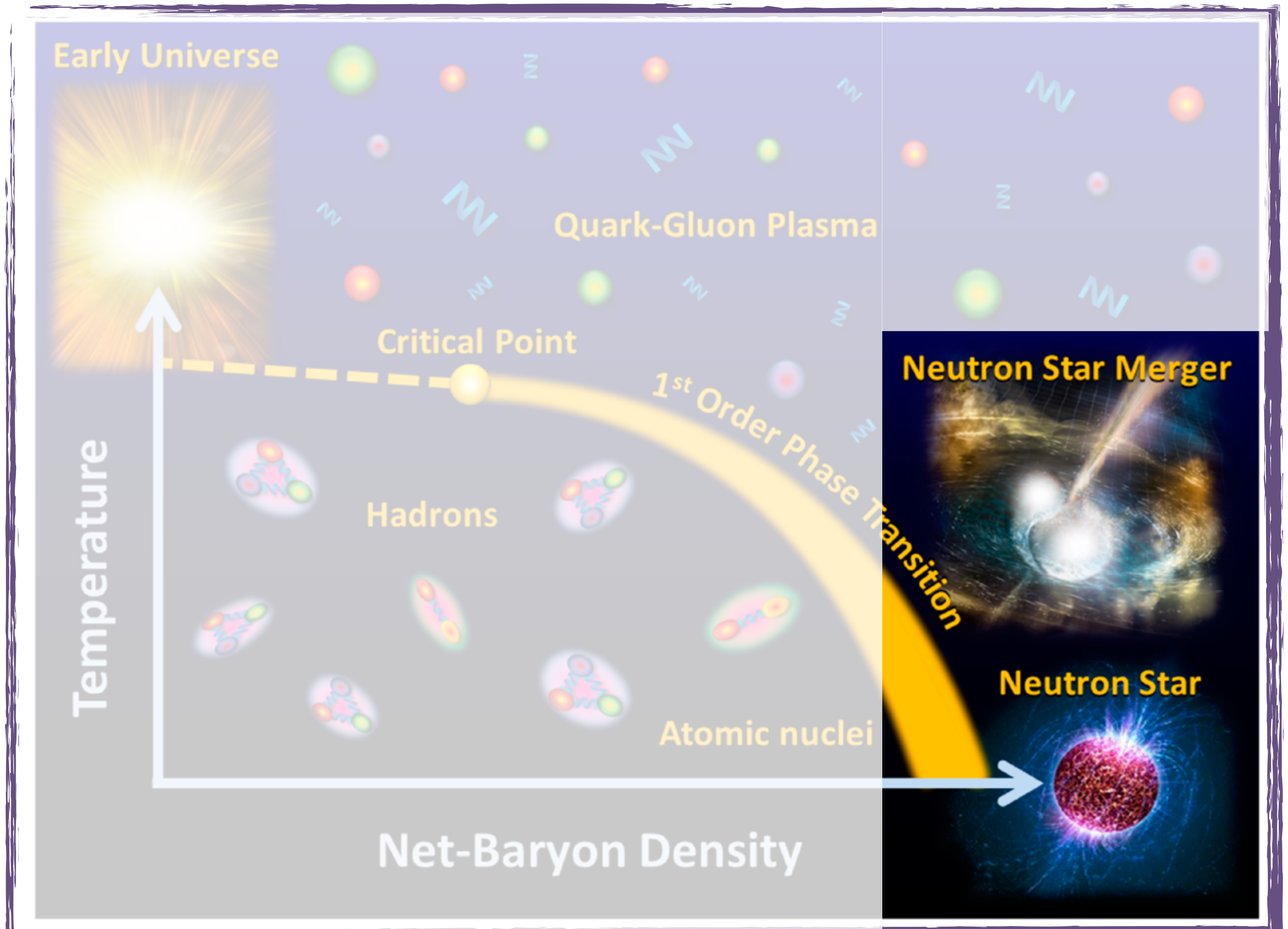
Scanning various μ_B

.. to study strongly interacting matter

.. to explore unknown QCD territory

Neutron star mergers

CBM and HADES future



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Temperature
T<50 MeV

Density
$n < 2 - 6n_0$

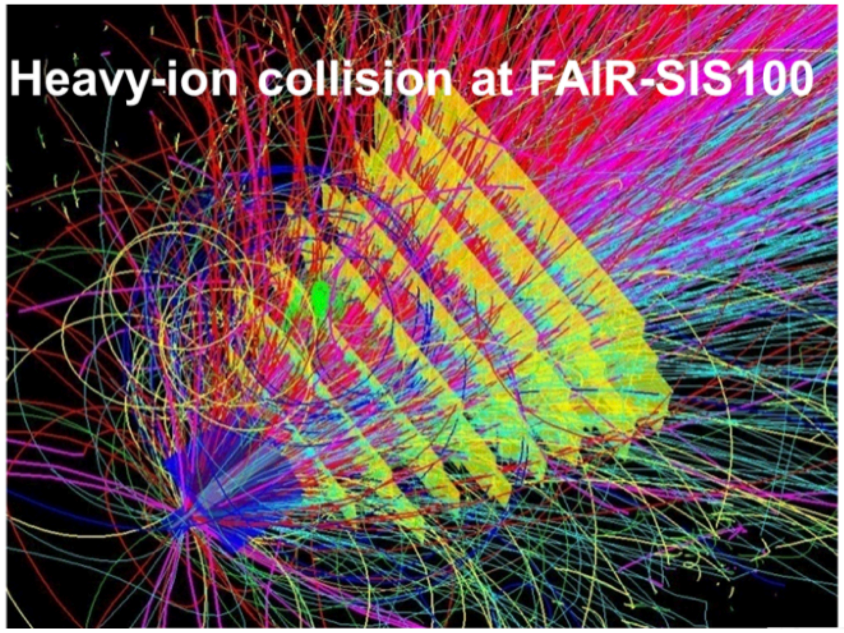
Reaction time
t ~ 10 ms
(GW170817)

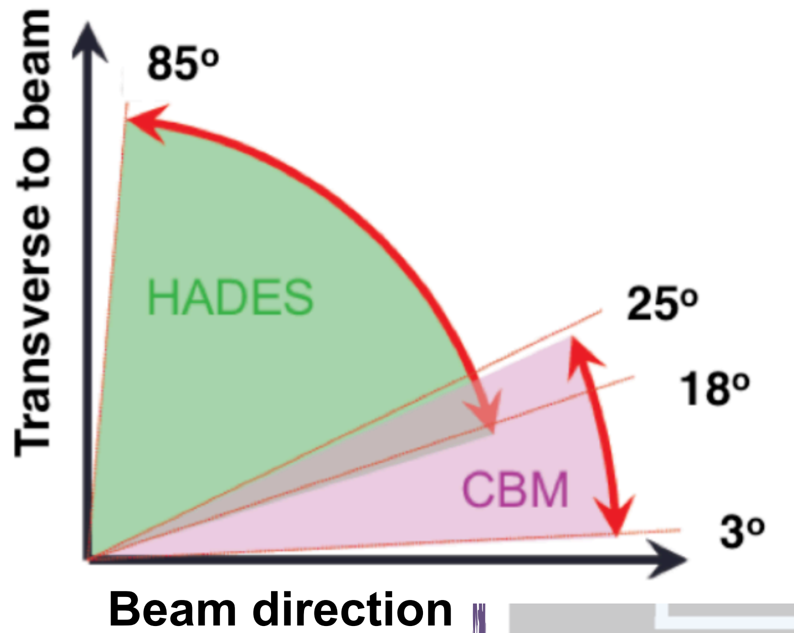
Temperature
T<10 MeV

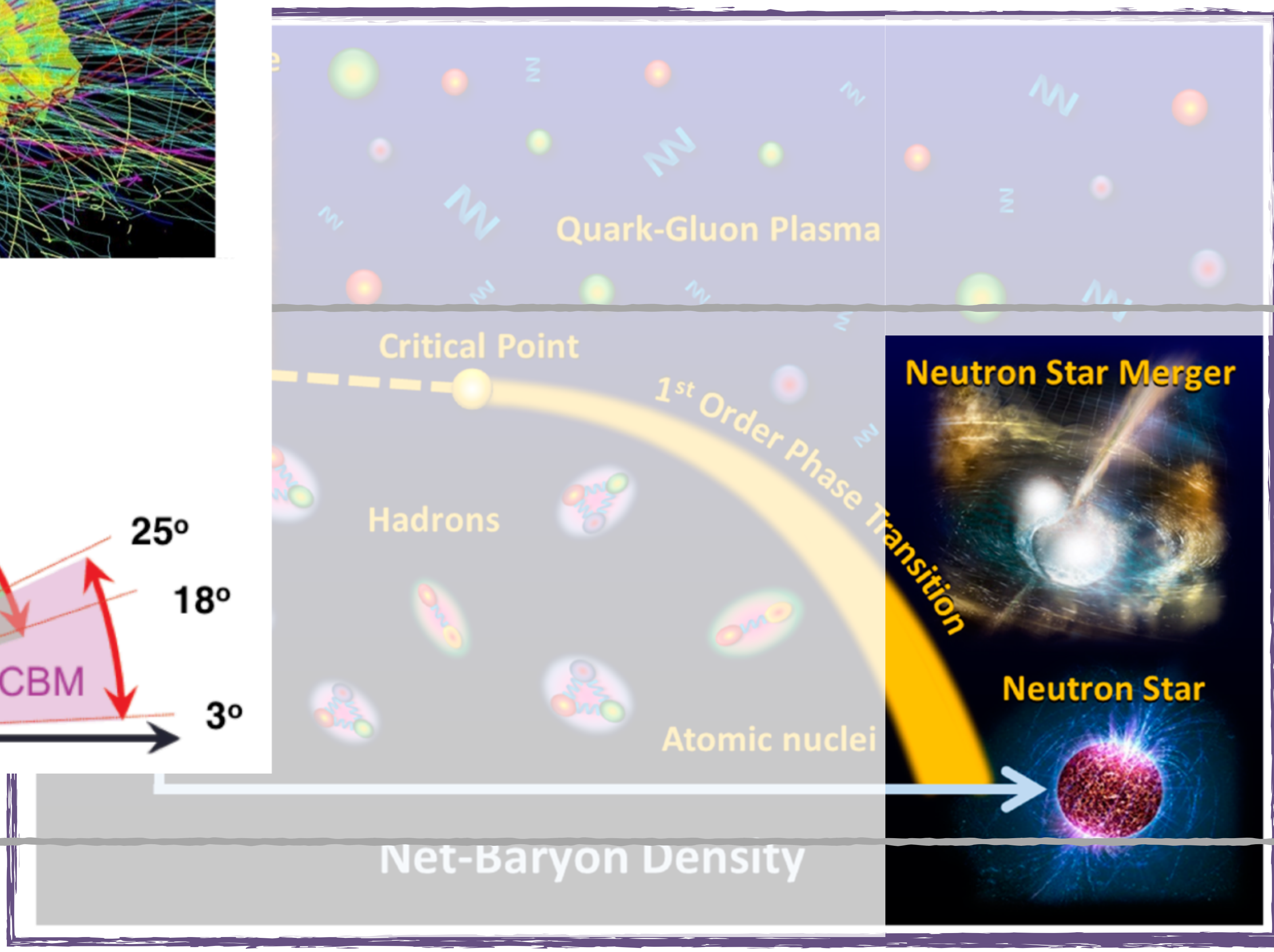
Density
$n < 10n_0$

Lifetime
t ~ infinity

CBM and HADES future







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(GW170817)

Temperature
T<10 MeV

Density
$n < 10n_0$

Lifetime
t ~ infinity

SIS-100: Temperature T<120 MeV | Density $n < 8n_0$ | Reaction time t $\sim 10^{-23}$ s

Neutron star puzzle

- **Hyperons**: expected in the core of neutron stars; **conversion of N into Y energetically favorable.**

- Appearance of Y: The relieve of Fermi pressure → **softer EoS → mass reduction (incompatible with observation).**

$M_{NS} \approx 1 \div 2\ M_{\odot}$

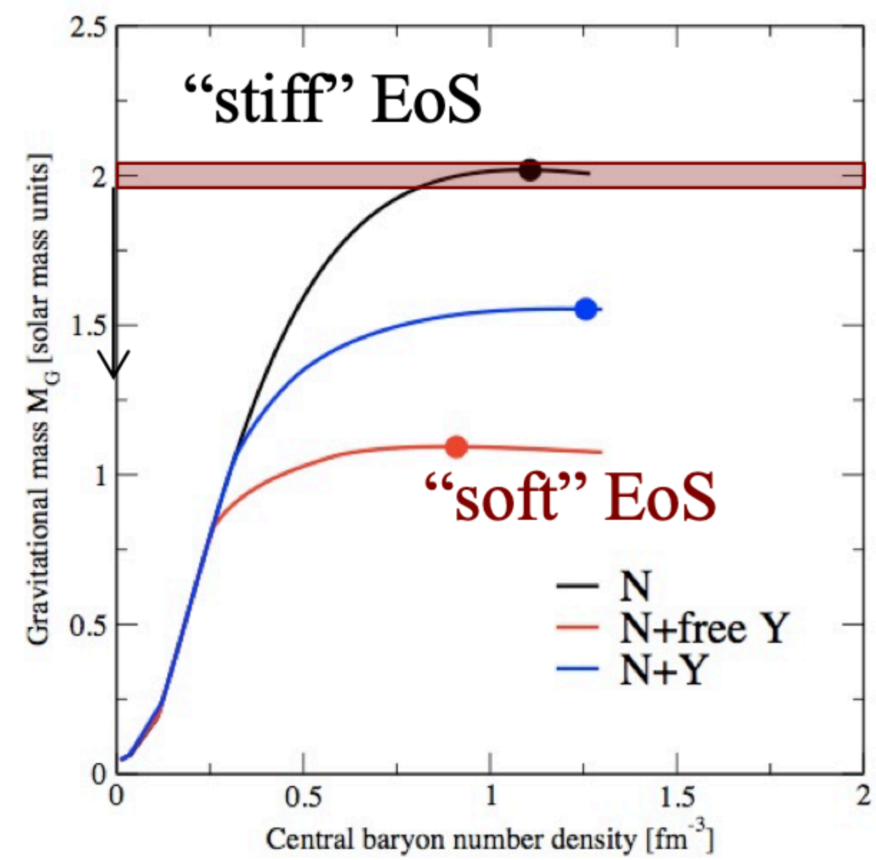
$R \approx 10\text{-}12$ km

$\rho \approx 3 \div 5\ \rho_0$

The solution requires a mechanism that could provide the **additional pressure** at high densities needed to make the EoS stiffer.

A few possible mechanisms, one of them:

- **Two-body YN & YY interactions**



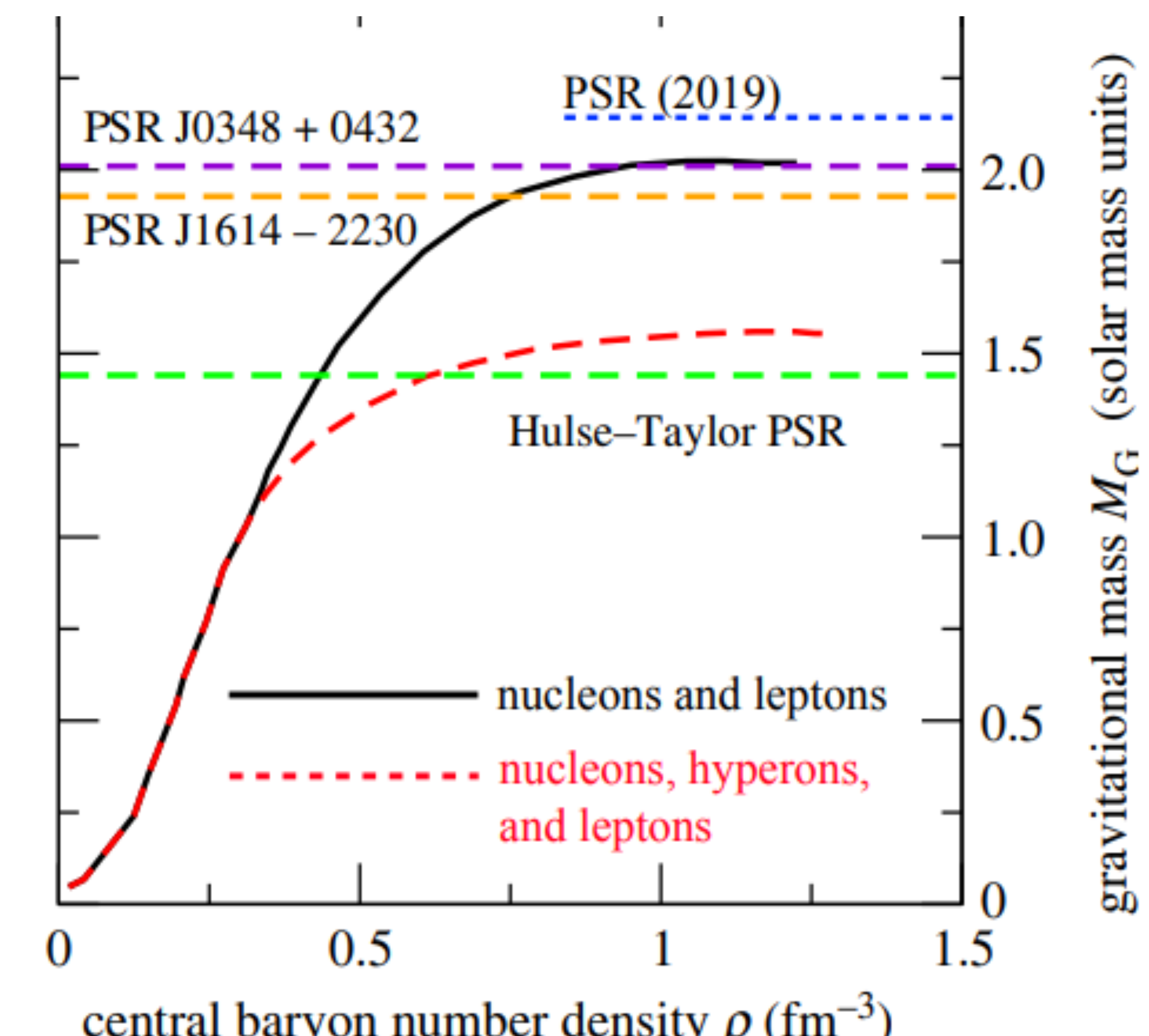
A lot of experimental and theoretical effort to understand:

- The **KN** interaction, governed by the presence of **Λ(1405)**
- The nature of **Λ(1405)**, the consequences of **KNN** formation
- **K** and $\bar{K}$ investigated to understand kaon condensation

$\rho_0 \approx 2.8 \times 10^{14}$ g/cm^3

Neutron star puzzle

https://royalsocietypublishing.org/doi/10.1098/rspa.2018.0145



Neutron stars (NS) are the remnants of the gravitational collapse of massive stars during supernova event.

Their masses and radii are of the order of $1 - 2\ M_{\odot}$ and $10 - 12$ km, respectively.

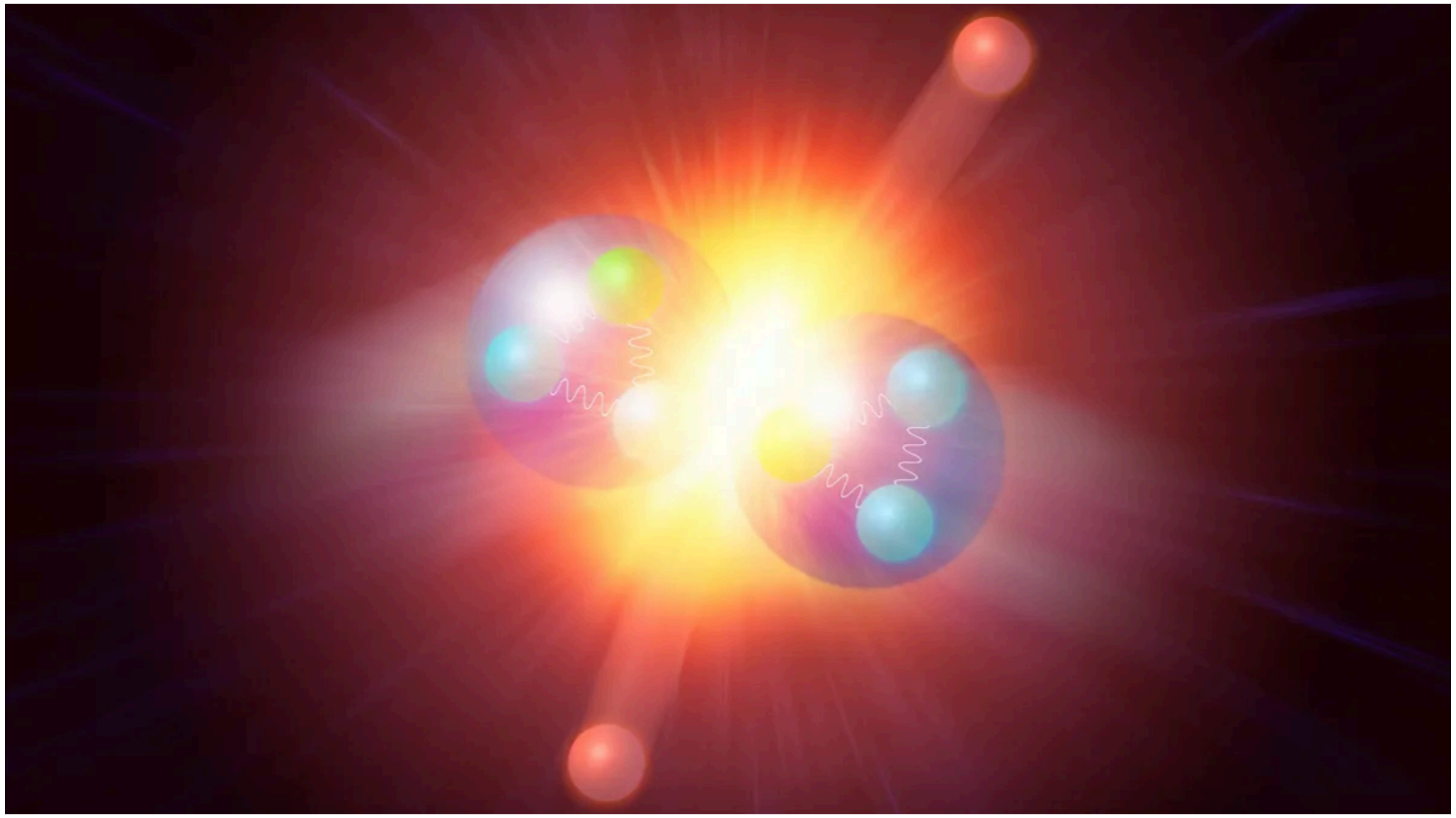
Central densities in the range of 4 – 8 times the normal nuclear matter saturation density, $\varepsilon_0 \sim 2.7 \times 10^{14}$ g/cm^3 ($\rho_0 \sim 0.16$ fm^{-3})

Best suitable theory takes hyperons into account,
Hyperons are expected to appear in the core of NS at $\rho \sim 2\text{-}\ 3\ \rho_0$

Hyperons soften the EoS —> Reduction on maximum NS mass

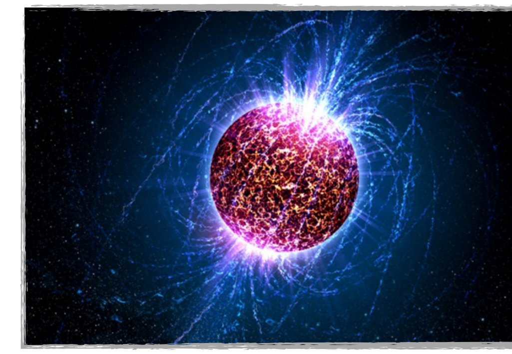
Observation of the NS with $M_G > 2M_S$ is incompatible with such soft EoS

Although the existence of hyperons is energetically favorable, their existence makes the EoS softer and is not consistent with the experimental results. This is the essence of the **hyperon puzzle.**



Final State Interactions
(+ other studies)

YN and YY interactions



- **Experiment**: More … and more!
  interest about YN and YY interactions!

- **Theory**: Major steps forward have been taken (Lattice QCD).

- **Numerous theoretical predictions** exist, **many experimental searches look for** evidence for **bound states**.

- The existence of **hypernuclei** (confirmed by attractive YN interaction) → indicates the possibility to bind Y to N.

- The measurement of the YN and YY interactions leads to important implications for the possible formation of **YN** or **YY bound states**.

- A precise knowledge of these interactions help to explore unknown structure of neutron stars.

HADES spectrometer

- SIS-18 beams: protons (1-4 GeV), nuclei (1-2 AGeV), pions (0.4-2 GeV/c) – secondary beam

- rare probes:(e^+, e^-), strangeness: $K^{+/-,0}$, Λ, Ξ^-, φ

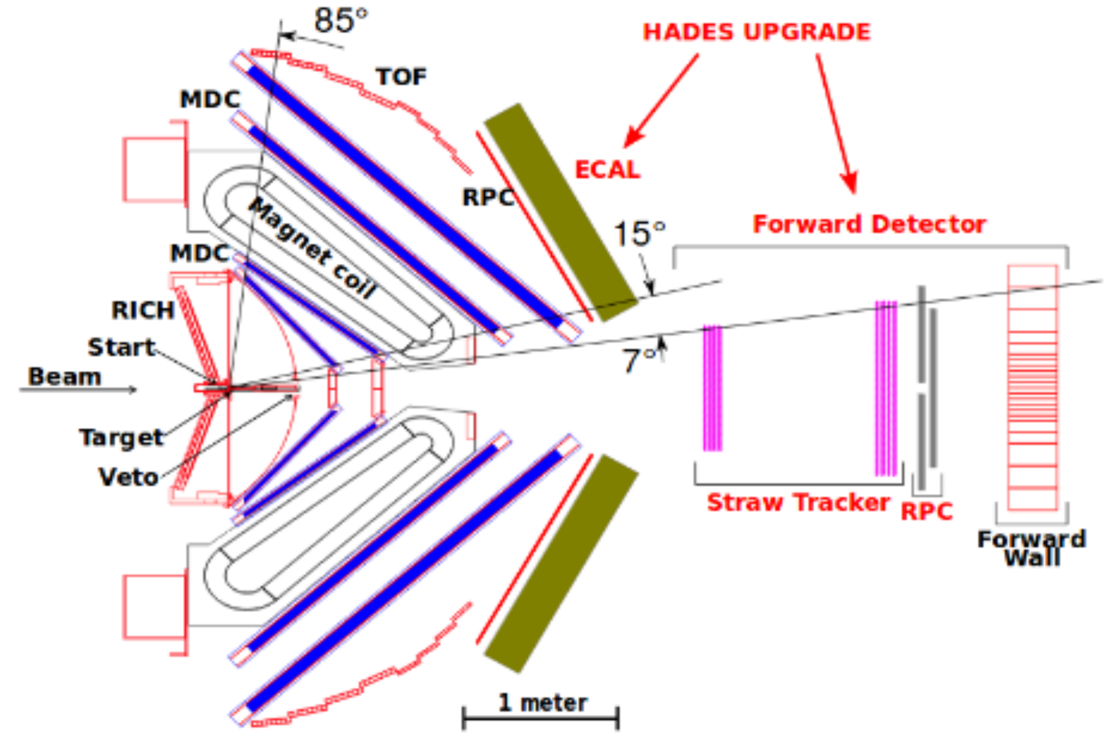
- PID : π/p/K – dE/dx (MDC) and TOF

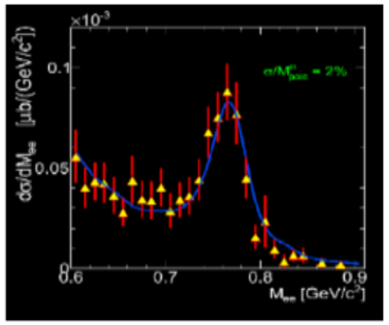
- electrons : RICH (hadron blind)

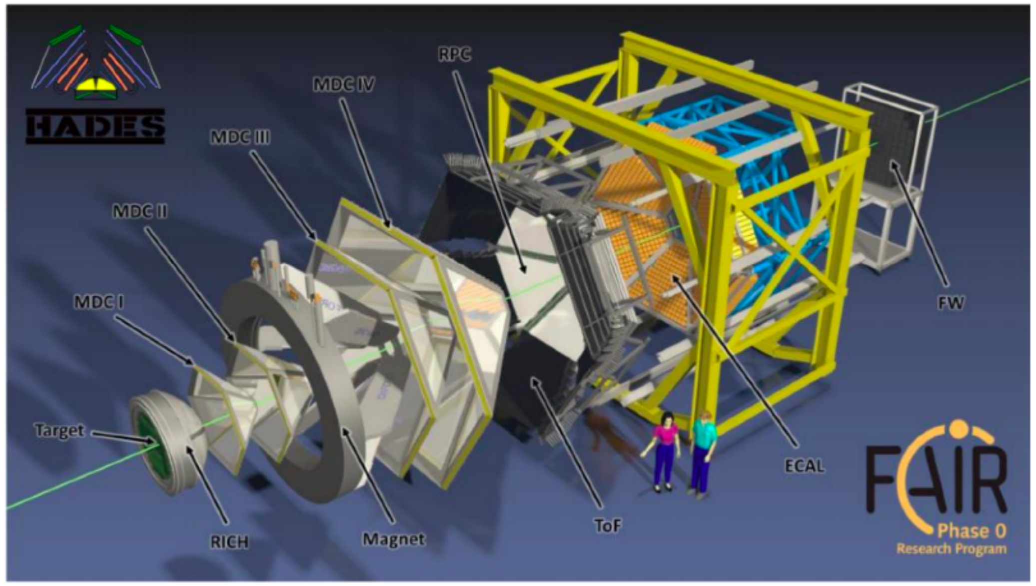
- neutral particles: ECAL

Geometry :

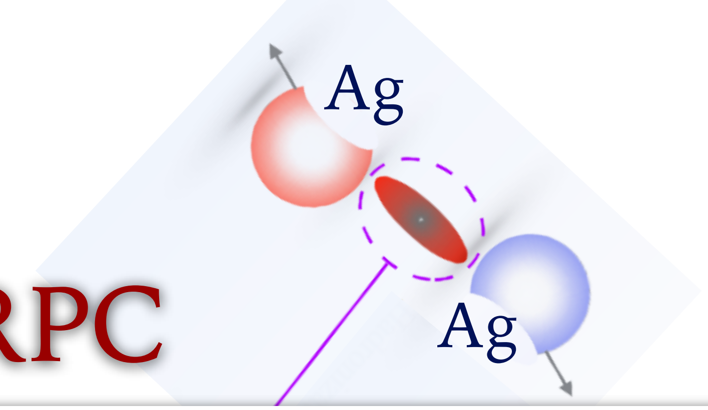
- full azimuthal, polar angles 18^o - 85^o

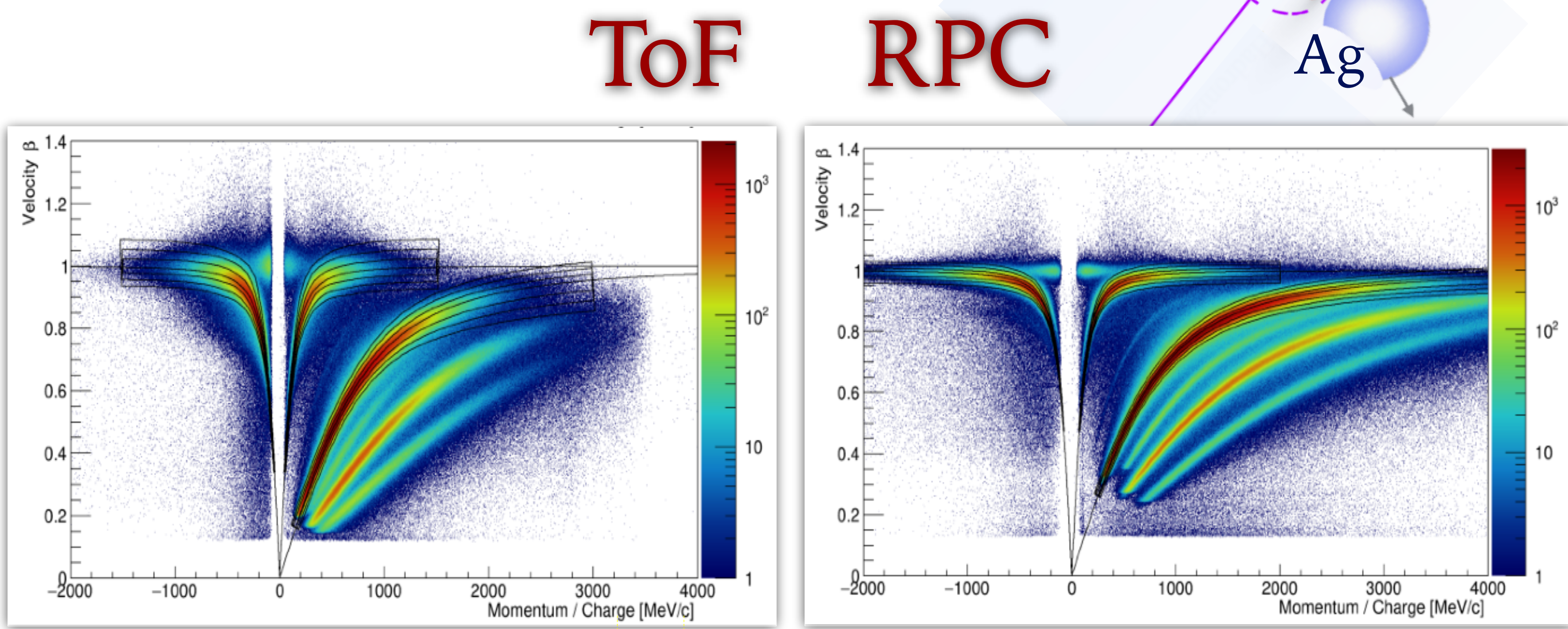


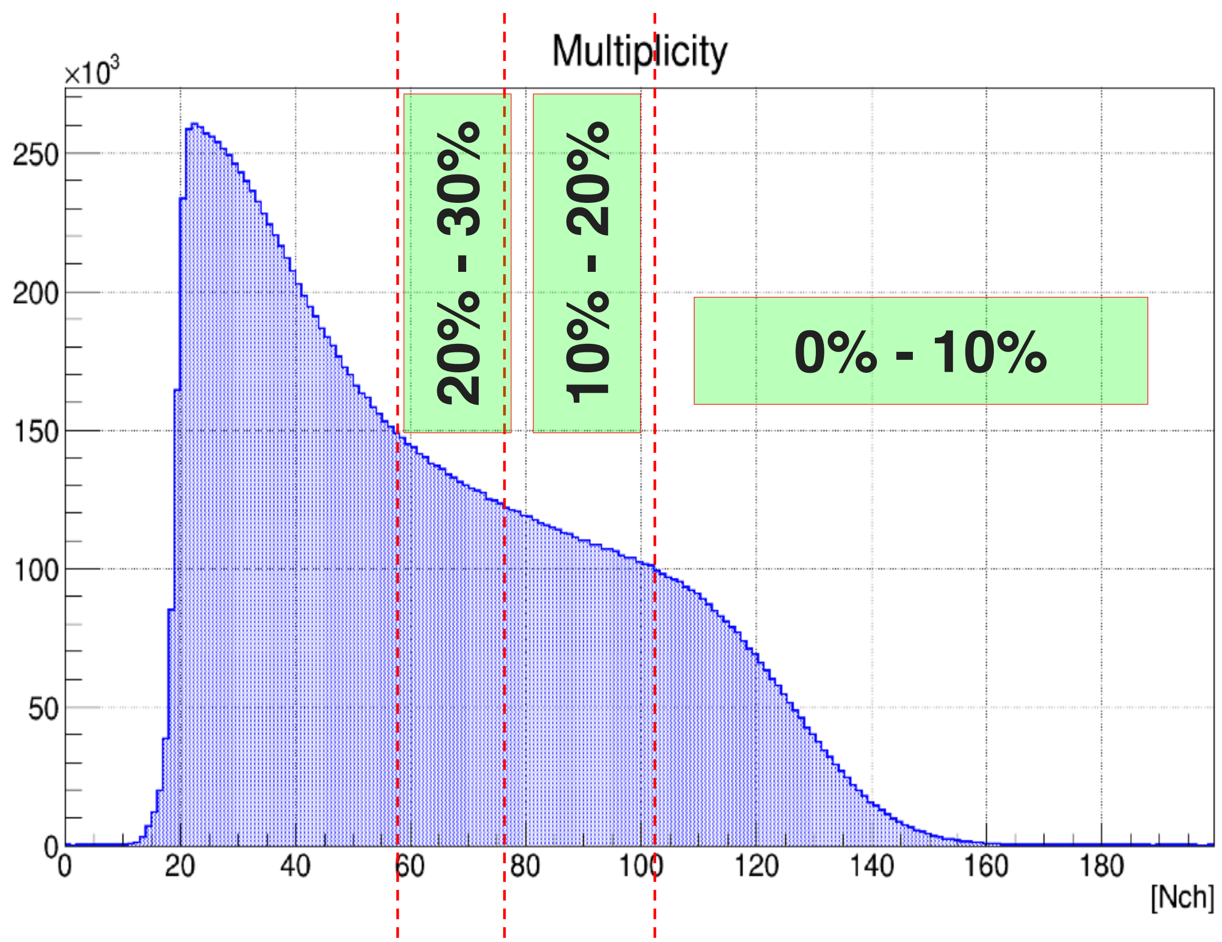




Particle identification







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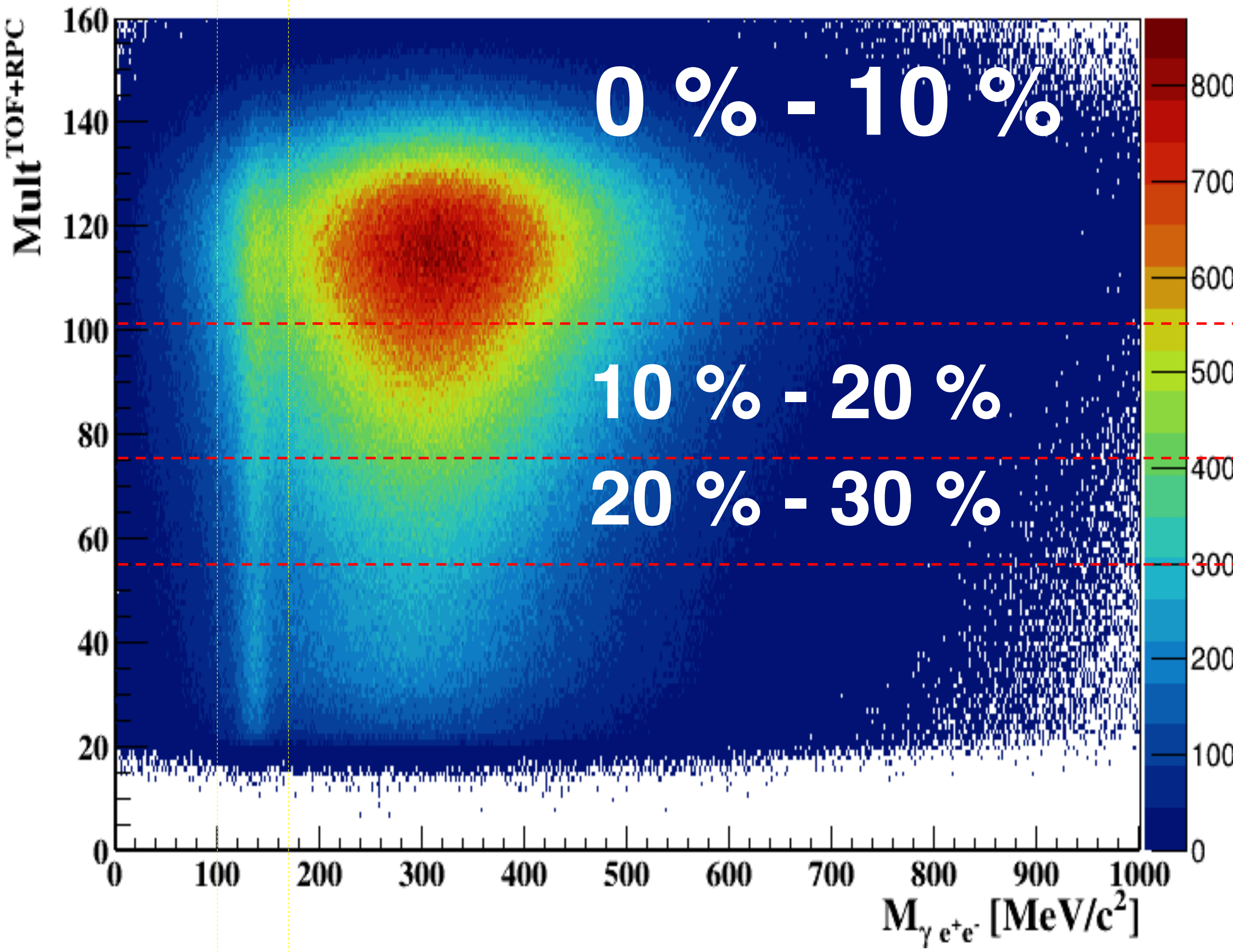
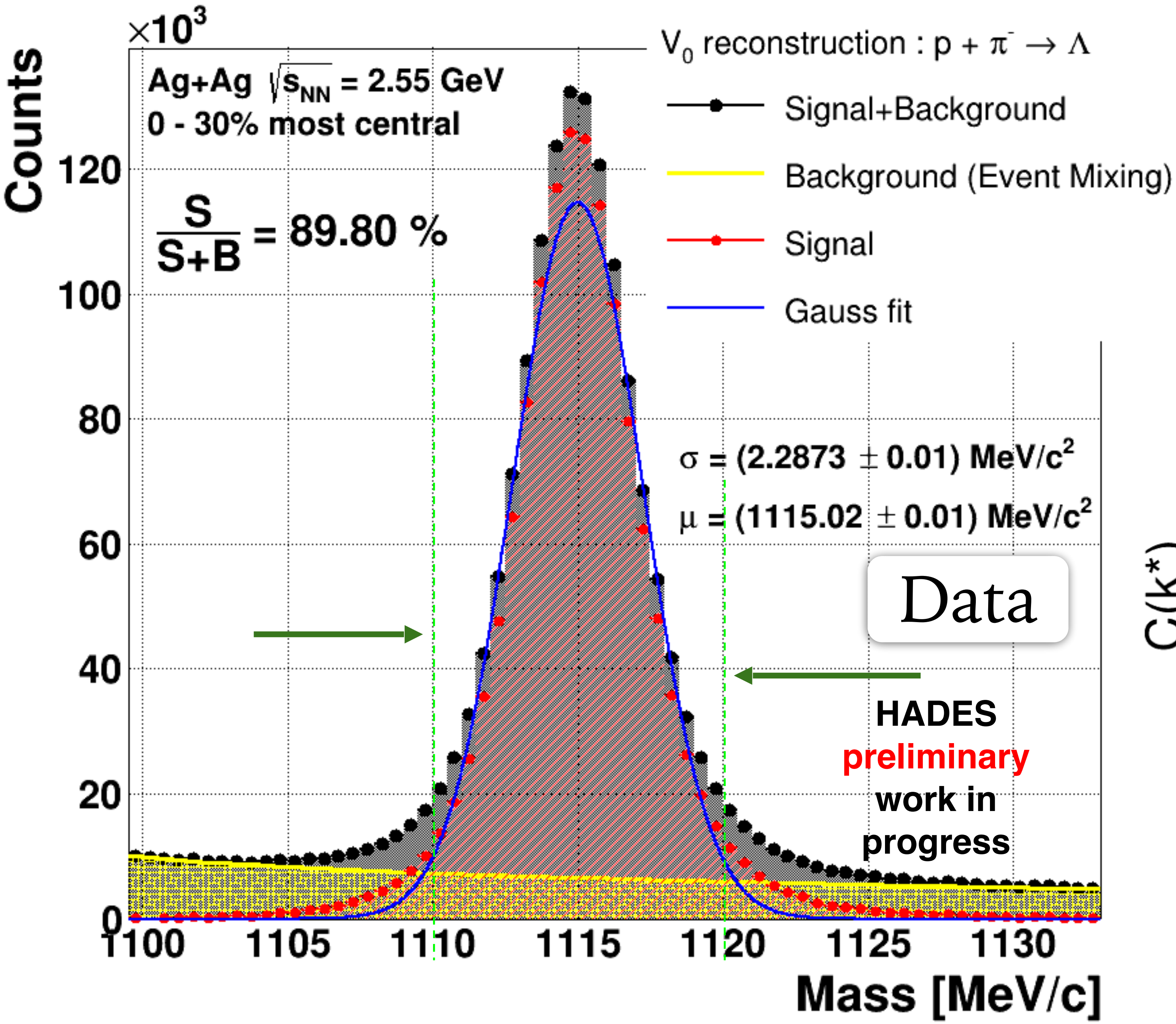
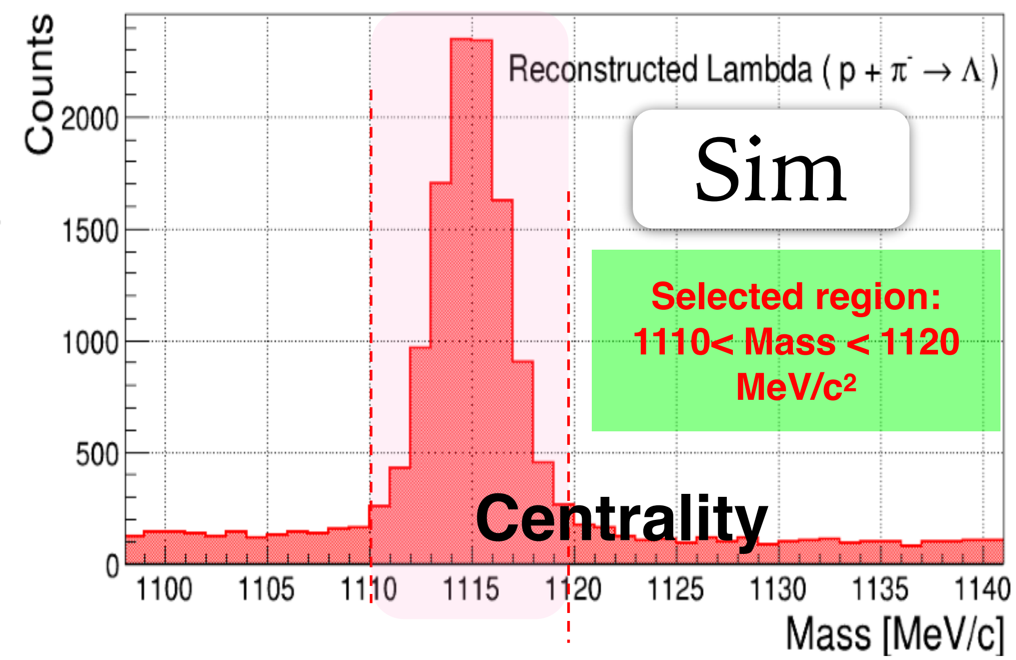
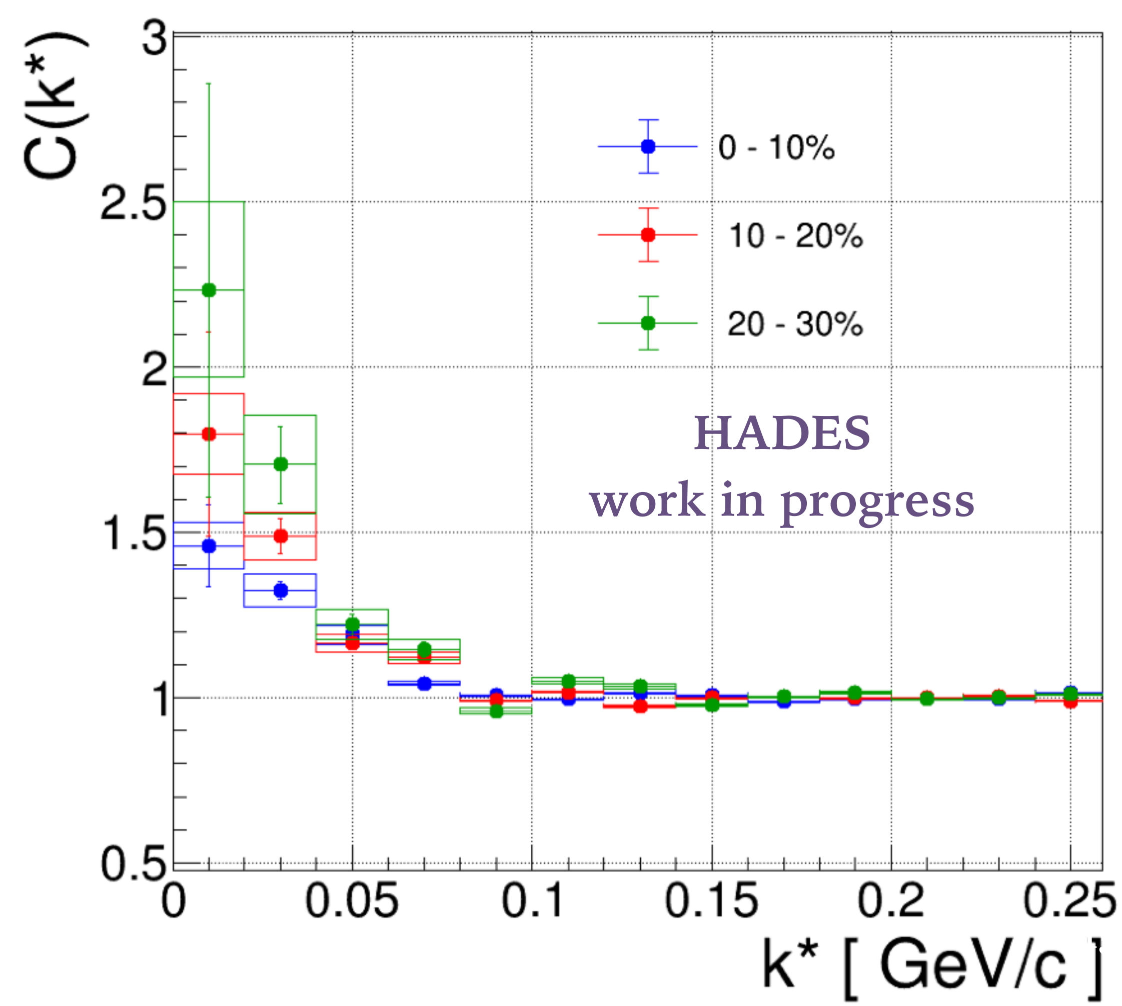


Signal reconstruction (p-Λ correlations)







Lednicky-Lyuboshitz model

The normalized pair separation distribution (source function) **S(r*)** is assumed to be Gaussian,

$$S(r^*) = (2\sqrt{\pi} r_0)^{-3} e^{-\frac{r^{*2}}{4r_0^2}},$$

Ref : Lednicky, Richard & Lyuboshits, V.L.. (1982). Sov. J. Nucl. Phys. (Engl. Transl.); (United States). 35:5.

The correlated function can be calculated analytically by averaging **Ψ^s** over the total spin S and the distribution of the relative distances **S(r*)**

$$C(k^*) = 1 + \sum_S \rho_s \left[\frac{1}{2}\left|\frac{f^S(k^*)}{r_0}\right|^2 \left(1 - \frac{d_0^S}{2\sqrt{\pi} r_0}\right) + \frac{2\mathbb{R} f^S(k^*)}{\sqrt{\pi} r_0} F_1(Q r_0) - \frac{\mathfrak{I} f^S(k^*)}{r_0} F_2(Q r_0)\right]$$

$$with\ F_1(z) = \int_0^z dx e^{x^2 - z^2}/z\ and\ F_2(z) = (1 - e^{-z^2})/z$$

Decomposition for spin channels :

$$C(k^*) = \frac{1}{4}(1 + \lambda C(k^*, s=0)) + \frac{3}{4}(1 + \lambda C(k^*, s=1))$$

f_0 and d_0 - parameters of strong interaction.

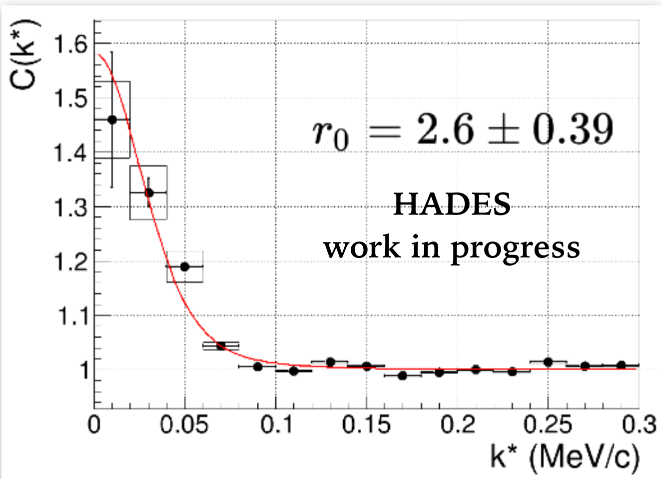
Theoretical correlation function (k*) depends on: R, f_0 and d_0.

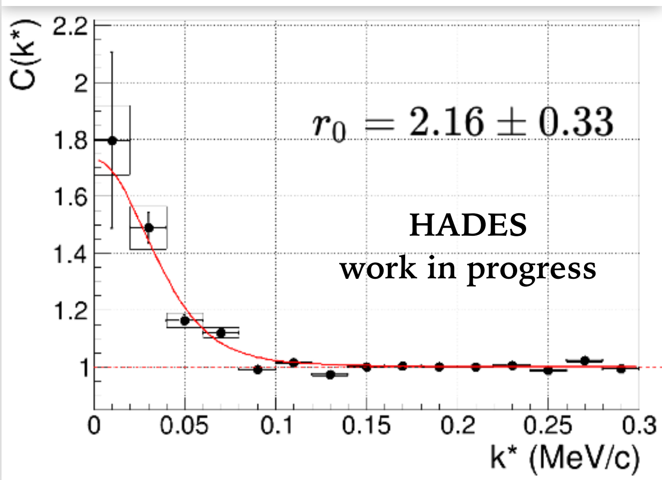
f_0 - the scattering length, determines low-energy scattering.

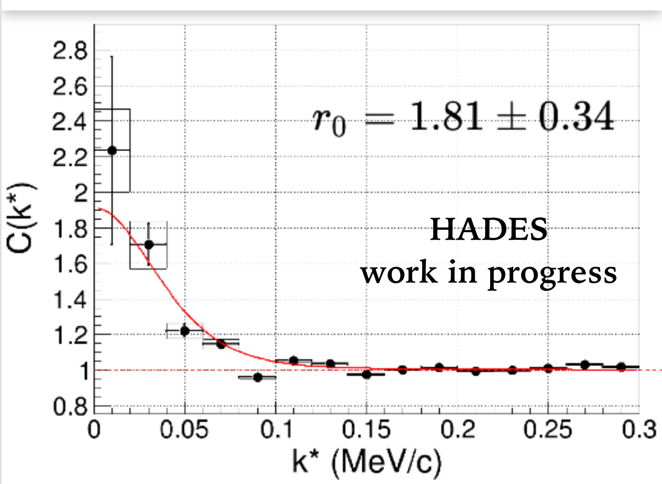
The elastic cross section, σ_e, (at low energies) determined by the scattering length, $\lim_{k \to 0} \sigma_e = 4\pi f_0^2$

d_0 - the effective range, corresponds to the range of the potential (simplified scenario - the square well potential.

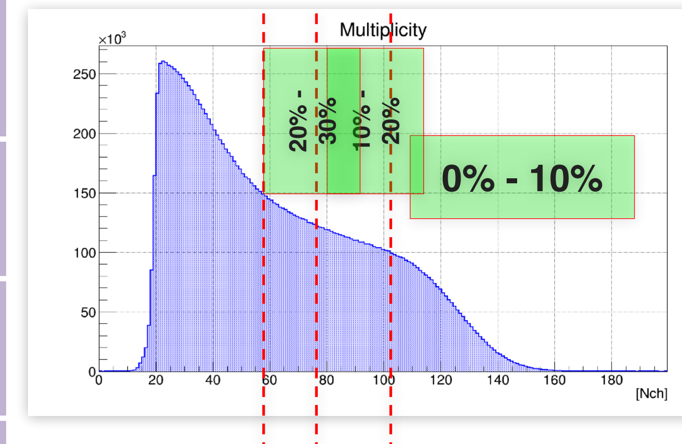
Results: centrality dependence

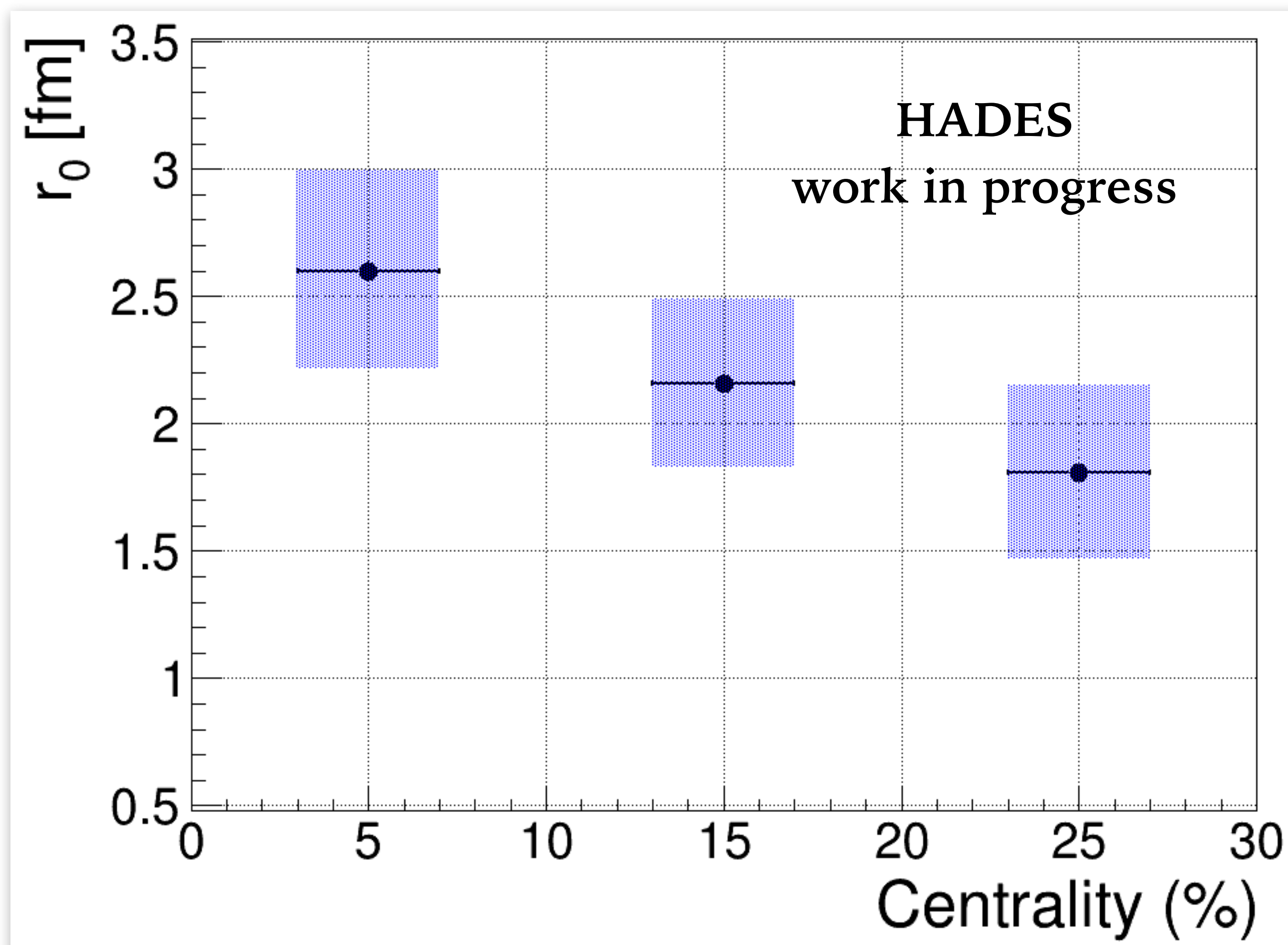






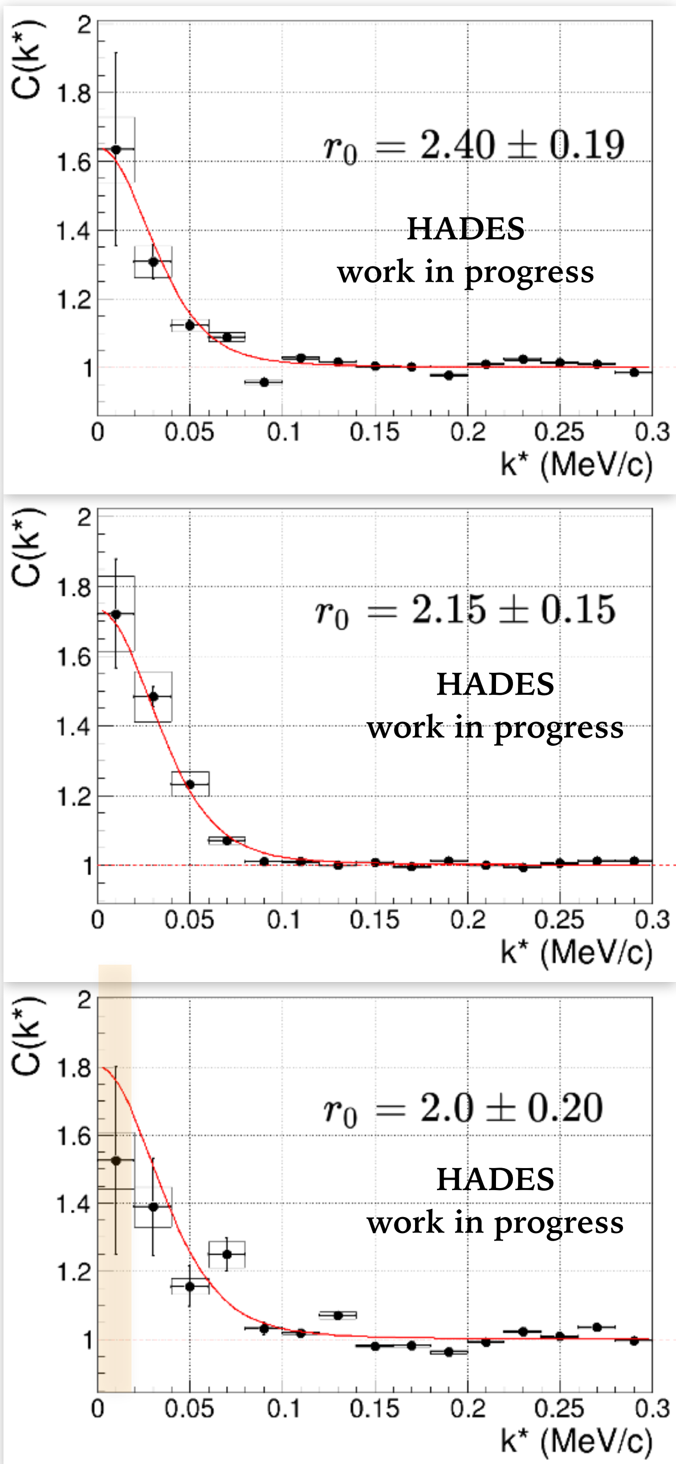
Centrality	Systematic Uncertainty
0 - 10 %	15.30 %
10 - 20 %	15.49 %
20 - 30 %	19.00 %

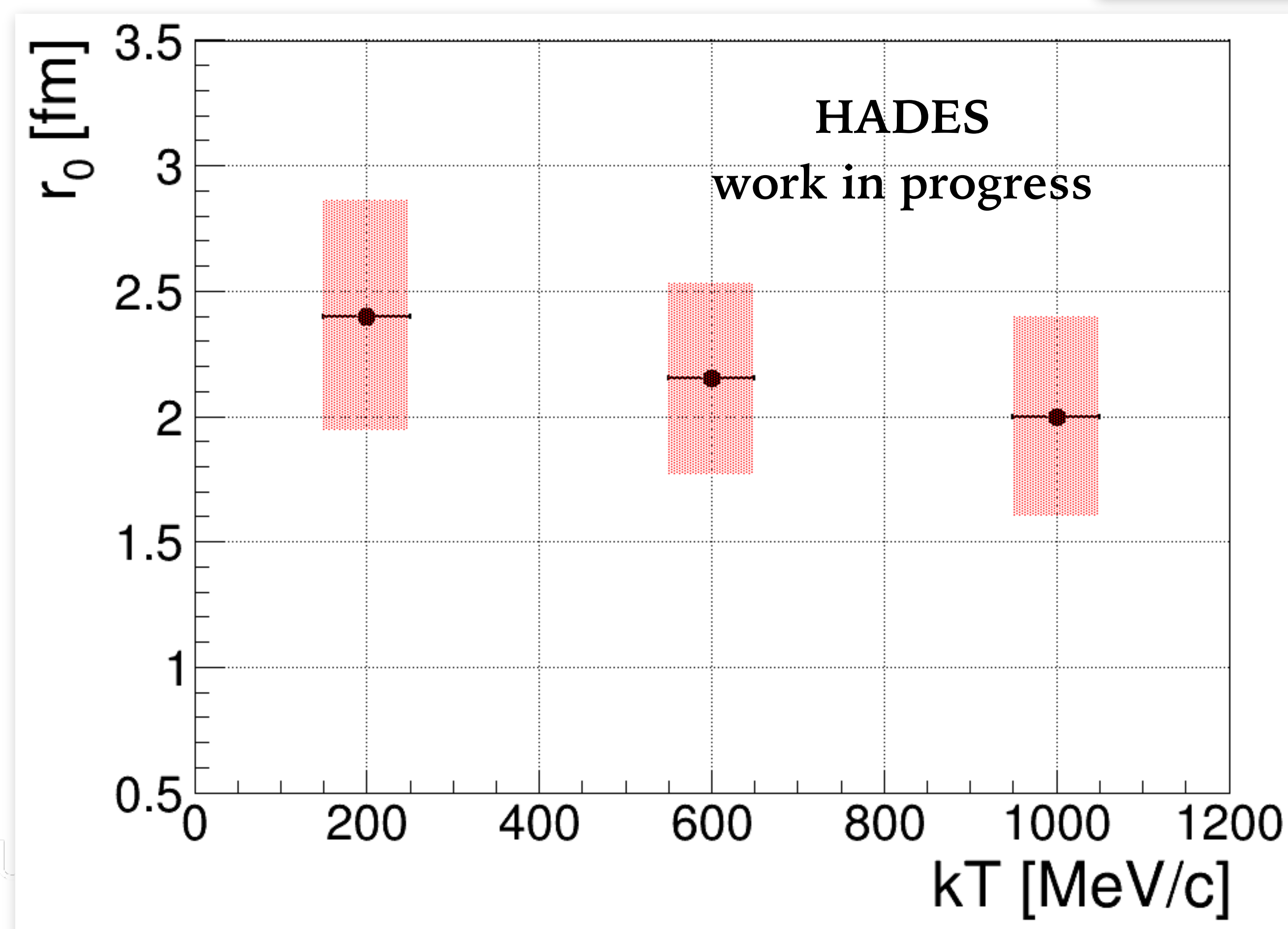


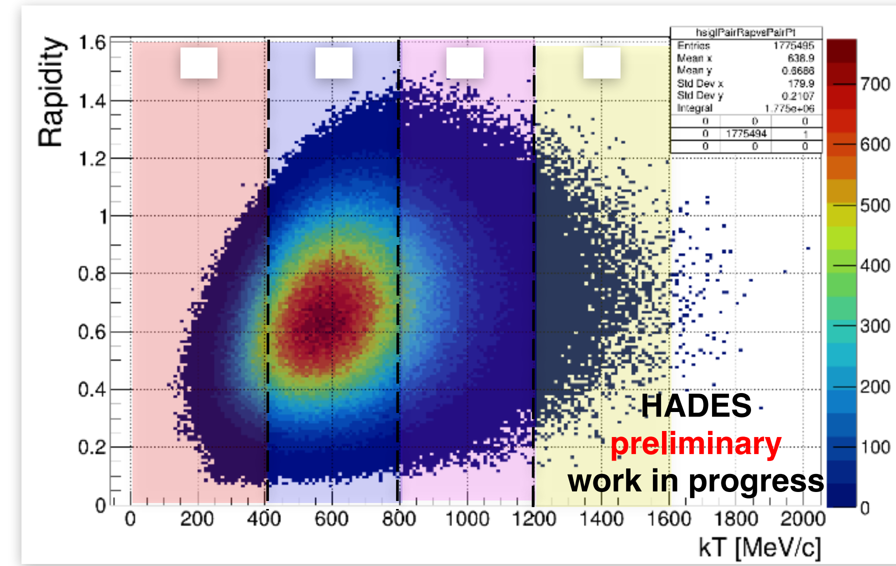


Results: k_T dependence

kT [GeV/c]	Systematic Uncertainty
0 - 400	19 %
400 - 800	15 %
800 - 1200	22 %







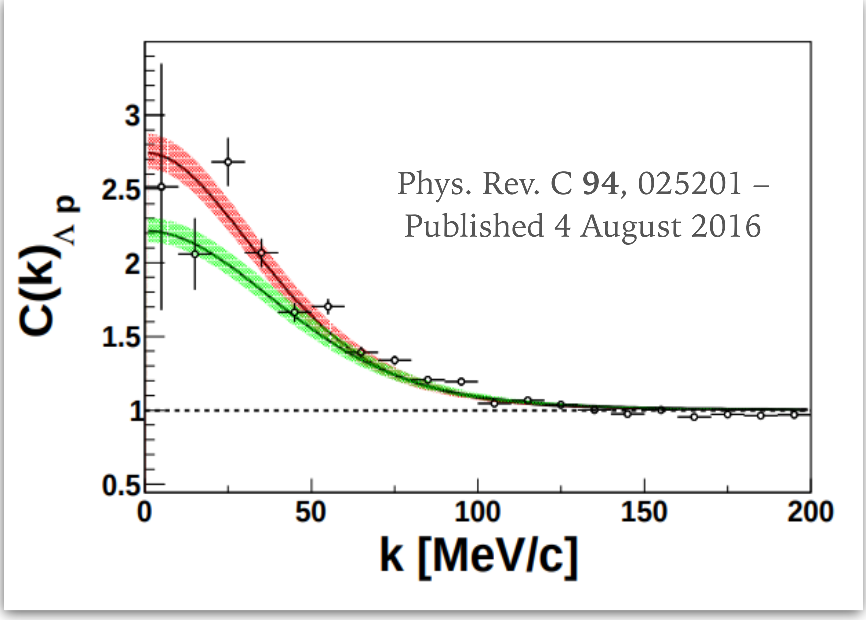
$$k_T = \frac{p_{T1} + p_{T2}}{2}$$

Lednicky-Lyuboshitz model

Model		$f_0^{S=0}$ (fm)	$f_0^{S=1}$ (fm)	$d_0^{S=0}$ (fm)	$d_0^{S=1}$ (fm)	n_σ
ND [77]		1.77	2.06	3.78	3.18	1.1
NF [78]		2.18	1.93	3.19	3.358	1.1
NSC89 [79]		2.73	1.48	2.87	3.04	0.9
NSC97 [80]	a	0.71	2.18	5.86	2.76	1.0
	b	0.9	2.13	4.92	2.84	1.0
	c	1.2	2.08	4.11	2.92	1.0
	d	1.71	1.95	3.46	3.08	1.0
	e	2.1	1.86	3.19	3.19	1.1
	f	2.51	1.75	3.03	3.32	1.0
ESC08 [81]		2.7	1.65	2.97	3.63	0.9
χEFT	LO [25]	1.91	1.23	1.4	2.13	1.8
	NLO [26]	2.91	1.54	2.78	2.72	1.5
Jülich	A [82]	1.56	1.59	1.43	3.16	1.0
	J04 [83]	2.56	1.66	2.75	2.93	1.4
	J04c [83]	2.66	1.57	2.67	3.08	1.1

S. Acharya *et al.* Phys. Rev. C 99, 024001 – Published 13 Feb 2019
https://doi.org/10.1103/PhysRevC.99.024001

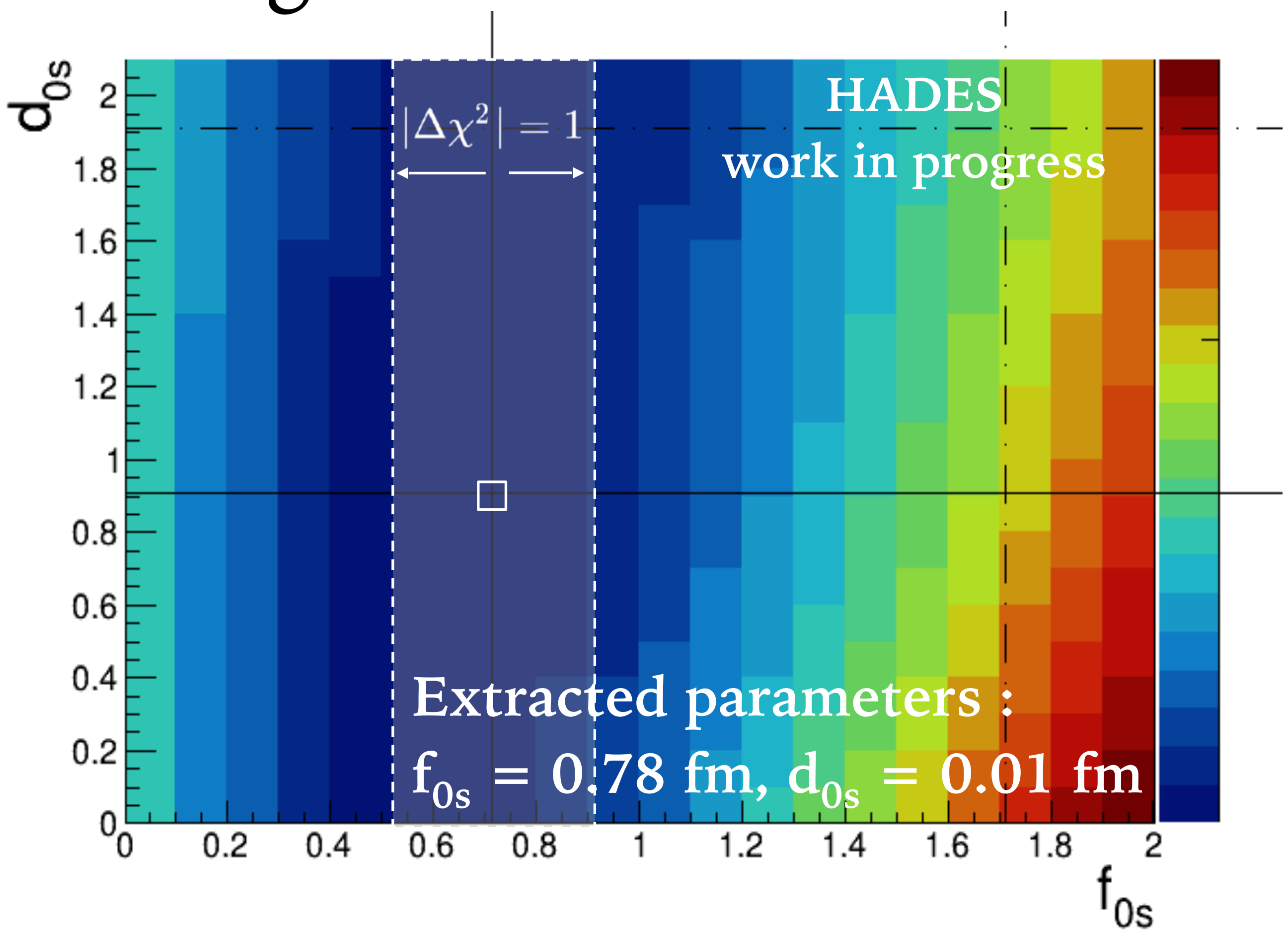
parameter scan boundaries : f_0 [0.01, 5.0], d_{0s} [0.01, 2.0] and d_{0t} [0.01, 5.0]



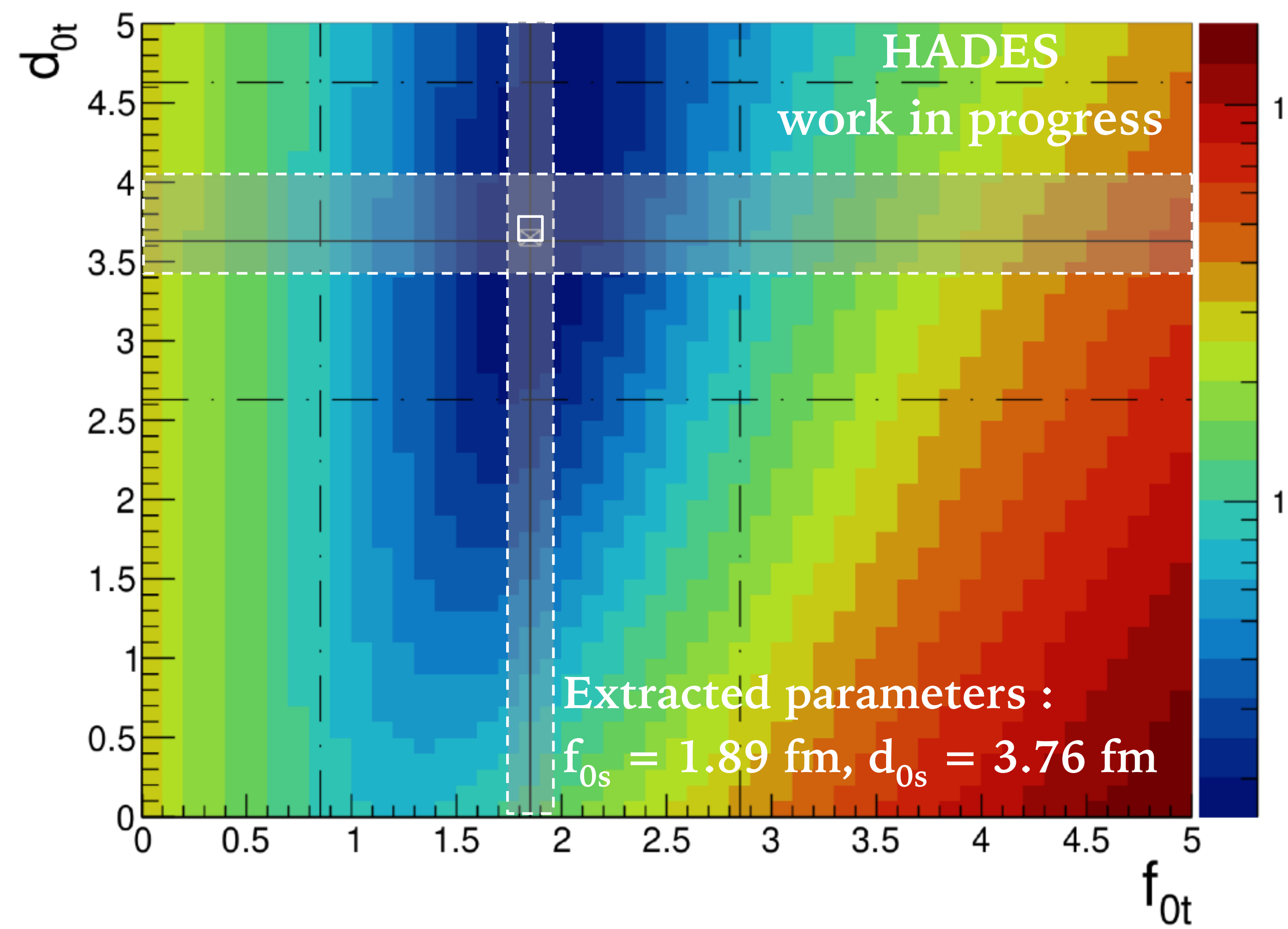
Parameters	p-Nb (LO)	p-Nb (NLO)
f_{0s}	1.91 fm	2.91 fm
d_{0s}	1.40 fm	2.78 fm
f_{0t}	1.23 fm	1.54fm
d_{0t}	2.13 fm	2.72fm
r_0	1.71±0.10	1.62±0.02

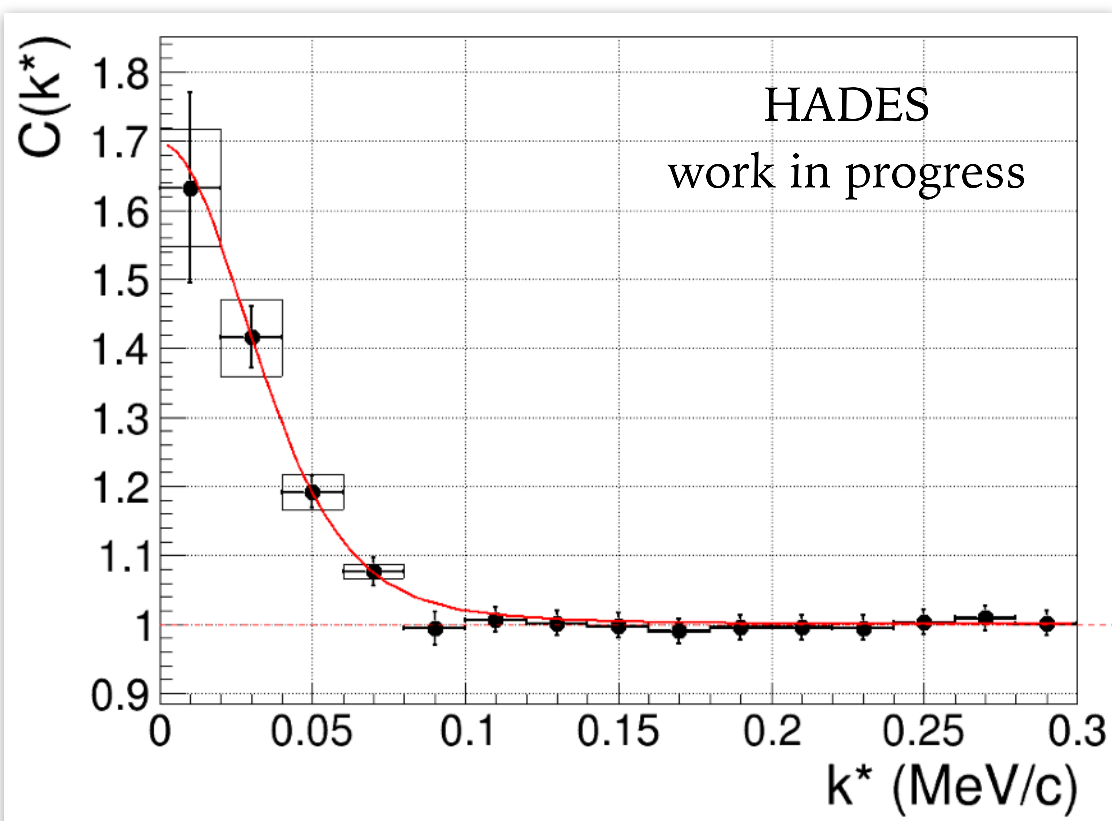
Lednicky-Lyuboshitz model

Singlet



Triplet





$\lambda = 0.74$

$f_{0s} = 0.80^{+0.39}_{-0.32}$

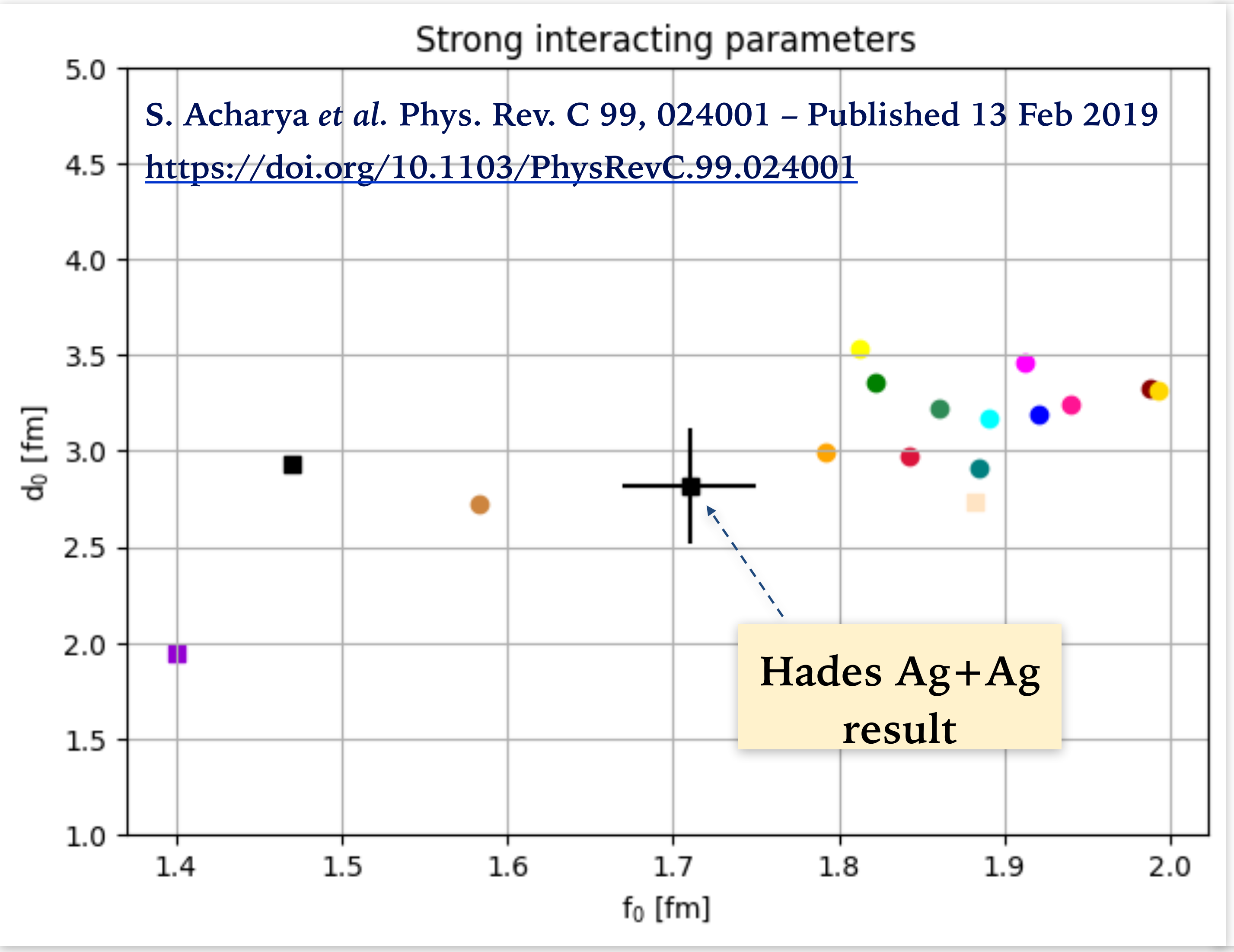
$d_{0s} = 0.01$

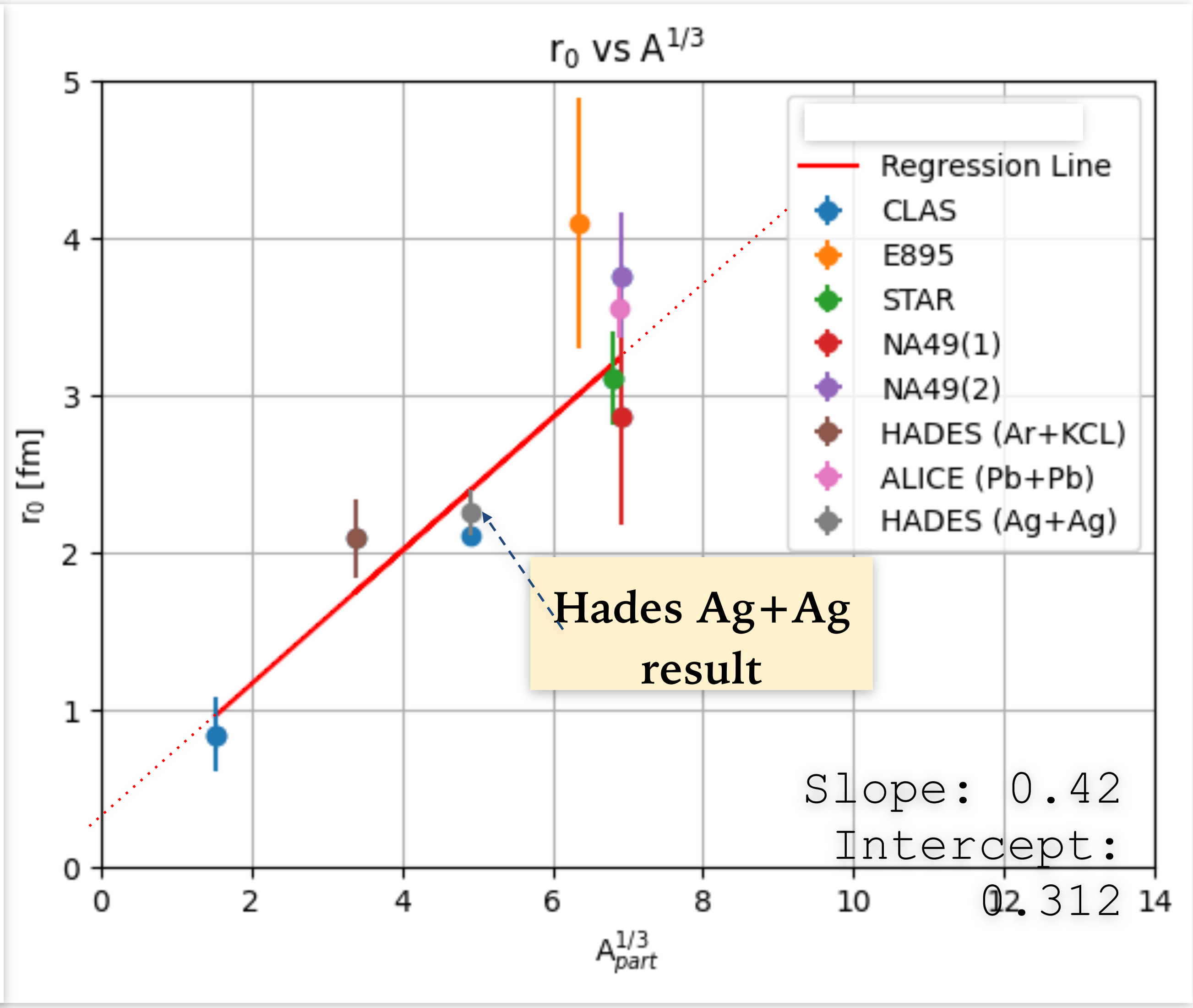
$f_{0t} = 1.89^{+0.10}_{-0.09}$

$d_{0t} = 3.76^{+0.27}_{-0.25}$

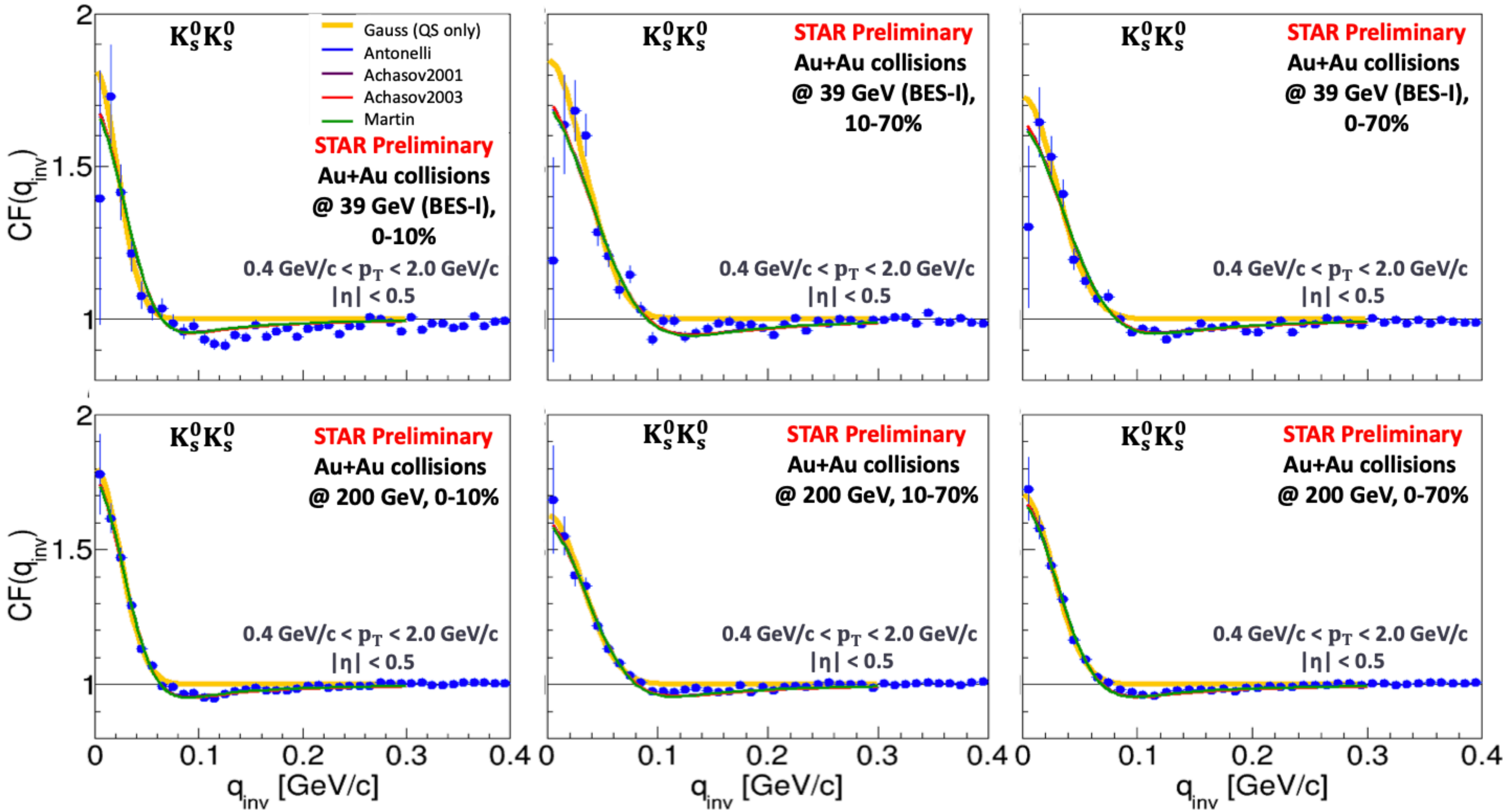
$r = 2.24^{+0.12}_{-0.11}$

Parameters scan





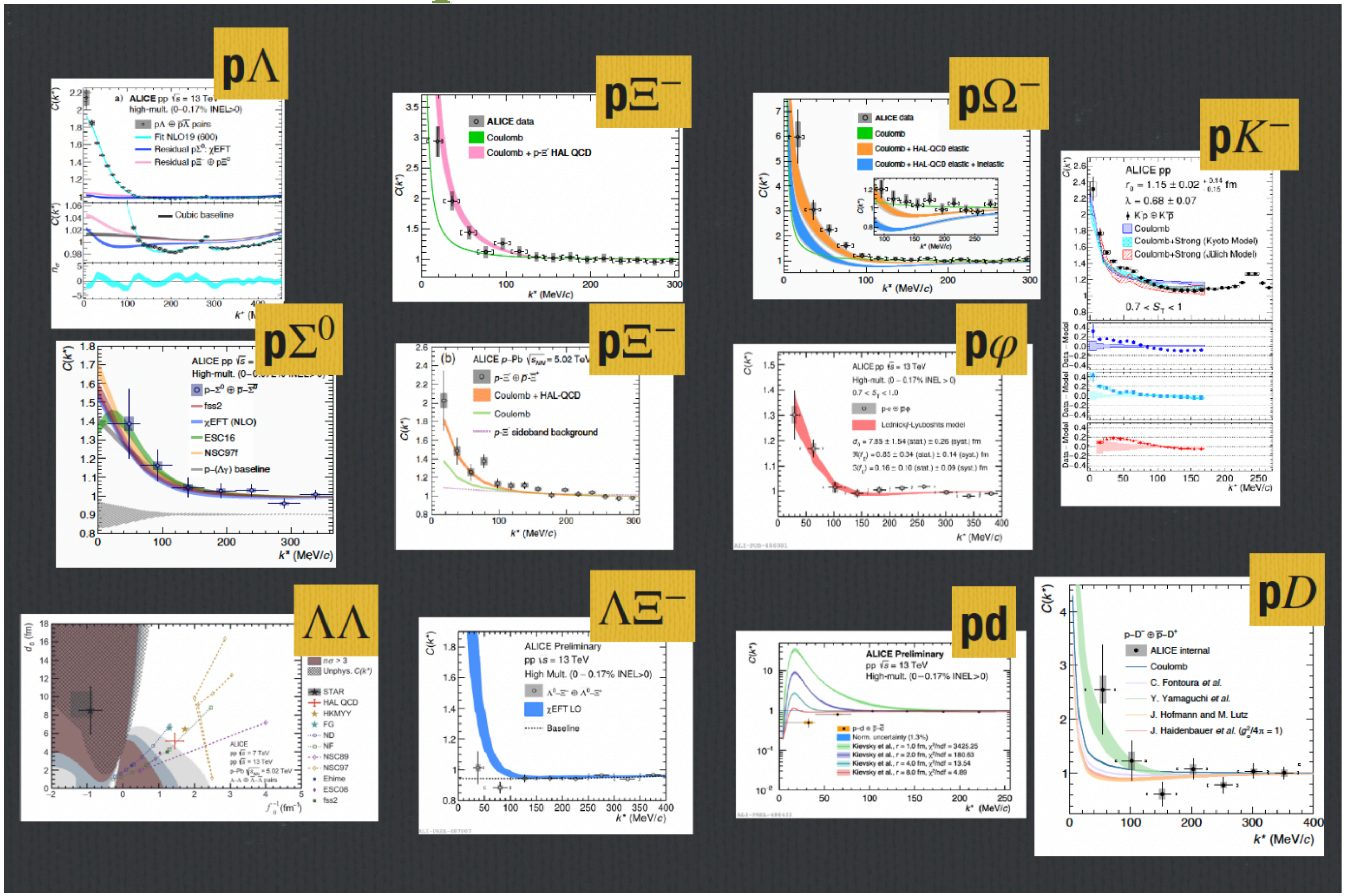
Femtoscopy with neutral kaons, planned at HADES



$q_{inv} = \sqrt{(\vec{p_1} - \vec{p_2})^2 - (E_1 - E_2)^2}$

4 parameterizations of SI FSI, scanning of parameters welcome!

Some examples from ALICE



CATS: EPJA 78 (2018)
Projector: EPJC 82 (2022)
Review 1: Prog.Part.Nucl.Phys. 112 (2020)
Review 2:Ann.Rev.Nucl.Part.Sci. 71 (2021)
p-ϕ bound state: arXiv:2212.12690
p-K: PRL 124 (2020) 092301
p-K: PLB 822 (2021), EPJC (2022)
p-p, p-Λ, Λ-Λ: PRC 99 (2019) 024001
Λ-Λ: PLB 797 (2019) 134822
p-Ξ⁻: PRL. 123 (2019)
p-Ξ⁻, p-Ω⁻: Nature 588 (2020) 232–238
p-Σ⁰: PLB 805 (2020) 135419
p-φ: PRL 127 (2021)
$p-\bar{p}, \Lambda-\bar{\Lambda}, p-\bar{\Lambda}$: PLB 829 (2022)
p-Λ: PLB 832 (2022) 137272
$\Lambda-\Xi$: PLB 137223 (2022)
D-p: PRD **106**, 052010 (2022)
ppp, ppΛ: arXiv:2206.03344

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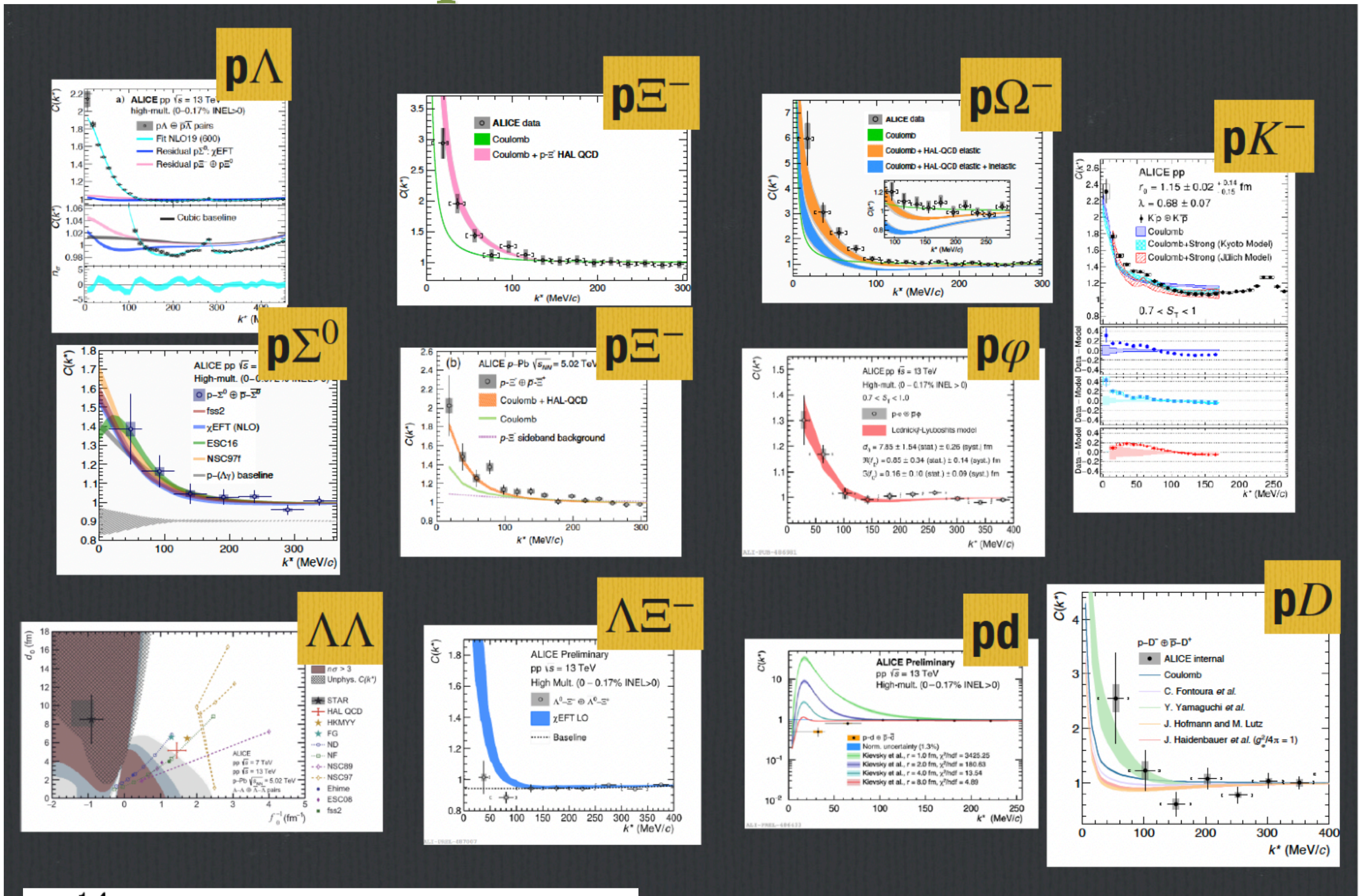

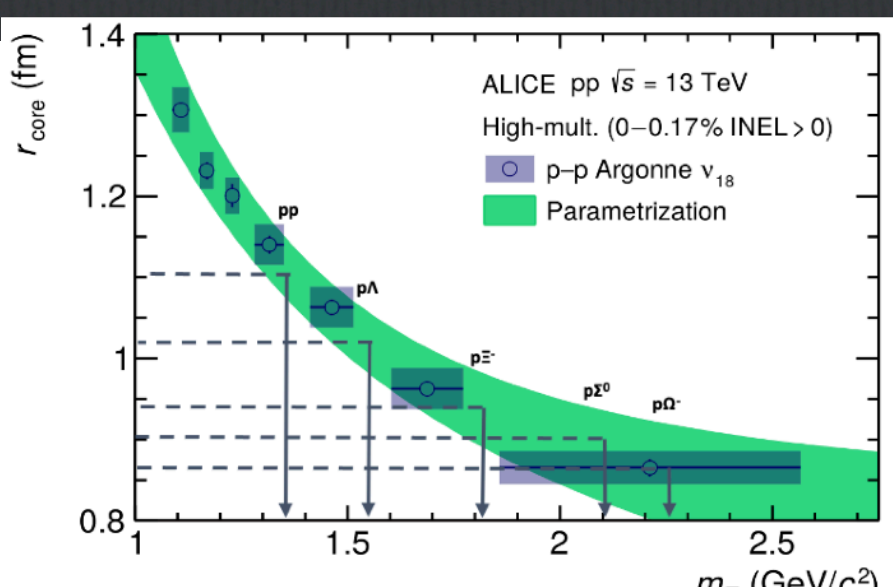

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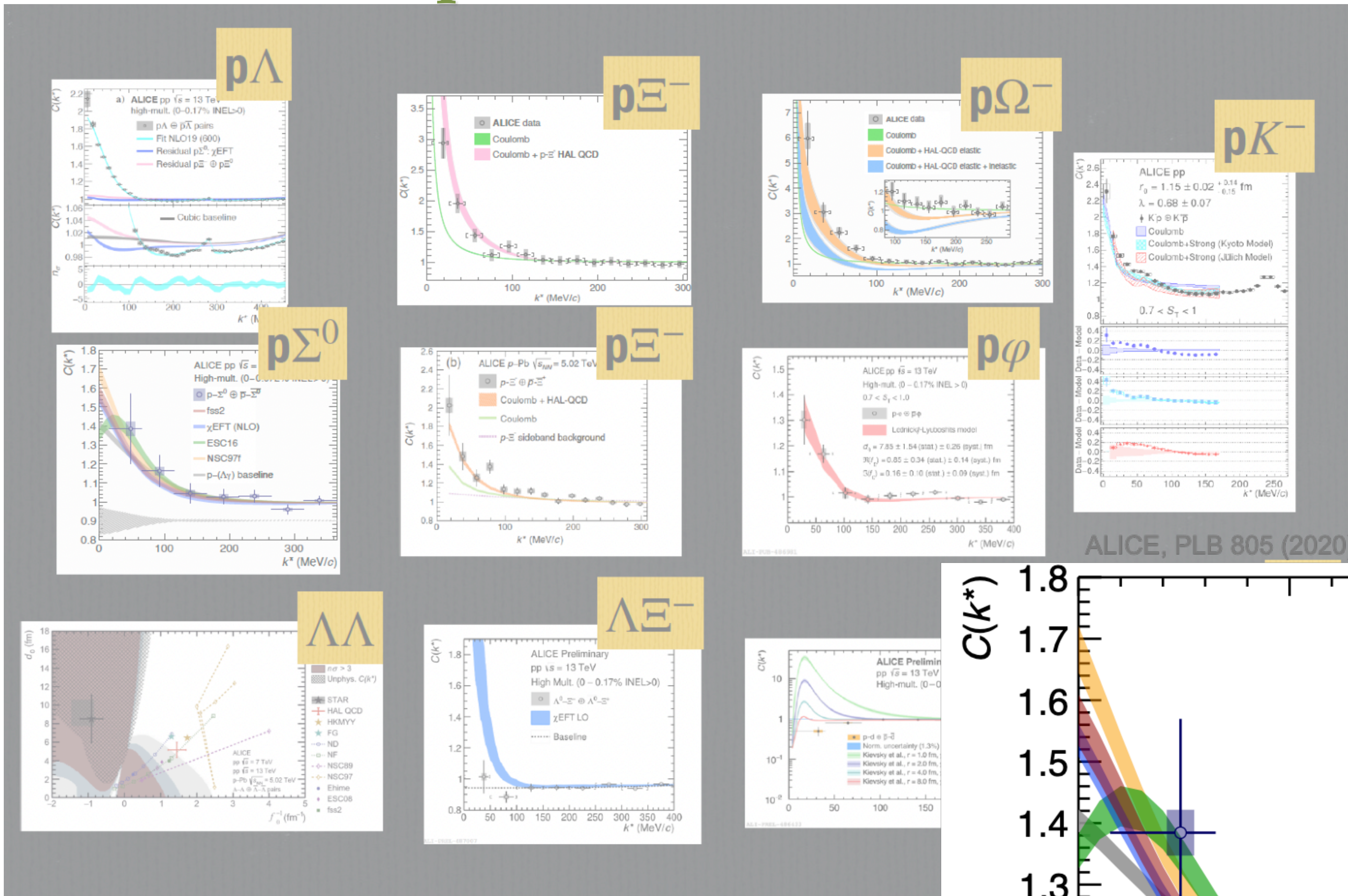
p-p, p-Λ, Λ-Λ: PRC 99 (2019) 024001
Λ-Λ: PLB 797 (2019) 134822
p-Ξ^-: PRL. 123 (2019)
p-Ξ^-, p-Ω^-: Nature 588 (2020) 232–238
p-Σ^0: PLB 805 (2020) 135419
p-φ: PRL 127 (2021)
$p-\bar{p}, \Lambda-\bar{\Lambda}, p-\bar{\Lambda}$: PLB 829 (2022)
p-Λ: PLB 832 (2022) 137272
$\Lambda-\Xi$: PLB 137223 (2022)
D-p: PRD **106**, 052010 (2022)
ppp, ppΛ: arXiv:2206.03344

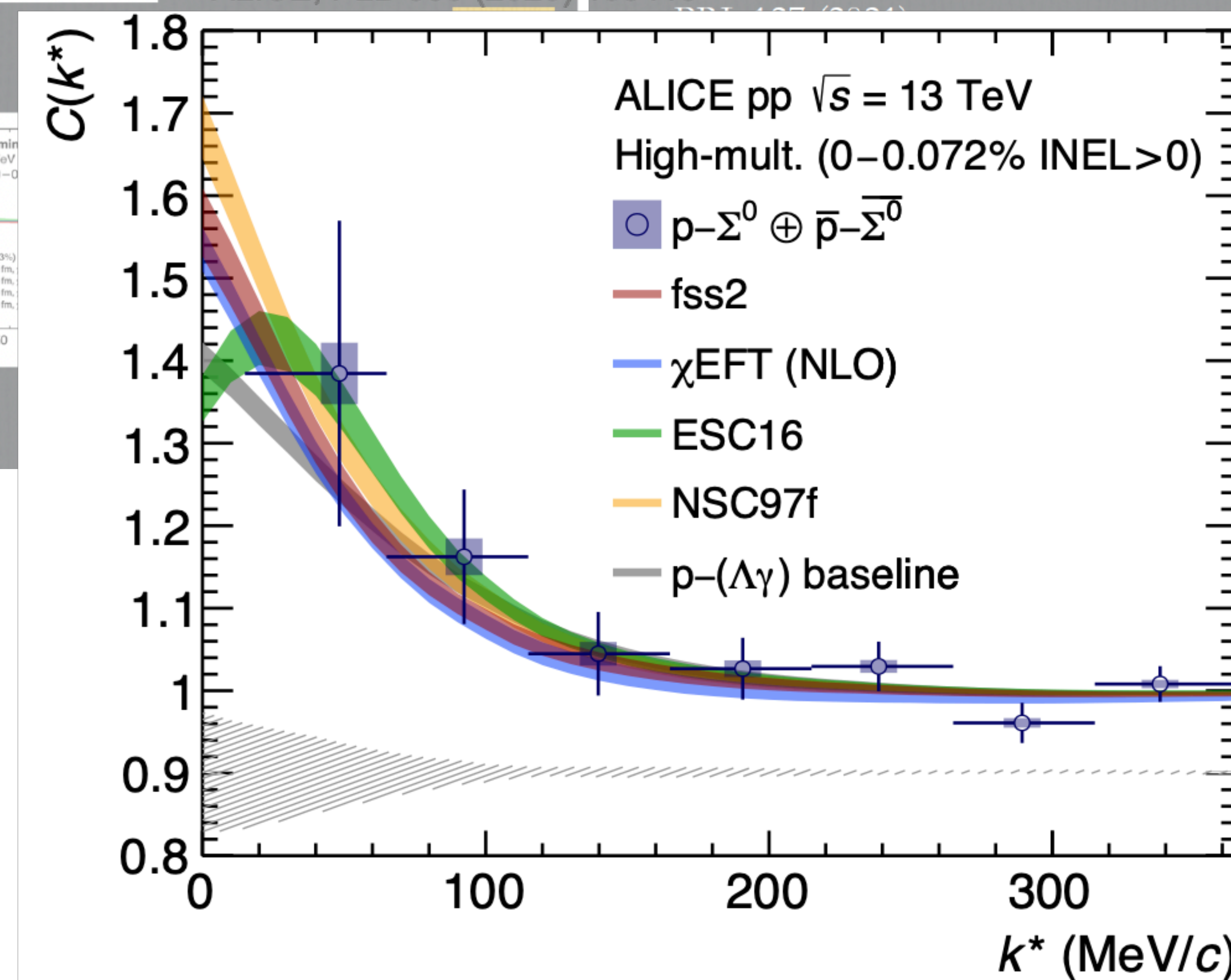
For lower collisions energy the **fraction of pure correlation** (contamination of feed-down contribution, residual correlations) is **higher that at LHC**.

Some examples from ALICE



CATS: EPJA 78 (2018)
Projector: EPJC 82 (2022)
Review 1: Prog.Part.Nucl.Phys. 112 (2020)
Review 2: Ann.Rev.Nucl.Part.Sci. 71 (2021)
p-ϕ bound state: arXiv:2212.12690
p-K: PRL 124 (2020) 092301
p-K: PLB 822 (2021), EPJC (2022)
p-p, p-Λ, Λ-Λ: PRC 99 (2019) 024001
Λ-Λ: PLB 797 (2019) 134822
p-Ξ⁻: PRL. 123 (2019)
p-Ξ⁻, p-Ω : Nature 588 (2020) 232–238
p-Σ⁰: PLB 805 (2020) 135419

ALICE, PLB 805 (2020) 135419



$$\Sigma^0 \rightarrow \Lambda + \gamma$$

$$\gamma \rightarrow e^+e^-$$

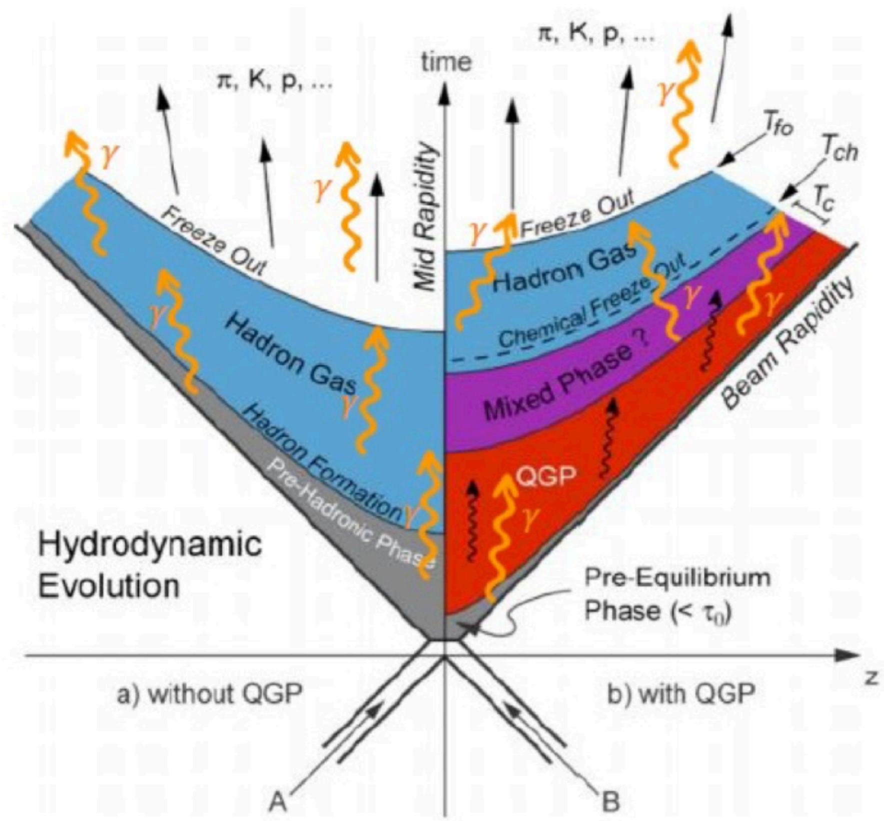
ECAL needed

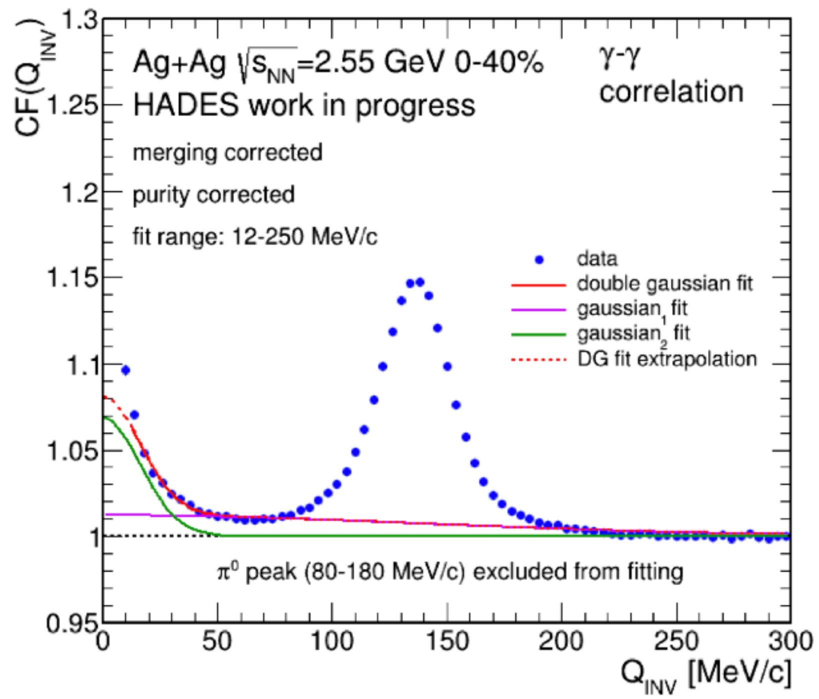
E-M probes with ECAL

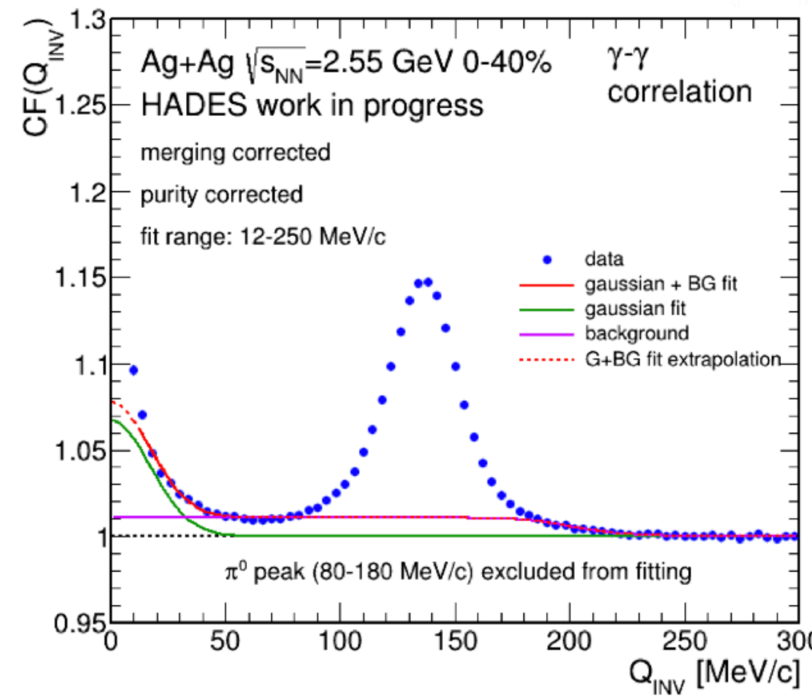
What if:
Non-interacting bosons → assuming 2 sources
(double gaussian or gaussian + some background)

$$CF(Q_{inv}) = 1 + \lambda_1 e^{(-Q_{inv}{}^2 \cdot R_1{}^2)} + \lambda_2 e^{(-Q_{inv}{}^2 \cdot R_2{}^2)}$$

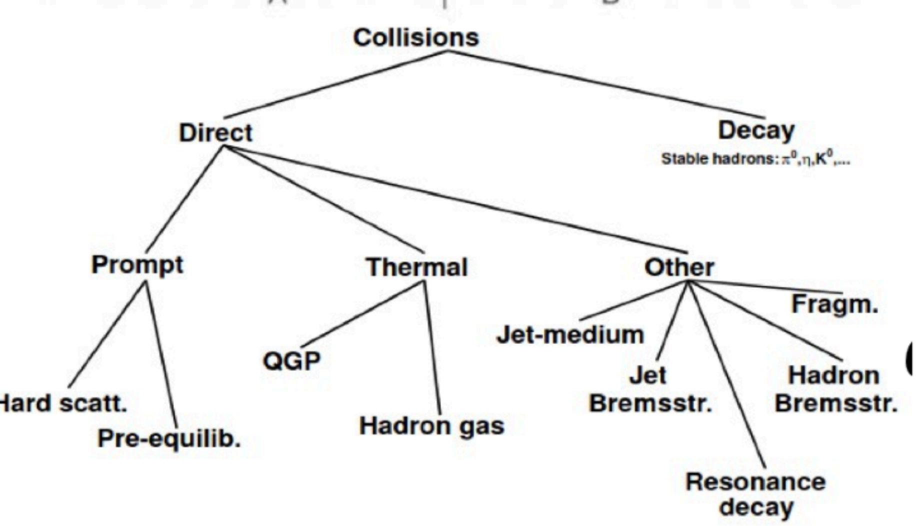
$$CF(Q_{inv}) = 1 + \lambda e^{(-Q_{inv}{}^2 \cdot R^2)} + \frac{a_0}{(1+(a_1 \cdot Q_{inv})^{a_2})}$$





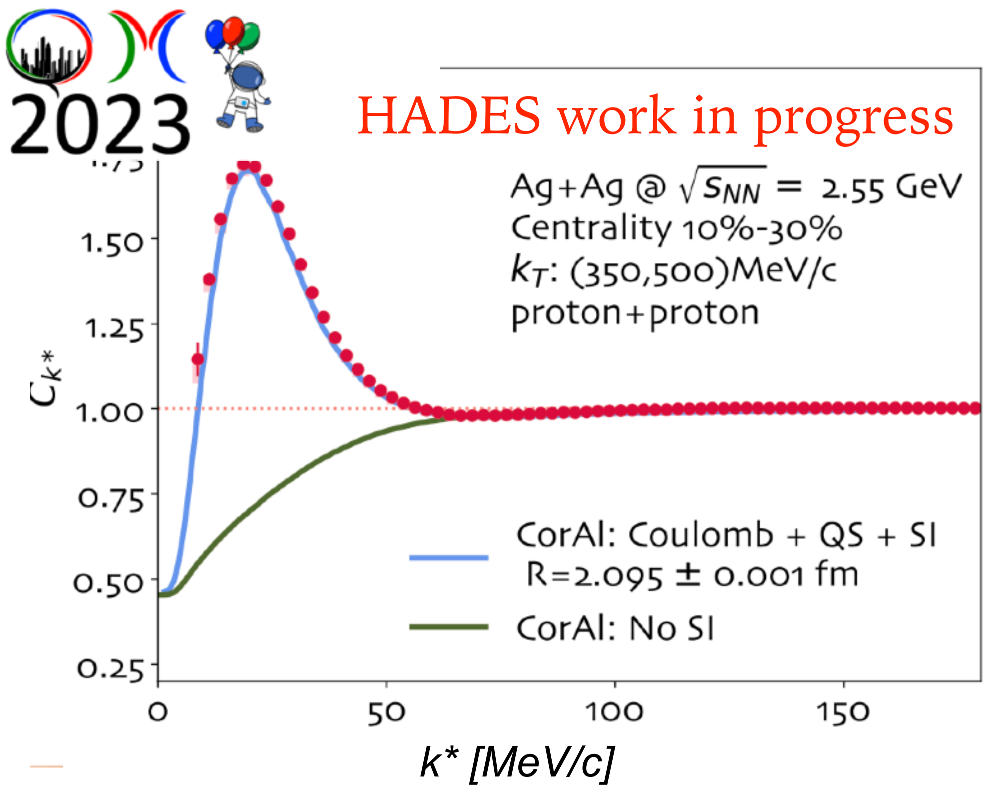


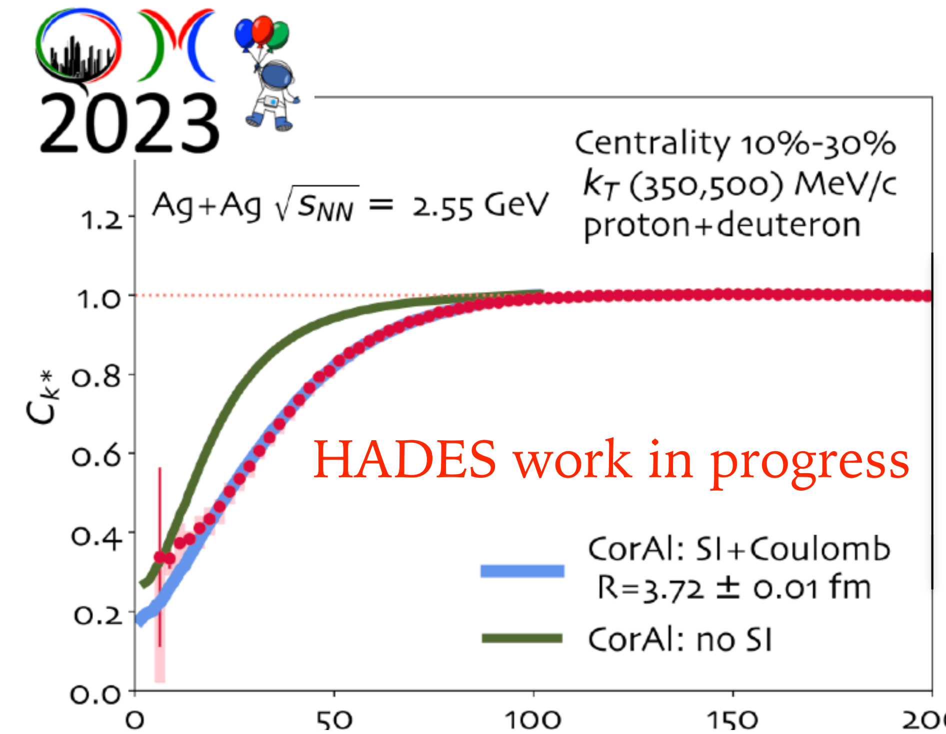
HADES work in progress



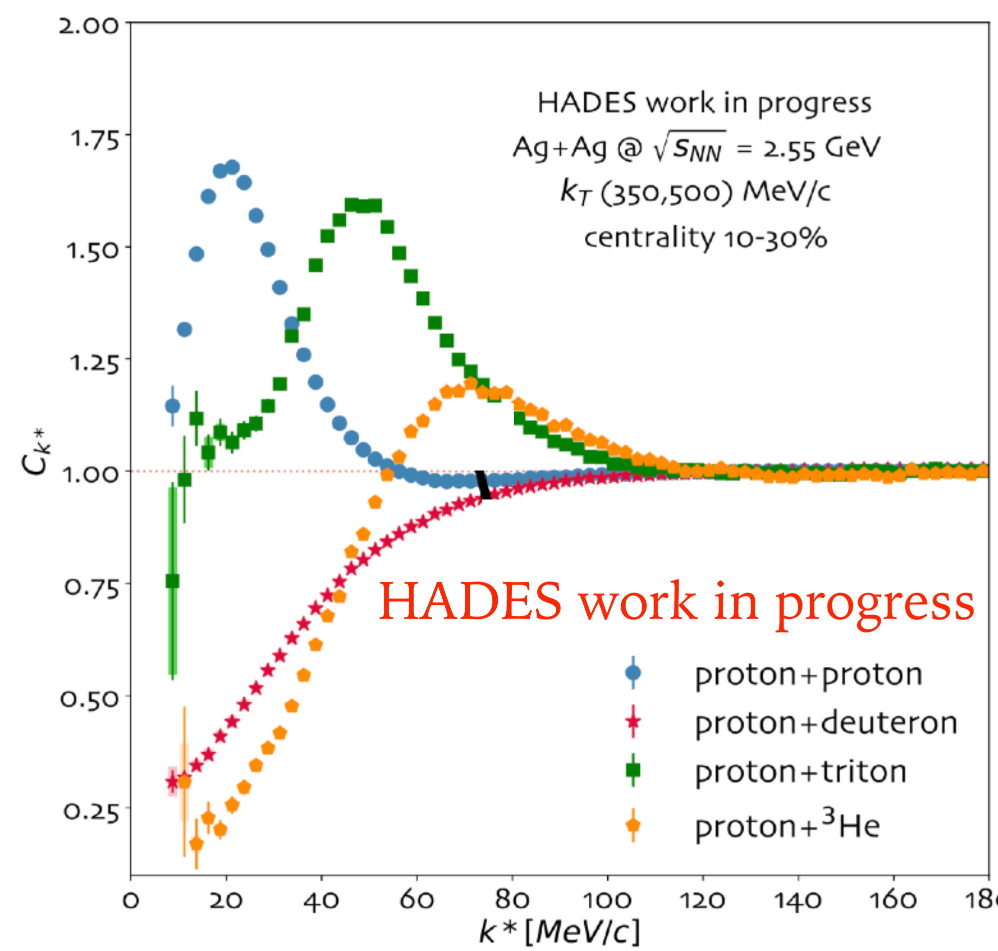
E-M probes signal various stages of the system's evolution;
Information about early stages
(inaccessible for hadrons);
Access before freeze-out era;
A way to hunt for direct photons.

Light nuclei (clusters) production at HADES





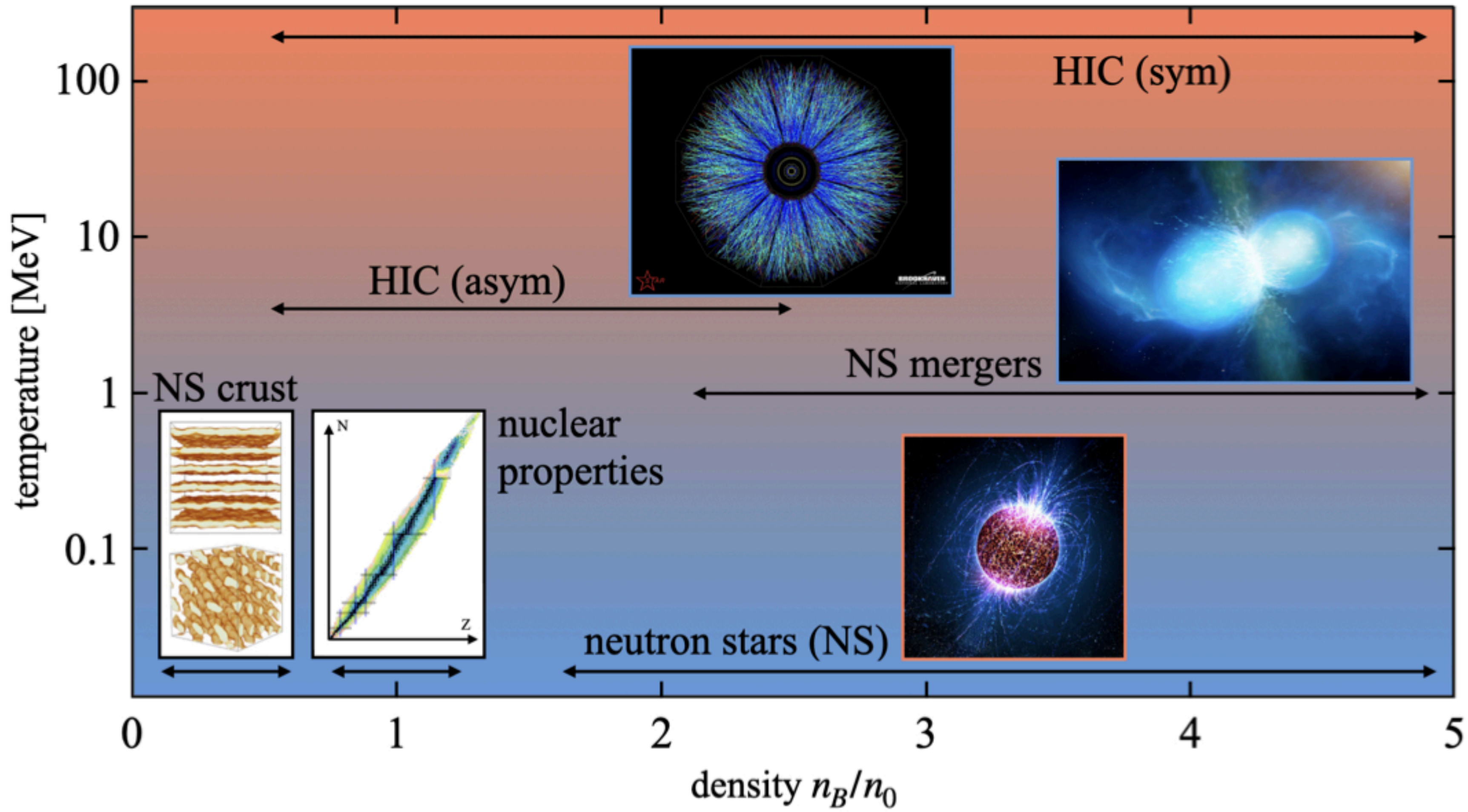
Door to study fundamental interactions, additional nucleons: $p \rightarrow d \rightarrow t$ and ^3_2He
What is the production mechanisms of nuclear fragments / clusters?
- thermal production?
- coalescence?



Proton-proton and proton-deuteron FSI described by potentials:
p-p: V.G.J.Stoks et al., Phys.Rev.C 49,2950 (1994)
p-d: T.C. Black et al., Phys.Lett.B 471, 103 (1999)

Summary

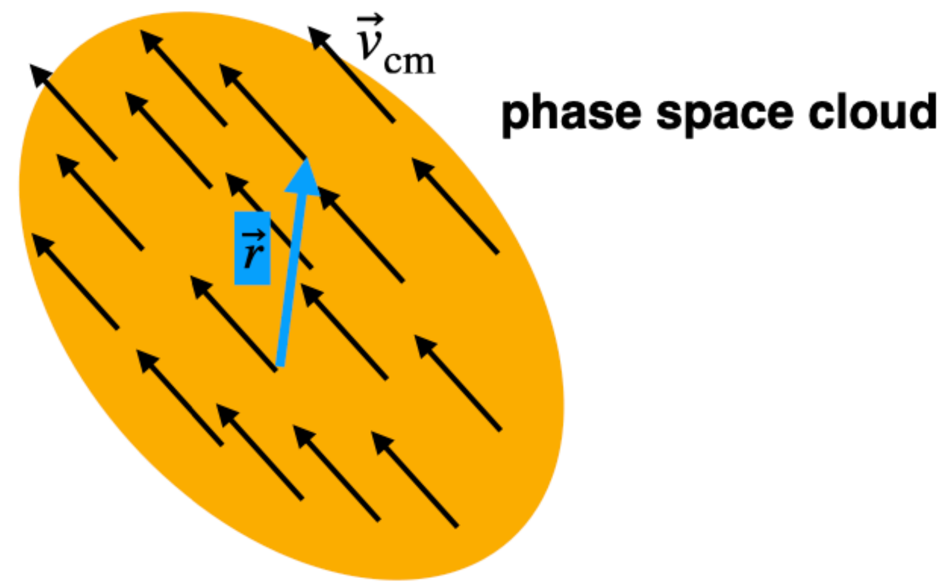
A. Sorensen, HZ et al.
arXiv:2301.13253 [nucl-th]
Prog.Part.Nucl.Phys. 134 (2024) 104080



Femtoscopy is a perfect tool to study two- and more particle interactions

Thank you

Smoothness approximation (from Scott Pratt)

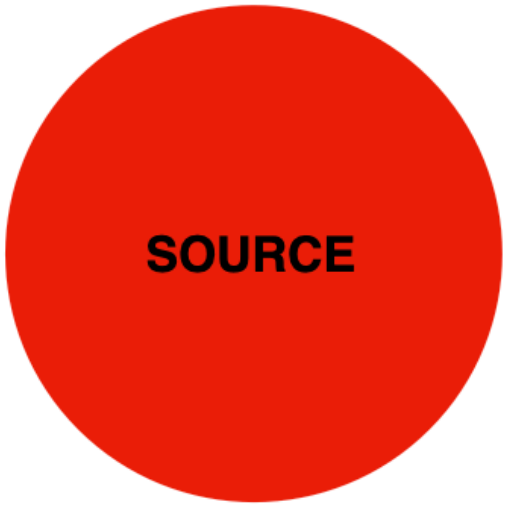


$$C(\vec{p}_1,\vec{p}_2) = \frac{P(\vec{p}_1,\vec{p}_2)}{P(\vec{p}_1)P(\vec{p}_2)}$$
$$= C(\vec{v}_{\rm cm},\vec{q})$$
$$= \int d^3r\, S(\vec{v}_{\rm cm},\vec{r})\, |\phi_{\vec{q}}(\vec{r})|^2,$$

source function — wave function

$$S(\vec{v}_{\rm cm},\vec{r}) = \frac{\int d^3r_1 d^3r_2\ f_{\rm cm}(\vec{v}_{\rm cm},\vec{r}_1,t) f_{\rm cm}(\vec{v}_{\rm cm},\vec{r}_2,t)\delta(\vec{r}_1-\vec{r}_2-\vec{r})}{\int d^3r_1 d^3r_2\ f_{\rm cm}(\vec{v}_{\rm cm},\vec{r}_1,t) f_{\rm cm}(\vec{v}_{\rm cm},\vec{r}_2,t)}$$

“SOURCE FUNCTION” measures phase space cloud, not source!!!

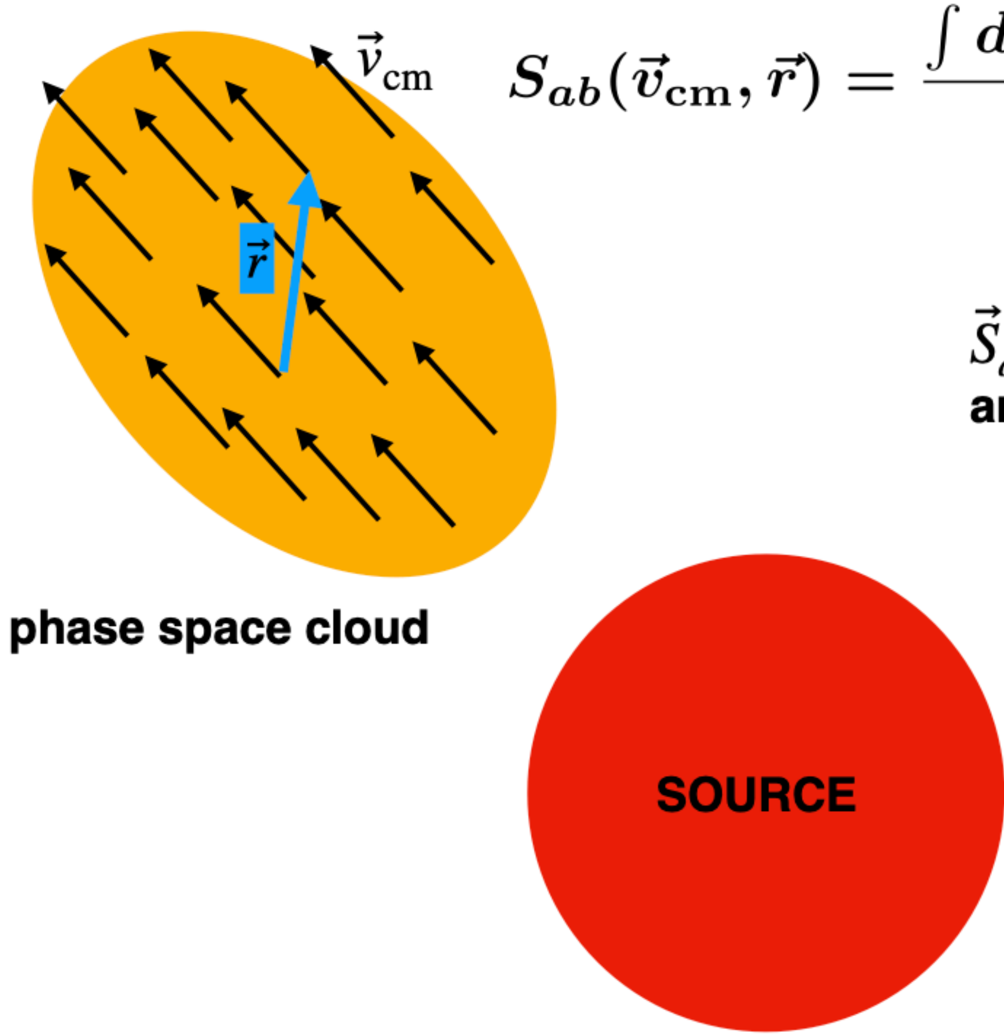


GOAL: Measure $C(\vec{p}_1,\vec{p}_2)$ **to infer** $S(\vec{v}_{\rm cm},\vec{r})$
For identical bosons: $|\phi|^2 = 1 + \cos(2\vec{q}\cdot\vec{r})$
Strong/Coulomb makes inversion more complicated

See S.Pratt talk during WPCF 2023
https://agenda.infn.it/event/33324/contributions/211990/attachments/112571/161133/wpcf_2023.pdf

Smoothness approximation (from Scott Pratt)

Both use same source function:



$$S_{ab}(\vec{v}_{\rm cm},\vec{r}) = \frac{\int d^3r_a d^3r_b\ f_a(\vec{v}_{\rm cm},\vec{r}_a,t) f_b(\vec{v}_{\rm cm},\vec{r}_b,t)\delta(\vec{r}_a-\vec{r}_b-\vec{r})}{\int d^3r_a d^3r_b\ f_a(\vec{v}_{\rm cm},\vec{r}_a,t) f_b(\vec{v}_{\rm cm},\vec{r}_b,t)}$$

$\vec{S}_{ab}(\vec{v}_{\rm cm},\vec{r})$ **is probability two particles of same** $\vec{v}$ **are separated by** $\vec{r}$

will visit later

"Smoothness" approximation:
Sometimes $\vec{v}_{\rm cm} \rightarrow \vec{v}_{\rm cm} \pm \delta\vec{v}$
$\delta\vec{v}$ sometimes related to Wigner transform of wave function,
sometimes from $\vec{p}_a - \vec{p}_b$,
sometimes just some finite width distribution for statistics…

Approximations

The Truth:

$$P(p_a, p_b) = \sum_{f'} \left| \int dx_a dx_b T_{f'}(x_a, x_b) \phi_{f'}(x_a, x_b; p_a, p_b,) \right|^2$$

Sum over all "remainder" states f'

APPROXIMATIONS

1) $\phi(x_a, x_b; p_a, p_b)$ does not depend on f'

a) fails if phase space density is high (identical particles)
multi-particle symmetrization is important
otherwise, must calculate for all momenta, then integrate over all other particles

b) fails if interaction with other particles lasts long time
at sufficiently small relative momentum, this is fine
Coulomb with other particles slowest other interaction

$$\phi_f(x_a, x_b; p_a, p_b) \to \phi(x_a - x_b; p_a, p_b)$$

After first approximation:

$$P(p_a, p_b) = \sum_{f'} \left| \int dx_a dx_b T_{f'}(x_a, x_b) \phi(x_a, x_b; p_a, p_b,) \right|^2$$

APPROXIMATIONS

2) Emission ($T-$matrix) is independent.
Sum over f' and $T-$matrices must factorize
Ignores other correlations (energy/momentum/charge) conservation…
good at small relative momentum (other sources have longer characteristic scales)

$$\sum_{f'} \to \sum_{f'_a} \sum_{f'_b},$$

$$T_{f'}(x_a, x_b) \to T_{f'_a}(x_a) T_{f'_b}(x_b)$$

Approximations

After approximations 1 & 2:

S.P. PRC 1997

Define:

$$s_a(x,p) \equiv \sum_{f'_a} \int d\delta x \; T^*_{f'_a}(x+\delta x/2) T_{f'_a}(x-\delta x/2) e^{ip\cdot\delta x}$$

this gives

$$P_{ab}(p_a,p_b) = \int dx_a dx_b d\delta x d\tilde{q} s_a(\bar{P}_a + \tilde{q}, x_a) s_b(\bar{P}_b - \tilde{q}, x_b)$$
$$e^{i\tilde{q}\delta x} \phi^*_q(x_a - x_b + \delta x/2) \phi^*_q(x_a - x_b - \delta x/2)$$

3) Smoothness approximation:

a) Ignore $\tilde{q}$ dependence in $s_a(\bar{P}_a + \tilde{q}, x_a)$ and $s_b(\bar{P}_b - \tilde{q}, x_b)$,

b) replace $s_a(\bar{P}_a + \tilde{q}, x_a) s_b(\bar{P}_b + \tilde{q}, x_b)$ with $s_a(p_a, x_b) s_b(p_b, x_b)$ or $s_a(E_a, \vec{p}_{a,\rm cm} s_b(E_b, \vec{p}_{b,\rm cm})$

— good when emissions are thermal and matrices are broad
— questionable if relative momentum is small
— necessary if you don't know off-shell behavior of $s(p,x)$
— for coalescence you can add $e^{B/T}$ factor

Last approximation: non-simultaneous wave functions

$\phi_q(x_1 - x_2) = \phi(\Delta t = 0, \vec{x}_1 - \vec{x}_2)$ **in pair frame**

4) Non-simultaneous emission
— no problem for pure HBT
— should be fine for small relative momentum

Sometimes interactions involve change of degrees of freedom:
No interaction through potential

Examples:
$K^+K^- \leftrightarrow \phi$
$\alpha d \leftrightarrow {}^6\mathrm{Li}$

Wavefunction paradigm questionable — but thermal equilibrium still applies

When are the approximations good?

<u>Femtoscopy</u>:
— Emission uncorrelated aside from FSI
— Relative motion is small, $q/\mu \lesssim 0.1$
— Phase space density not high
(as long as phase space densities $\lesssim 0.5$)
— Range of interaction smaller than source size
— Rearrangement interactions, e.g. $K^+K^- \leftrightarrow \phi$
where wave function paradigm is questionable

<u>Coalescence</u>:
— Same as above
— Wave function should not have high p components (low B)
— Should correct for binding energy: $e^{B/T}$

<u>Thermal</u>:
— Must be at freeze out!!
— Whenever wave function extent is $<<$ source size
— OK with rearrangement interactions

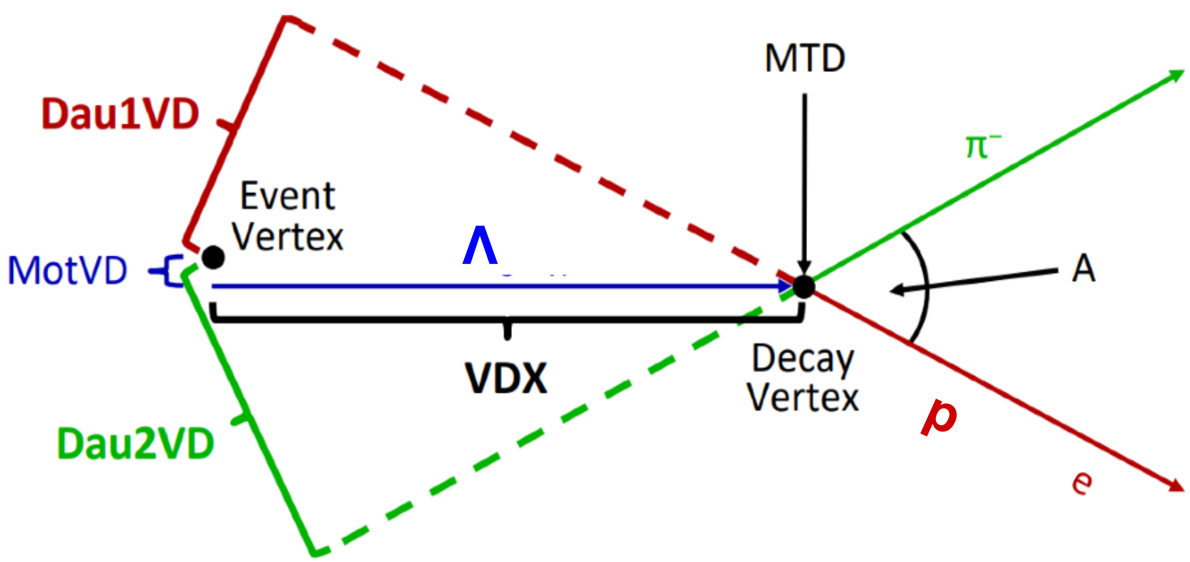
Central H.I. Collisions:
— Usually very solid

***pp* Collisions:**
— Be more careful

Rearrangment interactions
— Be careful

Signal reconstruction

Weak decay



Schematic depiction of the Off-Vertex-Decay-Topology of Λ decays.

- Distance of closest approach (DCA) between the daughter tracks and the primary vertex,

→ **Dau1VD = > 8 mm**

→ **Dau2VD = > 24 mm**

- DCA between reconstructed mother track and primary vertex (**Mot-VD**) = **< 5 mm**

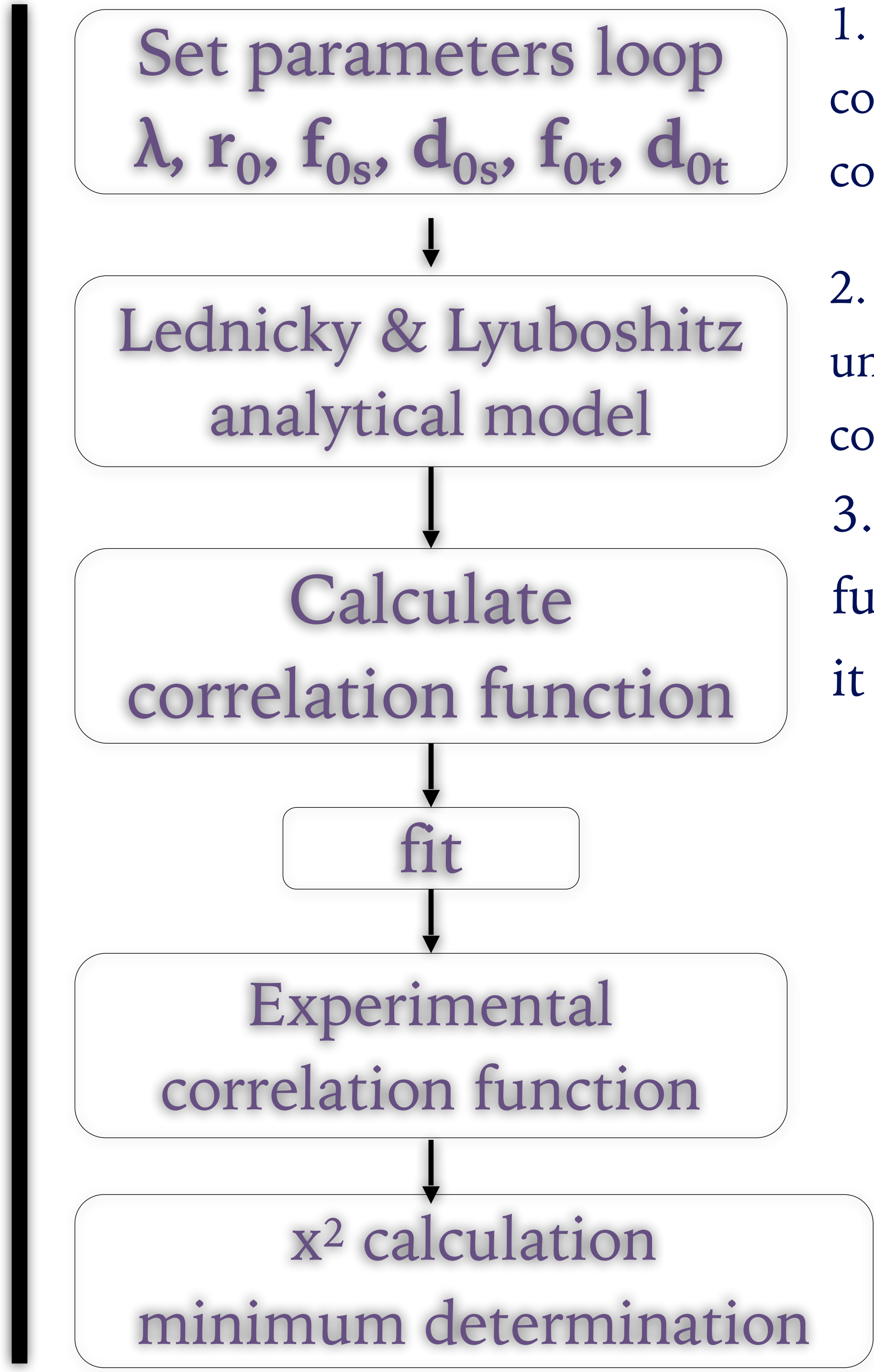
- Distance between the primary and secondary vertex (**VDX**) = **> 65 mm**

- DCA between the two daughter tracks (**MTD**)

= **< 6 mm**

- Opening angle between the two daughter tracks (**A**) = **> 15 ^{o}**

Lednicky-Lyuboshitz model



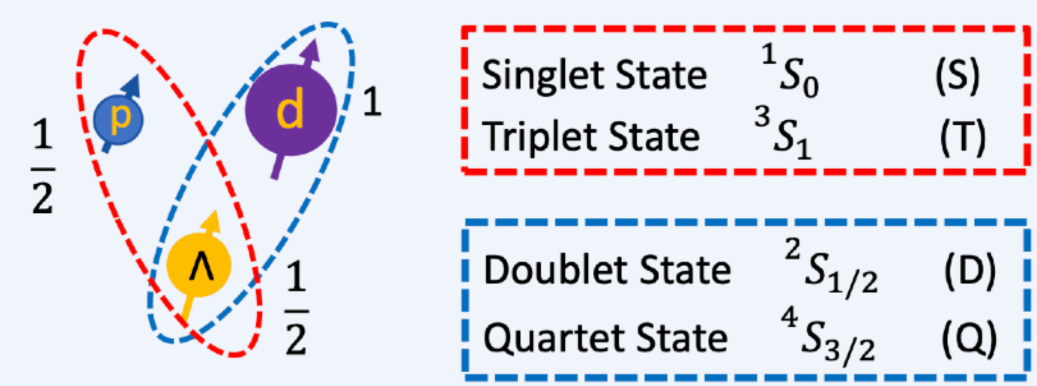
1. The Lednicky-Luboshitz semi-analytical model (utilized in CorrfitCumac codes) provides an immediate correlation function value but may be computationally intensive due to integral calculations.

2. The first fitter employs ROOT minimizers, offering precise statistical uncertainty estimation, but it operates on "continuous" maps with limited control over parameter steps.

3. The second fitter, Hal:Minimizer, accommodates "non-continuous" functions, allowing parameters to change in discrete steps. However, it provides only approximate uncertainty estimates.

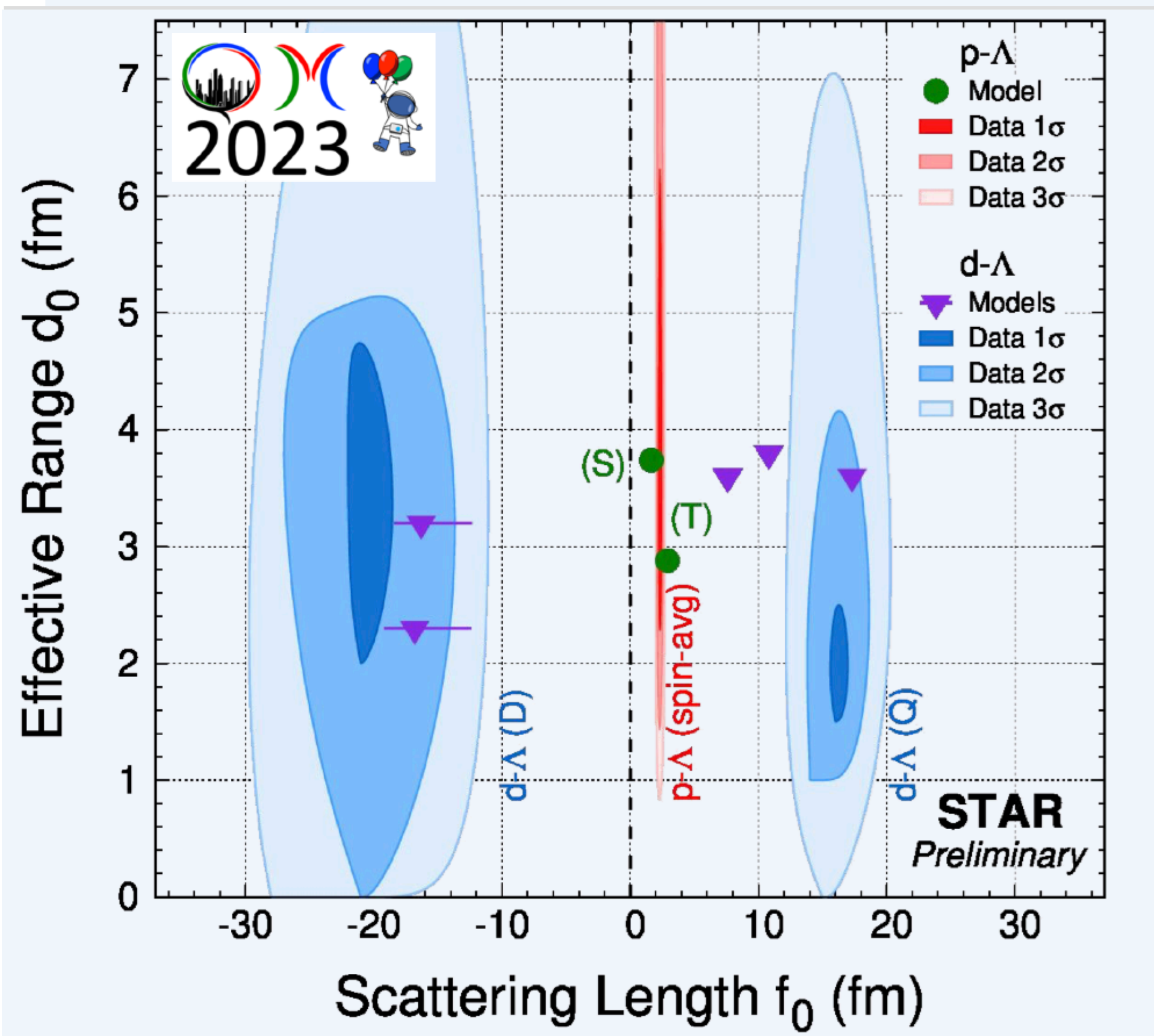
```
for( int λ = 0.6; λ < 0.8; λ+=0.1 )
    for( int r0 = 1.0; r0 < 4.0; r0+=0.1 )
        for( int f0 = 0.01; f0 < 5.0; f0+=0.1 )
            for( int d0 = 0.01; d0 <5.0; d0+=0.1)
                for( int ft = 0.01; ft < 5.0 ; ft += 0.1 )
                    for( int dt =0.01; dt < 5.0; dt+=0.1 )

                        Calculate Lednicky-Luboshitz
                            correlation function : fit data
                        χ2 : value is extracted : minimizer
```

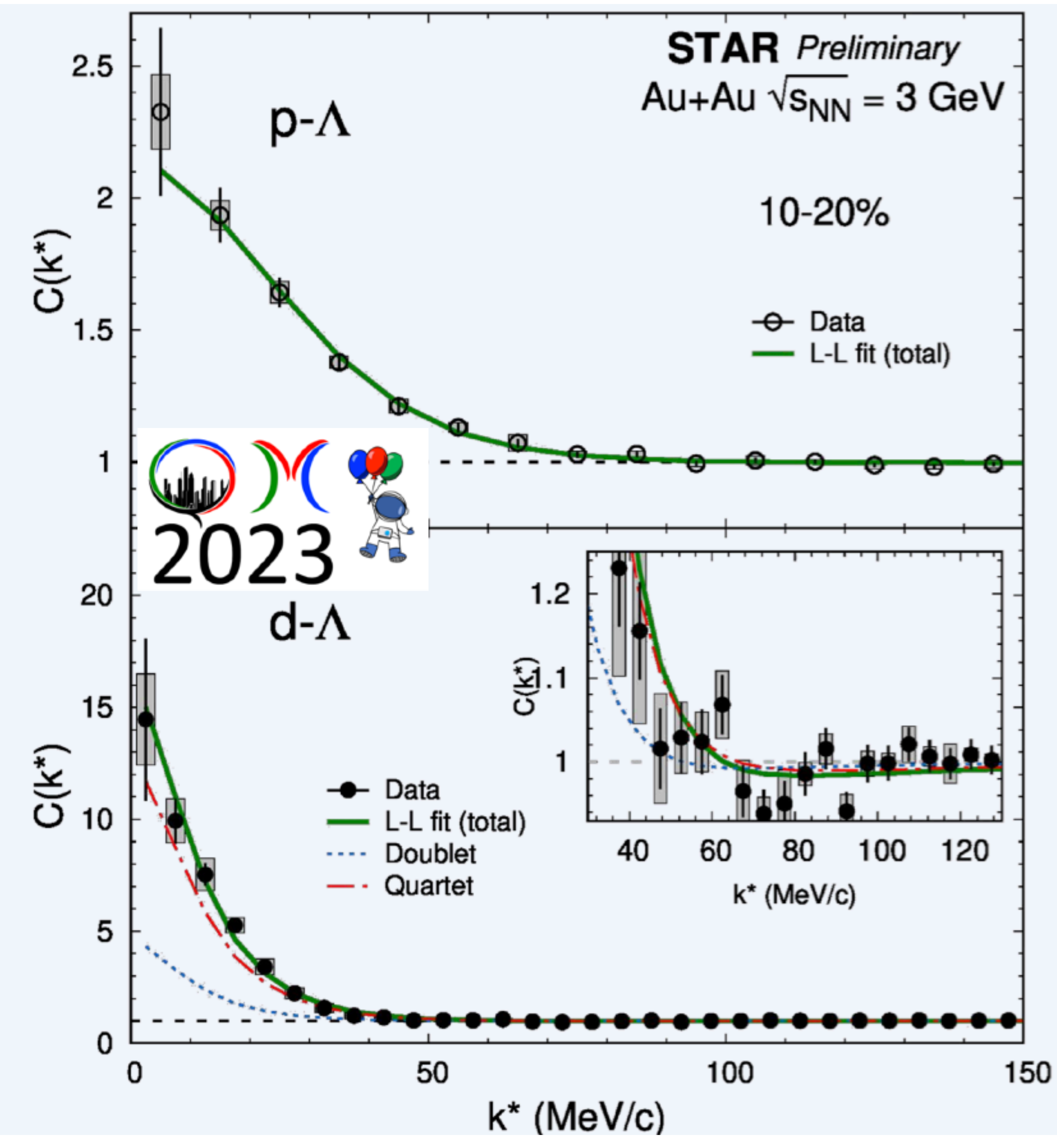
YN interactions at STAR



p-Λ: $|\psi(r,k)|^2 \rightarrow \frac{1}{4}|\psi_0(r,k)|^2 + \frac{3}{4}|\psi_1(r,k)|^2$

d-Λ: $|\psi(r,k)|^2 \rightarrow \frac{1}{3}|\psi_{1/2}(r,k)|^2 + \frac{2}{3}|\psi_{3/2}(r,k)|^2$





Different spin states with different FSI parameters

p-Λ correlation: currently spin-averaged fit

d-Λ correlation: spin-separated fit