

Perspectives of electroweak transition form factor studies with hyperons

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Understanding the strong interaction

model-independent methods to explore QCD (and in general QFT):

- perturbative QCD
 - works at high energies where strong interaction is weak
- lattice QCD
 - works best around Λ_{QCD} , m_s (hadronic scale ≈ 1 GeV)
 - light pion sees itself around the torus *if* volume is too small
 - but advantage: quark masses can be varied
- (chiral) effective field theory \rightsquigarrow works at low energies
- dispersion theory \rightsquigarrow works if there are only a few channels

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- experiment! \rightsquigarrow but quark masses fixed

Play it again, Sam

- we are all interested in exploring strongly interacting matter
- ↪ composite structures formed by quarks and gluons
- ↪ form factors are introduced to characterize composite objects



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- ↪ but we have heard all this several times ...



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- here is another motivation



Neutrino physics

- explore neutrino oscillations, CP violation in lepton sector, . . .
- want to count how many neutrinos one has
(of specific type and at specific distance)



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Neutrino physics

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- ↪ need to know how many react with matter (nuclei!)
 - at high energies: neutrinos “see” quarks
 - ↪ perturbative QCD
 - at lowest energies: neutrinos “see” nuclei
 - ↪ nuclear-structure physics
 - at low energies: neutrinos see “nucleons”
 - ↪ vector and axial-vector form factors of nucleon and pion production! \rightsquigarrow hyperons and $\Delta(1232)$

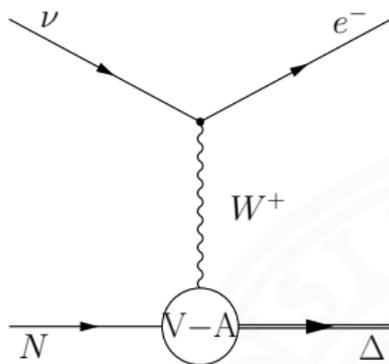
Neutrino physics

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 - ↪ vector and axial-vector form factors of nucleon and pion production! \rightsquigarrow hyperons and $\Delta(1232)$
- true for all energies: need to get stuff out from the nucleus
- ↪ transport theory \rightsquigarrow ask Ulrich Mosel

Why are Δ s and hyperons interesting?

Axial-vector (and vector) transition form factors

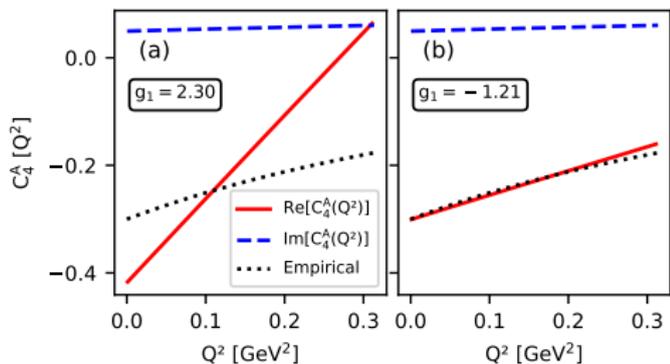
- interesting for scattering neutrino-nucleon to electron- Δ or muon- Δ or hyperon (Y) instead of Δ
- subsequently: $\Delta, Y \rightarrow \pi N$



NuSTEC Collaboration, L. Alvarez-Ruso et al., Prog. Part. Nucl. Phys. 100 (2018) 1;
 M. Hilt, T. Bauer, S. Scherer, L. Tiator, Phys. Rev. C 97 (2018) 3, 035205;
 M. Holmberg, SL, Phys. Rev. D 100 (2019) 11, 114001;
 Y. Ünal, A. Küçükarslan, S. Scherer, Phys. Rev. D 104 (2021) 9, 094014;
 S.K. Singh, M.J. Vicente Vacas, Phys. Rev. D 74 (2006) 053009

Δ couplings and Δ - N transition form factors

- coupling constant Δ - N - π known from decay width of $\Delta \rightarrow \pi N$
- but coupling constant Δ - Δ - π (g_1) unknown in size and sign
- (actually there are two couplings, p and f wave \rightsquigarrow here: p -wave coupling)



one of the axial-vector transition form factors; g_1 enters indirectly at loop level

Y. Ünal, A. Küçükarslan, S. Scherer, Phys. Rev. D 104 (2021) 9, 094014

- quark model and large- N_c QCD suggest **positive** g_1 (figure: left-hand side)
- recent analysis of π - N scattering suggests **negative** g_1
 \hookrightarrow ask Evgeny Epelbaum

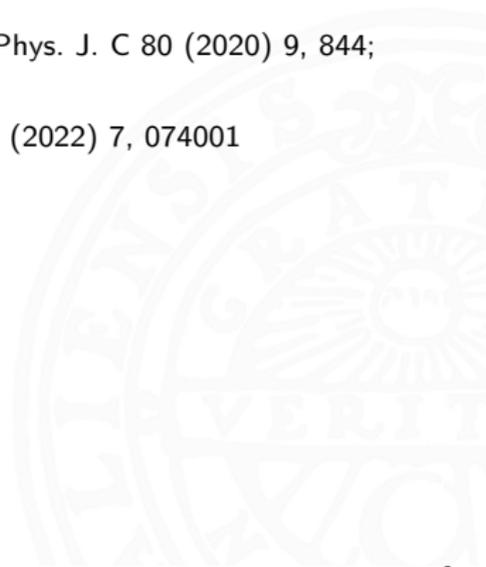
De-Liang Yao *et al.*, JHEP 05 (2016) 038

How about lattice QCD?

- currently under investigation using lattice QCD:
 - form factors of [stable baryons](#)
 - and their quark-mass dependence
 - interpretation of results by chiral effective field theory
- ↪ ask Matthias Lutz

M.F.M. Lutz, U. Sauerwein, R.G.E. Timmermans, Eur. Phys. J. C 80 (2020) 9, 844;
Phys. Rev. D 105 (2022) 5, 054005

see also F. Alvarado, L. Alvarez-Ruso, Phys. Rev. D 105 (2022) 7, 074001



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- but for transition form factors Δ - N :
 - much more complicated because Δ is **unstable**
 - ↪ essentially four-point function instead of three-point function
 - and even more complicated for coupling constant Δ - Δ - π

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- but for transition form factors Δ - N :
 - much more complicated because Δ is **unstable**
 - ↪ essentially four-point function instead of three-point function
 - and even more complicated for coupling constant Δ - Δ - π
- to circumvent problem: study (nearly) **stable** flavor partners
 - Ω - Ξ transition form factors
 - Ω - Ξ^* - K coupling constant
 - ↪ related to Ω - Ξ^* transition by Goldberger-Treiman relation

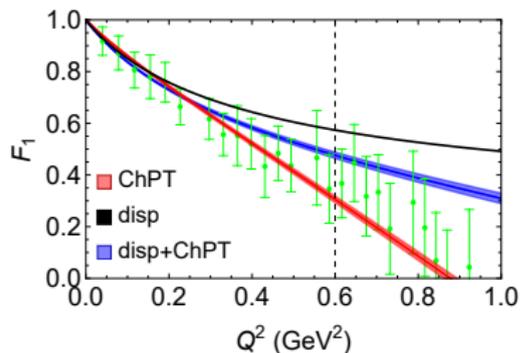
My suggestion for a research program

- 1 study Ω transition form factors in lattice QCD and experiment (and quark-mass dependence on lattice)
- 2 interpret results using (dispersively modified) chiral effective field theory ($\text{dim}\chi\text{EFT}$)
- 3 extrapolate to Δ transition form factors using ($\text{dim}\chi\text{EFT}$ and experimental data (**hyperons**))
- 4 obtain improved input for neutrino scattering (and obtain better understanding of structure of hadrons)

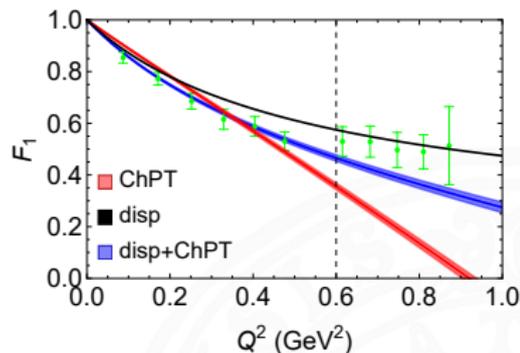
UU contribution: develop $\text{dim}\chi\text{EFT}$

Does all this make sense?

quark-mass and momentum dependence of nucleon Dirac form factor



$$M_\pi = 0.130 \text{ GeV}$$



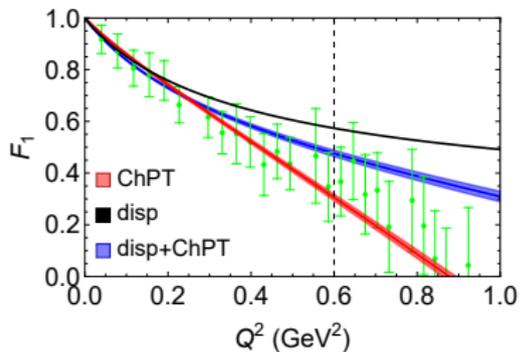
$$M_\pi = 0.223 \text{ GeV}$$

F. Alvarado, D. An, L. Alvarez-Ruso, SL, Phys. Rev. D 108 (2023) 11, 114021

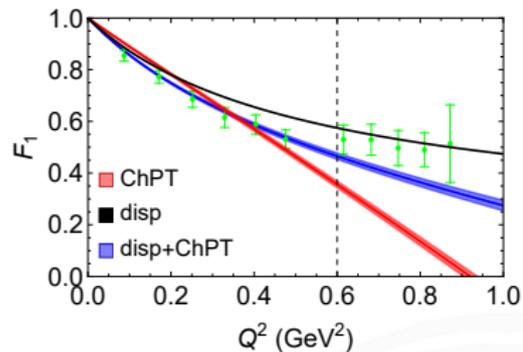
lattice data from Darmstadt-Edinburgh-Mainz group:

D. Djukanovic et al., Phys. Rev. D 103 (2021) 9, 094522

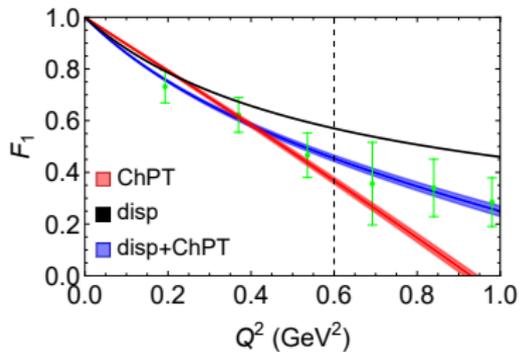
Dirac vector isovector form factor of nucleon



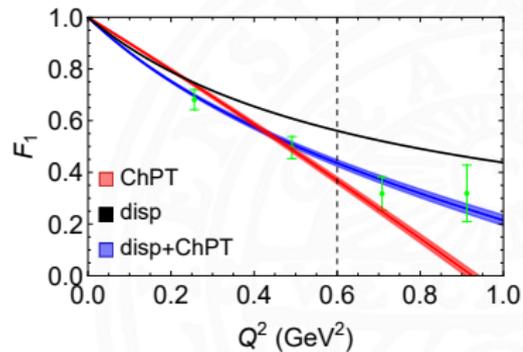
$M_\pi = 0.130$ GeV



$M_\pi = 0.223$ GeV



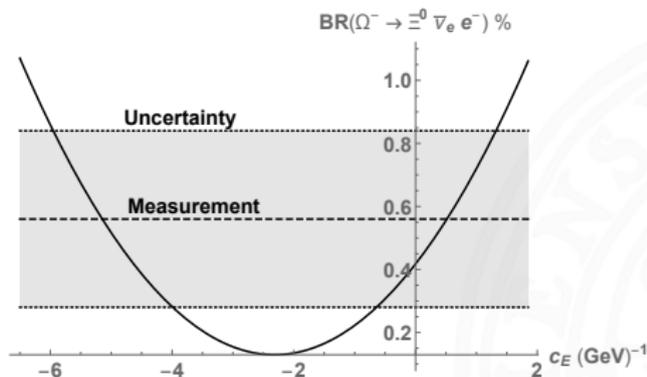
$M_\pi = 0.278$ GeV



$M_\pi = 0.353$ GeV

Branching ratio $\Omega^- \rightarrow \Xi^0 e^- \bar{\nu}_e$ (measured)

- so far only next-to-leading-order (NLO) calculation finished
 - contribution from LO Lagrangian ($\sim h_A$) related to $\Sigma^* \rightarrow \Sigma \pi$
 - contributions from NLO Lagrangian $\sim c_M, c_E$
- $\rightarrow |c_M|$ related to $\Sigma^{*0} \rightarrow \Lambda \gamma$
- \rightarrow get constraints on c_E from measured branching ratio:

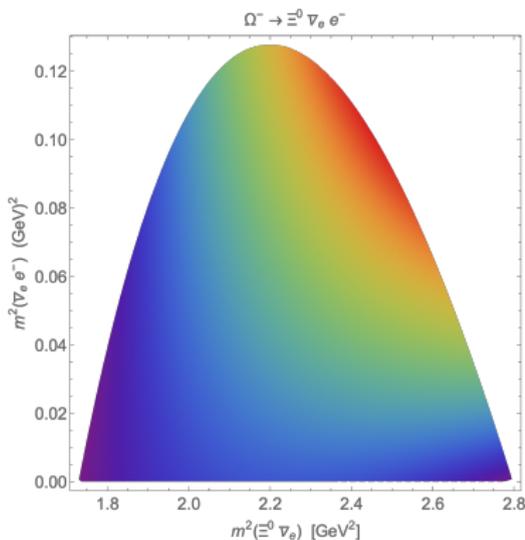


M. Holmberg, SL, Eur. Phys. J. A 54 (2018) 6, 103; Phys. Rev. D 100 (2019) 11, 114001
 C.J.G. Mommers, SL, Phys. Rev. D 106 (2022) 9, 093001

first steps beyond tree level: H. De Munck; M. Bertilsson, master theses UU 2023

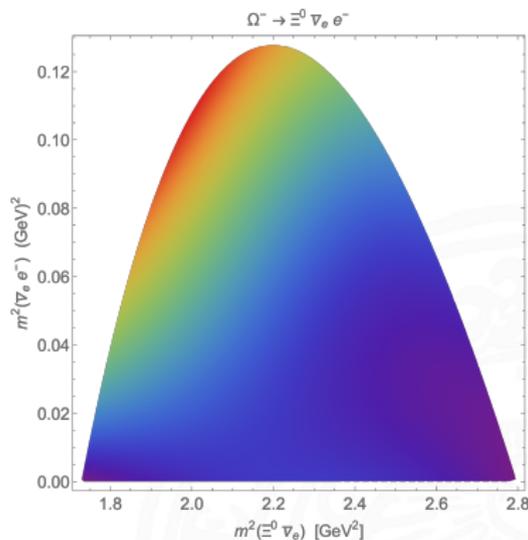
Dalitz plot $\Omega^- \rightarrow \Xi^0 e^- \bar{\nu}_e$ (not measured yet)

- different values for c_E and sign of c_M influence Dalitz plot:



$$c_E = 0.52 \text{ GeV}^{-1},$$

$$c_M = -1.92 \text{ GeV}^{-1}$$



$$c_E = -5.1 \text{ GeV}^{-1},$$

$$c_M = -1.92 \text{ GeV}^{-1}$$

- sign change of c_M flips plots right \leftrightarrow left

Decay $\Omega^- \rightarrow \Xi^{*0} \ell^- \bar{\nu}_\ell$ (not measured yet)

- provides access to **sign** and **size** of coupling constant $\Omega\text{-}\Xi^*(1530)\text{-}K$ via Golberger-Treiman relation
- flavor related to **sign** and **size** of coupling constant $\Delta\text{-}\Delta\text{-}\pi$ ($H_A = g_1$)
- so far only leading-order calculation for **branching ratio** and **forward-backward** (fb) asymmetry (Wu-type experiment)
(rest frame of dilepton, measuring angle between baryons and charged lepton)

	$\Gamma_{\Omega \rightarrow \Xi^* \ell \bar{\nu}_\ell} / \Gamma_{\Omega, \text{tot}}$	$\Gamma_{\text{fb}} / \Gamma_{\Omega \rightarrow \Xi^* \ell \nu}$
$\ell = e, H_A = +2$	$1.2 \cdot 10^{-4}$	$+0.011$
$\ell = e, H_A = 0$	$6.7 \cdot 10^{-5}$	-0.00043
$\ell = e, H_A = -2$	$1.2 \cdot 10^{-4}$	-0.012
$\ell = \mu, H_A = +2$	$4.3 \cdot 10^{-6}$	-0.23
$\ell = \mu, H_A = 0$	$2.5 \cdot 10^{-6}$	-0.33
$\ell = \mu, H_A = -2$	$4.3 \cdot 10^{-6}$	-0.25

- note: $\Xi^{*0}(1530)$ “easy” to reconstruct via sequence
 $\Xi^{*0} \rightarrow \Xi^- \pi^+, \Xi^- \rightarrow \Lambda \pi^-, \Lambda \rightarrow p \pi^-$

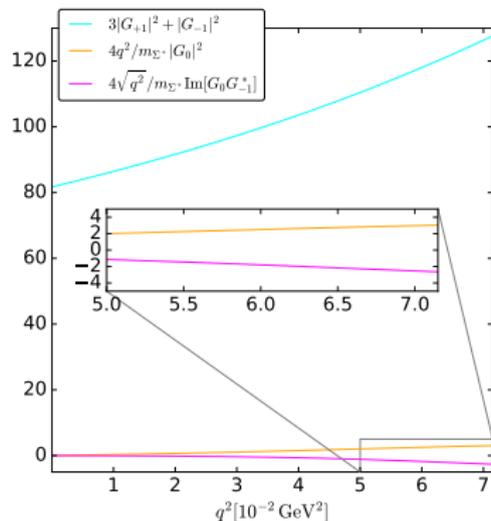
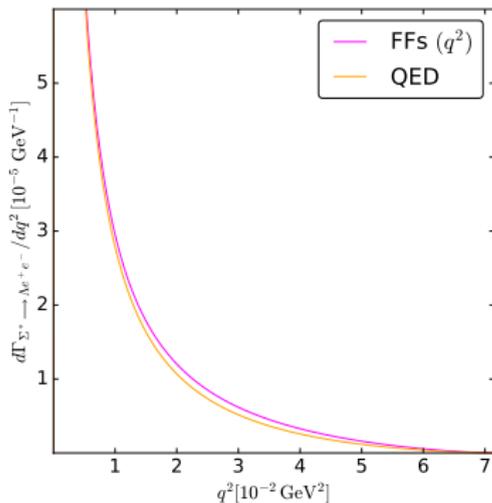
Radiative decays of $\Xi^*(1530)$ and $\Sigma^*(1385)$

- our calculation so far only tree level (NLO)
- our **predictions** in boldface
- blue: measured — red: predicted, but not measured yet

decay	$c/(c_M e)$	BR [%]	$c_M [\text{GeV}^{-1}]$
$\Delta \rightarrow N\gamma$	$2/\sqrt{3}$	0.60 ± 0.05	2.00 ± 0.03
$\Sigma^{*+} \rightarrow \Sigma^+\gamma$	$-2/\sqrt{3}$	0.70 ± 0.17	1.89 ± 0.08
$\Sigma^{*-} \rightarrow \Sigma^-\gamma$	0	< 0.024	—
$\Sigma^{*0} \rightarrow \Sigma^0\gamma$	$1/\sqrt{3}$	0.18 ± 0.01	—
$\Sigma^{*0} \rightarrow \Lambda\gamma$	-1	1.25 ± 0.13	1.89 ± 0.05
$\Xi^{*0} \rightarrow \Xi^0\gamma$	$-2/\sqrt{3}$	4.0 ± 0.3	—
$\Xi^{*-} \rightarrow \Xi^-\gamma$	0	< 4	—

Form factors in $\Sigma^{*0}(1385) \rightarrow \Lambda e^+ e^-$ (not measured yet)

- our method: dispersion relation (unsubtracted in lack of data)
O. Junker, SL, E. Perotti, T. Vitos, Phys. Rev. C 101 (2020) 1, 015206
- decay width about 3 keV



relevant combinations for

$$\Sigma^{*0}(1385) \rightarrow \Lambda e^+ e^- \rightarrow p \pi^- e^+ e^-$$

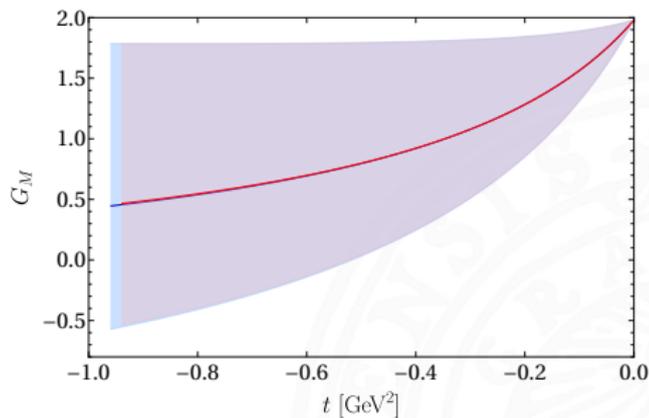
Magnetic form factor in $\Sigma^0 \rightarrow \Lambda e^+ e^-$ (not measured yet)

- our method: dispersion theory with 2-pion intermediate states

C. Granados, SL, E. Perotti, Eur. Phys. J. A 53 (2017) 6, 117

- extended to coupled channels: 2 pions and 2 kaons

Y.-H. Lin, H.-W. Hammer, U.-G. Meißner, Eur. Phys. J. A 59 (2023) 3, 54



↪ ask Hans-Werner Hammer

- prediction can be drastically improved by “just” measuring slope at $t = q^2 \approx +0$, i.e. from $\Sigma^0 \rightarrow \Lambda e^+ e^-$
- cross relation to transition form factors N - Λ and N - Σ

Summary and Outlook

- D** elta transition form factors are interesting for neutrino physics
- O** mega transition form factors are a better starting point
- L** attice QCD can tackle these (plus experimental guidance)
- C** hiral effective field theory can interpret results
- E** xtrapolation to Delta (EFT plus experimental guidance)



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DOLCE

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↪ **DOLCE**

interesting hyperon decays:

- $\Omega^- \rightarrow \Xi^0 \ell^- \bar{\nu}$

- $\Xi^{*0} \rightarrow \Xi^0 \gamma$

- $\Sigma^{*0} \rightarrow \Lambda e^+ e^-$

- $\Omega^- \rightarrow \Xi^{*0} \ell^- \bar{\nu}$

- $\Sigma^{*0} \rightarrow \Sigma^0 \gamma$

- $\Sigma^0 \rightarrow \Lambda e^+ e^-$

Spare slides

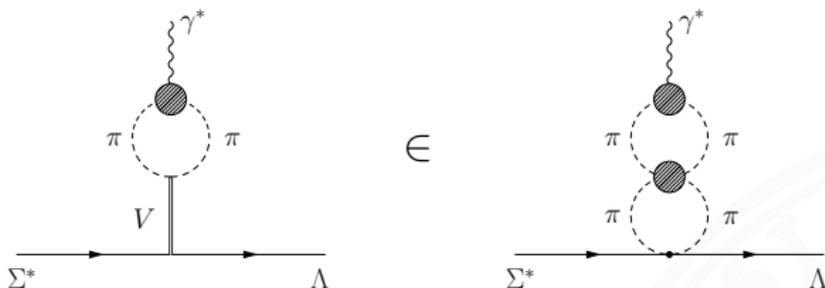


Form factors in $\Sigma^{*0}(1385) \rightarrow \Lambda e^+ e^-$ (not measured yet)

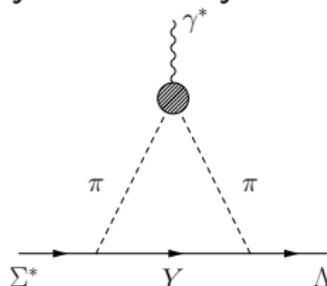
- our method: dispersion relation (unsubtracted in lack of data)

O. Junker, SL, E. Perotti, T. Vitos, Phys. Rev. C 101 (2020) 1, 015206

- ρ meson is included via pion phase shift (model independent)



- “our” triangles with baryons are beyond vector-dominance model



Unitarity and analyticity

- constraints from **local quantum** field theory: partial-wave amplitudes for reactions/decays must be
 - unitary**:

$$S S^\dagger = 1, \quad S = 1 + iT \quad \Rightarrow \quad 2 \operatorname{Im} T = T T^\dagger$$

↪ note that this is a matrix equation:

$$\operatorname{Im} T_{A \rightarrow B} = \sum_X T_{A \rightarrow X} T_{X \rightarrow B}^\dagger$$

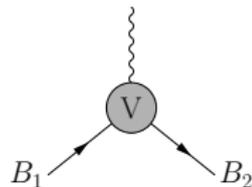
- analytic** (**dispersion relations**):

$$T(s) = \frac{1}{\pi} \int_{-\infty}^{\infty} ds' \frac{\operatorname{Im} T(s')}{s' - s - i\epsilon}$$

- ↪ can be used to calculate whole amplitude from imaginary part
 - practical limitation: too many states X at high energies
- ↪ in practice dispersion theory is a low-energy method ($\lesssim 1 \text{ GeV}$) or use resonance saturation

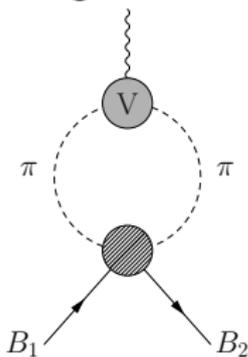
Example: electromagnetic baryon form factors

- how to obtain a form factor?



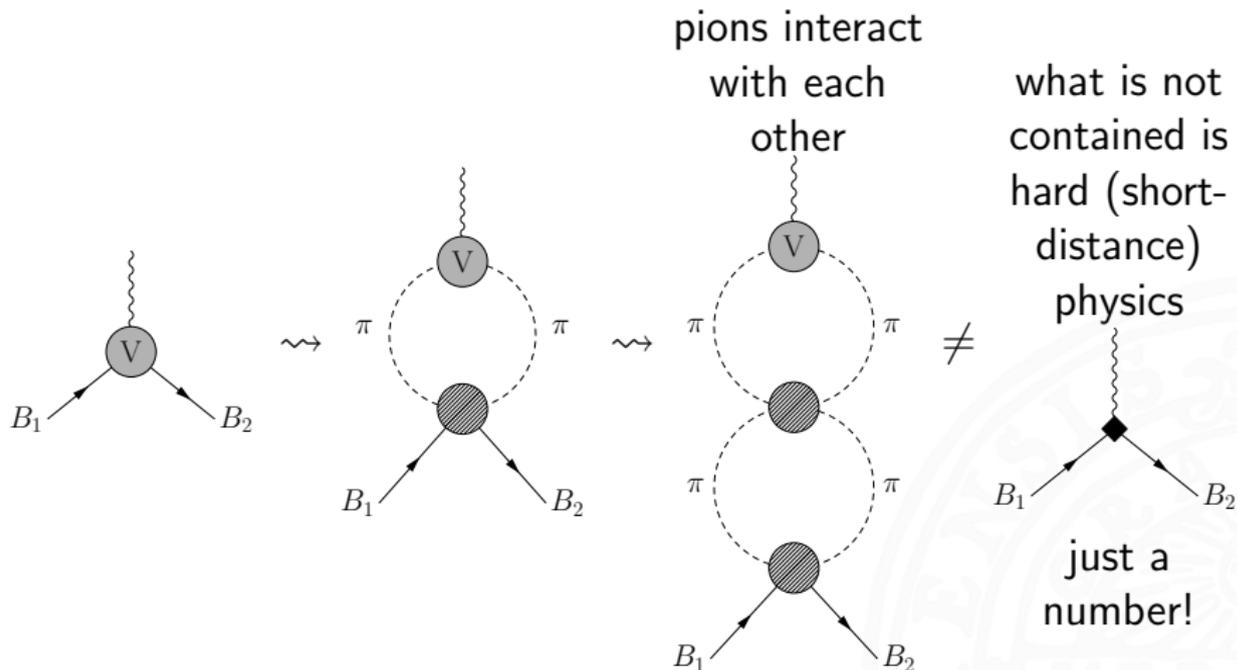
- need to resolve at least the finite size $\lesssim 1$ fm
- but inverse size of a hadron is larger than pion mass
- first one probes something universal (independent of $B_{1,2}$):

the “pion cloud”:



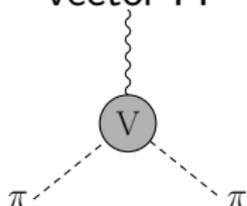
- now we are in the game with dispersion theory

Deconstruct a form factor



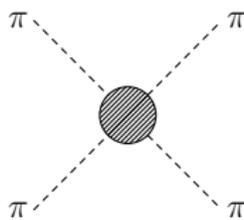
How to get the pion vector form factor?

apply same
logic to pion
vector FF

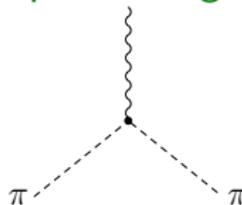


input:

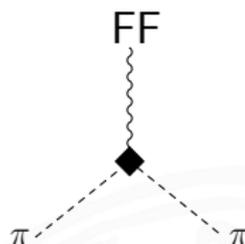
pion
scattering



pion charge



hard part of
pion vector
FF

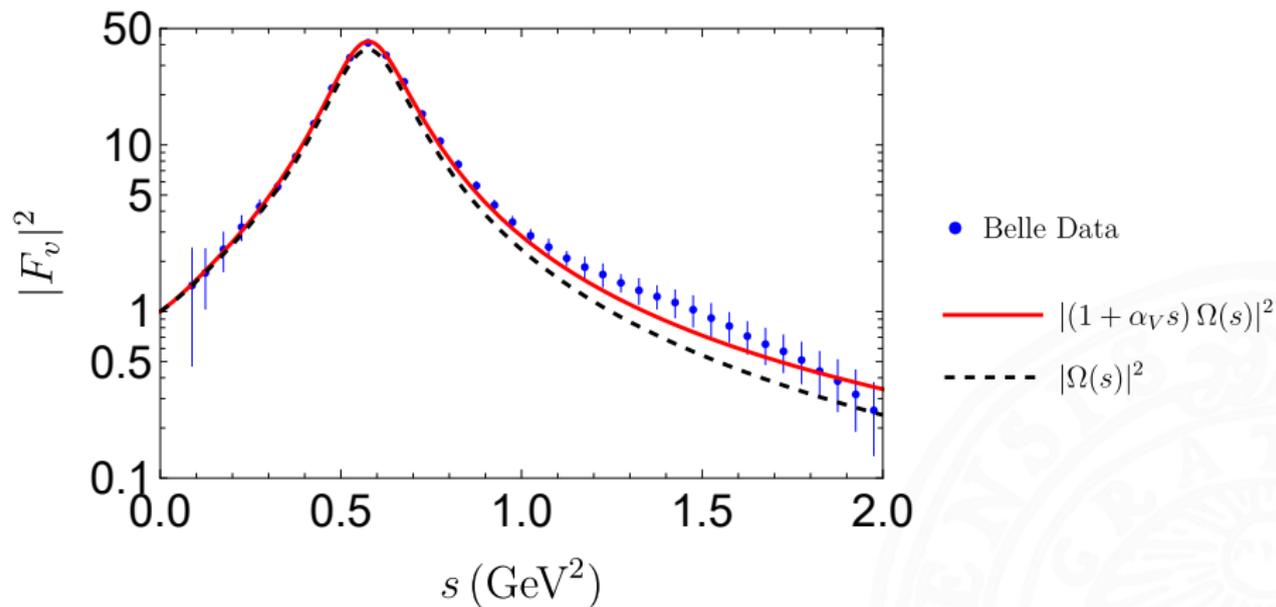


$$F_V(s) = (1 + \alpha_V s) \exp \left\{ s \int_{4m_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{\delta(s')}{s'(s' - s - i\epsilon)} \right\}$$

with pion phase shift δ

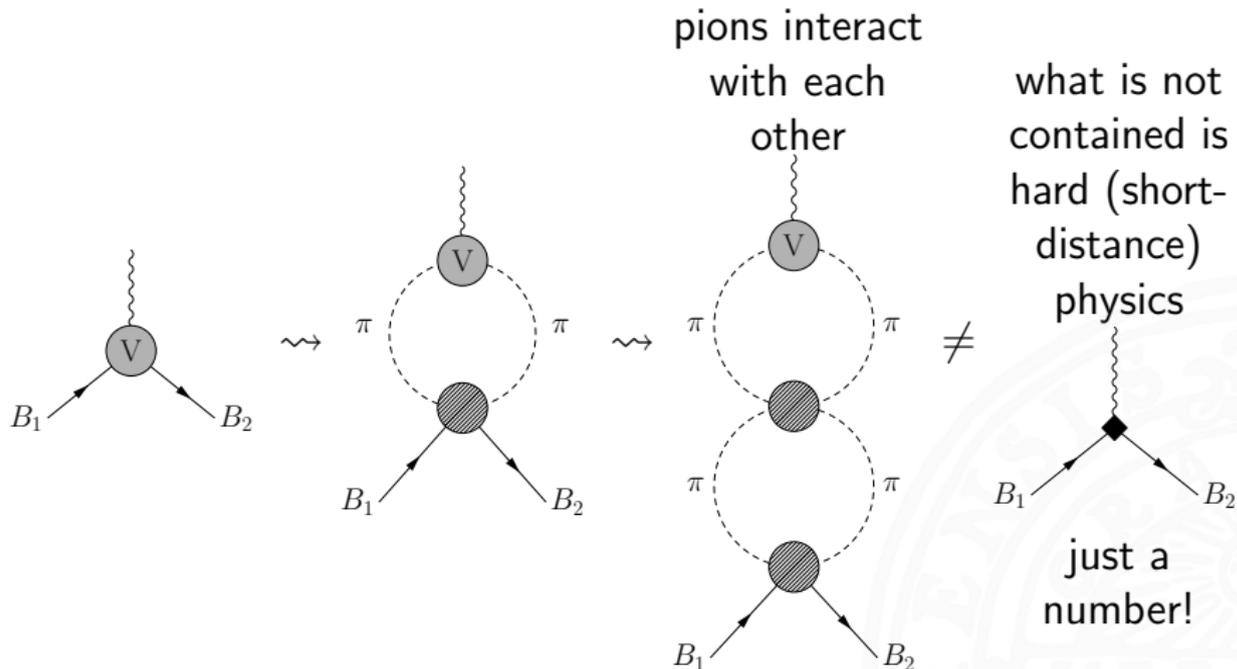
and $\alpha_V \approx 0.12 \text{ GeV}^{-2}$ (from fit to FF data)

Pion vector form factor and data



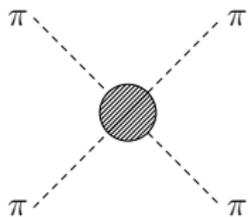
Alvarado/An/Alvarez-Ruso/SL, Phys. Rev. D 108 (2023) 11, 114021

Deconstruct a form factor

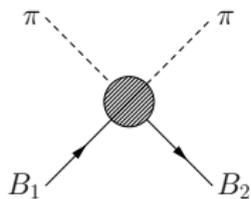


Scattering processes

from data
plus
dispersion
theory:

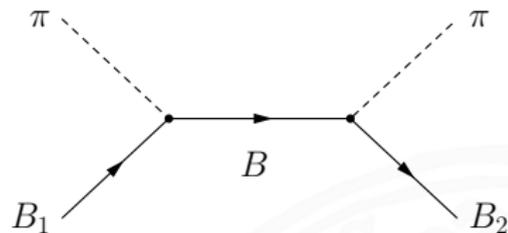


part that is
not pion
rescattering:

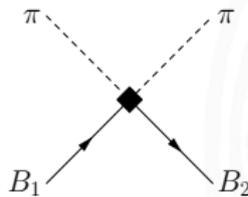


\rightsquigarrow

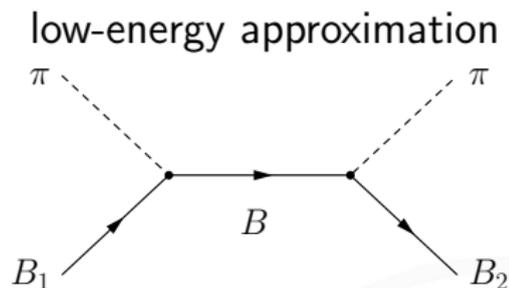
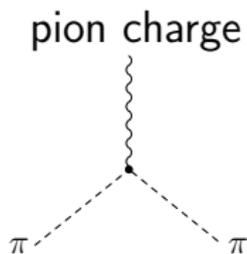
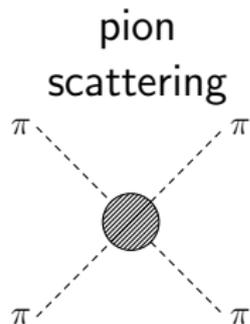
low-energy approximation:



what is not covered is hard physics
(contact terms)



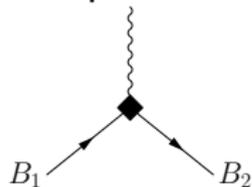
Known input



- baryon-pion coupling constants from decay widths
- ↪ sometimes only moduli known

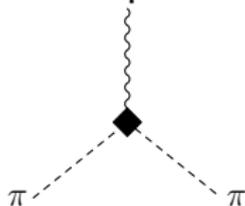
Unknown: some numbers

part without
two
intermediate
pions



↪ fit to data
(now)

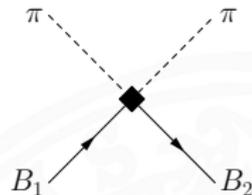
intermediate
state with
more than
two pions



or

calculate with quark-gluon based methods
(future)

intermediate
state that is
not one
baryon



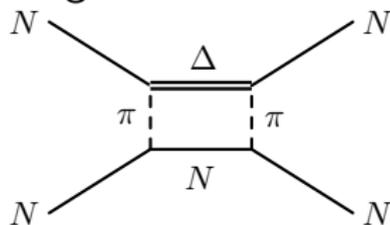
What can we learn here?

- the long-distance part is universal
 - ↪ needs to be understood once, not always new for each process
 - ↪ a lot is fixed by (chiral) symmetries
- the short-distance part is process dependent and sensitive to the details (dynamics) of QCD
- obtain information by combining hadron based effective field theories + dispersion theory (parametrization of short-distance physics) with quark based methods (determination of parameter values)

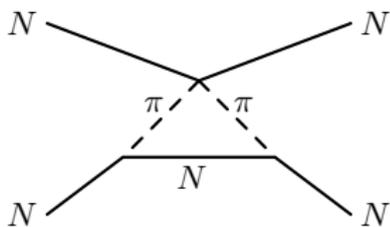
Importance of Δ in low-energy QCD?

consider nucleon-nucleon scattering:

- is it necessary to consider

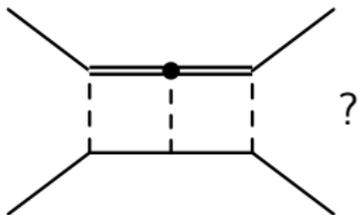


or is



sufficient?

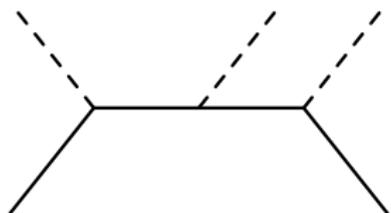
- how important is



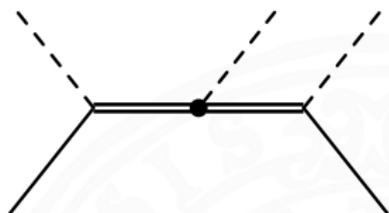
- coupling constant Δ - Δ - π • unknown!

What is unknown for Δ baryons?

- coupling constant Δ - Δ - π ● completely unknown; not even sign is clear
- further example: is interference of



and



constructive or destructive?

- one problem to determine Δ properties with better precision: Δ s are rather broad states ($\Gamma \approx 100$ MeV)
- ↪ worth to study strange siblings from decuplet
- ↪ e.g. Ω - Ξ^* - K coupling and related $\Omega \rightarrow \Xi^* \ell \nu$