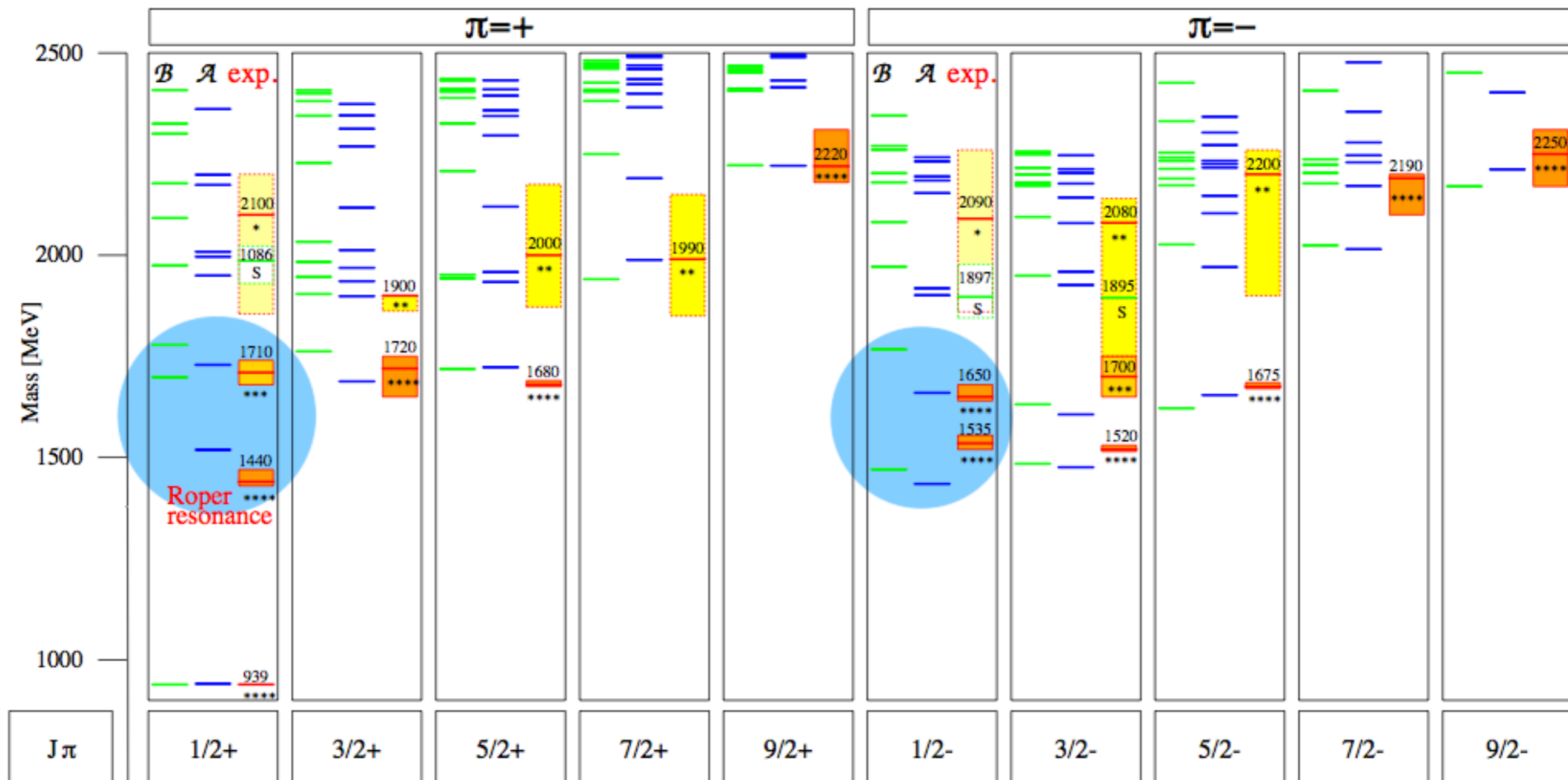


(Transition) Form factors of light and strange baryons

**Reviews: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]
Ramalho and Pena, to appear in PPNP, [2306.13900]**

Light baryon spectrum - quark model



Loring, Metsch, Petry, EPJA 10 (2001) 395

- ‘missing resonances’: three-body vs. quark-diquark

- level ordering: $N_{\frac{1}{2}+}$ vs. $N_{\frac{1}{2}-}$

Physics from form factors

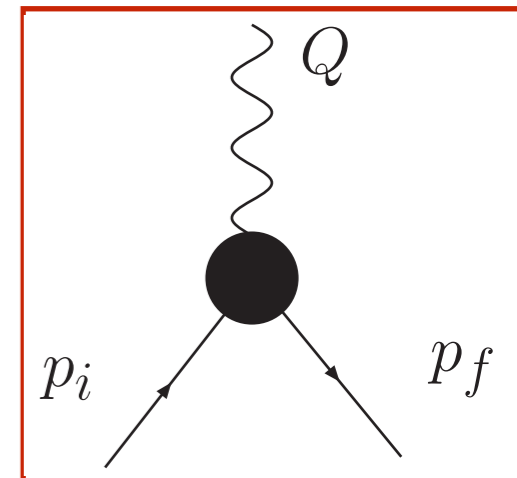
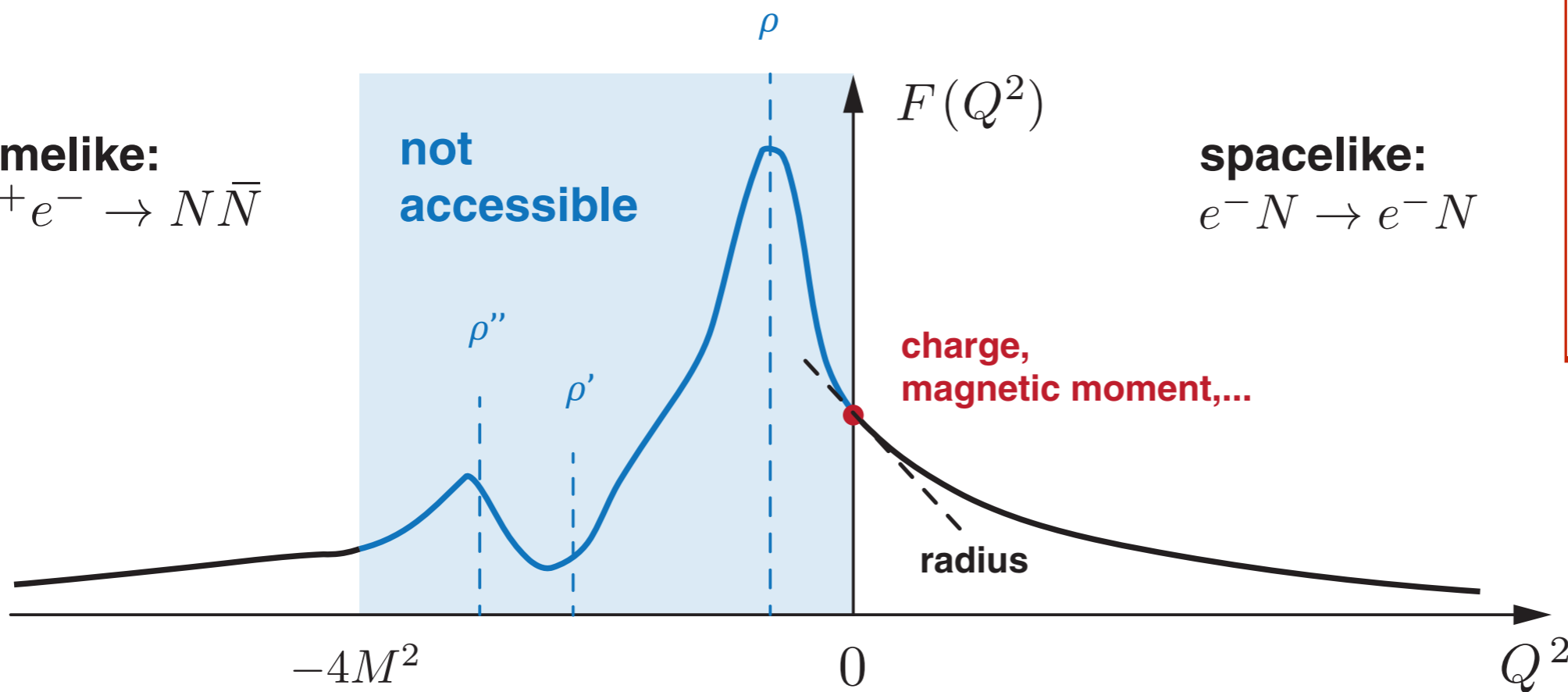
timelike:

$$e^+e^- \rightarrow N\bar{N}$$

**not
accessible**

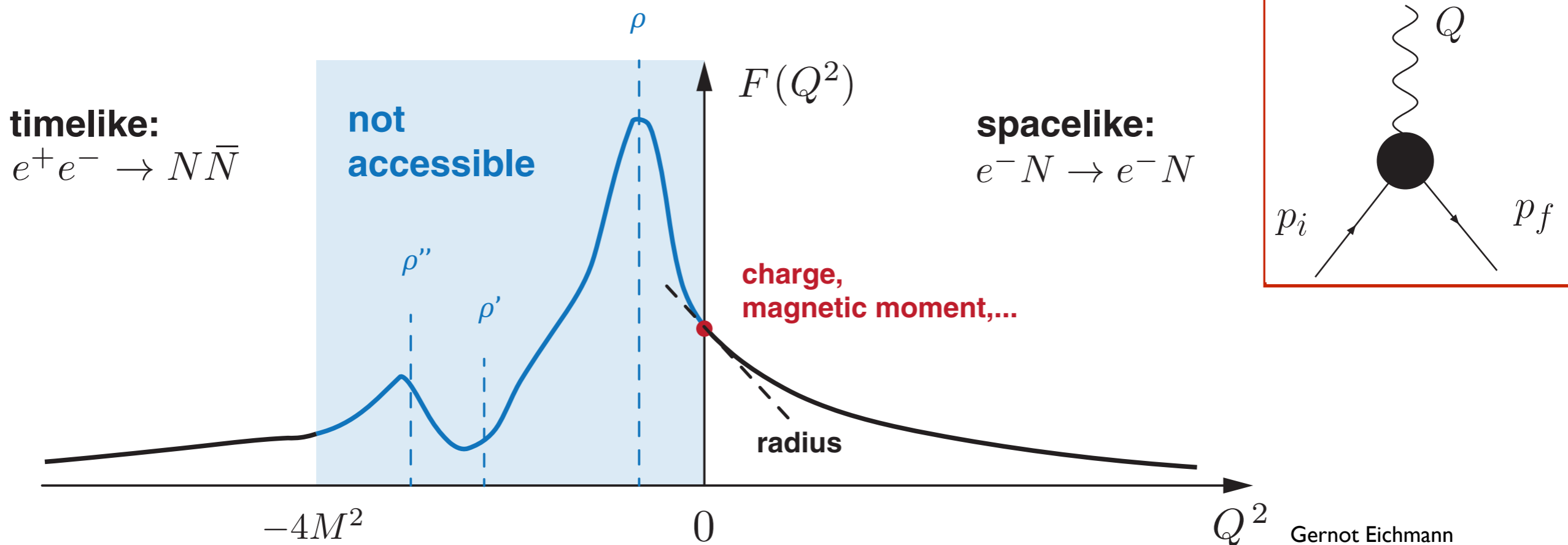
spacelike:

$$e^-N \rightarrow e^-N$$



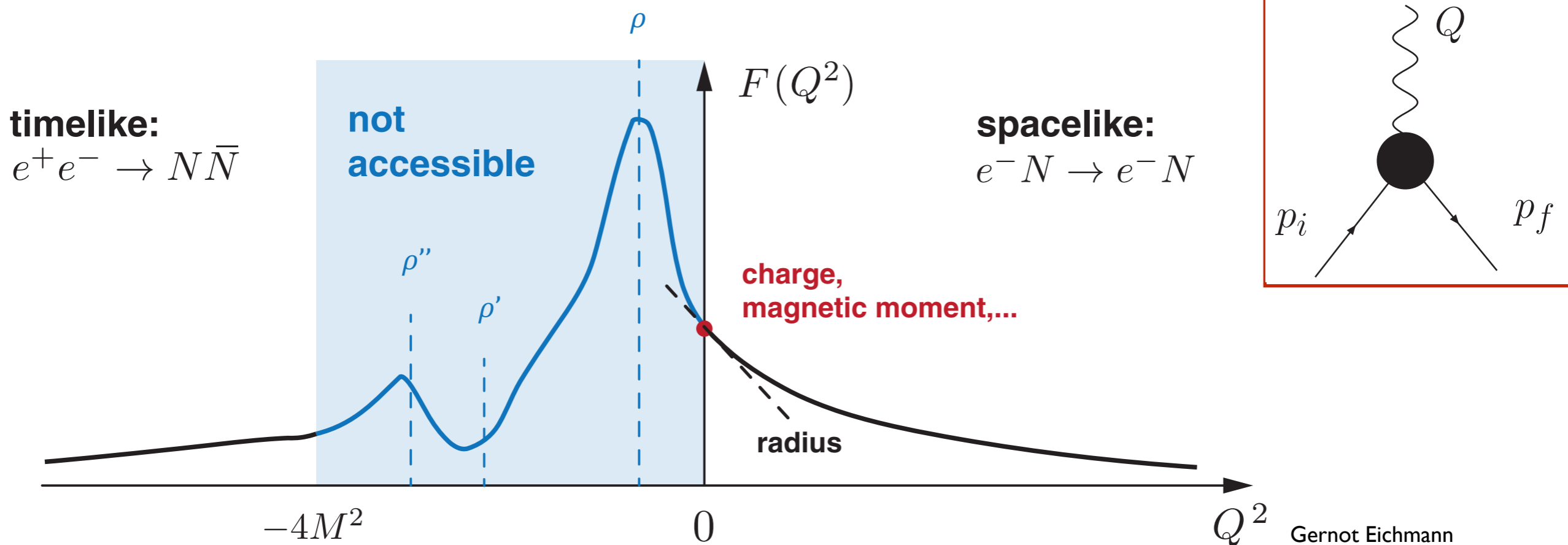
Gernot Eichmann

Physics from form factors



important info: charge, charge radii, magnetic moments
(proton radius puzzle....)

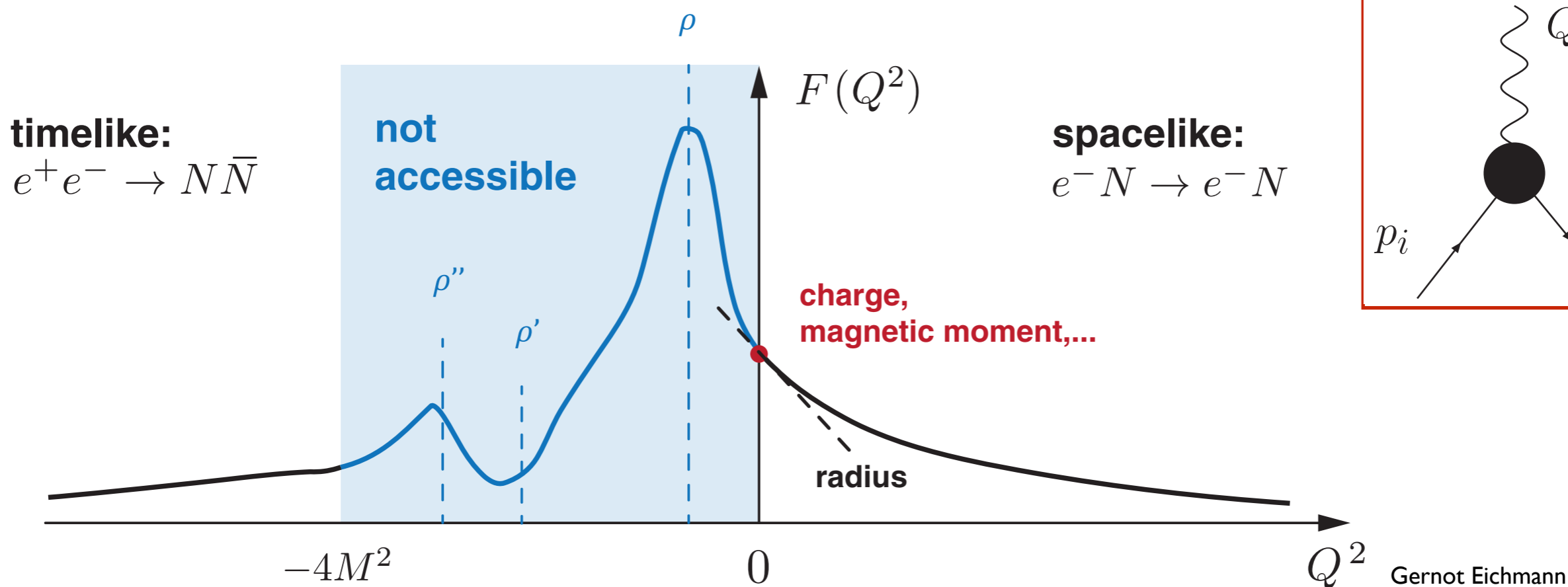
Physics from form factors



→
dispersion theory
ChPT

↑
important info: charge, charge radii, magnetic moments
(proton radius puzzle....)

Physics from form factors

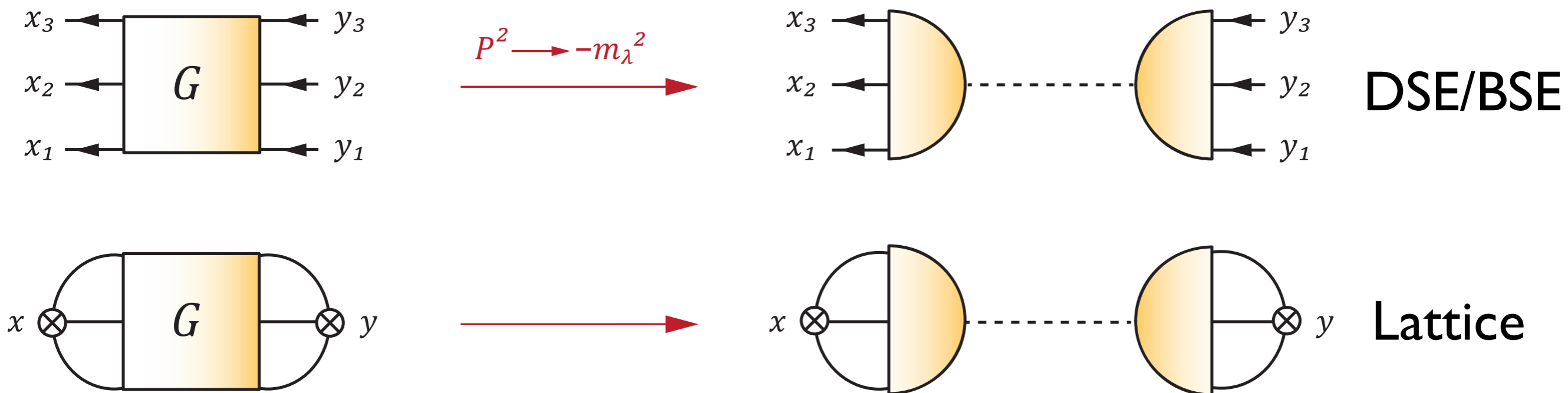


dispersion theory
ChPT

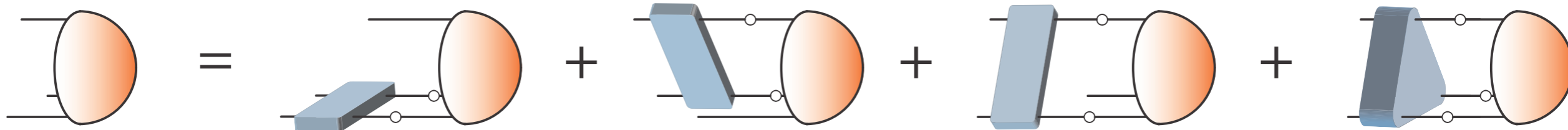
lattice QCD
functional methods
models

important info: charge, charge radii, magnetic moments
(proton radius puzzle....)

Extracting spectra from correlators



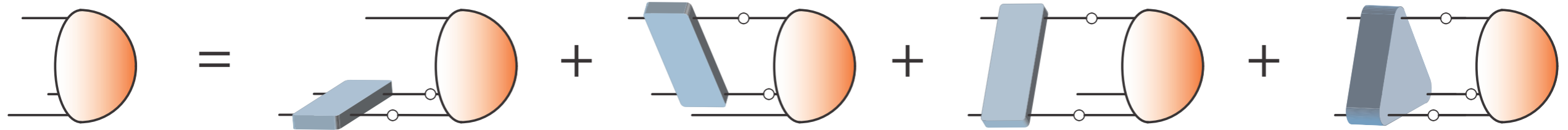
BSE for baryons (derived from equation of motion for G)



- exact equation for baryon 'wave function'

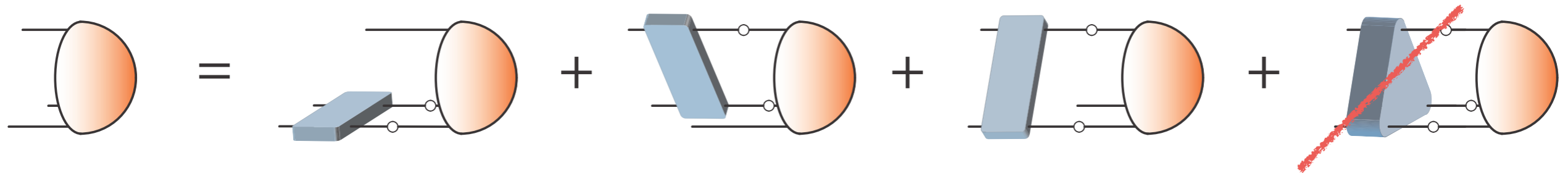
Three-body vs. Diquark-quark approximation

Bethe-Salpeter equation for baryons:



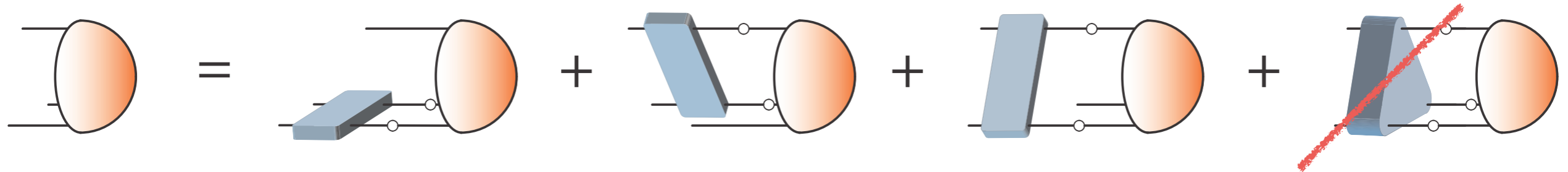
Three-body vs. Diquark-quark approximation

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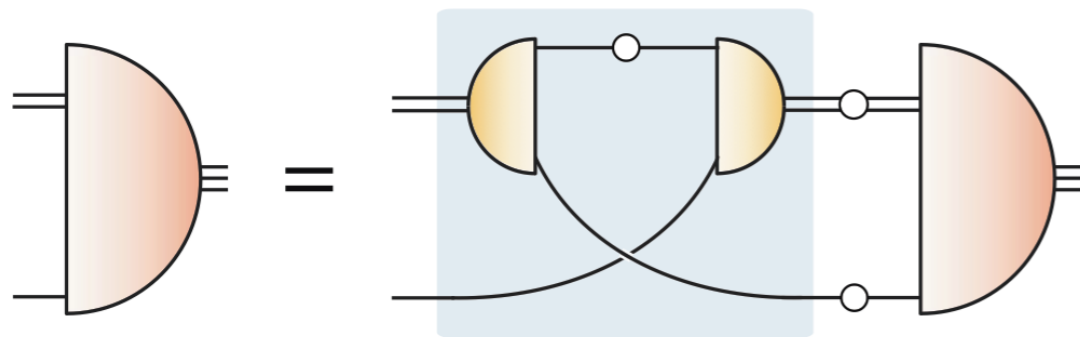


Three-body vs. Diquark-quark approximation

Bethe-Salpeter equation for baryons:



Diquark-quark approximation:



→ lattice: talk of Anthony Francis

Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91 (2017), [1606.09602]
Barabanov, ..., Eichmann, et al. PPNP, 116 (2021), [2008.07630].

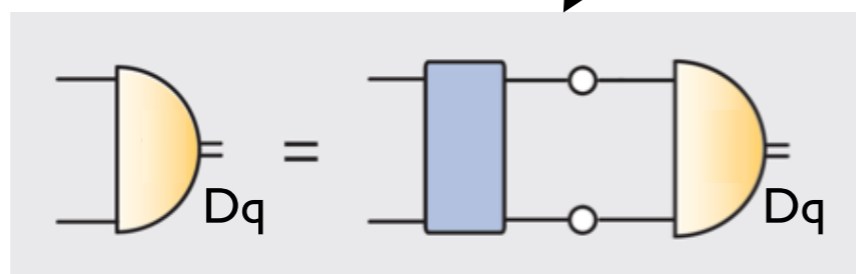
Three approximation schemes

Quark-diquark model

DSE (RL)

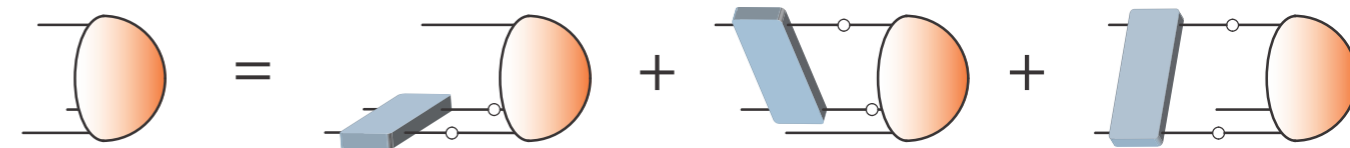
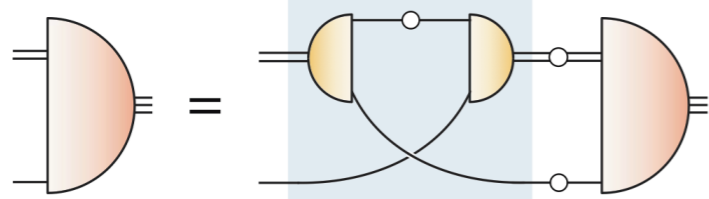


calc.



calc.

ansatze



Barabanov, ..., Eichmann, et al. PPNP, 116 (2021).

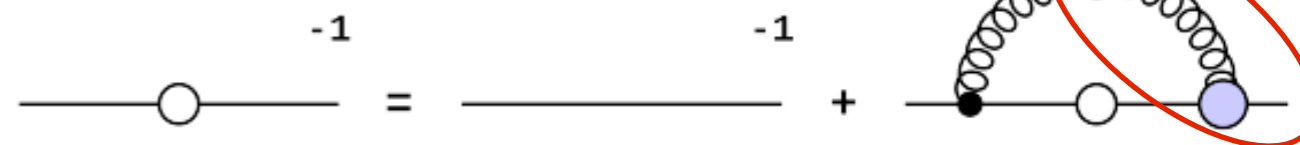
Lattice: Francis, de Forcrand, Lewis and Maltman, JHEP 05 (2022), 062

Three approximation schemes

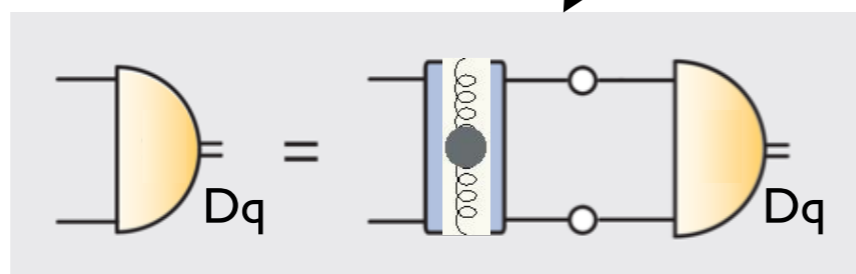
Quark-diquark model

DSE (RL)

ansatz

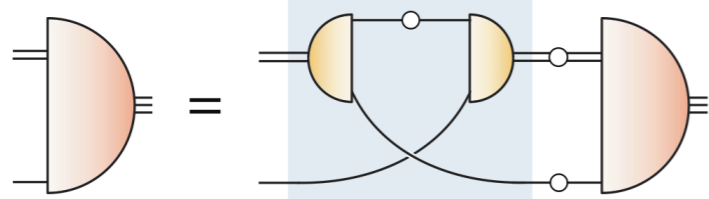


calc.

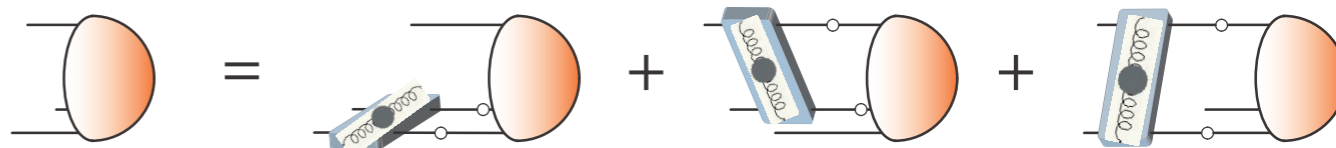


calc.

ansaetze



⋮



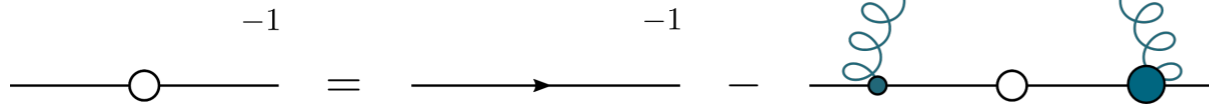
Barabanov, ..., Eichmann, et al. PPNP, 116 (2021).

Lattice: Francis, de Forcrand, Lewis and Maltman, JHEP 05 (2022), 062

Dyson-Schwinger equations - “3PI vs RL”

$$Z_{QCD} = \int \mathcal{D}[\Psi, A] \exp \left\{ - \int d^4x \left(\bar{\Psi} (i\not{D} - m) \Psi - \frac{1}{4} (F_{\mu\nu}^a)^2 \right) \right\}$$

propagators

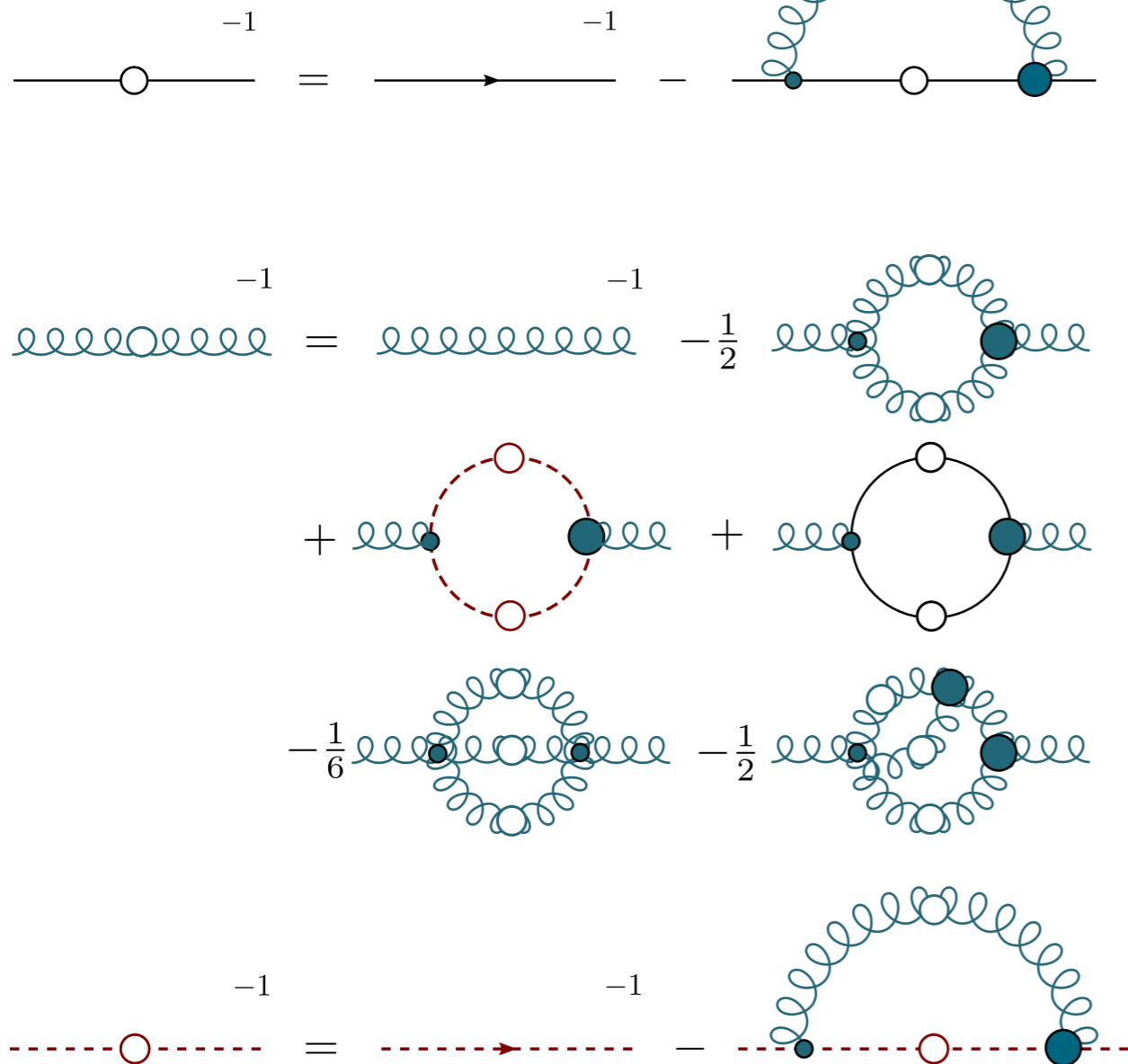


CF,Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

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propagators

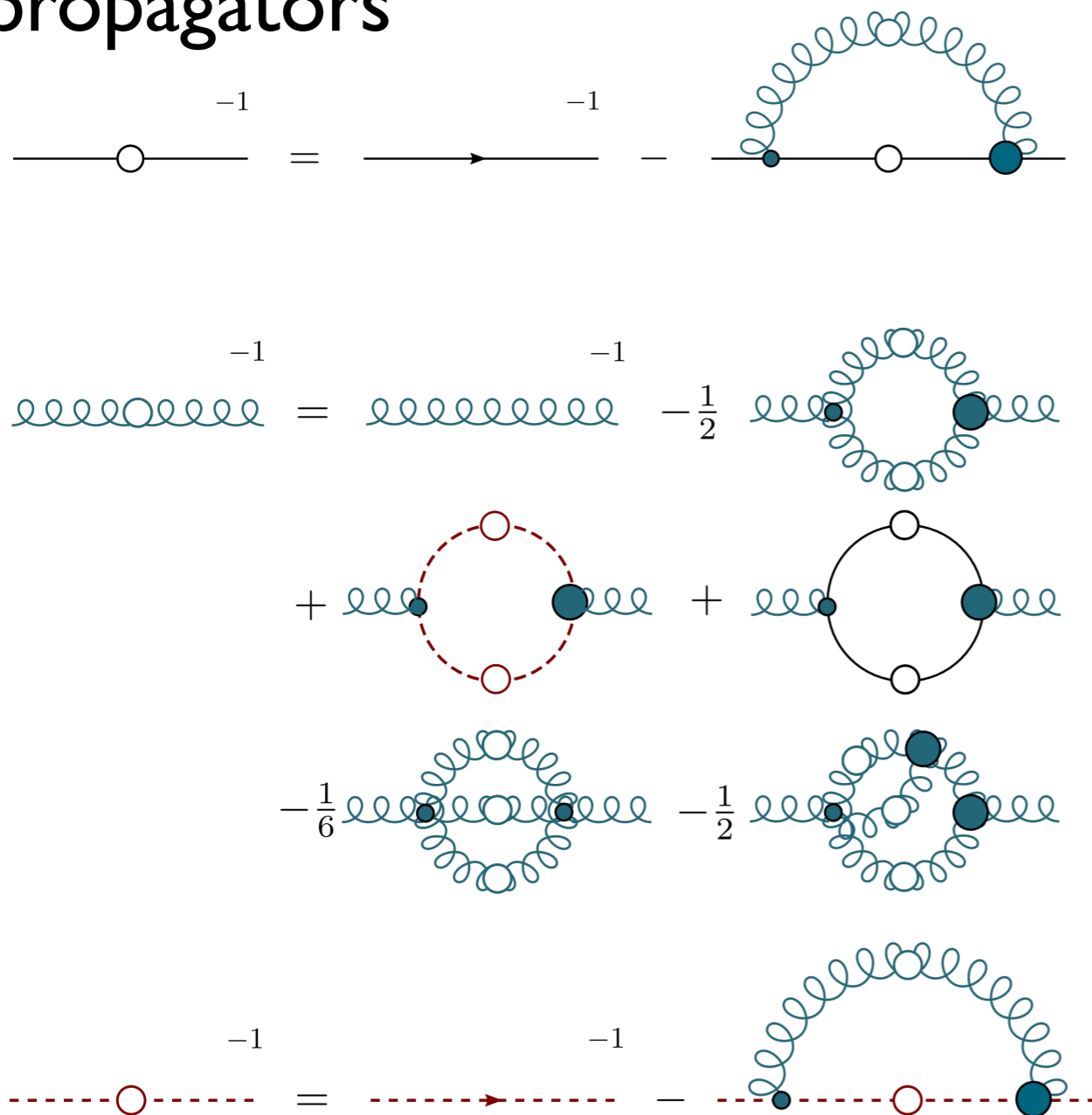


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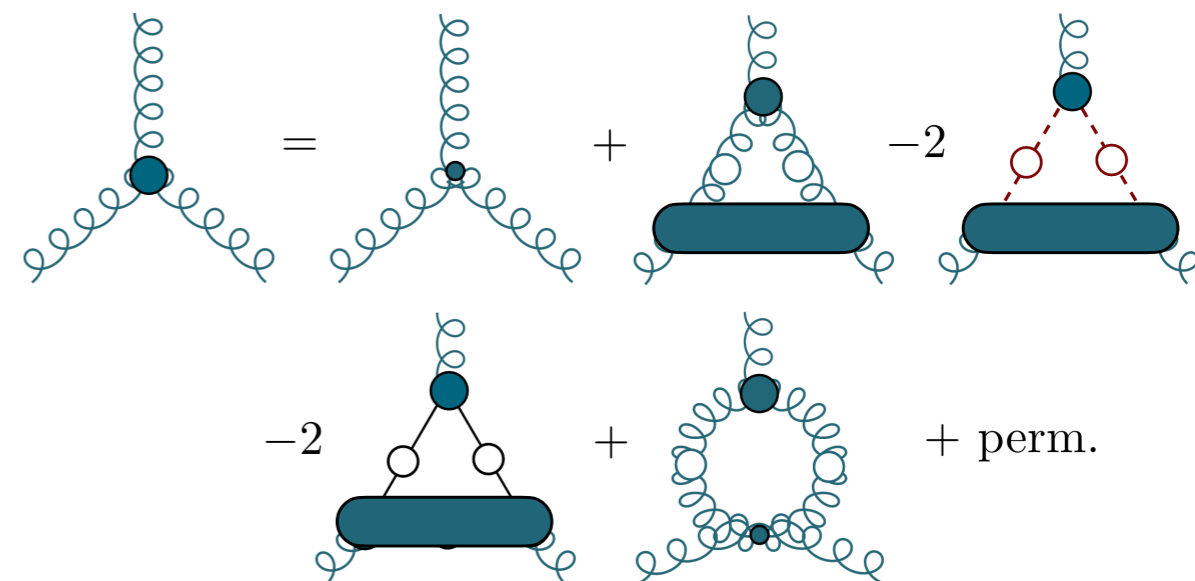
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propagators



vertices

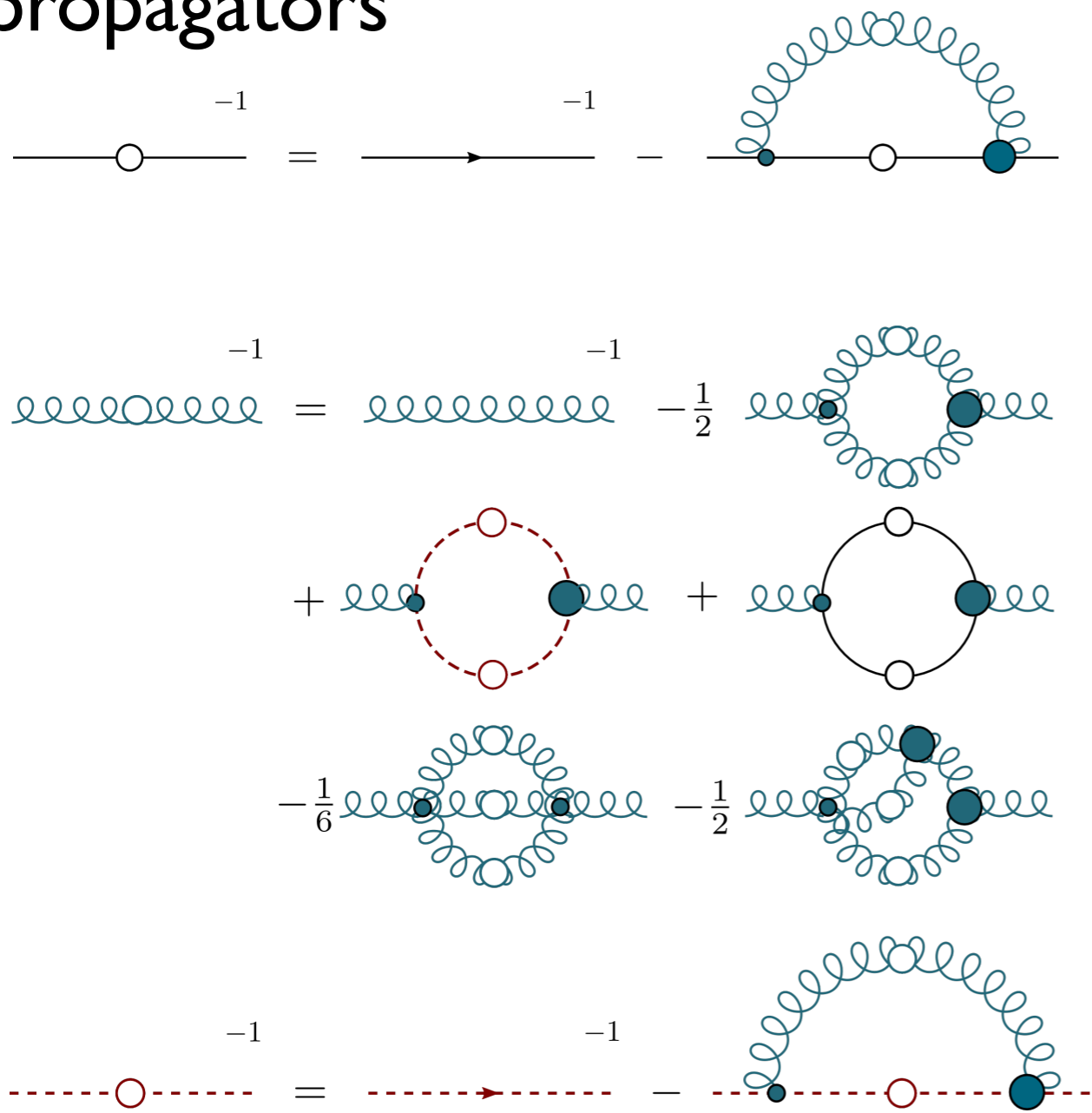


CF,Alkofer, PRD67 (2003) 094020
 Williams, CF,Heupel, PRD93 (2016) 034026
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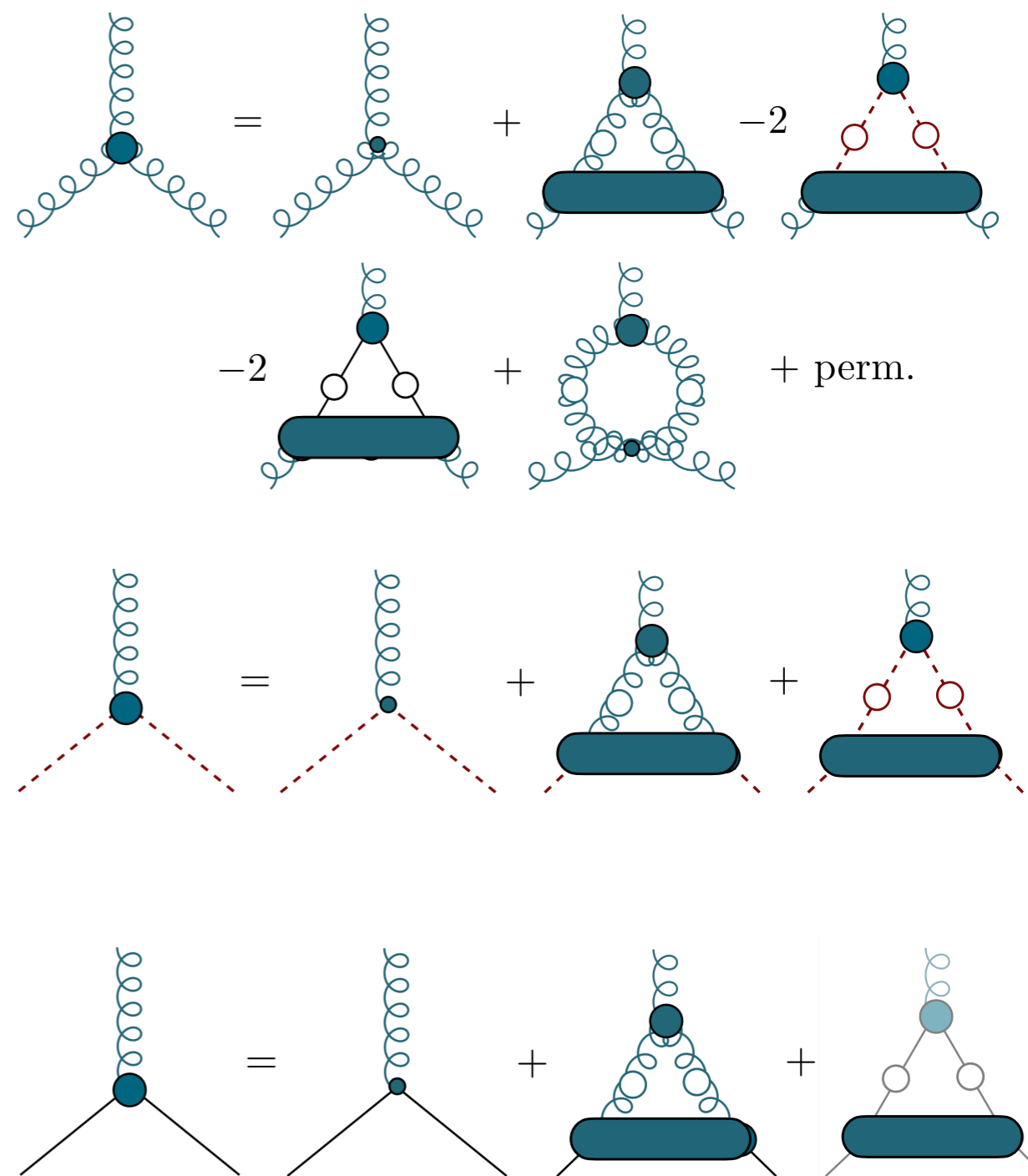
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propagators



vertices

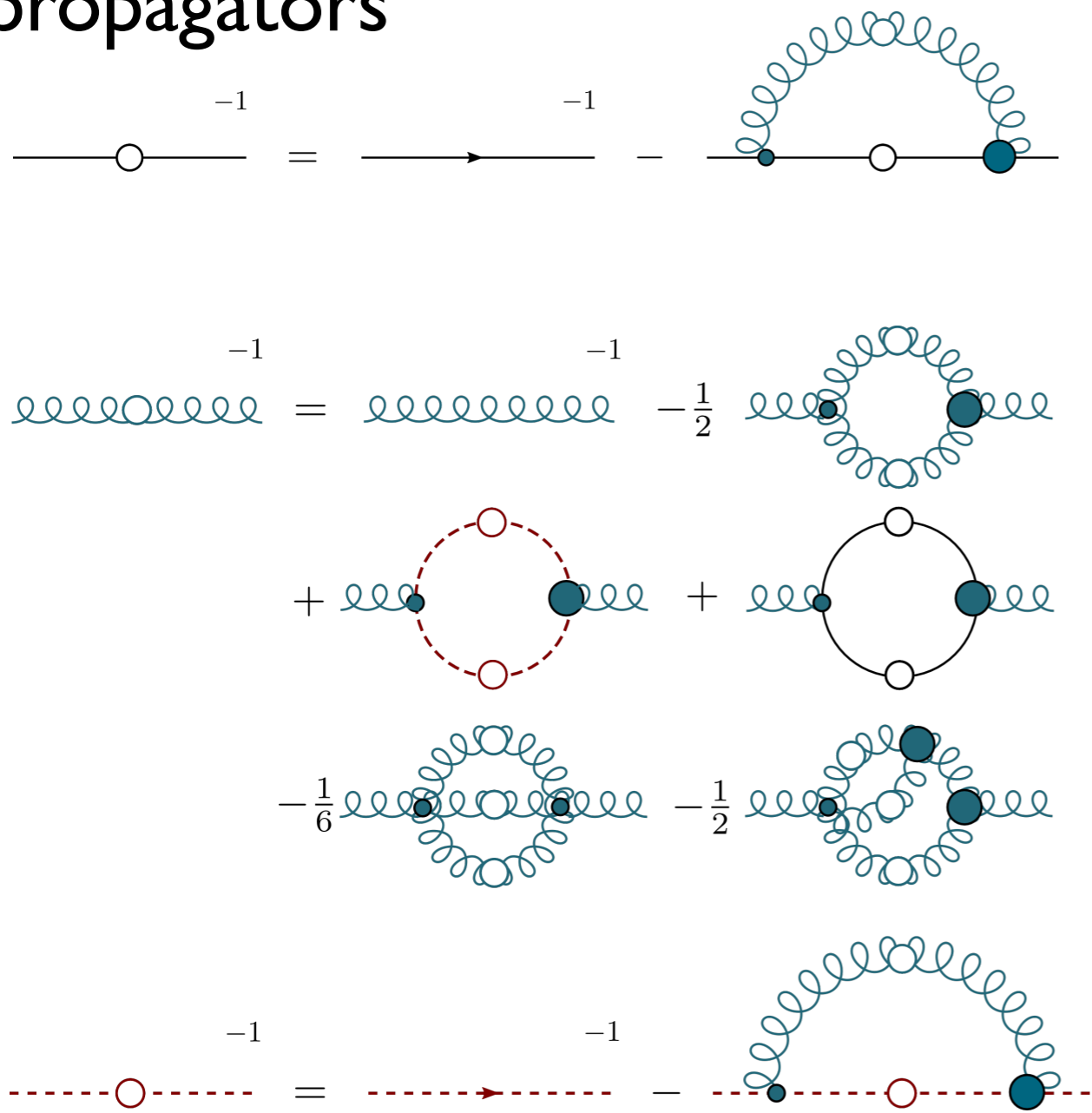


CF,Alkofer, PRD67 (2003) 094020
 Williams, CF,Heupel, PRD93 (2016) 034026
 Huber, PRD 101 (2020) 114009

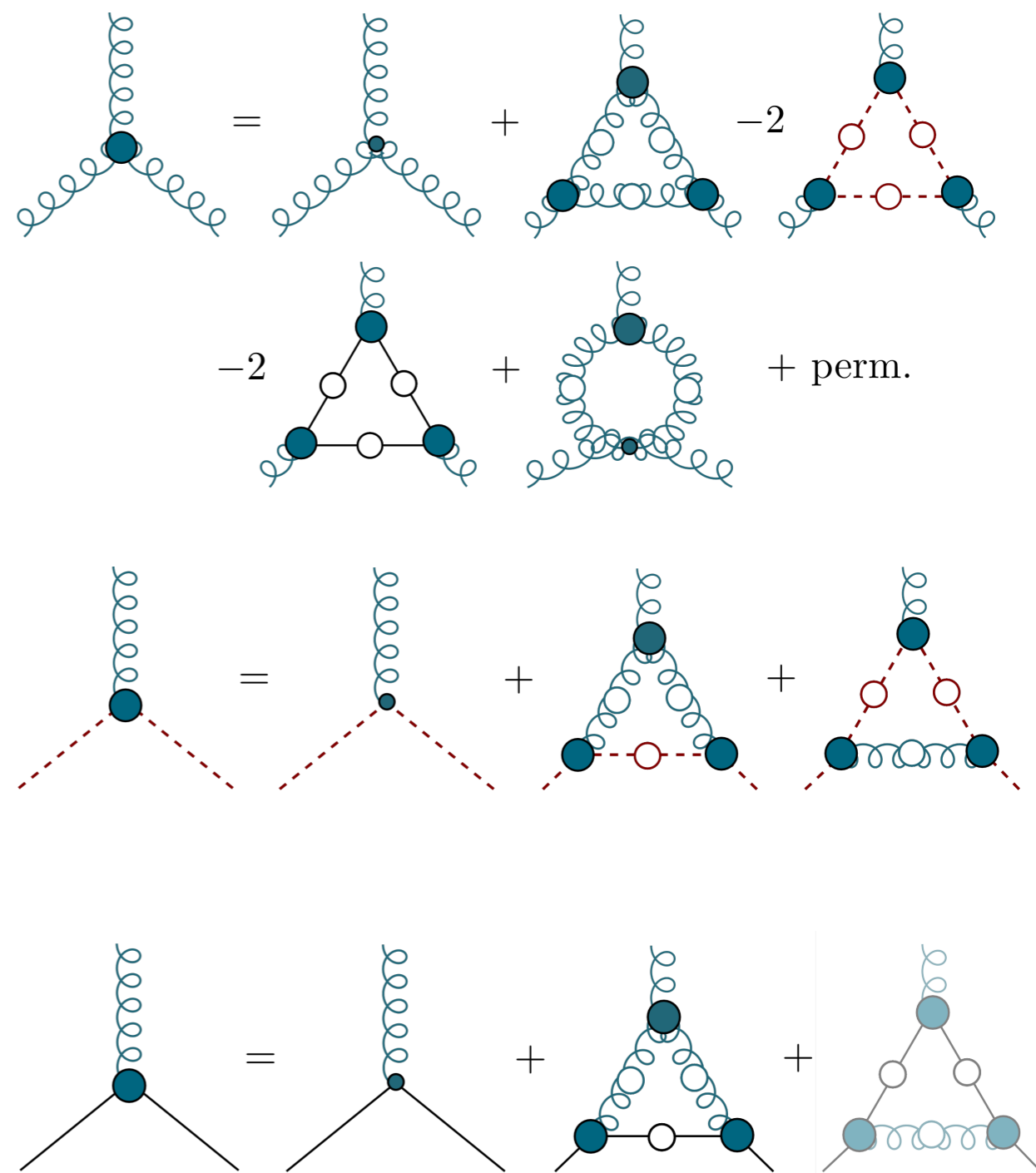
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propagators



vertices



CF,Alkofer, PRD67 (2003) 094020
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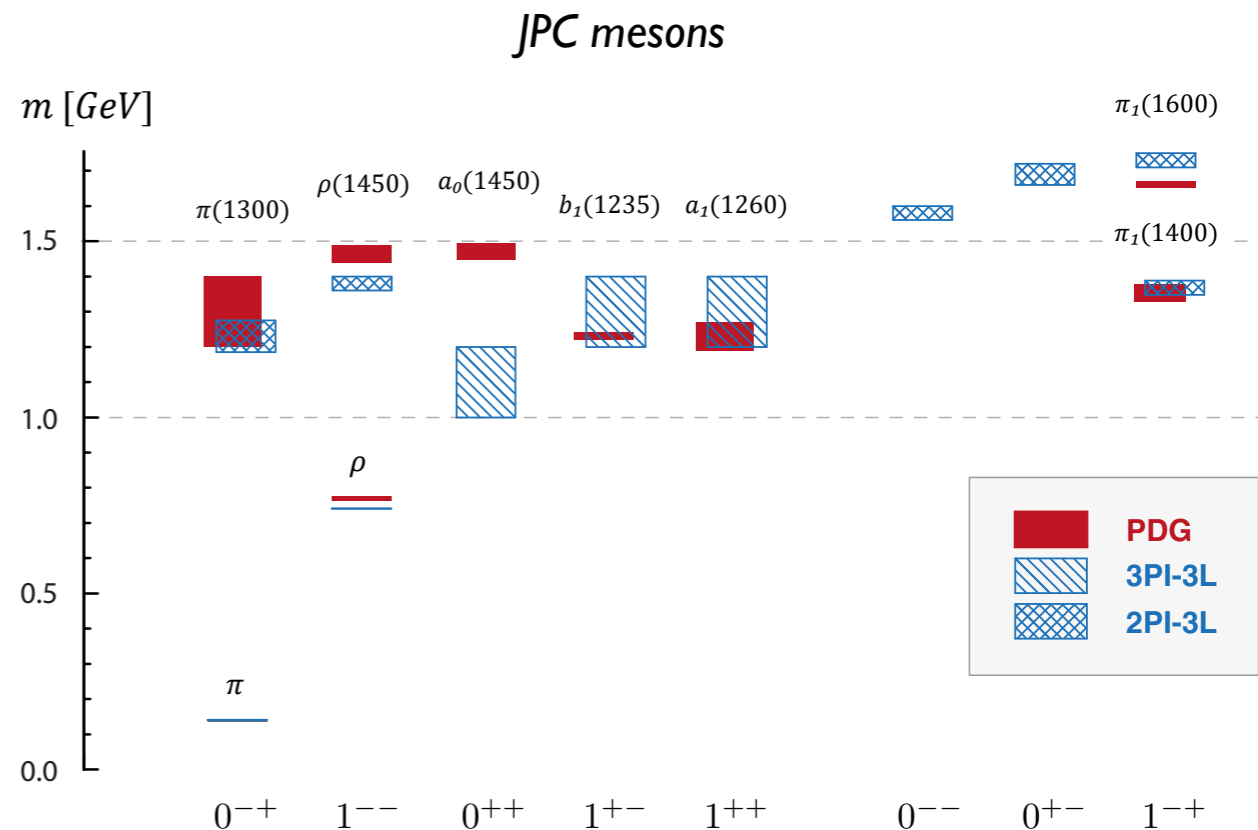
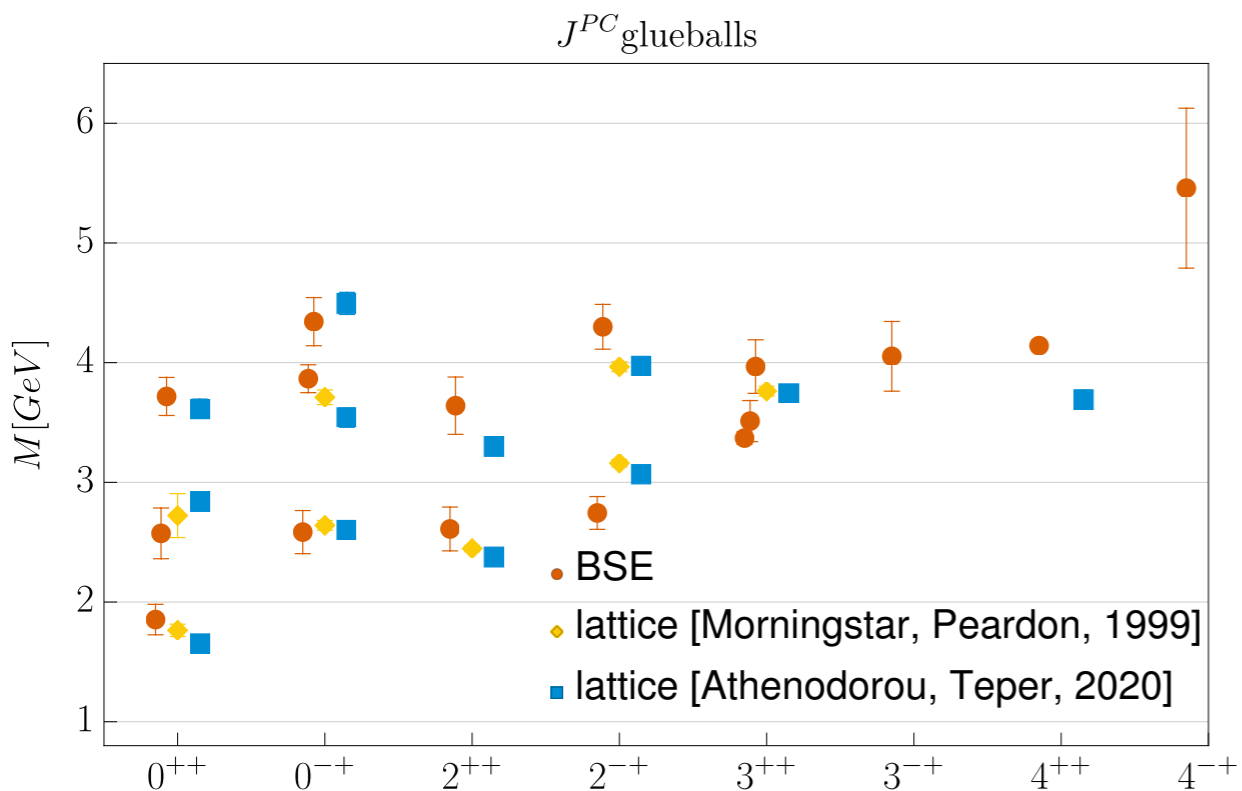
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propagators

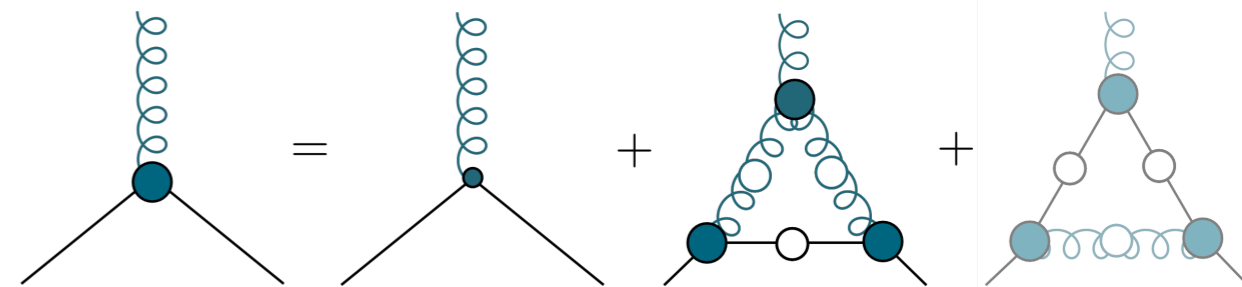
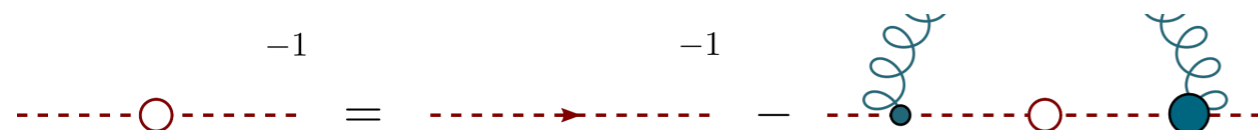


vertices



CF, Huber, Sanchis-Alepuz, EPJC 80 (2020) [arXiv:2004.00415]
 Huber, CF, Sanchis-Alepuz, EPJC 81 (2021) [arXiv:2110.09180]

Williams, CF, Heupel, PRD93 (2016) 034026

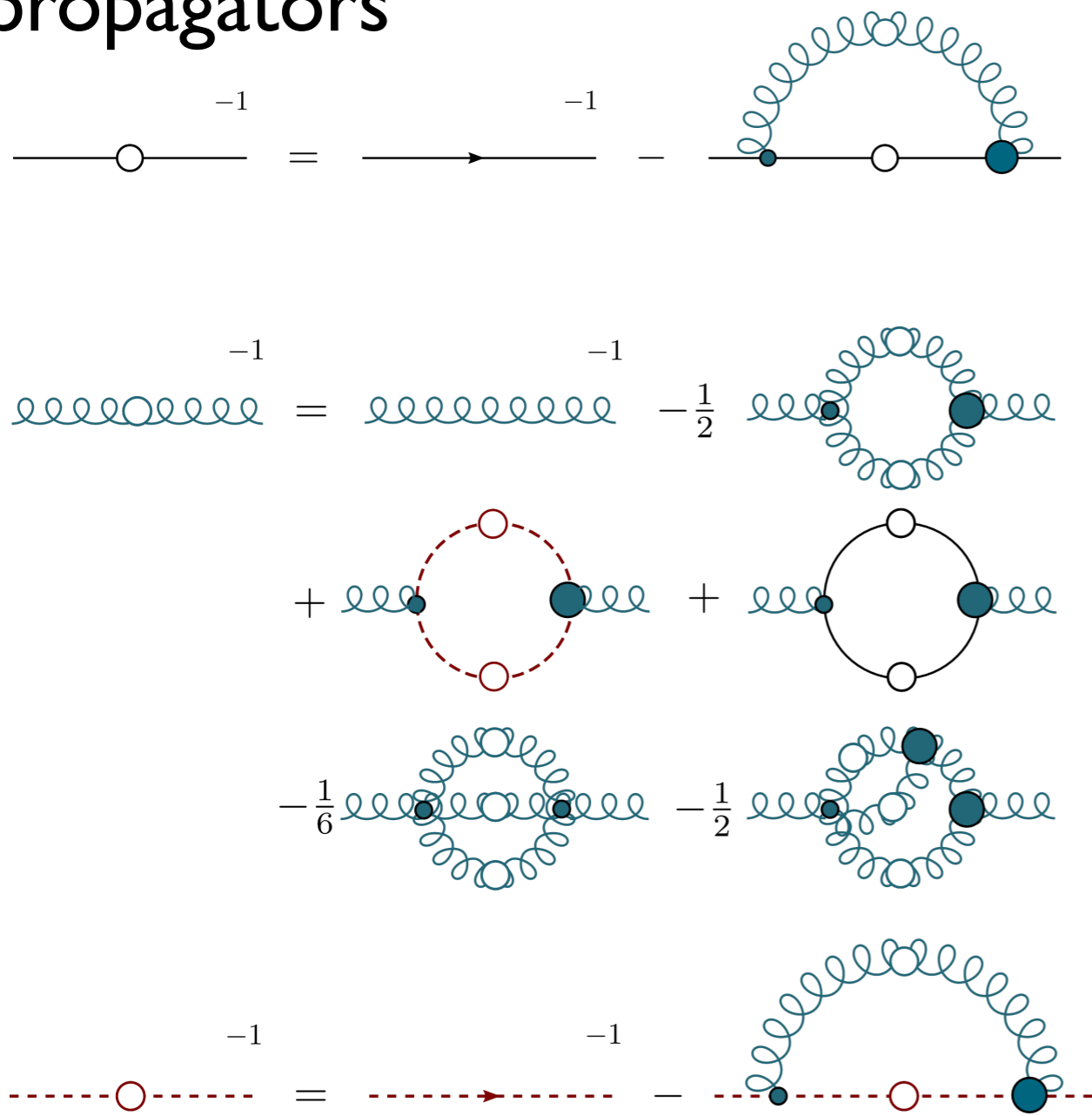


CF, Alkofer, PRD67 (2003) 094020
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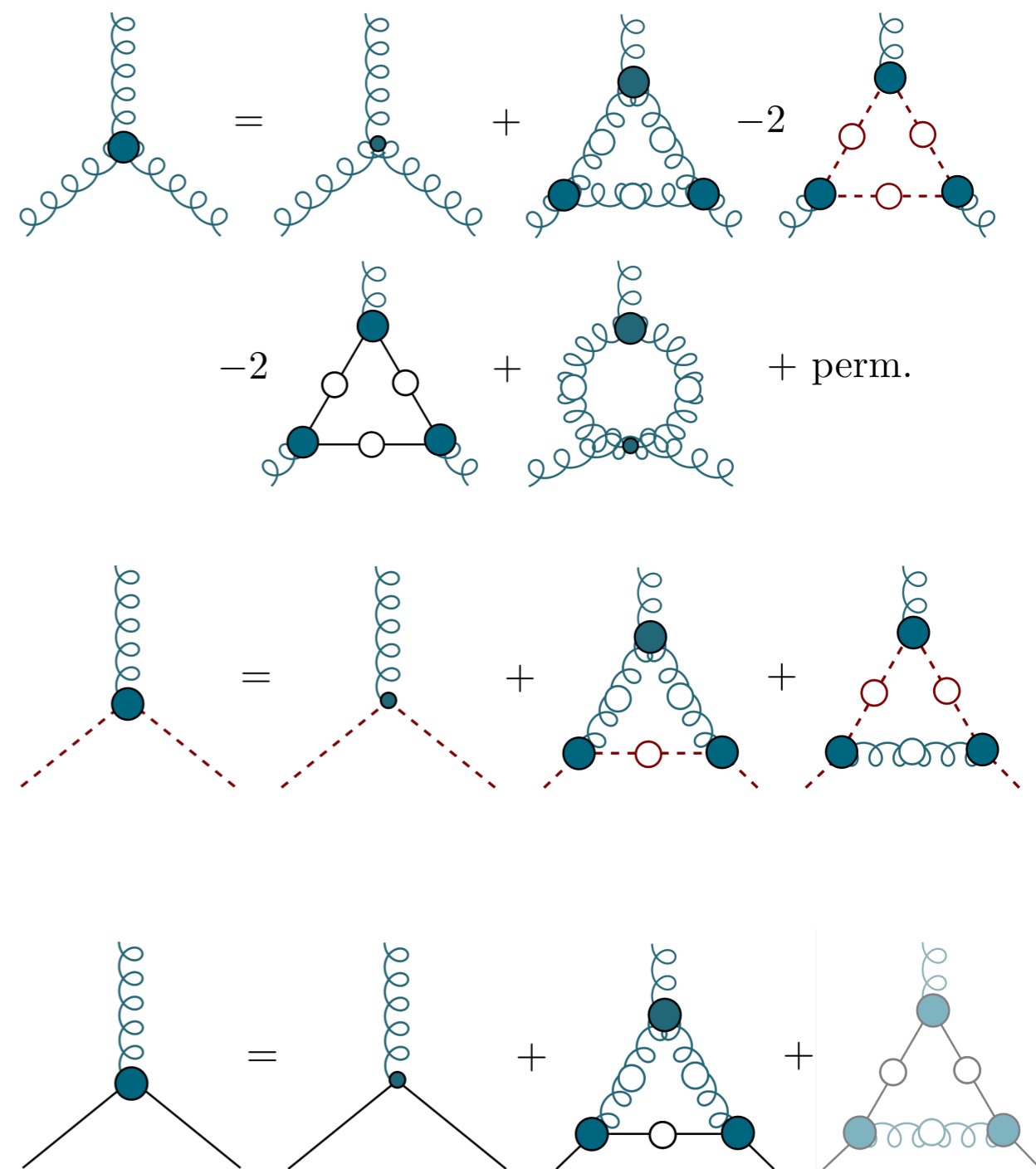
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propagators



vertices

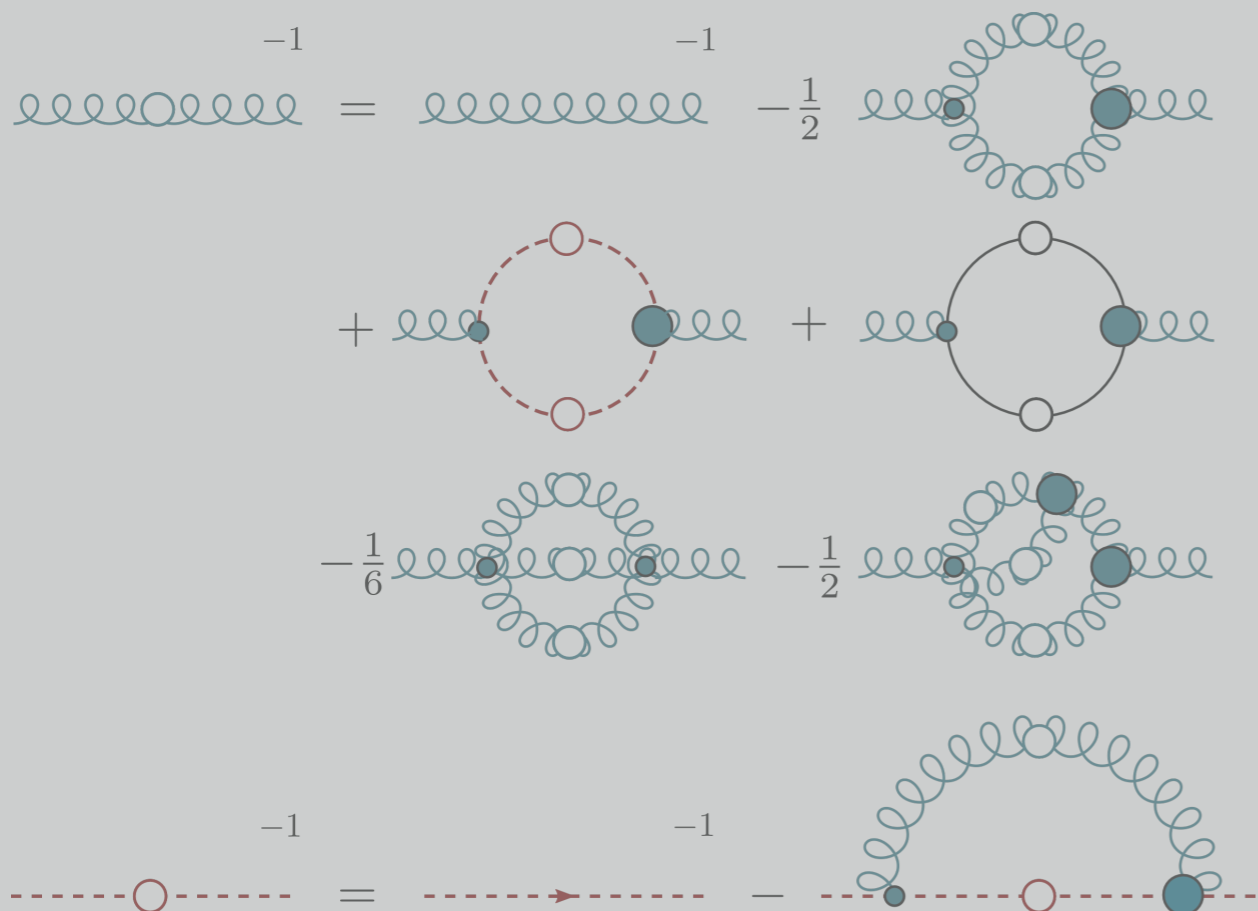
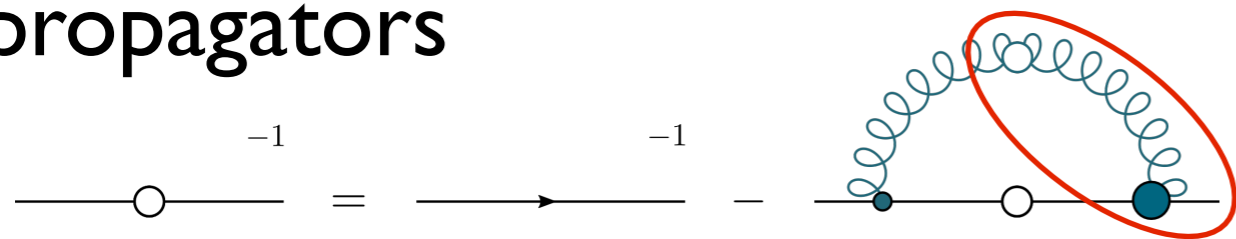


CF,Alkofer, PRD67 (2003) 094020
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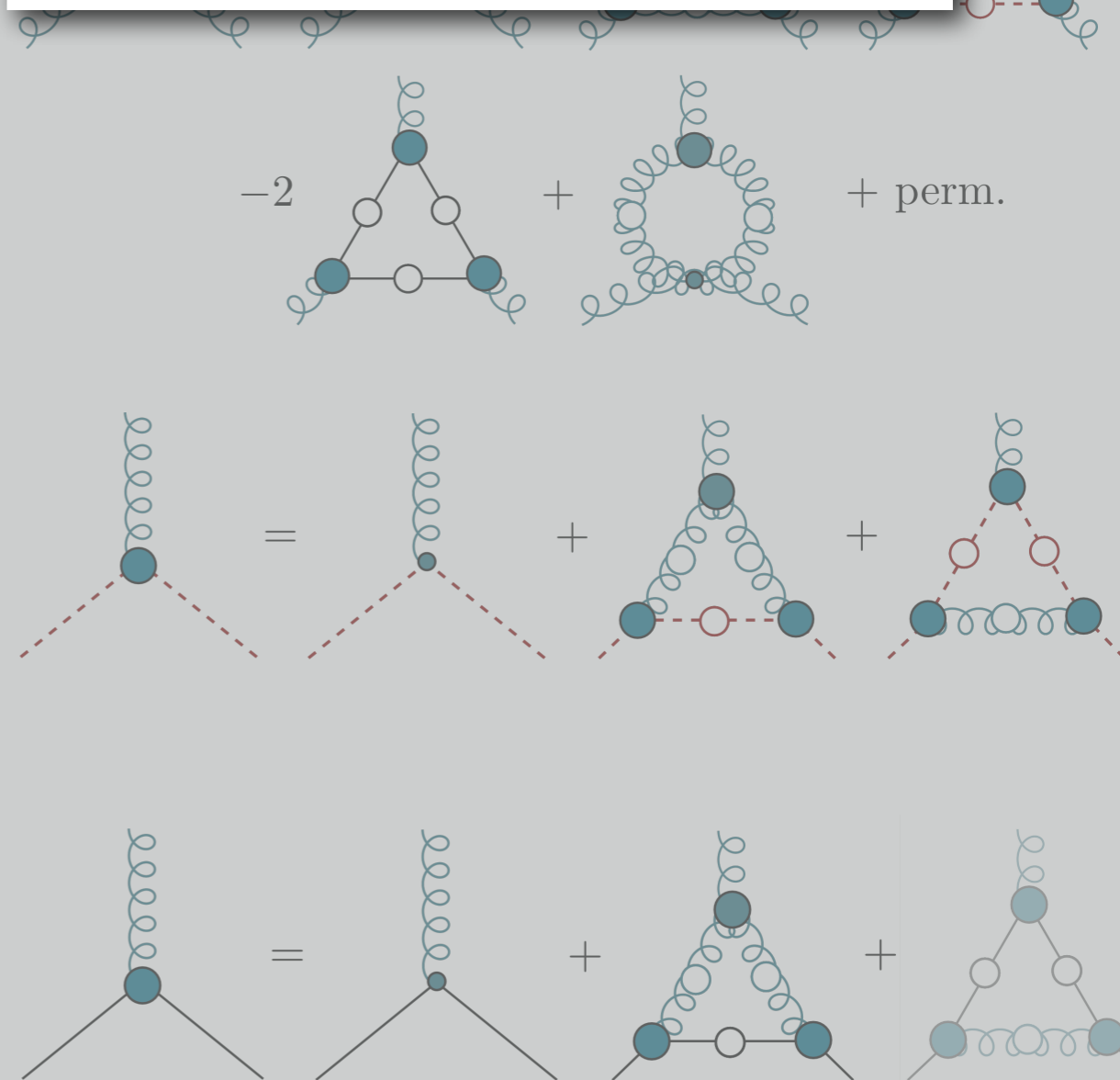
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propagators



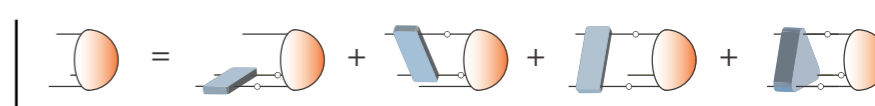
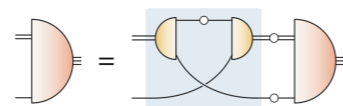
“rainbow-ladder” (RL) :
model for gluon+vertex



CF,Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

DSE/BSE/Faddeev landscape (2015)

level of complexity 



		I) NJL/contact interaction	II) Quark-diquark model	III) DSE (RL)		IV) DSE (bRL)
up/down	$P = \pm$	N, Δ masses	N, Δ masses	N, Δ masses	N, Δ masses	N, Δ masses
		N, Δ em. FFs	N, Δ em. FFs	N, Δ em. FFs	N, Δ em. FFs	N, Δ em. FFs
		$N \rightarrow \Delta \gamma$	$N \rightarrow \Delta \gamma$	$N \rightarrow \Delta \gamma$	$N \rightarrow \Delta \gamma$	$N \rightarrow \Delta \gamma$
	$P = +$	N^*, Δ^* masses	N^*, Δ^* masses			
		$\gamma N \rightarrow N^* / \Delta^*$	$\gamma N \rightarrow N^* / \Delta^*$			
	$P = -$	N^*, Δ^* masses	N^*, Δ^* masses			
		$\gamma N \rightarrow N^* / \Delta^*$	$\gamma N \rightarrow N^* / \Delta^*$			
strange		ground states	ground states			
		excited states	excited states			
		em. FF	em. FF			
		TFFs	TFFs			
c/b		ground states	ground states			
		excited states	excited states			

Cloet, Thomas, Roberts, Segovia, Chen, et al.

Oettel, Alkofer, Bloch, Roberts, Segovia, Chen, et al.

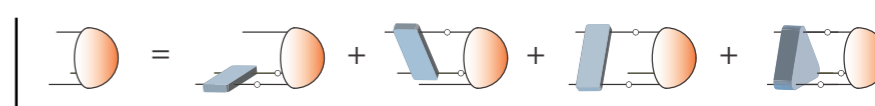
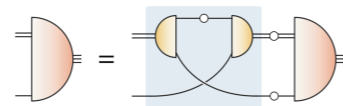
Eichmann, Alkofer, Krassnigg, Nicmorus, Sanchis-Alepuz, CF

Eichmann, Alkofer, Sanchis-Alepuz, CF, Qin, Roberts

Sanchis-Alepuz, Williams, CF

DSE/BSE/Faddeev landscape

level of complexity 



		I) NJL/contact interaction	II) Quark-diquark model	III) DSE (RL)		IV) DSE (bRL)
up/down	$P = \pm$	N, Δ masses				
		N, Δ em. FFs				
		$N \rightarrow \Delta \gamma$				
	$P = +$	N^*, Δ^* masses				
		$\gamma N \rightarrow N^* / \Delta^*$				
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strange		ground states				
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Cloet, Thomas, Roberts, Segovia, Chen, et al.

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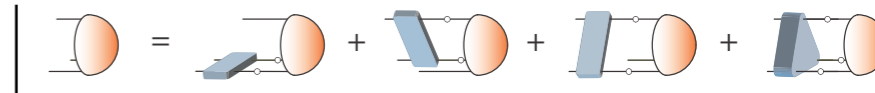
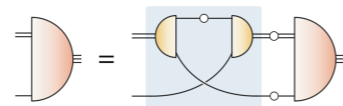
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Eichmann, Alkofer, Sanchis-Alepuz, CF, Qin, Roberts

Sanchis-Alepuz, Williams, CF

DSE/BSE/Faddeev landscape

level of complexity 



		I) NJL/contact interaction	II) Quark-diquark model	III) DSE (RL)		IV) DSE (bRL)
up/down	$P = \pm$	N, Δ masses	N, Δ masses	N, Δ masses	N, Δ masses	N, Δ masses
		N, Δ em. FFs	N, Δ em. FFs	N, Δ em. FFs	N, Δ em. FFs	N, Δ em. FFs
		$N \rightarrow \Delta \gamma$	$N \rightarrow \Delta \gamma$	$N \rightarrow \Delta \gamma$	$N \rightarrow \Delta \gamma$	$N \rightarrow \Delta \gamma$
	$P = +$	N^*, Δ^* masses	N^*, Δ^* masses	N^*, Δ^* masses	N^*, Δ^* masses	N^*, Δ^* masses
		$\gamma N \rightarrow N^* / \Delta^*$	$\gamma N \rightarrow N^* / \Delta^*$	$\gamma N \rightarrow N^* / \Delta^*$	$\gamma N \rightarrow N^* / \Delta^*$	$\gamma N \rightarrow N^* / \Delta^*$
	$P = -$	N^*, Δ^* masses	N^*, Δ^* masses	N^*, Δ^* masses	N^*, Δ^* masses	N^*, Δ^* masses
		$\gamma N \rightarrow N^* / \Delta^*$	$\gamma N \rightarrow N^* / \Delta^*$	$\gamma N \rightarrow N^* / \Delta^*$	$\gamma N \rightarrow N^* / \Delta^*$	$\gamma N \rightarrow N^* / \Delta^*$
strange		ground states	ground states	ground states	ground states	ground states
		excited states	excited states	excited states	excited states	excited states
		em. FF	em. FF	em. FF	em. FF	em. FF
		TFFs	TFFs	TFFs	TFFs	TFFs
c/b		ground states	ground states	ground states	ground states	ground states
		excited states	excited states	excited states	excited states	excited states

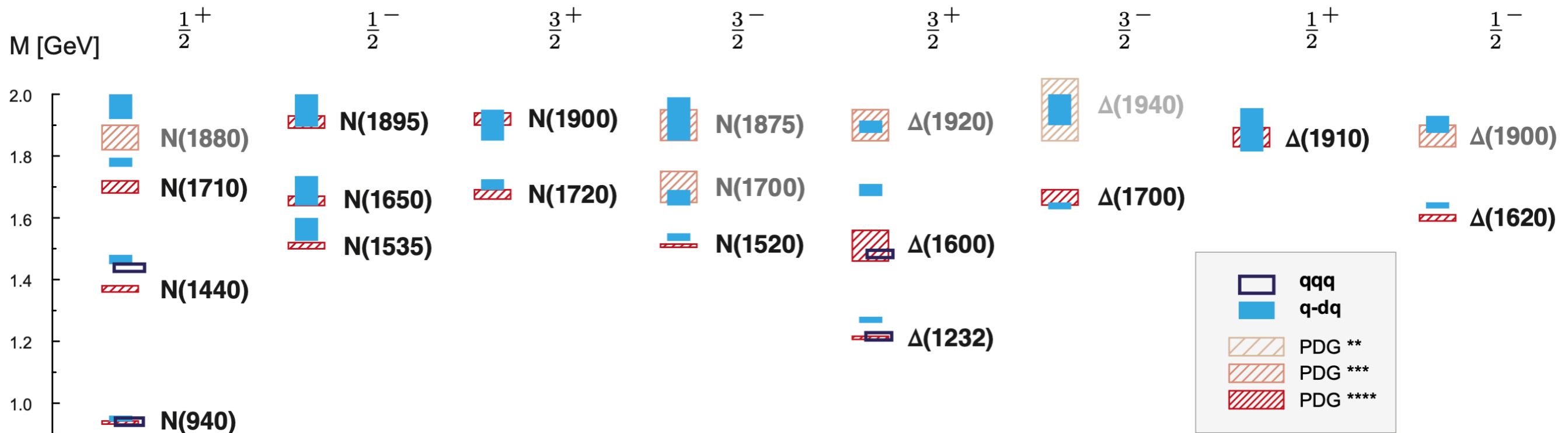
Cloet, Thomas, Roberts, Segovia, Chen, et al.

Oettel, Alkofer, Bloch, Roberts, Segovia, Chen, Liu, CF, et al.

Eichmann, Alkofer, Krassnigg, Nicmorus, Sanchis-Alepuz, CF

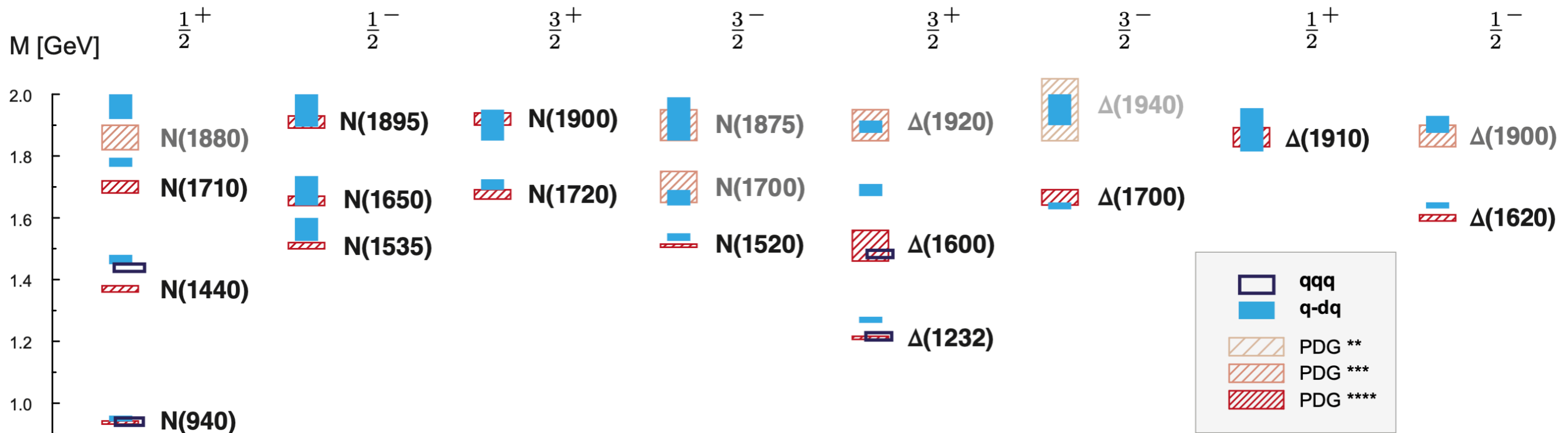
Eichmann, Alkofer, Sanchis-Alepuz, CF, Qin, Roberts

Sanchis-Alepuz, Williams, CF



Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]
 Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2
 Eichmann, Few Body Syst. 63 (2022) no.3,

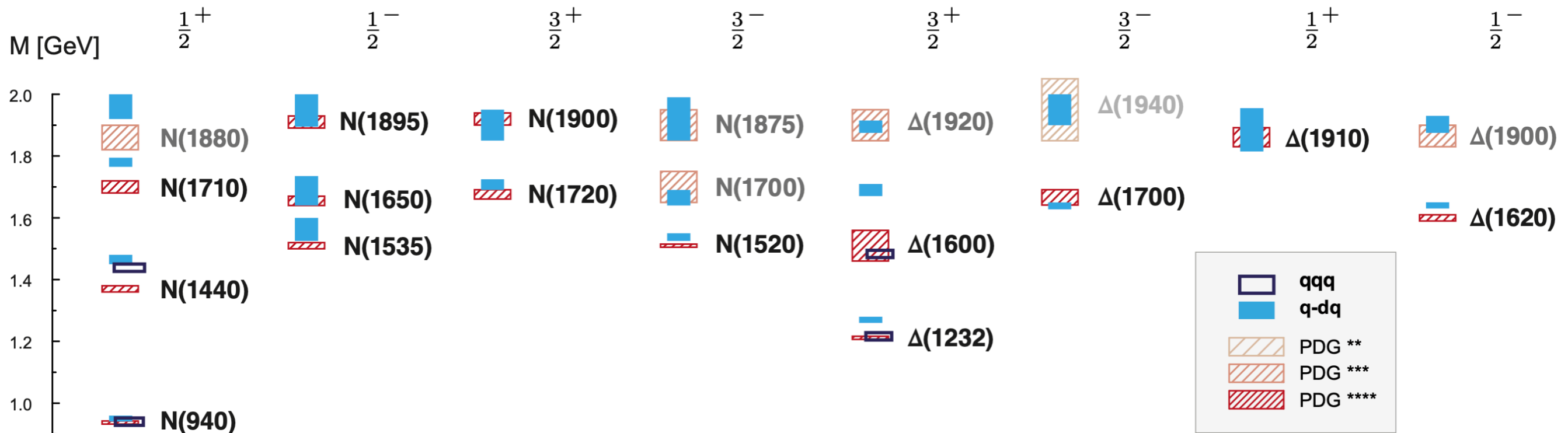
- spectrum in one to one agreement with experiment
- correct level ordering (without coupled channel effects...)
- three-body agrees with diquark-quark where applicable



need:
'good' scalar diquark

Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]
 Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2
 Eichmann, Few Body Syst. 63 (2022) no.3,

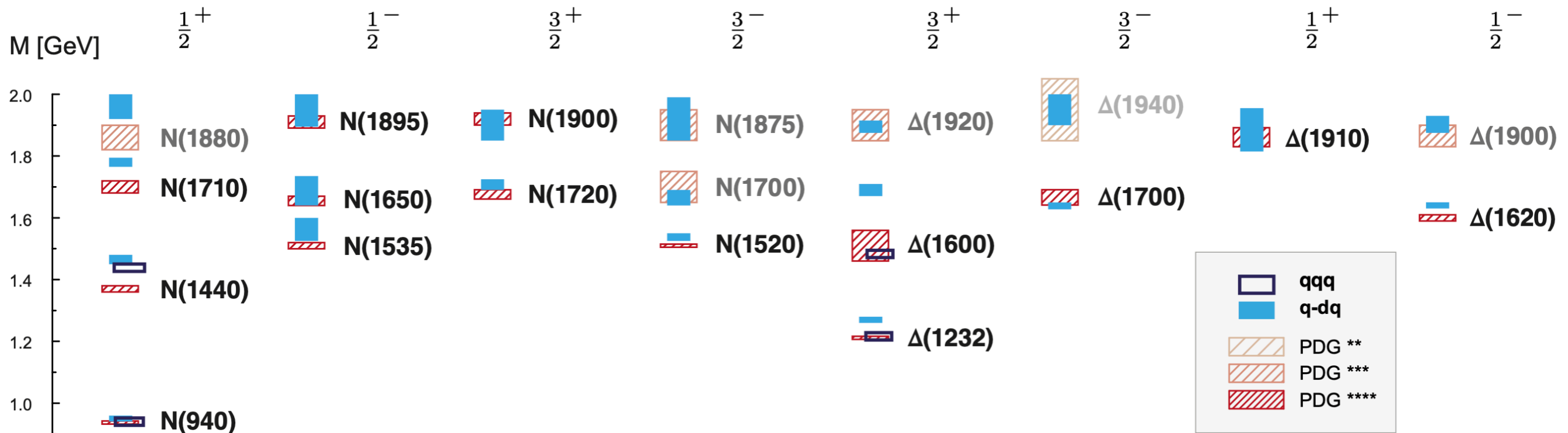
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need:
 'good' scalar diquark
 'bad' axialvector dq

Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]
 Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2
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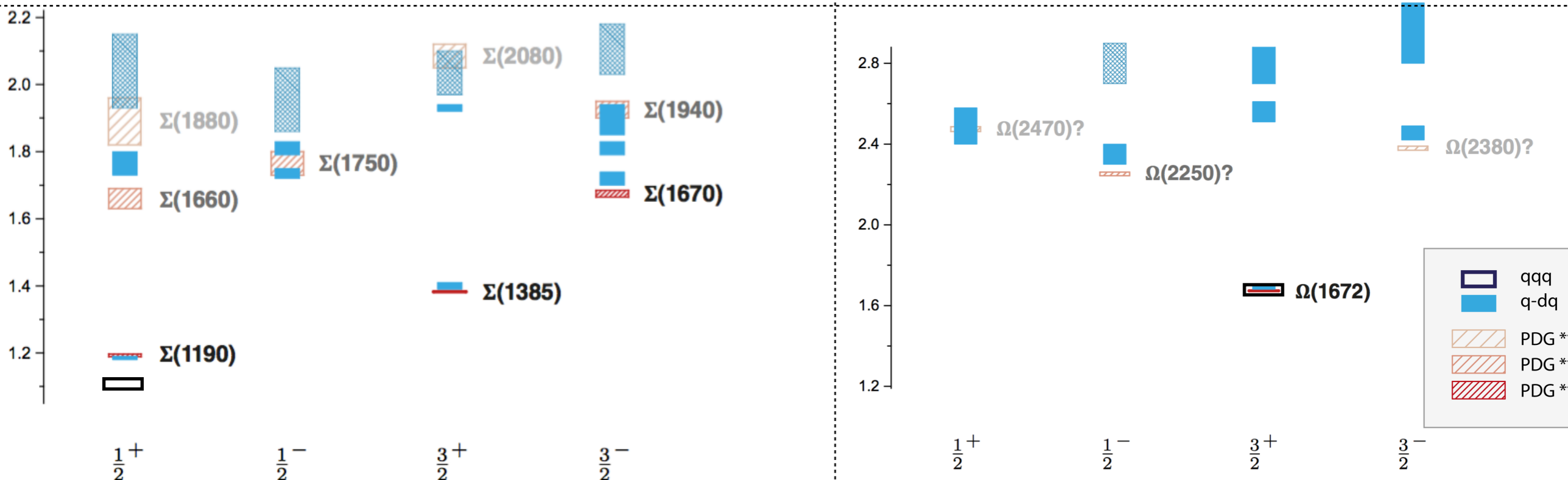
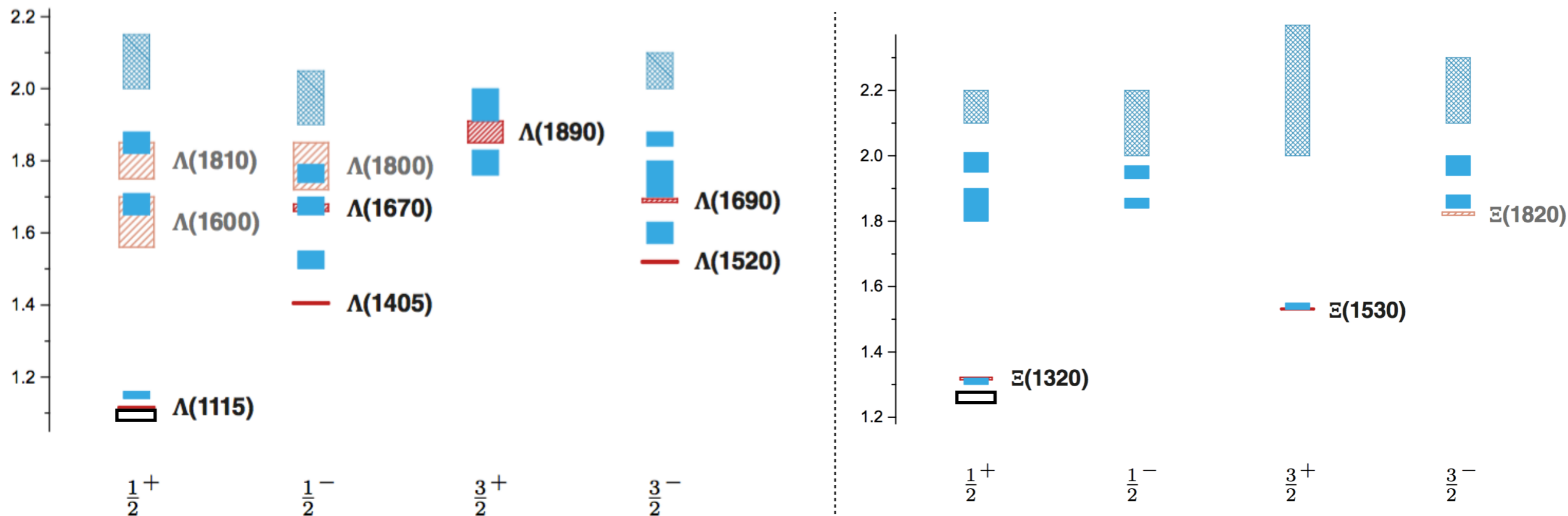


need:
 'good' scalar diquark
 'bad' axialvector dq
 'ugly' pseudoscalar dq

Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]
 Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2
 Eichmann, Few Body Syst. 63 (2022) no.3,

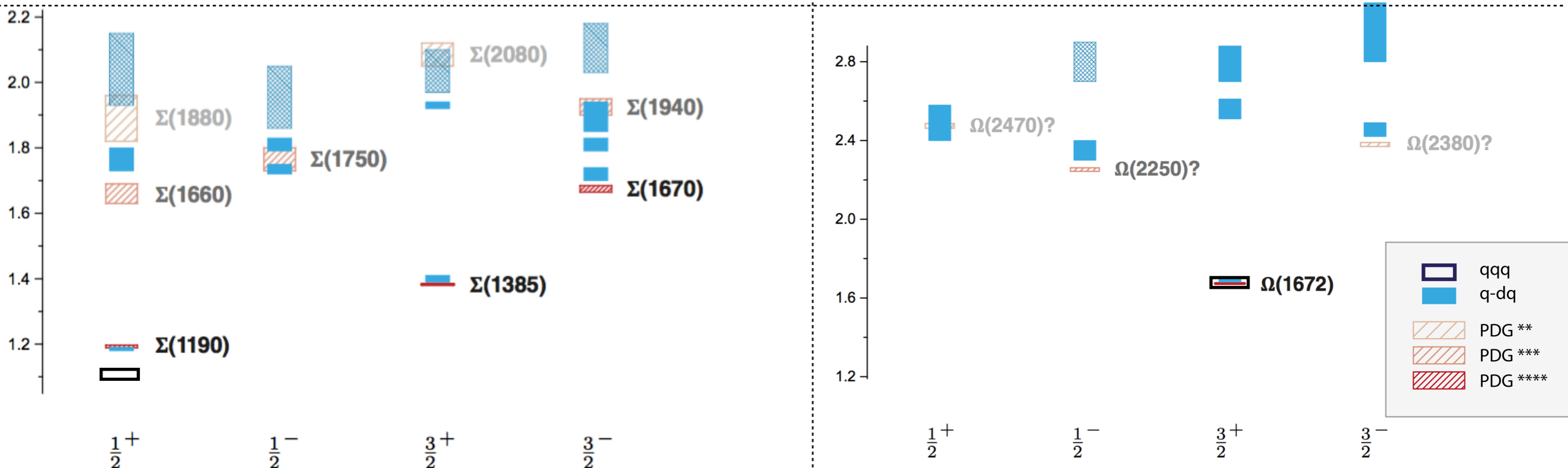
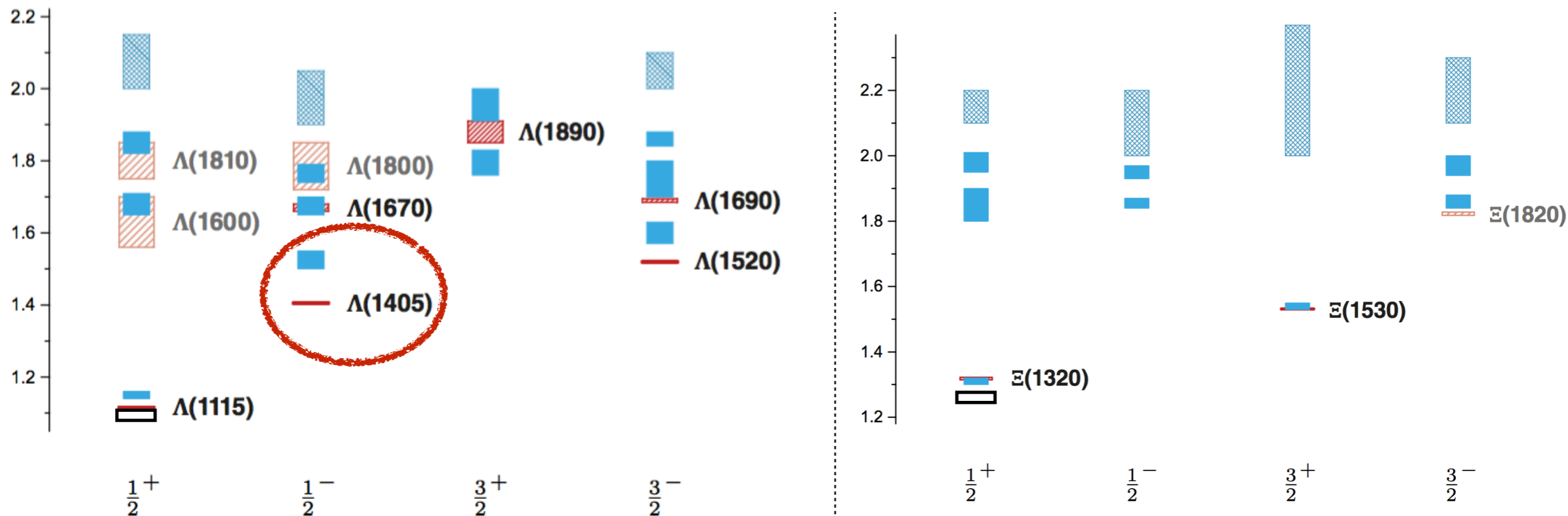
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- three-body agrees with diquark-quark where applicable

Strange baryon spectrum: DSE-RL (preliminary !)



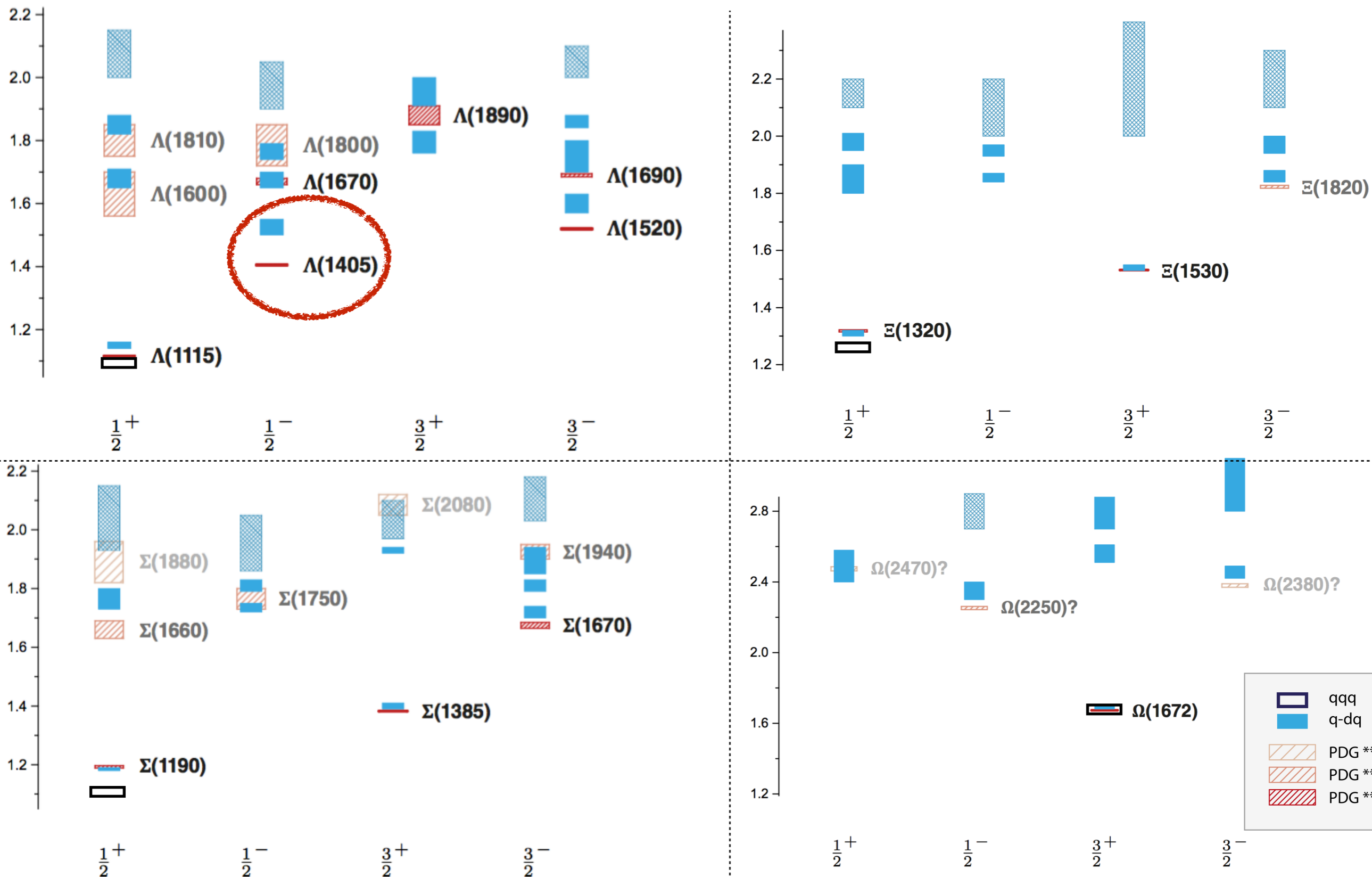
Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2
 CF, Eichmann PoS Hadron 2017 (2018) 007
 Sanchis-Alepuz, CF, PRD 90 (2014) 096001

Strange baryon spectrum: DSE-RL (preliminary !)



Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2
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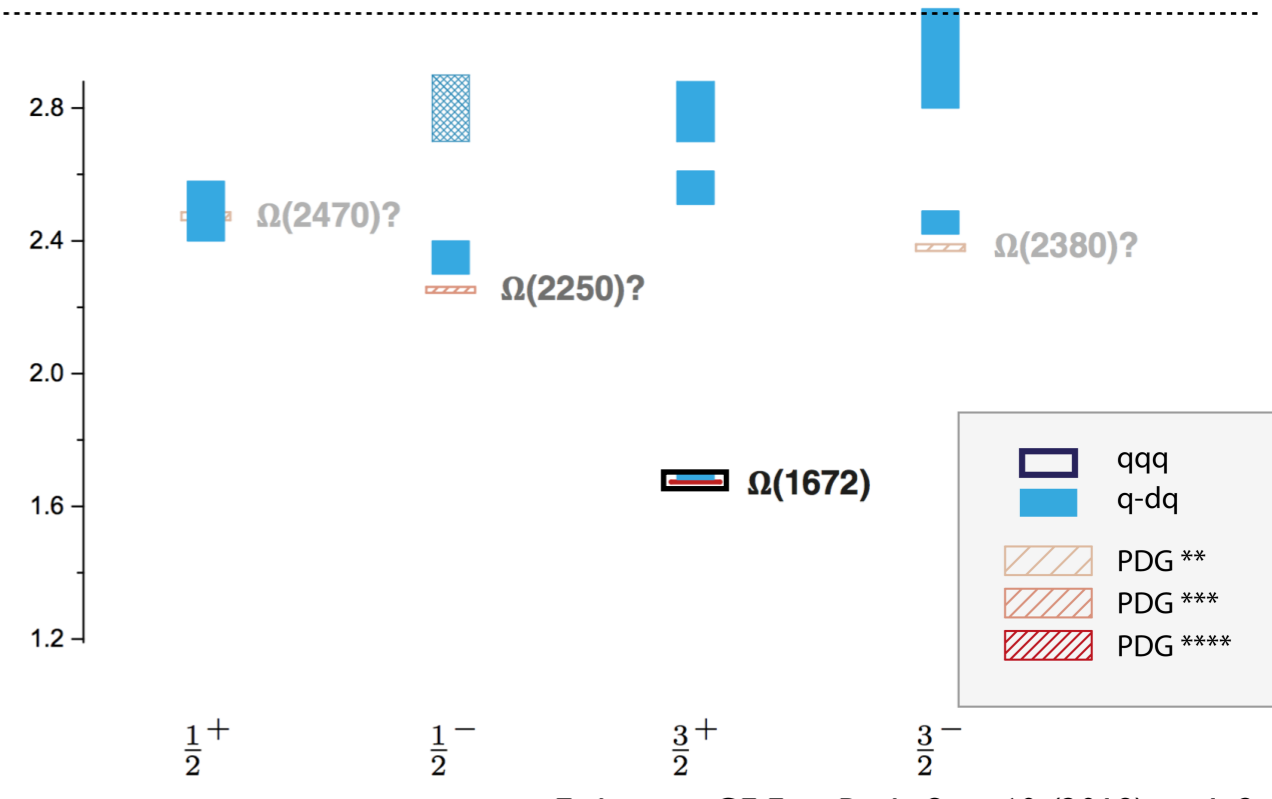
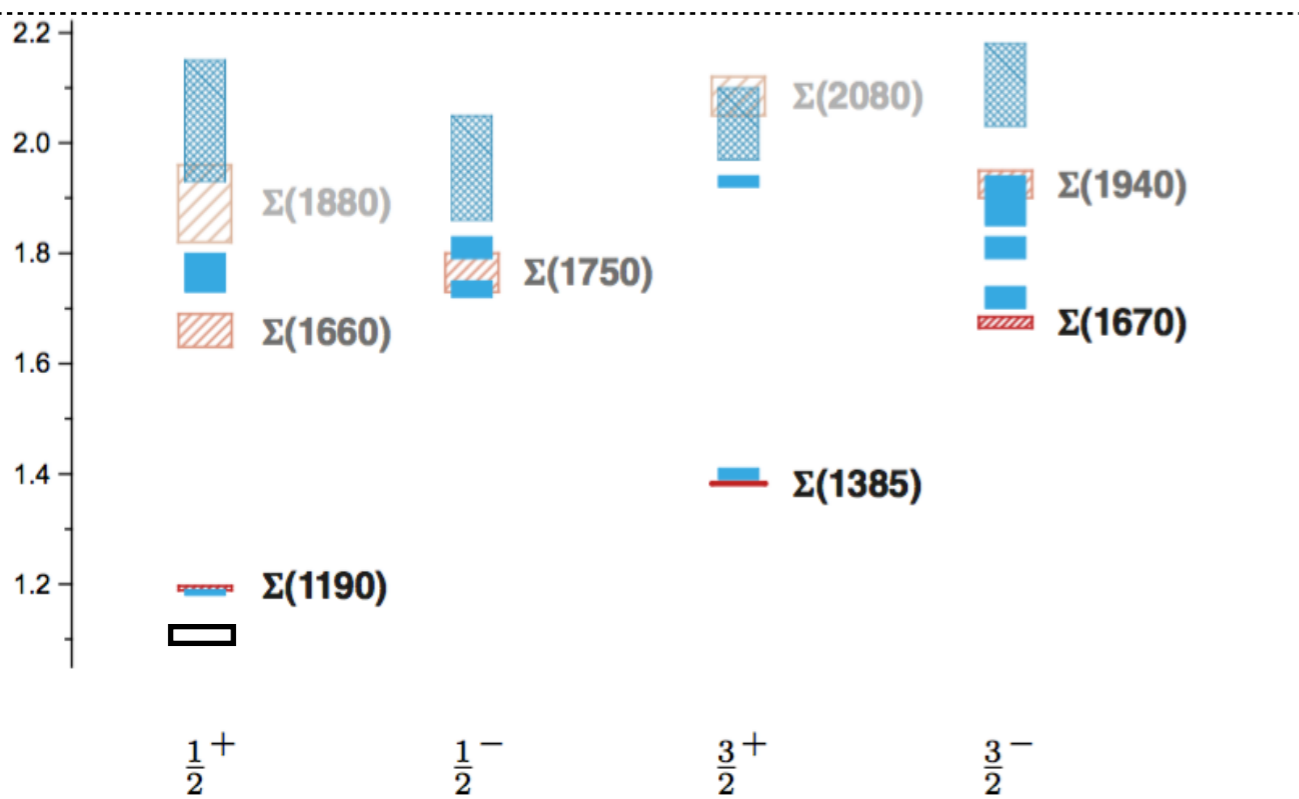
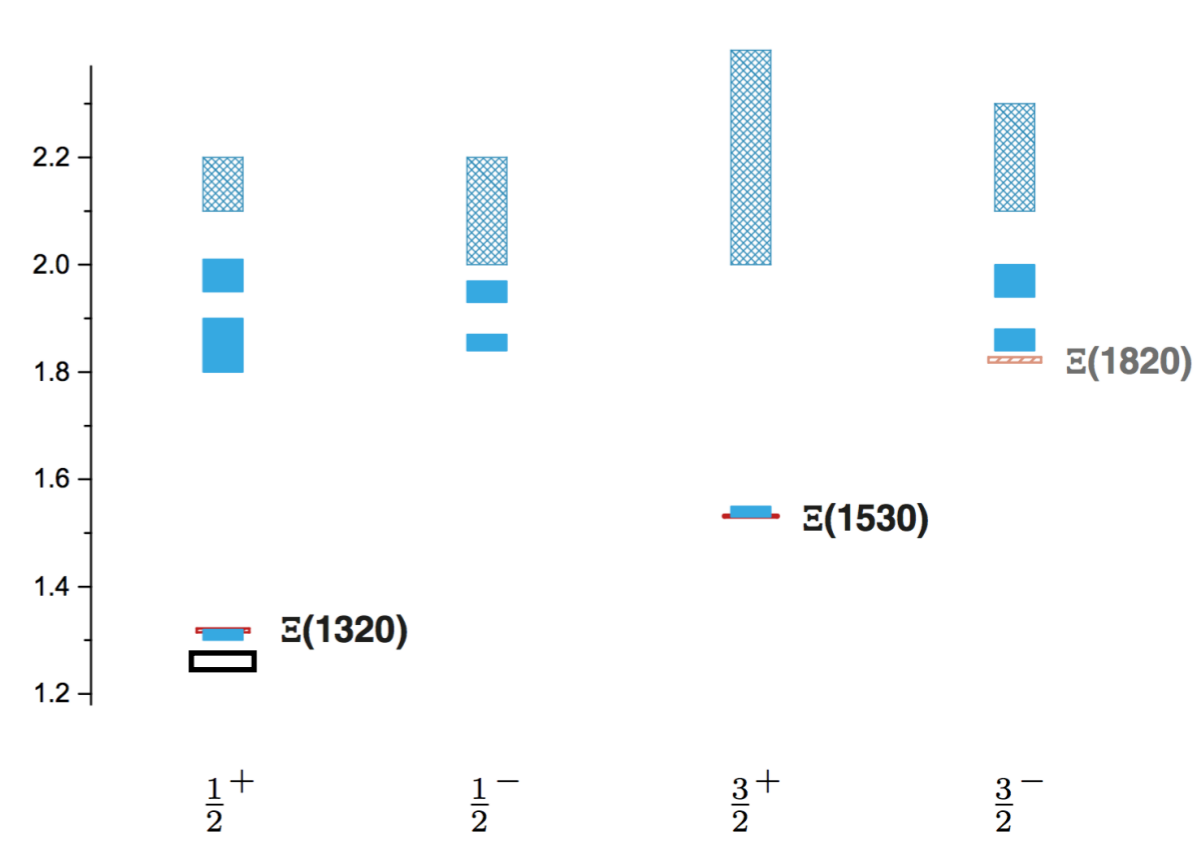
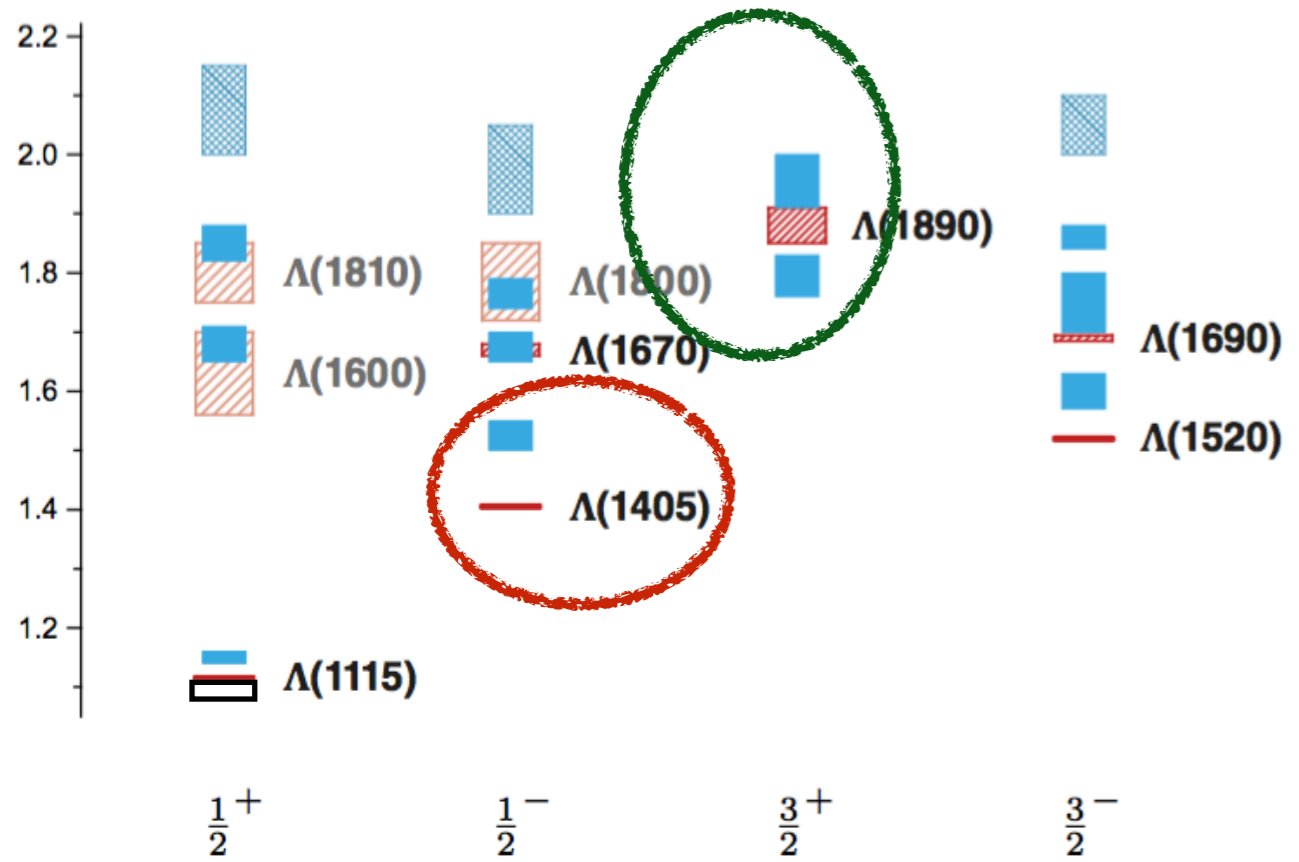
Strange baryon spectrum: DSE-RL (preliminary !)



Bonn-Gatchina (talk of M. Matveev, NSTAR 2019)
 Sarantsev, Matveev, et al EPJ A 55 (2019) 10, 180

Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2
 CF, Eichmann PoS Hadron 2017 (2018) 007
 Sanchis-Alepuz, CF, PRD 90 (2014) 096001

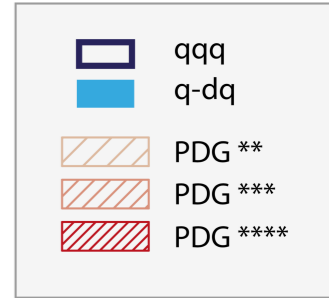
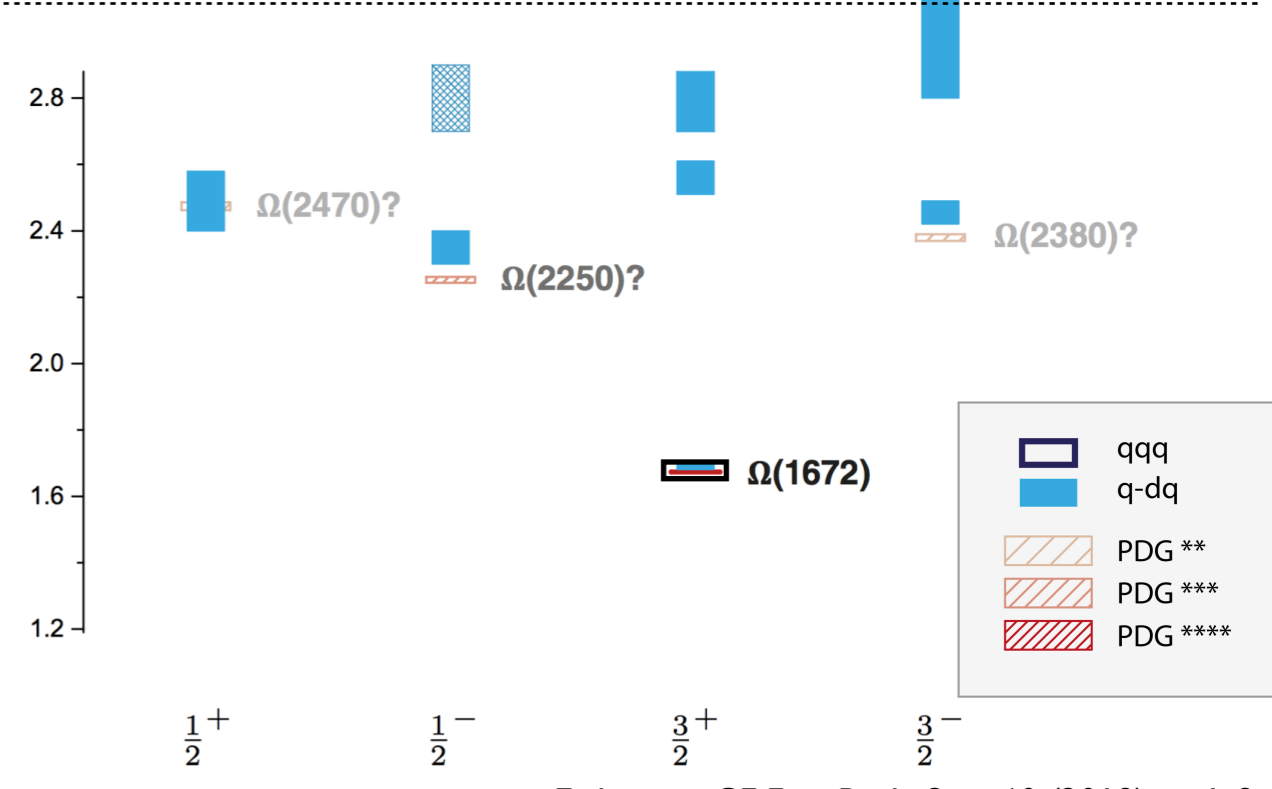
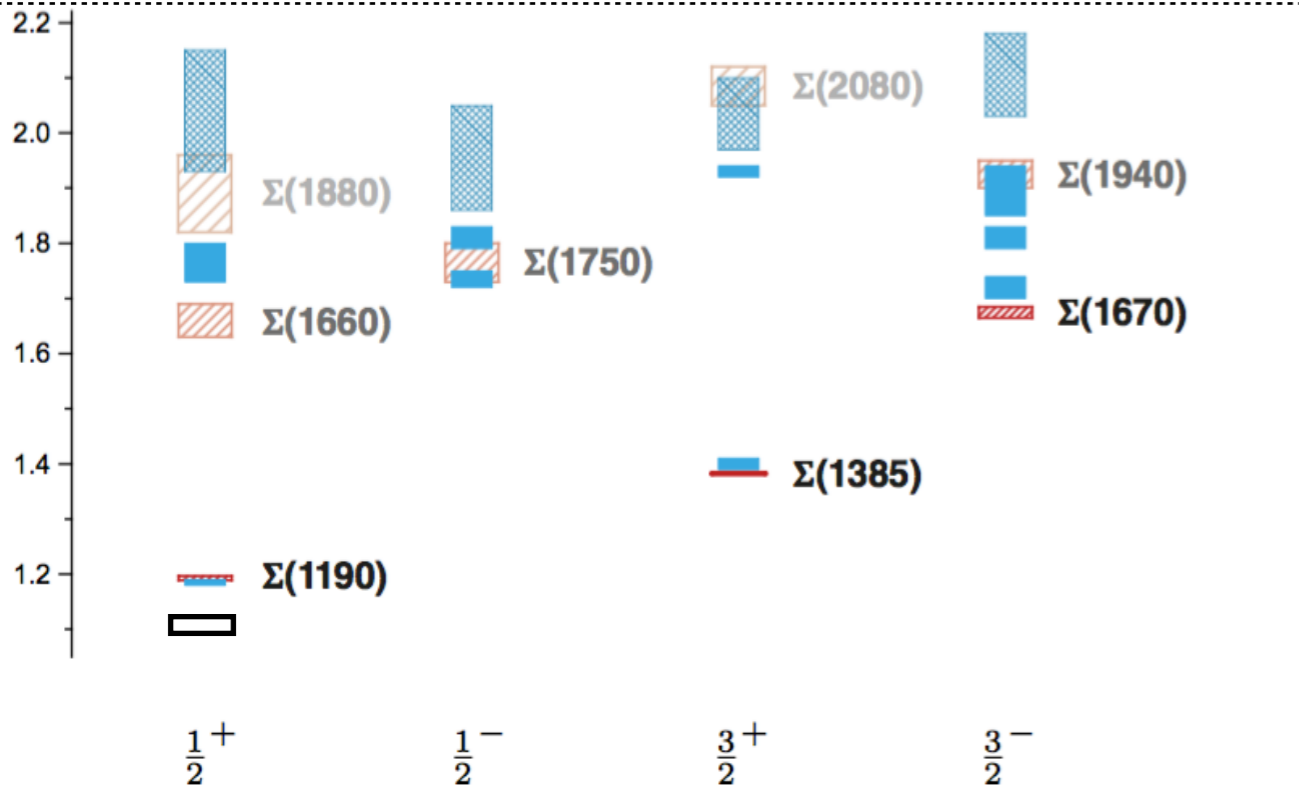
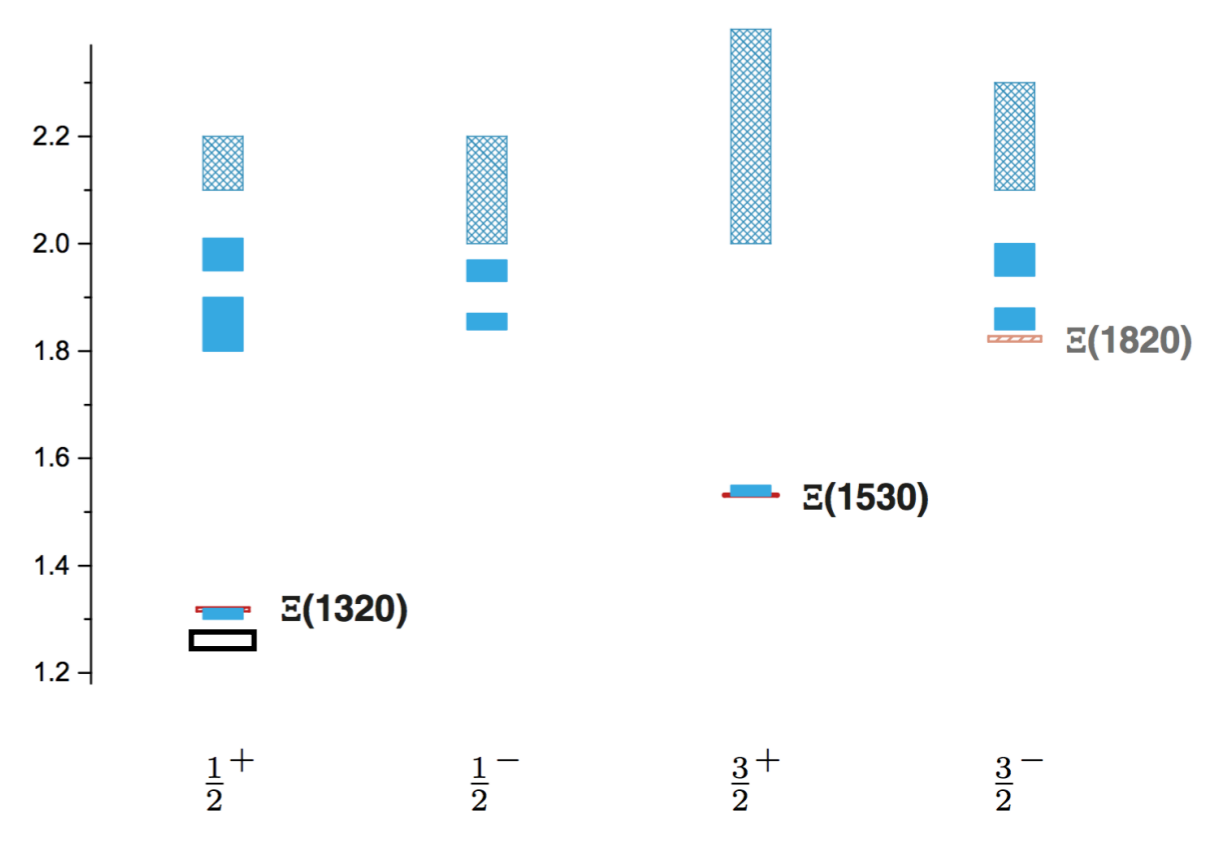
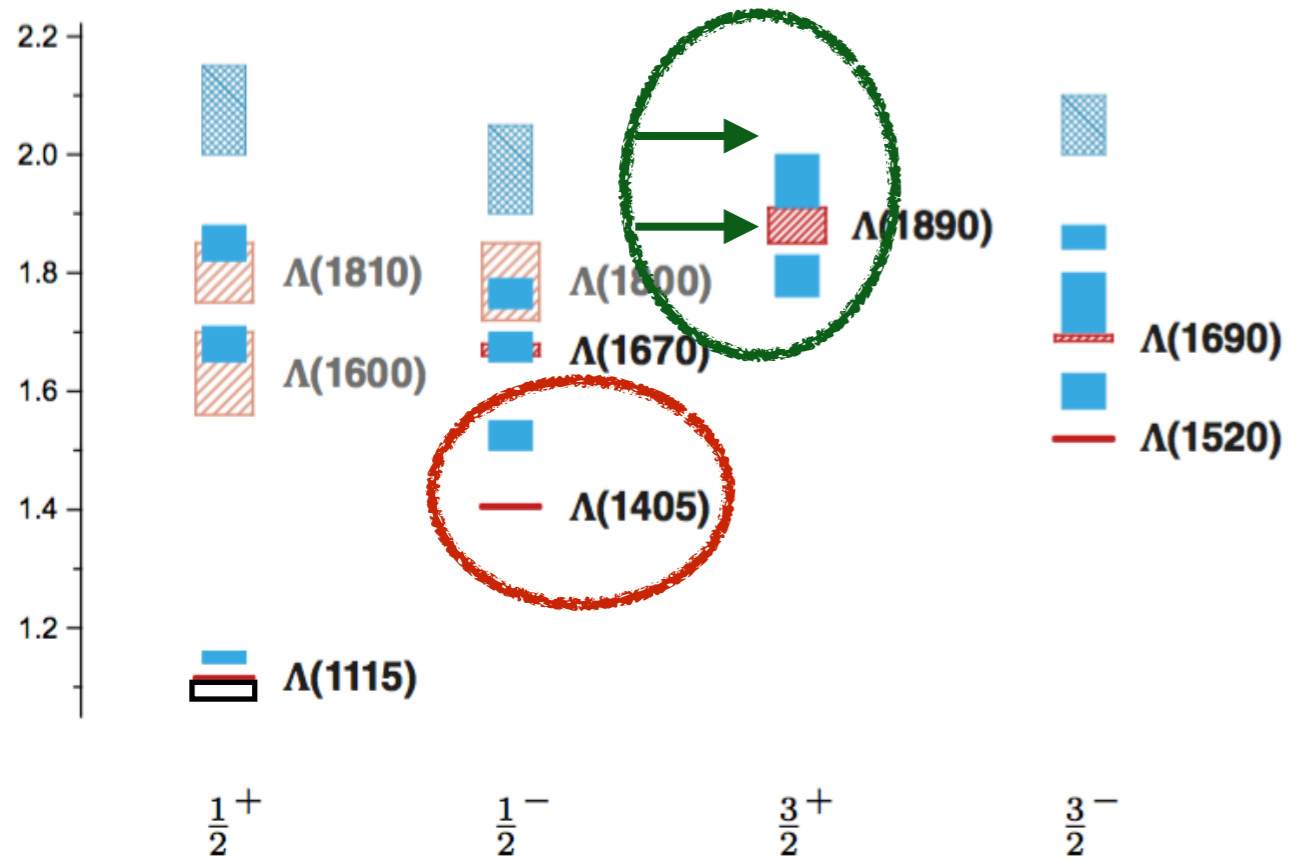
Strange baryon spectrum: DSE-RL (preliminary !)



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 Sarantsev, Matveev, et al EPJ A 55 (2019) 10, 180

Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2
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 Sanchis-Alepuz, CF, PRD 90 (2014) 096001

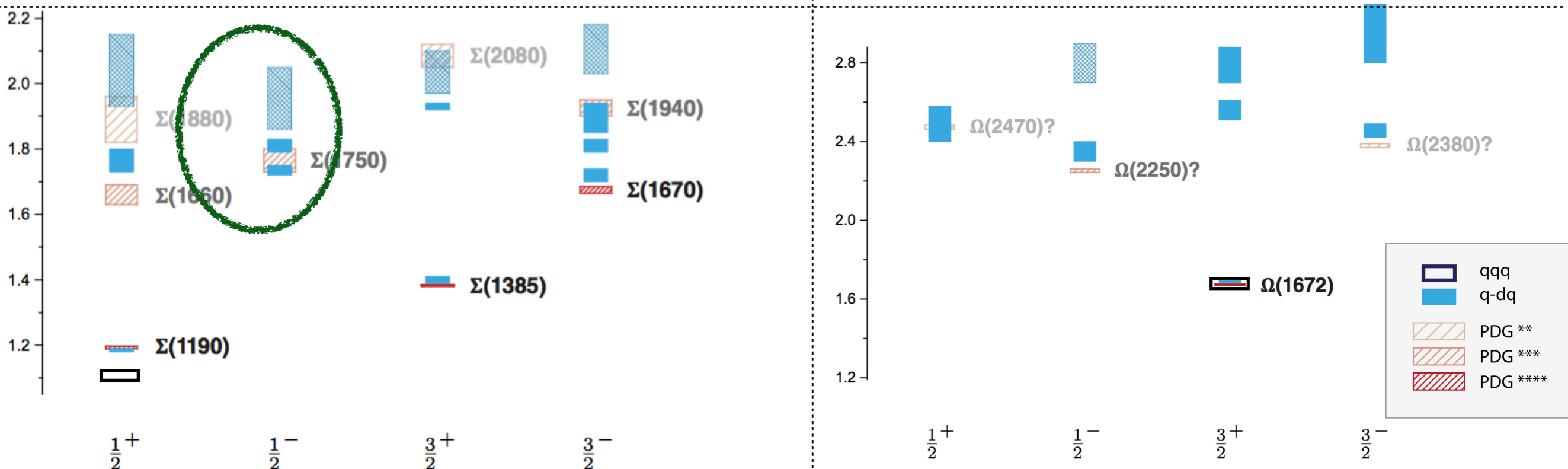
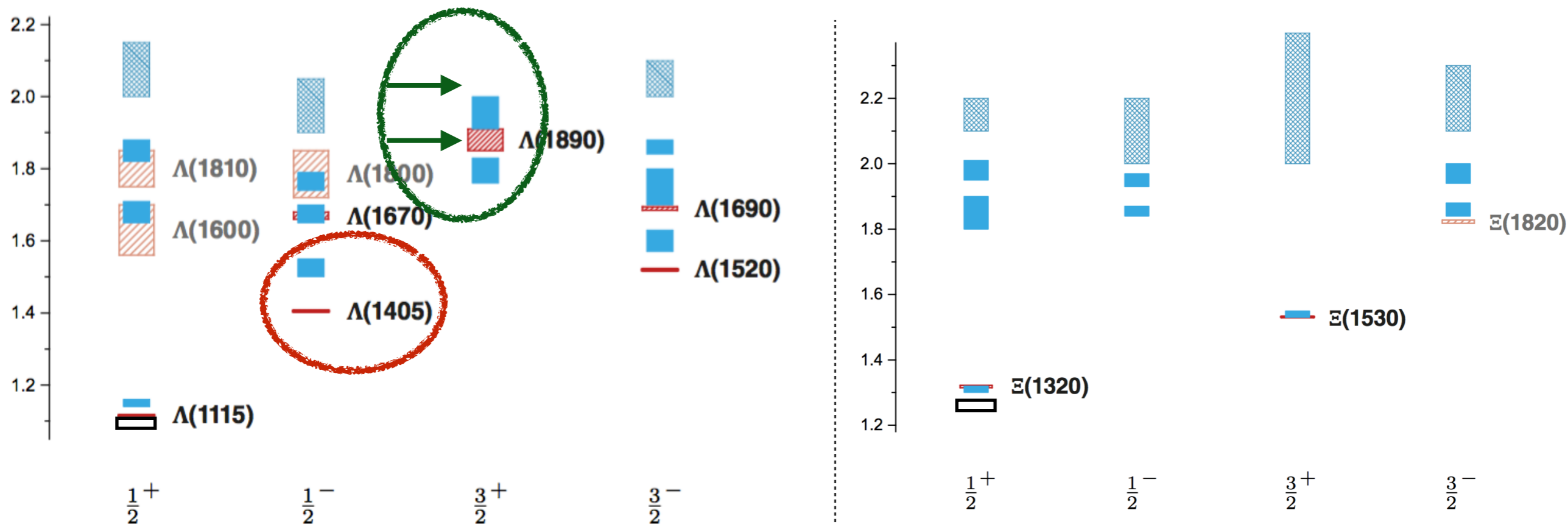
Strange baryon spectrum: DSE-RL (preliminary !)



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 Sarantsev, Matveev, et al EPJ A 55 (2019) 10, 180

Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2
 CF, Eichmann PoS Hadron 2017 (2018) 007
 Sanchis-Alepuz, CF, PRD 90 (2014) 096001

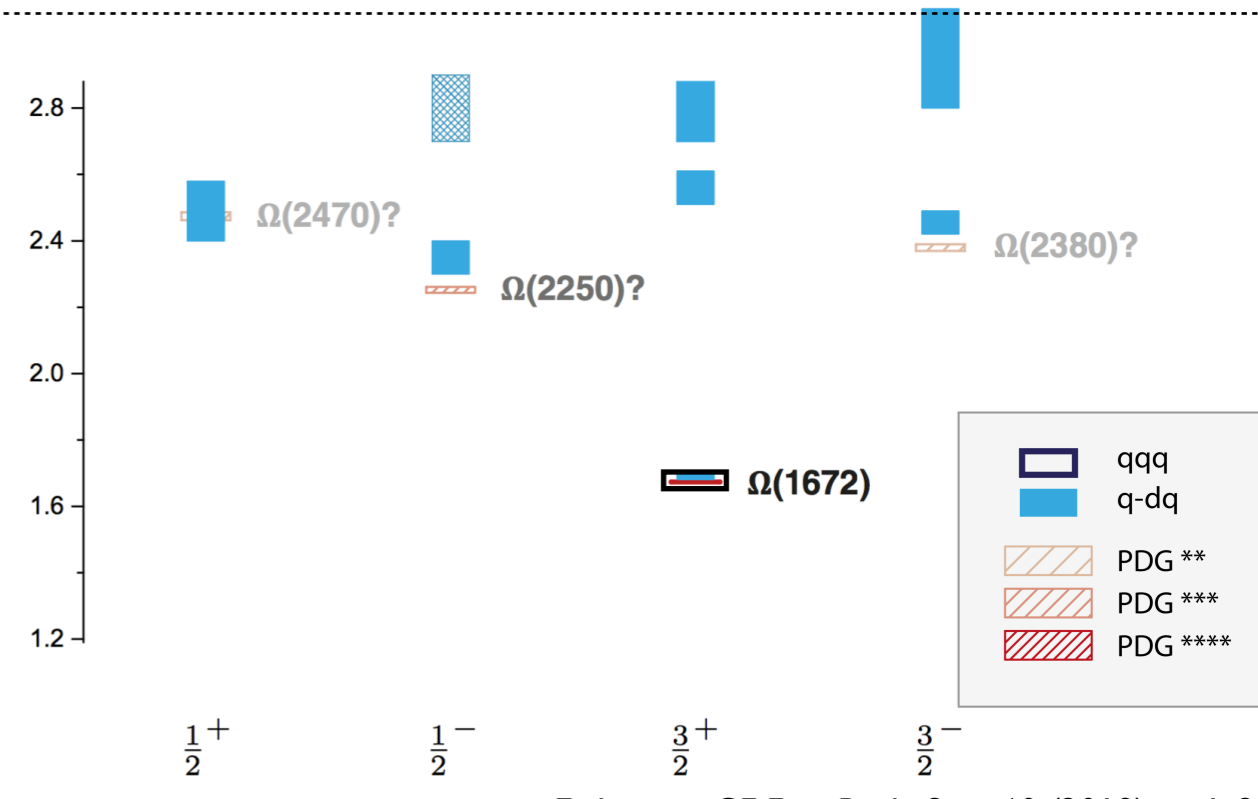
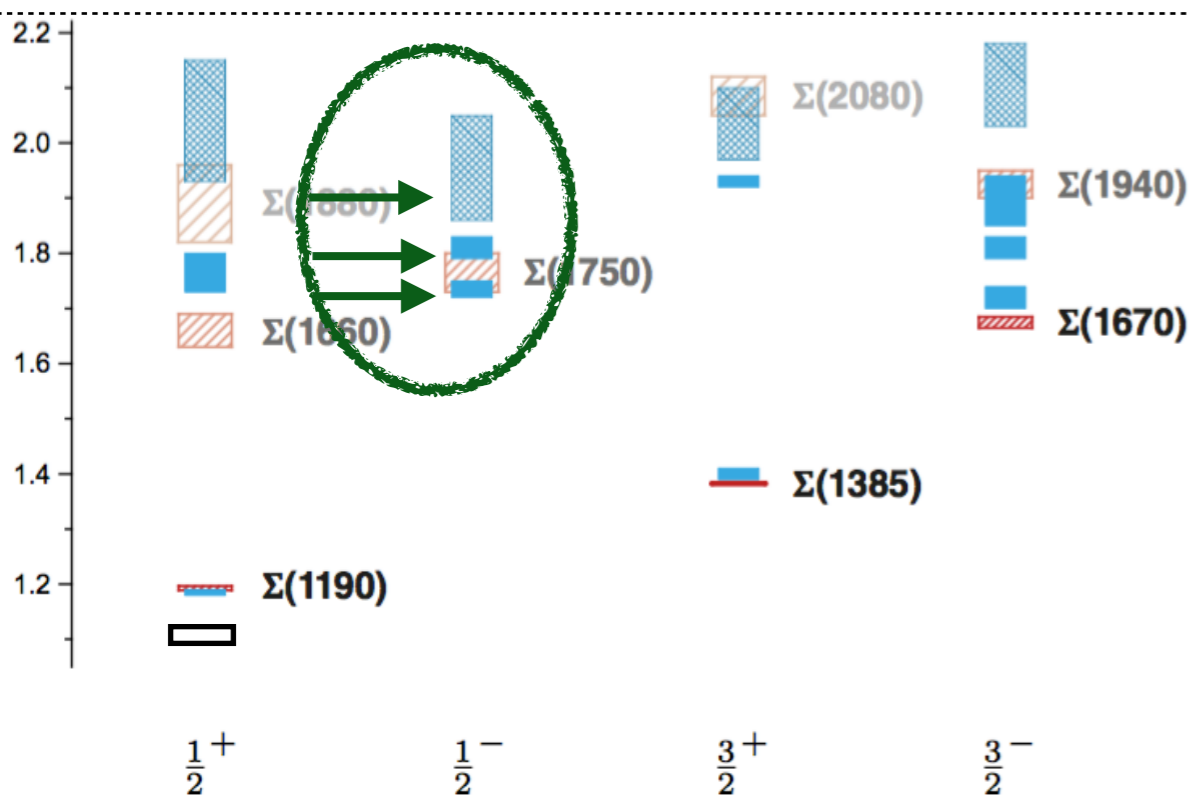
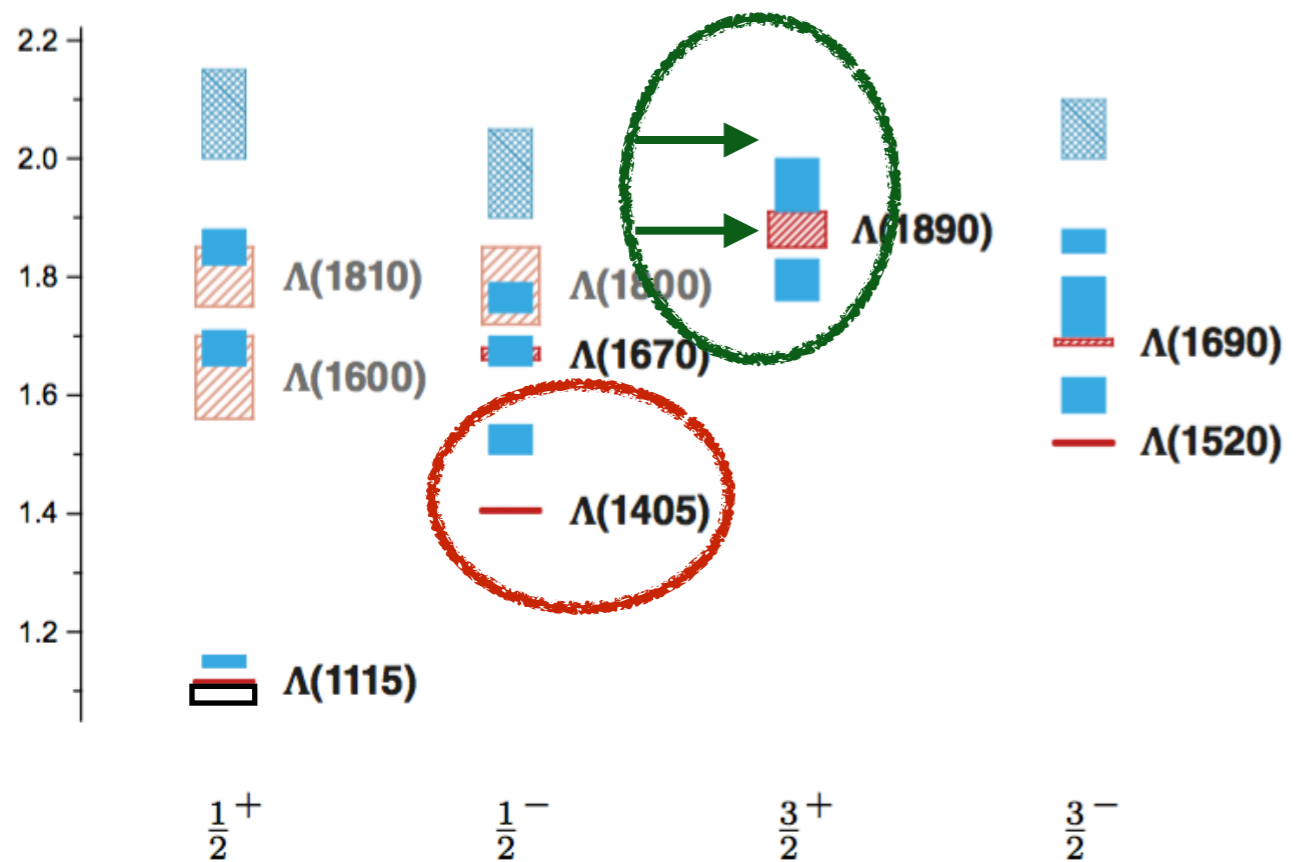
Strange baryon spectrum: DSE-RL (preliminary !)



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Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2
 CF, Eichmann PoS Hadron 2017 (2018) 007
 Sanchis-Alepuz, CF, PRD 90 (2014) 096001

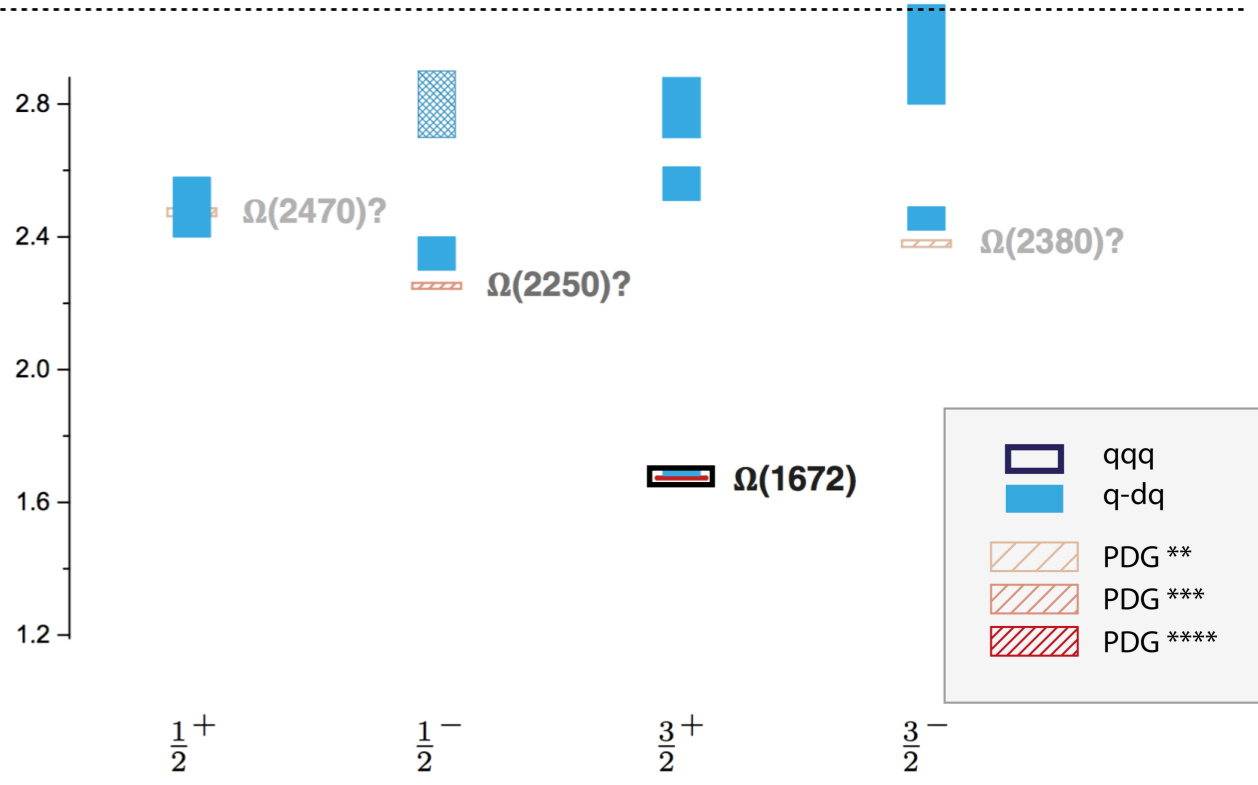
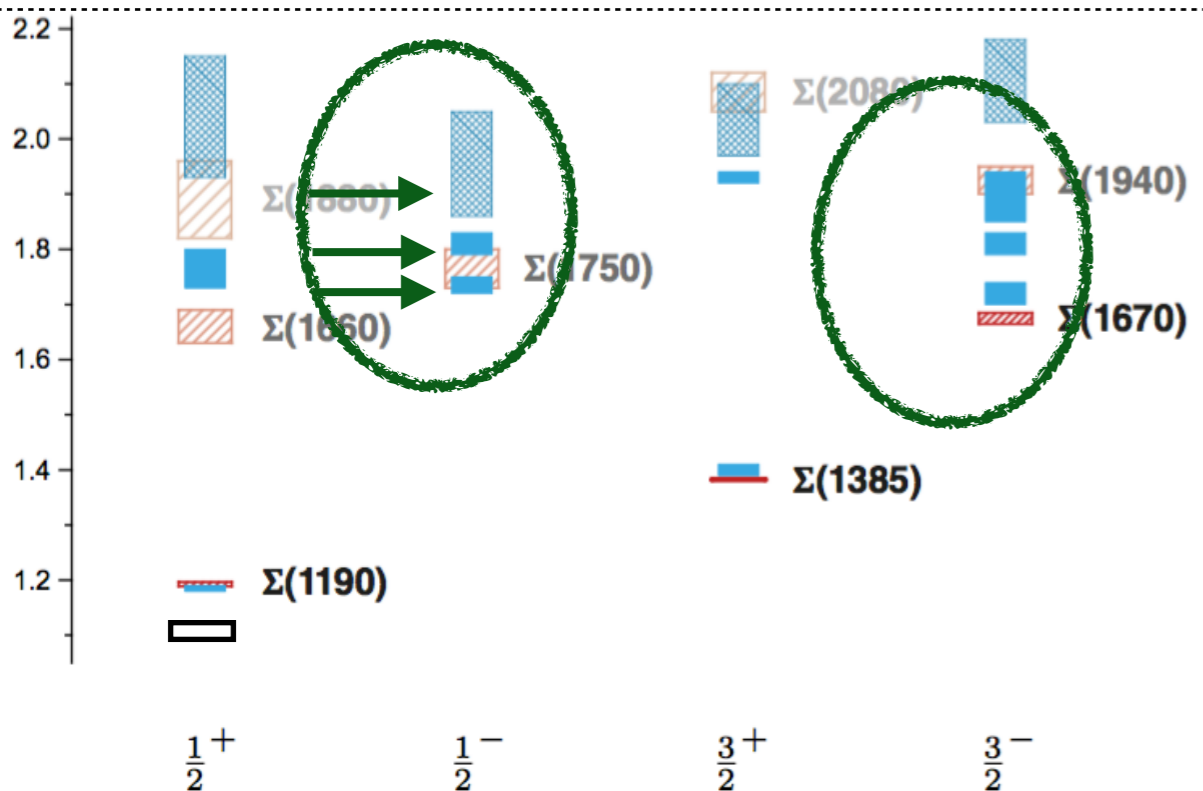
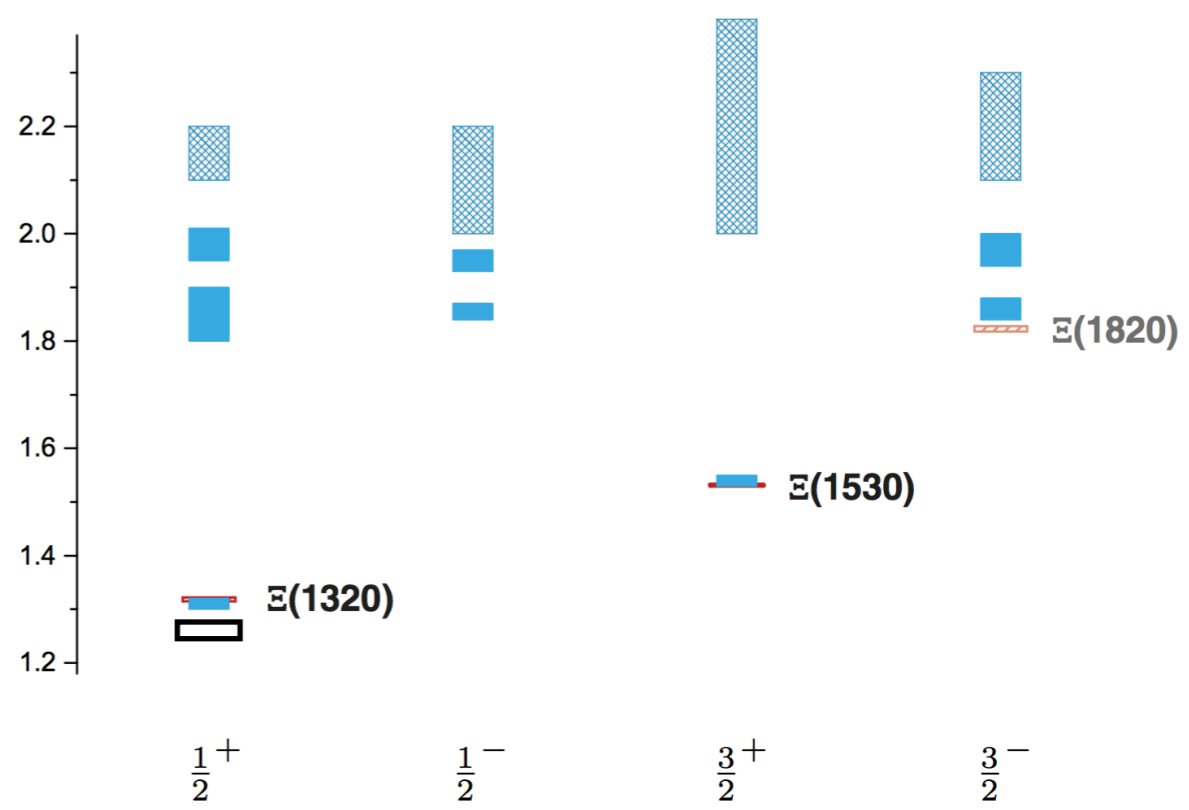
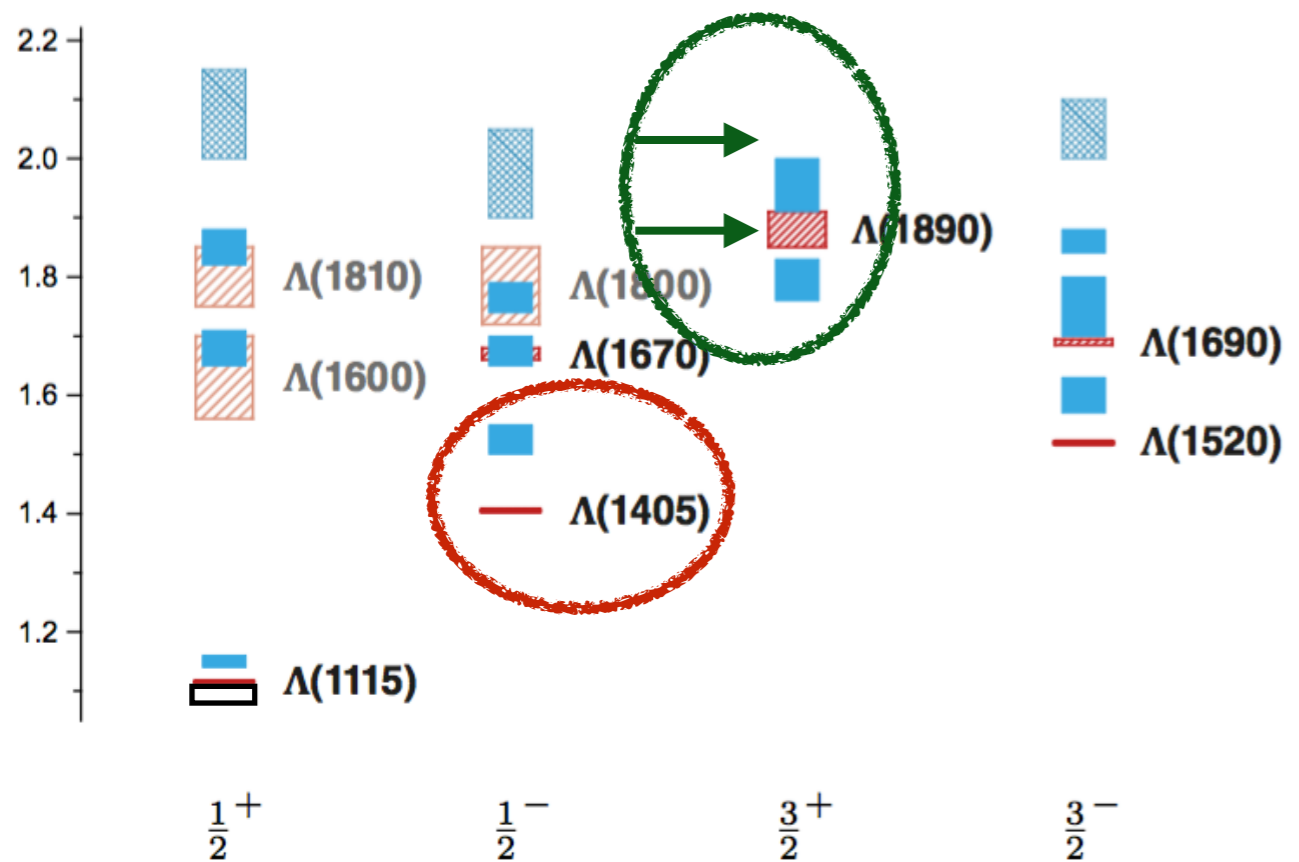
Strange baryon spectrum: DSE-RL (preliminary !)



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 Sarantsev, Matveev, et al EPJ A 55 (2019) 10, 180

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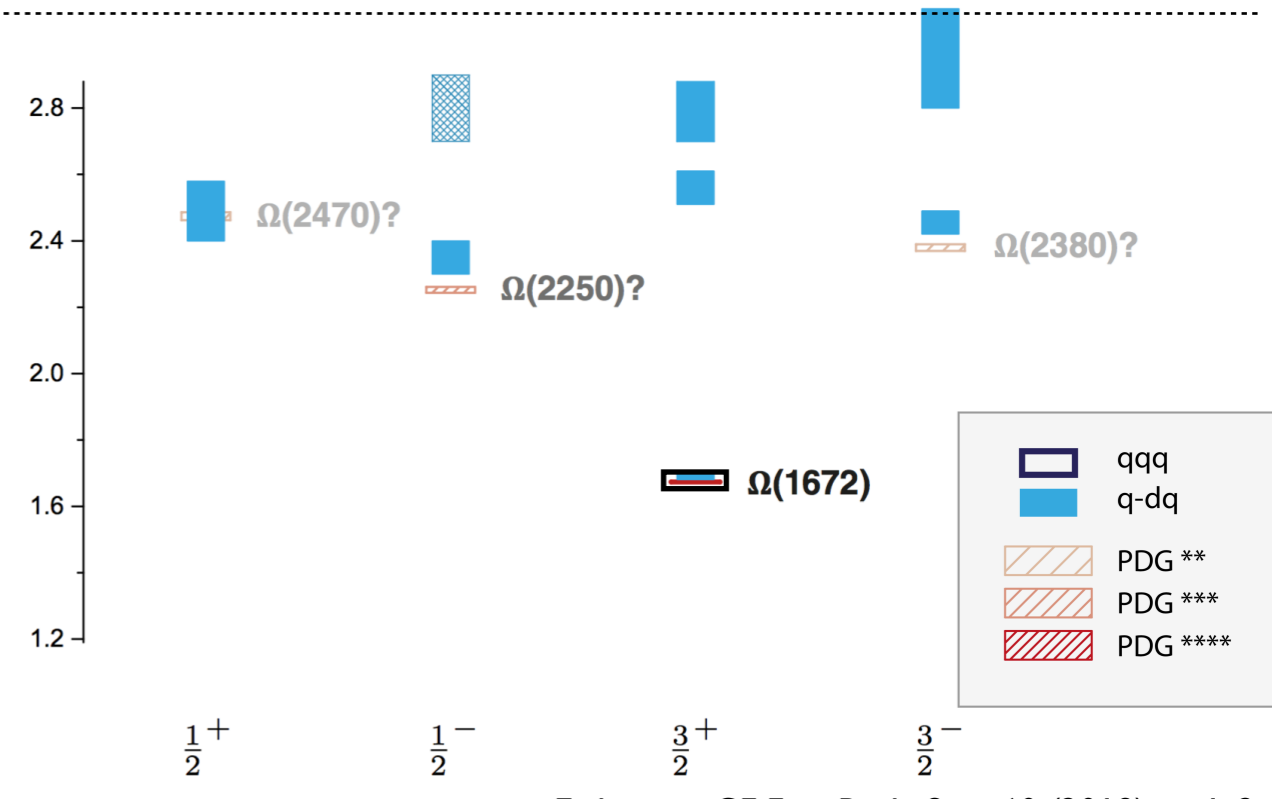
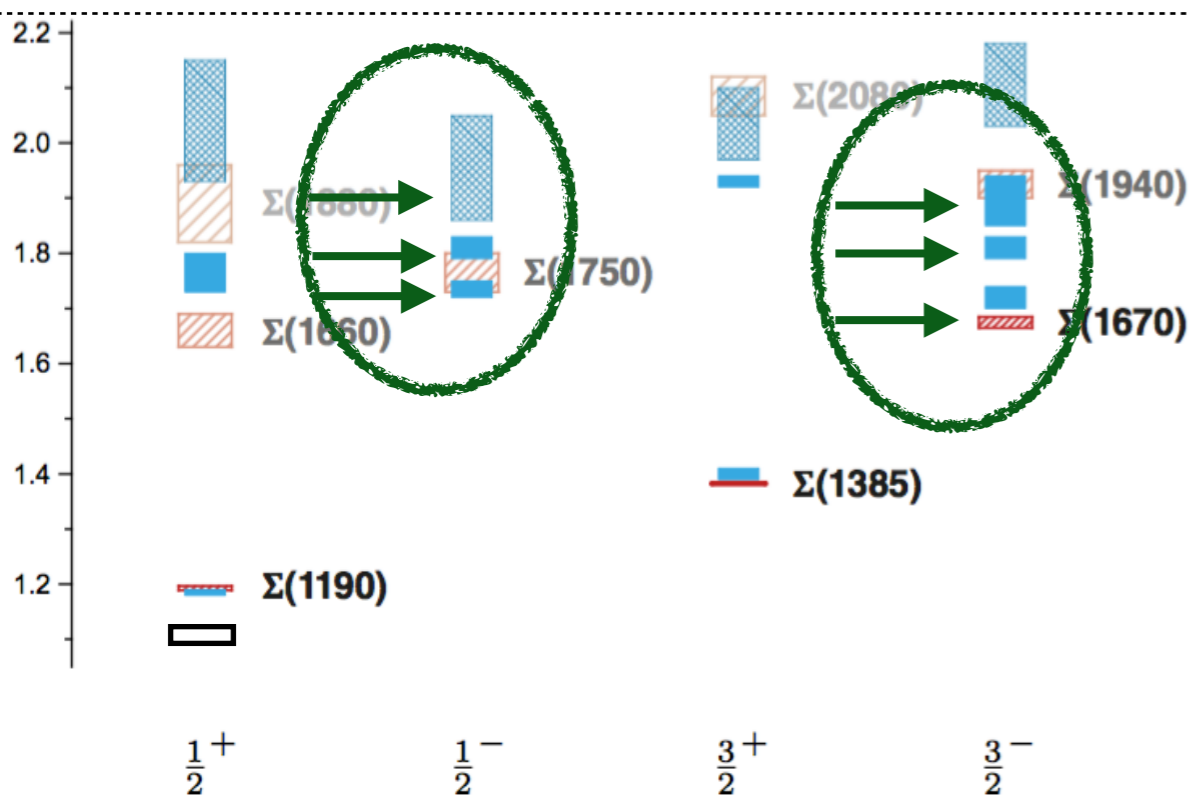
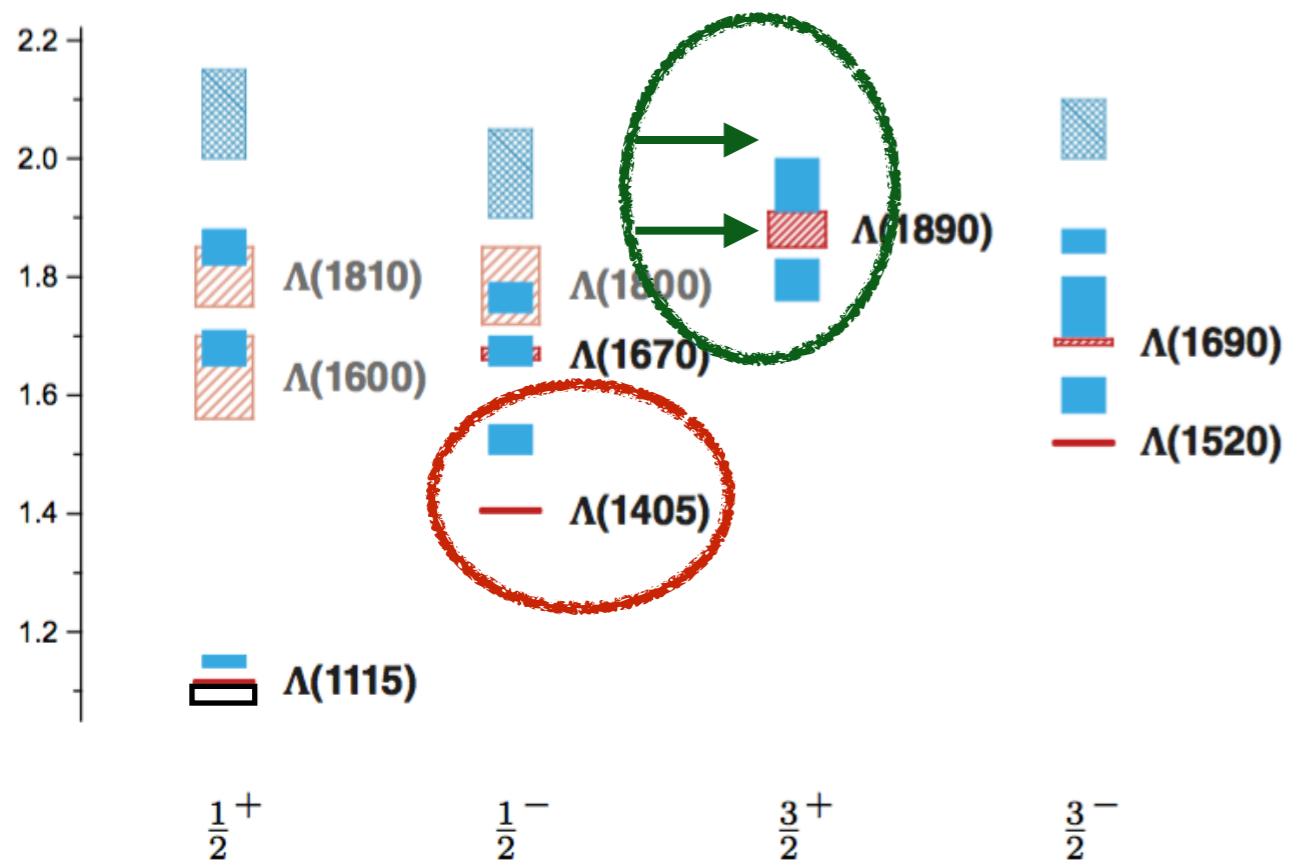
Strange baryon spectrum: DSE-RL (preliminary !)



Bonn-Gatchina (talk of M. Matveev, NSTAR 2019)
 Sarantsev, Matveev, et al EPJ A 55 (2019) 10, 180

Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2
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 Sanchis-Alepuz, CF, PRD 90 (2014) 096001

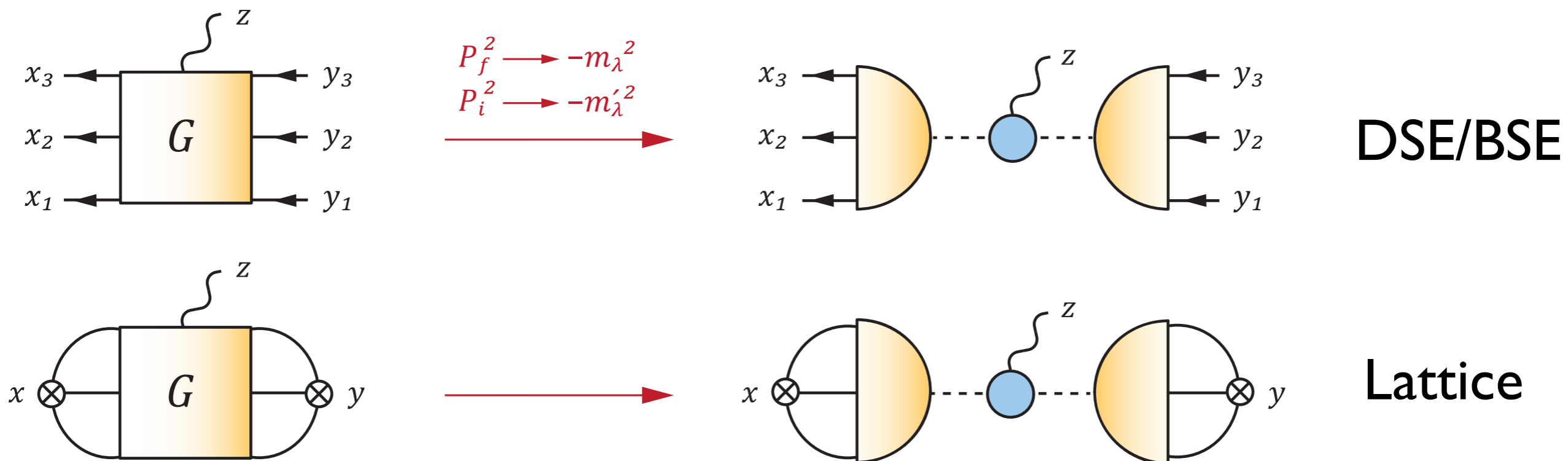
Strange baryon spectrum: DSE-RL (preliminary !)



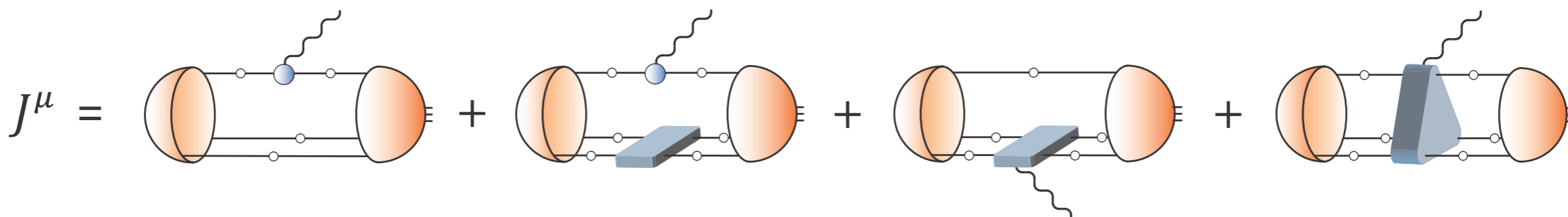
Bonn-Gatchina (talk of M. Matveev, NSTAR 2019)
 Sarantsev, Matveev, et al EPJ A 55 (2019) 10, 180

Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2
 CF, Eichmann PoS Hadron 2017 (2018) 007
 Sanchis-Alepuz, CF, PRD 90 (2014) 096001

Extracting form factors from correlators



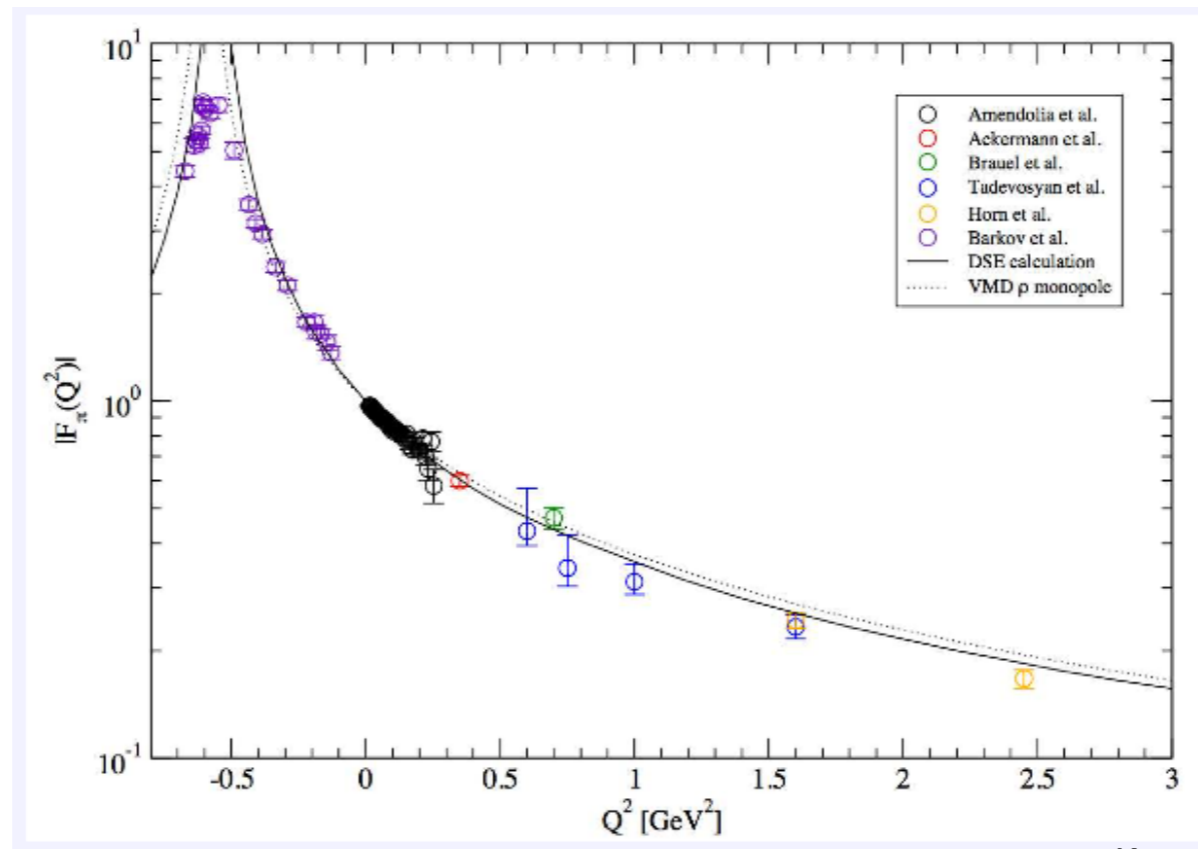
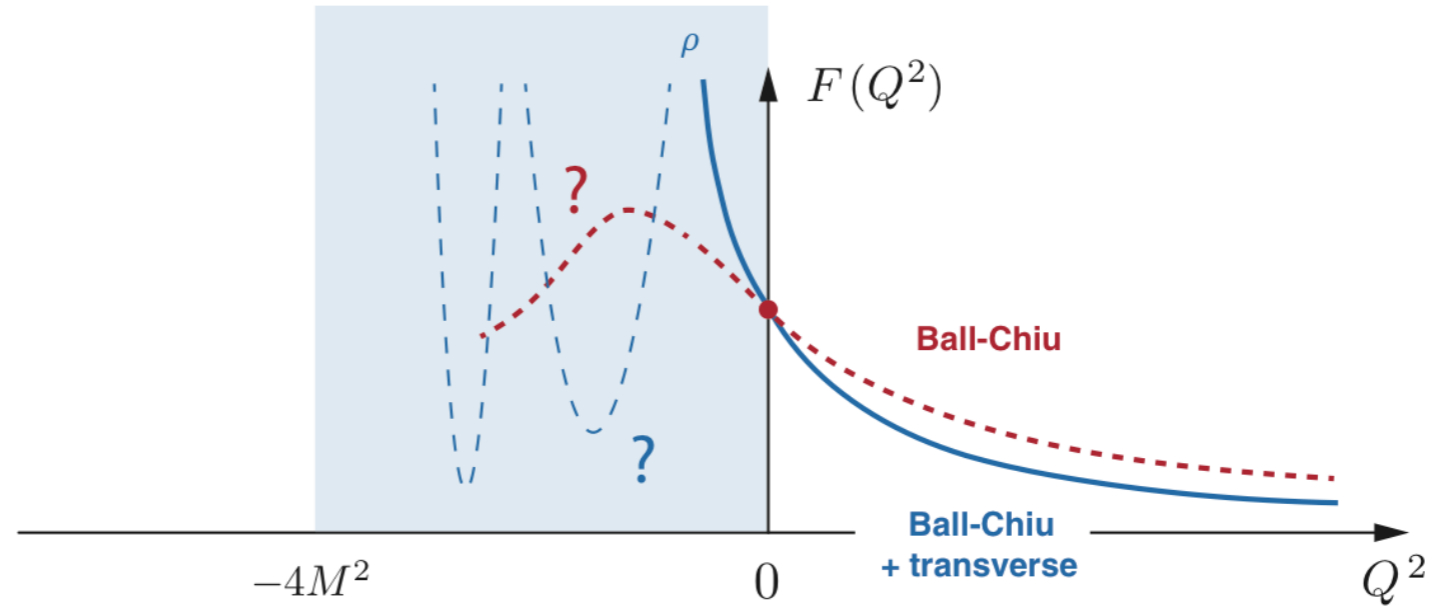
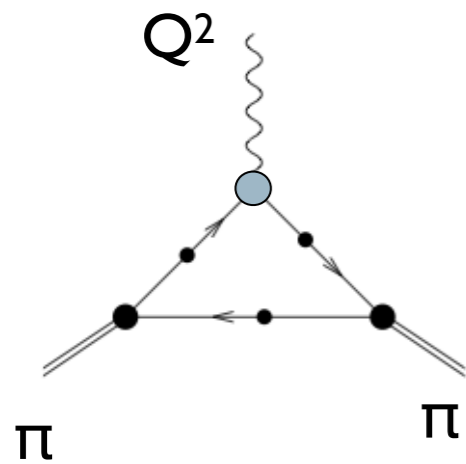
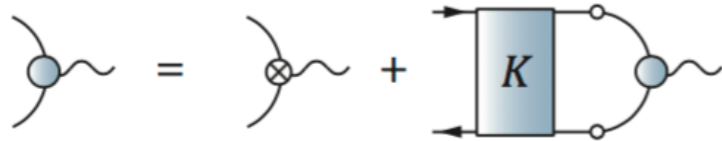
Form factor from BSEs (derived from equation of motion for G and 'gauging')



- exact equation for baryon form factors

Quark-photon vertex and pion form factors

Pion form factor:

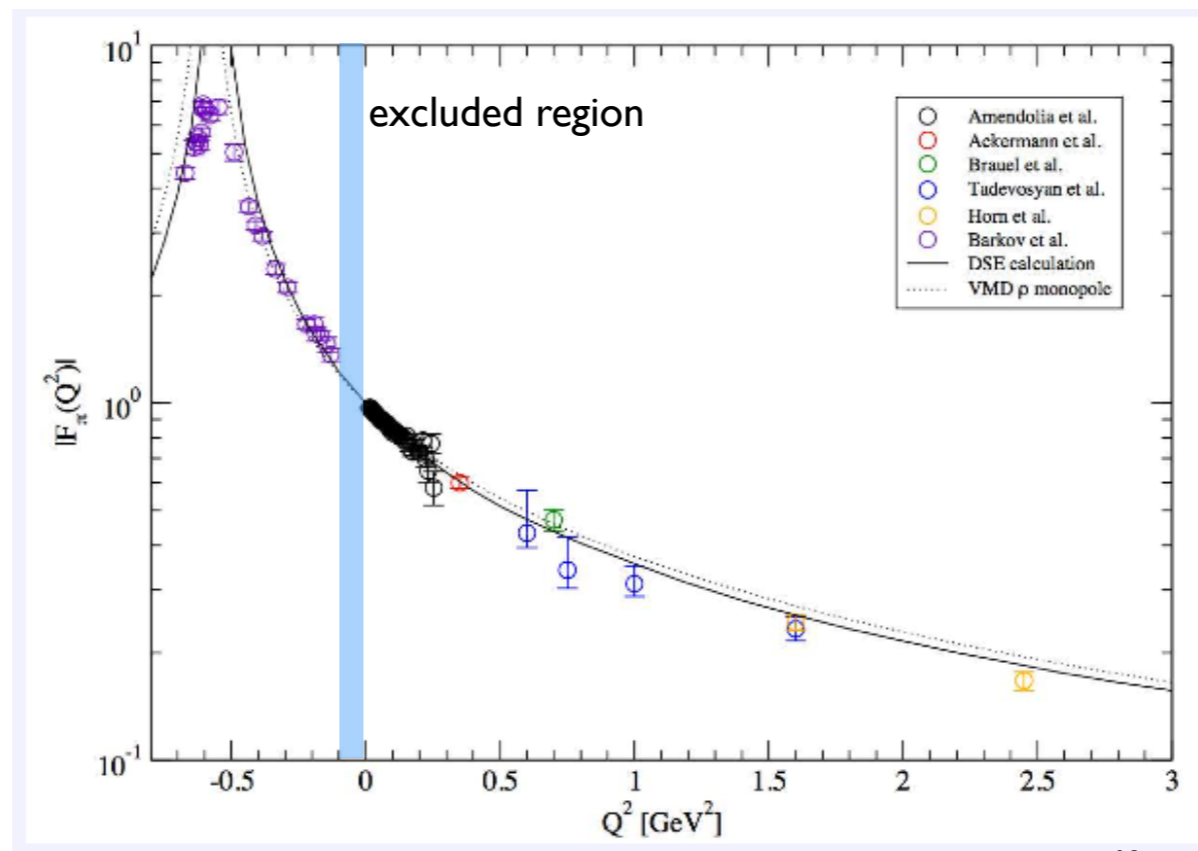
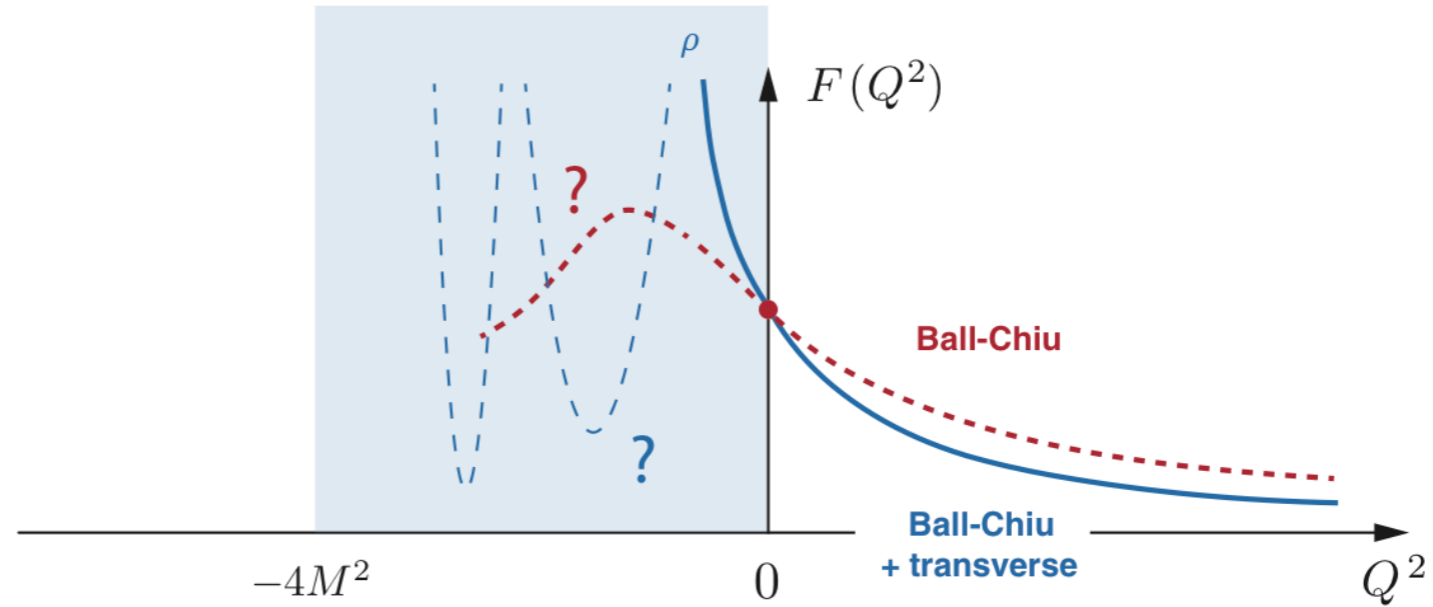
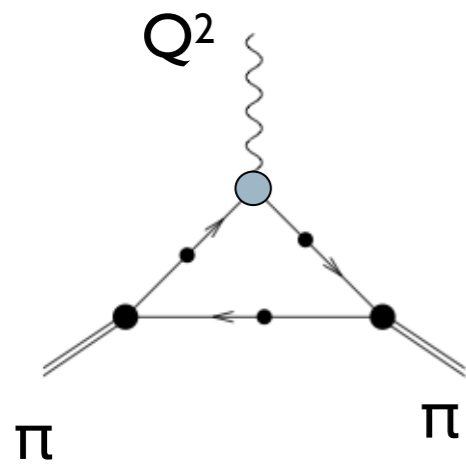
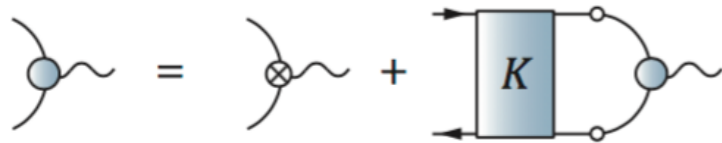


Krassnigg, Schladming 2011; Maris, Tandy NPPS 161, 2006

Vector meson poles dynamically generated!

Quark-photon vertex and pion form factors

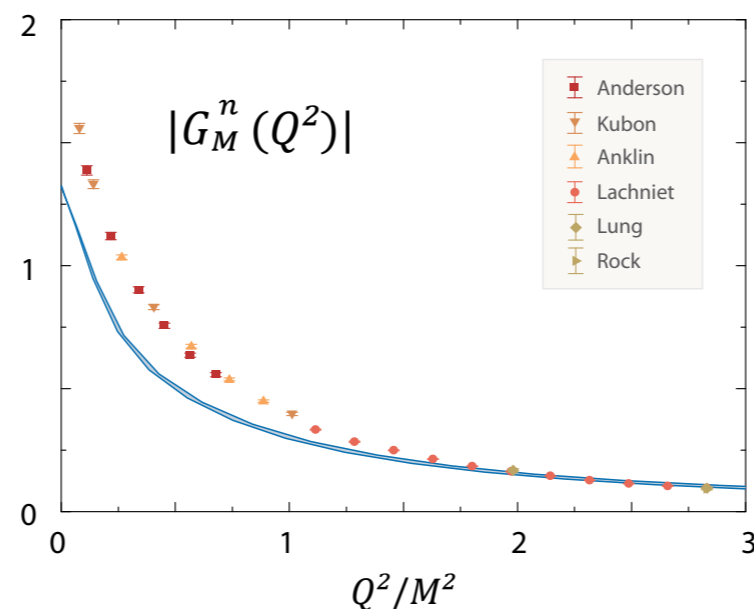
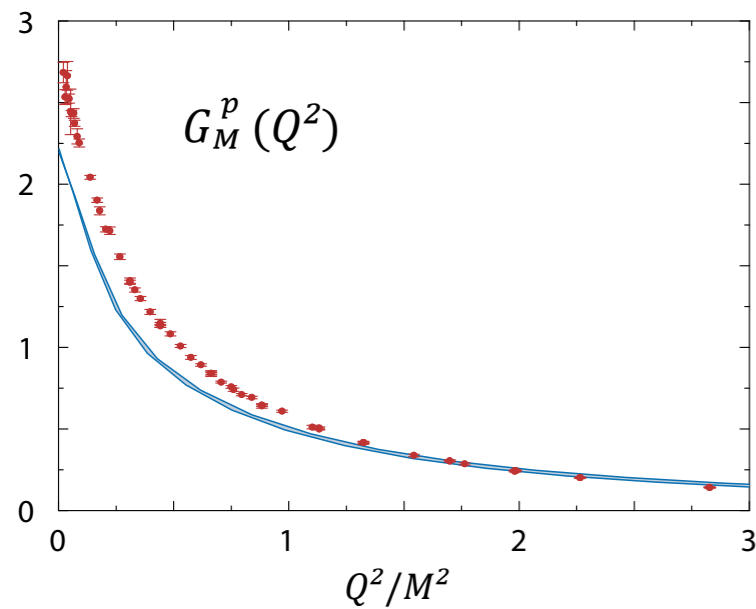
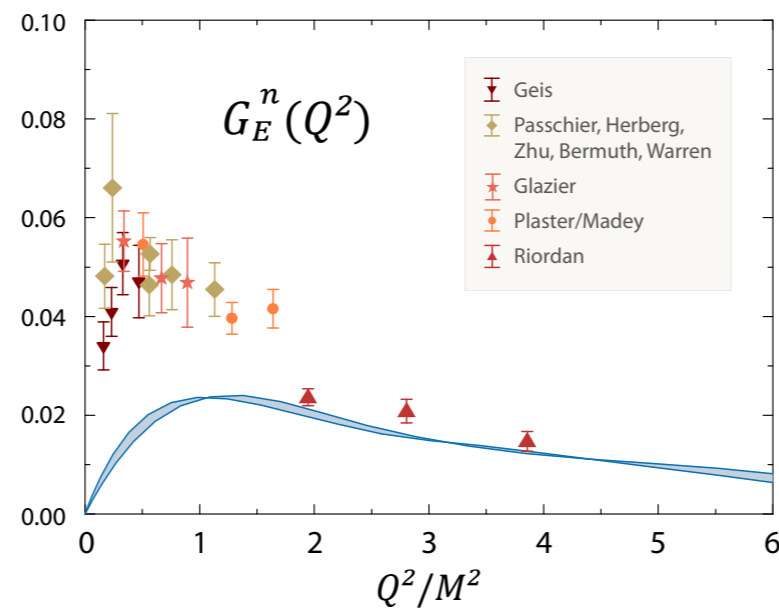
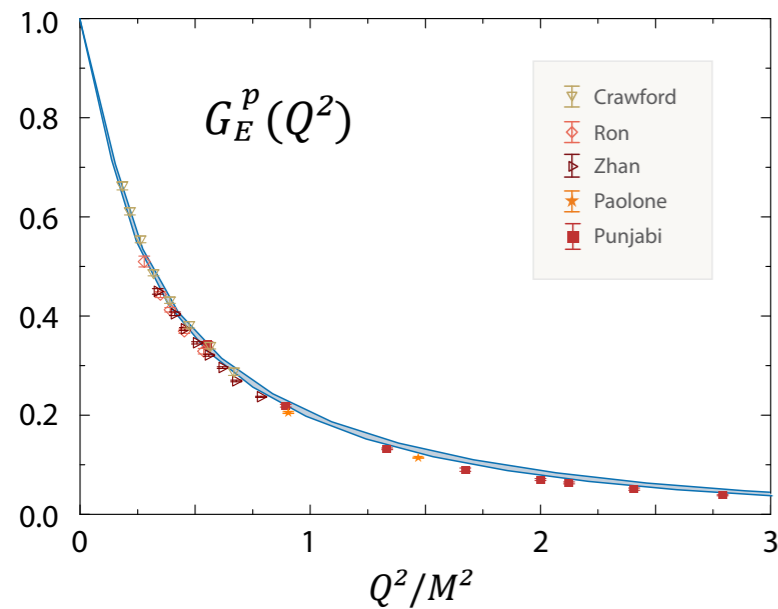
Pion form factor:



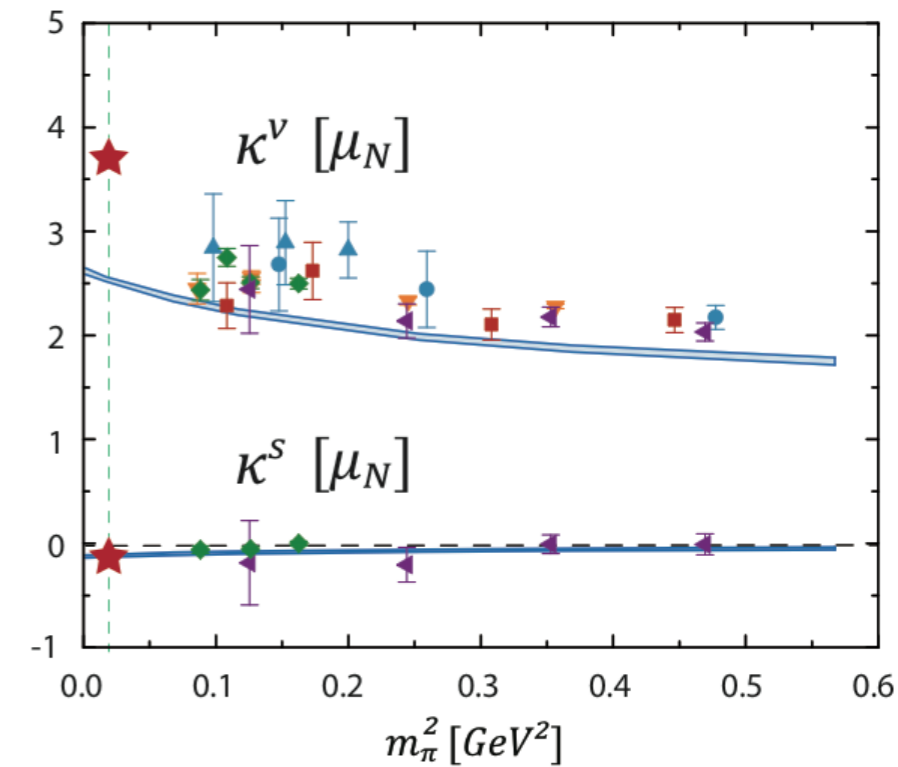
Krassnigg, Schladming 2011; Maris, Tandy NPPS 161, 2006

Vector meson poles dynamically generated!

Nucleon emFF and magnetic moments (three-body)



Isovector (p-n), isoscalar (p+n):



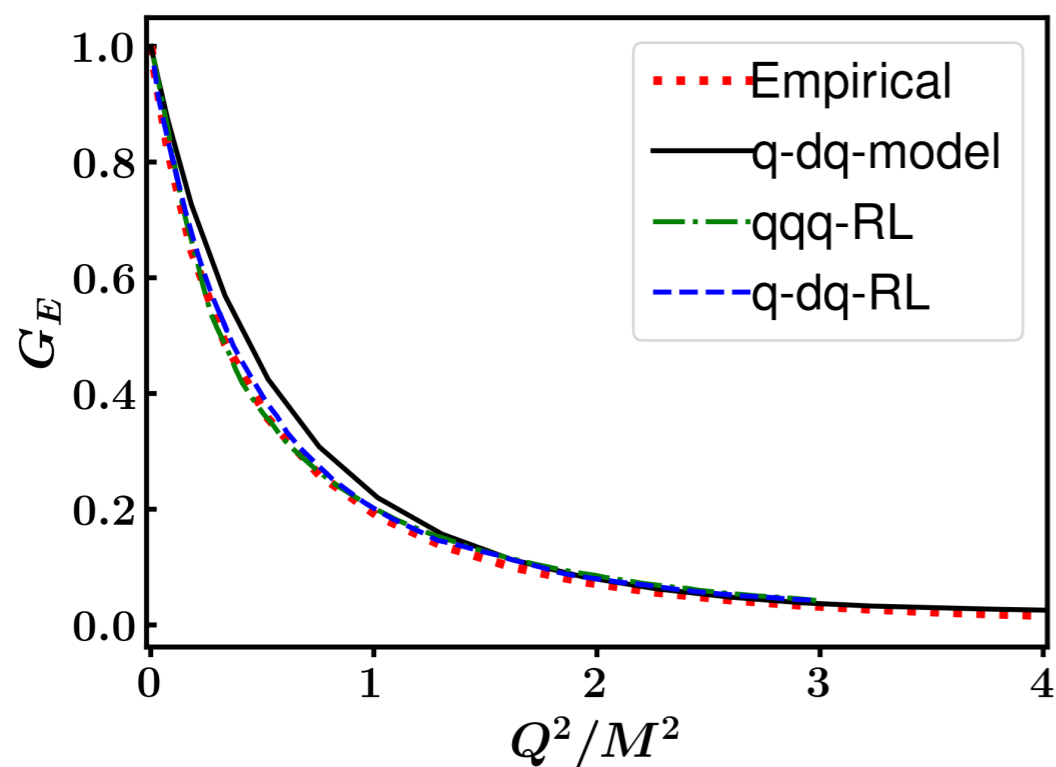
- missing **pion cloud** effects
- similar for axial form factors

Eichmann, PRD 84 (2011)

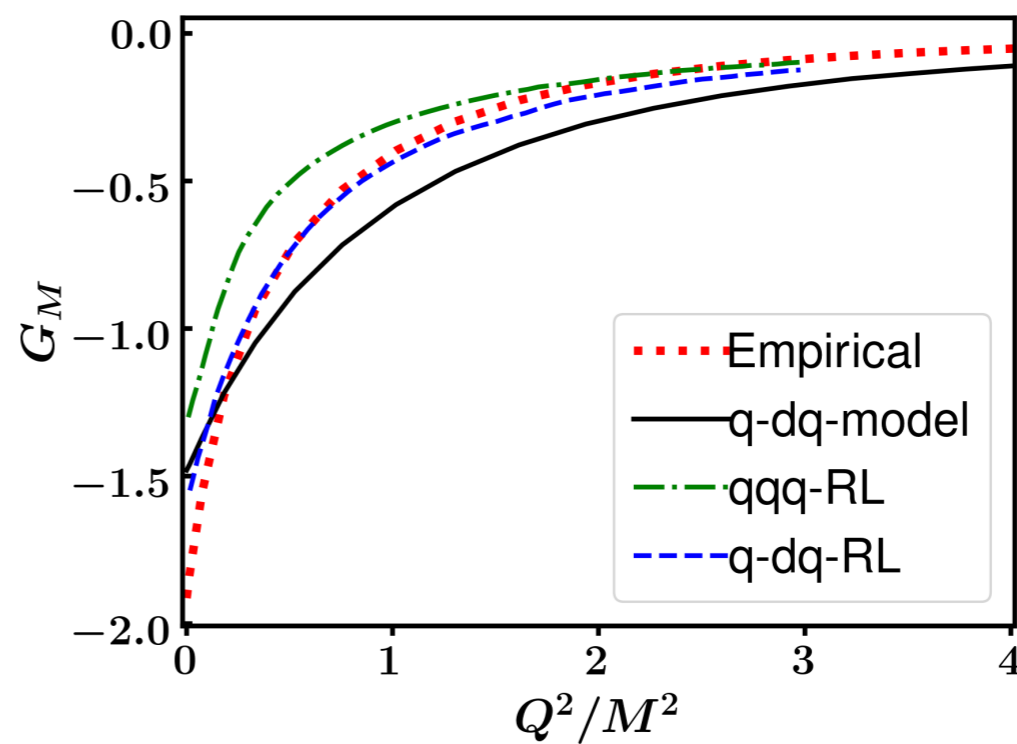
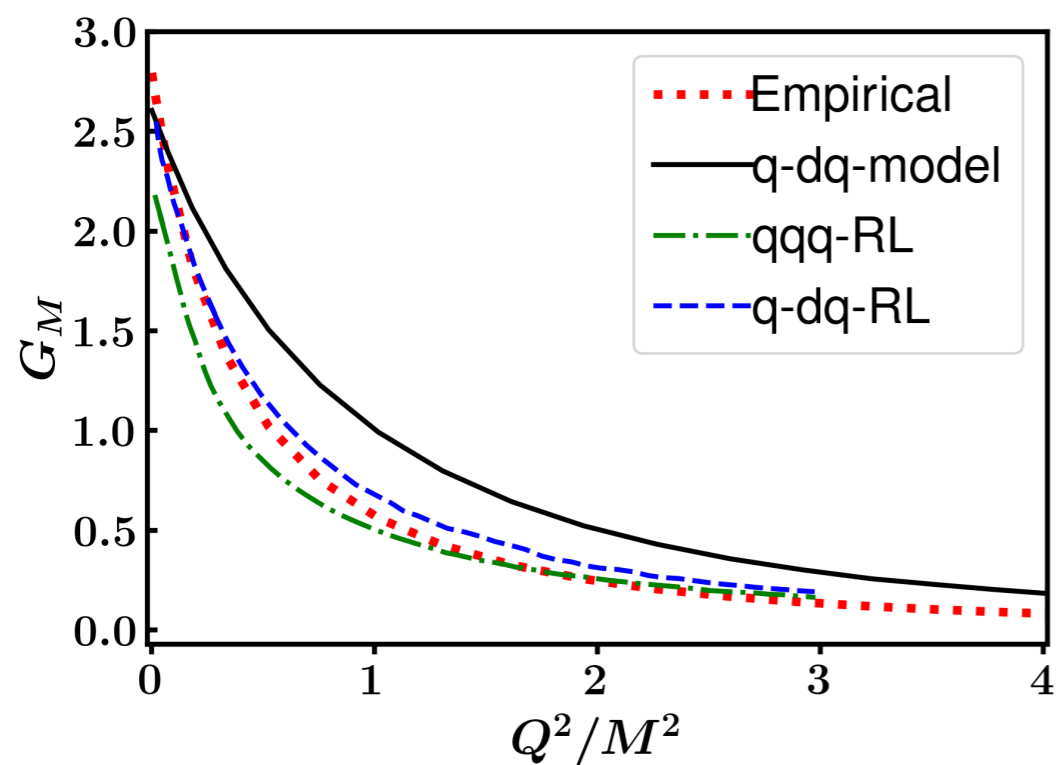
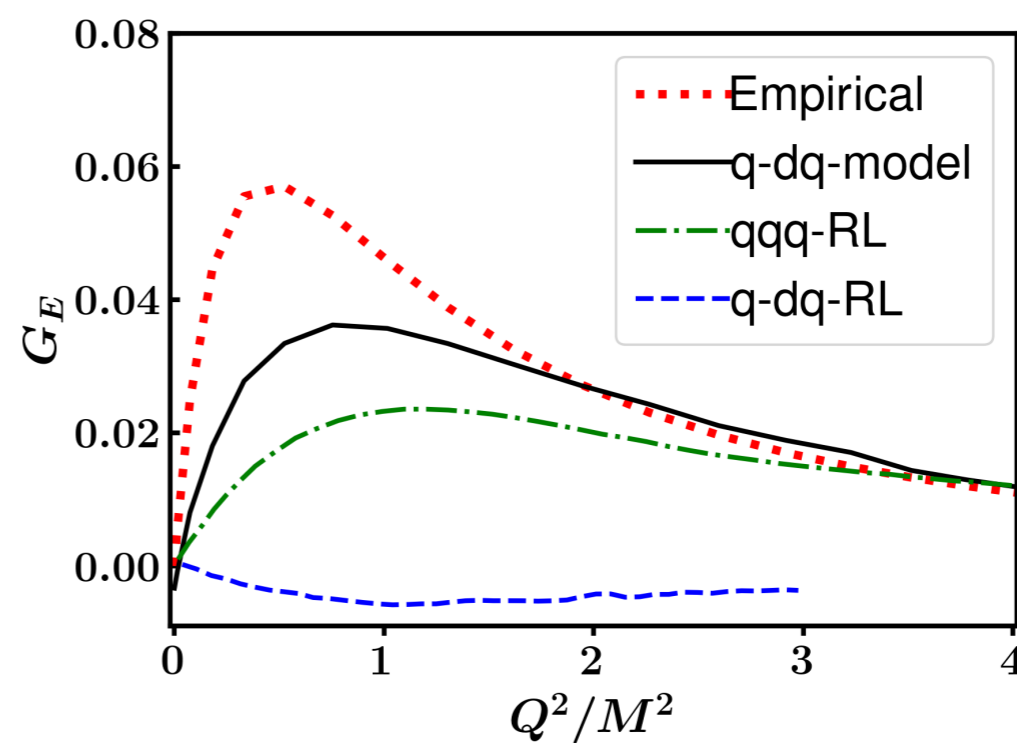
Eichmann and CF, EPJ A48 (2012) 9

Nucleon emFF (all approaches)

proton



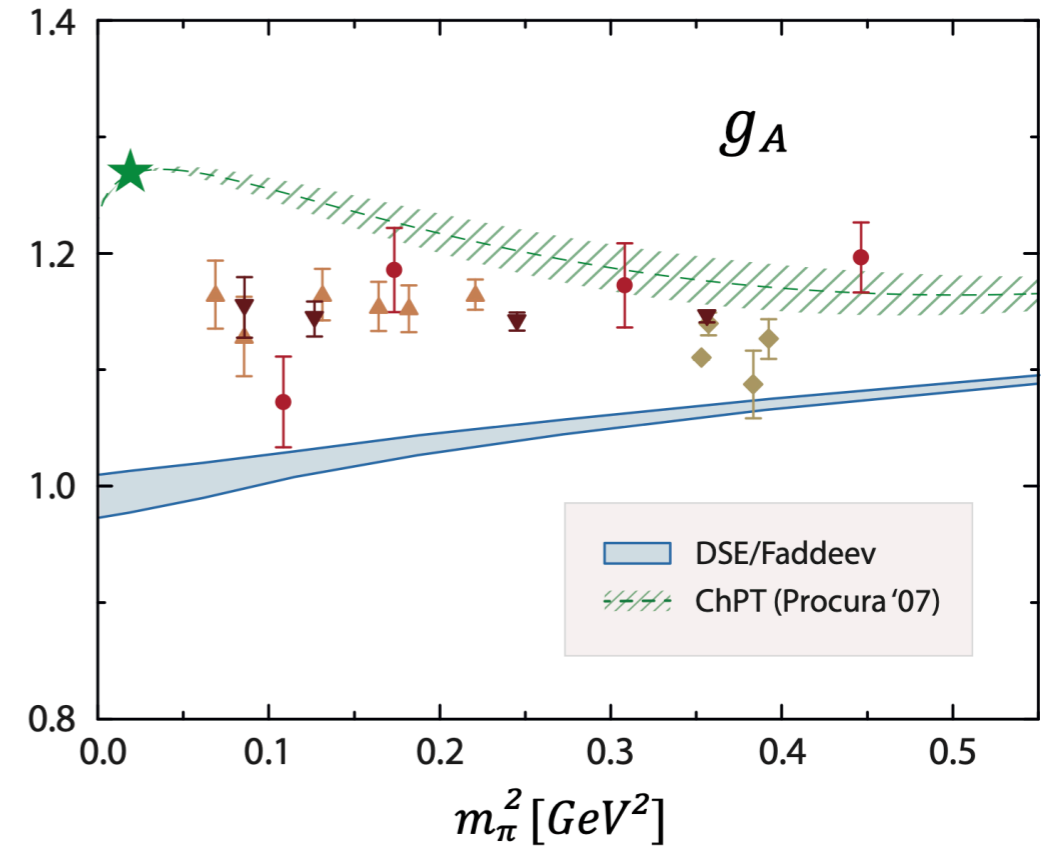
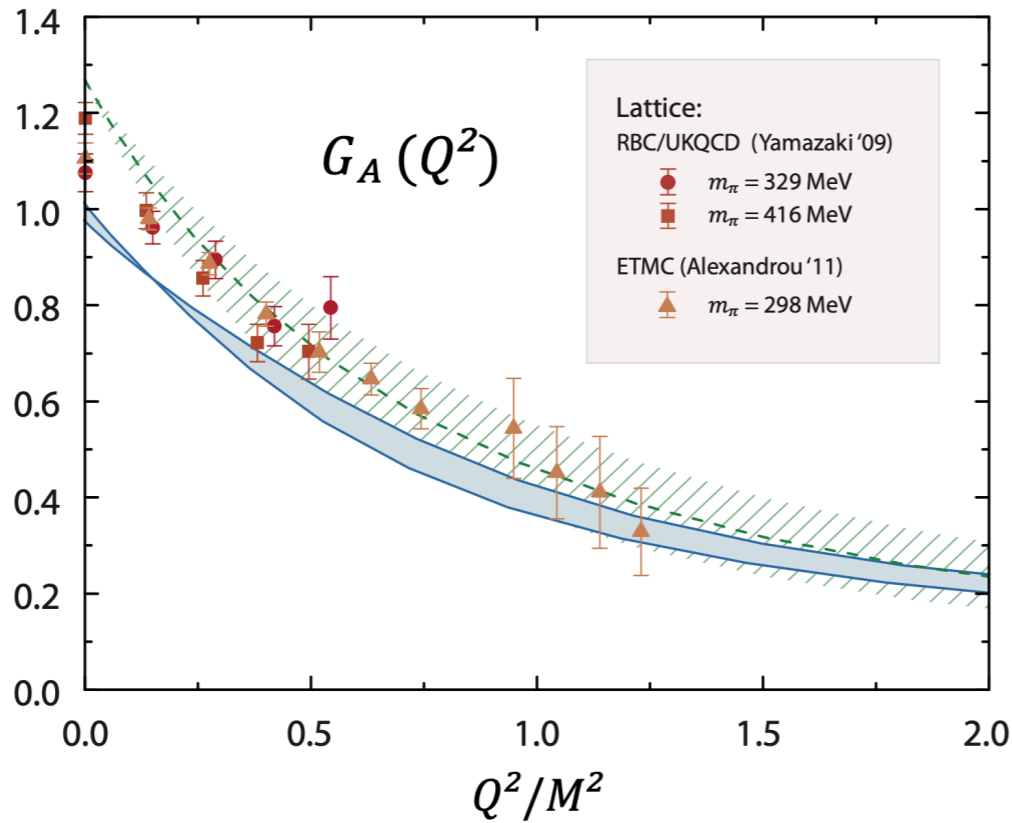
neutron



Liu and CF, arXiv:2311.13269

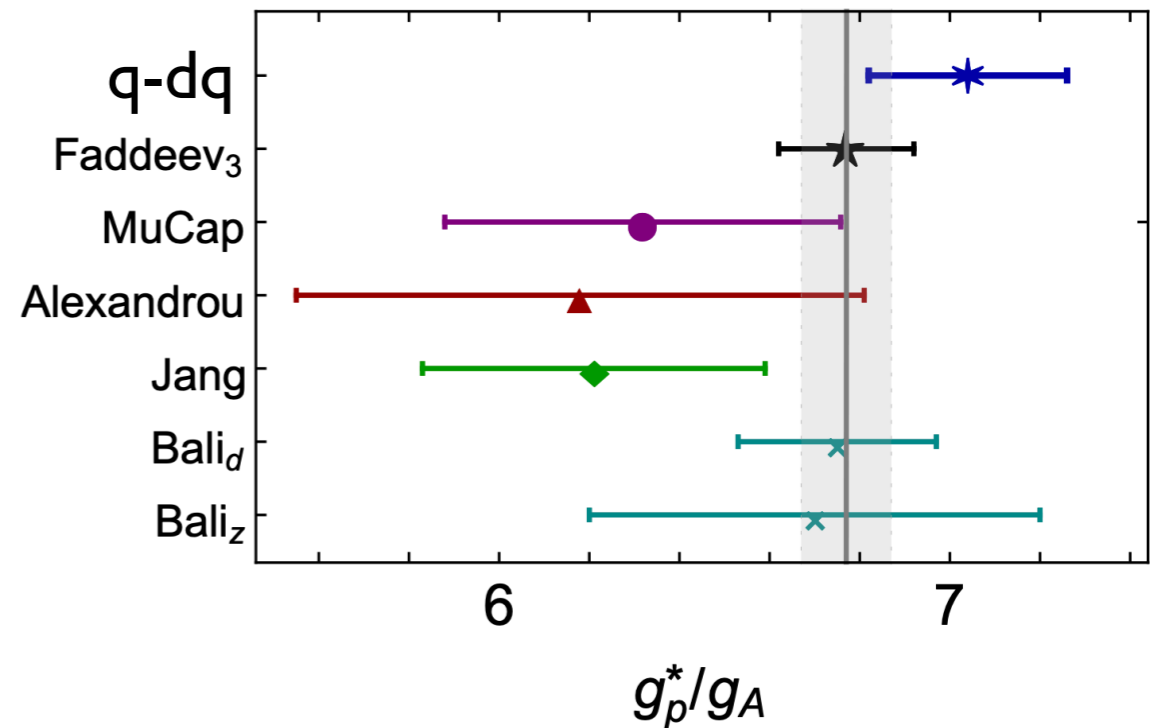
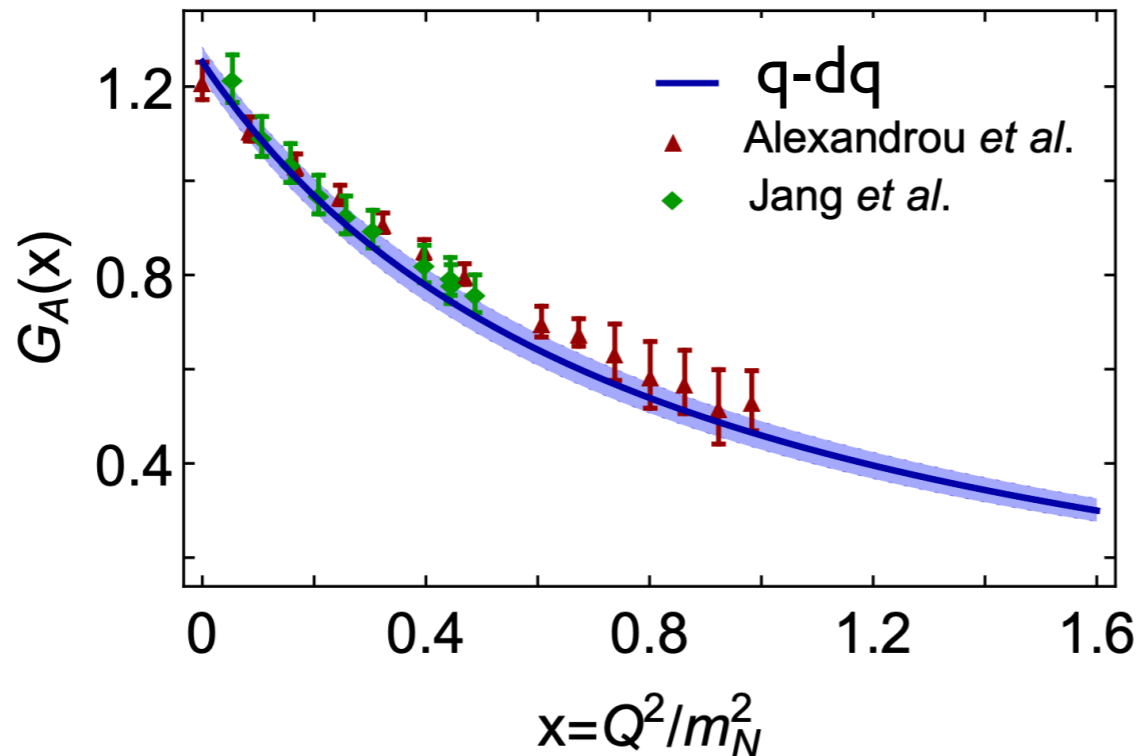
Nucleon axFF and g_A (three-body vs. q-dq-model)

three-body:



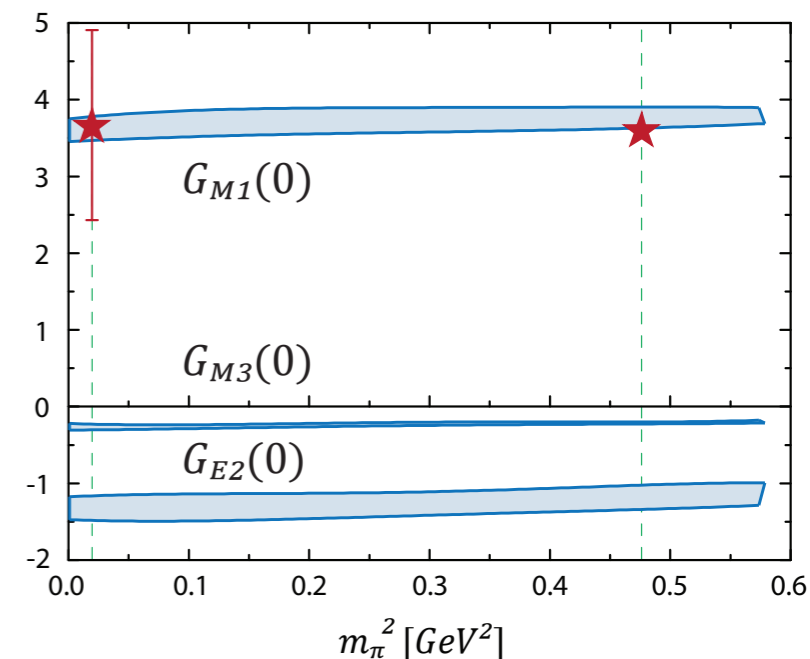
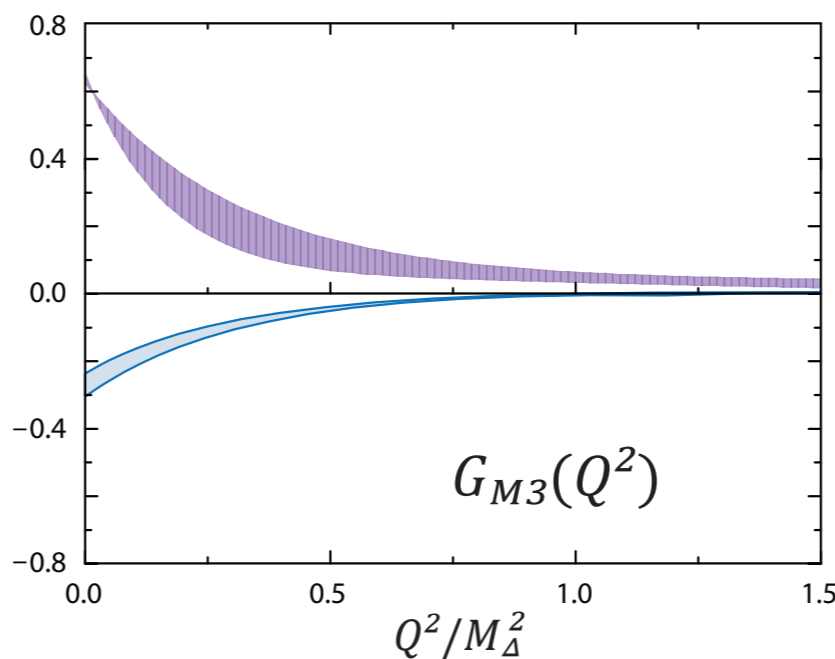
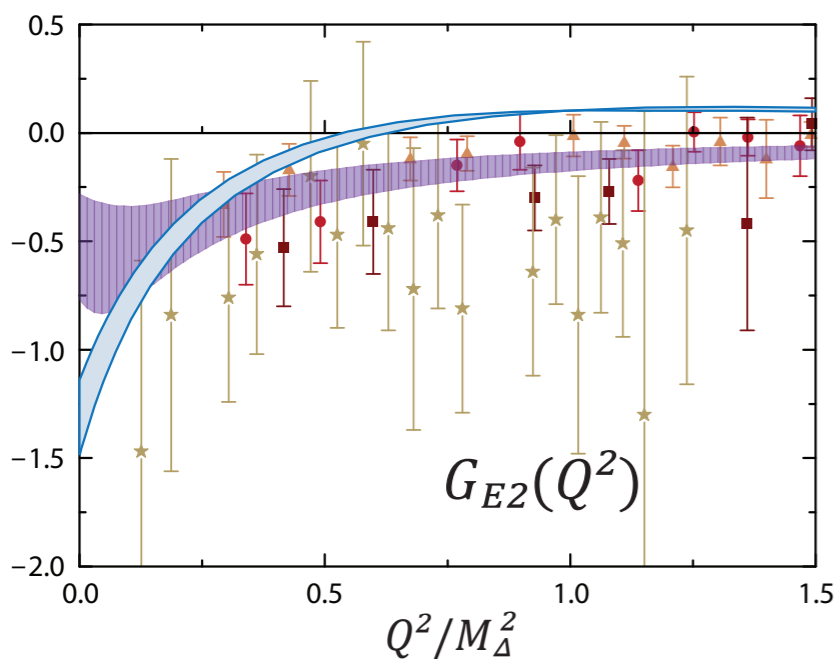
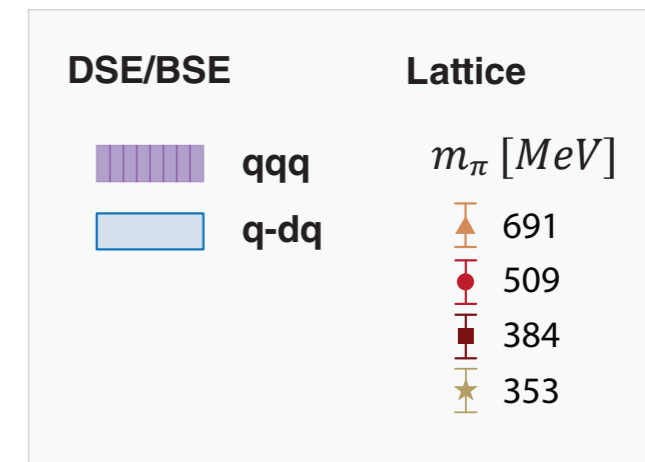
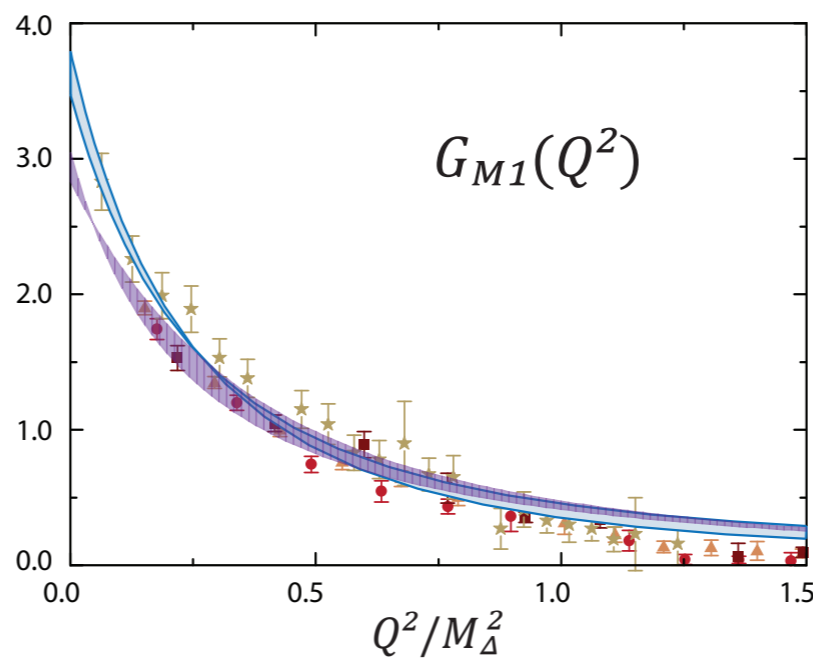
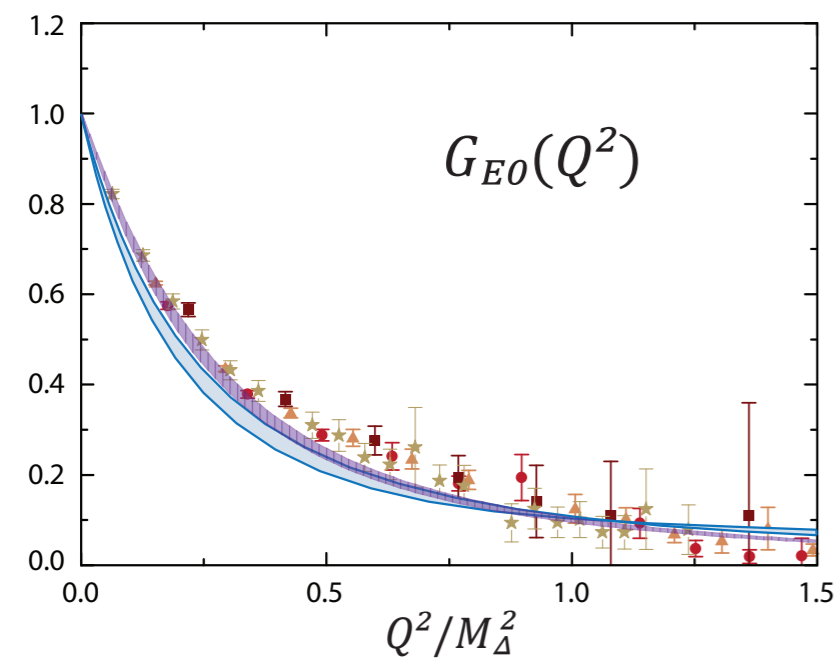
Eichmann and CF, EPJ A48 (2012) 9

q-dq-model:



Chen, CF, Roberts and Segovia, PLB 815 (2021), 136150, PRD 105 (2022) no.9, 094022

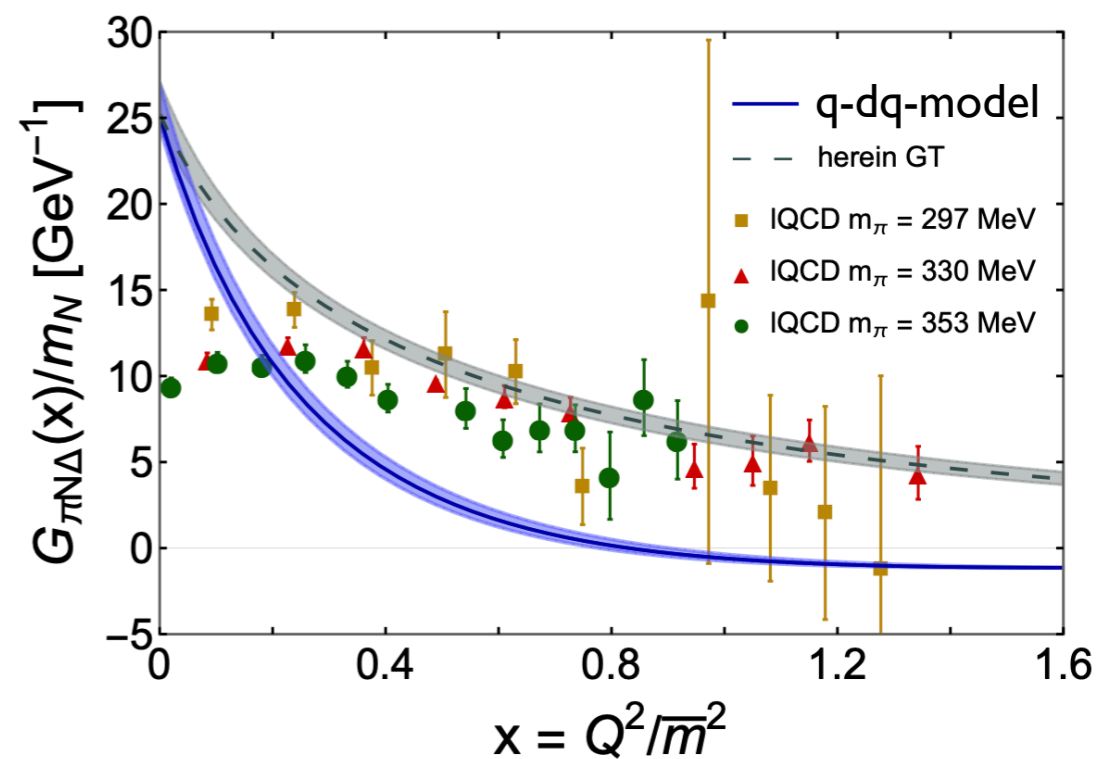
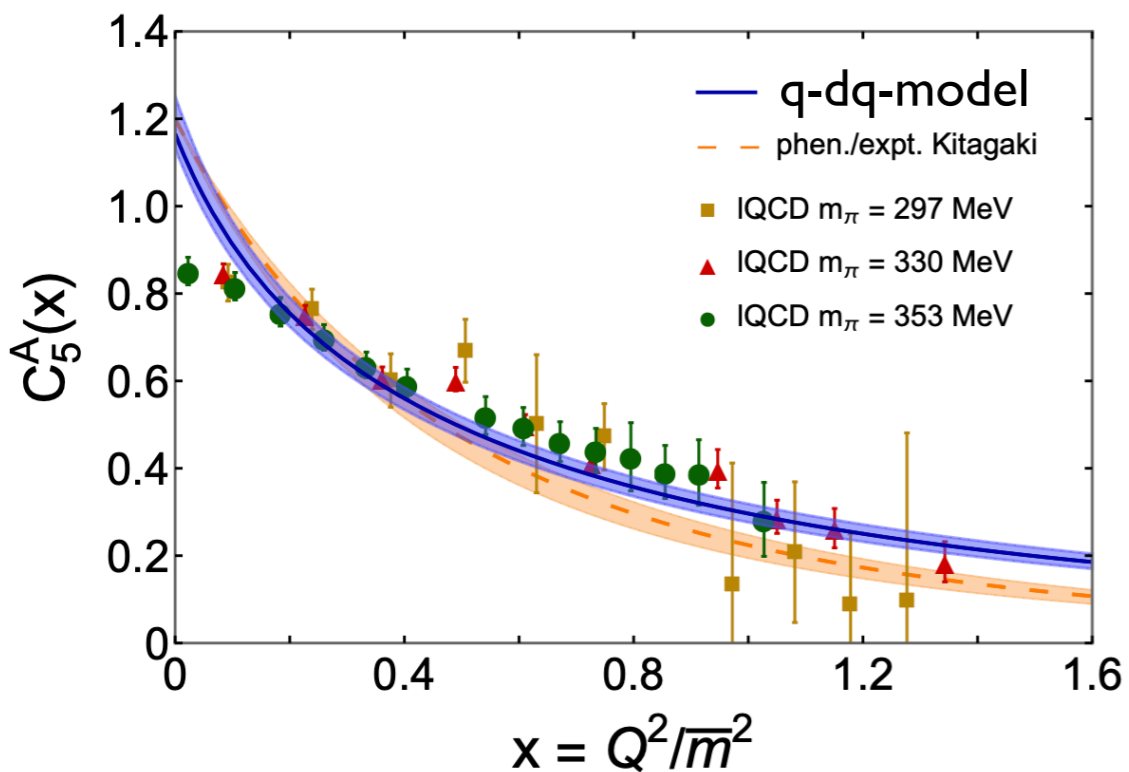
Δ : EM form factors



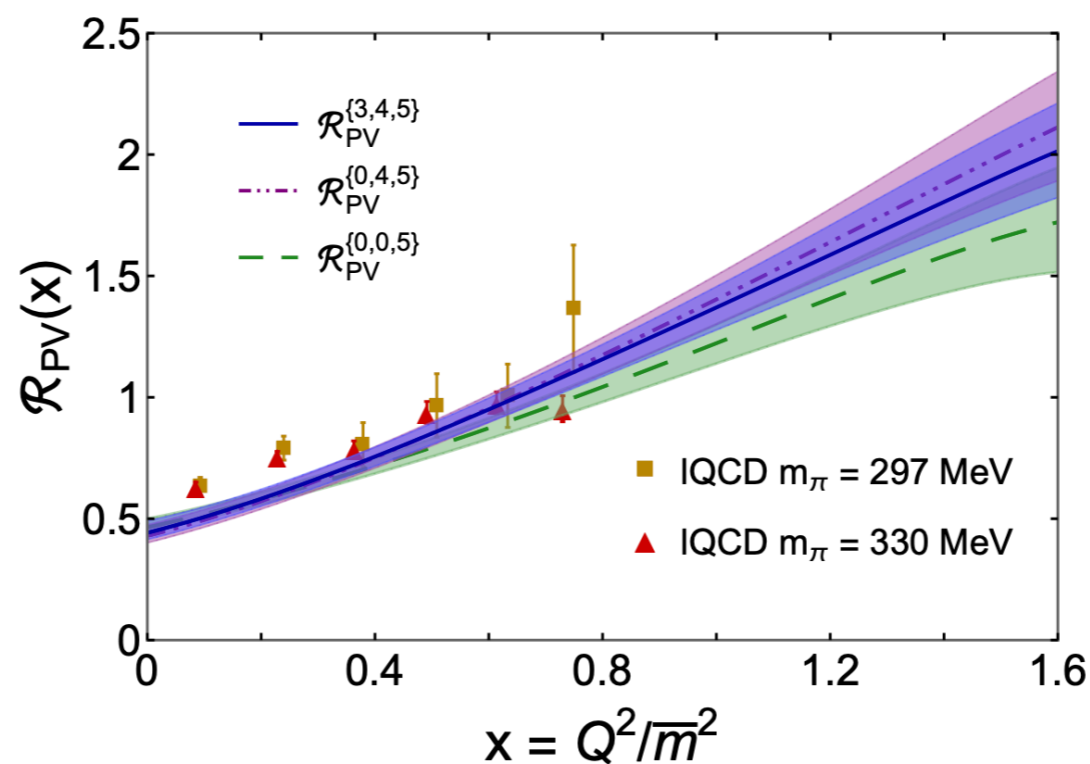
● may serve to distinguish between qqq and q-dq !

Sanchis-Alepuz, Williams, Alkofer, PRD87 (2013)
 Nicmorus, Eichmann, Alkofer, PRD82 (2010)

$N\Delta$: axial and pseudoscalar transition FF



parity violating asymmetry



● important application: inelastic scattering $\nu_l N \rightarrow l \Delta$

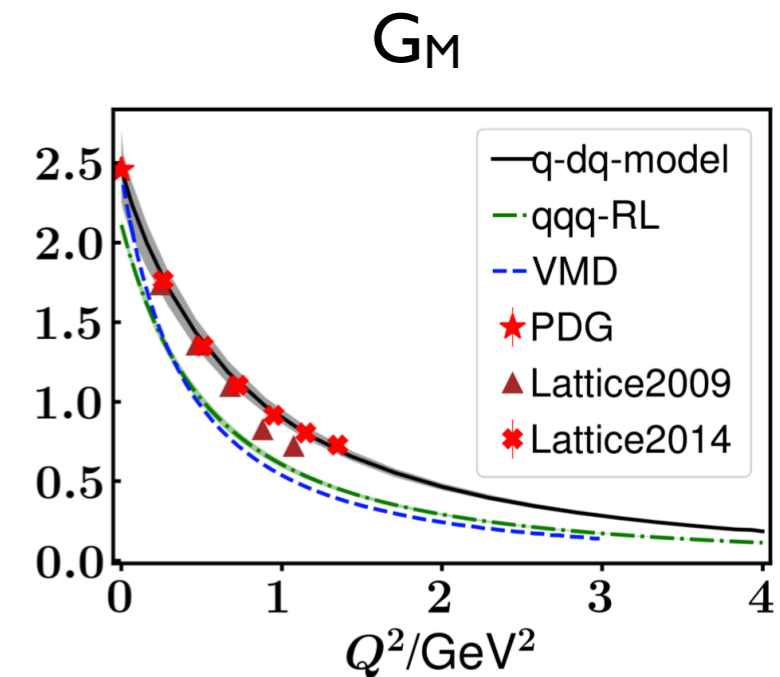
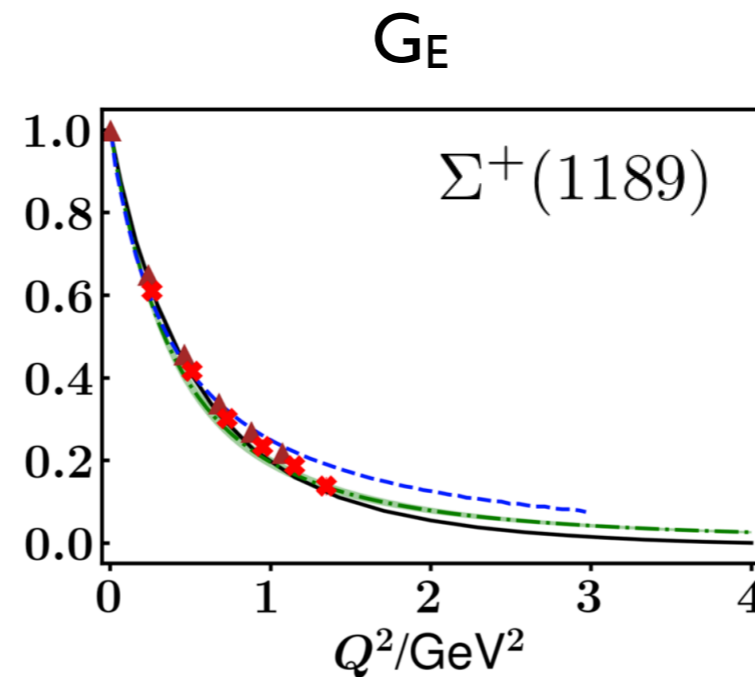
Chen, CF and Roberts, arXiv:2312.13724

Hyperon - charge radii and magnetic moments

	$\langle r_E^2 \rangle$	$\langle r_E^2 \rangle_{3b}$	$\langle r_E^2 \rangle_{\text{VMD}}$	$\langle r_E^2 \rangle_{\text{ChPT}}$	$\langle r_E^2 \rangle_{\text{disp}}$	$\langle r_E^2 \rangle_{\text{PDG}}$
Λ	0.036(14)	0.04(1)	0.012	0.11(2)	-	-
Σ^+	0.469(9)	0.56(3)	0.80(2)	0.60(2)	-	-
Σ^0	0.068(9)	0.057(8)	0.10(1)	-0.03(1)	-	-
Σ^-	0.353(26)	0.45(3)	0.70(2)	0.67(3)	-	0.61(15)

	$\langle r_M^2 \rangle$	$\langle r_M^2 \rangle_{3b}$	$\langle r_M^2 \rangle_{\text{VMD}}$	$\langle r_M^2 \rangle_{\text{ChPT}}$	$\langle r_M^2 \rangle_{\text{disp}}$
Λ	0.120(76)	0.21(1)	0.18	0.48(9)	0.464(2)
Σ^+	0.374(41)	0.43(2)	-	0.80(5)	-
Σ^0	0.201(169)	0.39(3)	-	0.45(8)	-
Σ^-	0.459(122)	0.50(1)	-	1.20(13)	-

	μ	μ_{3b}	μ_{PDG}
Λ	-0.390(3)	-0.435(5)	-0.613(4)
Σ^+	2.422(180)	1.82(2)	2.458(10)
Σ^0	0.630(48)	0.521(1)	-
Σ^-	-1.145(106)	-0.78(2)	-1.160(25)

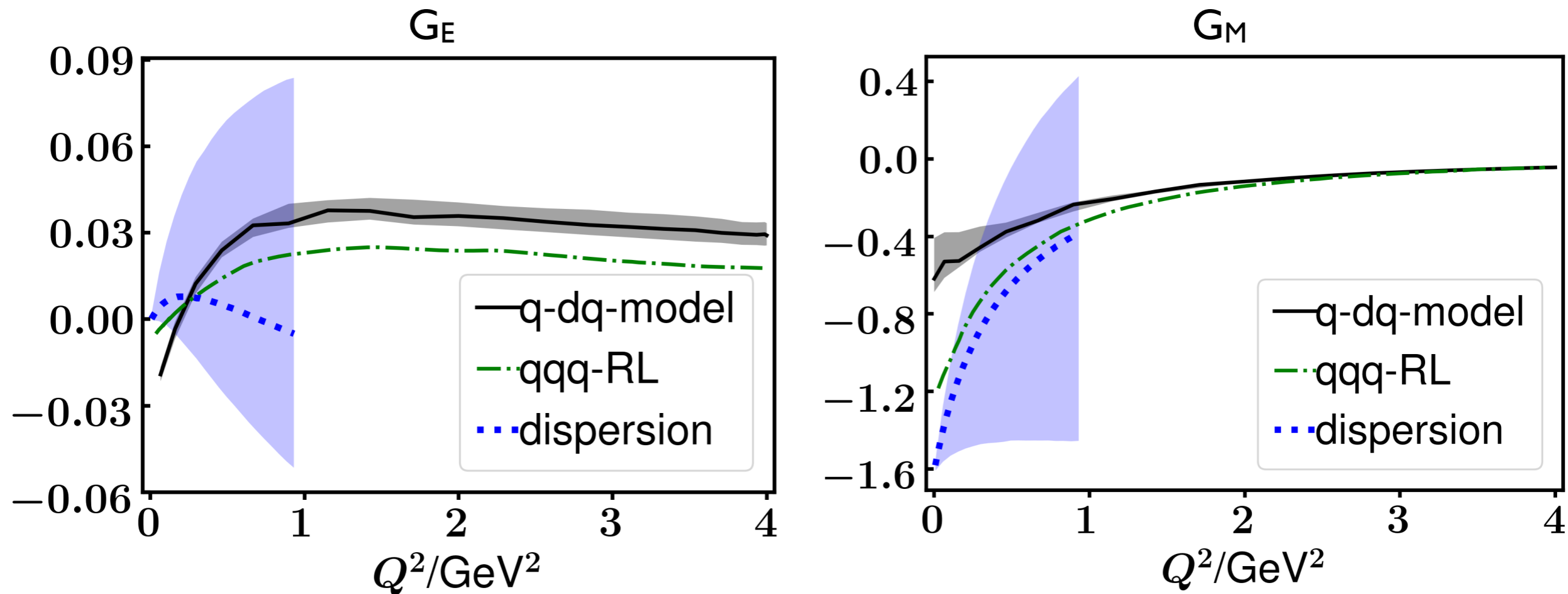


● first results in space-like momentum region

Liu and CF, arXiv:2311.13269

Strange transition emFF (three-body vs q-dq-model)

$\gamma^* \Sigma^0 \rightarrow \Lambda$ ● Considerable theoretical and experimental interest



Results:

- G_E is non-vanishing at finite Q^2 due to relativistic effects:
p-wave contributions!
(absent in quark model calculations $\rightarrow G_E=0$)

Sanchis-Alepuz, Alkofer and CF, EPJ A 54 (2018) no.3, 41
Liu and CF, arXiv:2311.13269

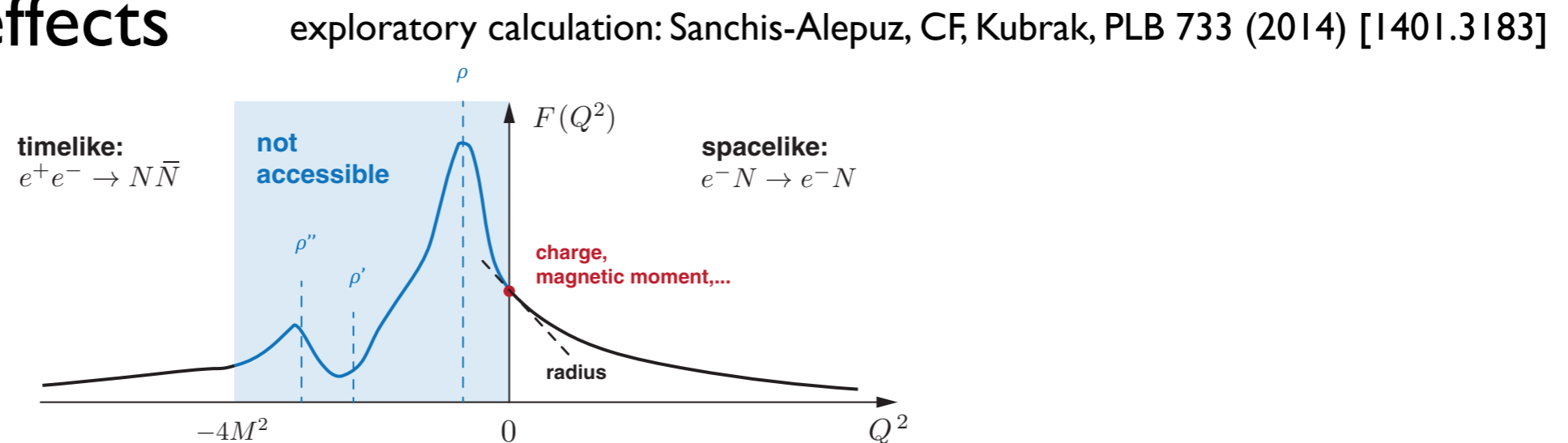
Summary and outlook

Summary

- Baryon spectrum: good agreement with experiment!
- Results for up/down, strange (and heavy) quarks
- Relativistic effects for all excited states, for some dominant!
- FF: meson cloud effects vs qqq vs quark-diquark
- Relativistic effects for some FF essential

Outlook

- Meson cloud effects
- Go time-like



Quantum numbers: non-relativistic vs relativistic

non-relativistic

relativistic

Mesons: $P = (-1)^{L+1}$

S	L	J^{PC}
0	0	0^{-+}
1	0	1^{--}
0	1	1^{+-}
1	1	0^{++}

Quantum numbers: non-relativistic vs relativistic

non-relativistic

relativistic

Mesons: $P = (-1)^{L+1}$

~~$P = (-1)^{L+1}$~~

S	L	J^{PC}
0	0	0^{-+}
1	0	1^{--}
0	1	1^{+-}
1	1	0^{++}

Bethe, Salpeter, Llewellyn-Smith 1950ies

$\Gamma_{\pi}(P, p) = \gamma_5 [$
 $F_1(P, p)$ **s-wave**
 $+ F_2(P, p)i\not{P}$
 $+ F_3(P, p)p\not{P}i\not{p}$ **p-wave**
 $+ F_4(P, p)[\not{p}, \not{P}]]$

Quantum numbers: non-relativistic vs relativistic

non-relativistic

relativistic

Mesons: $P = (-1)^{L+1}$

~~$P = (-1)^{L+1}$~~

S	L	J^{PC}
0	0	0^{-+}
1	0	1^{--}
0	1	1^{+-}
1	1	0^{++}

Bethe, Salpeter, Llewellyn-Smith 1950ies

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 $+ F_4(P, p)[\not{p}, \not{P}]]$

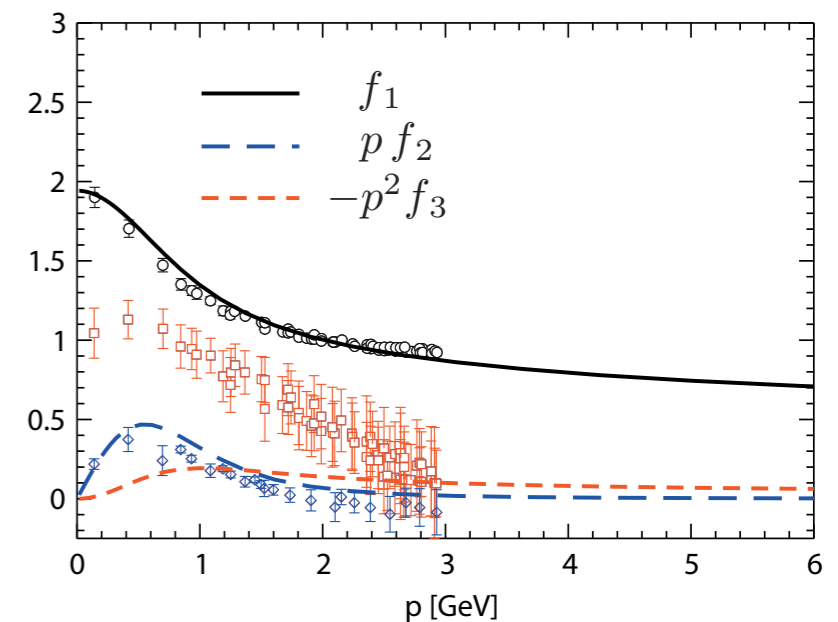
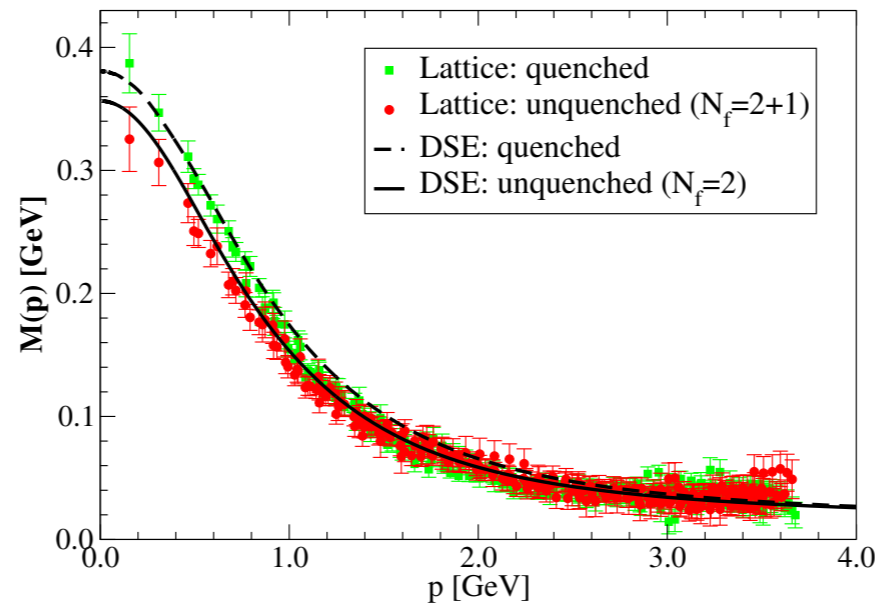
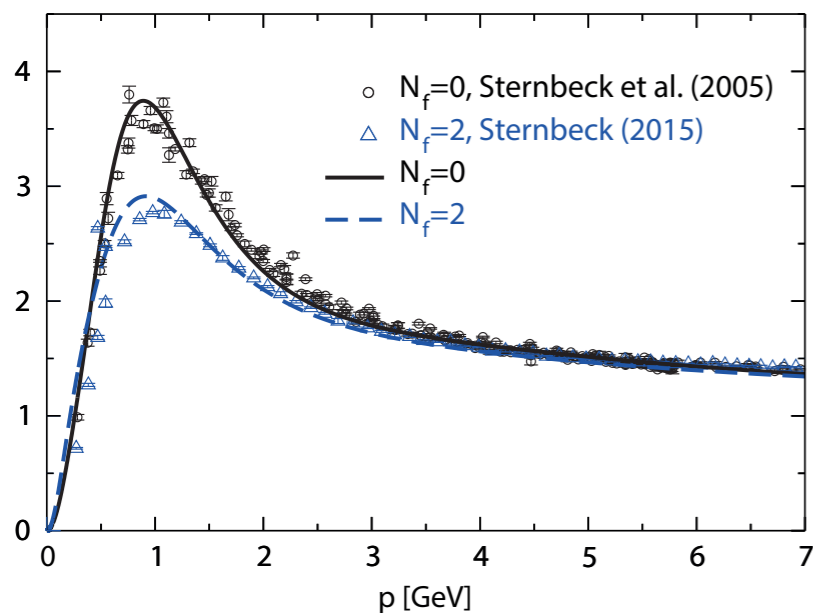
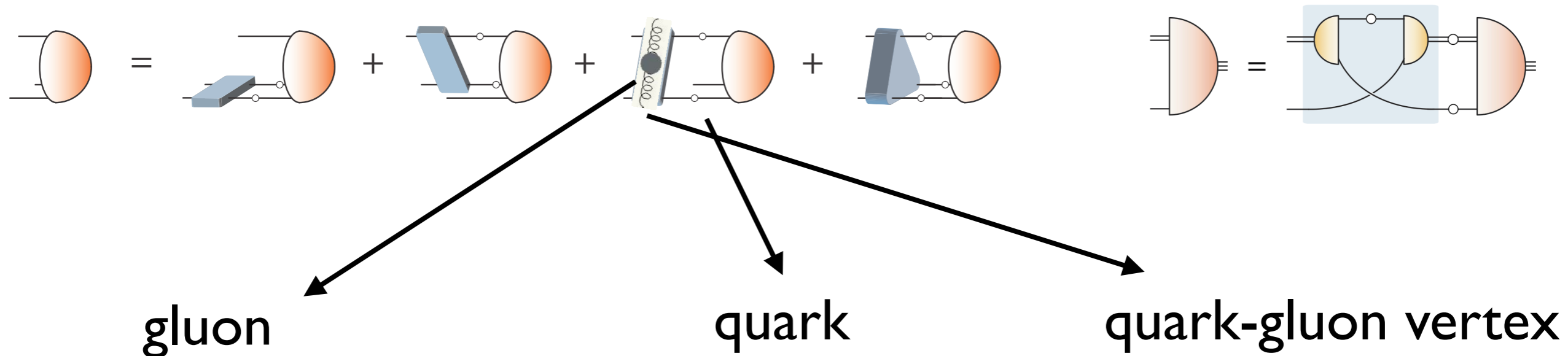
Baryons: $P = (-1)^L$

~~$P = (-1)^L$~~

S	L	J^P
1/2	0	$1/2^+$
3/2	2	

J^P	total	s-wave	p-wave	d-wave	f-wave
$1/2^+$	64	8	36	20	
$3/2^+$	128	4	36	60	28

Running quark mass ? - Running QCD !



Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

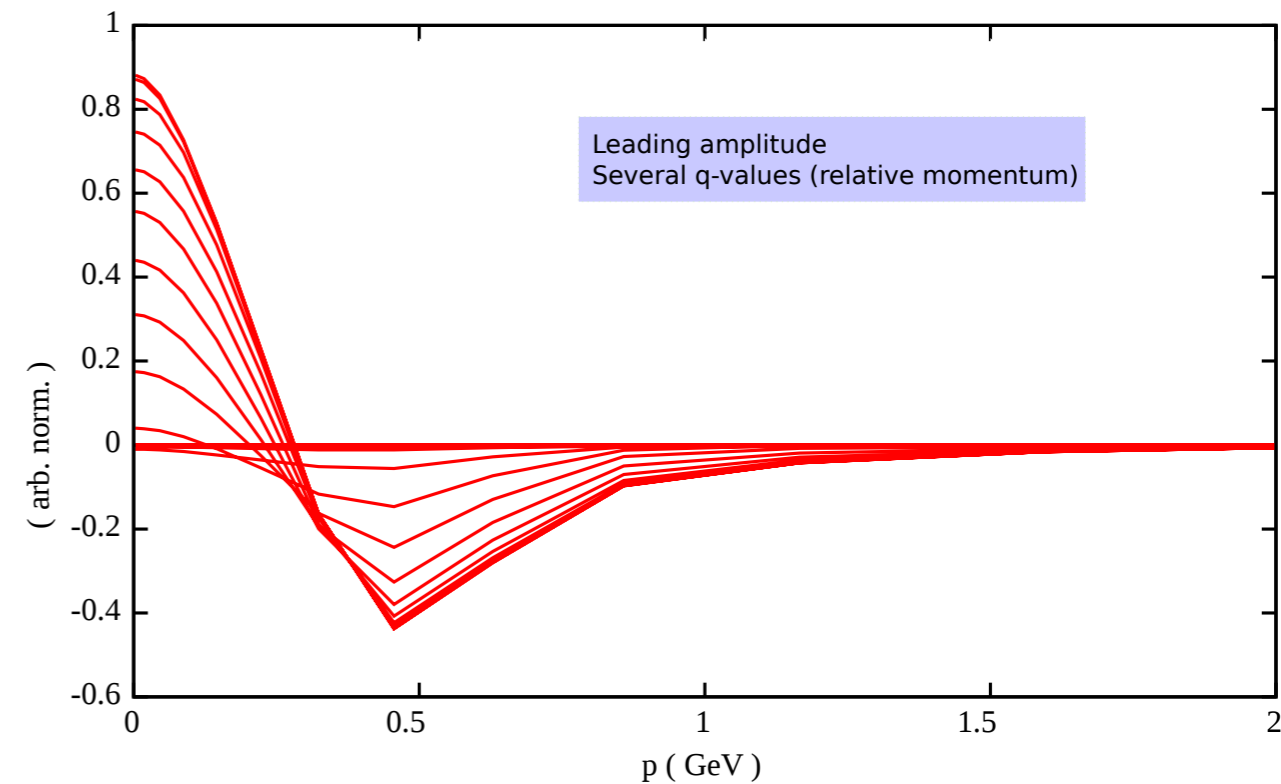
- Many running quantities go into calculation of observables

Cloet, Roberts and Thomas, PRL 111 (2013) 101803

Properties of the Roper

angular mom. decomposition

%	N	$N^*(1440)$	Δ	$\Delta^*(1600)$
s wave	66	15	56	10
p wave	33	61	40	33
d wave	1	24	3	41
f wave	—	—	< 0.5	16

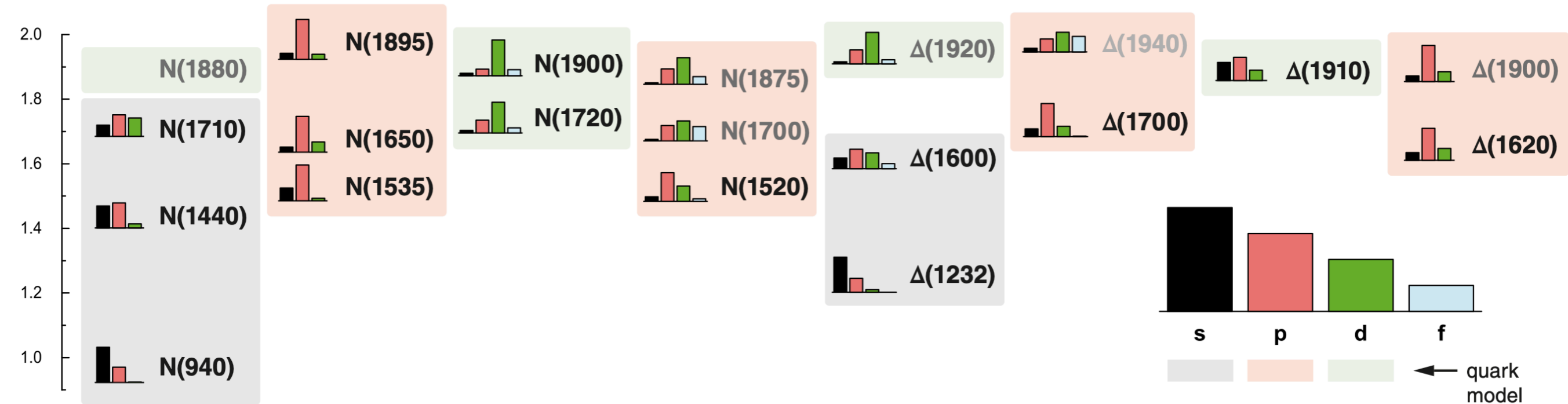
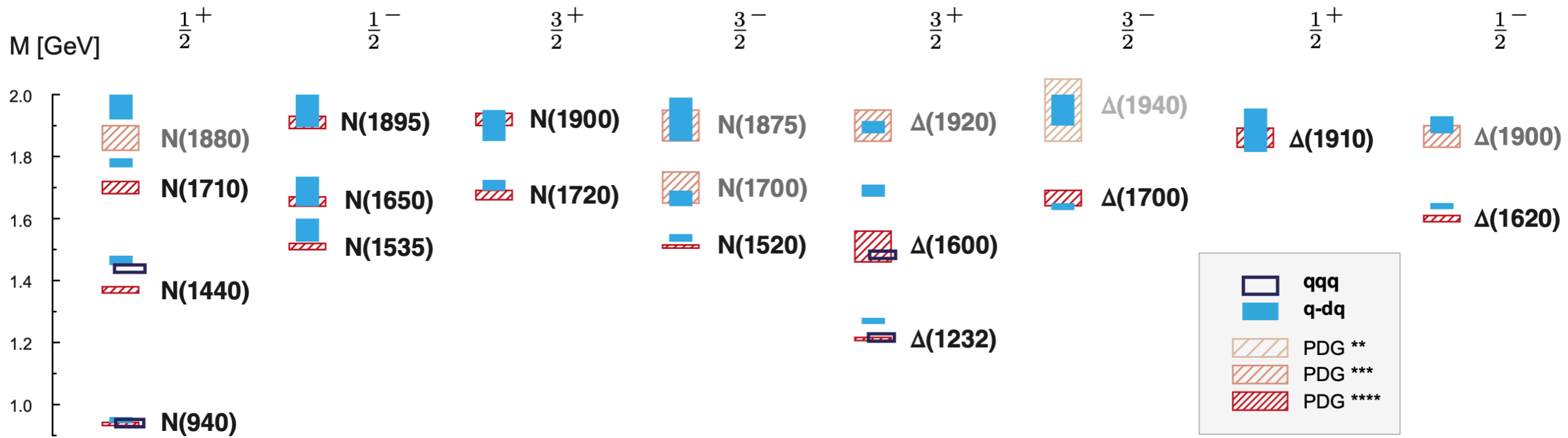


Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016)

- zero crossing of wave function: 2s-state
- every state is mixture of several partial waves !
- different internal structure of radial excitations

Light baryon spectrum: DSE-RL

3 parameters + $m_{u,d,s}$

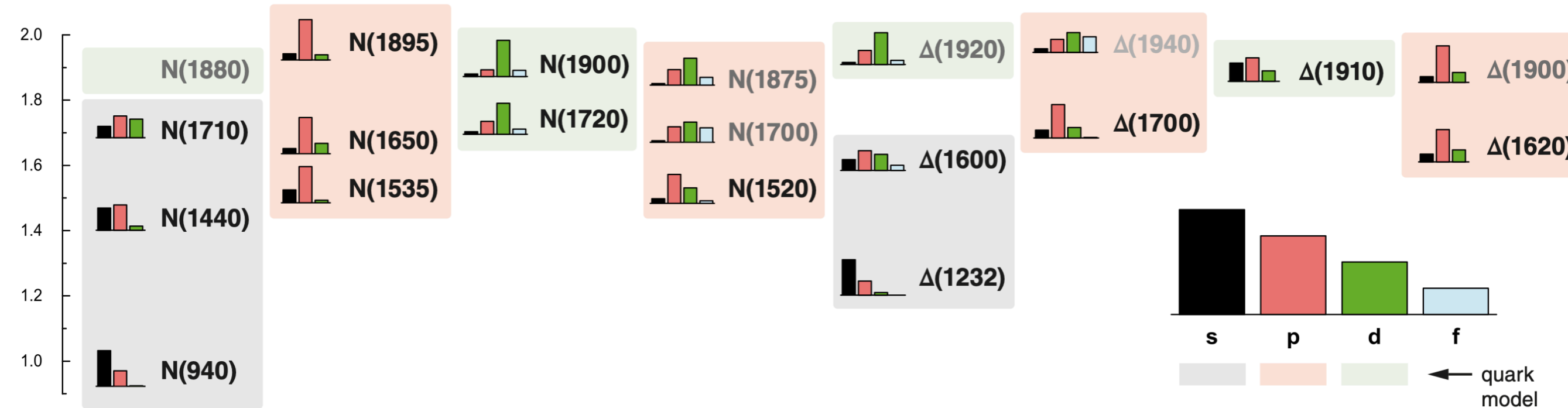
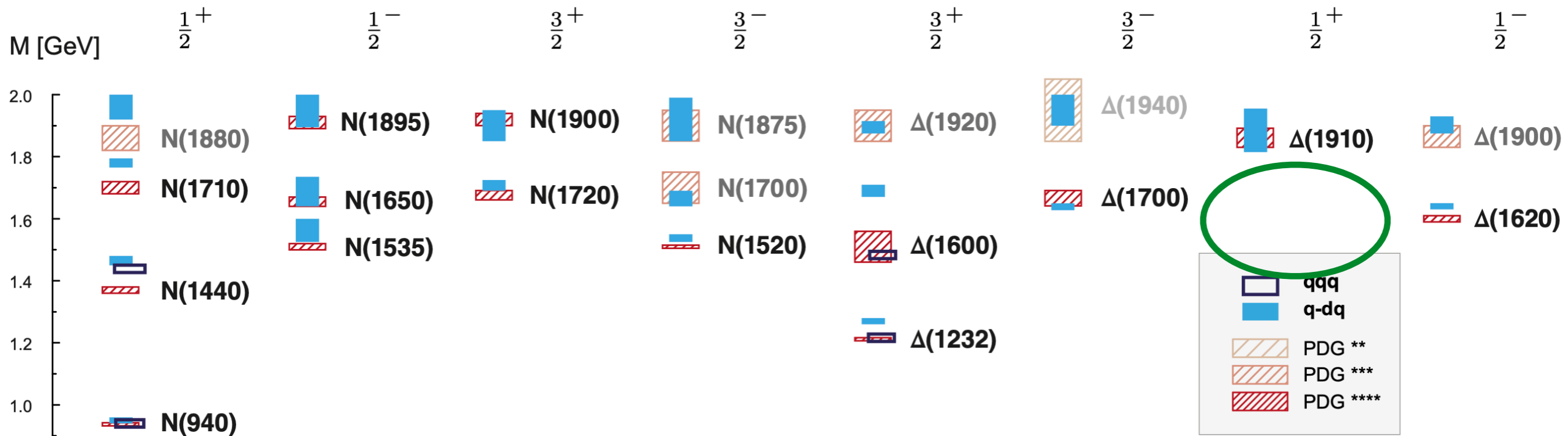


relativistic effects !

Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]
 Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2
 Eichmann, Few Body Syst. 63 (2022) no.3,

Light baryon spectrum: DSE-RL

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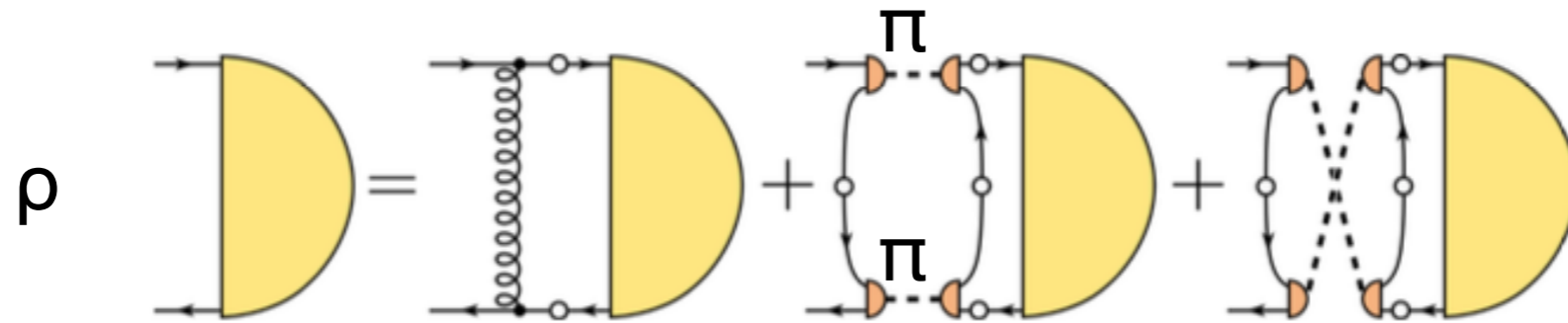


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Decays: $\rho\pi\pi$

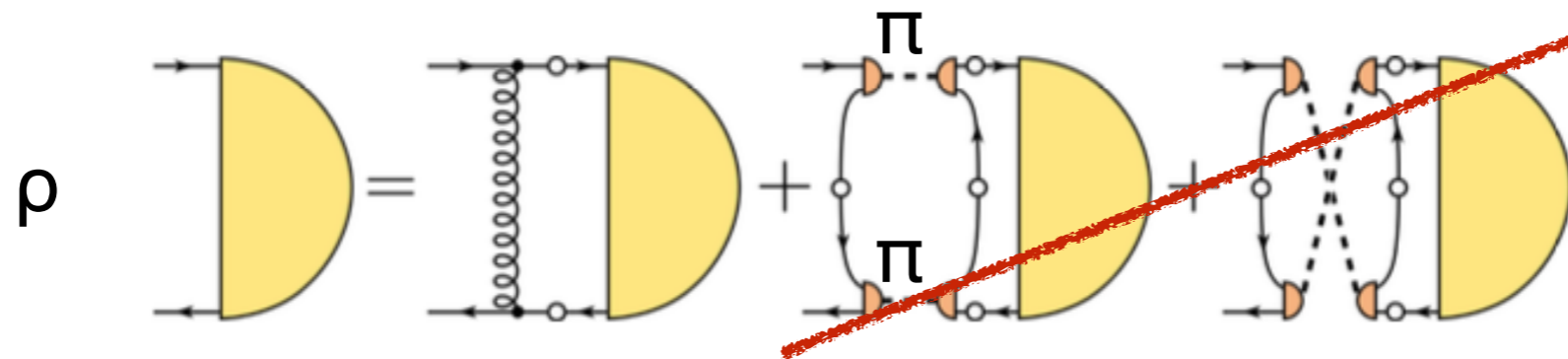
Beyond rainbow-ladder: pion contributions in BSE-kernel:



Williams, arXiv:1804.11161

Decays: $\rho\pi\pi$

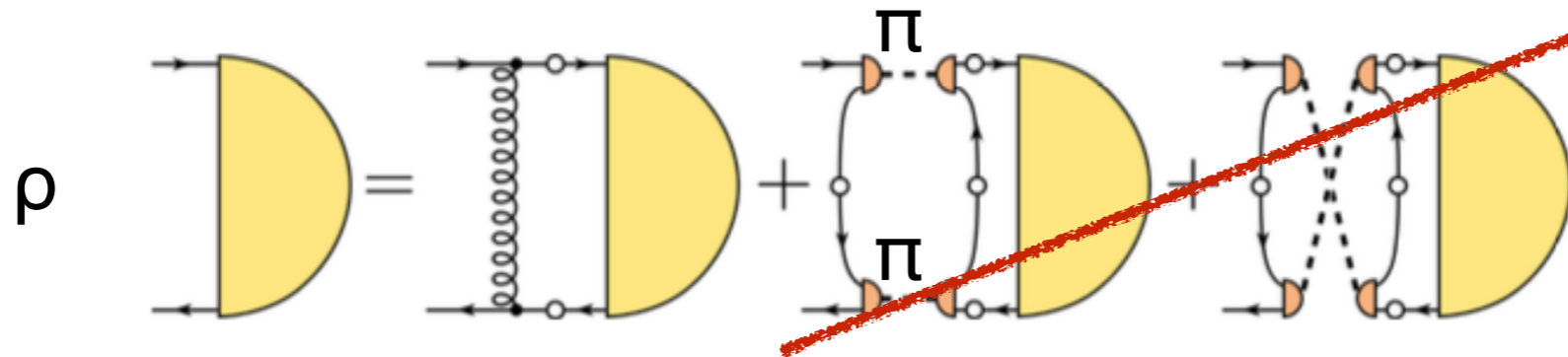
Beyond rainbow-ladder: pion contributions in BSE-kernel:



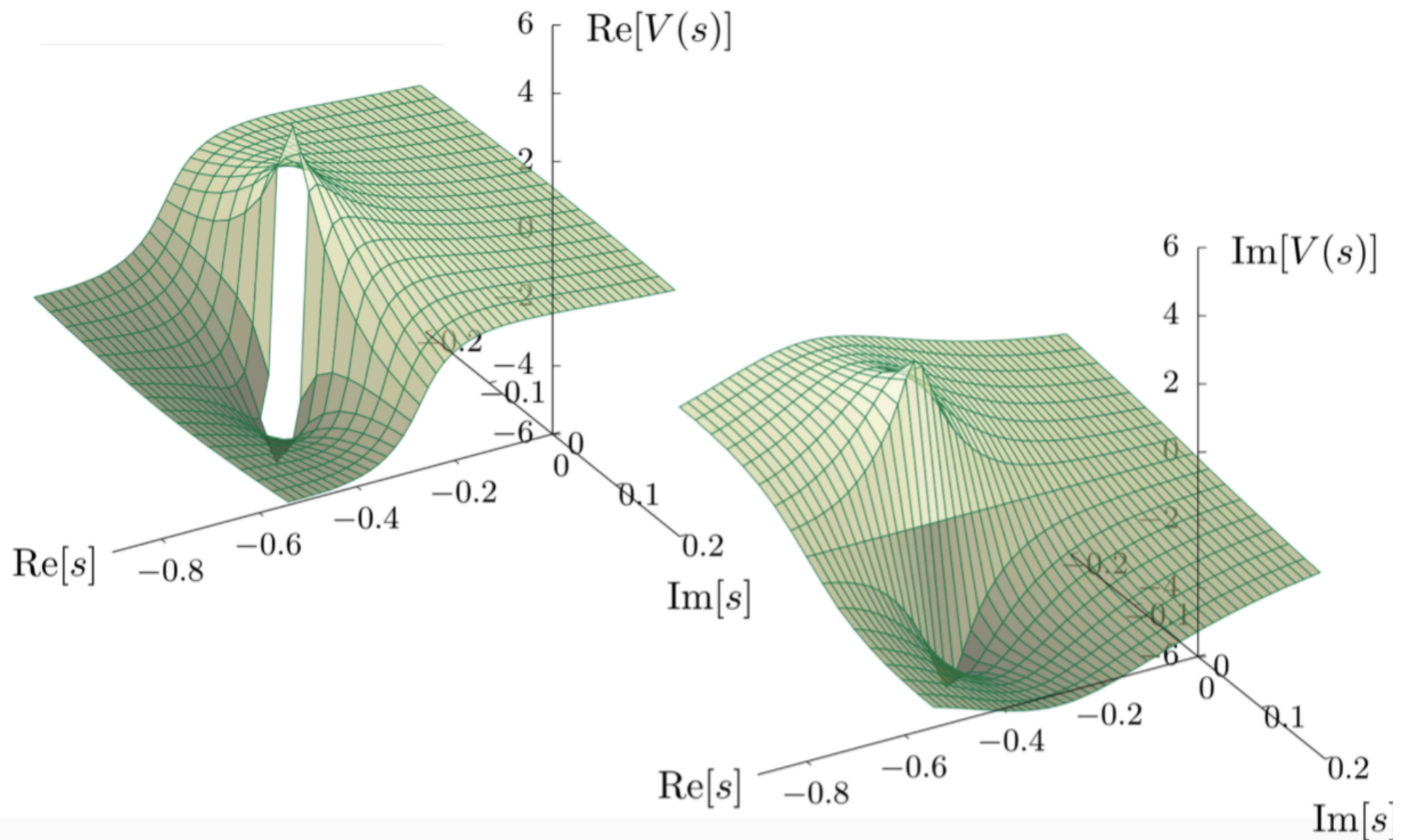
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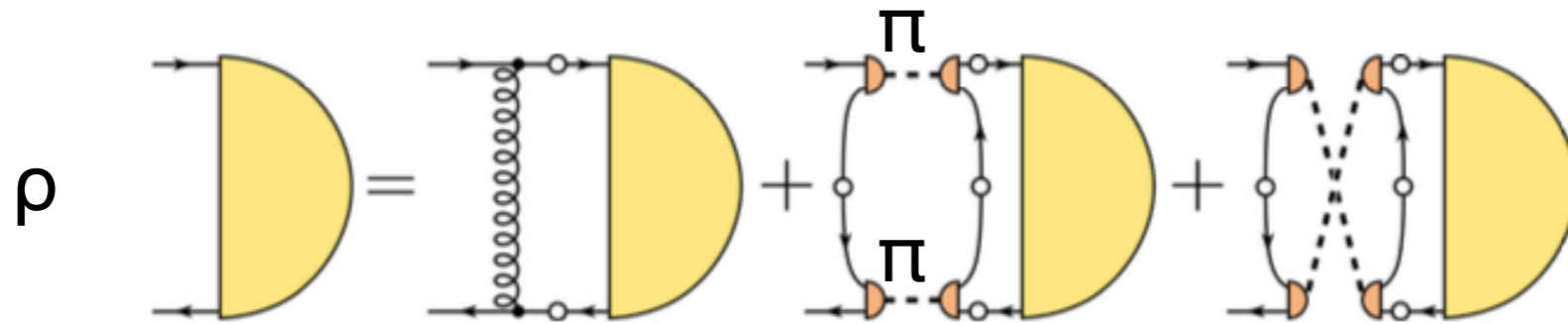


Williams, arXiv:1804.11161

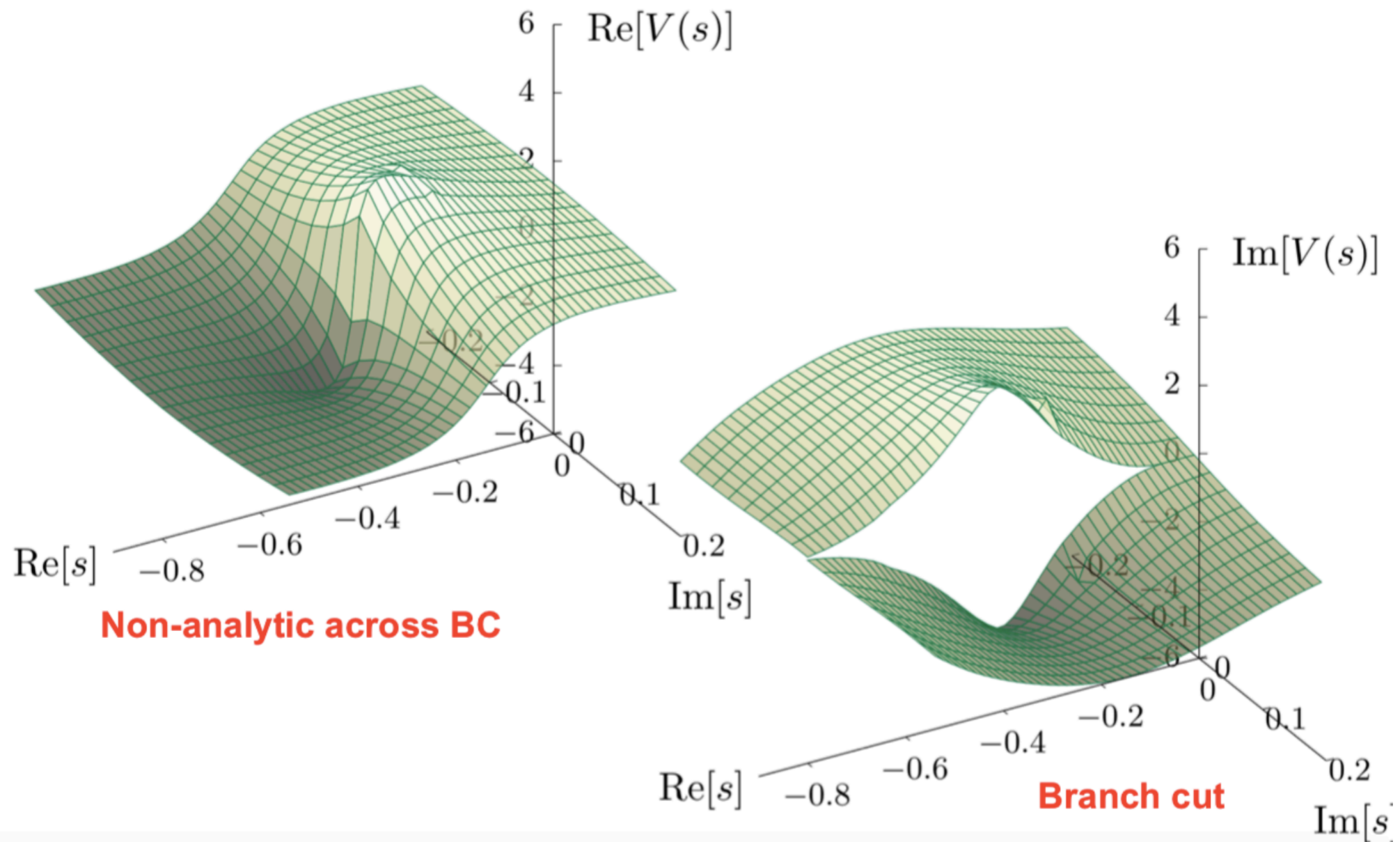


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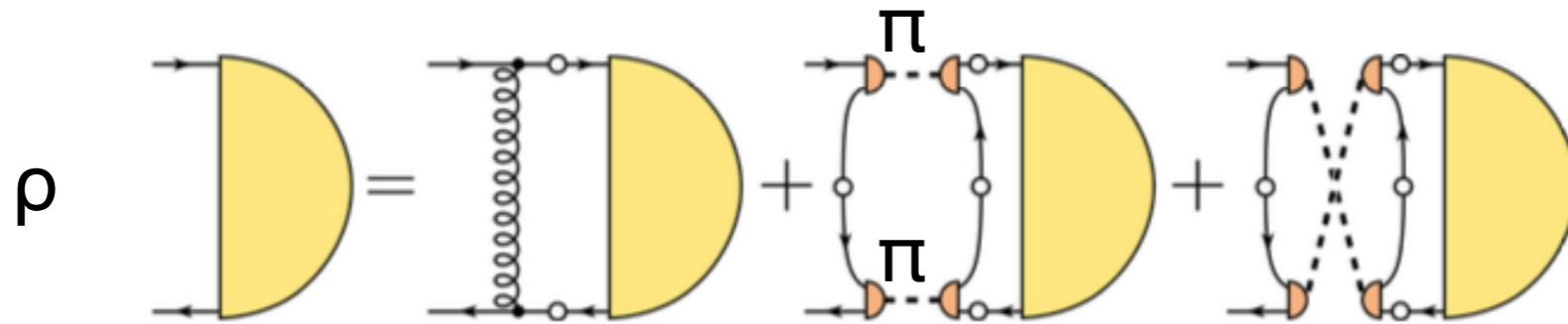


Williams, arXiv:1804.11161

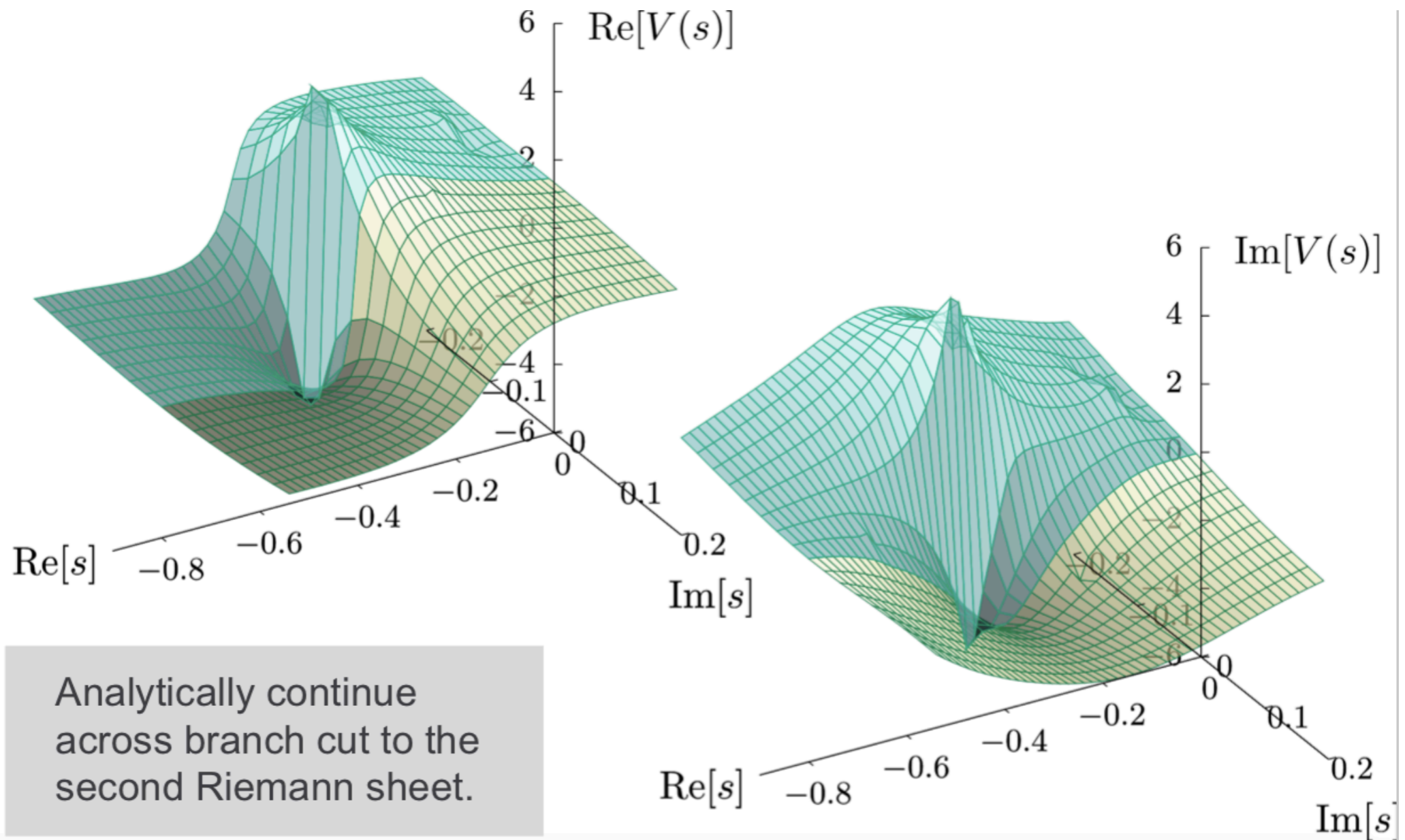


Decays: $\rho\pi\pi$

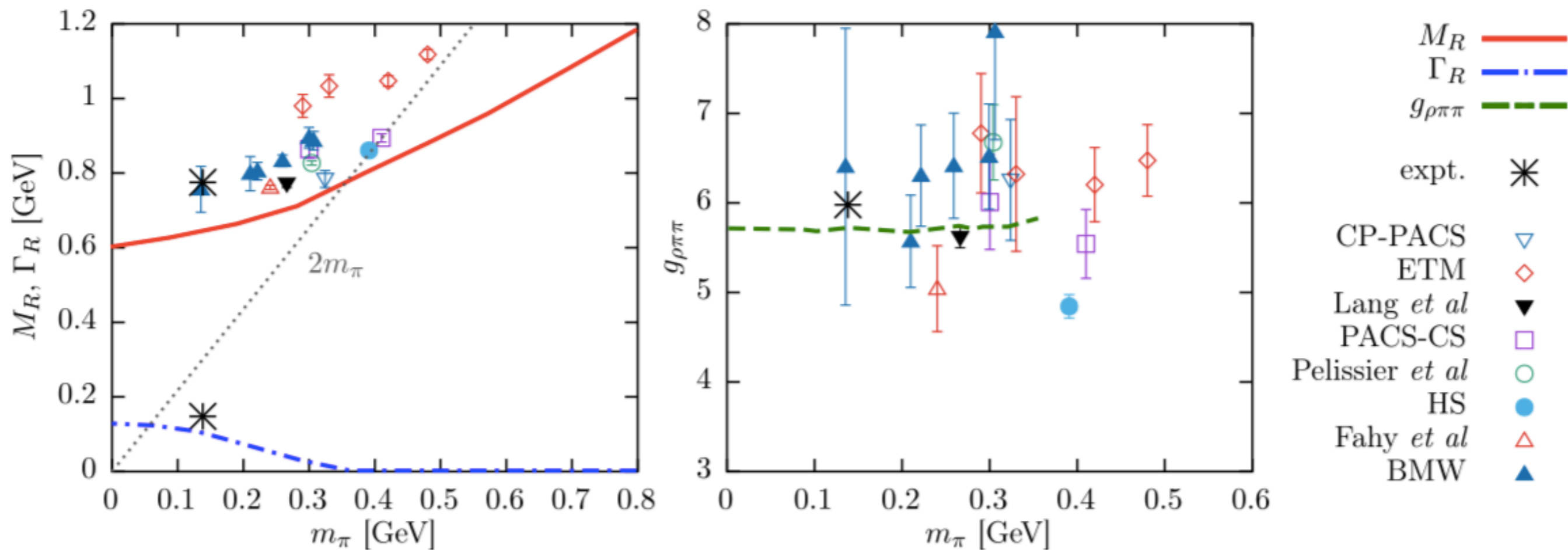
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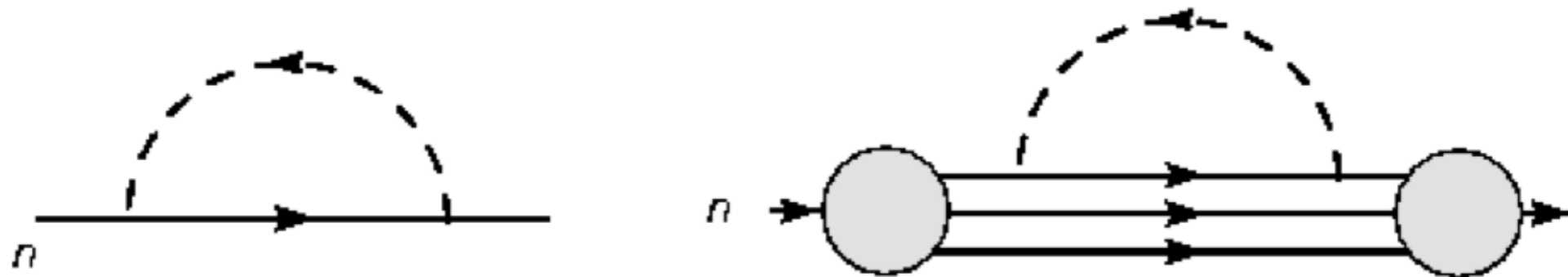


Williams, arXiv:1804.11161

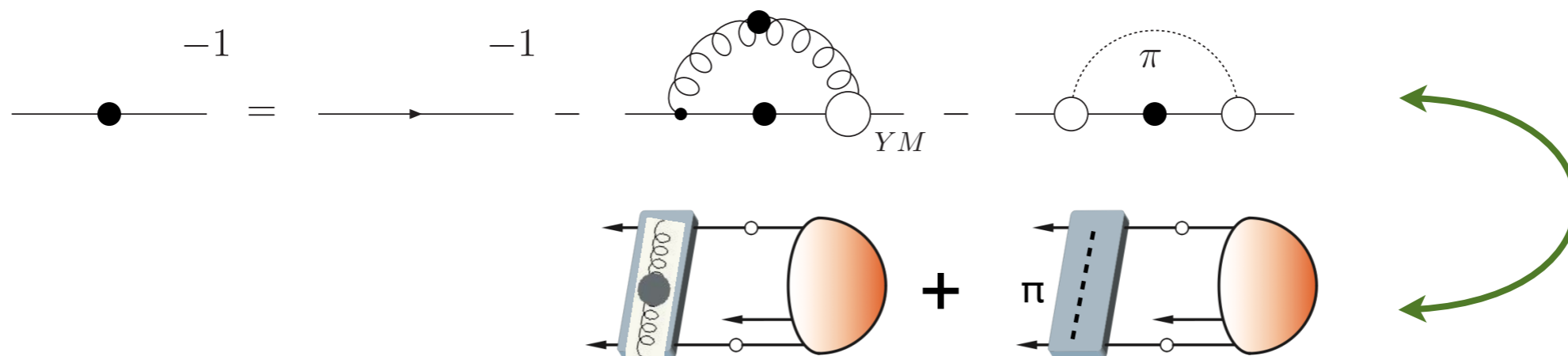
- Additional corrections known to increase mass by $O(100)$ MeV

CF and Williams, PRL 103 (2009), 122001

Pion cloud effects



- Hadron level: πN -contributions to nucleon self-energy
- Quark-level: π -contributions to quark self-energy and interactions

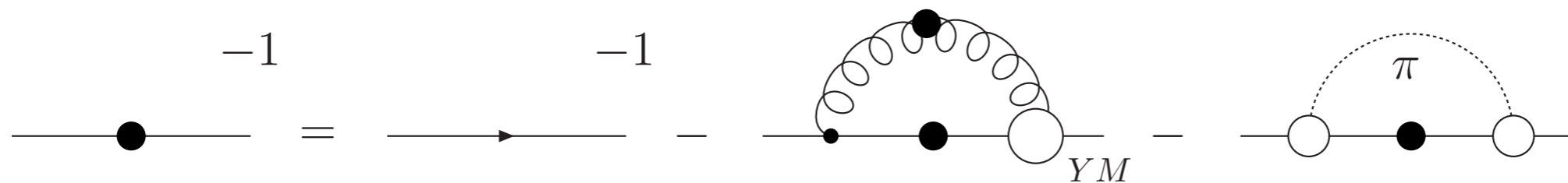


Pion not an elementary field \longrightarrow BSE !

Setup derived from DSE for quark-gluon interaction!

CF, Nickel and Wambach, PRD 76 (2007) 094009

Pion cloud effects in light mesons



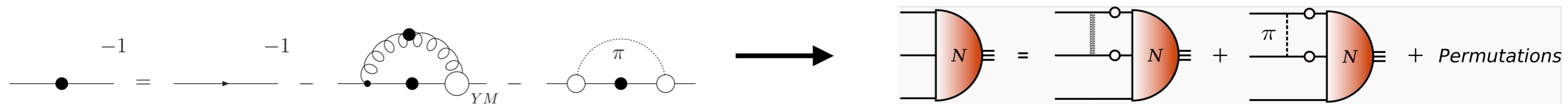
	RL	RL+3g	RL+3g+ π	PDG
M_π	138	138	138	138
M_ρ	758	881	805	776

CF,Williams, PRL 103 (2009), PRD 78 (2008)

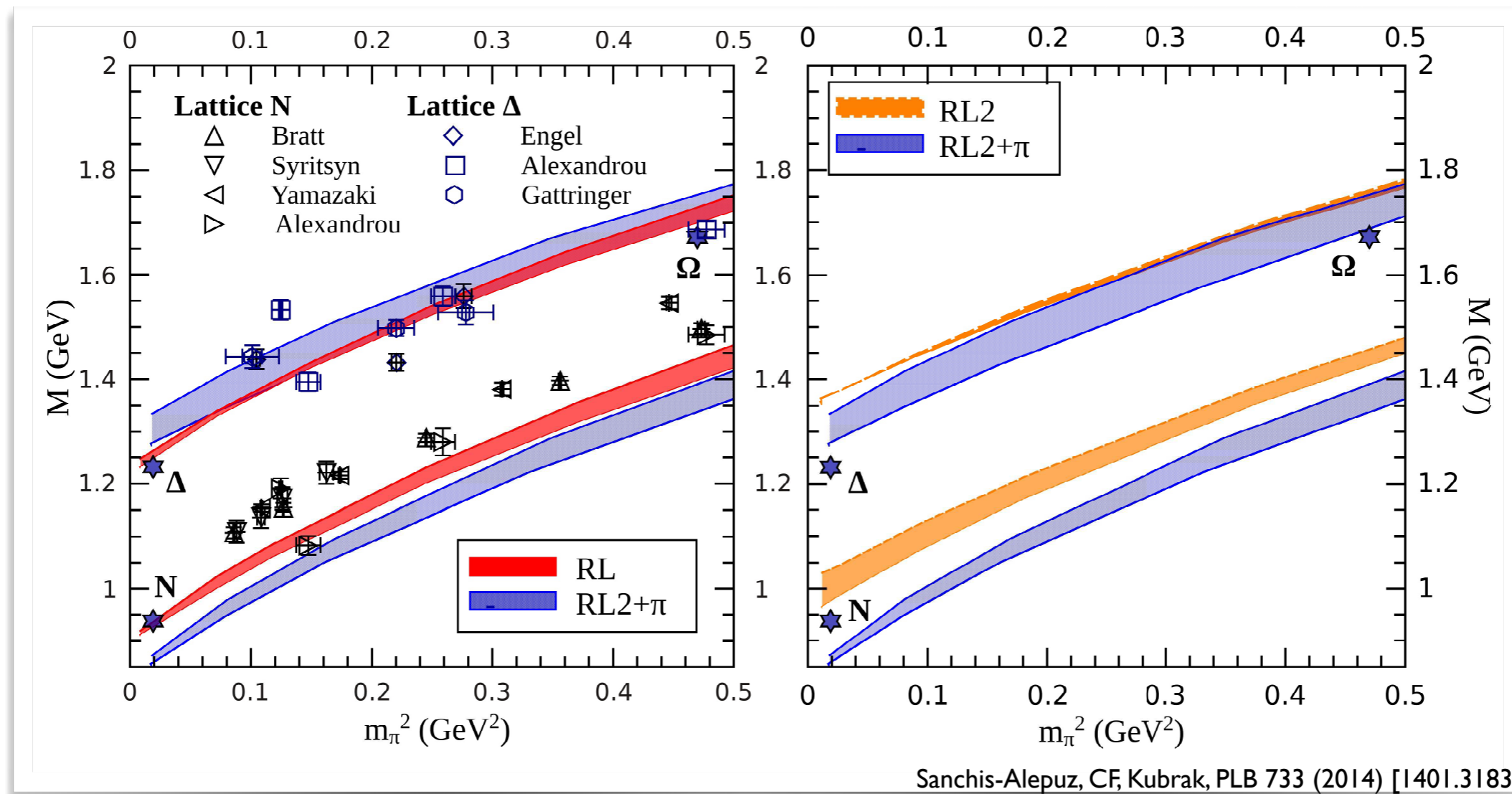
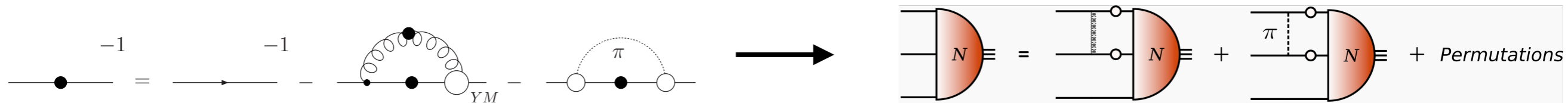
- Attractive effects of pion cloud
- Furthermore: generate decay $\rho \rightarrow \pi\pi$

Williams, accepted by PLB, arXiv:1804.11161

Pion cloud effects in baryons



Pion cloud effects in baryons



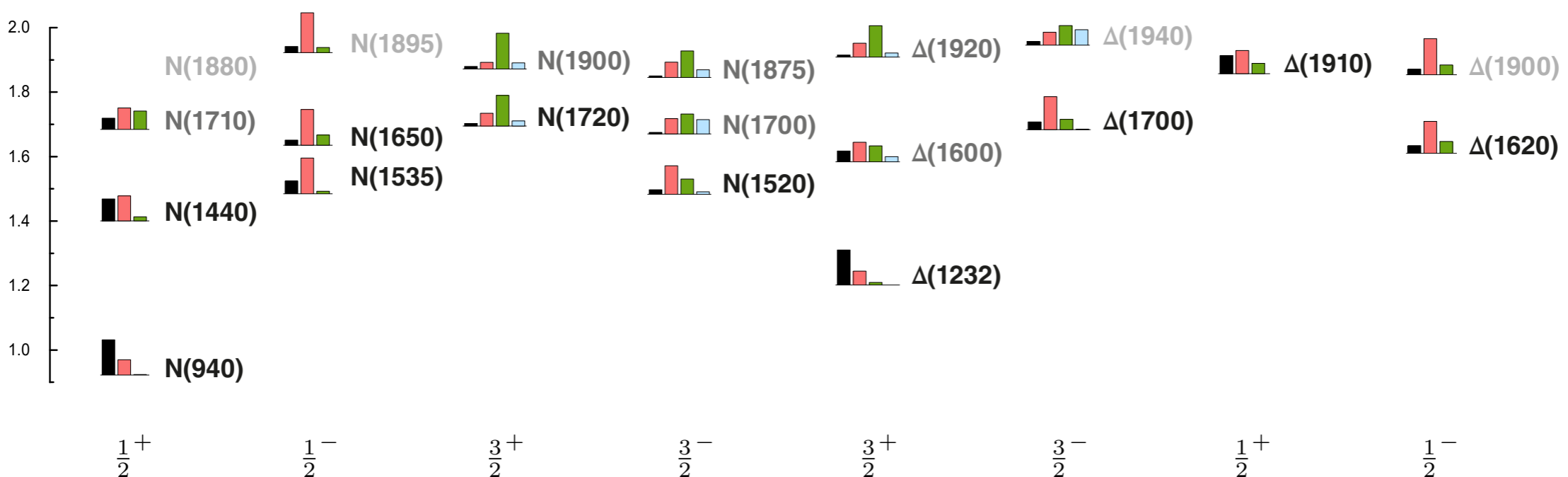
- fix Λ by f_π , vary η s.t. f_π still ok
- effects of the order of 50-100 MeV
- missing: gluon self-interaction effects

$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2} \right) e^{-\eta^2 \left(\frac{k^2}{\Lambda^2} \right)} + \alpha_{UV}(k^2)$$

Baryon spectrum

Quark-diquark with reduced pseudoscalar + vector diquarks: [GE, FBS 58 \(2017\)](#)

M [GeV]

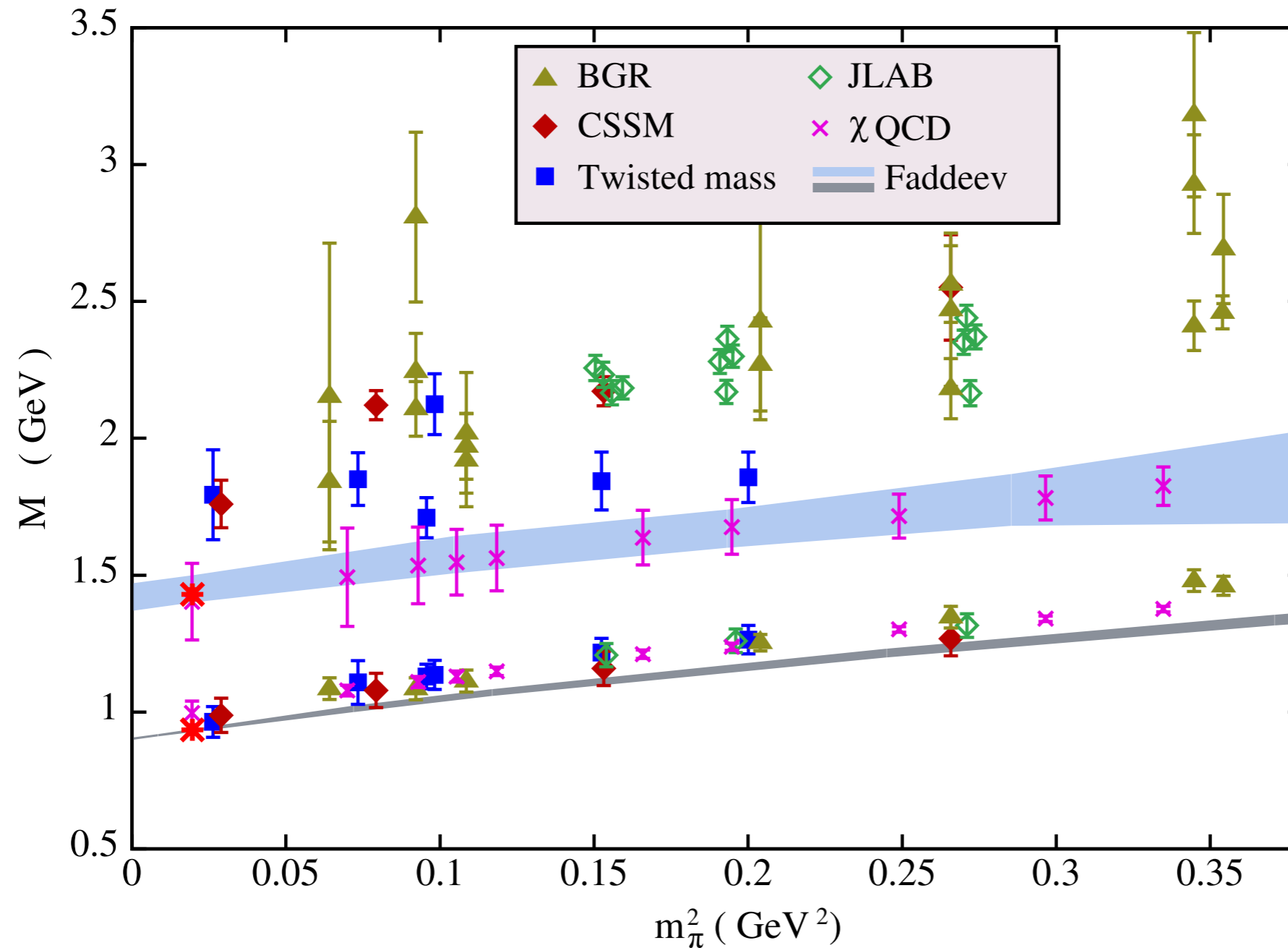


Orbital angular momentum content:



- in nonrelativistic quark model:
N, Δ ~ **s waves**, negative-parity states ~ **p waves**, etc.
- Here: ‘quark-model forbidden’ contributions are always present, e.g. **Roper: dominated by p waves** \Rightarrow **relativity is important!**

Mass evolution



Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]

- Mass evolution as expected for three-body state...

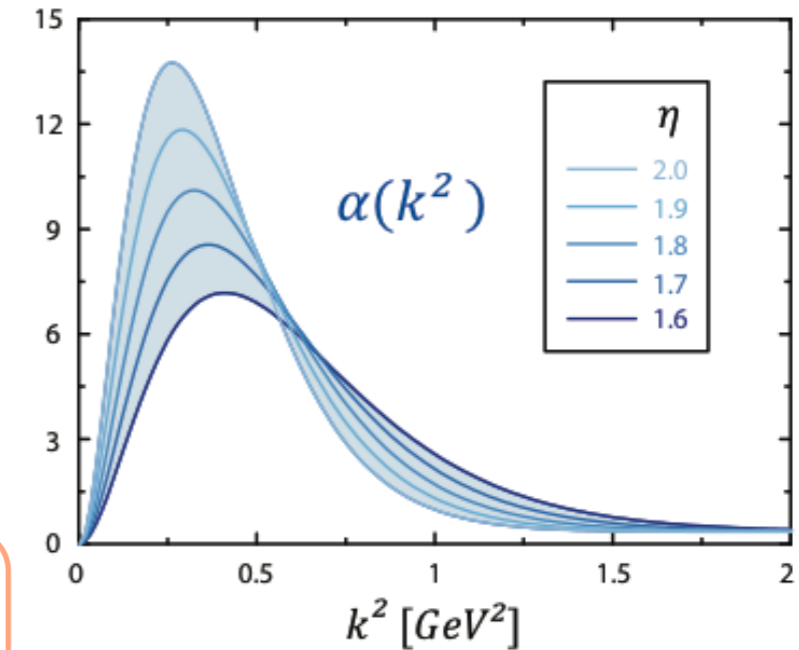
Rainbow-ladder model for quark-gluon interaction



Combine **gluon** with **quark-gluon vertex**:

effective coupling

$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2} \right) e^{-\eta^2 \left(\frac{k^2}{\Lambda^2} \right)} + \alpha_{UV}(k^2)$$



Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999)

- scale Λ from f_π , masses $m_u=m_d$, m_s from m_π, m_K
- α_{UV} from perturbation theory
- parameter η : band of results

Binosi, Chang, Papavassiliou and Roberts, PLB 742 (2015) 183

Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

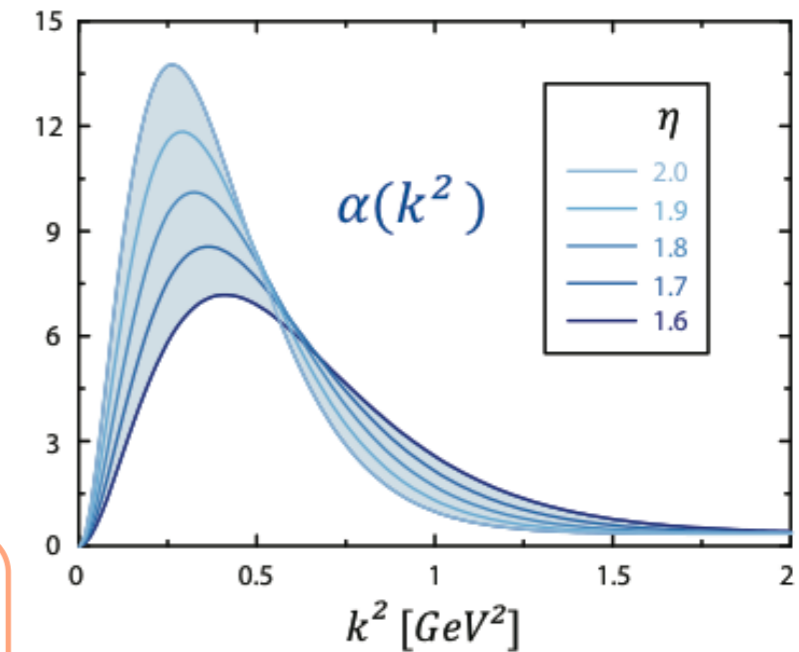
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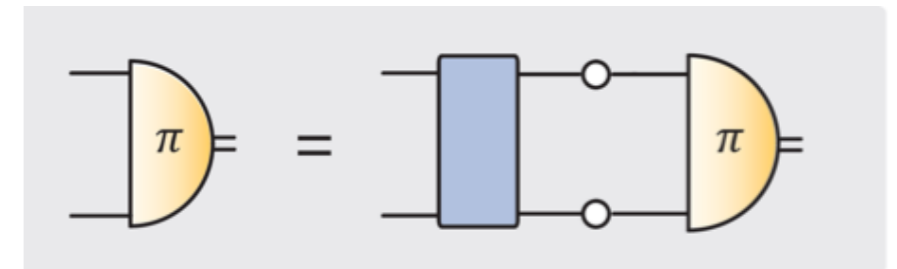
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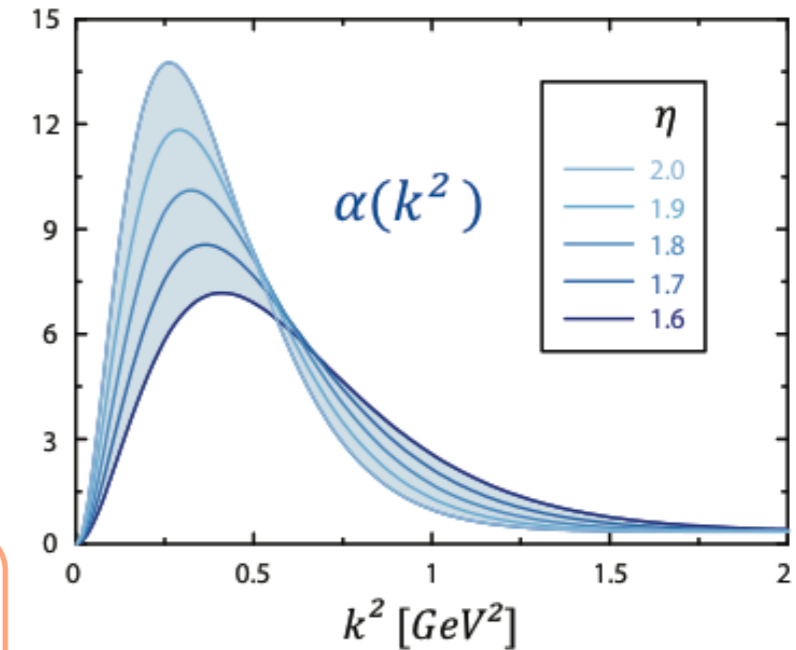
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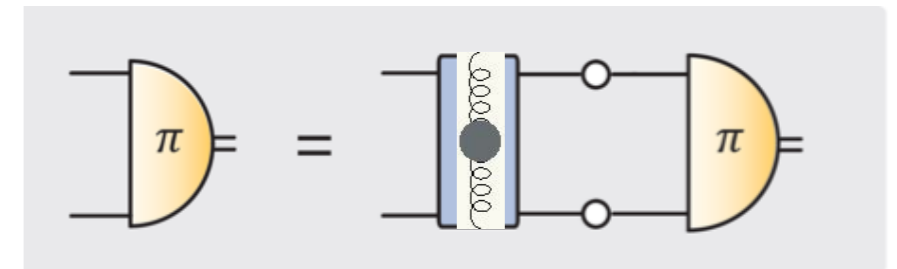
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Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999)

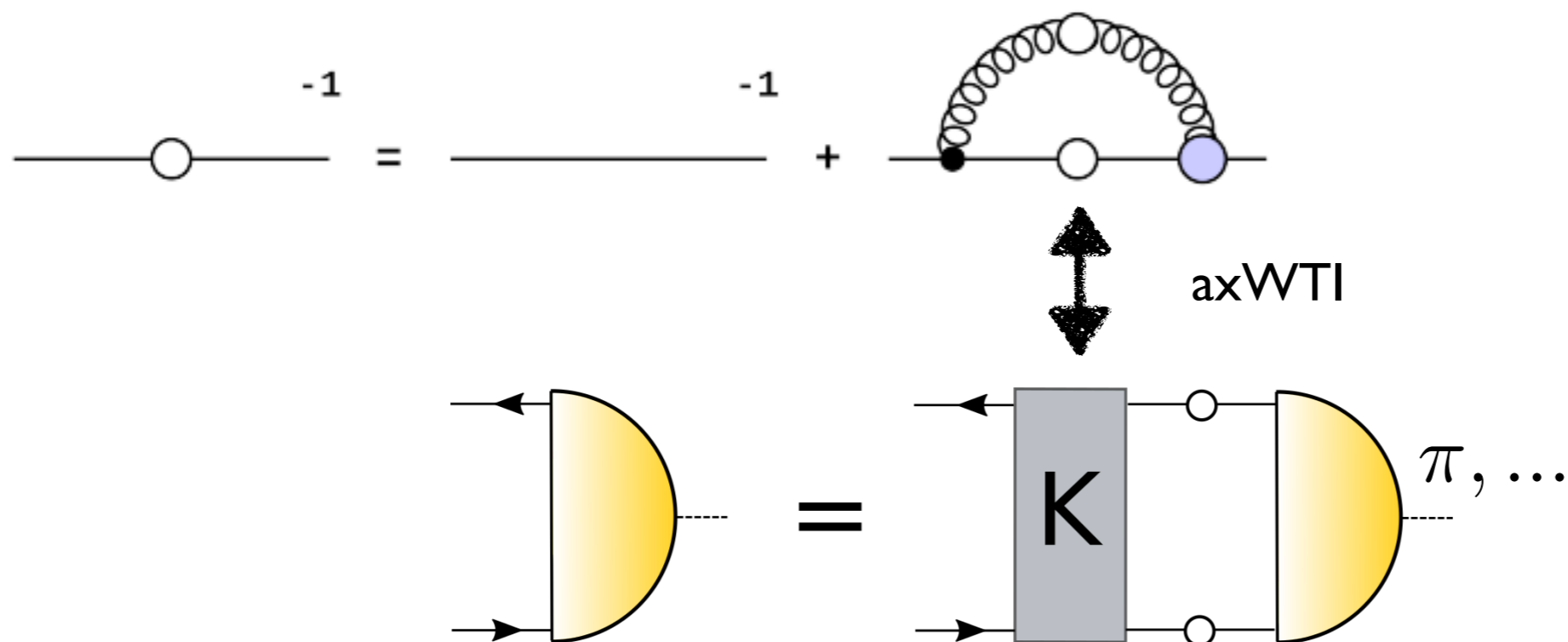
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Binosi, Chang, Papavassiliou and Roberts, PLB 742 (2015) 183

Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

DSEs and Bethe-Salpeter equation

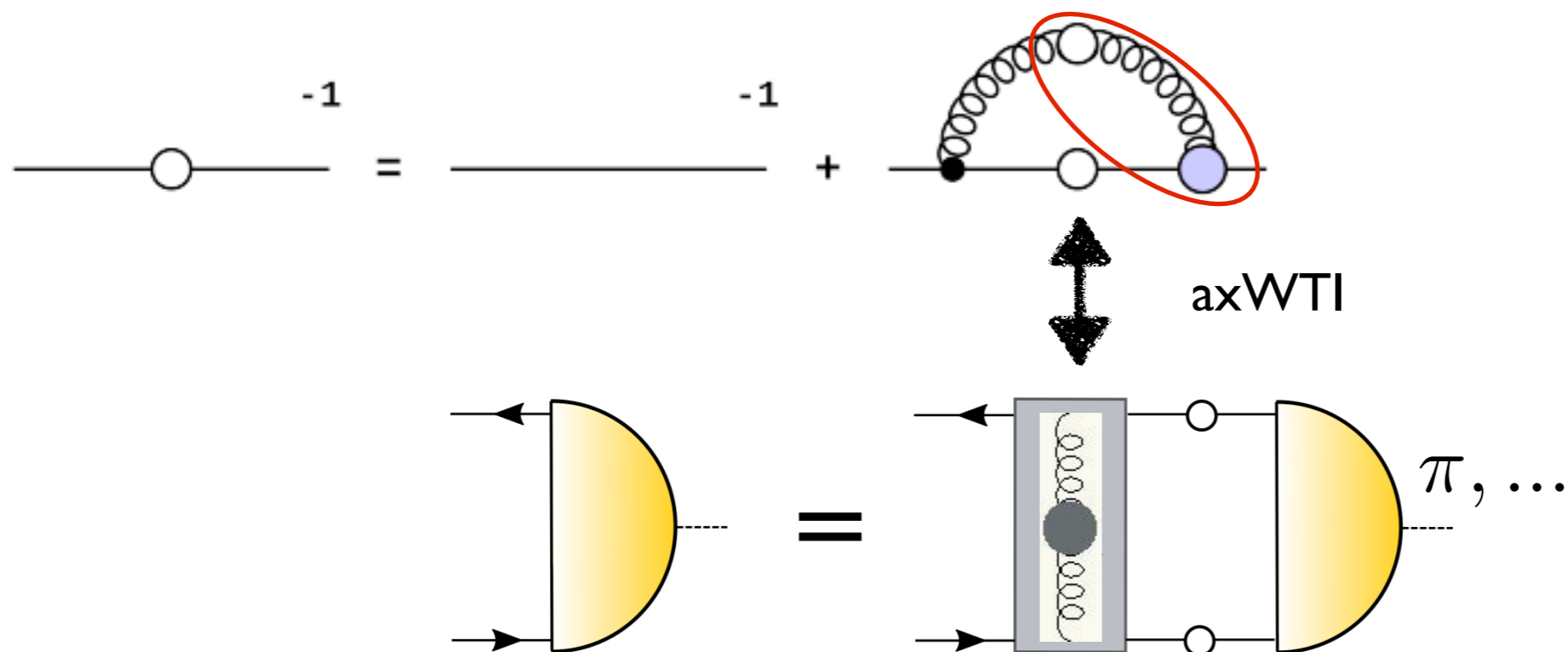


Kernel K uniquely related to quark-DSE via axialvector Ward-Takahashi-Identity (axWTI):

$$-i \int (K \gamma_5 S_- + K S_+ \gamma_5) = \int \gamma_\mu S_+ D_{\mu\nu} \Gamma_\nu \gamma_5 + \int \gamma_5 \gamma_\mu S_- D_{\mu\nu} \Gamma_\nu$$

→ Pion is bound state **and** Goldstone boson

DSEs and Bethe-Salpeter equation



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→ Pion is bound state **and** Goldstone boson

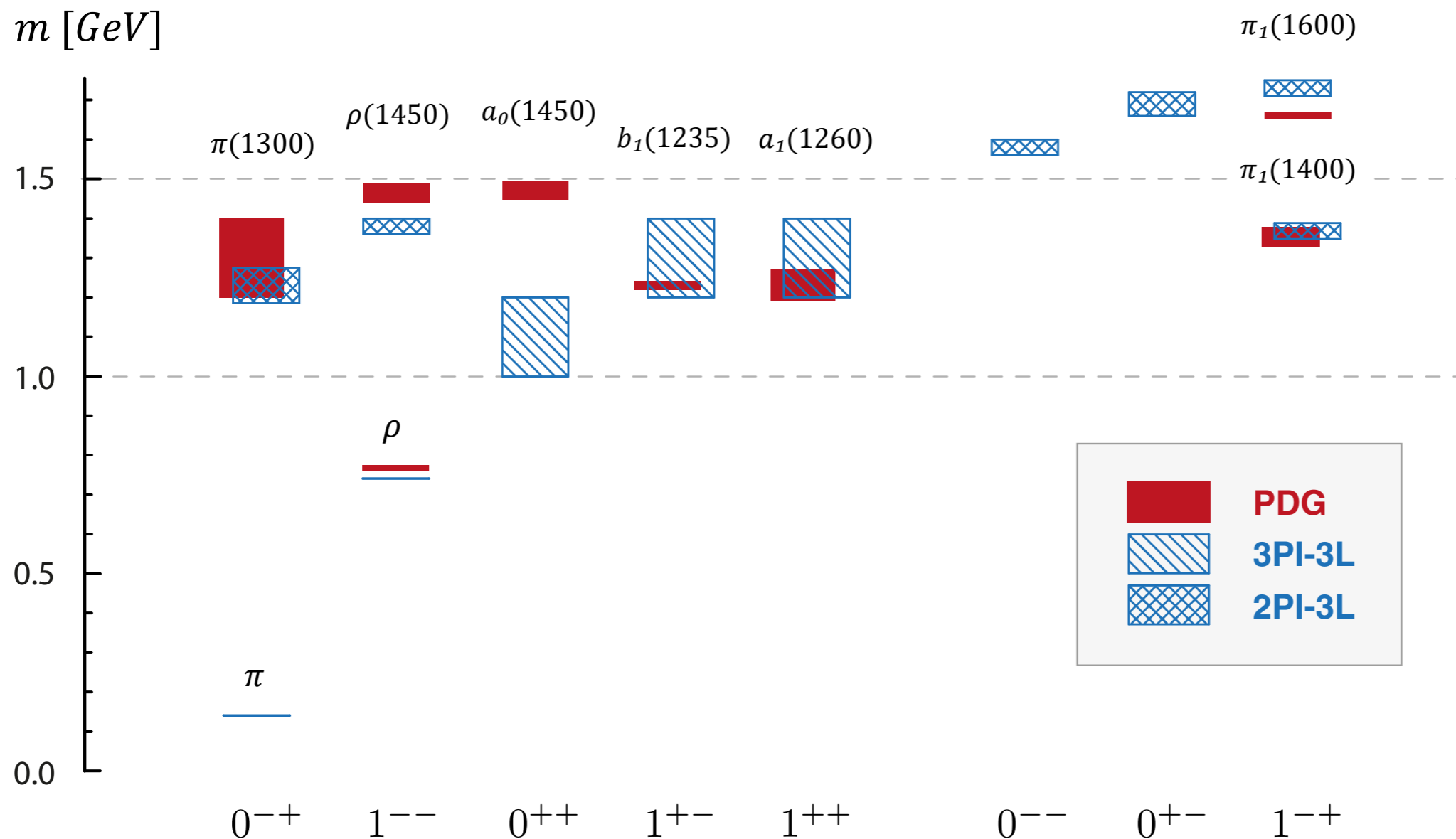
Maris, Roberts, Tandy, PLB 420 (1998) 267

CF, Kubrak, Williams, EPJA 50 (2014) 126

Williams, CF, Heupel, PRD93 (2016) 034026

- nice agreement with experiment (up to scalar)
- exotics as relativistic quark-antiquark states
- **drastic improvement beyond rainbow-ladder !**

Light meson spectrum (bRL)



CF, Kubrak, Williams, EPJA 50 (2014) 126

Williams, CF, Heupel, PRD93 (2016) 034026

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