XYZ Exotic Mesons

Physics Opportunities with Proton beams at SIS100

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Exotic Hadron

- Traditional quark model classification:
  mesons and baryons   Gell-Mann & Zweig 1964
- Exotic states: XYZ mesons (heavy-quark sector)

  ✓ States that don't fit traditional $Q\bar{Q}$ spectrum.

  ✓ Exotic quantum numbers:

  - $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}$ etc. are exotic
  - Charged Z_c and Z_b states: minimal 4-quark state:

    $Z_c(4430)^{\pm}$ $Z_b(10650)^{\pm}$ **Tetraquarks**

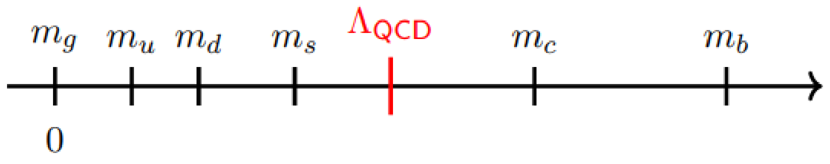
  For review see Brambilla et al. Phys. Reports. 873 (2020)

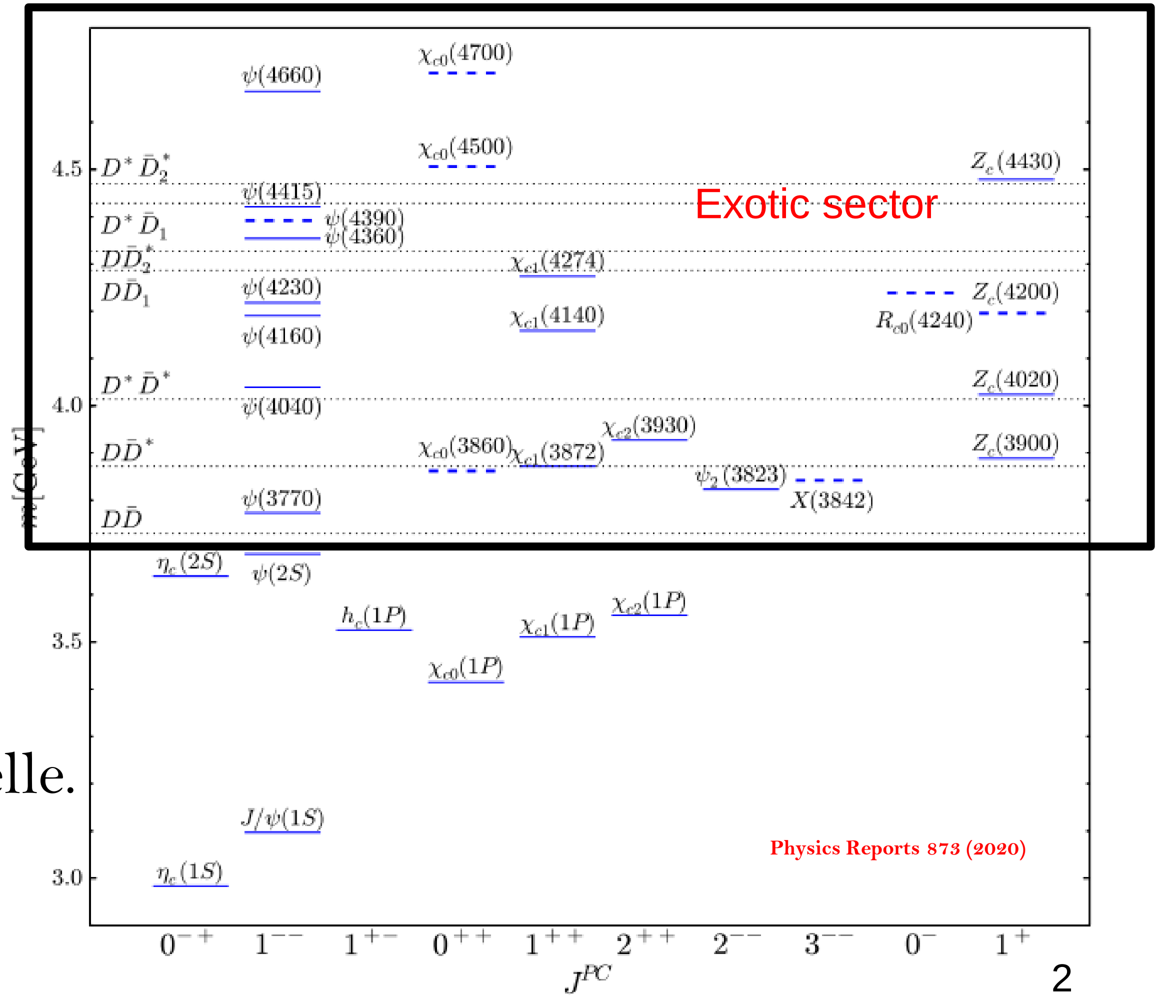
- X(3872): First exotic state discovered in 2003 by Belle.

  Phys. Rev. Lett. 91, 262001 (2003)

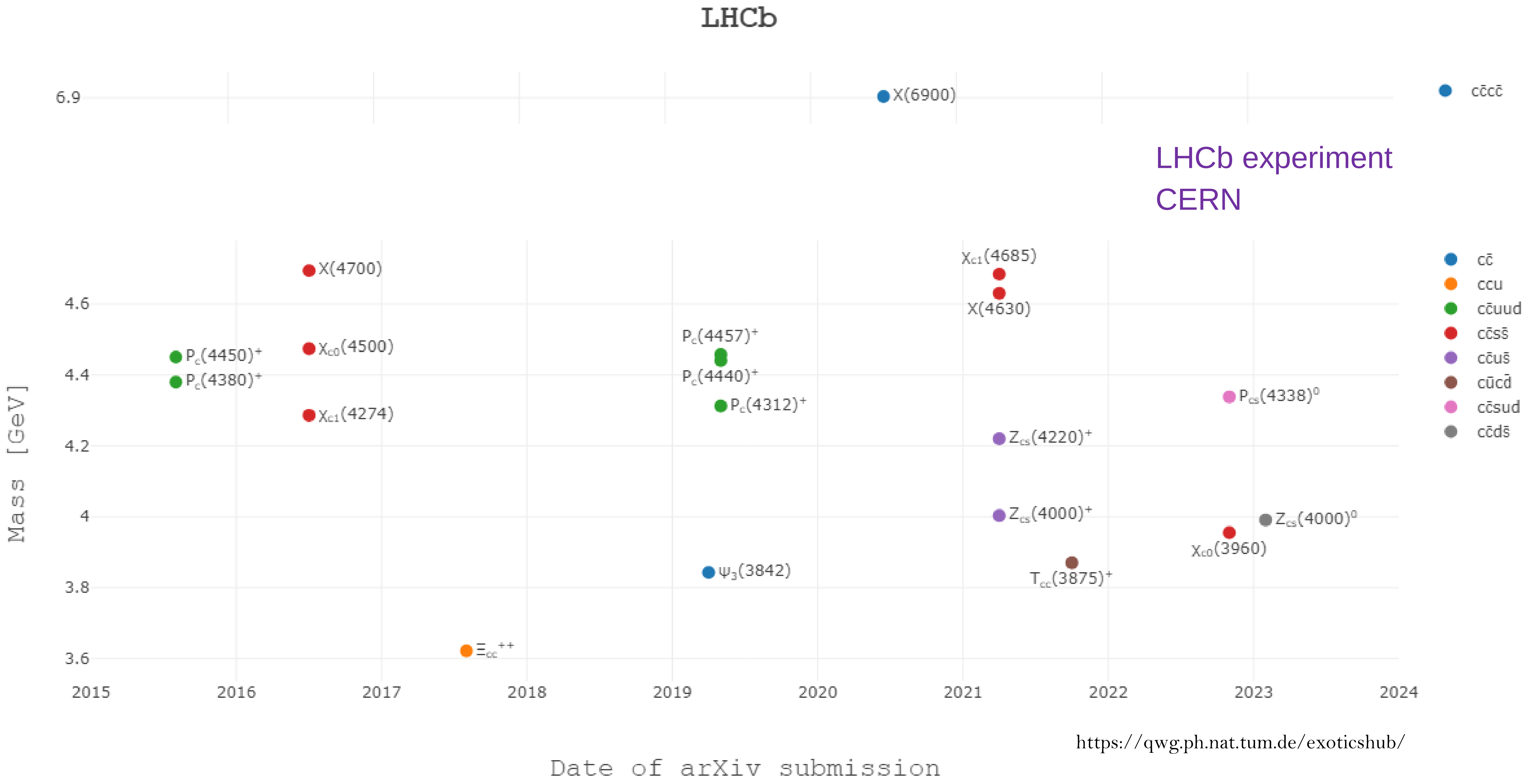
- Dozens of XYZ mesons discovered since 2003.

$m_c \approx 1.5\,\mathrm{GeV}$ $m_b \approx 5\,\mathrm{GeV}$





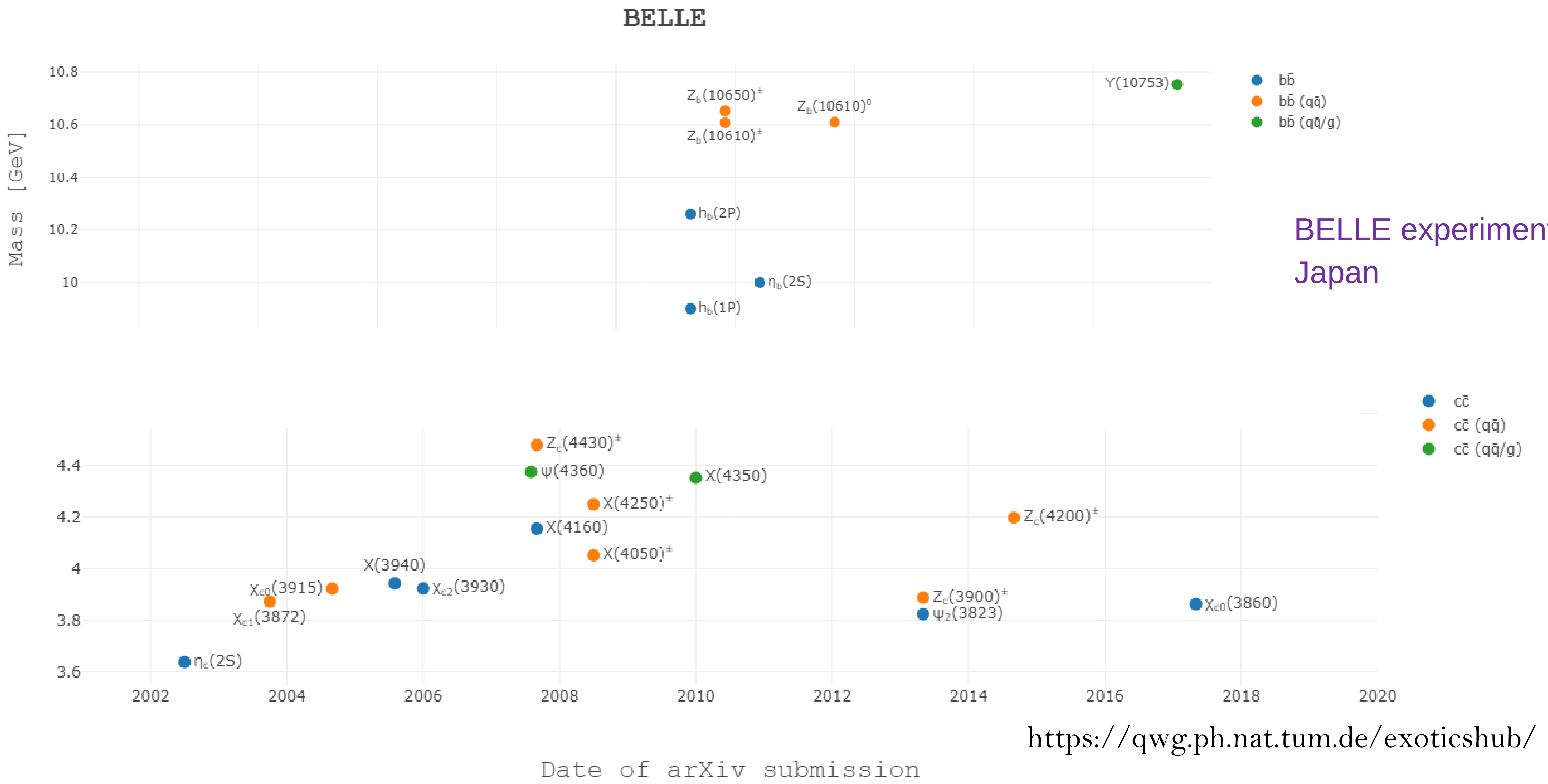
New Hadrons: XYZ mesons



LHCb experiment
CERN

https://qwg.ph.nat.tum.de/exoticshub/

New Hadrons: XYZ mesons



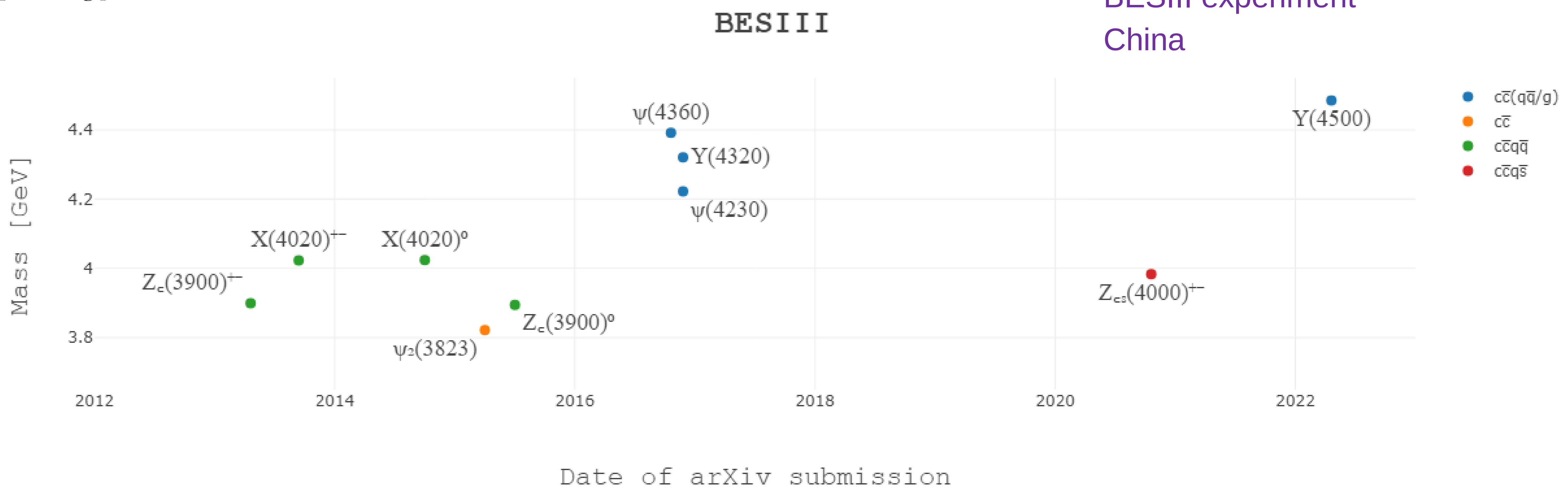
BELLE experiment
Japan

https://qwg.ph.nat.tum.de/exoticshub/

New Hadrons: XYZ mesons

https://qwg.ph.nat.tum.de/exoticshub/

BESIII experiment
China



Observation of many newer heavy hadrons are expected in the near future !!

BIG QUESTION:

How to understand XYZ states ? Can we theoretically predict the spectrum ??

Exotic Hadron

- Multiple Models for Exotics:

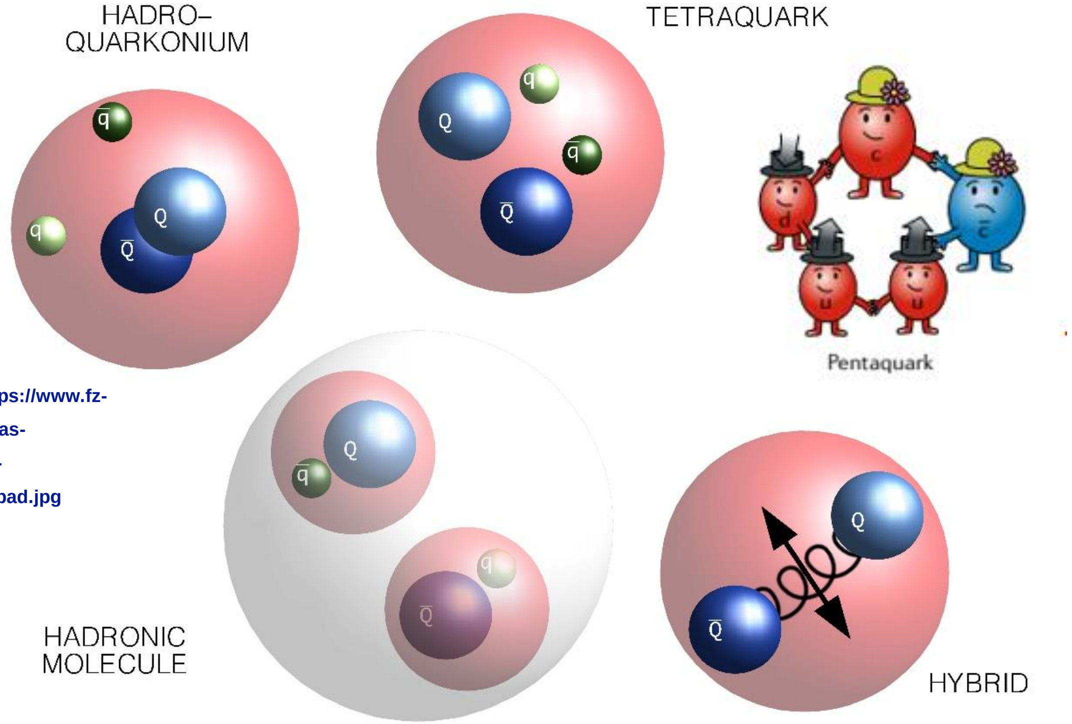


Figure from https://www.fz-juelich.de/en/ias/ias-4/research/exotic-hadrons/exotics_pad.jpg

Major **challenge** to understanding of **QCD spectrum!!!**

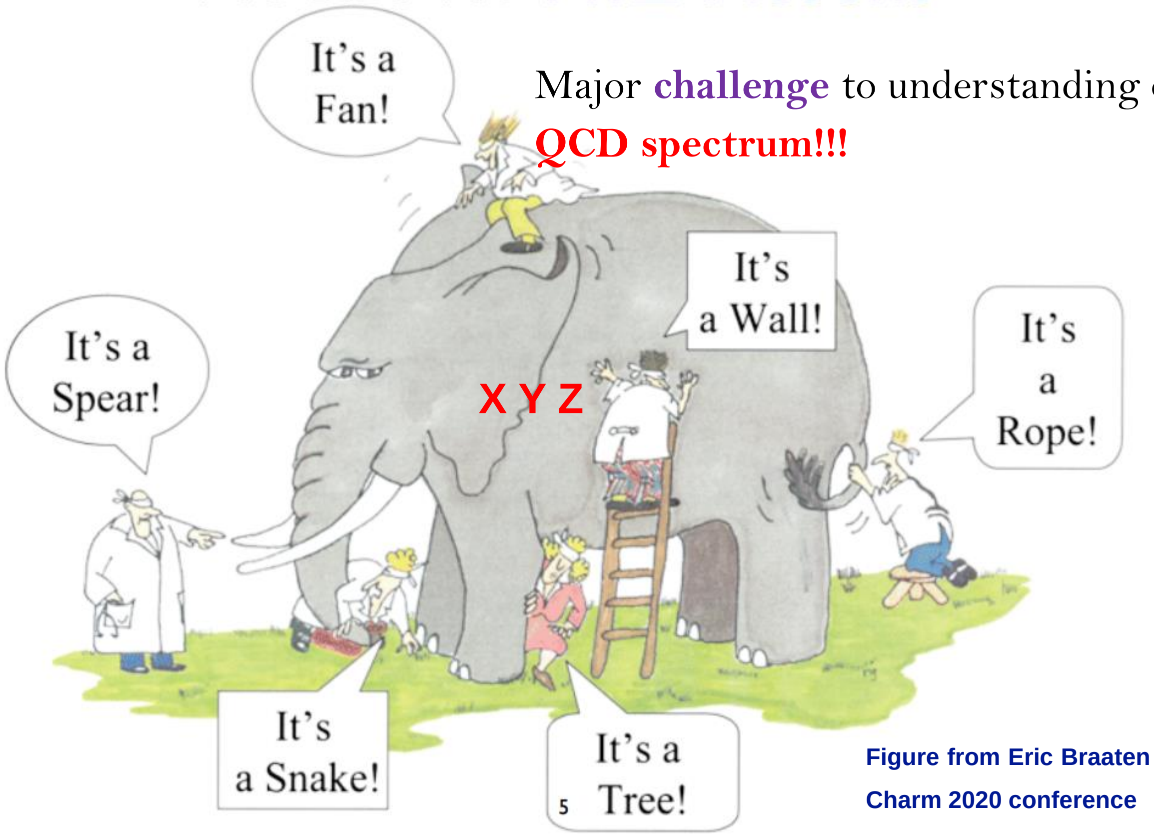


Figure from Eric Braaten talk: Charm 2020 conference

....

- Individual success in describing some XYZ hadrons. No success in revealing general pattern.

BIG QUESTION:

Is there a **coherent comprehensive framework** based on QCD for all X Y Z hadrons ???

Exotic Hadron

- Multiple Models for Exotics:

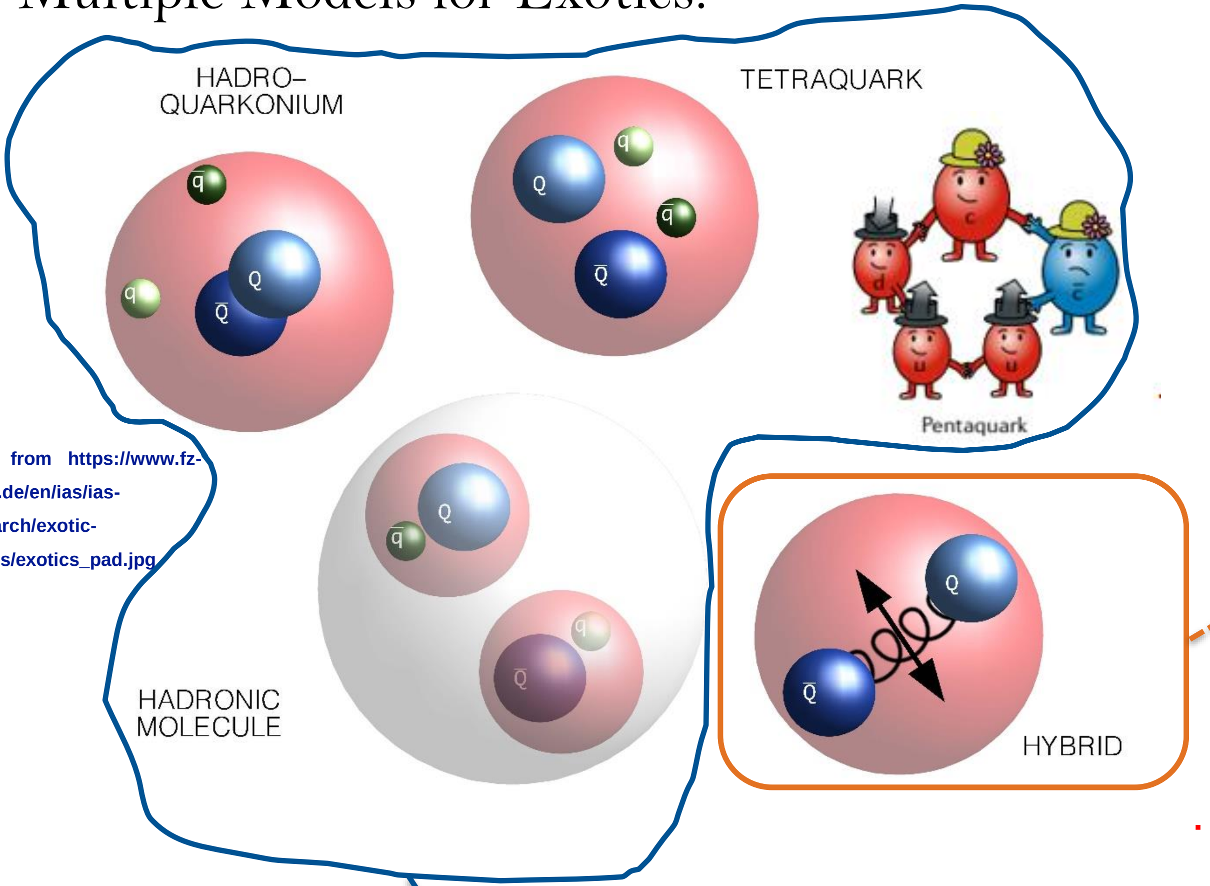


Figure from https://www.fz-juelich.de/en/ias/ias-4/research/exotic-hadrons/exotics_pad.jpg

Hybrids ($Q\bar{Q}g$): Isospin scalar exotic state.

Use EFT + lattice for describing hybrid

Brambilla, Lai, AM, Vairo Phys. Rev. D 107, 054034 (2023)

Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92, (2015), 114019

....

Non-zero isospin states. Can be described in the EFT.
However, lack of lattice inputs on the static energies for these states

Brambilla, AM, Vairo arXiv 2402.xxxxx

EFT Roadmap for $Q\bar{Q}/QQ$ systems

QCD → **NRQCD** → **potential-NRQCD (pNRQCD)**

$m \gg mv \gg mv^2 \sim \Lambda_{\rm QCD}$

(weak coupling regime)

Brambilla, Pineda, Soto and Vairo, Nucl. Phys. B 566, (2000) 275

Brambilla, Pineda, Soto and Vairo, Phys. Rev. D 63, (2000) 014023

$m \gg mv \sim \Lambda_{\rm QCD} \gg mv^2$

(nonperturbative dynamics: Strong coupling)

Weak-coupling regime:

- ✓ $Q\bar{Q}$ pair: color singlet & color octet fields
- ✓ Photons of energy scale $m_Q v^2$ mediate interactions
- ✓ Matching coefficients are potential which can be computed perturbatively
- ✓ Relevant for low-lying $Q\bar{Q}$ states Ex. $J/\psi, \Upsilon(1S)$.

Strong-coupling regime:

- ✓ Only color singlet fields.
- ✓ Matching coefficients are potential which are non-perturbative. Use Lattice inputs
- ✓ Relevant for any $Q\bar{Q}$ states.
- ✓ **Generalized to Exotic Hadrons: Born-Oppenheimer EFT.**

- Power counting will be unique since only one energy scale $m_Q v^2$.

BOEFT: Exotic Hadron

- Exotic hadron ($Q\bar{Q}X, QQX, \ldots\ldots$), X: any combination of light quark and gluons to obtain color singlet hadron

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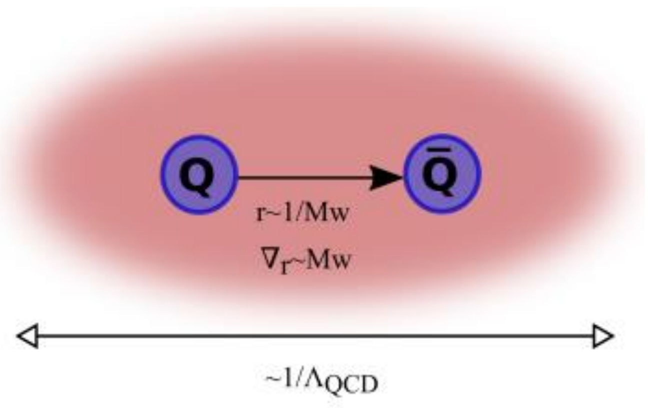
Example: Quarkonium hybrids $Q\bar{Q}g$, Tetraquarks $Q\bar{Q}q\bar{q}$



- Hierarchy of scales:

$$m \gg mv \gtrsim \Lambda_{\rm QCD} \gg mv^2$$

Heavy quark: slow-degrees of freedom X: fast-degrees of freedom

BOEFT: Effective theory based on Born-Oppenheimer Approximation

BOEFT: EFT focused at energy scale mv^2

$$\rm QCD \to NRQCD \to pNRQCD/BOEFT$$

- Time-scale for dynamics of $\rm Q\bar{Q}$: $\sim \frac{1}{\rm mv^2} \gg \frac{1}{\Lambda_{\rm QCD}}$

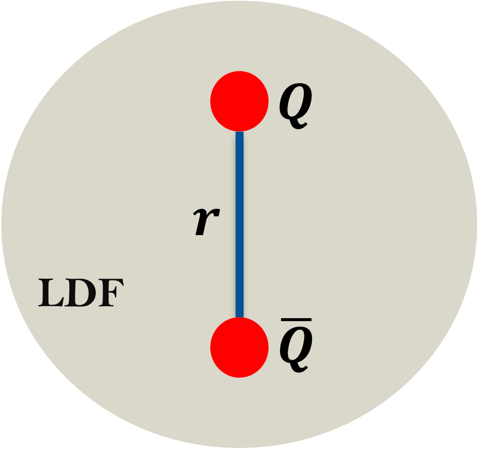
Born-Oppenheimer (BO) Approximation

Castellà , Soto Phys. Rev. D. 102, (2020)

Brambilla, AM, Vairo arXiv 2402.xxxxx

Braaten, Langmack, Smith Phys. Rev. D. 90, 014044 (2014)

BO-Quantum #'s



- **Static limit (m⟶ ∞):** heavy quarks are fixed in position.

  Cylindrical symmetry due to preferred quark-antiquark axis

- **BOEFT potentials ($V_\Gamma(r)$):** Potential between 2 heavy quarks due to energy of LDF (light quarks, gluons) known as **static energies**

- **$V_\Gamma(r)$** : **Γ** labelled by cylindrical symmetry representation of diatomic molecules:

  ✓ Absolute value of component of angular momentum of light d.o.f

  $$|\boldsymbol{r} \cdot \boldsymbol{K}_{\text{light}}| \equiv \Lambda = 0, 1, 2, \ldots \ldots \ldots (\text{or } \Sigma, \Pi, \Delta, \Phi, \ldots \ldots)$$

  ✓ Product of charge conjugation and parity ($\boldsymbol{CP}$):

  $$\boldsymbol{\eta} = +1 \ (\mathbf{g}), -1 \ (\boldsymbol{u})$$

  ✓ $\boldsymbol{\sigma}$: Eigenvalue of reflection about a plane containing static sources.

  $$\sigma = P\,(-1)^{K_{\text{light}}} = \pm 1$$

  Braaten, Langmack, Smith Phys. Rev. D. 90, 014044 (2014)

$$\Gamma \equiv \Lambda_\eta^\sigma$$

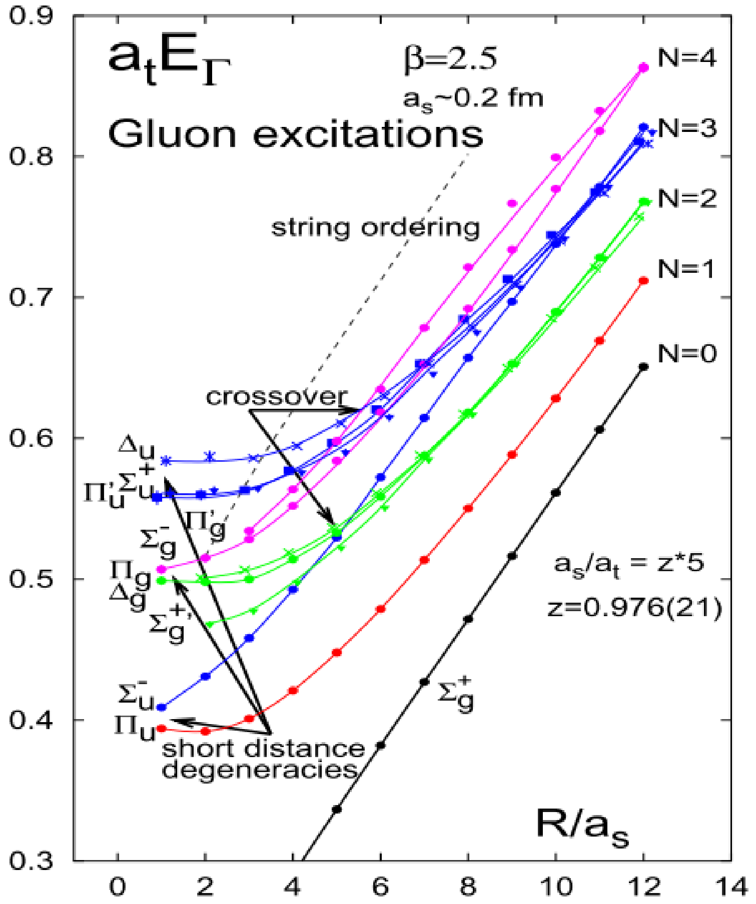
- r⟶ 0: **Spherical symmetry restored**: Labelled by LDF quantum #'s $\kappa = K^{PC}$.

Brambilla, Pineda , Soto, and Vairo Nucl. Phys. B566 (2000)   Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92, (2015), 114019

Static Energies Examples:

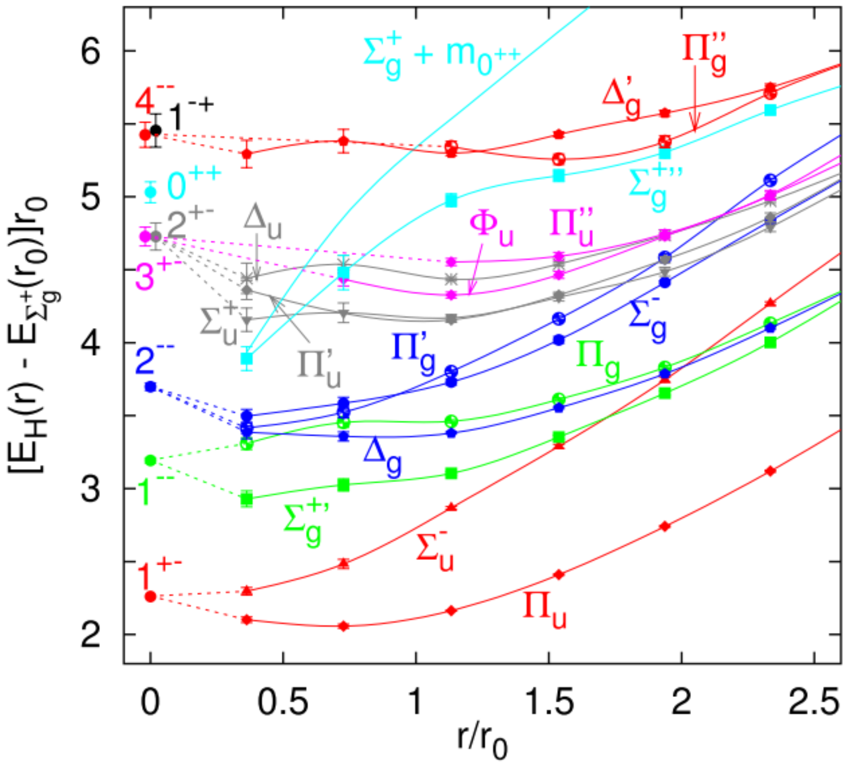
- **Static limit ($m \to \infty$):** heavy quarks are fixed in position. Interquark potential given by gluon configuration.

K. Juge, J. Kuti, C. Morningstar, Phys. Rev. Lett. 90 (2003)

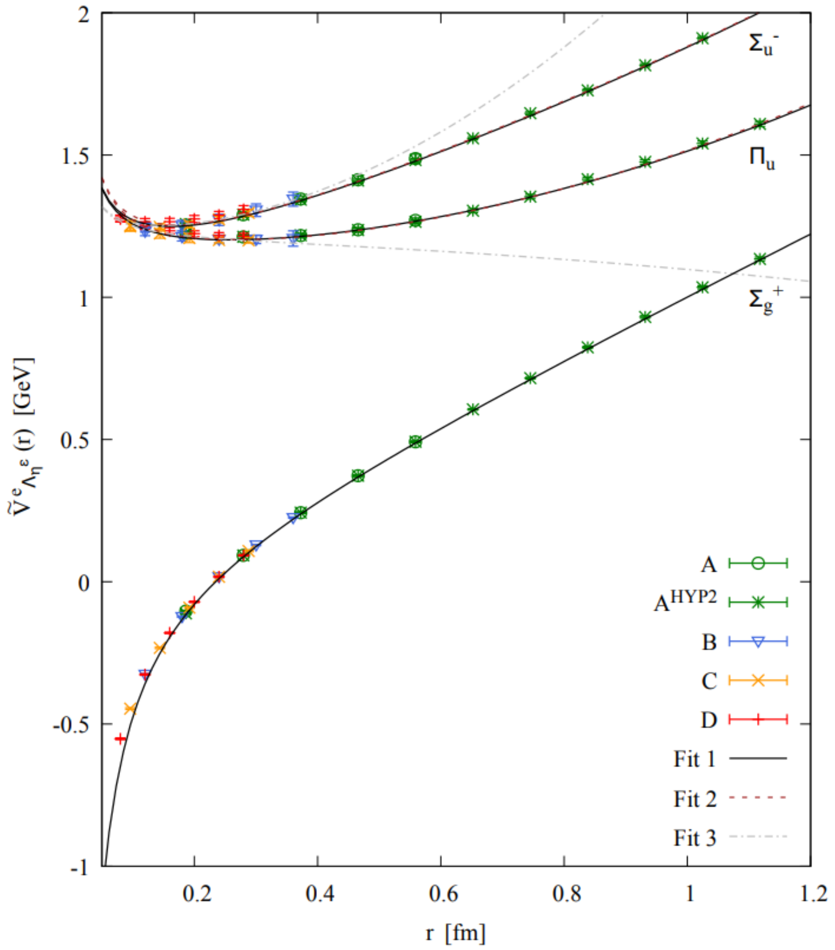


Results of **nonperturbative** Gluonic Static energies from lattice:

Bali and Pineda Phys. Rev. D. 69, (2004)



Schlosser and Wagner Phys. Rev. D. 105, (2022)

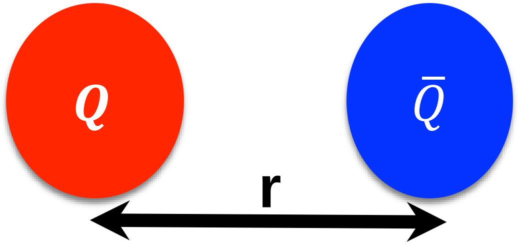


Exotic Hadron

Brambilla, AM, Vairo
arXiv 2402.xxxxx

- Exotic hadron ($Q\bar{Q}X, QQX, \ldots\ldots$), X is light d.o.f.

$Q\bar{Q}X$



color: $3 \otimes \bar{3} = 1 \oplus 8$

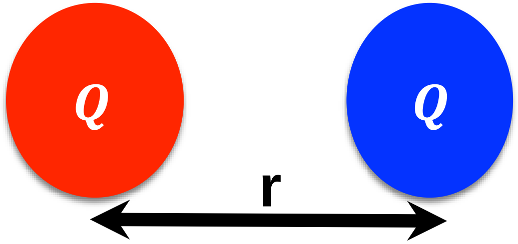
X= gluon → **Hybrid**

X= $q\bar{q}$ → **Tetraquark / Molecule**

X= qqq → **Pentaquark / Molecule** and so on

BOEFT can address all these states with inputs from Lattice QCD on BOEFT potentials

QQX



color: $3 \otimes \bar{3} = \bar{3} \oplus 6$

X= q → **Double heavy baryon**

X= $\bar{q}\bar{q}$ → **Tetraquark**

X= $q\bar{q}q$ → **Pentaquark** and so on

$$\mathcal{O}_K(t,\, \mathbf{r},\, \mathbf{R}) \longrightarrow Z_{\Psi_K}(r,\, \Lambda_{\rm QCD})\Psi_K(t,\, \mathbf{r},\, \mathbf{R})$$

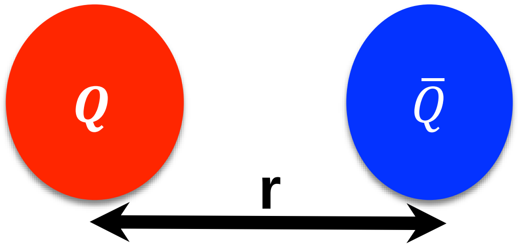
NRQCD BOEFT

Matching condition discussed in Brambilla, AM, Vairo arXiv 2402.xxxxx.

Light quark operators essential for determinig BO-potentials $V_{\Lambda_\eta^\sigma}(r)$

pNRQCD/BOEFT

- Exotic hadron ($\boldsymbol{Q\bar{Q}X}, \boldsymbol{QQX}, \ldots\ldots$), $\boldsymbol{X}$ is light d.o.f.



spin: $1/2 \otimes 1/2 = 0 \oplus 1$

color: $3 \otimes \bar{3} = 1 \oplus 8$

Quantum number of light d.o.f $\boldsymbol{X}$ includes spin $\mathbf{K}$, isospin, color etc….

NRQCD operator (gauge invariant) for exotic hadron $\boldsymbol{Q\bar{Q}X}$:

$$\mathcal{O}_K(t,\,\mathbf{r},\,\mathbf{0}) = \chi^\dagger(\mathbf{t},\,\mathbf{r}/2)\phi(\mathbf{t},\,\mathbf{r}/2,\mathbf{0})\mathbf{H}_\mathbf{K}(\mathbf{t},\,\mathbf{0})\phi(\mathbf{t},\,\mathbf{0},-\mathbf{r}/2)\psi(\mathbf{t},\,-\mathbf{r}/2)$$

$H_K(t,\,\mathbf{0})$: Operator that characterizes the light d.o.f $\boldsymbol{X}$ corresponding to quantum # $\mathbf{K}$, isospin, color etc..

Let's say $\boldsymbol{Q\bar{Q}}$ pair in octet color, then the operator characterizing $\boldsymbol{Q\bar{Q}X}$

$$\underbrace{\mathcal{O}_K(t,\,\mathbf{r},\,\mathbf{R})}_{\text{NRQCD}} \longrightarrow \underbrace{\mathbf{Z}_{\mathbf{H_K}}(\mathbf{r})\mathbf{O^a}(\mathbf{t},\,\mathbf{r},\,\mathbf{R})\mathbf{H^a_K}(\mathbf{t},\,\mathbf{R})}_{\text{Weakly coupled pNRQCD}} \longrightarrow \underbrace{\mathbf{Z}_{\mathbf{\Psi_K}}(\mathbf{r},\,\mathbf{\Lambda}_{\mathrm{QCD}})\mathbf{\Psi_K}(\mathbf{t},\,\mathbf{r},\,\mathbf{R})}_{\text{BOEFT}}$$

Castellà , Soto Phys. Rev. D. 102, (2020) Brambilla, AM, Vairo arXiv 2402.xxxxx

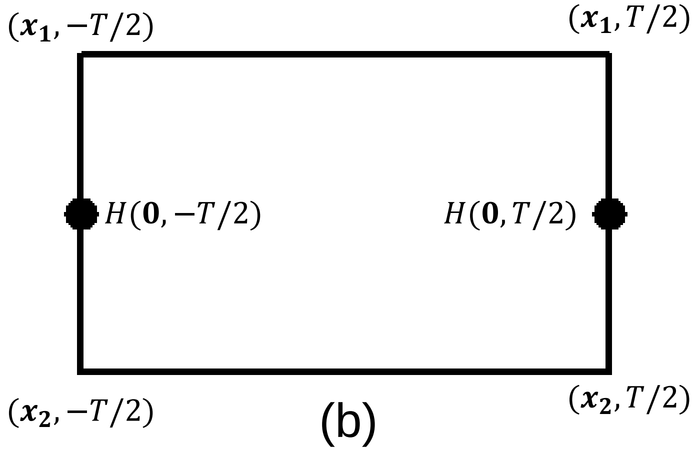
pNRQCD/BOEFT: Potentials

$$E_\kappa^{(0)}(r) = \lim_{T\to\infty} \frac{i}{T} \log\langle X_\kappa, T/2 | X_\kappa, -T/2\rangle$$

$$\kappa = K^{PC}$$ (Light d.o.f)

$$|X_\kappa; t\rangle = \mathcal{O}_\kappa^\dagger\,(t, \mathbf{r}, \mathbf{0})\,|\text{vac}\rangle$$

(a)



(b)

$$W_\square \equiv \langle 1 \rangle_\square$$

Wilson loop for quarkonium

$$\langle H\,(\mathbf{0}, T/2)\, H\,(\mathbf{0}, -T/2)\rangle_\square$$

Wilson loop for exotics

Short-distance behavior of BO-Potentials:

$$E_{\Sigma_g^+}(r) = V_s(r) + b_{\Sigma_g^+}\, r^2 + \ldots,$$
$$E_{\Sigma_u^-,\Pi_u}(r) = V_o(r) + \Lambda + b_{\Sigma,\Pi} r^2 + \ldots$$

Long-distance behavior of BO-Potentials:

- String behavior (pure SU(3) gauge theory)

$$E_N(r) = \sqrt{\sigma^2 r^2 + 2\pi\sigma\,(N - 1/12)}$$

- Mixing with pair of heavy-light states based on BO-quantum numbers or $\boldsymbol{\Lambda_\eta^\sigma}$ representations

Brambilla, AM, Vairo arXiv 2402.xxxxx

K. Juge, J. Kuti, C. Morningstar, Phys. Rev. Lett. 90 (2003)

Bali et al (SESAM collaboration), Phys. Rev. D. 71 (2005)

Bulava et al, Phys. Lett. B. 793 (2019)

Gluonic operators $\mathbf{H}_{\mathbf{K}^{\mathbf{PC}}}$ in lattice characterizing Hybrids **$Q\bar{Q}g$**

$$\mathbf{H}_{1^{+-}}(t,\boldsymbol{x}) = \boldsymbol{B}(t,\boldsymbol{x})$$
$$\mathbf{H}_{1^{--}}(t,\boldsymbol{x}) = \boldsymbol{E}(t,\boldsymbol{x})$$

Light quark operators $\mathbf{H}_{\mathbf{K}^{\mathbf{PC}}}$ relevant for lattice computation of static energies for tetraquarks **$Q\bar{Q}q\bar{q}$**

$$\mathbf{H}_{0^{++}}(t,\boldsymbol{x}) = [\bar{q}(t,\boldsymbol{x})T^a q(t,\boldsymbol{x})]\, T^a$$
$$\mathbf{H}_{0^{-+}}(t,\boldsymbol{x}) = \left[\bar{q}(t,\boldsymbol{x})\gamma^5 T^a q(t,\boldsymbol{x})\right] T^a$$
$$\mathbf{H}_{1^{++}}(t,\boldsymbol{x}) = \left[\bar{q}(t,\boldsymbol{x})\boldsymbol{\gamma}\gamma^5 T^a q(t,\boldsymbol{x})\right] T^a$$
$$\mathbf{H}_{1^{--}}(t,\boldsymbol{x}) = [\bar{q}(t,\boldsymbol{x})\boldsymbol{\gamma} T^a q(t,\boldsymbol{x})]\, T^a$$
$$\mathbf{H}_{1^{+-}}(t,\boldsymbol{x}) = [\bar{q}(t,\boldsymbol{x})\,(\boldsymbol{\gamma}\times\boldsymbol{\gamma})\, T^a q(t,\boldsymbol{x})]\, T^a$$

Light quark operators $\mathbf{H}_{\mathbf{K}^{\mathbf{PC}}}$ relevant for lattice computation of static energies for pentaquarks **$Q\bar{Q}qqq$**

Brambilla, AM, Vairo arXiv 2312.xxxxx

$$H^{\alpha}_{I_3,(1/2)^+}(t,\boldsymbol{x}) =$$
$$\Big[\left(\delta_{\alpha\beta_1}\sigma^2_{\beta_2\beta_3}+\delta_{\alpha\beta_2}\sigma^2_{\beta_1\beta_3}+\delta_{\alpha\beta_3}\sigma^2_{\beta_1\beta_2}\right)\left(\delta_{I_3f_1}\tau^2_{f_2f_3}+\delta_{I_3f_2}\tau^2_{f_1f_3}+\delta_{I_3f_3}\tau^2_{f_1f_2}\right)(T_2)^a_{l_1,l_2,l_3}$$
$$+\left(\delta_{\alpha\beta_1}\sigma^2_{\beta_2\beta_3}+\delta_{\alpha\beta_2}\sigma^2_{\beta_3\beta_1}+\delta_{\alpha\beta_3}\sigma^2_{\beta_2\beta_1}\right)\left(\delta_{I_3f_1}\tau^2_{f_2f_3}+\delta_{I_3f_2}\tau^2_{f_3f_1}+\delta_{I_3f_3}\tau^2_{f_2f_1}\right)(T_3)^a_{l_1,l_2,l_3}$$
$$+\left(\delta_{\alpha\beta_1}\sigma^2_{\beta_3\beta_2}+\delta_{\alpha\beta_2}\sigma^2_{\beta_3\beta_1}+\delta_{\alpha\beta_3}\sigma^2_{\beta_1\beta_2}\right)\left(\delta_{I_3f_1}\tau^2_{f_3f_2}+\delta_{I_3f_2}\tau^2_{f_3f_1}+\delta_{I_3f_3}\tau^2_{f_1f_2}\right)(T_1)^a_{l_1,l_2,l_3}\Big]$$
$$(P_+q_{l_1f_1}(t,\boldsymbol{x}))^{\beta_1}\,(P_+q_{l_2f_2}(t,\boldsymbol{x}))^{\beta_2}\,(P_+q_{l_3f_3}(t,\boldsymbol{x}))^{\beta_3}\,T^a\,. \quad (53)$$

Similar operator list can be written for Doubly heavy tetraquark **$QQ\bar{q}\bar{q}$** and Pentaquark states **$QQqq\bar{q}$**. List of operators will be addressed in Brambilla, AM, Vairo arXiv 2402.xxxxx

Hybrids

BOEFT: Hybrids

Brambilla, Lai, AM, Vairo
Phys. Rev. D 107, 054034 (2023)

- BOEFT Lagrangian:

$$L_{\mathrm{BOEFT}} = L_{\Psi} + L_{\Psi_{\kappa\lambda}} + L_{\mathrm{mixing}},$$

Brambilla, Krein, Castellà , Vairo Phys. Rev. D. 97, (2018)

Quarkonium: $$L_{\Psi} = \int d^3\mathbf{R} \int d^3\mathbf{r}\, \mathrm{Tr}\left[\Psi^{\dagger}(\mathbf{r},\,\mathbf{R},\,t)\left(i\partial_t + \frac{\nabla_r^2}{m_Q} - V_{\Psi}(r)\right)\Psi(\mathbf{r},\,\mathbf{R},\,t)\right]$$

Trace over spin indices.

Hybrid: $$L_{\Psi_{\kappa\lambda}} = \int d^3\mathbf{R} \int d^3\mathbf{r} \sum_{\kappa\lambda\lambda'} \mathrm{Tr}\left\{\Psi^{\dagger}_{\kappa\lambda}(\mathbf{r},\,\mathbf{R},\,t)\left[i\partial_t - V_{\kappa\lambda\lambda'}(r) + P^{i\dagger}_{\kappa\lambda}\frac{\nabla_r^2}{m_Q}P^{i}_{\kappa\lambda'}\right]\Psi_{\kappa\lambda'}(\mathbf{r},\,\mathbf{R},\,t)\right\}$$

r: relative coordinate
R: COM coordinate

Hybrid potential: $$V_{\kappa\lambda\lambda'}(r) \equiv P^{i\dagger}_{\kappa\lambda}V^{ij}_{\kappa}(r)P^{j}_{\kappa\lambda'} = V^{(0)}_{\kappa\lambda}(r)\delta_{\lambda\lambda'} + \frac{V^{(1)}_{\kappa\lambda\lambda'}(r)}{m_Q} + \dots$$

Static potential — Spin-dependent potential

Brambilla, Lai, Segovia, Castellà, Vairo Phys. Rev. D. 101, (2020)

Brambilla, Lai, Segovia, Castellà, Vairo Phys. Rev. D. 99, (2019)

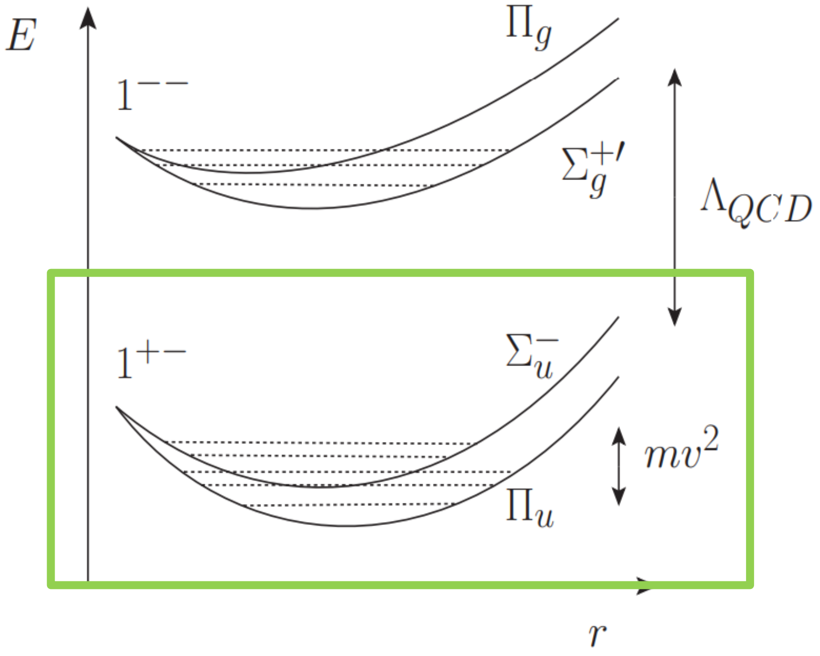
- Hybrid spin-dependent potentials: at order $1/m_Q$ (contrary to quarkonium $O(1/m_Q^2)$)

Soto, Valls, arXiv 2302.01765

Hybrid-Quarkonium mixing: $$L_{\mathrm{mixing}} = -\int d^3\mathbf{R} \int d^3\mathbf{r} \sum_{\kappa\lambda} \mathrm{Tr}\left[\Psi^{\dagger}\, V^{\mathrm{mix}}_{\kappa\lambda}\,\Psi_{\kappa\lambda} + \mathrm{h.c.}\right]$$

- No lattice calculations on mixing potential. Current work, ignore mixing, $V^{\mathrm{mix}}_{\kappa\lambda} = 0$

BOEFT: Hybrids



Brambilla, Lai, AM, Vairo Phys. Rev. D 107, 054034 (2023)

Coupled Schrödinger equation can be derived from general principles of rigid-body dynamics considering the pair of heavy quarks as linear Rigid rotor. Derivation in Brambilla, AM, Vairo arXiv 2402.xxxxx.

- Degeneracy at short distances $r \to 0$, mixes hybrid states corresponding to Σ_u^- and Π_u potential

- Coupled Schrödinger Eq: due to angular part of the kinetic term.

Schrödinger equation

$$\left[-P^{i\dagger}_{\kappa\lambda}\frac{\nabla_r^2}{m}P^i_{\kappa\lambda'} + V_{\kappa\lambda\lambda'}(r)\right]\Psi^n_{\kappa\lambda'}(\boldsymbol{r}) = E_n\Psi^n_{\kappa\lambda}(\boldsymbol{r})$$

$\kappa = 1^{+-}$

$\lambda = 0, \pm 1$

Hybrid Spectrum:

Multiplet	J^{PC}	$M_{c\bar{c}g}$	$M_{b\bar{b}g}$
H_1	$\{1^{--}, (0,1,2)^{-+}\}$	4155	10786
H_1'		4507	10976
H_1''		4812	11172
H_2	$\{1^{++}, (0,1,2)^{+-}\}$	4286	10846
H_2'		4667	11060
H_2''		5035	11270
H_3	$\{0^{++}, 1^{+-}\}$	4590	11065
H_3'		5054	11352
H_3''		5473	11616
H_4	$\{2^{++}, (1,2,3)^{+-}\}$	4367	10897
H_5	$\{2^{--}, (1,2,3)^{-+}\}$	4476	10948

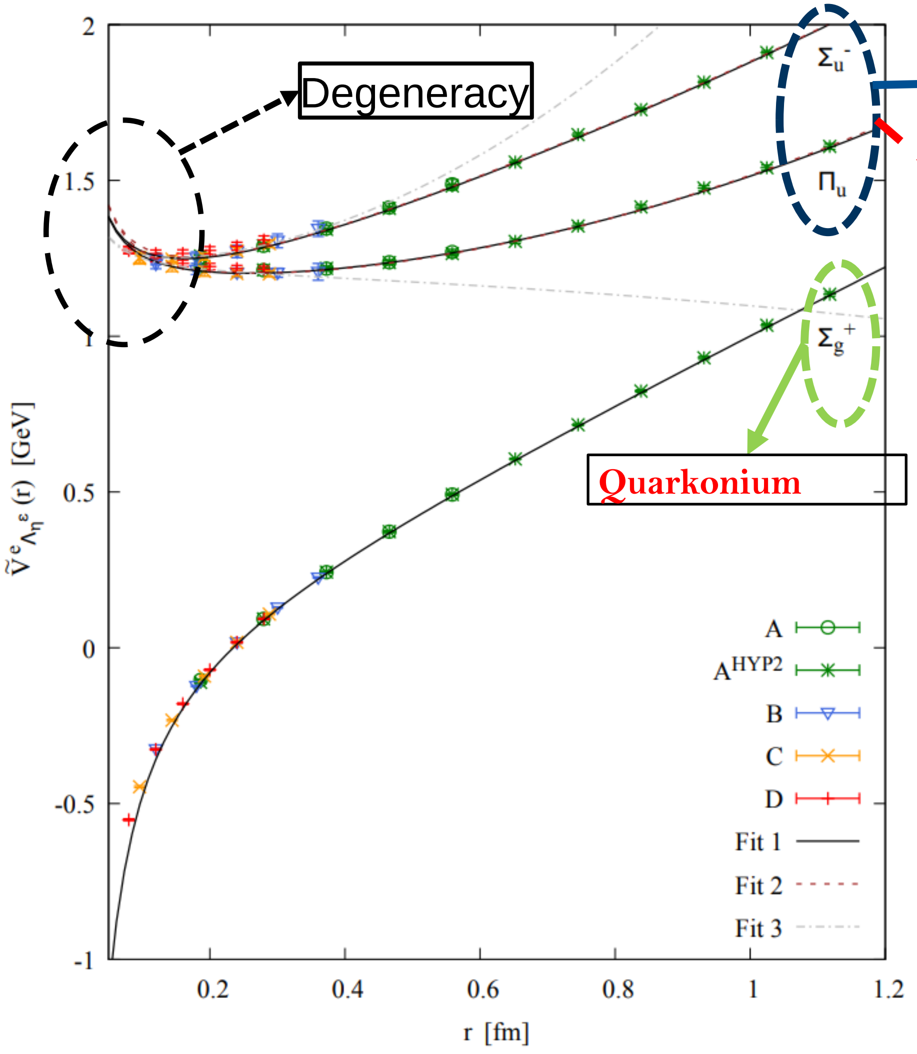
Λ- doubling: opposite parity states non-degenerate.

Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92, (2015)

Static Energies for hybrids

- **Static limit (m→ ∞):** heavy quarks are fixed in position. Interquark potential given by LDF energy.

Schlosser and Wagner Phys. Rev. D. 105, (2022)



Gluonic static energies for Hybrid

Gluonic static energies as a function of the heavy quark-antiquark distance ***r***

Use these as potentials between heavy quark pairs to solve the Schoedinger equation for hybrids.

Foster and Micheal (UKQCD collaboration), Phys. Rev. D 59, 094509 (1999)

Brambilla, Pineda, Soto and Vairo, Rev. Mod. Phys 77, (2005)

Exotic: Hybrid candidates

- Hybrids ($Q\bar{Q}g$): Color singlet state of color octet $Q\bar{Q}$ + gluon. (Q = c, b)

  ✓ **Isoscalar neutral mesons (Isospin=0)**

  ✓ Examples of candidates based on **mass and quantum numbers:**

Charm sector:

$c\bar{c}g$: $\chi_{c1}(4140), X(4160), \psi(4230), X(4350), \psi(4360), \psi(4390), Y(4500), Y(4710), \ldots\ldots$

Bottom sector:

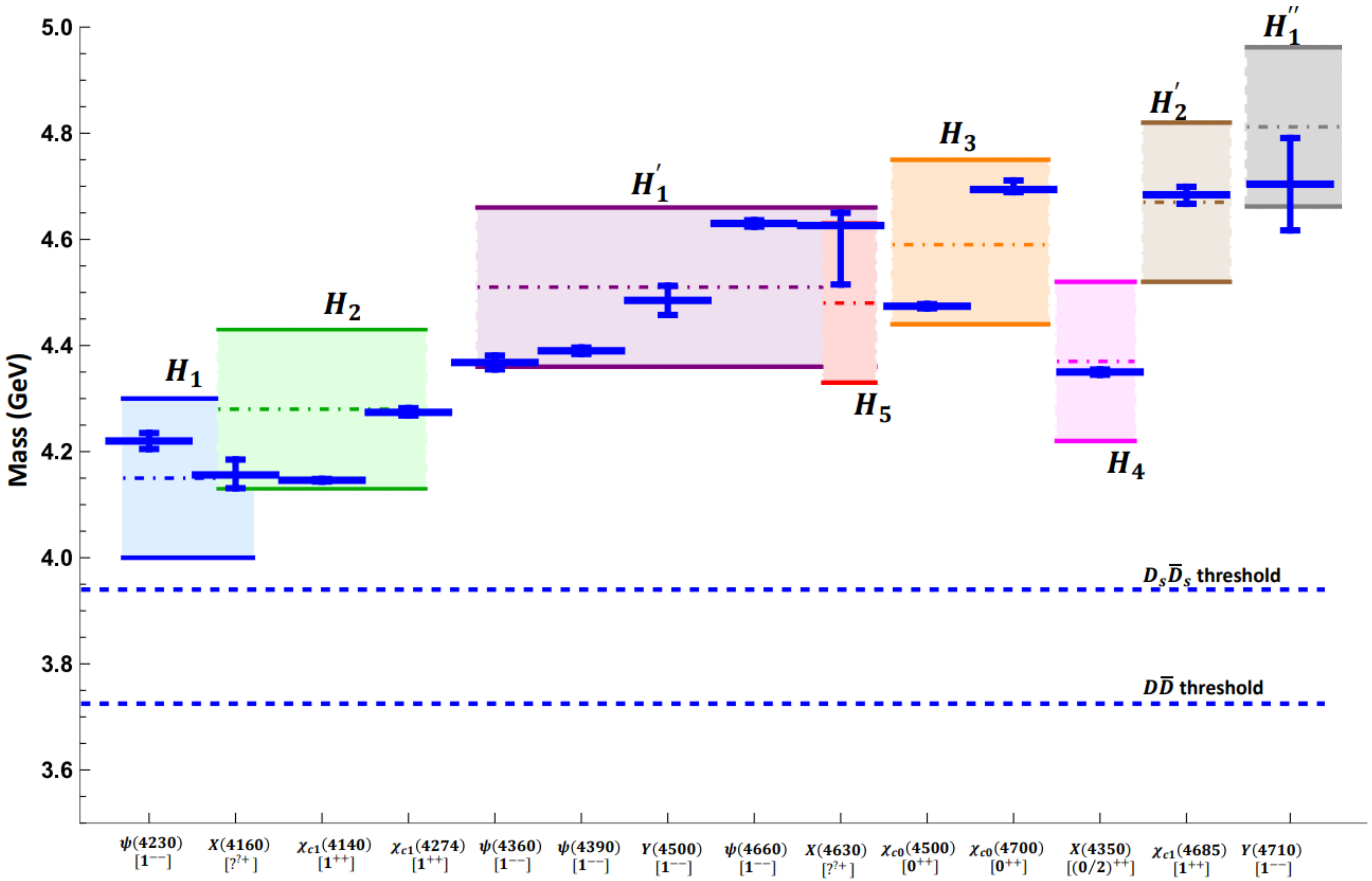
$b\bar{b}g$: $\Upsilon(10753), \Upsilon(10860), \Upsilon(11020)$

✓ Studying **decays** can **rule out** or **favor** hybrid interpretation.

Brambilla, Lai, AM, Vairo
Phys. Rev. D 107, 054034 (2023)

BOEFT: Hybrids

- **Charmonium hybrids**: comparison with experimental results:



	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

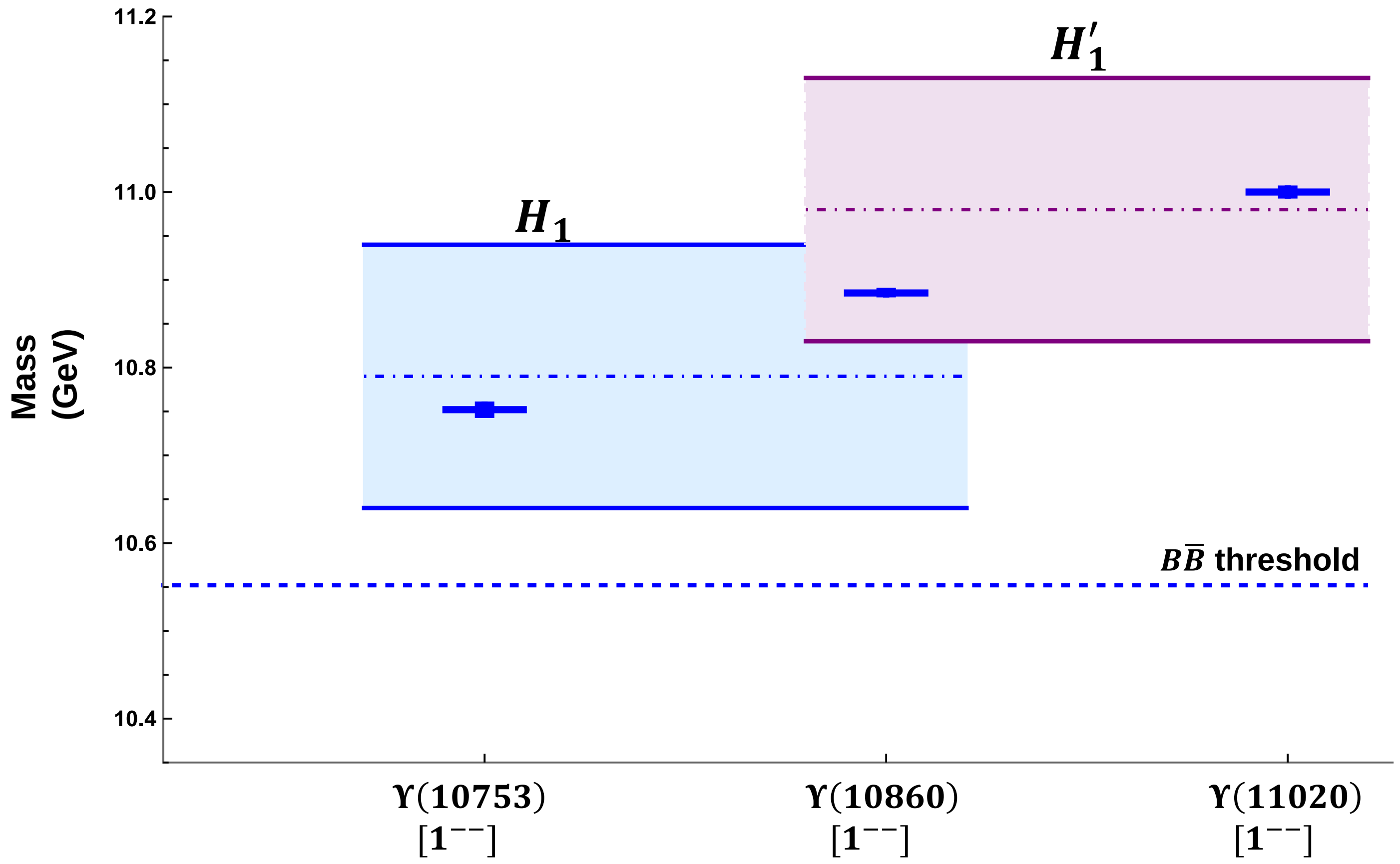
PDG 2022

Brambilla, Lai, AM, Vairo

Phys. Rev. D 107, 054034 (2023)

BOEFT: Hybrids

- **Bottomonium hybrids**: comparison with experimental results:



PDG 2022

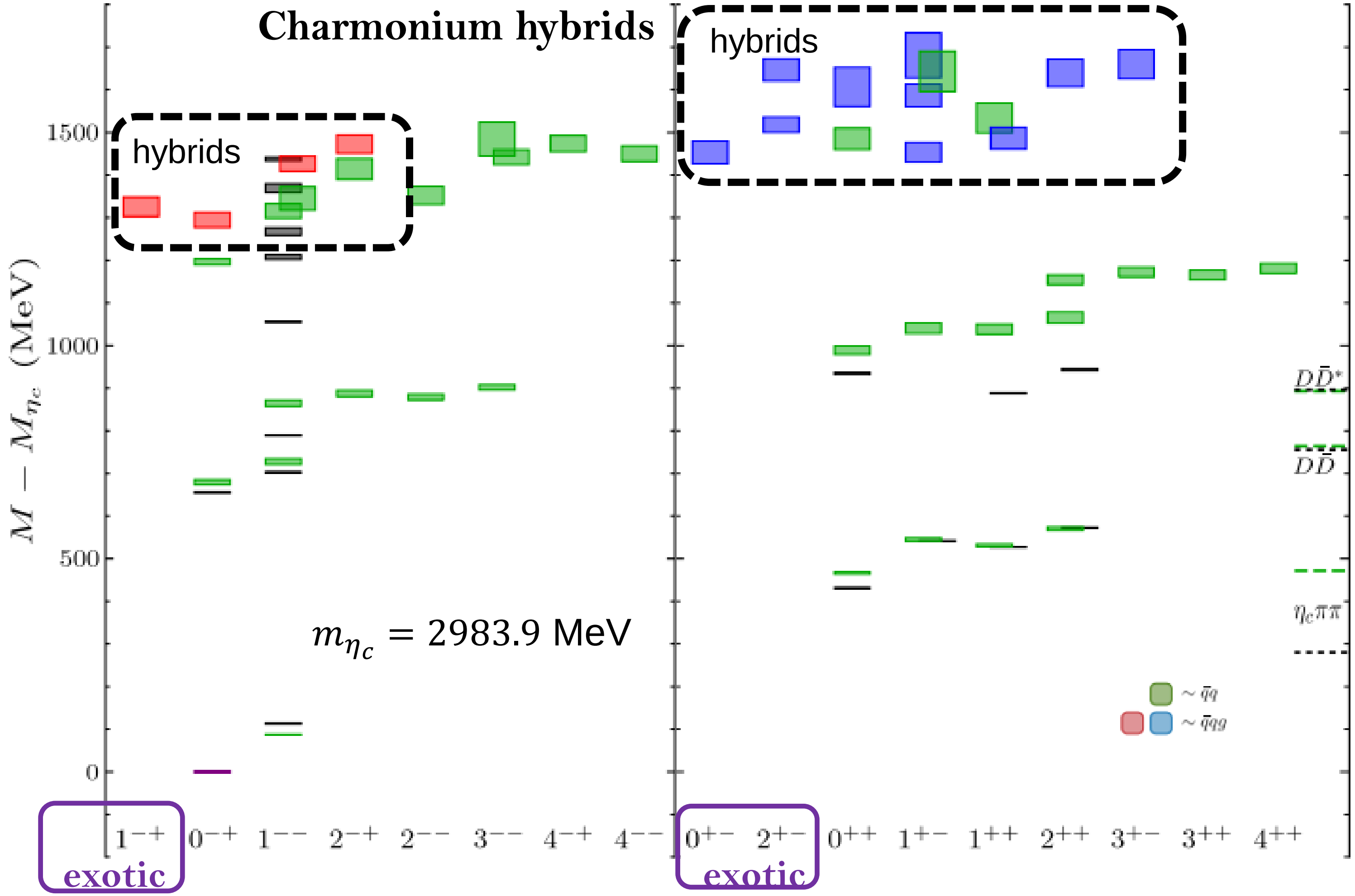
	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

Brambilla, Lai, AM, Vairo arXiv:2212.09187

BOEFT: Hybrids

- Lattice results for charm hybrids ($m_\pi \approx 240$ MeV) :

Results agree within error bars



Lattice data from
Hadron Spectrum collaboration JHEP 12 (2016) 89

Box represents uncertainties in lattice computations

- J^{PC}= 0^{--}, 0^{+-}, 1^{-+}, 2^{+-} etc. are exotic quantum #'s

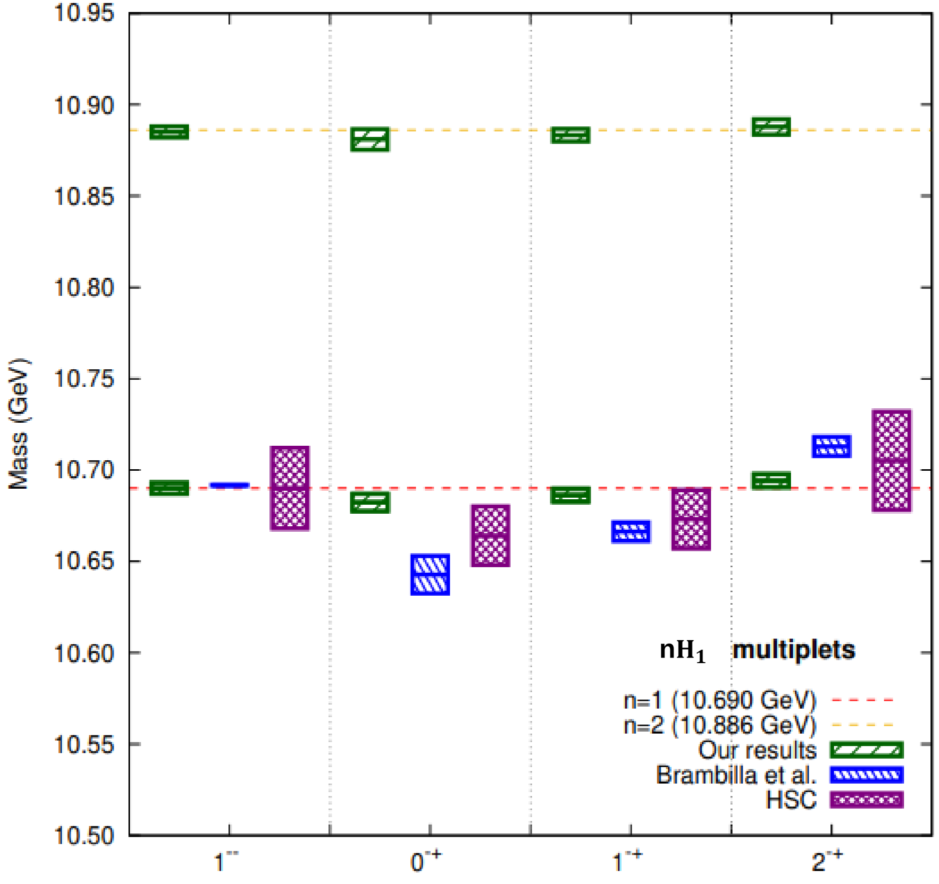
	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

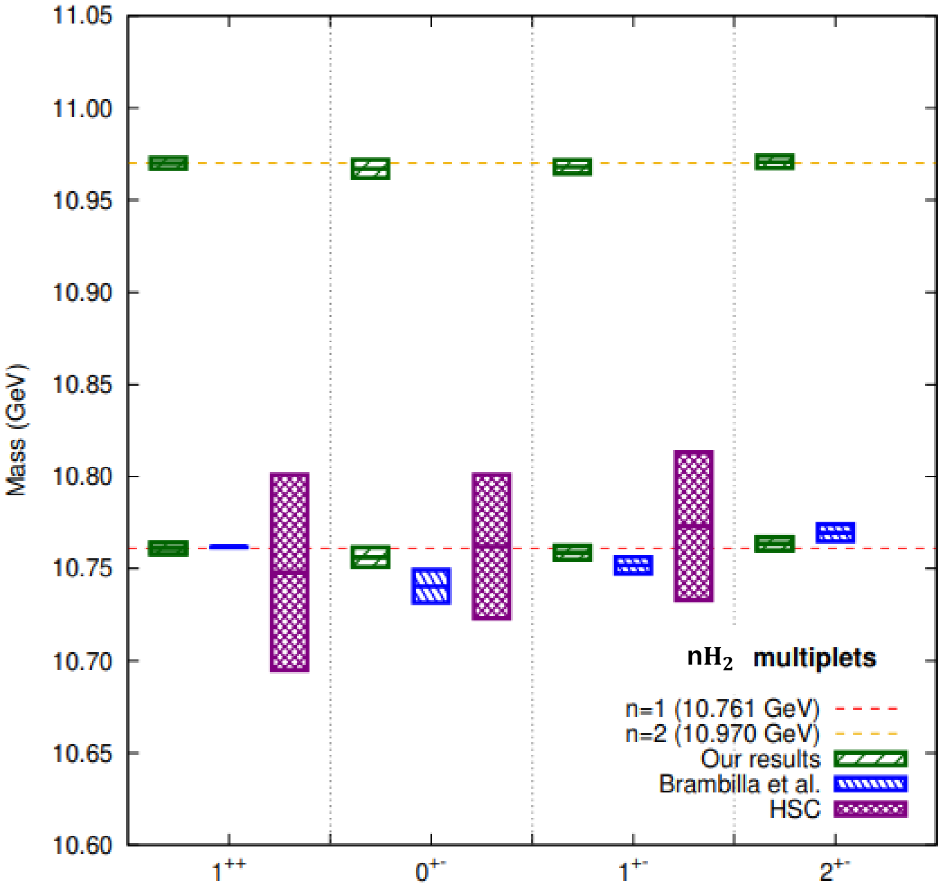
Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92, (2015)

Brambilla, Lai, AM, Vairo arXiv:2212.09187

BOEFT: Hybrids

- Including spin-dependent hybrid potentials:





	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

Our results refer to Soto & Valls arXiv 2302.01765

Brambilla, Lai, Segovia, Castellà, Vairo Phys. Rev. D. 101, (2020)

Brambilla, Lai, Segovia, Castellà, Vairo Phys. Rev. D. 99, (2019)

Soto, Valls, arXiv 2302.01765

Lattice data from Hadron Spectrum collaboration JHEP 02 (2021) 214

BOEFT: Hybrid Decays

- Semi-inclusive process: $\boldsymbol{H_m \to Q_n + X}$; H_m: low-lying hybrid, Q_n: low-lying quarkonium (states below threshold) and X: light hadrons.

✓ ΔE: Large energy difference ⇒ $\Delta E \equiv \boldsymbol{E_{H_m} - E_{Q_n} \gtrsim 1}$ **GeV**.

✓ Assume hierarchy of scales: $\Lambda_r \gg \Delta E \gg \Lambda_{QCD} \gg mv^2$

Energy scale related to decay

$$\Lambda_r^{-1} \equiv |\langle Q_n|\mathbf{r}|H_m\rangle|$$

- In BOEFT, all energy scales above $\mathbf{mv^2}$ are integrated out. So, scale ΔE must be integrated out. This gives imaginary contribution to hybrid potential:

Optical theorem: $$\sum_n \Gamma(H_m \to Q_n) = -2\,\mathrm{Im}\,\langle H_m|V|H_m\rangle$$

DISCLAIMER!!!
Decay to open-flavor threshold states not accounted here.

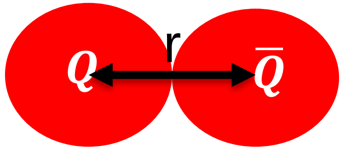
- Imaginary piece of hybrid potential: determined from matching **pNRQCD** and **BOEFT** effective theories.

Brambilla, Lai, AM, Vairo arXiv:2212.09187

Hybrid Decays

Brambilla, Lai, AM, Vairo Phys. Rev. D 107, 054034 (2023)

- Color configuration of $Q\bar{Q}$ pair ($\mathbf{r} \to \mathbf{0}$): quarkonium and hybrid in short-distance limit



Quarkonium -----> Singlet

Hybrid -----> Octet

- Quarkonium and Hybrid fields in short-distance limit $\mathbf{r} \to \mathbf{0}$ (matching condition)

Fields:

$$S\left(\mathbf{r},\mathbf{R},t\right) \to Z_{\Psi}^{1/2}(\mathbf{r})\,\Psi(\mathbf{r},\mathbf{R},t),$$

$$P^{i\dagger}_{\kappa\lambda} O^{a}\left(\mathbf{r},\mathbf{R},t\right) G^{ia}_{\kappa}(\mathbf{R},t) \to Z_{\kappa}^{1/2}(\mathbf{r})\,\Psi_{\kappa\lambda}(\mathbf{r},\mathbf{R},t)$$

singlet (S) and octet (O)

G_{κ}^{ia}: **Gluon fields**

Potentials:

$$E_{\Sigma_g^+}(r) = V_s(r) + b_{\Sigma_g^+}\, r^2 + \ldots,$$

$$E_{\Sigma_u^-,\Pi_u}(r) = V_o(r) + \Lambda + b_{\Sigma,\Pi} r^2 + \ldots$$

V_s & V_o: singlet and octet potential

Λ: gluelump mass

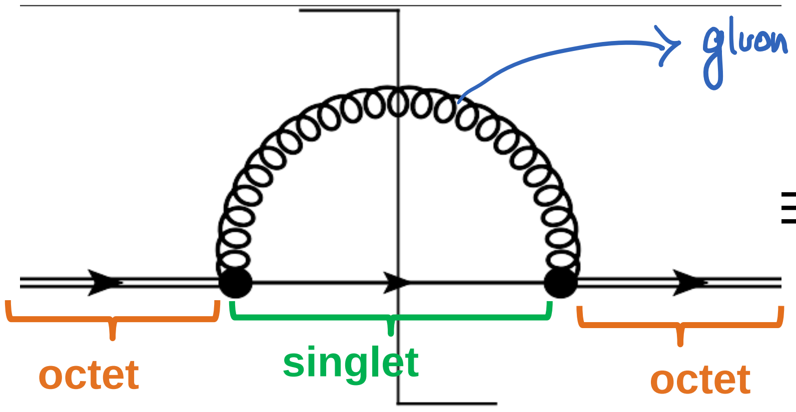
For decay rate computation, **start with effective theory of singlet and octet fields** and match to **BOEFT of quarkonium and hybrid fields**

Hybrid Decays

Brambilla, Lai, AM, Vairo Phys. Rev. D 107, 054034 (2023)

✓ Hierarchy of scales: $\Delta E \gg \Lambda_{QCD} \gg mv^2$. Integrate out the scale ΔE perutbatively.

matching **pNRQCD** and **BOEFT:**



pNRQCD

$\equiv$ Im

BOEFT

Virtual gluon resolves color structure of $Q\bar{Q}$ pair ($\mathbf{r} \to \mathbf{0}$) in quarkonium and hybrid in short-distance limit

Quarkonium ---> Singlet

Hybrid ---> Octet

Weakly-coupled pNRQCD Lagrangian

$$L_{\rm pNRQCD} = \int d^3R \Big\{ \int d^3r \Big(\mathrm{Tr}\left[\mathrm{S}^\dagger \left(i\partial_0 - h_s\right) \mathrm{S} + \mathrm{O}^\dagger \left(iD_0 - h_o\right) \mathrm{O} \right]$$

$$+ g\mathrm{Tr}\left[\mathrm{S}^\dagger \boldsymbol{r}\cdot\boldsymbol{E}\,\mathrm{O} + \mathrm{O}^\dagger \boldsymbol{r}\cdot\boldsymbol{E}\,\mathrm{S} + \frac{1}{2}\mathrm{O}^\dagger \boldsymbol{r}\cdot\{\boldsymbol{E},\mathrm{O}\} \right] + \frac{g}{4m}\mathrm{Tr}\left[\mathrm{O}^\dagger \boldsymbol{L}_{Q\bar{Q}}\cdot[\boldsymbol{B},\mathrm{O}] \right]$$

$$+ \frac{gc_F}{m}\mathrm{Tr}\left[\mathrm{S}^\dagger (\boldsymbol{S}_1 - \boldsymbol{S}_2)\cdot\boldsymbol{B}\,\mathrm{O} + \mathrm{O}^\dagger (\boldsymbol{S}_1 - \boldsymbol{S}_2)\cdot\boldsymbol{B}\,\mathrm{S} + \mathrm{O}^\dagger \boldsymbol{S}_1\cdot\boldsymbol{B}\,\mathrm{O} - \mathrm{O}^\dagger \boldsymbol{S}_2\mathrm{O}\cdot\boldsymbol{B} \right] - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu\,a} \Big\}$$

- **Spin preserving decays $[O(r^2)]$**
- **Spin flipping decays $[O(1/m^2)]$**

Hybrid Decays

Brambilla, Lai, AM, Vairo Phys. Rev. D 107, 054034 (2023)

- Spin-conserving decay due to $\boldsymbol{r} \cdot \boldsymbol{E}$ term :

$|S_H = 1 > ---\rightarrow |S_Q = 1 >$
$|S_H = 0 > ---\rightarrow |S_Q = 0 >$

$$\Gamma(H_m \to Q_n) = \frac{4\alpha_s(\Delta E)\, T_F}{3N_c} T^{ij}\, (T^{ij})^\dagger\, \Delta E^3$$

DISCLAIMER!!!
Decay to open-flavor threshold states not accounted here.

$$T^{ij} \equiv \langle H_m | r^j | Q_n \rangle = \int d^3\mathbf{r}\, \Psi^{i\dagger}_{(m)}(\mathbf{r})\, r^j\, \Phi^{Q\bar{Q}}_{(n)}(\mathbf{r})$$

$\Psi^i_{(m)}$: Hybrid wf

Φ^Q_n : Quarkonium wf

R. Oncala, J. Soto, Phys. Rev. D96, 014004 (2017).

J. Castellà, E. Passemar, Phys. Rev. D104, 034019 (2021)

$$\langle H_m | \boldsymbol{r} | Q_n \rangle = \sqrt{T^{ij}(T^{ij})^\dagger}$$

- Spin-flipping decay due to $\boldsymbol{S}.\boldsymbol{B}$ term:

$|\chi_H\rangle$: Hybrid spin wf

$|\chi_Q\rangle$: Quarkonium spin wf

$|S_H = 1 > ---\rightarrow |S_Q = 0 >$
$|S_H = 0 > ---\rightarrow |S_Q = 1 >$

$$T^{ij} \equiv \langle H_m | \left(S_1^j - S_2^j\right) | Q_n \rangle = \left[\int d^3\mathbf{r}\, \Psi^{i\dagger}_{(m)}(\mathbf{r})\, \Phi^Q_{(n)}(\mathbf{r})\right] \langle \chi_H | \left(S_1^j - S_2^j\right) | \chi_Q \rangle$$

Depends on overlap of quarkonium and hybrid wavefunctions.

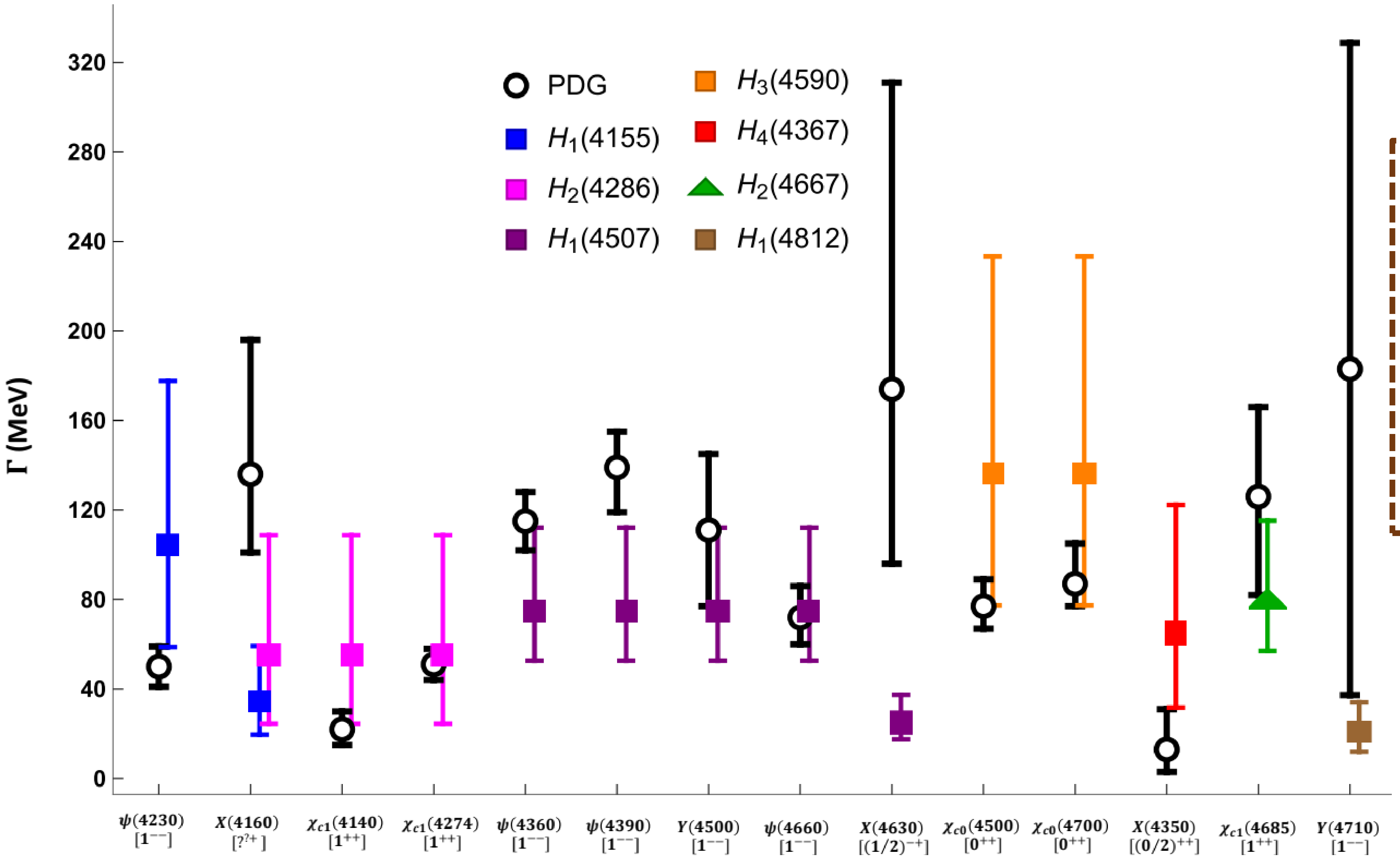
Hybrid-to-Quarkonium transition decay rate
= spin-conserving + spin-flipping decay rates.

Our estimate of decay rate are lower-bounds for the **total width of hybrids**

Results

Brambilla, Lai, AM, Vairo Phys. Rev. D 107, 054034 (2023)

- Comparison: charm exotic states with corresponding charmonium hybrid state:



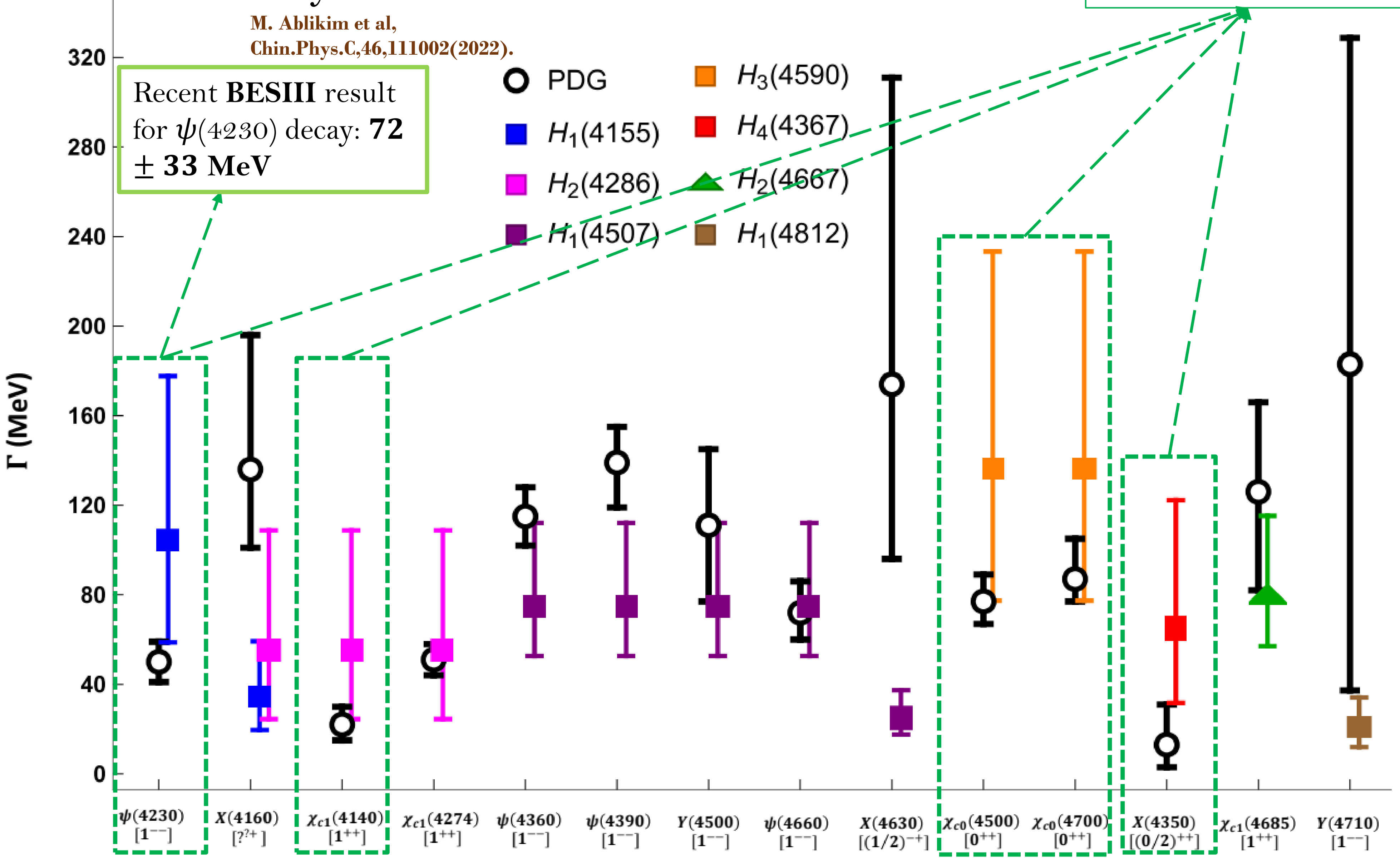
$$H_m \to Q_n + X$$

spin-conserving + spin-flipping decays
=
lower bound on the total decay widths of hybrids which is compared with inclusive rate of physical states in PDG.

Results

- Comparison: charm exotic states with corresponding charmonium hybrid state:

Disfavors hybrid interpretation !



	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--},(0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++},(0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++},1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++},(1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--},(1,2,3)^{-+}\}$	Π_u

$\chi_{c0}(4500)$ & $\chi_{c0}(4700)$

Decay to $\chi_c(1P)$ not seen !!!. Major contribution to theoretical estimate from this decay channel.

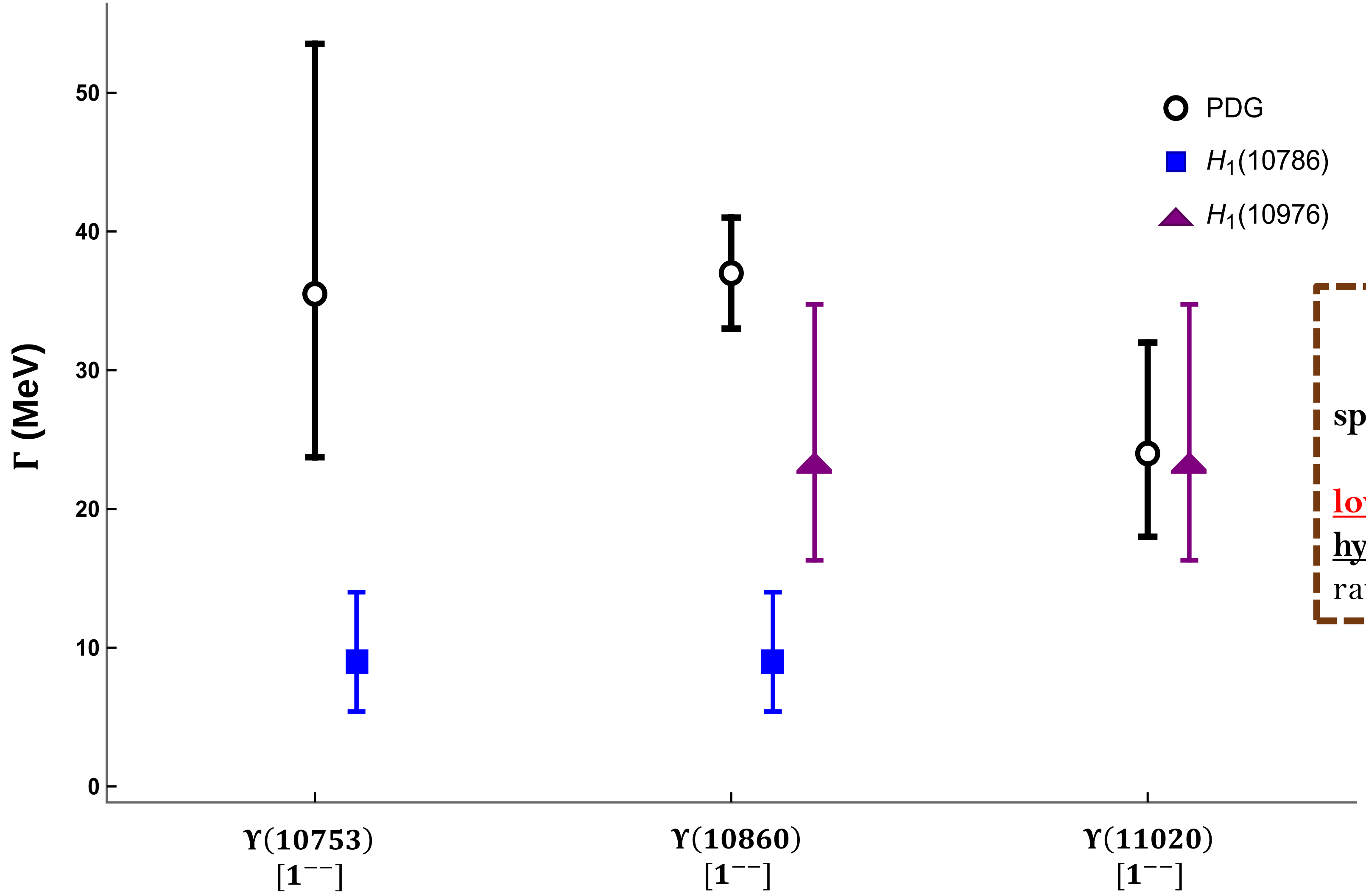
Brambilla, Lai, AM, Vairo

Phys. Rev. D 107, 054034 (2023)

Results

Brambilla, Lai, AM, Vairo Phys. Rev. D 107, 054034 (2023)

- Comparison: bottom exotic states with corresponding bottomonium hybrid state:



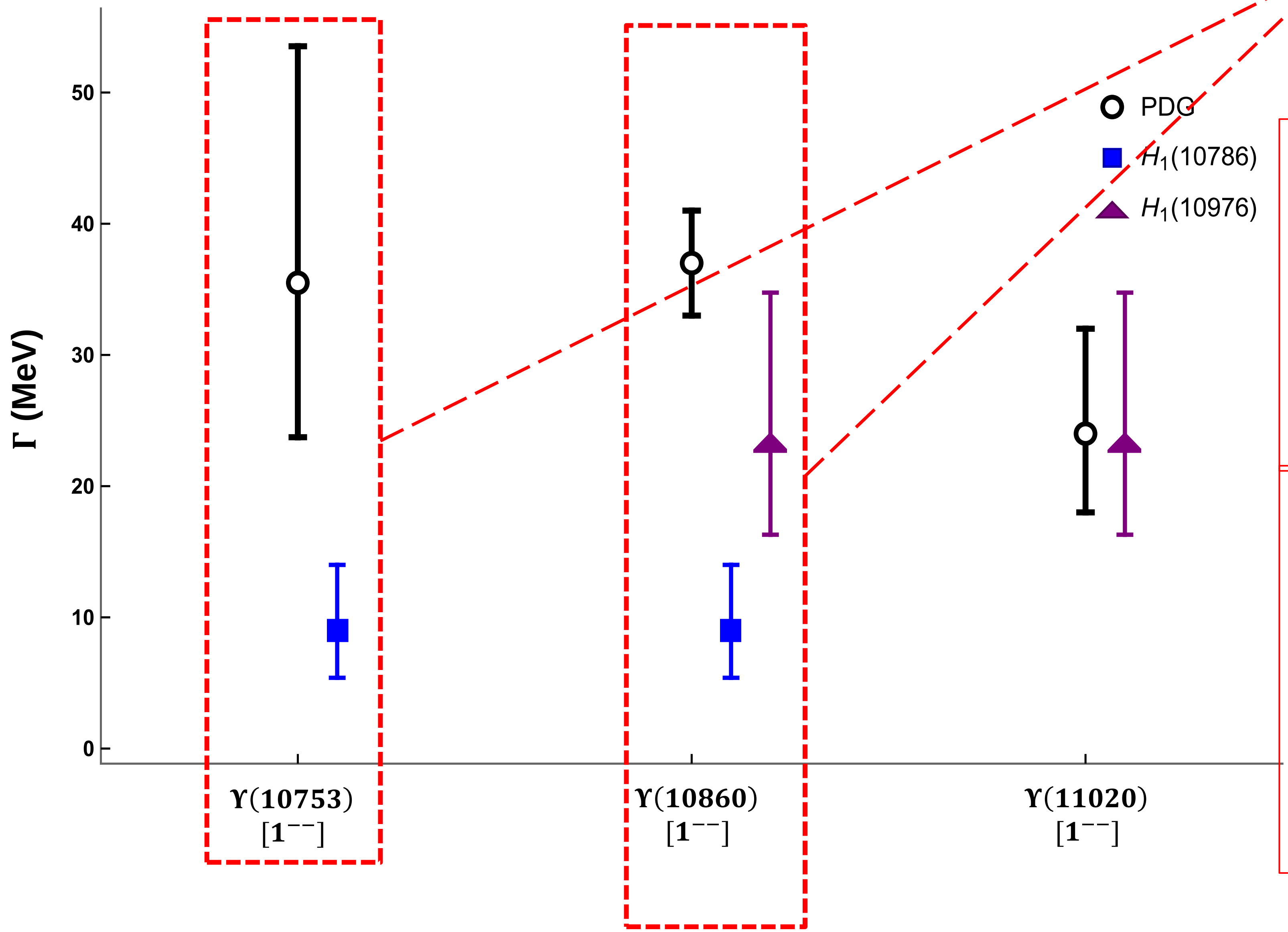
$$H_m \to Q_n + X$$

spin-conserving + **spin-flipping decays**
=
lower bound on the total decay widths of hybrids which is compared with inclusive rate of physical states in PDG.

Results

Brambilla, Lai, AM, Vairo Phys. Rev. D 107, 054034 (2023)

- Hybrid-to-quarkoniunium transition widths:



Could have a significant hybrid component.

Υ(10860)

- ✓ Inclusive rate: **37 ± 4** MeV. **Branching fraction for decay to open-bottom mesons**: **76.6%**. Decay rate to quarkonium: $\mathbf{8.8^{+2.6}_{-1.8}}$ MeV. Good agreement with our lower-bound estimate for $H_1[1^{--}](10786)$.
- ✓ Candidate for $\Upsilon(5S)$ state

$\Upsilon(11020)$ & $\Upsilon(10753)$

Large branching fraction for decays to open-bottom mesons expected. Different quark model calculations predict large branching fraction. Need experimental input on branching fraction. See Hüsken, Mitchell & Swanson, Phys Rev D 106 (2022).

$\Upsilon(10753)$: Candidate for $\Upsilon(3D)$ state.

Hybrid Decays

Brambilla, AM, Vairo arXiv 2312.xxxxx

- **Hybrid decays to meson-pair threshold states:** $\Delta E \lesssim \Lambda_{QCD}$

Conventional Wisdom: Hybrid decays to two S-wave mesons forbidden! $H_m \nrightarrow D^{(*)}\bar{D}^{(*)}$

Kou & Pene, Phys Lett B 631 (2005) Page, Phys Lett B 407 (1997) Farina, Tecocoatzi, Giachino, Santopinto & Swanson, Phys Rev D 102 (2020)

Decay to two s-wave mesons allowed if BO-quantum numbers for hybrids and meson-pair are same.

Hybrid

Light spin K^{PC}	Static energies $D_{\infty h}$	l	J^{PC} $\{S_Q=0, S_Q=1\}$	Multiplets
1^{+-}	$\{\Sigma_u^-, \Pi_u\}$	1	$\{1^{--}, (0,1,2)^{-+}\}$	H_1
	$\{\Pi_u\}$	1	$\{1^{++}, (0,1,2)^{+-}\}$	H_2
	$\{\Sigma_u^-\}$	0	$\{0^{++}, 1^{+-}\}$	H_3
	$\{\Sigma_u^-, \Pi_u\}$	2	$\{2^{++}, (1,2,3)^{+-}\}$	H_4
	$\{\Pi_u\}$	2	$\{2^{--}, (1,2,3)^{-+}\}$	H_5

Meson-antimeson threshold

$K_{\bar{q}}^P \otimes K_q^P$	K^{PC}	Static energies $D_{\infty h}$
$(1/2)^- \otimes (1/2)^+$	0^{-+}	$\{\Sigma_u^-\}$
	1^{--}	$\{\Sigma_g^+, \Pi_g\}$
$(1/2)^- \otimes (1/2)^-$	0^{++}	$\{\Sigma_g^+\}$
	1^{+-}	$\{\Sigma_u^-, \Pi_u\}$
$(1/2)^- \otimes (3/2)^-$	1^{+-}	$\{\Sigma_u^-, \Pi_u\}$
	2^{++}	$\{\Sigma_g^+, \Pi_g, \Delta_g\}$

s-wave+s-wave Ex. $\mathbf{D\bar{D}}$ threshold

s-wave+p-wave Ex. $\mathbf{D_1\bar{D}}$ threshold

Σ_u^- component in hybrids mix with Σ_u^- component in s-wave+s-wave threshold!!!!

Bruschini 2306.17120 Bruschini & Gonzalez, 1912.07337 J. Castella 2401.13393

Recent lattice computation for $c\bar{c}$ hybrid 1^{-+} **decay** to

$D_1\bar{D}$: 258(133) MeV

$D^*\bar{D}$: 88(18) MeV

$D^*\bar{D}^*$: 150(118) MeV

Shi et al 2306.12884

Computing these decays of hybrid to threshold states in BOEFT framework ??

Hybrid-Quarkonium mixing and impact on hybrid interpretation of Exotics ?

Brambilla, AM, Vairo, Wagner, Schlosser (in progress)

- Hybrid states in the same energy range and with same quantum #'s as quarkonium can mix.

- Mixing impacts spectrum and decay properties of hybrid. **Implications for exotic hadrons !!**.

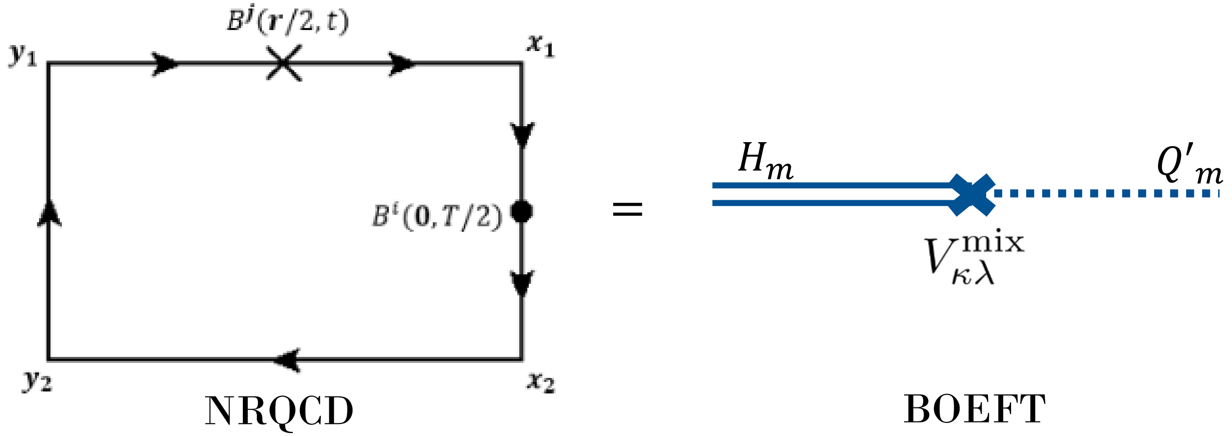
Oncala & Soto, PRD (2017).

Ex. $H_1\left[1^{--}\right](4155) \leftrightarrow c\bar{c}\left[1^{--}\right](3S)$ Effect on decay: $H_m \leftrightarrow Q_m^{'} \rightarrow (\eta_c, J/\psi, \cdots) + (\gamma, \cdots)$

- Hybrid-quarkonium mixing through heavy-quark spin dependent operator. Mixing potential at $O(1/m)$ in BOEFT.

$$L_{\rm BOEFT} = L_{Q\bar{Q}} + L_{Q\bar{Q}g} + L_{\rm mixing}, \qquad L_{\rm mixing} = -\int d^3\mathbf{R}\int d^3\mathbf{r}\sum_{\kappa\lambda}{\rm Tr}\left[\Psi^\dagger\, V_{\kappa\lambda}^{\rm mix}\,\Psi_{\kappa\lambda} + {\rm h.c.}\right]$$

Matching two-point correlators in **NRQCD** and **BOEFT:**



Expression after matching:

$$V_{|\lambda|}^{\rm mix} = -\frac{gc_F}{2m_Q}\;{}^{(0)}_{\lambda}\langle 1|B^j\left(\mathbf{r}/2,0\right)|0\rangle^{(0)} P^j_\lambda,$$

Above expression can be computed on lattice if we identify:

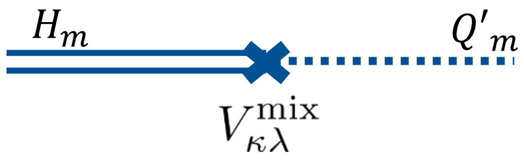
$$|0\rangle^{(0)} = |\Sigma_g^+\rangle$$

$$|1\rangle^{(0)}_{\lambda=0} = |\Sigma_u^-\rangle,\ |1\rangle^{(0)}_{|\lambda|=1} = |\Pi_u\rangle$$

Hybrid-Quarkonium mixing and impact on hybrid interpretation of Exotics (cont'd) ?

Questions to be answered:

- Computing the mixing potential on lattice ??



- Computing hybrid spin-dependent potential (also at $O(1/m)$) on lattice ??
- Solve coupled Schrodinger equations (mixing potential included) of hybrid and quarkonium to obtain the **new spectrum.**
- Study effect of mixing on hybrid **decays**.

Munich-Frankfurt collaboration:

N, Brambilla, AM, Vairo, M. Wagner, C. Schlosser

Takeaway:

A physical state could be a mixture of both quarkonium and hybrid !!!

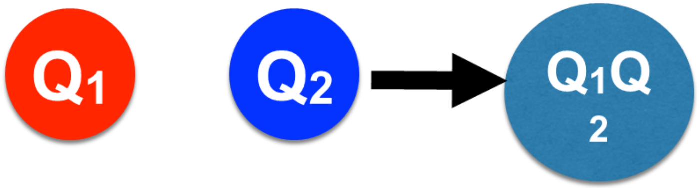
Tetraquarks & Pentaquarks

BOEFT: $QQ\bar{q}\bar{q}$ multiplets

Brambilla, AM, Vairo arXiv 2402.xxxx

doubly heavy core

spin: $1/2 \otimes 1/2 = 0 \oplus 1$



color: $3 \otimes 3 = 6 \oplus 3^*$

flavor: $\{Q_1Q_2\}$ $[Q_1Q_2]$

J^P: 1^+ 0^+

light antiquarks

q q'

$\{qq'\}$, 1^+ $[qq']$, 0^+

Defines the Born-Oppenheimer static potentials $\Sigma_g^+, \{\Sigma_g^-, \Pi_g\}$

doubly heavy tetraquarks

QQ color state	Light spin K^{PC}	Static energies	Isospin I	l	J^P $S_Q = 0$	J^P $S_Q = 1$
anti-triplet $\bar{3}$	0^+	$\{\Sigma_g^+\}$	0	0	—	1^+
				1	1^-	—
	1^+	$\{\Sigma_g^-, \Pi_g\}$	1	0	0^-	—
				1	1^-	$(0,1,2)^+$

J^P for T_{cc}^+

Limited lattice inputs available on Born-Oppenheimer static potentials $\Sigma_g^+, \{\Sigma_g^-, \Pi_g\}$

Bicudo, Cichy, Peters, & Wagner
PRD 93, 034501 (2016)

BOEFT: $Q\bar{Q}q\bar{q}$ multiplets

Brambilla, AM, Vairo arXiv 2402.xxxx

$Q\bar{Q}$ color state	Light spin K^{PC}	Static energies	l	J^{PC} $\{S_Q = 0, S_Q = 1\}$	Multiplets
Octet	0^{-+}	$\{\Sigma_u^-\}$	0	$\{0^{++}, 1^{+-}\}$	T_1^0
			1	$\{1^{--}, (0,1,2)^{-+}\}$	T_2^0
			2	$\{2^{++}, (1,2,3)^{+-}\}$	T_3^0
	1^{--}	$\{\Sigma_g^{+\prime}, \Pi_g\}$	1	$\{1^{+-}, (0,1,2)^{++}\}$	T_1^1
		$\{\Sigma_g^{+\prime}\}$	0	$\{0^{-+}, 1^{--}\}$	T_2^1
		$\{\Pi_g\}$	1	$\{1^{-+}, (0,1,2)^{--}\}$	T_3^1
		$\{\Sigma_g^{+\prime}, \Pi_g\}$	2	$\{2^{-+}, (1,2,3)^{--}\}$	T_4^1

J^{PC} for neutral partner of Z_c, Z_b states. Probably mixing between both channels required ?

J^{PC} for $X(3872)$

Limited lattice inputs available on Born-Oppenheimer static potentials $\Sigma_u^-, \{\Sigma_g^+, \Pi_g\}$

Prelovsek, Bahtiyar, & Petkovic
Phys. Lett. B 805 (2020)

Mixing of BO-potentials with pair of heavy-light states relevant for states near threshold. More on this mixing see Brambilla, AM, Vairo arXiv 2402.xxxx
Braaten, AM, Bruschini (in preparation)

BOEFT: Pentaquark multiplets

Brambilla, AM, Vairo arXiv 2402.xxxx

$Q\bar{Q}qqq$

$Q\bar{Q}$ color state	Light spin $\boldsymbol{K}^P$	Static energies	l	J^P $\{S_Q=0, S_Q=1\}$
Octet	$(1/2)^+$	$(1/2)_g$	$1/2$	$\{1/2^-, (1/2, 3/2)^-\}$
	$(3/2)^+$	$(3/2)_g$	$3/2$	$\{3/2^-, (1/2, 3/2, 5/2)^-\}$

No lattice inputs available on Born-Oppenheimer static potentials for pentaquarks

$QQqq\bar{q}$

QQ color state	Light spin $\boldsymbol{K}^P$	heavy spin	
		$S_Q=0$	$S_Q=1$
sextet	$(1/2)^-$	$\{(1/2)^-\}$	$\{(1/2, 3/2)^+, (1/2, 3/2, 5/2)^+\}$
	$(3/2)^-$	$\{(3/2)^-\}$	$\{(1/2, 3/2)^+\}, \{(1/2, 3/2, 5/2)^+\},$ $\{(3/2, 5/2, 7/2)^+\}$
antitriplet	$(1/2)^-$	$\{(1/2)^+, (3/2)^+\}$	$\{(1/2, 3/2)^-\}$
	$(3/2)^-$	$\{(1/2)^+\}, \{(3/2)^+\}, \{(5/2)^+\}$	$\{(1/2, 3/2, 5/2)^-\}$

Coupled Schrödinger equation for These pentaquark states derived in Brambilla, AM, Vairo arXiv 2402.xxxxx.

Takeaway message

Brambilla, Lai, AM, Vairo Phys. Rev. D 107, 054034 (2023)
Brambilla, AM, Vairo arXiv 2402.xxxxx

- BOEFT provides a model-independent & systematic way to study heavy quark exotics.
- Specific to **Hybrids**: BOEFT computation for decay to quarkonium involves hierarchy of energy scales :

$$\Delta E \gg \Lambda_{\rm QCD} \gg m_Q v^2$$

pNRQCD and BOEFT matching

Neglect hybrids of higher gluonic excitations and mixing.

- Our results for hybrid-to-quarkonium transition widths

Hybrid-to-Quarkonium transition decay rate = **spin-conserving** + **spin-flipping** decay rates.

- Our analysis disfavors: $\psi(4230)$, $\chi_{c1}(4140)$, $\chi_{c0}(4500)$, $\chi_{c0}(4700)$, and $X(4350)$ as pure hybrid states.

DISCLAIMER!!!
All the above interpretation can differ accounting for decays to meson-pair threshold states and hybrid-quarkonium mixing.

- Our analysis suggests:
  - $\boldsymbol{X(4160)}$: could be **charm hybrid** $\boldsymbol{H_1[2^{-+}](4155)}$.
  - $\mathbf{X(4630)}$: could be **charm hybrid** $\boldsymbol{H_1[(1/2^{-+})](4507)}$.
  - $\boldsymbol{\Upsilon(10753)}$: could be **bottom hybrid** $\boldsymbol{H_1[(1^{--})](10786)}$.
  - $\boldsymbol{\psi(4390)}$: could be **charm hybrid** $\boldsymbol{H_1[1^{--}](4507)}$.
  - $\boldsymbol{\psi(4710)}$: could be **charm hybrid** $\boldsymbol{H_1[(1^{--})](4812)}$.

- Obtained new results for tetraquark and pentaquark multiplets based on BOEFT.

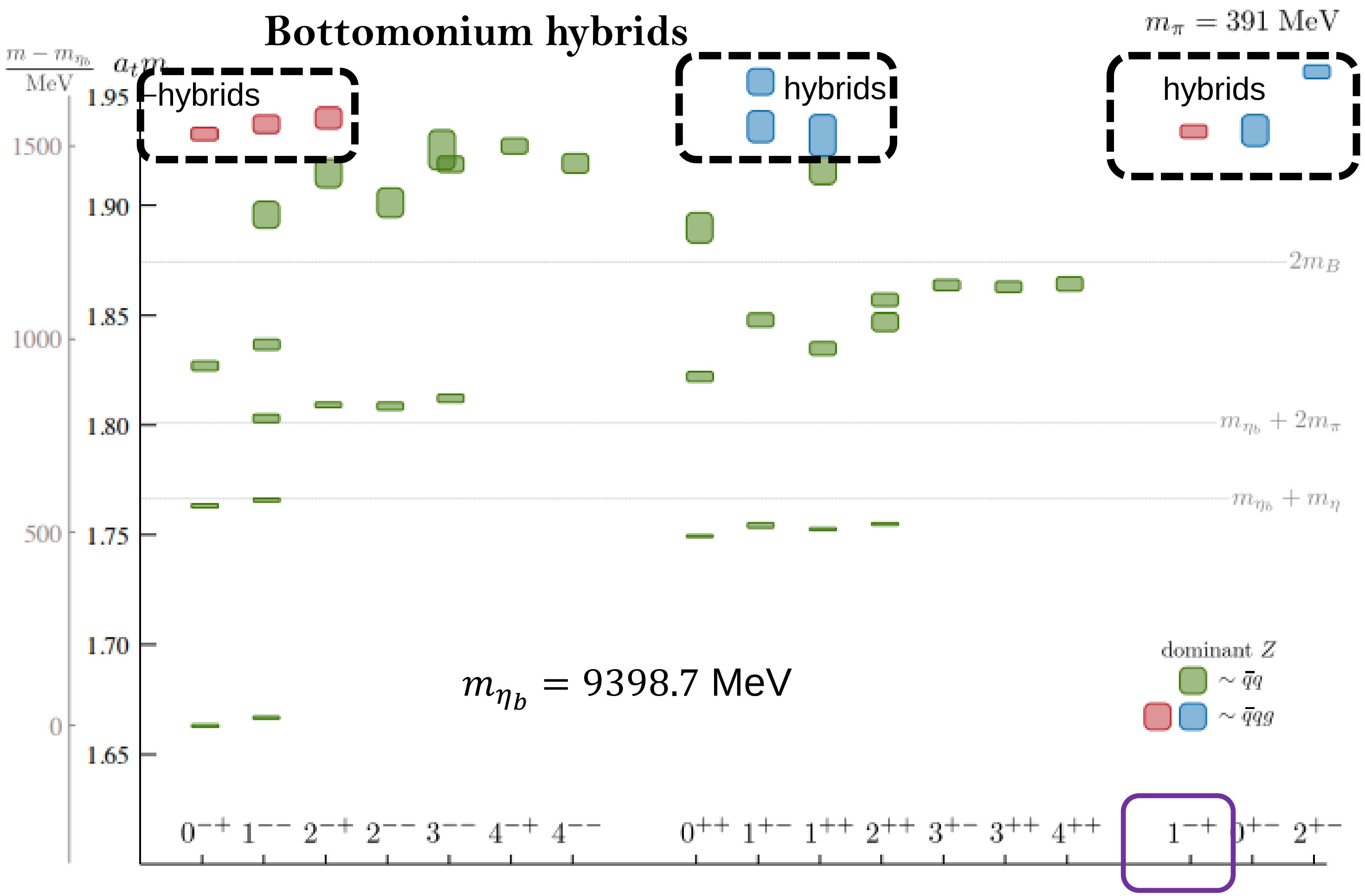
Thank you!!

Backup Slides

BOEFT: Hybrids

- Lattice results for bottom hybrids ($m_\pi \approx 391$ MeV):

Results agree within error bars



Lattice data from
Hadron Spectrum collaboration JHEP 02 (2021) 214

Box represents uncertainties in lattice computations

- $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}$ etc. are exotic quantum #'sv

	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92, (2015)

Brambilla, Lai, AM, Vairo arXiv:2212.09187

Indication of new pentaquark state with isospin = 3/2 (?)

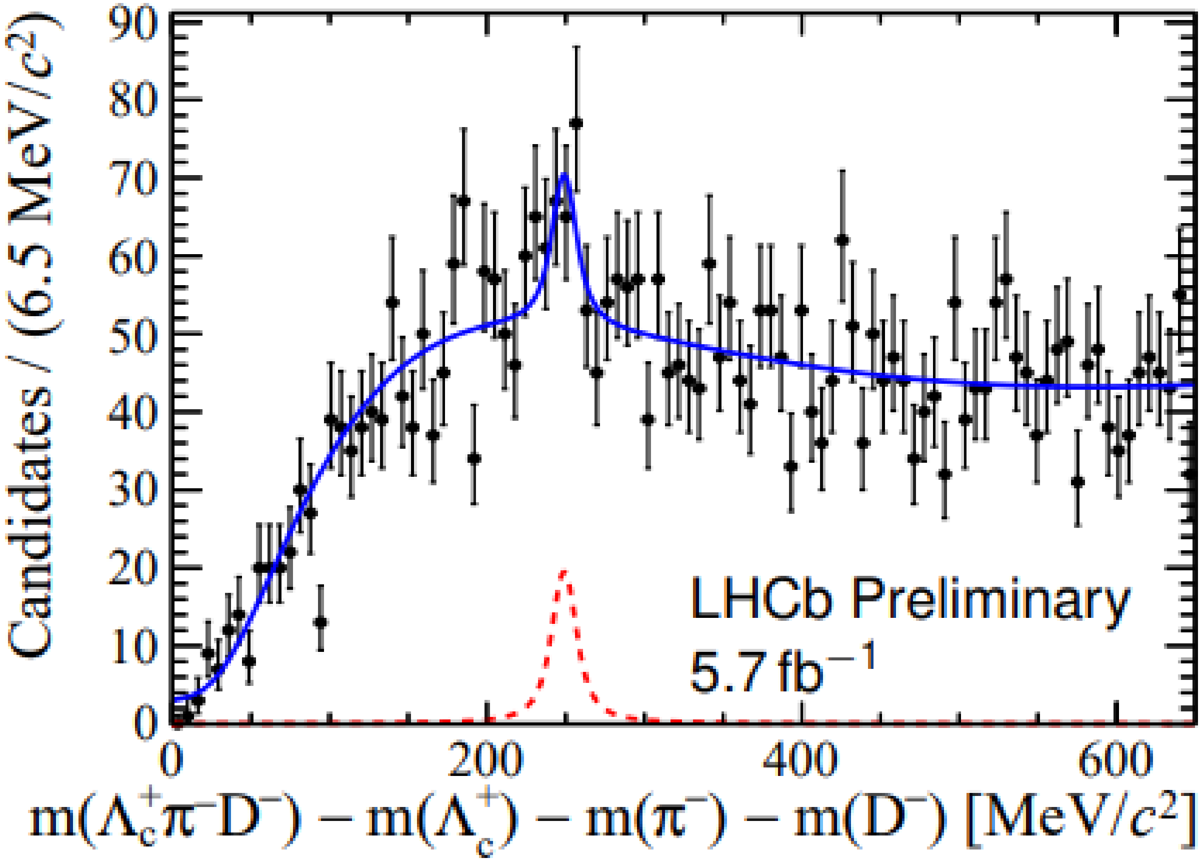


Figure from Robertson talk:

LHCb 2023 conference

Not yet confirmed by LHCb.

Virial theorem: Nonrelativistic Bound State

- Non relativistic because $v \sim \alpha \ll 1$.

Def. p and v are the particle momentum and velocity in the center of mass reference frame.

One can see this from the virial theorem applied to a central potential:

$$\begin{cases} \left\langle \frac{p^2}{2m_{red}} \right\rangle = \frac{1}{2}\langle r V'(r)\rangle = \frac{1}{2}\left\langle \frac{\alpha}{r}\right\rangle \\ \text{for } V(r) = -\frac{\alpha}{r} \text{ (Coulomb potential)} \\ E_n = -\frac{m_{red}\alpha^2}{2n^2} = \left\langle \frac{p^2}{2m_{red}}\right\rangle - \left\langle \frac{\alpha}{r}\right\rangle = -\frac{1}{2}\left\langle\frac{\alpha}{r}\right\rangle \end{cases} \Rightarrow \quad \left\langle \frac{\alpha}{r}\right\rangle = \frac{m_{red}\alpha^2}{n^2}, \quad \left\langle \frac{p^2}{2m_{red}}\right\rangle = \frac{m_{red}\alpha^2}{2n^2}$$

$m_{red} \equiv$ reduced mass $\equiv m_1 m_2/(m_1+m_2)$

In particular this implies

$$\left\langle \frac{1}{r}\right\rangle = \frac{m_{red}\,\alpha}{n^2} \quad \text{and} \quad \sqrt{\langle p^2\rangle} = \frac{m_{red}\,\alpha}{n}$$

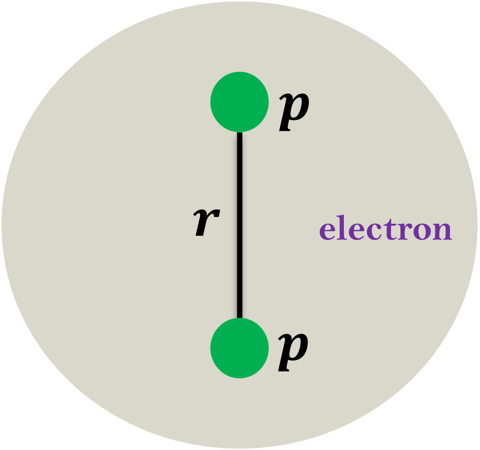
EFT lectures by A. Vairo

- if $m_1 = m_2 = m$, $m_{red} = \frac{m}{2}$; $m v^2 \equiv \left\langle \frac{p^2}{m}\right\rangle = \frac{m\alpha^2}{4n^2} \Rightarrow v = \frac{\alpha}{2n}$
- if $m_1 = \infty$, $m_2 = m$, $m_{red} = m$; $\frac{1}{2} m v^2 \equiv \left\langle \frac{p^2}{2m}\right\rangle = \frac{m\alpha^2}{2n^2} \Rightarrow v = \frac{\alpha}{n}$

Born-Oppenheimer Philosophy

- Sharp difference between time or energy scales of heavy & light degrees of freedom.

Ex. H_2^+ molecule: 2 protons & 1 electron. $m_p \sim 1\,\mathrm{GeV} \gg m_e \sim 0.5\,\mathrm{MeV}$



Protons (nuclei) move very slowly compared to electrons and can be considered **static** (fixed) when considering the motion of the electrons

Electrons instantaneously adjust as **r** changes

1. Solve electron Schrödinger eq. for fixed **r**

$$H_{\mathrm{el}}(\mathbf{r})\,|\psi_{\mathrm{el}}^i;\mathbf{r}\rangle = E_{\mathrm{el}}^i(r)\,|\psi_{\mathrm{el}}^i;\mathbf{r}\rangle$$

2. Solve nuclei (proton) Schrödinger eq. with $\boldsymbol{E_{\mathrm{el}}^i(r)}$ as **potential**.

QCD states with 2-heavy quarks (XYZ mesons): analogous of molecules in atomic systems !!!

Heavy quarks ↔ nuclei **Gluons & light quarks ↔ electrons**

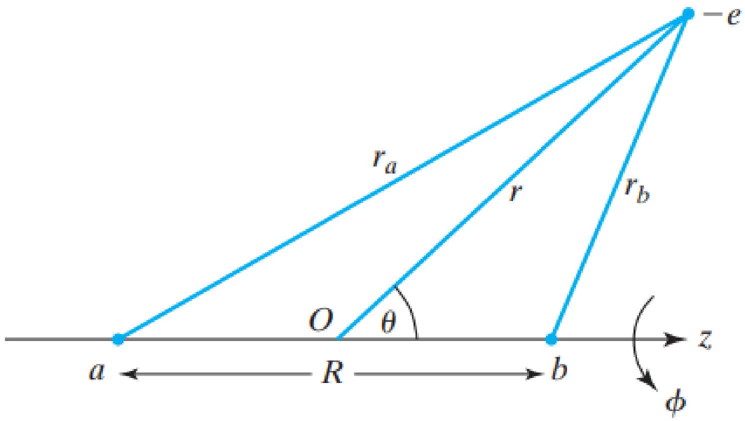
Born-Oppenheimer Philosophy

Full Hamiltonian of the system:

$$\hat{H} = -\tfrac{1}{2}\sum_{I}^{nuclei}\frac{1}{M_I}\nabla_I^2 - \tfrac{1}{2}\sum_{i}^{electrons}\nabla_i^2 + \tfrac{1}{2}\sum_{IJ}^{nuclei}\frac{Z_I Z_J}{R_{IJ}} + \tfrac{1}{2}\sum_{ij}^{electrons}\frac{1}{r_{ij}} - \sum_{I,i}\frac{Z_I}{|\mathbf{r}_i - \mathbf{R}_I|}$$

Nuclear Kinetic Energy — Electronic Kinetic Energy — Nuclear-Nuclear Repulsion — Electron-Electron Repulsion — Electron-Nuclear Attraction

H_2^+ ion



Electron Hamiltonian: $$\hat{H}_{el} = -\frac{\hbar^2}{2m_e}\nabla^2 - \frac{e^2}{4\pi\varepsilon_0 r_a} - \frac{e^2}{4\pi\varepsilon_0 r_b}$$

1. Solve electron Schrödinger eq. for fixed **R**

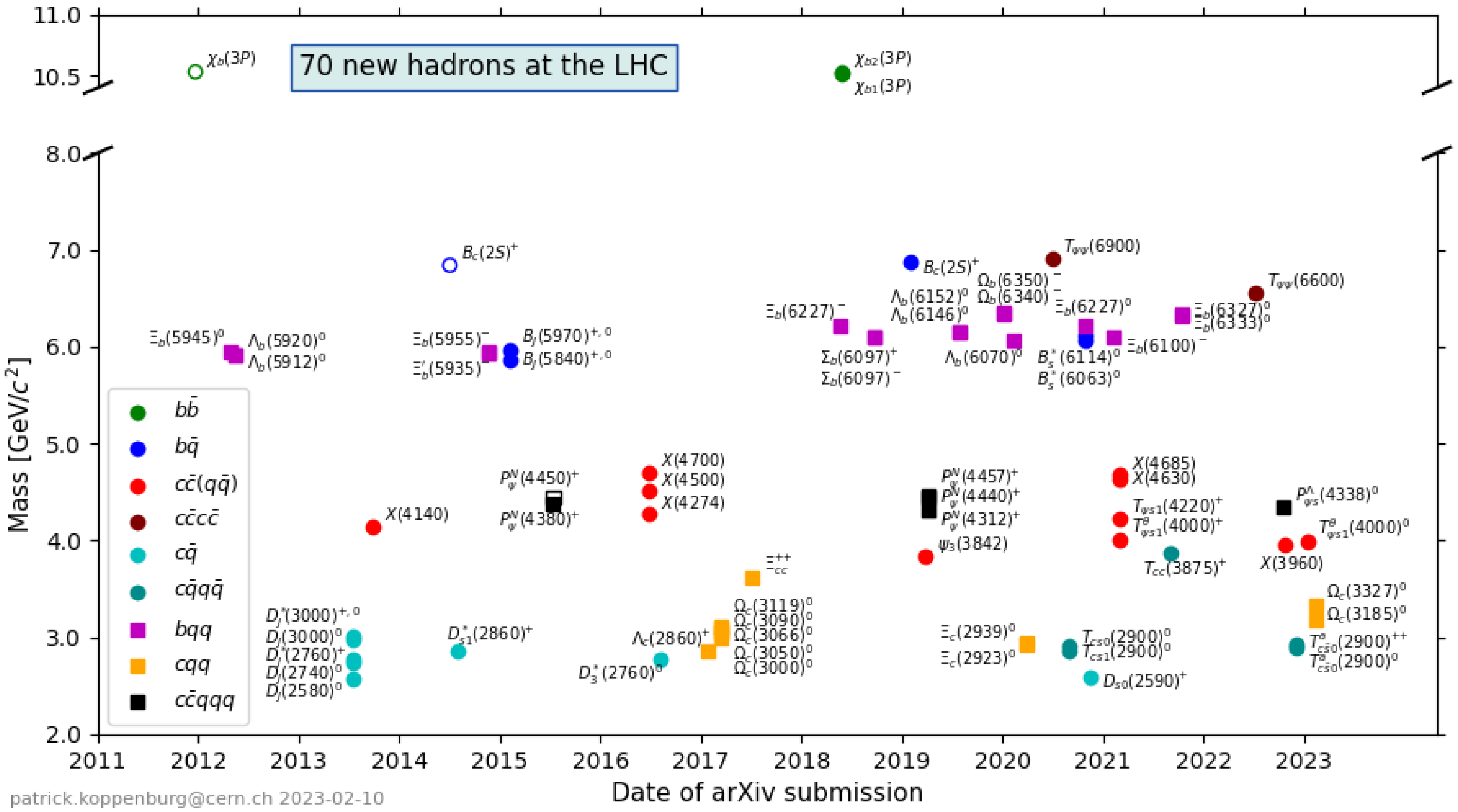
$$(\hat{H}_{el} + V_{NN})\psi_{el} = U\psi_{el}$$

2. Solve nuclei (proton) Schrödinger eq. with U(***R***) as **potential**.

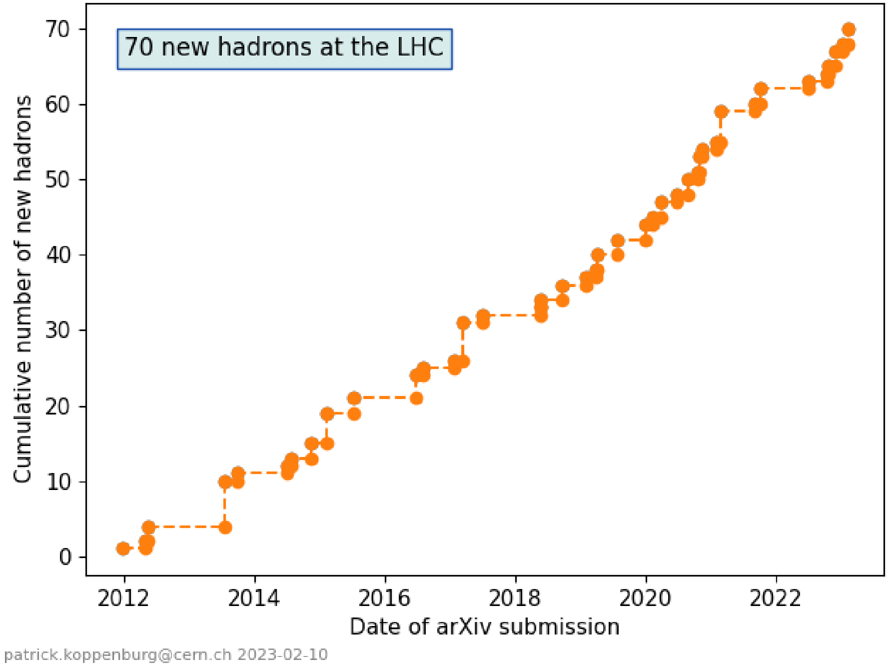
$$\left[-\frac{\hbar^2}{2m_\alpha}\nabla_\alpha^2 - \frac{\hbar^2}{2m_\beta}\nabla_\beta^2 + U(R)\right]\psi_N = E\psi_N$$

50+ New Hadrons seen at LHC

70 new hadrons with at least one heavy quark (charm or bottom) observed till now!



Here mesons not included from other experiments BESIII, Belle etc.



More heavy hadrons are expected in the near future !!

https://www.nikhef.nl/~pkoppenb/particles.html

pNRQCD

$$S_{pNRQCD} = \int d^3\mathbf{x}_1\, d^3\mathbf{x}_2\, dt\, \mathrm{tr}\Bigg(\Psi^\dagger(\mathbf{x}_1, \mathbf{x}_2, t)$$

$$\left\{ iD_0 + \frac{\mathbf{D}^2_{\mathbf{x}_1}}{2m} + \frac{\mathbf{D}^2_{\mathbf{x}_2}}{2m} \right\} \Psi(\mathbf{x}_1, \mathbf{x}_2, t) \Bigg)$$

$$+ \frac{\alpha_s}{|\mathbf{x}_1 - \mathbf{x}_2|} \mathrm{tr}\left(T^a \Psi(\mathbf{x}_1, \mathbf{x}_2, t) T^a \Psi^\dagger(\mathbf{x}_1, \mathbf{x}_2, t) \right)$$

Upon making the local field redefinition,

$$\Psi(\mathbf{x}_1, \mathbf{x}_2, t) = P\left[e^{ig \int_{\mathbf{x}_2}^{\mathbf{x}_1} \mathbf{A} d\mathbf{x}}\right] S(\mathbf{x}, \mathbf{X}, t)$$

$$+ P\left[e^{ig \int_{\mathbf{X}}^{\mathbf{x}_1} \mathbf{A} d\mathbf{x}}\right] O(\mathbf{x}, \mathbf{X}, t) P\left[e^{ig \int_{\mathbf{x}_2}^{\mathbf{X}} \mathbf{A} d\mathbf{x}}\right]$$

$$\mathbf{x} = \mathbf{x}_1 - \mathbf{x}_2, \quad \mathbf{X} = (\mathbf{x}_1 + \mathbf{x}_2)/2$$

pNRQCD

Under gauge transformations,

$$\Psi(\mathbf{x}_1,\mathbf{x}_2,t) \to g(\mathbf{x}_1,t)\Psi(\mathbf{x}_1,\mathbf{x}_2,t)g^{-1}(\mathbf{x}_2,t)$$

$$S(\mathbf{x},\mathbf{X},t) \to S(\mathbf{x},\mathbf{X},t)$$

$$O(\mathbf{x},\mathbf{X},t) \to g(\mathbf{X},t)O(\mathbf{x},\mathbf{X},t)g^{-1}(\mathbf{X},t)$$

Upon multipole expanding (up to O($\mathbf{x}^2$)),

$$\mathcal{L}_{pNRQCD} = \int d^3\mathbf{x}\; tr\Big\{ S^\dagger\Big\{ i\partial_0 - \frac{\mathbf{p}^2}{m} + \frac{C_f\alpha_s}{|\mathbf{x}|}\Big\}S + O^\dagger\Big\{ iD_0 - \frac{\mathbf{p}^2}{m} - \frac{1}{2N_c}\frac{\alpha_s}{|\mathbf{x}|}\Big\}O$$

$$+g\mathbf{x}O\mathbf{E}(\mathbf{X},t)S^\dagger + g\mathbf{x}O^\dagger\mathbf{E}(\mathbf{X},t)S + \frac{g}{2}\mathbf{x}OO^\dagger\mathbf{E}(\mathbf{X},t) + \frac{g}{2}\mathbf{x}O^\dagger O\mathbf{E}(\mathbf{X},t)\Big\}$$

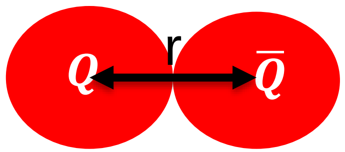
pNRQCD/BOEFT

- Behavior of heavy quark-antiquark pair at short ($r \to 0$) and large ($r \to \infty$) distance:

Consider example of quarkonium:

Short-distance ($r \to 0$) :

✓ The 2 heavy quarks are close together in color singlet configuration.



$$\chi_a^\dagger(\mathbf{r},t)\psi_a(0,t) \to Z_s^{1/2} S(\mathbf{r},t) \to Z_\Psi^{1/2}(\mathbf{r})\,\Psi(\mathbf{r},t),$$

What does this tell us???

NRQCD

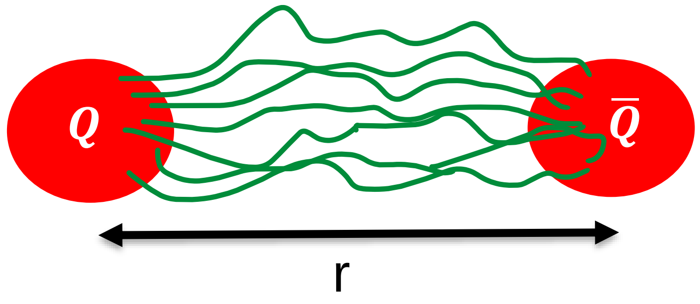
Weakly coupled pNRQCD

strongly coupled pNRQCD / BOEFT

a=1,2,3: color index

Large-distance ($r \to \infty$):

✓ The 2 heavy quarks are far apart connected by a flux tube.



$$\chi^\dagger(\mathbf{r},t)\phi(\mathbf{r},\mathbf{0};t)\psi(\mathbf{0},t) \longrightarrow Z_\Psi^{1/2}(\mathbf{r})\,\Psi(\mathbf{r},t)$$

NRQCD

strongly coupled pNRQCD / BOEFT

BOEFT

Brambilla, Lai, AM, Vairo arXiv:2212.09187

- Quarkonium static potential: $V_\Psi(r) = E_{\Sigma_g^+}(r)$
- Hybrid static potential: $V_{10}(r) = E_{\Sigma_u^-}(r)\,,$ $V_{1\pm1}(r) = E_{\Pi_u}(r)$

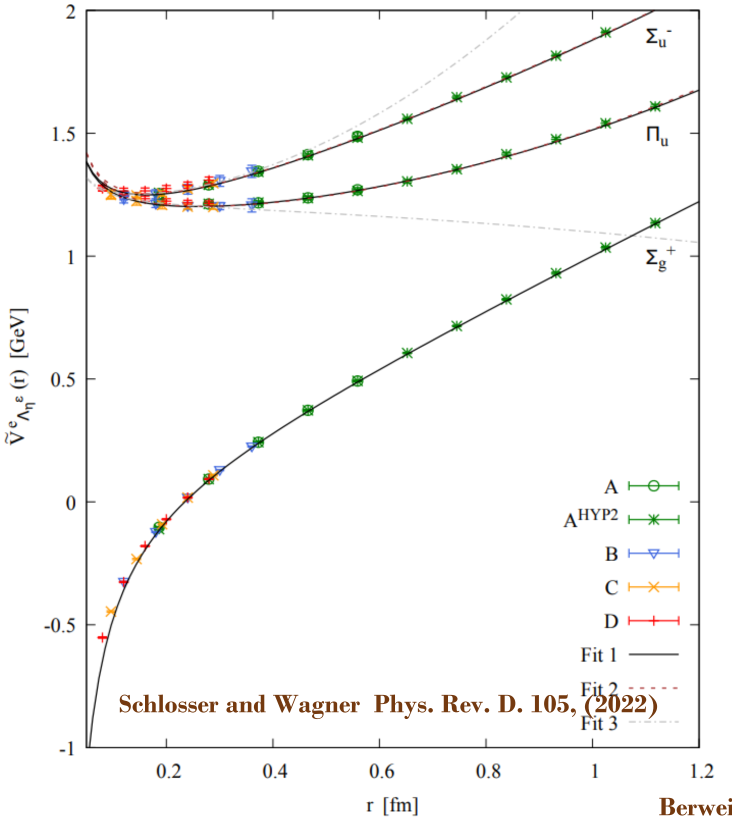
Quarkonium Potential:

$$V_{\Sigma_g^+}(r) = -\frac{\kappa_g}{r} + \sigma_g r + E_g^{Q\bar{Q}}$$

$m_c^{RS} = 1.477\,(40)$ GeV
$m_b^{RS} = 4.863\,(55)$ GeV

$\kappa_g = 0.489\,, \quad \sigma_g = 0.187\,\mathrm{GeV}^2 \qquad E_g^{c\bar{c}} = -0.254\,\mathrm{GeV},\ E_g^{b\bar{b}} = -0.195\,\mathrm{GeV},$

Gluonic Static energies from lattice:



Schlosser and Wagner Phys. Rev. D. 105, (2022)

Hybrid Potential:

$$E_{\Sigma_u^-,\Pi_u}(r) = \begin{cases} V_o^{\mathrm{RS}}(\nu_f) + \Lambda_{\mathrm{RS}}(\nu_f) + b_{\Sigma,\Pi} r^2, & r < 0.25\ \mathrm{fm} \\ \frac{a_1^{\Sigma,\Pi}}{r} + \sqrt{a_2^{\Sigma,\Pi} r^2 + a_3^{\Sigma,\Pi}} + a_4^{\Sigma,\Pi}, & r > 0.25\ \mathrm{fm} \end{cases}.$$

$a_1^\Sigma = 0.000\,\mathrm{GeV fm}\,, \quad a_2^\Sigma = 1.543\,\mathrm{GeV}^2/\mathrm{fm}^2, \quad a_3^\Sigma = 0.599\,\mathrm{GeV}^2, \quad a_4^\Sigma = 0.154\,\mathrm{GeV}\,,$

$a_1^\Pi = 0.023\,\mathrm{GeV fm}\,, \quad a_2^\Pi = 2.716\,\mathrm{GeV}^2/\mathrm{fm}^2, \quad a_3^\Pi = 11.091\,\mathrm{GeV}^2, \quad a_4^\Pi = -2.536\,\mathrm{GeV}\,,$

$b_\Sigma = 1.246\,\mathrm{GeV}/\mathrm{fm}^2, \quad b_\Pi = 0.000\,\mathrm{GeV}/\mathrm{fm}^2 \quad \Lambda_{\mathrm{RS}}$: 0.87 (15) GeV

Gluelump mass definition:

$$\langle 0|G_{1+-}^{ia}(\mathbf{R},T/2)\phi^{ab}(T/2,-T/2)G_{1+-}^{jb}(\mathbf{R},-T/2)|0\rangle = \delta^{ij} e^{-i\Lambda T}$$

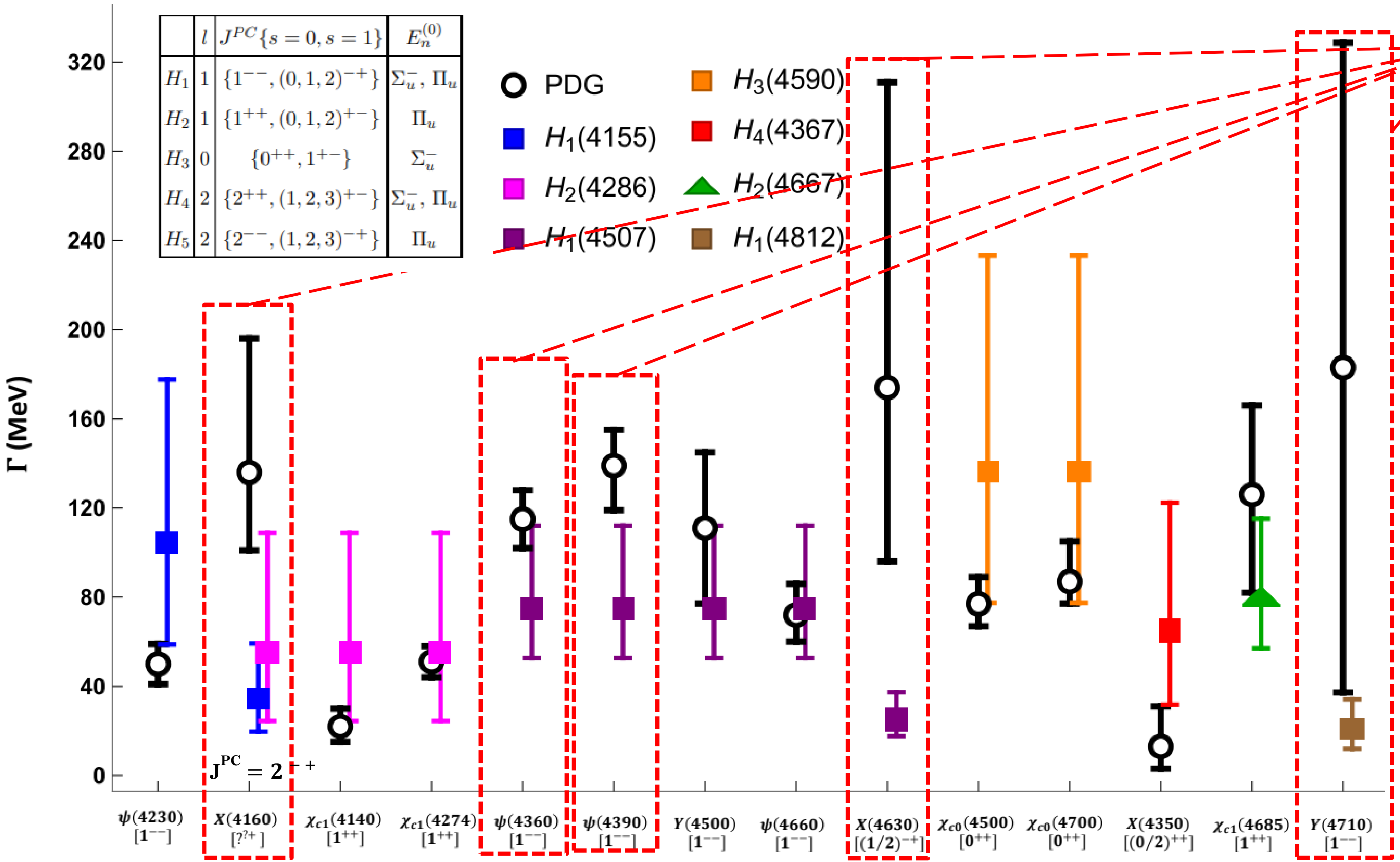
✓ Perturbative RS-scheme potentials V_o^{RS} upto order α_s^3.

Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92, (2015) Bali and Pineda Phys. Rev. D. 69, (2004)

Results

Brambilla, Lai, AM, Vairo arXiv:2212.09187

- Hybrid-to-quarkoniunium transition widths:



	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

Could have a significant hybrid component.

- ✓ Decay to open-threshold states could be significant or expected to be large (?).
- ✓ See Farina, Tecocoatzi, Giachino, Santopinto & Swanson, Phys Rev D 102 (2020) for 1^{--} hybrid decay to threshold states.
- ✓ No information on branching ratio (BR) for decays to threshold states in PDG.

$QQ\bar{q}\bar{q}$

First doubly charmed tetraquark seen by LHCb

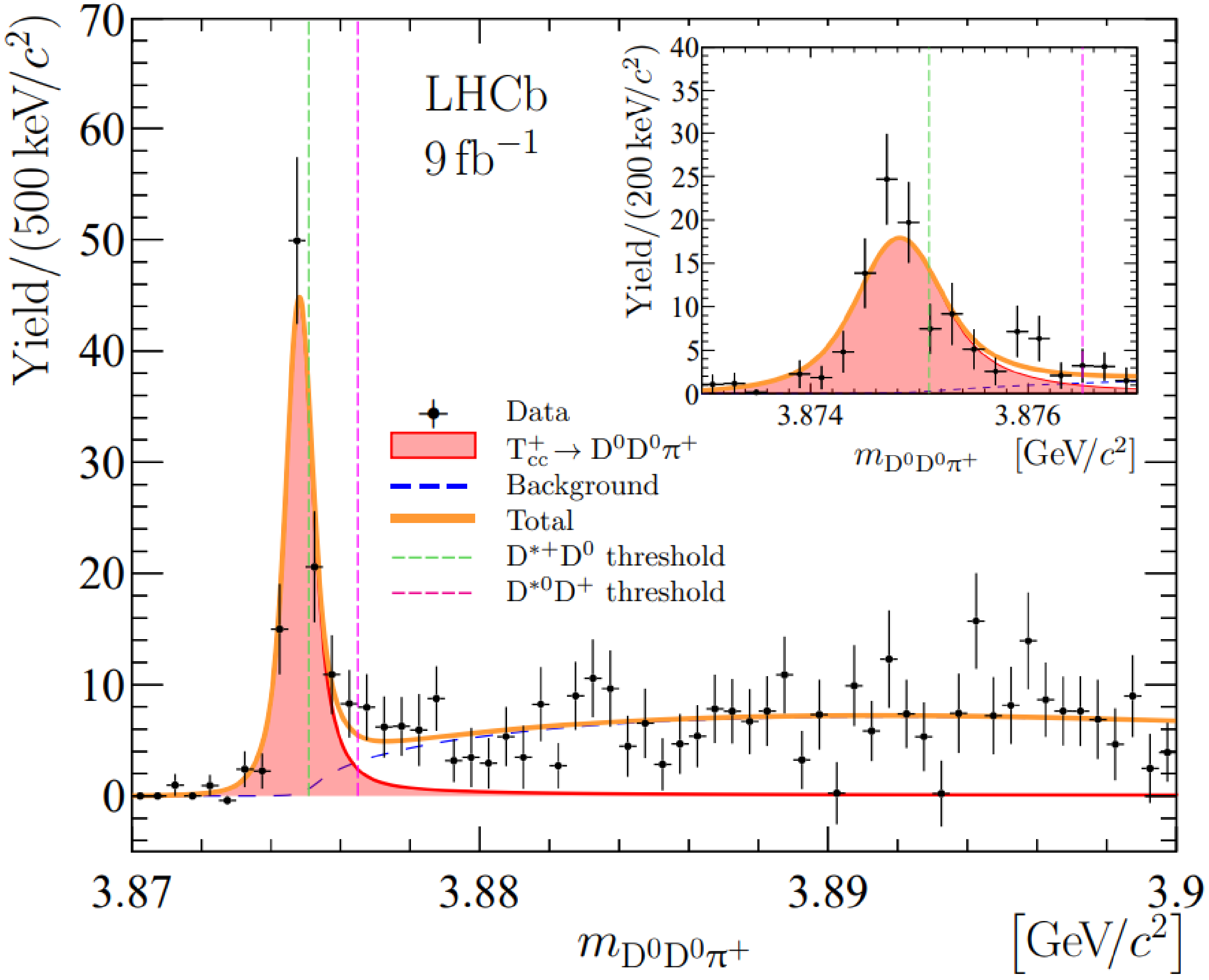
$$T_{cc}^{+}(3875) \to D^0 D^0 \pi^+$$

- Exotic quark content $cc\bar{u}\bar{d}$
- Consistent with **isoscalar** with **J^P=1$^+$**

Mass below $D^{*+}D^0$ threshold and very narrow

$$m_{T_{cc}^+} - (m_{D^{*+}} + m_{D^0}) = -0.27 \pm 0.06 \text{ MeV}.$$

$T_{cc}^{+}(3875)$



LHCb (Nature Phys. 18 (2022) 7, 751; Nature Comm. 13 (2022) 3351)

Results

Brambilla, Lai, AM, Vairo arXiv:2212.09187

- Spin-flipping transitions: suppressed by powers of the heavy-quark mass due to the heavy-quark spin symmetry;
- Relative comparison between **spin-conserving** and **spin-flipping decays**: $H_m \to Q_n + X$:
  - ✓ Size of energy gap **ΔE**: final quarkonium states are different in both the decay process.
  - ✓ Depends on relative magnitude of matrix element (radial): $|\langle Q_n|r|H_m\rangle|$ & $|\langle Q_n|H_m\rangle|/m$

Ratio : $m\,|\langle Q_n|\mathbf{r}|H_m\rangle|/|\langle Q_n|H_m\rangle|$

No obvious hierarchy relation between the two-decay process.

  - ✓ For bottom hybrids: spin-flipping transitions are smaller compared to spin-conserving.
  - ✓ For charm hybrids: spin-flipping transitions are not necessarily small: $m\,|\langle Q_n|\mathbf{r}|H_m\rangle|/|\langle Q_n|H_m\rangle| \sim 1$

Spin-flipping ~ spin-conversing: indicating heavy-quark spin-symmetry violations!

- Quarkonium transition: $Q_m \to Q_n + X$:spin-flipping decay suppressed by $O(v)^2$ compared to spin- conserving.

$$(|\langle Q_n|Q_m\rangle|/m\,|\langle Q_n|\mathbf{r}|Q_m\rangle|)^2 \sim v^2 \ll 1$$

Exotic Hadron

- **conventional quarkonium**, which consists of a color-singlet heavy quark-antiquark pair: $(Q\bar{Q})_1$,
- **quarkonium hybrid meson**, which consists of a color-octet $Q\bar{Q}$ pair to which a gluonic excitation is bound: $(Q\bar{Q})_8 + g$,
- **compact tetraquark** [7], which consists of a $Q\bar{Q}$ pair and a light quark q and antiquark $\bar{q}$ bound by inter-quark potentials into a color singlet: $(Q\bar{Q}q\bar{q})_1$,
- **meson molecule** [8], which consists of color-singlet $Q\bar{q}$ and $\bar{Q}q$ mesons bound by hadronic interactions: $(Q\bar{q})_1 + (\bar{Q}q)_1$,
- **diquark-onium** [9], which consists of a color-antitriplet Qq diquark and a color-triplet $\bar{Q}\bar{q}$ diquark bound by the QCD color force: $(Qq)_{\bar{3}} + (\bar{Q}\bar{q})_3$,
- **hadro-quarkonium** [10], which consists of a color-singlet $Q\bar{Q}$ pair to which a color-singlet light-quark pair is bound by residual QCD forces: $(Q\bar{Q})_1 + (q\bar{q})_1$. An essentially equivalent model is a quarkonium and a light meson bound by hadronic interactions.
- **quarkonium adjoint meson** [11], which consists of a color-octet $Q\bar{Q}$ pair to which a light quark-antiquark pair is bound: $(Q\bar{Q})_8 + (q\bar{q})_8$.

Braaten, Langmack, and Smith Phys. Rev. D90, 014044 (2014)

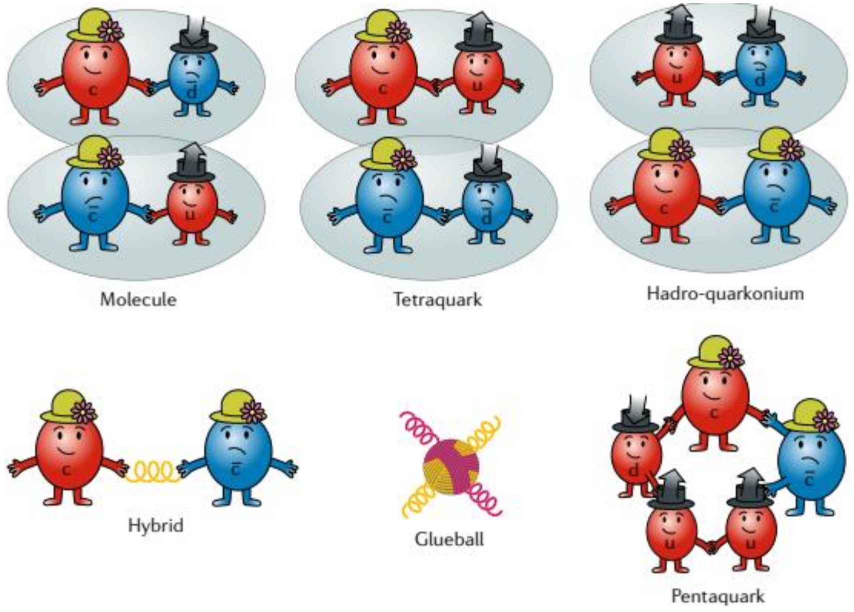


Figure from Nat Rev Phys 1, 480-494 (2019)

....

<u>"Common Names" *(with exceptions)*</u>

Y(mass): produced in $e^+e^- \to Y$
Z(mass): has non-zero isospin
X(mass): everything else

Lattice: Static energies

- Based on spectral decomposition:

Any state $|X_\Gamma\rangle = \sum_n c_n \, |\underline{n}\rangle$

$|\underline{n}\rangle$: Eigenstate of static NRQCD Hamiltonian ($m \to \infty$).

- In large time limit, the NRQCD correlator (Heisenberg picture) gives:

$$E_\Gamma(r) = \lim_{T\to\infty} \frac{i}{T} \log\langle X_\Gamma, T/2 | X_\Gamma, -T/2\rangle$$

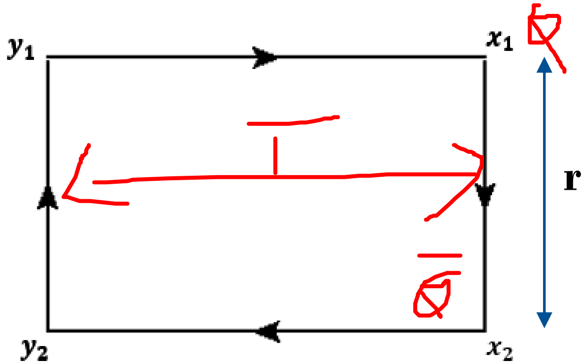
Γ: quantum #'s corresponding to representation of $D_{\infty h}$: $\boldsymbol{\Lambda_\eta^\sigma}$

Quarkonium

Color singlet $Q\bar{Q}$ pair

$$|X_\Gamma\rangle = \chi(\mathbf{x}_2)\phi(\mathbf{x}_2, \mathbf{x}_1)\psi^\dagger(\mathbf{x}_1)|\Omega\rangle$$

$$T^a P^a_\Gamma = \mathbb{I}, \;\; \Gamma = \Sigma_g^+$$



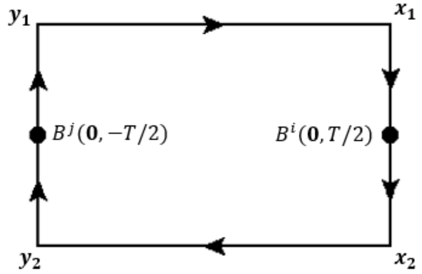
Hybrid

Color octet $Q\bar{Q}$ pair + gluon

$$|X_\Gamma\rangle = \chi(\mathbf{x}_2)\phi(\mathbf{x}_2, \mathbf{R})T^a G^a_\Gamma(\mathbf{R})\phi(\mathbf{R}, \mathbf{x}_1)\psi^\dagger(\mathbf{x}_1)|\Omega\rangle$$

gluonic operator with quantum numbers Γ

$$G^a_\Gamma = \hat{\mathbf{r}} \cdot \mathbf{B}^a, \hat{\mathbf{r}} \times \mathbf{B}^a \quad \text{for } \Gamma = \Sigma_u^-, \Pi_u$$



Wilson line:

$$\phi(\mathbf{y}, \mathbf{x}; t) \equiv \mathrm{P}\exp\left\{ ig \int_0^1 ds \, (\mathbf{y} - \mathbf{x}) \cdot \mathbf{A}(\mathbf{x} - s(\mathbf{x} - \mathbf{y}), t) \right\}$$

Hybrid Decays

- Connection with non-perturbative fields: quarkonium and hybrid in $\mathbf{r} \to \mathbf{0}$

Fields:

$$S\,(\mathbf{r},\mathbf{R},t) \to Z_\Psi^{1/2}(\mathbf{r})\,\Psi(\mathbf{r},\mathbf{R},t),$$

$$P^{i\dagger}_{\kappa\lambda} O^a\,(\mathbf{r},\mathbf{R},t)\,G^{ia}_\kappa(\mathbf{R},t) \to Z_\kappa^{1/2}(\mathbf{r})\,\Psi_{\kappa\lambda}(\mathbf{r},\mathbf{R},t)$$

Potentials:

$$E_{\Sigma_g^+}(r) = V_s(r) + b_{\Sigma_g^+} r^2 + \ldots,$$

$$E_{\Sigma_u^-,\Pi_u}(r) = V_o(r) + \Lambda + b_{\Sigma,\Pi} r^2 + \ldots$$

κ: denotes quantum numbers

Gluelump mass definition:

$$\langle 0|G^{ia}_{1^{+-}}(\mathbf{R},T/2)\phi^{ab}(T/2,-T/2)G^{jb}_{1^{+-}}(\mathbf{R},-T/2)|0\rangle = \delta^{ij} e^{-i\Lambda T}$$

$$\phi^{ab}(T/2,-T/2) = \mathrm{P}\exp\left\{-ig\int_{-T/2}^{T/2} dt\, A_0^{adj}(\mathbf{R},t)\right\}^{ab}$$

Eigenvalue of static NRQCD Hamiltonian

$$H_0 G^{ia}_\kappa(\boldsymbol{R},t)|0\rangle = \Lambda_\kappa G^{ia}_\kappa(\boldsymbol{R},t)|0\rangle\,,$$

$$H_0 = \int d^3R\,\frac{1}{2}\left[\boldsymbol{E}^a\cdot\boldsymbol{E}^a + \boldsymbol{B}^a\cdot\boldsymbol{B}^a\right].$$

QCD spectrum: Hadrons

- "Confinement conjecture": only color singlet states exist in nature that we term "Hadrons"
- Hadrons: **Color singlet** bound states of quarks and gluons bound by strong interactions.
- Quark Model: Classified all hadrons (states) as Mesons or Baryons   Gell-Mann & Zweig

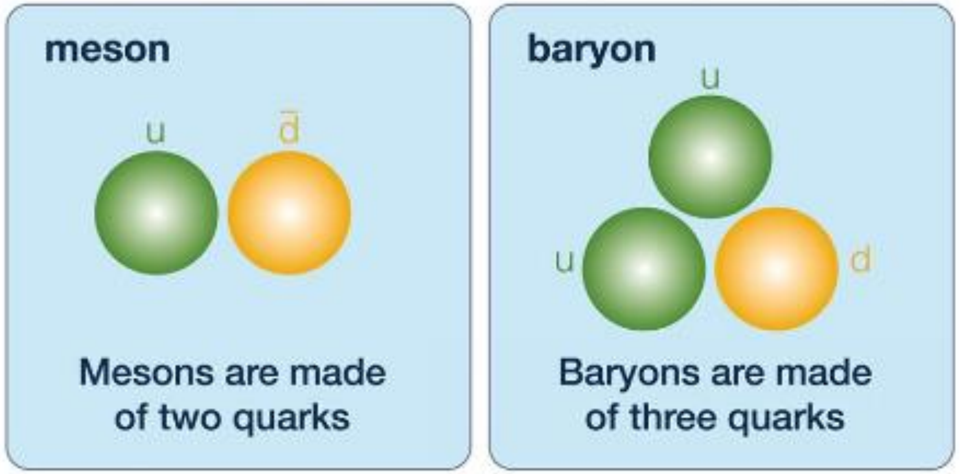


Fig taken from https://www.eurekalert.org/multimedia/911829

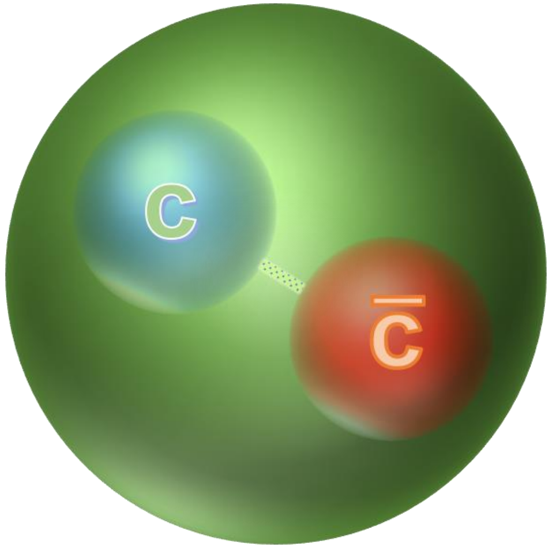
Constituents	Combinations	Naming convention (quark model)
$3 \otimes \bar{3}$	$1 \oplus 8$	Meson
$3 \otimes 3 \otimes 3$	$1 \oplus 8 \oplus 8 \oplus 10$	Baryon
$8 \otimes 8$	$1 \oplus 8 \oplus 8 \oplus 10 \oplus 10 \oplus 27$	Glueball
$\bar{3} \otimes 8 \otimes 3$	$1 \oplus 8 \oplus 8 \oplus 8 \oplus 10 \oplus 10 \oplus 27$	Hybrid
$\bar{3} \otimes \bar{3} \otimes 3 \otimes 3$	$1 \oplus 1 \oplus 8 \oplus 8 \oplus 8 \oplus 8 \oplus 10 \oplus 10 \oplus 27$	Tetraquark/molecule
$3 \otimes 3 \otimes 3 \otimes 3 \otimes \bar{3}$	$1 \oplus 1 \oplus 1 \oplus 8 \oplus 8 \oplus 8 \oplus 8 \oplus 8 \oplus 8 \oplus 8 \oplus 8 \oplus 10 \oplus 10 \oplus 27 \oplus 35 + \cdots$	Pentaquark
....	....	?

A constituent model of hadrons

Focus of this talk are hadrons involving 2-heavy quarks. !!!

Quarkonium

- Quarkonium: Color singlet bound state of $Q\bar{Q}$ (Q = c, b).



- Hierarchy of scales in Quarkonium:
  - ❖ Heavy quark mass: $\boldsymbol{m}$
  - ❖ Heavy quark K.E scale: mv^2
  - ❖ Relative separation between heavy quarks: $r \sim 1/mv$
  - ➤ **Nonrelativistic bound-state system: $\boldsymbol{v \ll 1}$**
  - ➤ $\Lambda_{\text{QCD}} \sim \mathbf{0.4}$ GeV

$$m \gg mv \gg mv^2 \sim \Lambda_{\text{QCD}}$$

(perturbative dynamics)

⬇

Heirarchy for low-lying states (far-away from threshold)

Ex. $\boldsymbol{J/\psi, \Upsilon(1S)}$.

Brambilla, Pineda, Soto and Vairo, Nucl. Phys. B 566, (2000) 275

$$m \gg mv \sim \Lambda_{\text{QCD}} \gg mv^2$$

(nonperturbative dynamics: Strongly Coupled)

⬇

Hierarchy for all other cases.

Estimates

	$c\bar{c}$	$b\bar{b}$
$\langle r \rangle \gtrsim$	$0.4\ fm$	$0.2\ fm$
$v^2 \sim$	$1/3$	$1/9$

- Potential-NRQCD (pNRQCD): EFT for quarkonium. **Describes physics at the scale $\boldsymbol{mv^2}$.**

Schrödinger description for quarkonium states.

Mehen and Fleming, Phys. Rev. D73, 034502 (2005) Bodwin, Braaten & Lepage (1994) Luke, Manohar & Rothstein (2000) Brambilla, Pineda, Soto and Vairo, Rev. Mod. Phys 77, (2005)