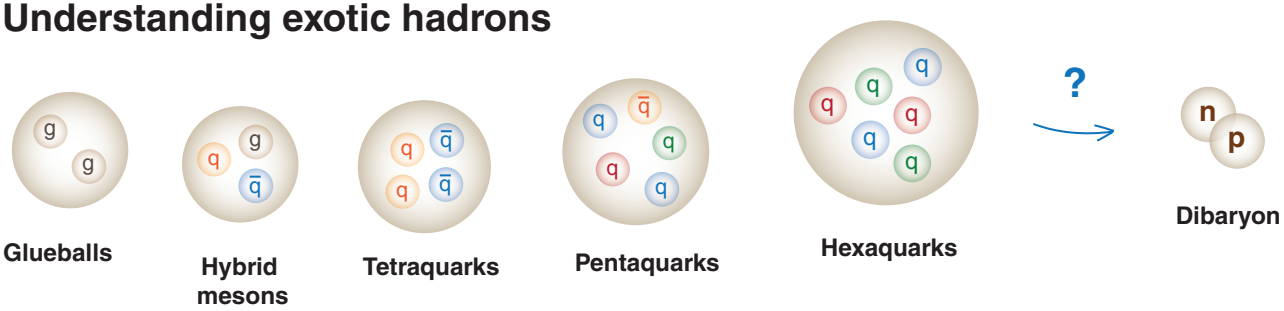
From baryons to dibaryons with functional methods

Gernot Eichmann
University of Graz

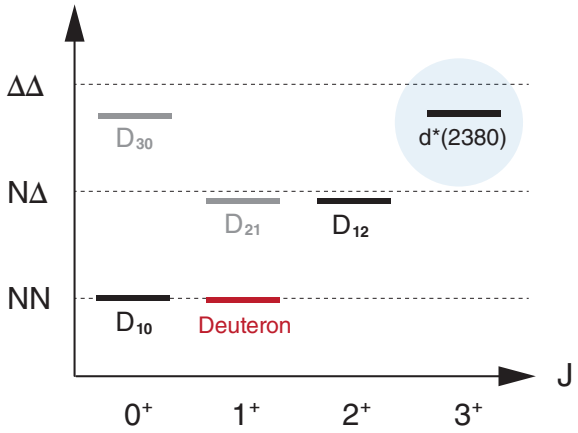
Physics opportunities with proton beams at SIS100
Wuppertal, Feb 8, 2024

Motivation

- **Understanding exotic hadrons**

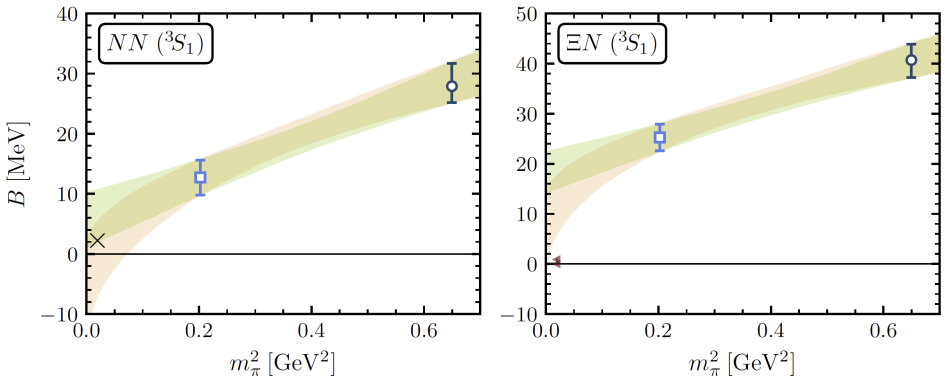


- **Light dibaryons** Dyson, Xuong 1964



Adlarson et al., PRC 106 (2011), PRC 112 (2014),
Bashkanov, Brodsky, Clement, PLB 727 (2013),
Gal, Garcilazo, PRC 111(2013), . . .

- **Strange dibaryons** NPLQCD, HALQCD, USQCD, PACS-CS, . . .



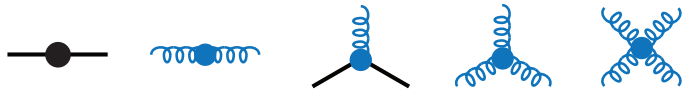
Illa et al., PRD 103 (2021)

- **Nucleons in nuclei?**
  Short-range correlations, EMC effect?

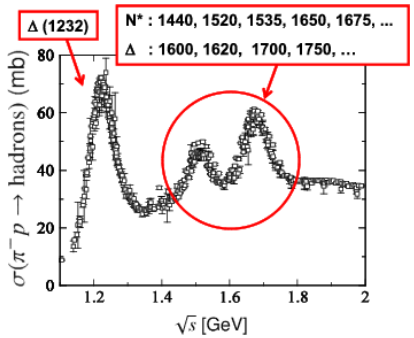
CLAS: Duer et al., Nature 560 (2018), Schmidt et al., Nature 578 (2020)

Theory tools

Functional methods
(DSEs & BSEs, FRG, ...)

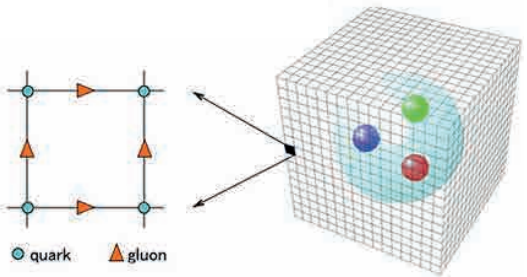


Amplitude analyses

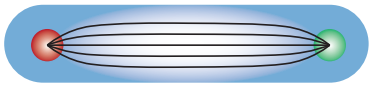


ExoHad
Exotic Hadrons Topical Collaboration

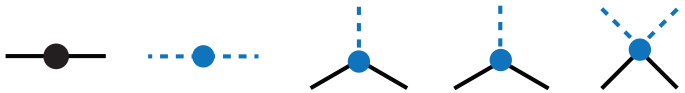
Lattice QCD



Phenomenological models

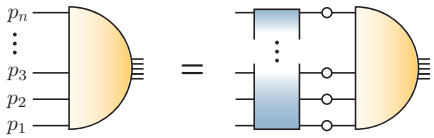


Effective theories
(ChPT, ...)



Functional methods

- Hadronic **bound-state equations** (Bethe-Salpeter & Faddeev eqs)

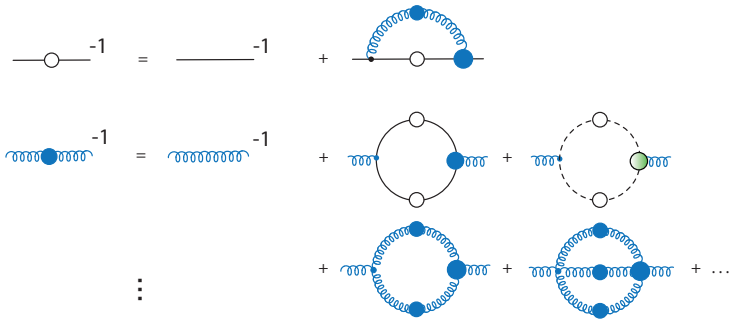


"QFT analogue of Schrödinger eq."

→ hadron masses & "wave functions"

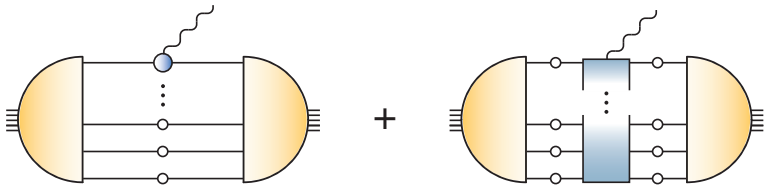
→ **spectroscopy calculations**

- Ingredients: **QCD's n-point functions,** Satisfy quantum eqs. of motion (DSEs)

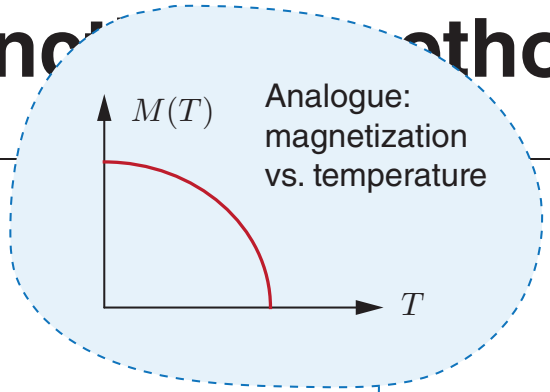


→ Dynamical mass generation, gluon mass gap, confinement, ...

- **Structure calculations:** form factors, PDFs, GPDs, TMDs, two-photon processes, ...



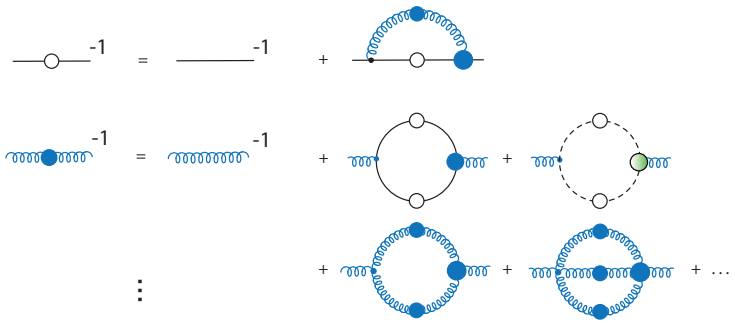
Functional methods



Analogue:
magnetization
vs. temperature

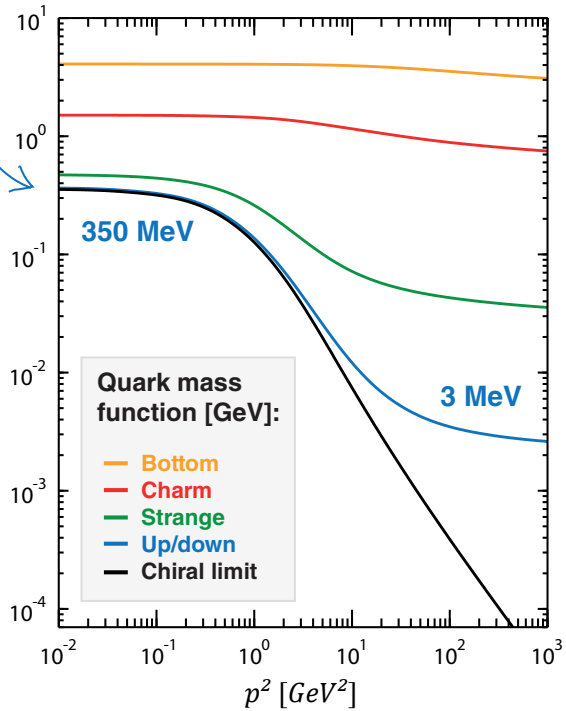
- Ingredients: **QCD's n-point functions,**
  Satisfy quantum eqs. of motion (DSEs)

Quark mass



→ Dynamical mass generation,
gluon mass gap, confinement, ...

"constituent-quark mass"

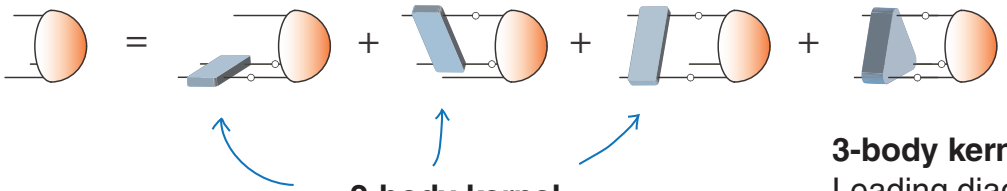


current-quark mass

Baryons

Three-quark BSE (Faddeev equation) for baryons:

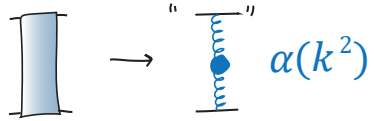
GE, Alkofer, Nicmorus, Krassnigg, PRL 104 (2010)

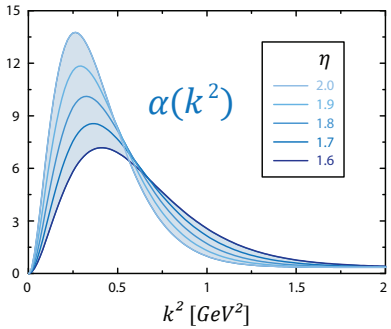


2-body kernel:
fixed in meson sector

3-body kernel:
Leading diagram (3-gluon vertex) vanishes by color trace, higher-order diagrams small (?)
2-quark correlations dominant?

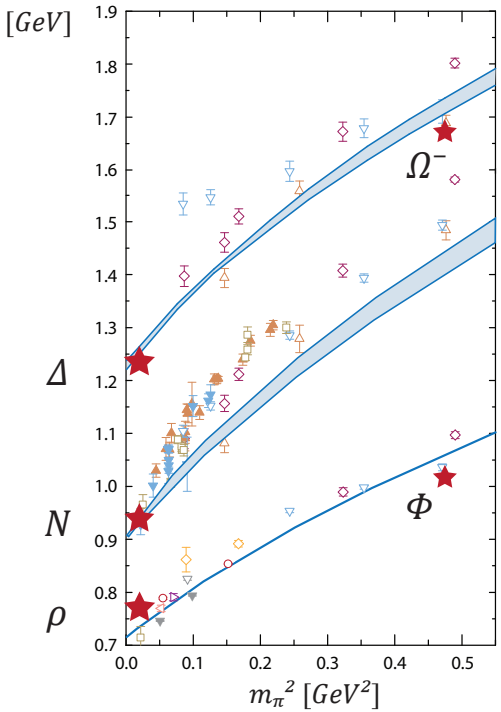
Rainbow-ladder





Scale set by f_π,
shape parameter → bands

Maris, Tandy, PRC 60 (1999)



- Analogous results for many **form factors**

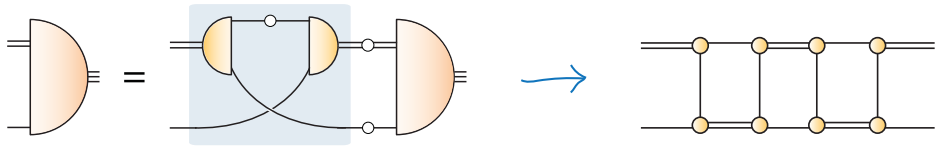
  Review: GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, Prog. Part. Nucl. Phys. 91 (2016)

- Relativistically, nucleon also has **p waves!**

see also:
Qin, Roberts, Schmidt, PRD 97 (2018)

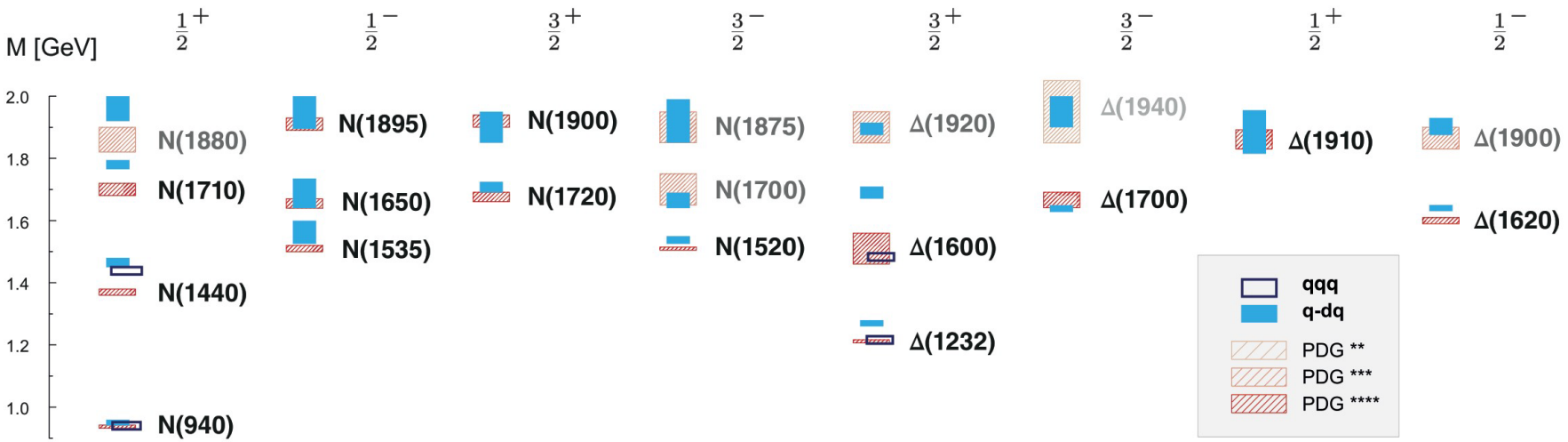
Diquark correlations

- **Quark-diquark** (two-body) equation
  Oettel et al., PRC 58 (1998), GE et al., Ann. Phys. 323 (2008), Cloet et al., FBS 46 (2009), Segovia et al., PRL 115 (2015)



Diquark clustering in baryons?
Barabanov et al., Prog. Part. Nucl. Phys. 116 (2021)

- **Three-quark** and **quark-diquark** results very similar
  GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)



Diquark correlations

- **Quark-diquark** (two-body) equation
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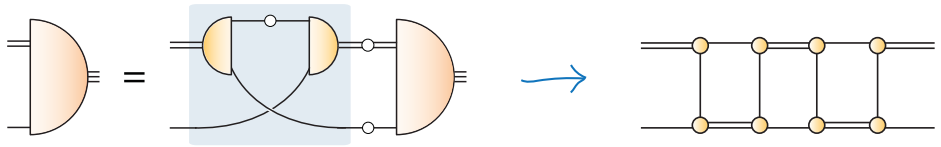
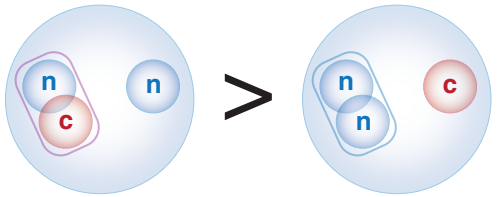
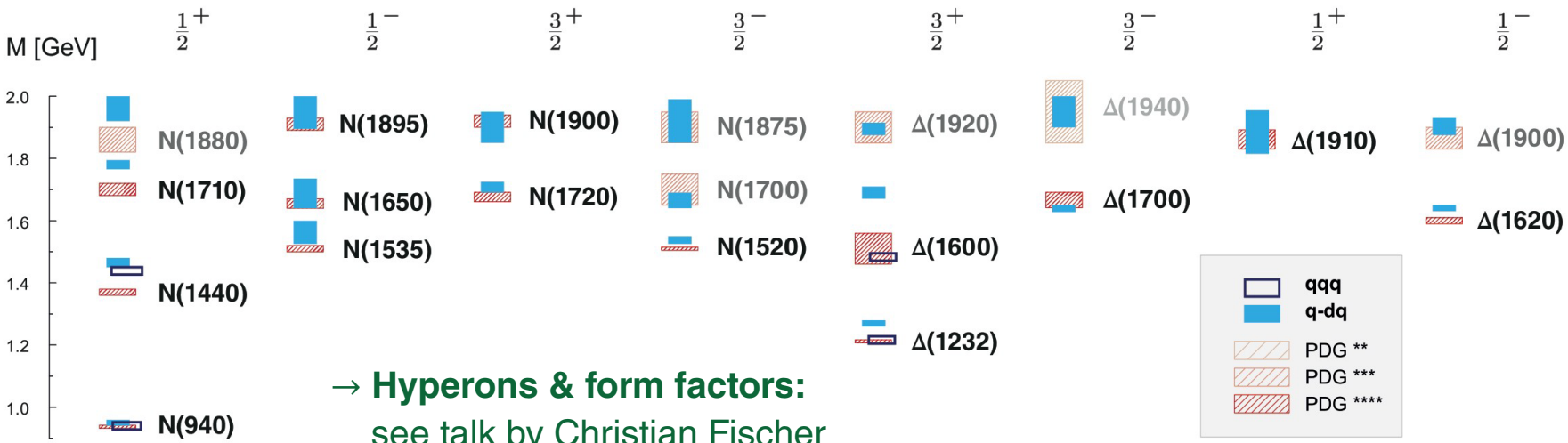


Heavy baryons
Torcato, Arriaga, GE, Peña, FBS 64 (2023)



~60% ~40%

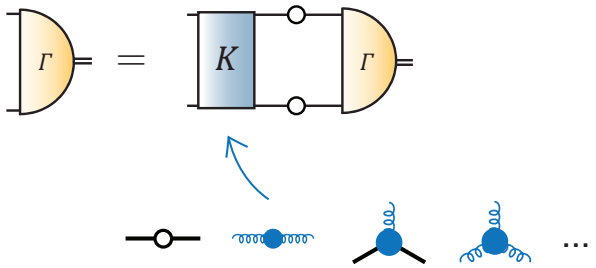
- **Three-quark** and **quark-diquark** results very similar
  GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)



→ **Hyperons & form factors:**
see talk by Christian Fischer

Towards ab-initio

- **Goal:** go towards ab-initio calculations by calculating **higher n-point functions**



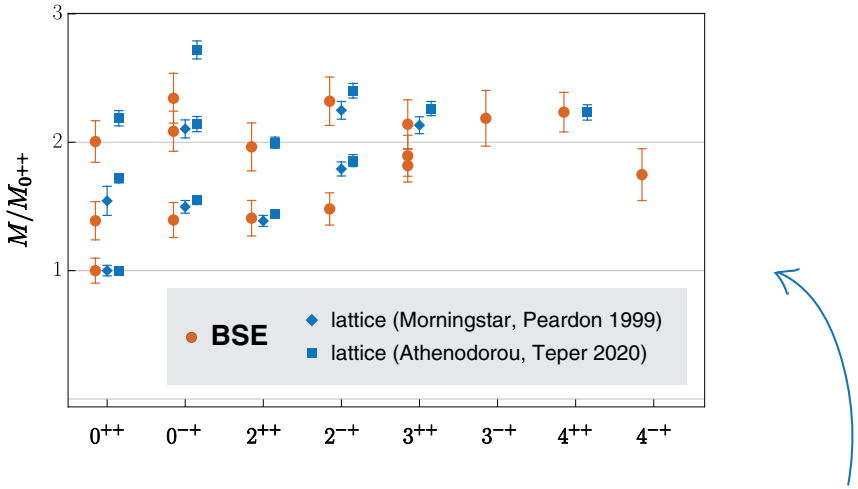
. . . ,
Williams, Fischer, Heupel, PRD 93 (2016),
Cyrol et al., PRD 97 (2018),
Oliveira, Silva, Skullerud, Sternbeck, PRD 99 (2019),
Aguilar et al., EPJ C 80 (2020),
Huber, PRD 101 (2020),
Qin, Roberts, Chin. Phys. Lett. 38 (2021),
GE, Pawlowski, Silva, PRD 104 (2021),
. . .

truncation error:

1 60% 2 10% 3 4%

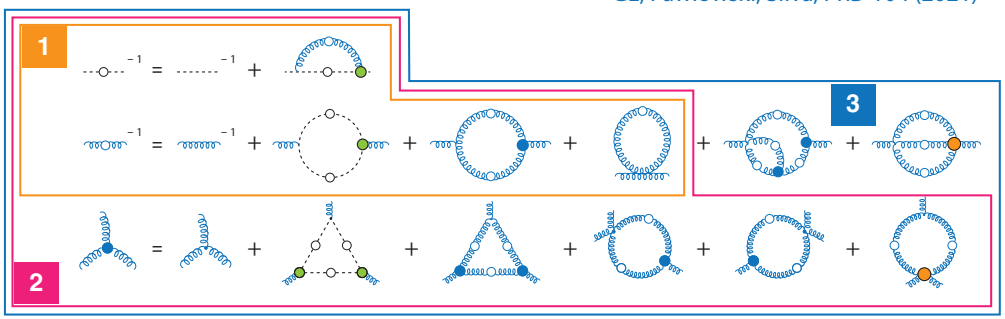
- **Glueball spectrum** agrees with lattice QCD

Huber, Fischer, Sanchis-Alepuz, EPJ C 80 (2020), EPJ C 81 (2021)

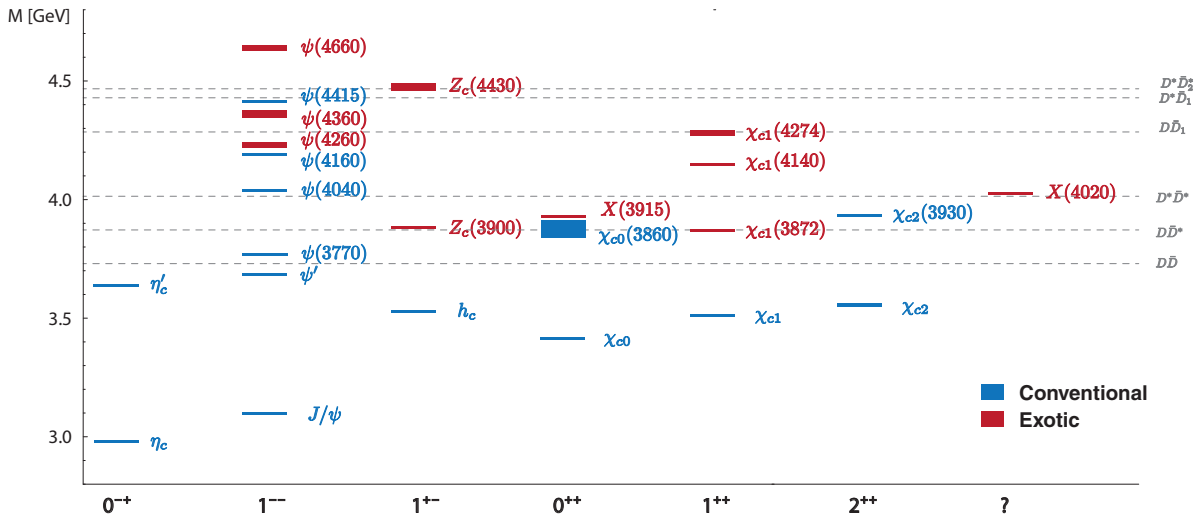


- Coupled **Yang-Mills DSEs**

Huber, PRD 101 (2020),
GE, Pawlowski, Silva, PRD 104 (2021)



Exotic mesons



- Several tetraquark candidates in **charmonium spectrum**: X(3872), X(3915), Zc(3900), ...
- Z states cannot be $c\bar{c}$ since they carry charge
- Recent additions: all-charm X(6900), open-charm T_{cc}^{+}, ...
- Oldest tetraquark candidates: **light scalar mesons**

Reviews:

Chen, Chen, Liu, Zhu,
Phys. Rept. 639 (2016), 1601.02092

Lebed, Mitchell, Swanson
PPNP 93 (2017), 1610.04528

Esposito, Pilloni, Polosa,
Phys. Rept 668 (2017), 1611.07920

Guo, Hanhart, Meißner et al.,
Rev. Mod. Phys. 90 (2018), 1705.00141

Ali, Lange, Stone,
PPNP 97 (2017), 1706.00610

Olsen, Skwarnicki, Zieminska,
Rev. Mod. Phys. 90 (2019), 1708.04012

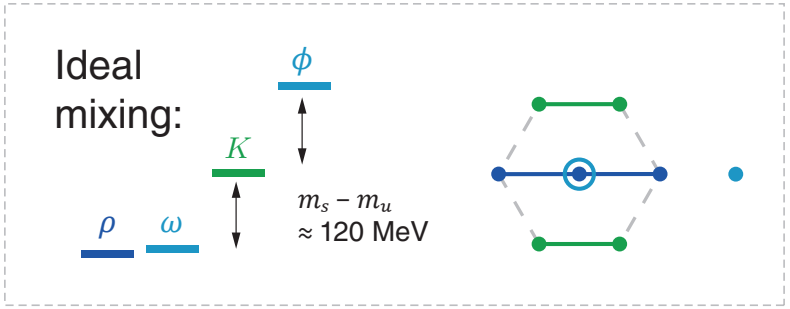
Liu, Chen, Chen, Liu, Zhu,
PPNP 107 (2019), 1903.11976

Brambilla, Eidelman, Hanhart et al.,
Phys. Rept. 873 (2020)

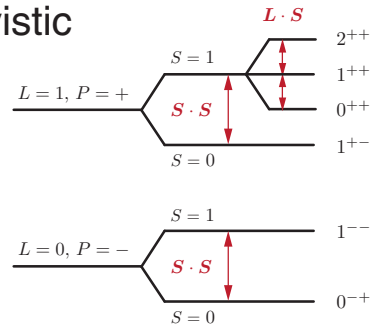
...

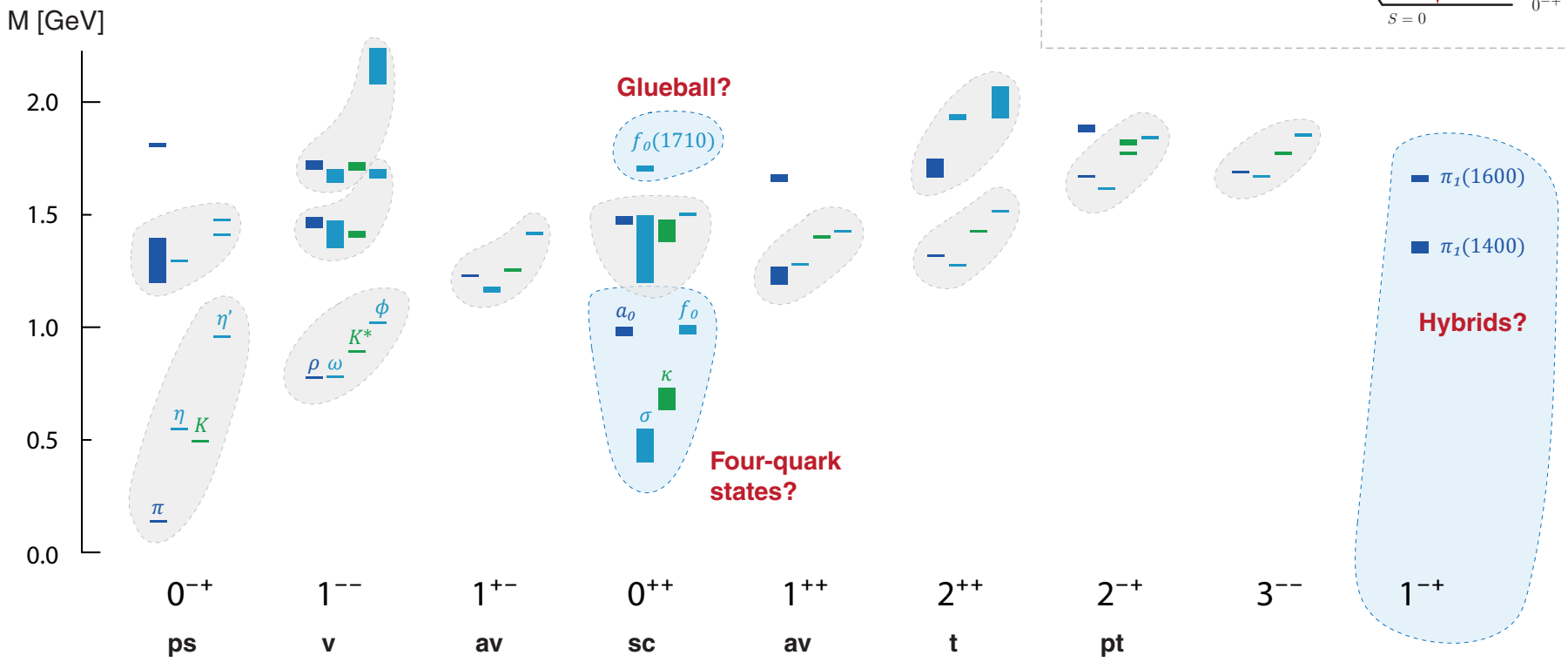
Light exotic mesons

Light meson spectrum
(PDG 2020)



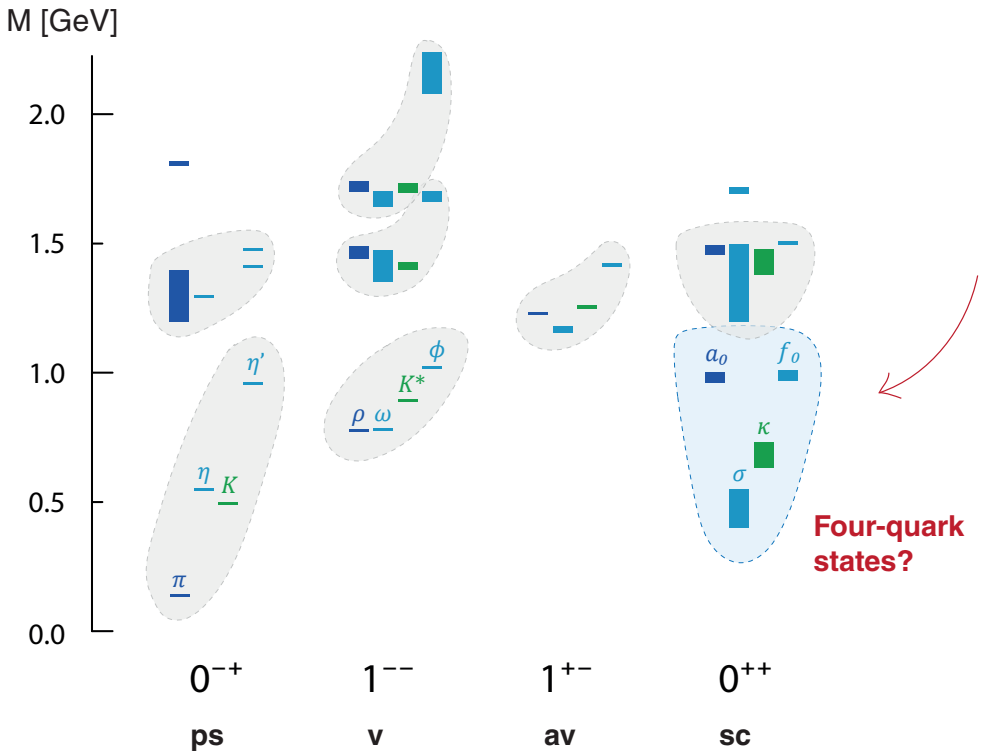
Non-relativistic level ordering



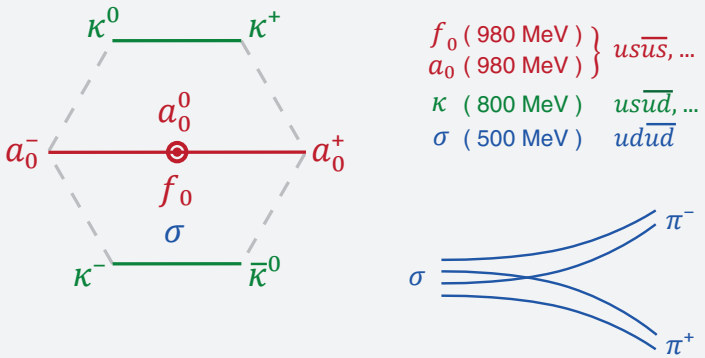


Light exotic mesons

Light meson spectrum
(PDG 2020)



- **Diquark-antidiquark**?
  Explains mass ordering & decay widths
  Jaffe 1977, Close, Tornqvist 2002,
  Maiani, Polosa, Riquer 2004

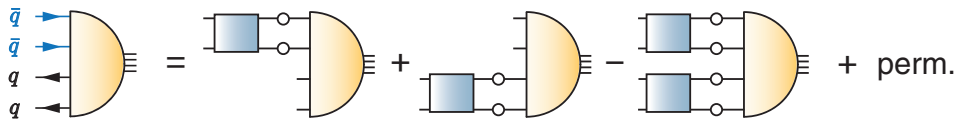


- **Meson molecules?**
  Weinstein, Isgur 1982, 1990; Close, Isgur, Kumano 1993

- **Non-$q\bar{q}$ nature** supported by various approaches
  Pelaez, Phys. Rept. 658 (2016)

Four-quark states

- Light scalar mesons (σ, κ, a_0, f_0) as **four-quark states:**
  GE, Fischer, Heupel, PLB 753 (2016)



$$\Gamma(p,q,k,P) = \sum_i f_i\left(p^2,q^2,k^2,\{\omega_j\},\{\eta_j\}\right)\,\tau_i(p,q,k,P) \;\otimes\; \text{Color} \;\otimes\; \text{Flavor}$$

9 Lorentz invariants:

$p^2,\quad q^2,\quad k^2,\quad P^2 = -M^2$

$\omega_1 = q\cdot k \qquad \eta_1 = p\cdot P$
$\omega_2 = p\cdot k \qquad \eta_2 = q\cdot P$
$\omega_3 = p\cdot q \qquad \eta_3 = k\cdot P$

256 Dirac-Lorentz tensors

2 Color tensors:

$3\otimes\bar{3},\quad 6\otimes\bar{6}$ or
$1\otimes 1,\quad 8\otimes 8$
(Fierz-equivalent)

$K\,\psi_i = \lambda_i\,\psi_i$

	dim K	memory
Mesons	10^3	20 MB
Baryons	10^8	10^7 GB
Tetraquarks	10^{13}	10^{18} GB

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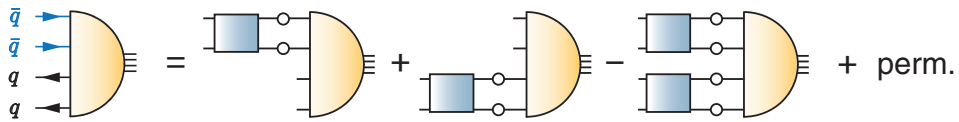

$$\Gamma(p,q,k,P) = \sum_i f_i\left(p^2, q^2, k^2, \{\omega_j\}, \{\eta_j\}\right) \tau_i(p,q,k,P) \otimes \text{Color} \otimes \text{Flavor}$$

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$\omega_2 = p \cdot k \qquad \eta_2 = q \cdot P$
$\omega_3 = p \cdot q \qquad \eta_3 = k \cdot P$

256 Dirac-Lorentz tensors

2 Color tensors:

$3 \otimes \bar{3}, \quad 6 \otimes \bar{6}$ or
$1 \otimes 1, \quad 8 \otimes 8$
(Fierz-equivalent)

- Group momentum variables into multiplets of **permutation group S4:** can switch off groups of variables without destroying symmetries
  GE, Fischer, Heupel, PRD 92 (2015)

$f_i\left(\mathcal{S}_0, \bigtriangledown, \diamondsuit, \bigcirc\right)$

Four-quark states

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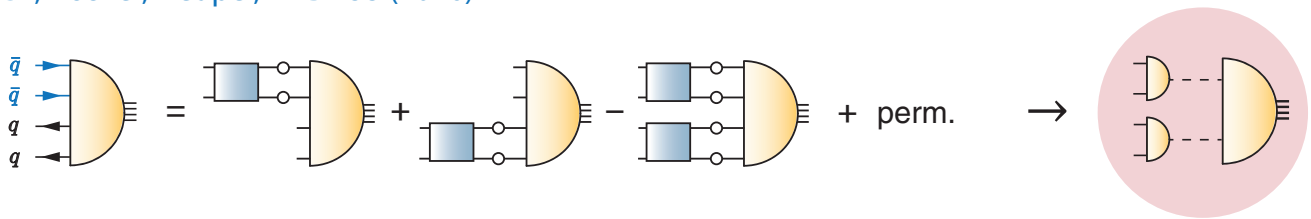
  GE, Fischer, Heupel, PLB 753 (2016)



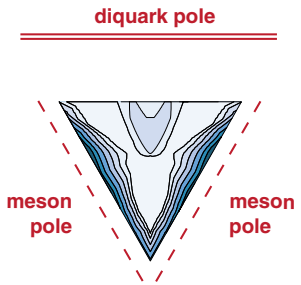
- BSE dynamically generates **meson poles** in BS amplitude:

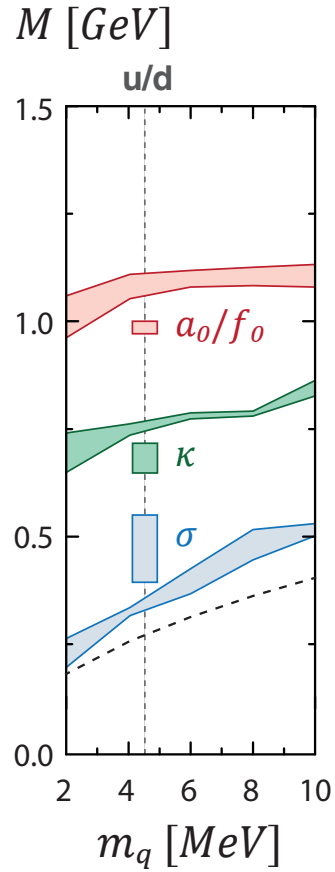
$f_i(\mathcal{S}_0, \triangledown, \triangle, \bigcirc) \rightarrow$ 1500 MeV

$f_i(\mathcal{S}_0, \triangledown, \triangle, \bigcirc) \rightarrow$ 1500 MeV

$f_i(\mathcal{S}_0, \triangledown, \triangle, \bigcirc) \rightarrow$ 1200 MeV

$f_i(\mathcal{S}_0, \triangledown, \triangle, \bigcirc) \rightarrow$ **350 MeV !**



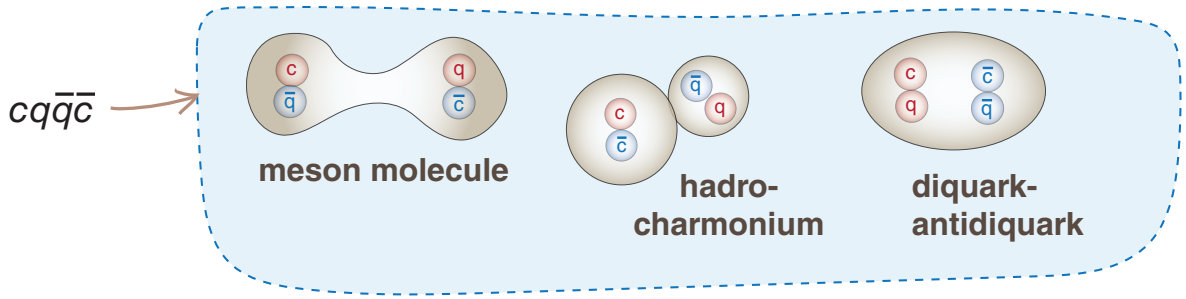


- "Light scalar mesons" look like **meson molecules,**
  diquark-antidiquark components almost negligible.
  Lightness is inherited from pseudoscalar Goldstone bosons!

Four-quark states

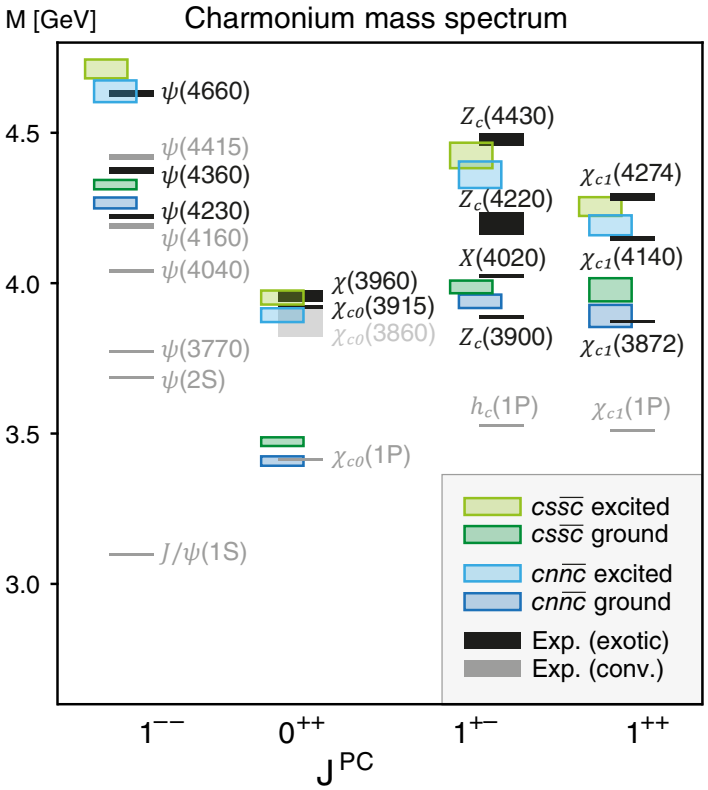
Heavy-light **4q states**

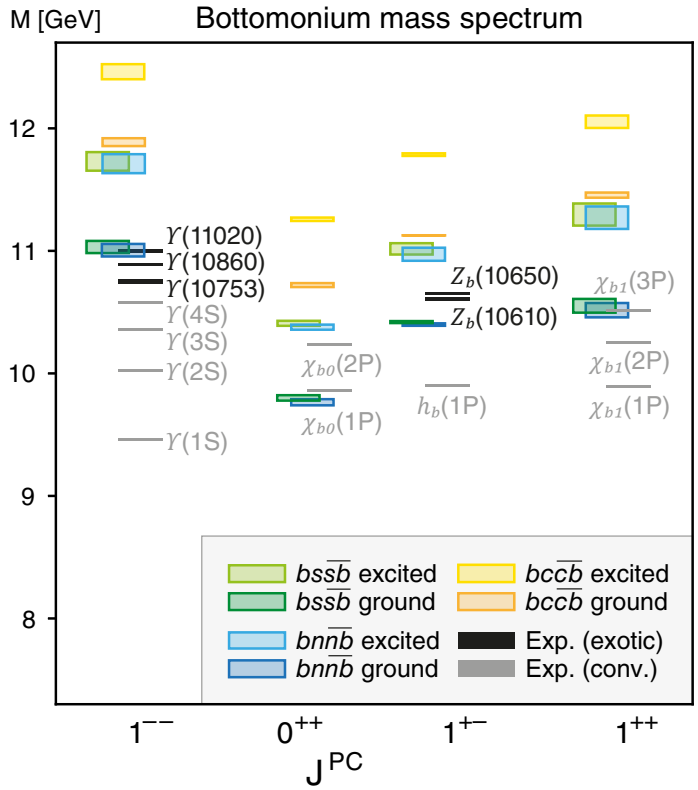
Wallbott, GE, Fischer,
PRD 100 (2019), PRD 102 (2020)



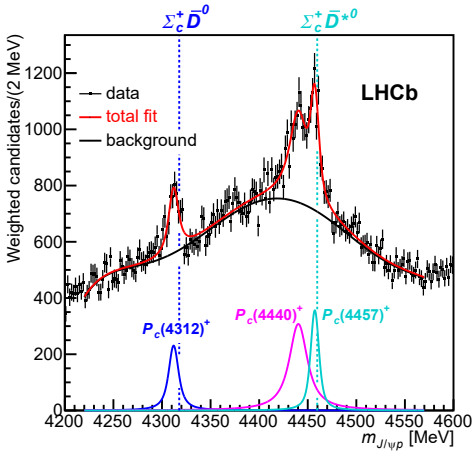
preliminary

Hoffer, GE, Fischer,
in preparation



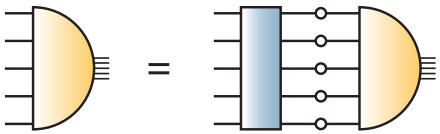


Pentaquarks?

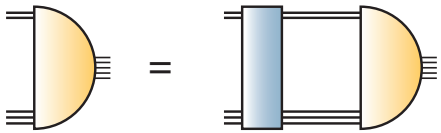


Aaij et al., PRL 112 (2019)

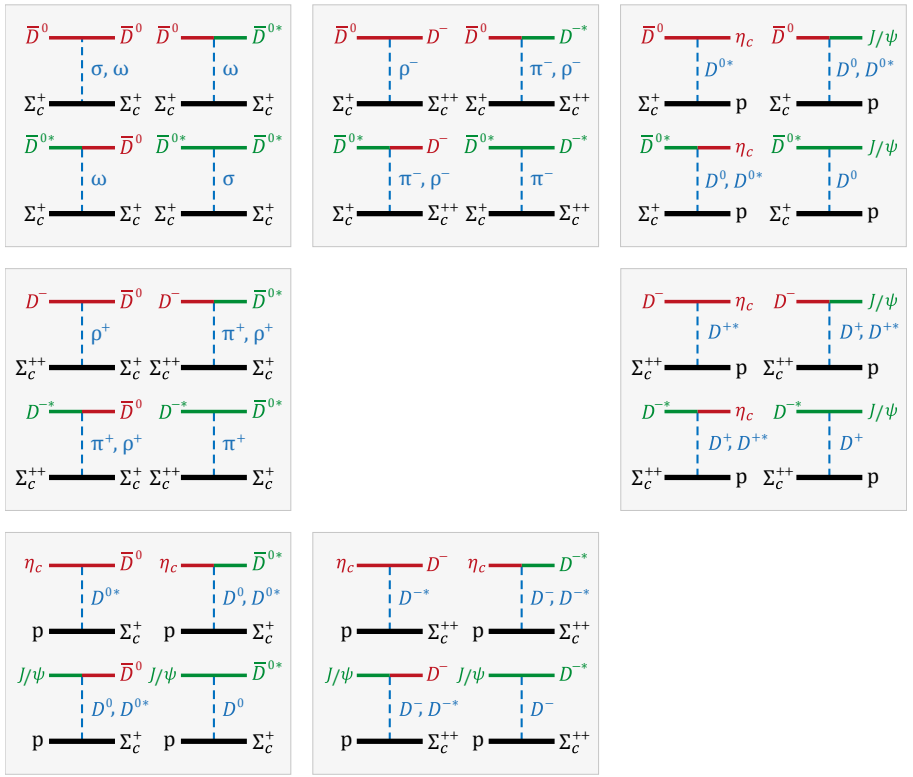
- **5-body equation:** in progress
  GE, Peña, Torres, in preparation



- Meson-baryon equation with hadronic exchanges
  GE, Lourenco, Peña, Stadler, Torres, in preparation

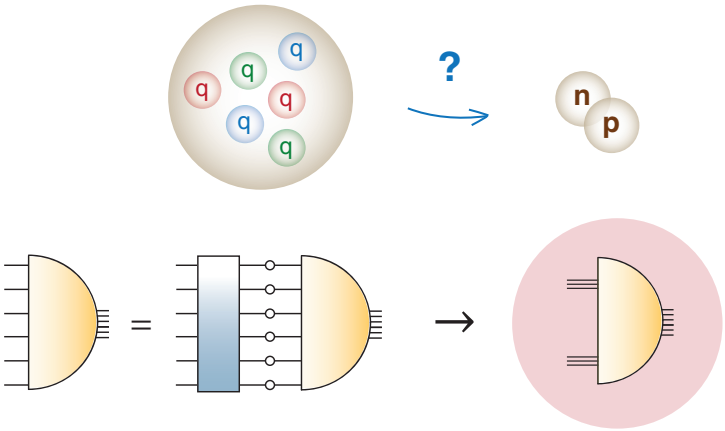


... all couplings calculated dynamically



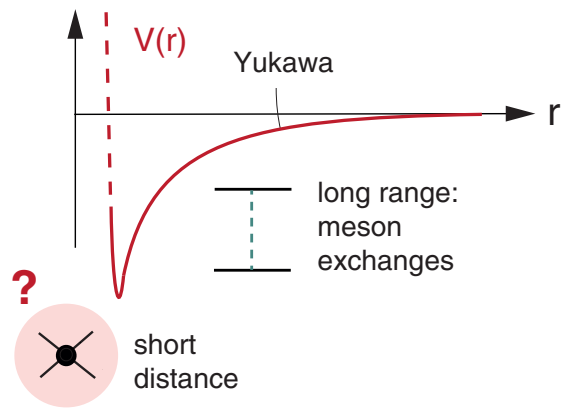
Nucleons in nuclei?

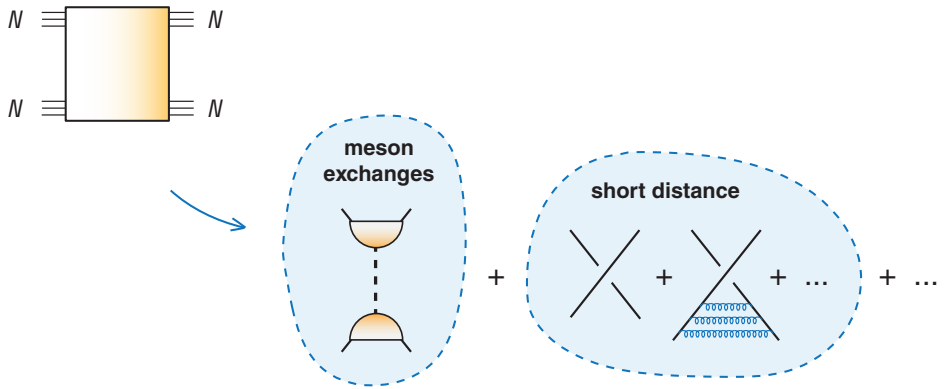
Transition from quarks & gluons to **light nuclei:**



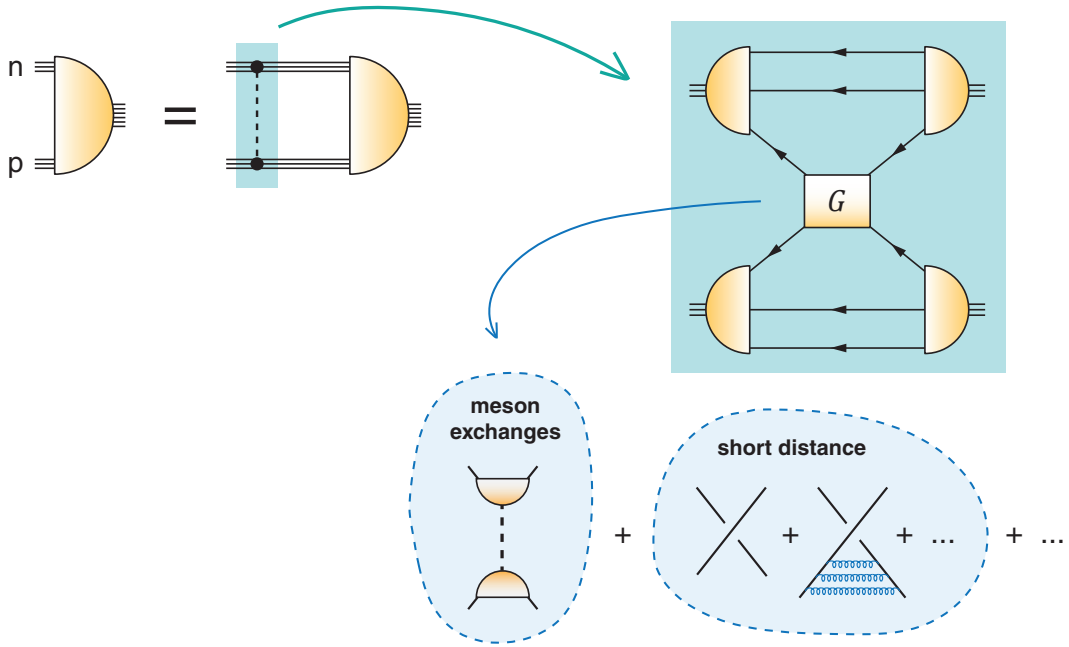
- Relativistic structure of **deuteron?**
- Exotic dibaryons, hypernuclei, short-range correlations, EMC effect ...

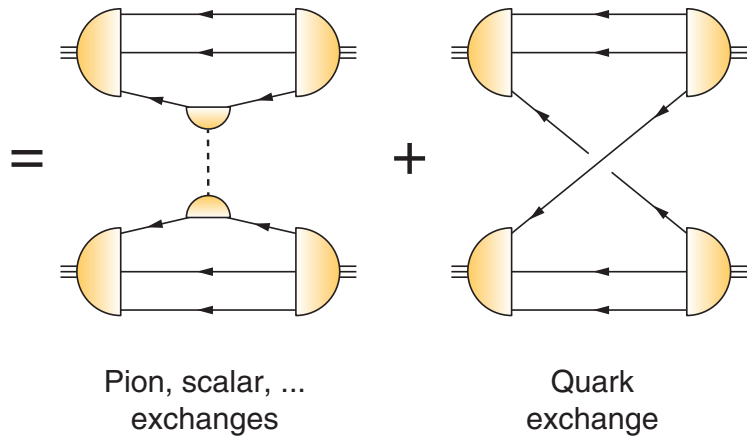
Microscopic origins of **short-range nuclear force?**





Deuteron

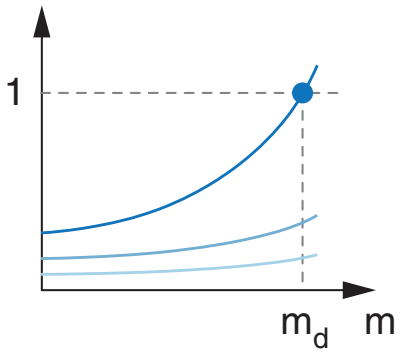




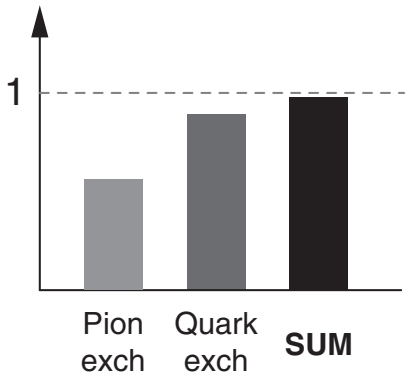
preliminary

Arriaga, GE, Nunes, Peña, in preparation

Eigenvalue (sketch)



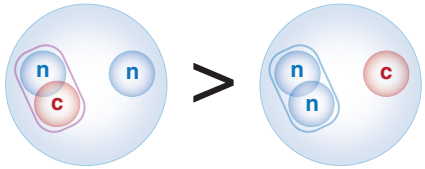
Eigenvalue at $m = m_d$



OAM composition:

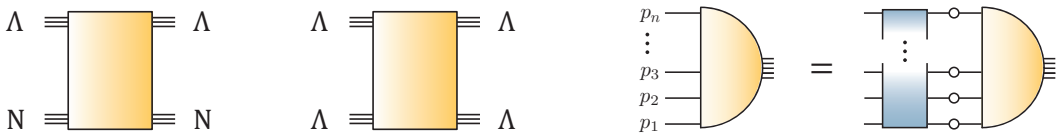
Outlook

- **Hyperons** and **charmed baryons:** Spectroscopy, form factors, structure



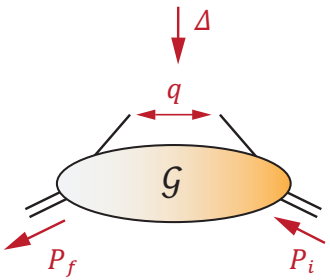
~60% ~40%

- **Dibaryons & baryon-baryon interactions**

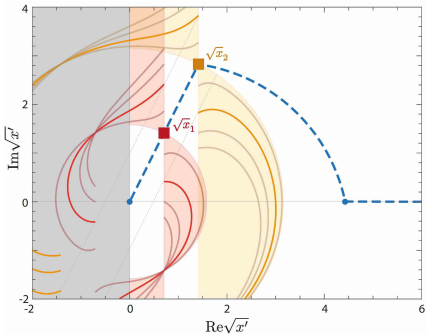


Spectroscopy, form factors, PDFs, scattering amplitudes

- **Hadron structure:** PDFs, GPDs, TMDs



$$\mathcal{G}(z, P, \Delta) = \langle P_f | \, \mathsf{T} \, \Phi(z) \, \mathcal{O} \, \Phi(0) \, | P_i \rangle$$



New method to compute light-front wave functions via contour deformations

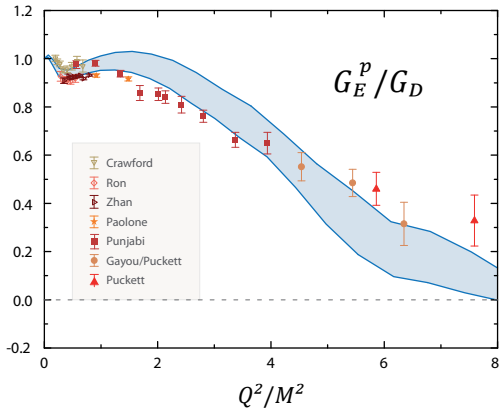
Editors' Suggestion:
GE, Ferreira, Stadler, PRD 105 (2022)

Thank you!

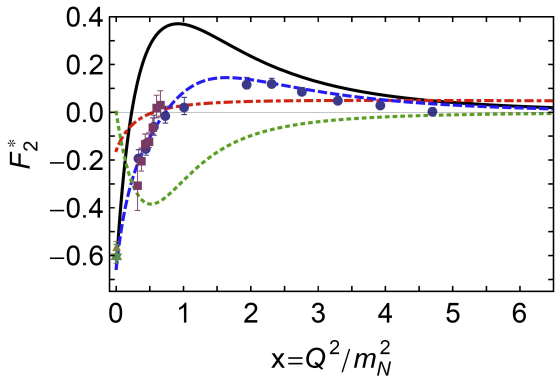
Backup slides

Baryon structure

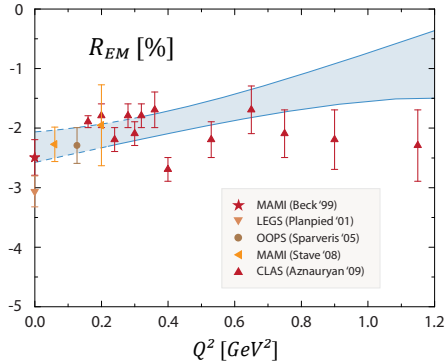
Nucleon electromagnetic FFs
GE, PRD 84 (2011)



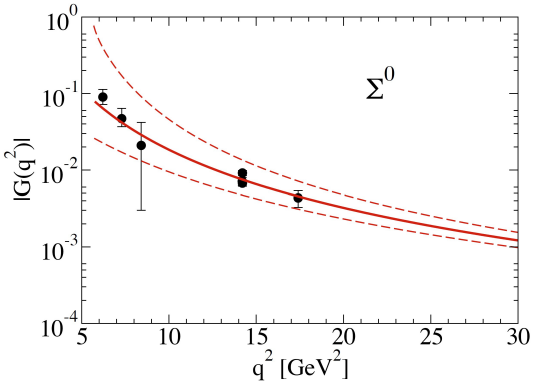
Roper em. transition FFs
Segovia et al., PRL 115 (2015)



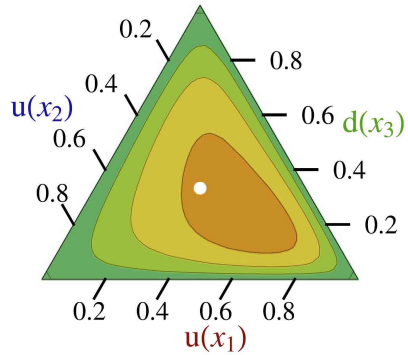
Δ em. transition FFs
GE, Nicmorus, PRD 85 (2012)



Timelike em. strangeness FFs
Ramalho, Peña, PRD 101 (2020)



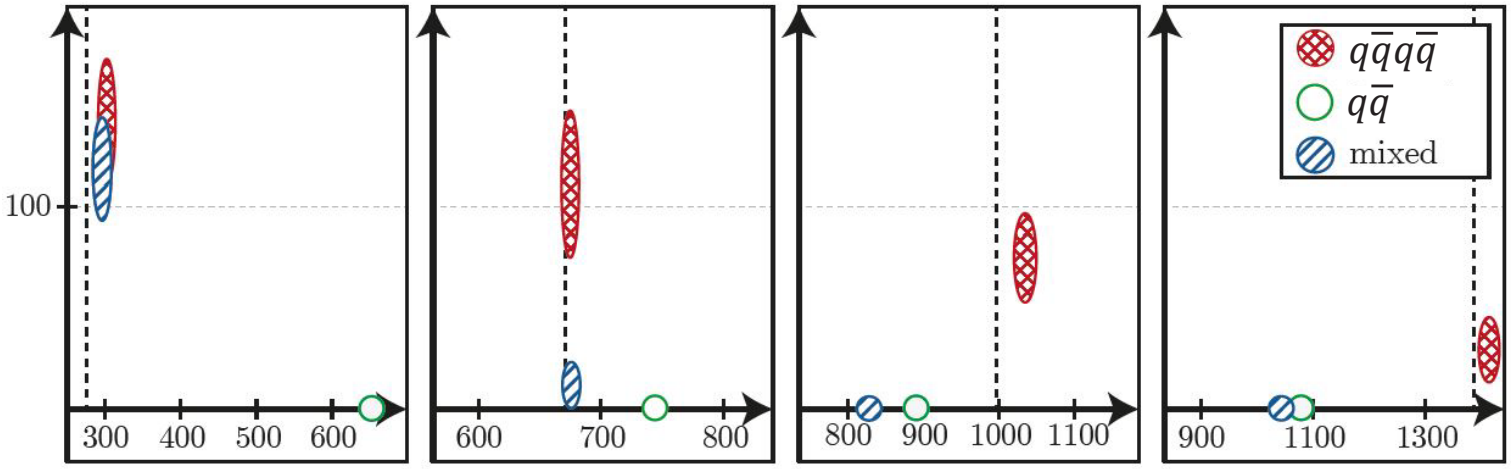
Distribution amplitudes
Mezrag, Segovia, Chang, Roberts, PLB 783 (2018)



Four-quark states

Four-quark vs. $q\bar{q}$ dominance

Santowsky, Fischer, PRD 105 (2022)



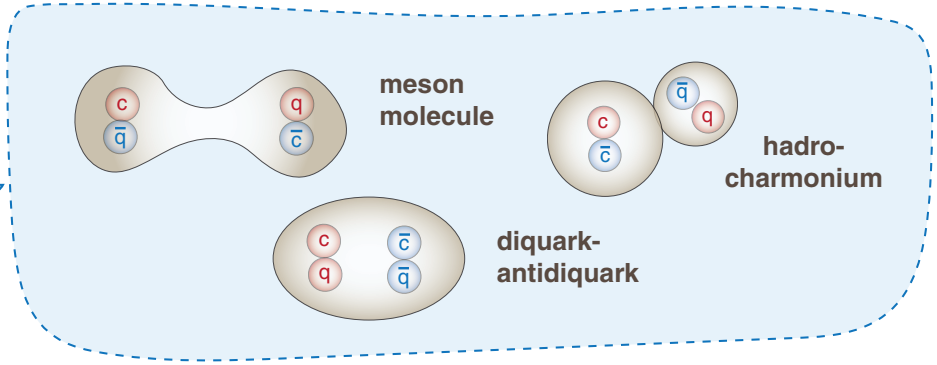
Light quarks:
$\pi\pi$ dominant

Strange quarks:
$q\bar{q}$ dominant

Four-quark states

- Heavy-light **four-quark states:** what is their internal decomposition?

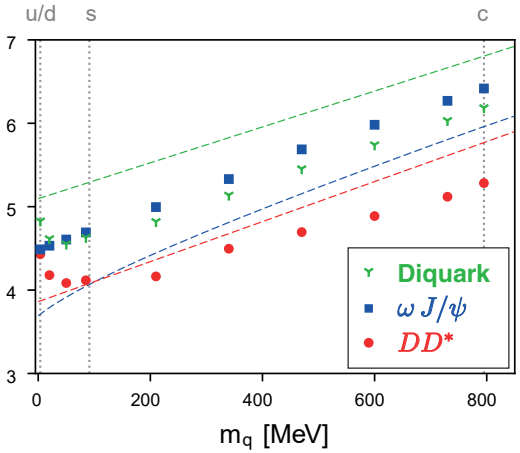
$cq\bar{q}\bar{c}$

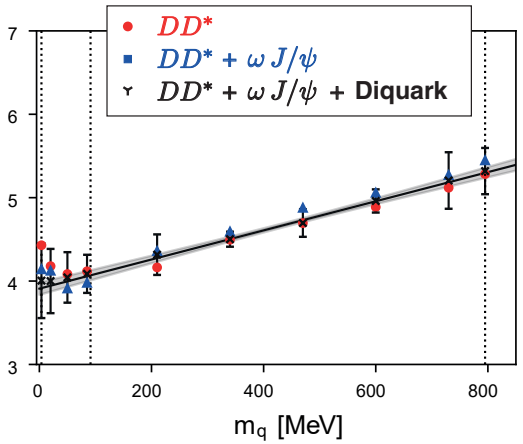


- **Four-quark BSE:** all mix together

$M_{cq\bar{q}\bar{c}}\,[\mathrm{GeV}]$ in X(3872) channel

$m_S + m_A$
$m_\omega + m_{J/\psi}$
$m_D + m_{D^*}$



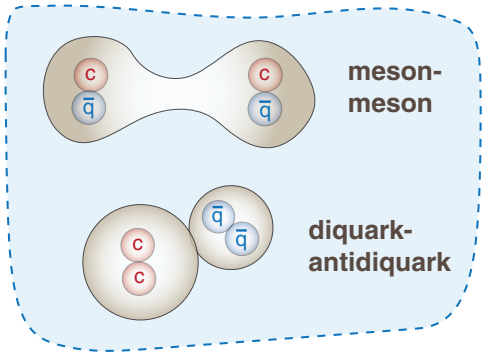


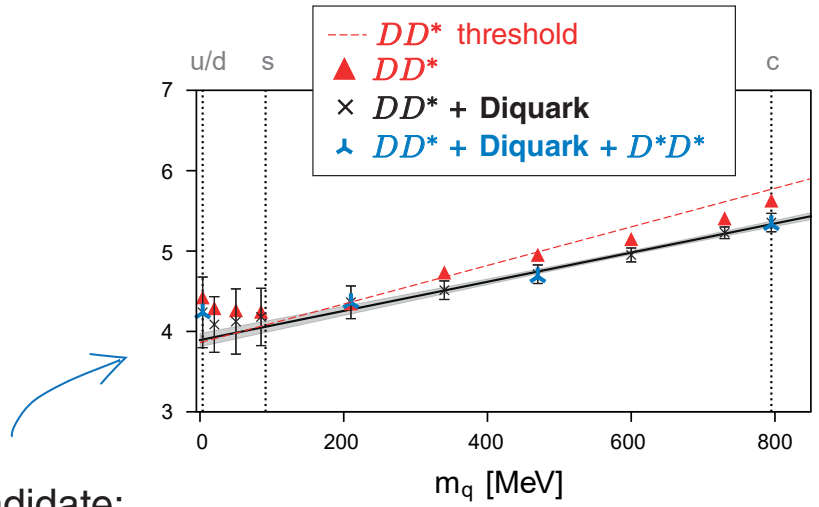
Wallbott, GE, Fischer, PRD 100 (2019), PRD 102 (2020)

$cq\bar{q}\bar{c}$ → strong meson-meson component: DD* for X(3872), Zc(3900)

Four-quark states

- **Open-charm states:** $cc\bar{q}\bar{q}$





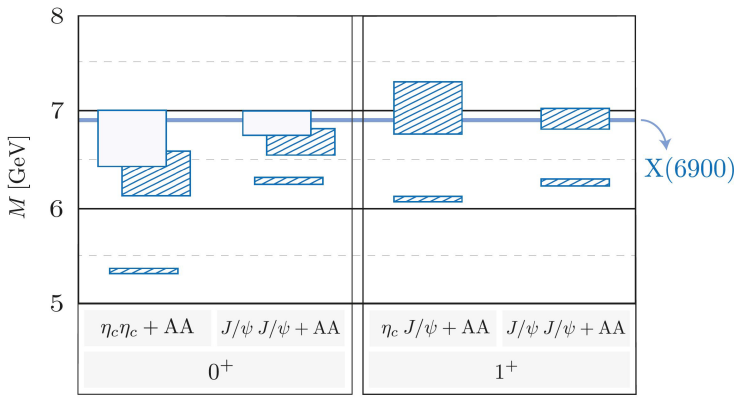
Wallbott, GE, Fischer, PRD 102 (2020)

Experimental candidate:
T_{cc}^{+}, $0(1^{+})$, 3875 MeV

- **All-charm state:** $cc\bar{c}\bar{c}$
  X(6900)

  Results so far available in two-body approach, 1st radial excitation?

  Santowsky, Fischer, EPJC 82 (2022)

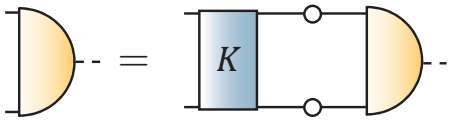


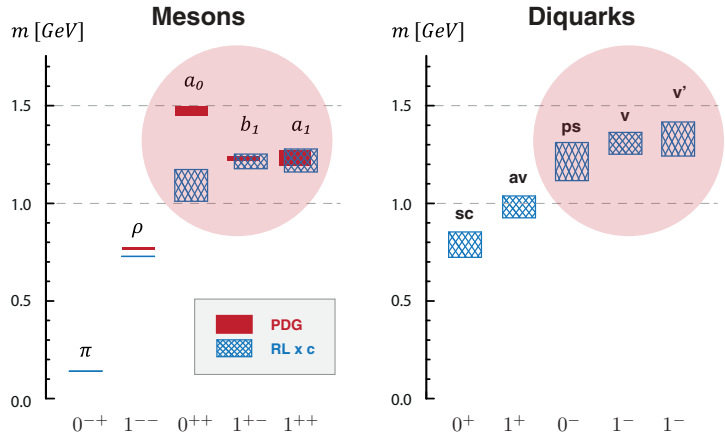
meson-meson dominated

significant diquark admixture

Diquark correlations

Mesons and diquarks closely related through BSE
Maris, FBS 32 (2002)





Lowest-lying diquarks are dominant for ground-state octet & decuplet baryons

pseudoscalar mesons	⇔	**scalar diquarks** (~0.8 GeV)
vector mesons	⇔	**axialvector diquarks** (~1 GeV)

Higher-lying diquarks are subleading, but contribute to excited states & remaining channels

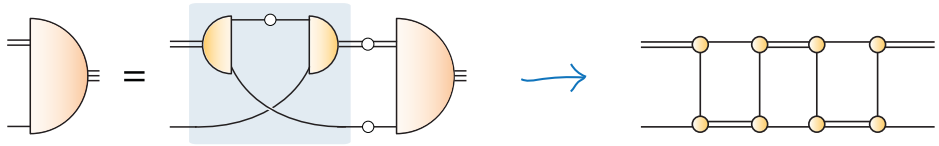
scalar mesons	⇔	**pseudoscalar diquarks** (~1.2 GeV)
axialvector mesons	⇔	**vector diquarks** (~1.3 GeV)

In RL, these are too strongly bound; simulate beyond-RL effects by (one) strength parameter c

Roberts, Chang, Cloet, Roberts, FBS 51 (2011)
GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)

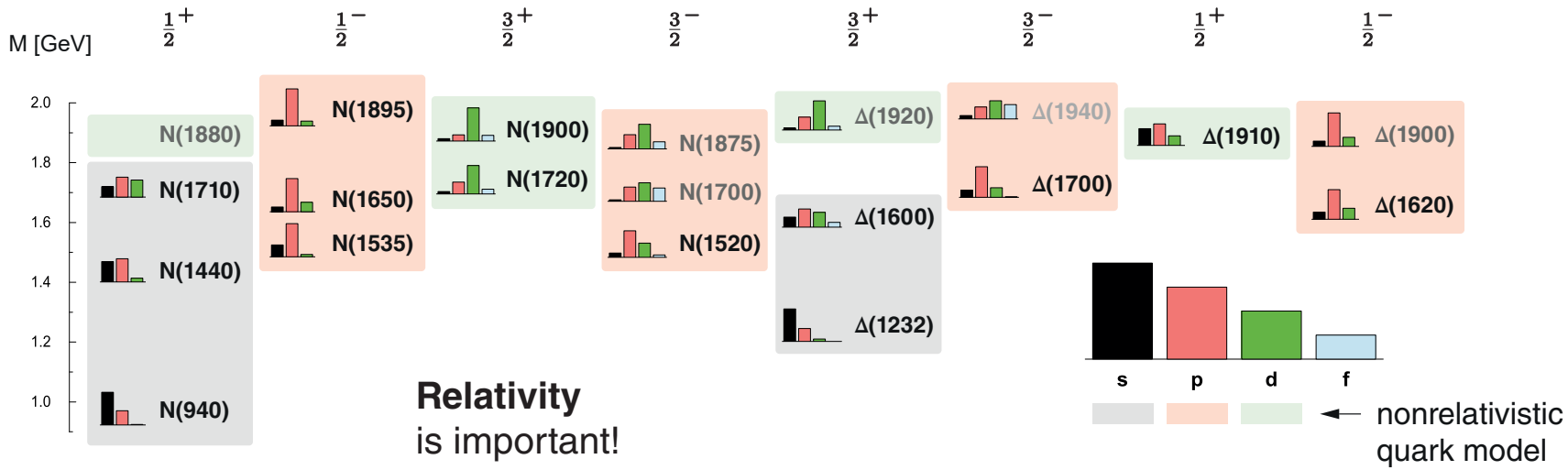
Diquark correlations

- **Quark-diquark** (two-body) equation
  Oettel et al., PRC 58 (1998), GE et al., Ann. Phys. 323 (2008), Cloet et al., FBS 46 (2009), Segovia et al., PRL 115 (2015), Chen et al., PRD 97 (2018)



Diquark clustering in baryons?
Barabanov et al., Prog. Part. Nucl. Phys. 116 (2021)

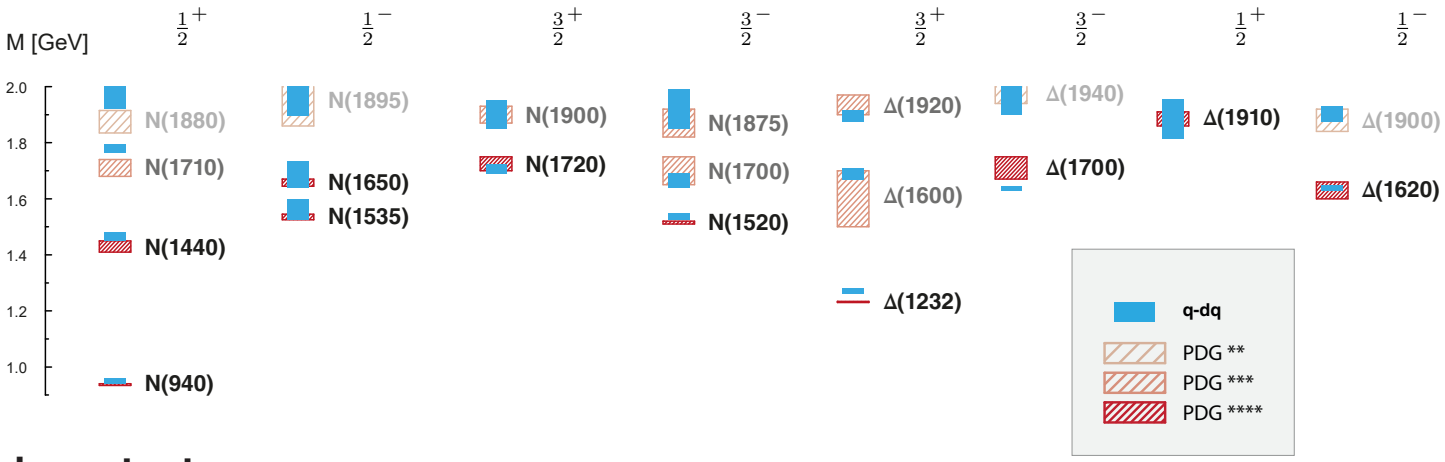
- **Three-quark** and **quark-diquark** results very similar
  GE, Fischer, Sanchis-Alepuz, PRD 94 (2016), GE, FBS 63 (2022)



Diquark correlations

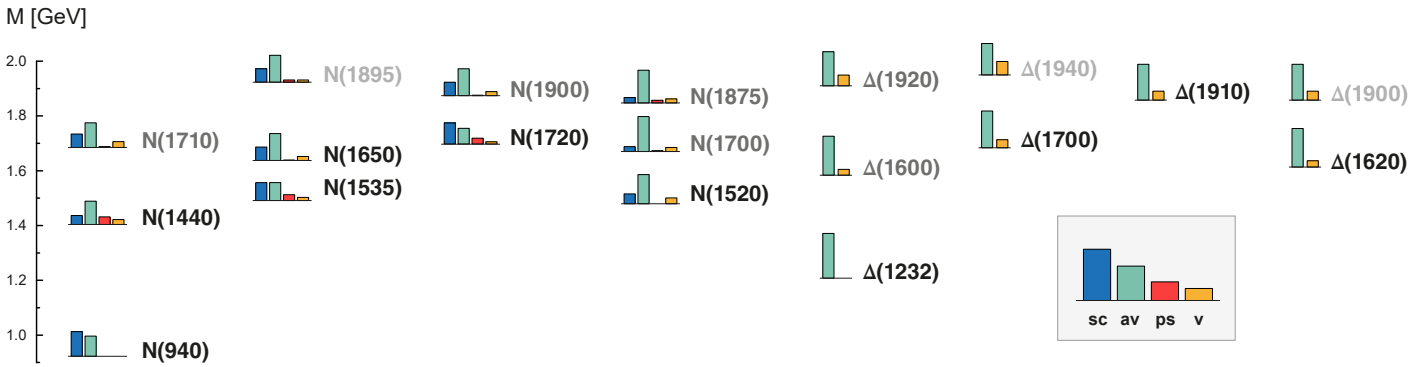
Light baryon spectrum

GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)



Diquark content:

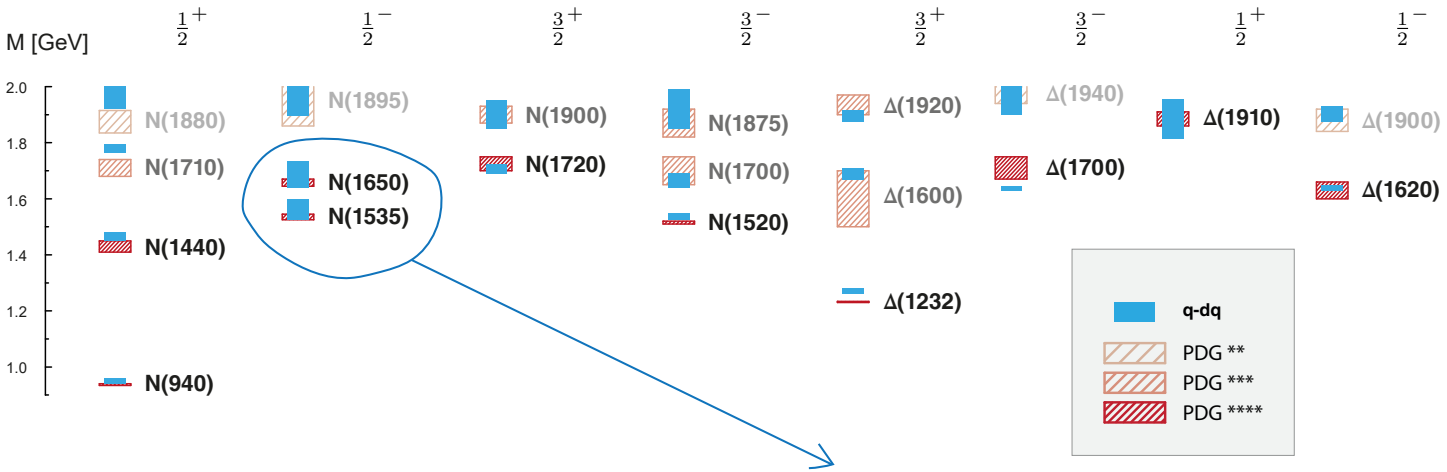
Barabanov et al., PPNP 116 (2021)



Diquark correlations

Light baryon spectrum

GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)



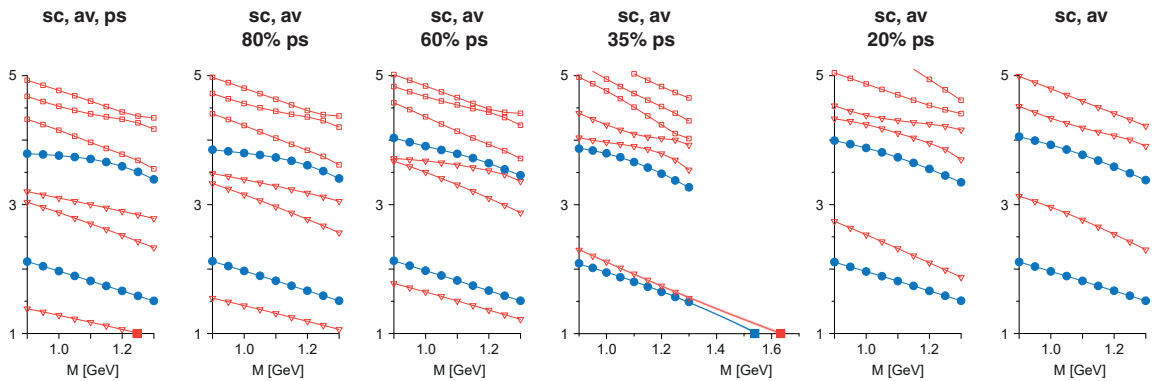
RL, all diquarks:
"N(1535)" too low

"Beyond RL":
N(1535), N(1650)

RL, sc+av only:
"N(1650)" too high

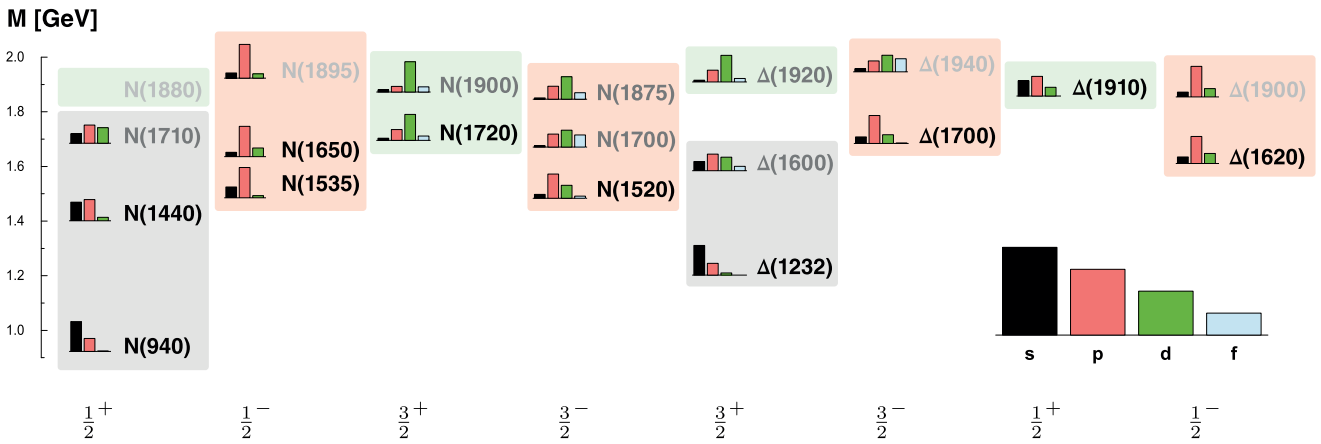
- **Level ordering** determined by diquark dynamics
- Diquarks are not pointlike, also here **rich spectrum!**

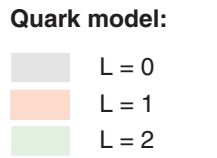
Barabanov et al., PPNP 116 (2021)



Relativistic effects

Orbital angular momentum: clear traces of nonrelativistic quark model, but strong relativistic effects (in some cases even dominant)

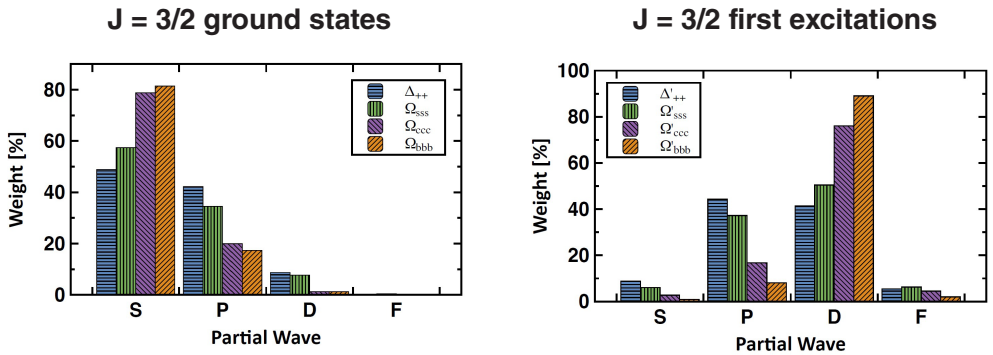




GE, FBS 58 (2017)

Relativistic contributions
even up to bottom baryons!

Qin, Roberts, Schmidt, PRD 97 (2018)



Towards ab-initio

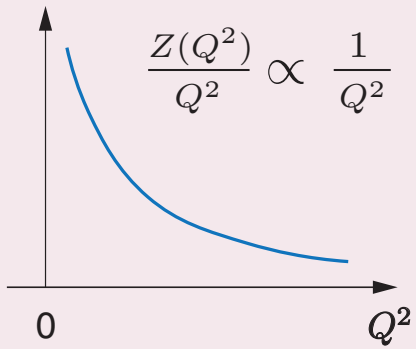
Gluon propagator:

transverse dressing

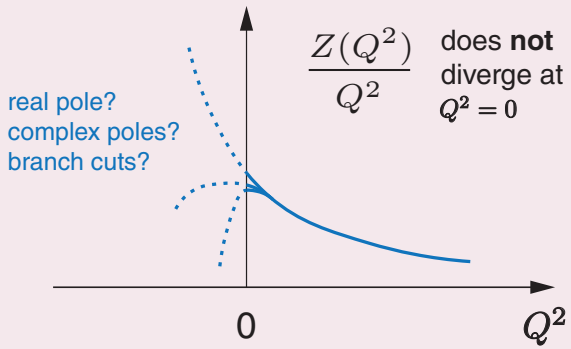
longitudinal dressing = 1

$$D^{\mu\nu}(Q) = \frac{Z(Q^2)}{Q^2}\left(\delta^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2}\right) + \xi\,\frac{L(Q^2)}{Q^2}\,\frac{Q^\mu Q^\nu}{Q^2}$$

- **Perturbation theory:**
  Massless gluon pole



- **Nonperturbative calculations:**
  Massless pole disappears!



Family of **"decoupling"** solutions, also seen in lattice QCD

Cucchieri, Maas, Mendes, PRD 77 (2008)
Boucaud et al., JHEP 06 (2008)
Bogolubsky et al., PLB 676 (2009)
Fischer, Maas, Pawlowski, Ann. Phys. 324 (2009)
Duarte, Oliveira, Silva, PRD 94 (2016)
Aguilar et al., EPJ C 80 (2020)

Endpoint is **"scaling"** solution, confinement manifest

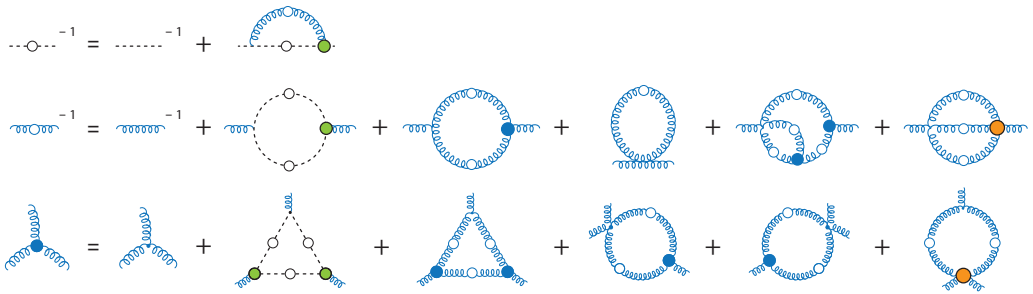
Lerche, Smekal, PRD 65 (2002)
Fischer, Alkofer, PLB 536 (2002)
Alkofer, Fischer, Llanes-Estrada, MPLA 23 (2008)

All solutions show **gluon mass gap**

$$\lim_{r\to\infty} \int \frac{d^3Q}{(2\pi)^3}\,\frac{Z(Q^2)}{Q^2}\,e^{i\,\boldsymbol{x}\cdot\boldsymbol{Q}} \propto e^{-m_{\rm gap}\,r}$$

Coupled **Yang-Mills DSEs**
GE, Pawlowski, Silva, PRD 104 (2021)

→ Test confinement
in hadron observables!

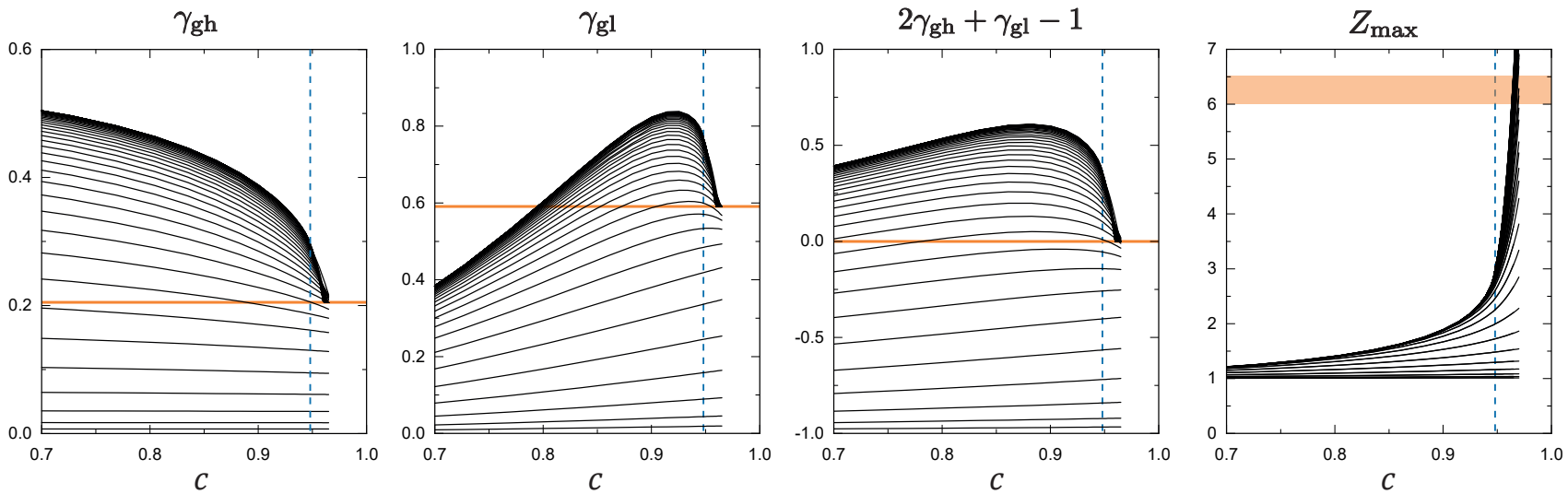


Truncation error

- Set $Z_{3g} \to c\, Z_{3g}$ ... quantifies deviation from STI (without truncation: $c = 1$), same effect from "over-renormalizing" 3-gluon vertex
- YM system only converges up to $c_{\max} < 1$
- Anomalous dimensions reproduced for
  - 1 c ~ 0.4
  - 2 c ~ 0.9
  - 3 c ~ 0.96

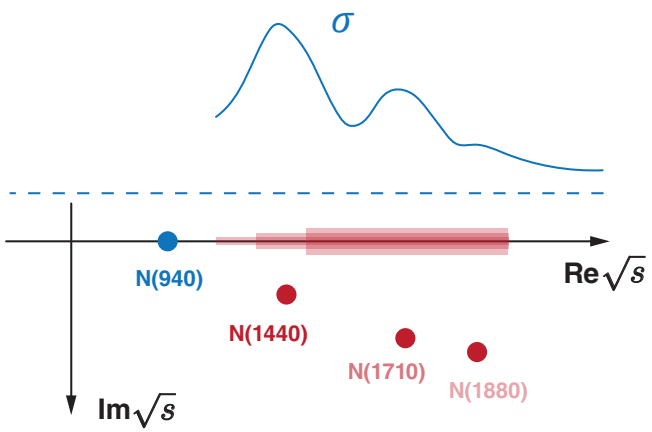
  $\Rightarrow$ identifies "physical point" for each truncation

  GE, Pawlowski, Silva, PRD 104 (2021)



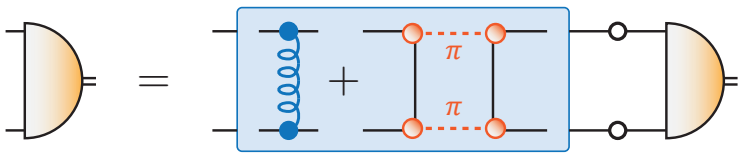
Resonances

- Most hadrons are **resonances** and decay ⇔ poles in complex momentum plane



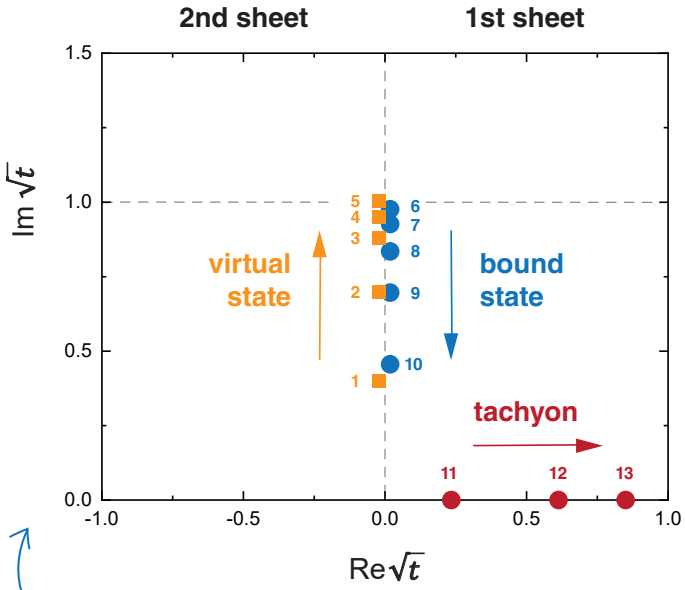
- BSE kernel must include decay channels: ρ meson becomes resonance

  Williams, PLB 798 (2019), Miramontes, Sanchis-Alepuz, EPJA 55 (2019), Santowsky, GE, Fischer, Wallbott, PRD 102 (2020), Miramontes, Sanchis-Alepuz, Alkofer, PRD 103 (2021)

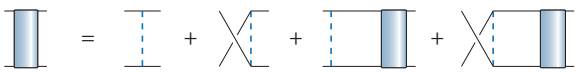


- **Contour deformations** as tool to go beyond thresholds

  GE, Duarte, Peña, Stadler, PRD 100 (2019)

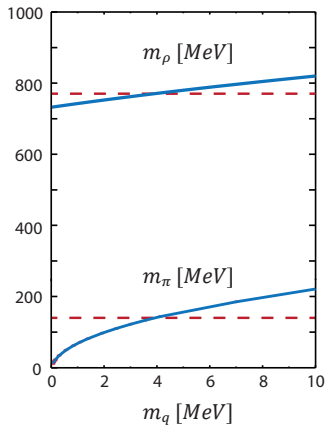


Scattering equation for 4-point function

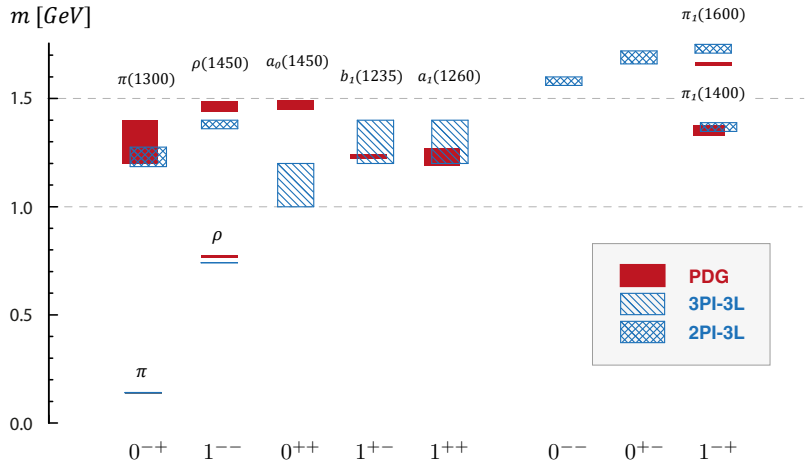


Mesons

- Pion is **Goldstone boson:** $m_\pi{}^2 \sim m_q$



- **Light meson spectrum** beyond rainbow-ladder

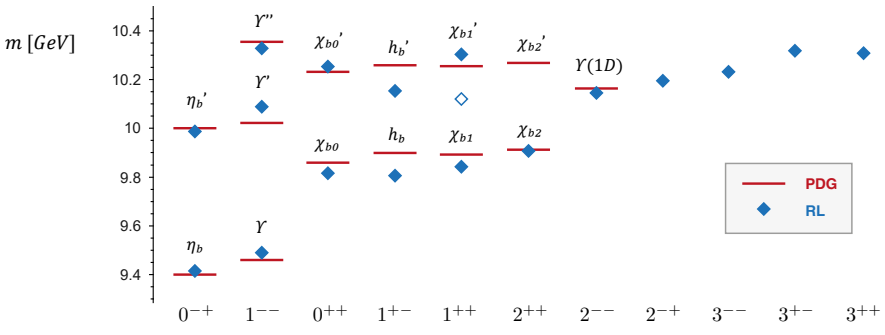


Williams, Fischer, Heupel, PRD 93 (2016)

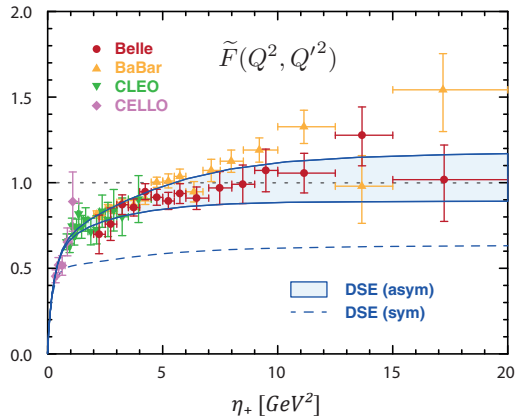
GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016)

- **Bottomonium spectrum**

Fischer, Kubrak, Williams, EPJ A 51 (2015)

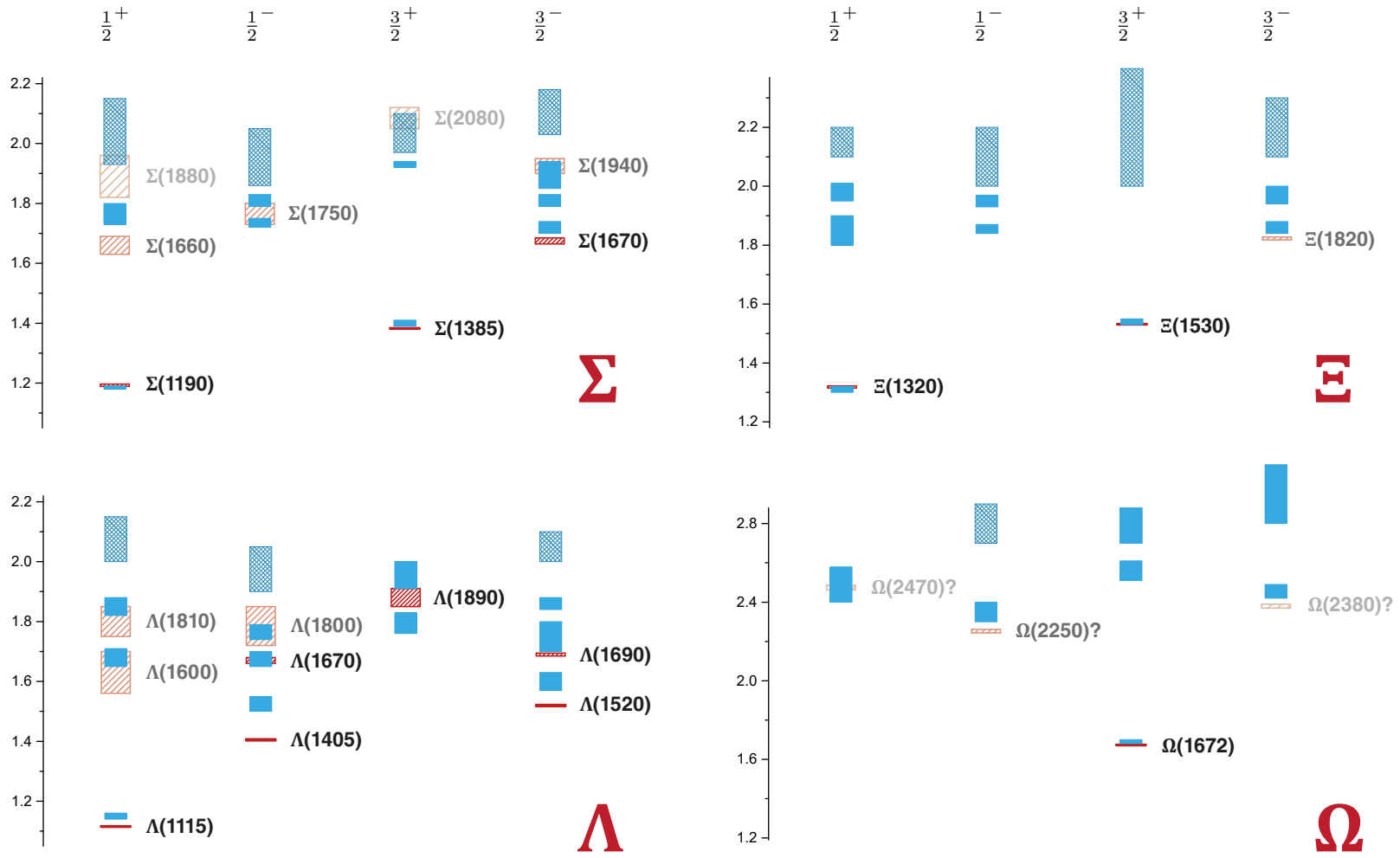


- **Pion transition form factor**



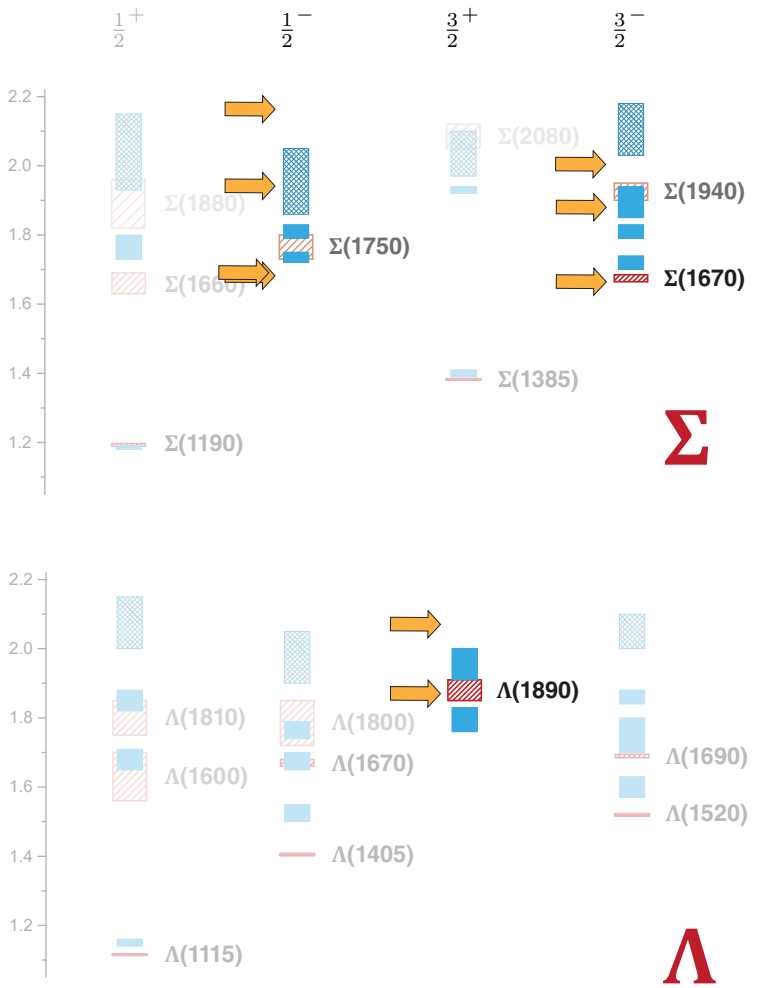
GE, Fischer, Weil, Williams, PLB 774 (2017)

Strange baryons



GE, Fischer, FBS 60 (2019), Fischer, GE, PoS Hadron 2017

Strange baryons



New states from Bonn-Gatchina

Sarantsev et al., 1907.13387 [nucl-ex]

GE, Fischer, FBS 60 (2019), Fischer, GE, PoS Hadron 2017