HADRON-HADRON SCATTERING LENGTHS FROM PRODUCTION REACTIONS

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MOTIVATION

A systematic study of final state interaction effects allows one to

- investigate the interaction of unstable particles for very small relative momenta.

  E.g. hyperon-nucleon and hyperon-hyperon scattering lengths, addressing

  $\rightarrow$ Flavor–$SU(3)$ breaking pattern?
  $\rightarrow$ Structure of neutron stars?

- nature of particles: bound system vs. elementary state

  $\rightarrow$ encoded in effective range parameters

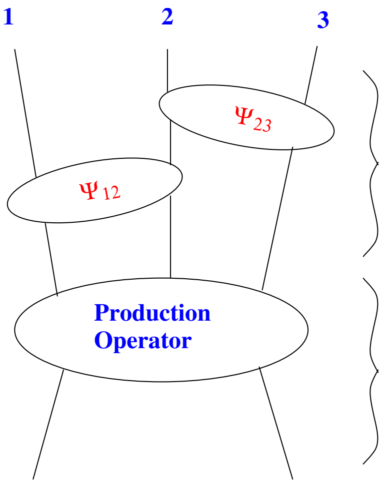
- study meson–nucleus interactions: Bound states or not?

DIFFERENT OPTIONS

Production from	
small momentum transfer (femtoscopy)	large momentum transfer (this talk)
e.g. heavy ion or *pp* collisions	e.g. meson production in *pp* coll.
weak dependence from production	sizeable dep. from production
uncertainty difficult to quantify	controllable uncertainties
spin states with known weights	admixture of spin states unknown

In any case: Two methods with very different systematics

GENERALITIES FOR *pp* INDUCED



Final state interaction

strongly energy dependent

sensitive to interactions of all subsystems; for more than 2 final particles: Dalitz plot analysis!

Production operator

weakly energy dependent;

selection rules!

(isospin, parity, Pauli principle ...)

$\rightarrow \; d\sigma \propto |fM|^2$, where $M \simeq const.$ and $f = f[\Psi_{ij}]$.

SCALES

Variation of M controlled by typical momentum transfer p_t

Variation of f relevant momentum range of subsystem p'

For $p' \ll p_t$ FSI is universal

Use initial momentum at threshold p for p_t; $1/a$ for p'

Examples: Meson production in

- pp: $|\vec{p}\,| = \sqrt{M_N m_x + m_x^2/4}$ (m_x = mass produced)

  e.g. $pp \to pK\Lambda \;\Rightarrow\; |\vec{p}\,| \sim 860$ MeV $\Rightarrow\; (p'/p)^2 \sim 0.05$

  $pd \to \eta^3\text{He} \;\Rightarrow\; |\vec{p}\,| \sim 870$ MeV $\Rightarrow\; (p'/p)^2 \sim 0.02$
- γA: $|\vec{p}\,| = (M_A m_x + m_x^2/2)/(M_A + m_x)$

  e.g. $\gamma^3\text{He} \to \eta^3\text{He} \;\Rightarrow\; |\vec{p}\,| \sim 500$ MeV $\Rightarrow\; (p'/p)^2 \sim 0.05$

Expansion parameter decreases as system produced gets heavier!

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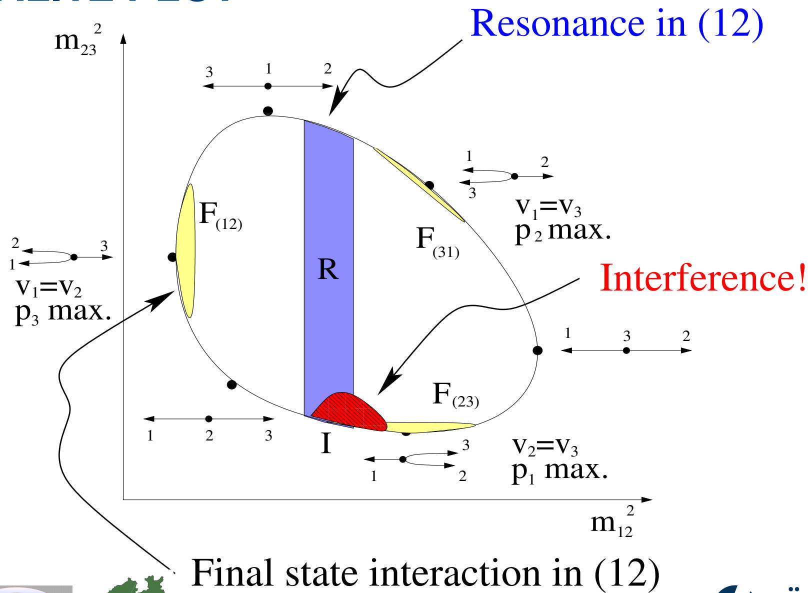
DALITZ PLOT



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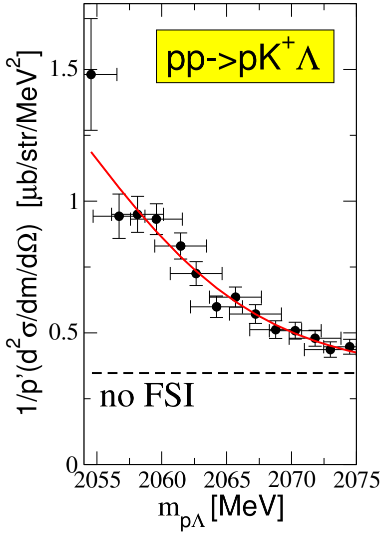
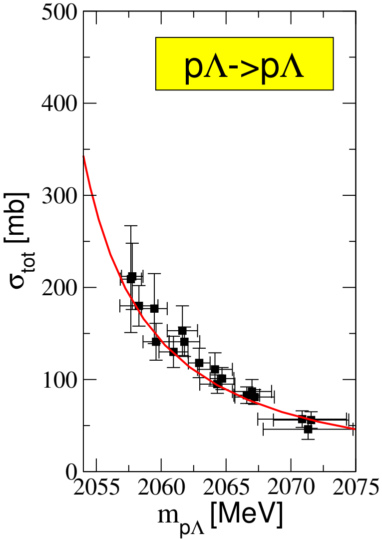
GENERALITIES II: PROD. VS. SCATTERING





R. Siebert et al. (1994); G. Alexander et al. (1968)

ELASTIC INTERACTIONS

We see that production and scattering are related, but not equal

Goal:

reliable extraction of scattering parameters for unstable particles
particles from production reactions with large momentum transfer

This demands:

- Error estimation
  - Residual effect of production operator
  - Effects of resonances
  - Effects of inelastic channels
- Proper theoretical framework

Use dispersion relations!

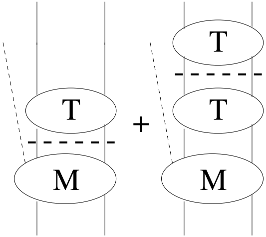
DISPERSION INTEGRAL I

$$A(s,t,m^2) = \underbrace{\frac{1}{\pi}\int_{-\infty}^{\tilde{m}^2} \frac{D(s,t,m'^2)}{m'^2 - m^2}\, dm'^2}_{left-hand\ cut: production} + \underbrace{\frac{1}{\pi}\int_{m_0^2}^{\infty} \frac{D(s,t,m'^2)}{m'^2 - m^2}\, dm'^2}_{right-hand\ cut: FSI},$$

where m_0 =production threshold, $\tilde{m}$=start of left-hand cut and

$D(s,t,m^2) = \mathrm{Disc}(A(s,t,m^2))$ with

$$D(s,t,m^2 > m_0^2) =$$



$$= i p' A T^{*}_{on-shell}$$

Muskhelishvili (1953), Omnes (1958), ...

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DISPERSION INTEGRAL II

For this equation a solution exists:

$$A(s,t,m^2) = \exp\left[\frac{1}{\pi}\int_{m_0^2}^{\infty} \frac{\delta(m'^{\,2})}{m'^{\,2} - m^2 - i0} dm'^{\,2}\right] \Phi(s,t,m^2),$$

- large momentum transfer $\rightarrow$ Φ is at most weakly m^2 dependent.

  $\Longrightarrow$ included into uncertainty estimate

- The FSI effect in terms of the (elastic) scattering phaseshift.

  The factor can be interpreted as wavevfunction at the origin (inverse Jost function).

  $\Longrightarrow$ large m' region into uncertainty estimate

FSI enhancement $\leftrightarrow$ elastic phase-shift

THREE STRATEGIES

Three different strategies to proceed:

1. Assume that phaseshifts are given by effective range expansion;
   $p' \operatorname{ctg}(\delta(m^2)) = 1/a + (1/2)rp'^{\,2}$ (sign!) $\Rightarrow$
   $$A(m^2) = \frac{(p'^{\,2} + \alpha^2)r/2}{1/a + (r/2)p'^{\,2} - ip'}\Phi(m^2)\ ,$$
   $\alpha = 1/r(1 + \sqrt{1 + 2r/a})$ (*Jost–function method*)
   Sibirtsev et al. (1996, 2004), Shyam et al. (2001), ...

2. Ignore numerator (*Watson method*) Goldberger, Watson 1964

3. Invert the equation and express phaseshifts through observables
   Geshkenbein (1969)
   and restrict integration range! (*Integral*) Gasparyan et al. (2004)ff

This is the most systematic approach in line with goal

SCATTERING LENGTH

It is possible to invert the Omnes-function: Geshkenbein (1969), Gasparyan et al. (2004)

$$\delta_S(m^2) = -\frac{1}{2\pi}\int_{m_0^2}^{m_{max}^2} \frac{dm'^2}{m'^2-m^2}\sqrt{\frac{(m_{max}^2-m^2)(m^2-m_0^2)}{(m_{max}^2-m'^2)(m'^2-m_0^2)}}\log\left\{\frac{1}{p'}\left(\frac{d^2\sigma_S}{dm'^2dt}\right)\right\}$$

with $\lim_{m^2\to m_0^2}\delta_S(s) = a_S p(s)$ and S denoting a specified spin state

we chose: $m_{max} - m_0 \simeq 1/(2\mu a^2)$

Estimates for uncertainties: $\delta a^{(th)} = \delta a^{(lhc)} + \delta a^{m_{max}}$ with

$$|\delta a^{m_{max}}| = \frac{2}{\pi p'_{max}}\left|\int_0^\infty \frac{\delta(y)dy}{(1+y^2)^{(3/2)}}\right| \le \frac{2}{\pi p'_{max}}|\delta_{max}| \sim 0.2 \text{ fm}$$

$$\delta a^{(lhc)} \sim (p'_{max}/p^2) \sim 0.05 \text{ fm}$$

using $\delta_{max} = 0.4$ rad (for ΛN - see next pages) and $p' \sim 1/a$

TESTING THE METHOD I

Gasparyan, Haidenbauer, CH PRC72(2005)034006

We want to test the three methods (Integral, Jost, Watson);

Strategy:

- Produce $d\sigma/dm^2$ for various models for ΛN (and NN).
- Extract scattering length.
- Compare to exact value.

Note: any working method should work for any realistic model

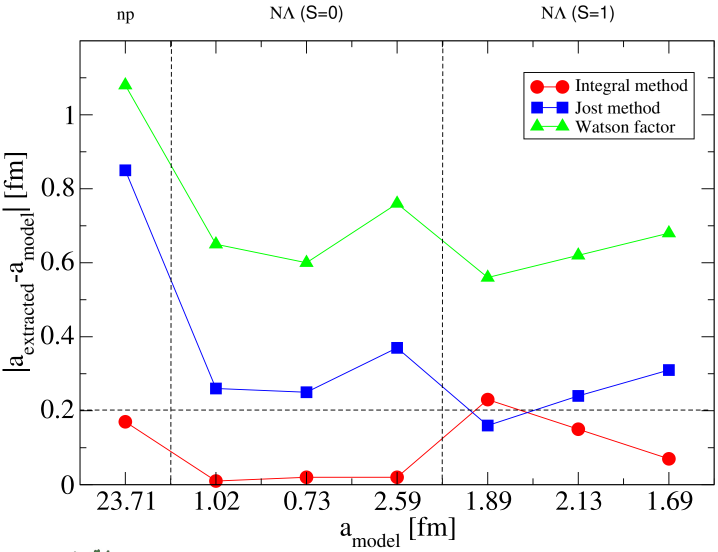
We use $S = 0$ & $S = 1$ for YN from (where in green: scattering lengths in fm)

NSC97a (0.73,2.13), NSC97f (2.59,1.69), Jülich ('94) (1.02,-1.89)

and the Argonne potential for pn with $S = 0$ (-23.71).

Perform calculations for pointlike production operator!

TESTING THE METHOD II



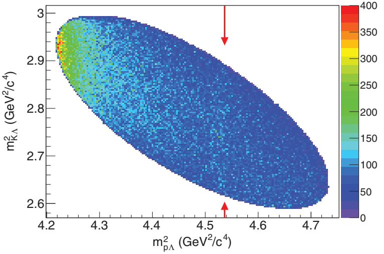
SPIN TRIPLET $a_{\Lambda p}$ FROM $\vec{p}p \to pK\Lambda$

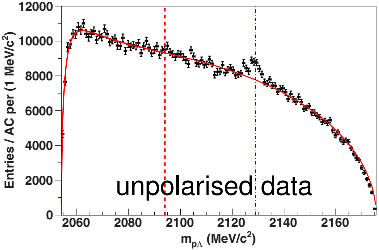
F. Hauenstein et al., PRC95(2017)034001

S=1 and S=0 possible in final state with unknown relative weight

However, S=1 can be isolated from analysing power

Gasparyan, Haidenbauer, CH PRC72(2005)034006





Procedure: Fit $m_{p\Lambda}$ spectrum with $\exp\{C_0 + C_1/(m^2_{p\Lambda} - C_2)\}$, then

$$a_t = \frac{C_1}{2}\sqrt{\frac{m_0^2(m^2_{max} - m_0^2)}{m_p m_\Lambda (m^2_{max} - C_2)(m_0^2 - C_2)^3}} \Longrightarrow 2.55^{+0.72}_{-1.39}\,_{\text{stat.}} \pm 0.6_{\text{syst.}} \pm 0.3_{\text{theo.}}\ \text{fm}$$

OPPORTUNITIES WITH PROTONS AT SIS100

The high initial energy ($\sqrt{s_{\max}} = 7.5$ GeV) promises access to

- $pp \to ppJ/\psi$ and the pJ/ψ interaction $\sqrt{s} > 5$ GeV

  $\Longrightarrow$ discovery channel of $\bar{c}c$ pentaquarks & role of $\Lambda_c D^{(*)}$ channels

- $pp \to p\Sigma_c^{(*)}\bar{D}^{(*)}$ and the $\Sigma_c^{(*)}\bar{D}^{(*)}$ interaction $\sqrt{s} > 5.6$ GeV

  $\Longrightarrow$ formation of $\bar{c}c$ pentaquarks

- $pp \to \bar{K}^0\bar{K}^0\Sigma^+\Sigma^+$ and the $\Sigma^+\Sigma^+$ interaction (S=0 only!) $\sqrt{s} > 3.4$ GeV

  $\Longrightarrow$ closely SU(3) related to pp scattering

- .... certainly many more

Note: Measurements need to be well above threshold, but not too high ...

Challenge: Needs high resolution for small relative momenta and
high statistics

SUMMARY

For elastic interactions dispersion integrals allow one to connect

scattering data to FSI effects.

Method allows for error estimates.

Important to keep in mind:

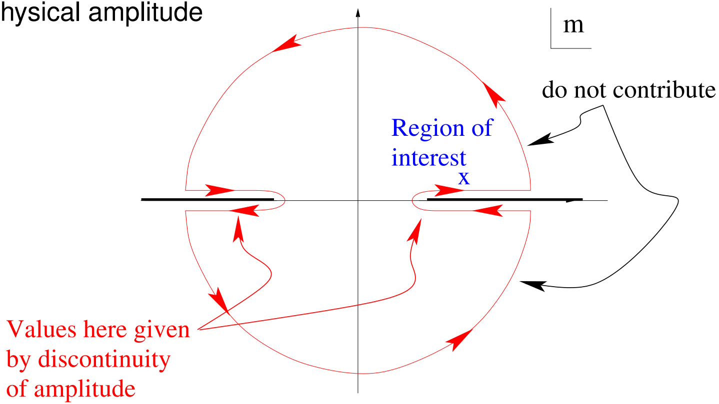
- Dalitz plot required to control crossed channels.
- employ polarization observables to project on spin states;
  e.g. in case of $pp \to pK\Lambda$ and $\gamma d \to K^+\Lambda n$ single spin observables sufficient to isolate spin triplet.
- repulsive Coulomb interaction can be accounted for
  allows a study of $\Sigma^+\Sigma^+$ channel

Back-up slides

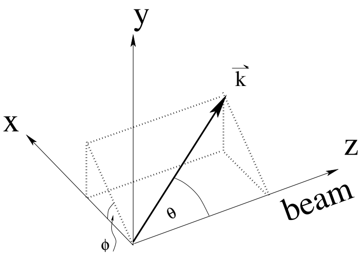
INSERTION: DISPERSION THEORY

Cauchys Theorem states: $f(z) = \frac{1}{2\pi i} \oint_C dz' \frac{f(z')}{z'-z}$

For a physical amplitude



POLARIZATION OBSERVABLES



$$\sigma(\xi, \vec{P}_b, \vec{P}_t, \vec{P}_f) = \sigma_0(\xi) \times \left[1 + \sum_i ((P_b)_i A_{i0}(\xi) + (P_f)_i D_{0i}(\xi)) + \sum_{ij} (P_b)_i (P_t)_j A_{ij}(\xi) + \ldots\right]$$

$$A_{0y}\sigma_0 = -\frac{1}{4} k^2 \beta \sin(2\theta)\cos(\phi) + \sin(\theta)\cos(\phi)(\text{spin triplet only}) ,$$

exploiting θ–dependence of A_{0y} allows one to isolate S=1!