Hadronic interaction studies in three-body systems using femtoscopy at the LHC

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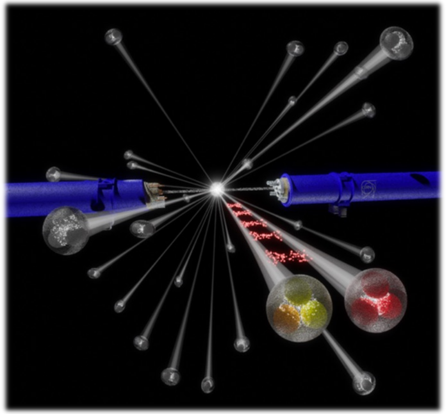

7th February 2024, Wuppertal University, Wuppertal, Germany

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Hadronic interactions in many-body systems

- Properties of nuclei and hypernuclei cannot be described satisfactorily with two-body forces only
  H. Hammer, A. Nogga, A. Schwenk, RMP 85, 197 (2013)
  L.E. Marcucci et al., Front. Phys. 8:69 (2020)

- NNN interaction contributes ~10% to the binding energies of ^{3}H and ^{4}He

- Many-body scattering requires three-body calculations (e.g. **neutron-deuteron**)
  L. Girlanda et al., PRC 102, 064003 (2020)
  E. Epelbaum et al., PRC 99, 024313 (2019)

^{3}H and ^{4}He binding energies and n-d scattering length

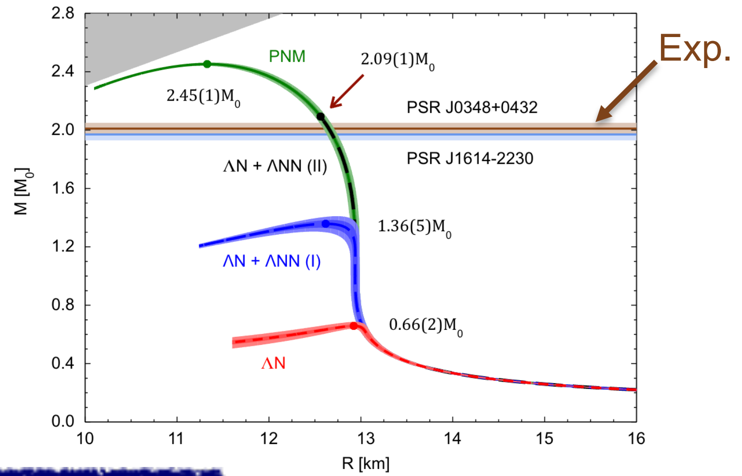
Potential(NN)	^{3}H[MeV]	^{4}He[MeV]	$^2a_{nd}$[fm]
AV18	7.624	24.22	1.258
CDBonn	7.998	26.13	
N3LO-Idaho	7.854	25.38	1.100
Potential(NN+NNN)			
AV18/UIX	8.479	28.47	0.590
CDBonn/TM	8.474	29.00	
N3LO-Idaho/N2LO	8.474	28.37	0.675
Exp.	8.48	28.30	0.645±0.010

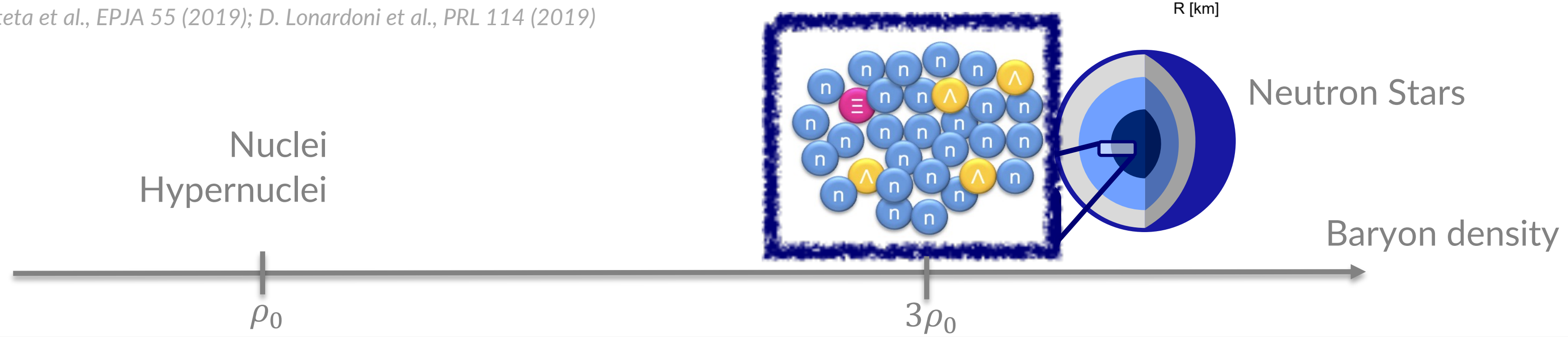
L.E. Marcucci et al., Front. Phys. 8:69 (2020)

Hadronic interactions in many-body systems

- Production of hyperons energetically favourable in neutron stars (NS) around 2-3 ρ_0
  L. Tolos, L. Fabbietti, PPNP 112 (2020) 103770
- Only two-body ΛN
  → Too soft equation of state (EoS)
- Introduction of three-body ΛNN forces
  → Stiffens EoS, model-dependent
  → Need for additional experimental constraints
  D. Logoteta et al., EPJA 55 (2019); D. Lonardoni et al., PRL 114 (2019)

D. Lonardoni et al., PRL 114 (2019)

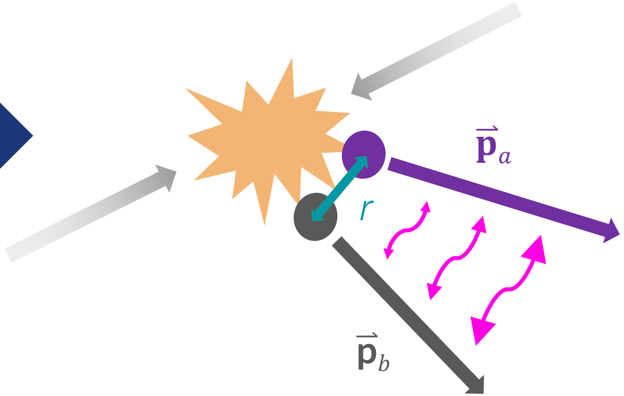




The femtoscopy technique at the LHC

ALICE at the LHC

Hadronic interaction

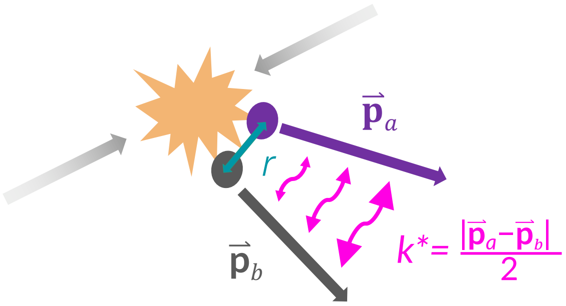


Femtoscopy technique

Correlation function

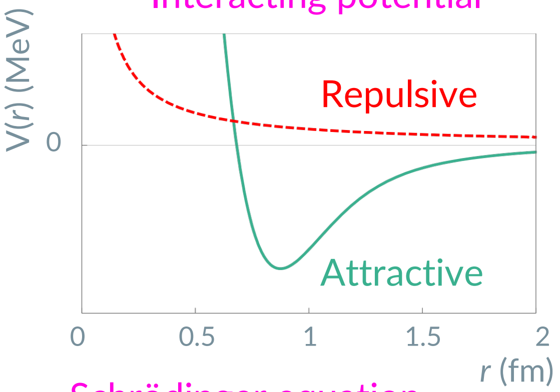
$$C(\vec{\mathbf{p}}_a, \vec{\mathbf{p}}_b) \equiv \frac{P(\vec{\mathbf{p}}_a, \vec{\mathbf{p}}_b)}{P(\vec{\mathbf{p}}_a)\, P(\vec{\mathbf{p}}_b)}$$

The femtoscopy technique at the LHC

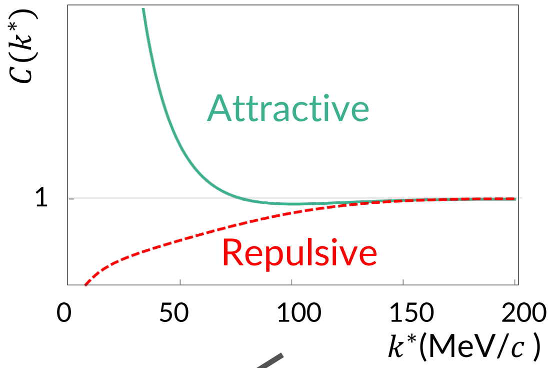


Emission source $S(\vec{r}\,)$

Interacting potential



Correlation function



Schrödinger equation

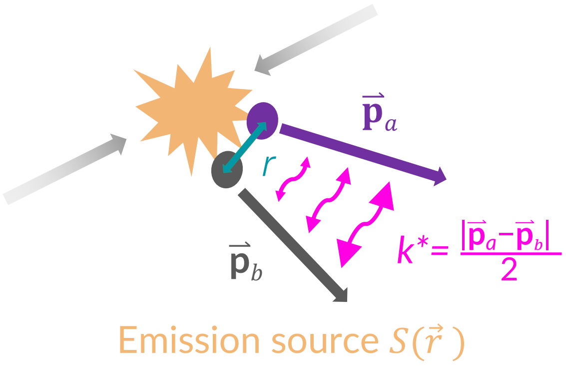
CATS Framework: D. Mihaylov et al., Eur. Phys. J. C78 (2018) 394

Two-particle wave function

$$C(k^*) = \int S(\vec{r}\,) \left|\psi(\vec{k}^*, \vec{r}\,)\right|^2 d^3\vec{r} \; = \mathcal{N}(k^*) \frac{N_{\mathrm{same}}(k^*)}{N_{\mathrm{mixed}}(k^*)}$$

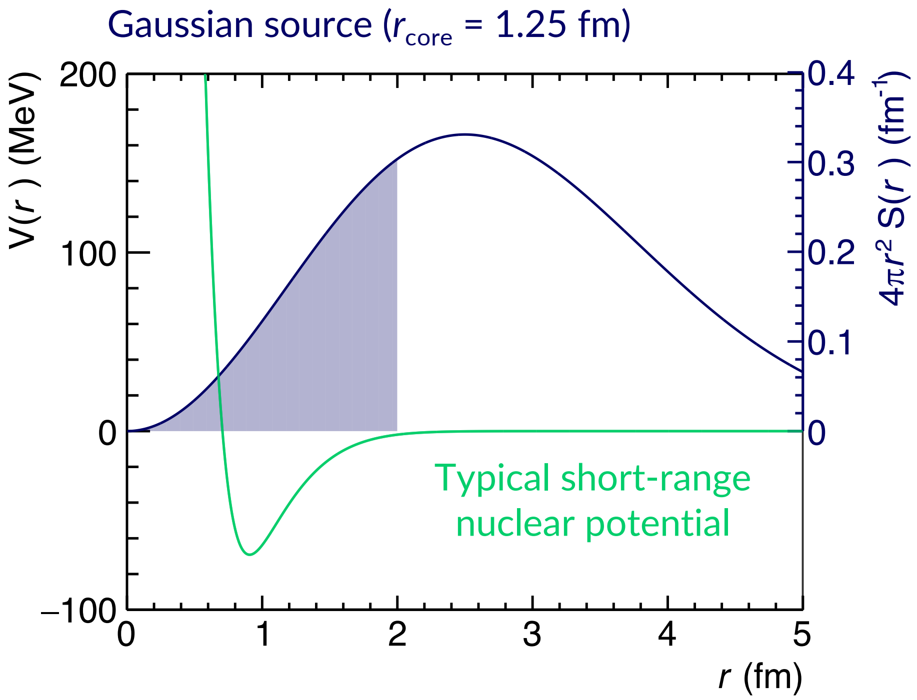
Measuring $C(k^*)$, fixing the source $S(\vec{r}\,)$, study the interaction

The femtoscopy technique at the LHC



Small particle-emitting source created in pp and p–Pb collisions at the LHC:

- **Essential ingredient for detailed studies of the strong interaction**
- Source function determined using p-p correlation functions



Proton-deuteron correlations

- Point-like particle models anchored to scattering experiments

	$S = 1/2$		$S = 3/2$	
	f_0(fm)	d_0(fm)	f_0(fm)	d_0(fm)
Van Oers et al.(1967)	$-1.30^{+0.20}_{-0.20}$	—	$-11.40^{+1.20}_{-1.80}$	$2.05^{+0.25}_{-0.25}$
Arvieux (1973)	$-2.73^{+0.10}_{-0.10}$	$2.27^{+0.12}_{-0.12}$	$-11.88^{+0.10}_{-0.40}$	$2.63^{+0.01}_{-0.02}$
Huttel et al. (1983)	-4.0	—	-11.1	—
Kievsky et al. (1997)	-0.024	—	-13.7	—
Black et al. (1999)	$0.13^{+0.04}_{-0.04}$	—	$-14.70^{+2.30}_{-2.30}$	—

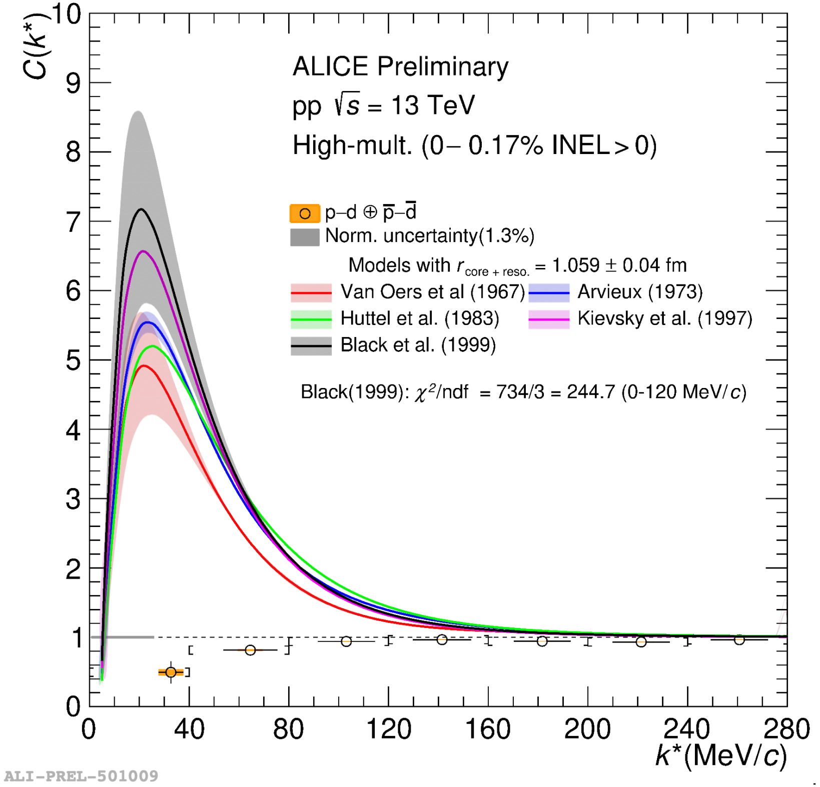
W. T. H. Van Oers, & K. W. Brockman Jr, NPA 561 (1967);
J. Arvieux et al., NPA 221 (1973); E. Huttel et al., NPA 406 (1983);
A. Kievsky et al., PLB 406 (1997); T. C. Black et al., PLB 471 (1999);

- Coulomb + strong interaction using the Lednický model

Lednický, R. Phys. Part. Nuclei 40, 307–352 (2009)

- Only s-wave interaction
- Source radius evaluated using the hadron-hadron universal mT scaling

Point-like particle description doesn't work for p-d

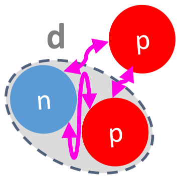


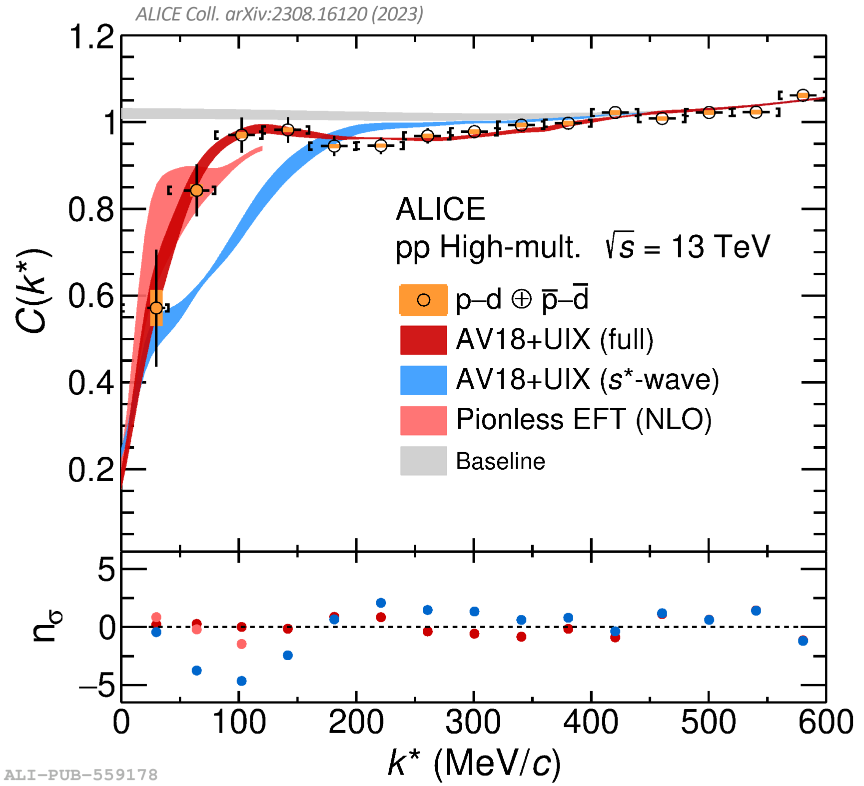
Proton-deuteron correlation

The measured p-d correlation function reflects the full three-nucleon dynamics:

Coulomb + strong interaction (NN and NNN) + Quantum Statistics

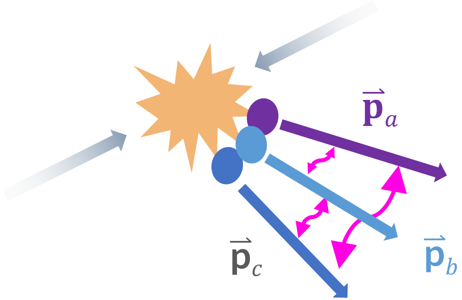
M. Viviani et al., PRC 108 (2023) 6, 064002





- Sensitivity to the short inter-particle distances
- Hadron-nuclei correlations at the LHC can be used to study many-body dynamics

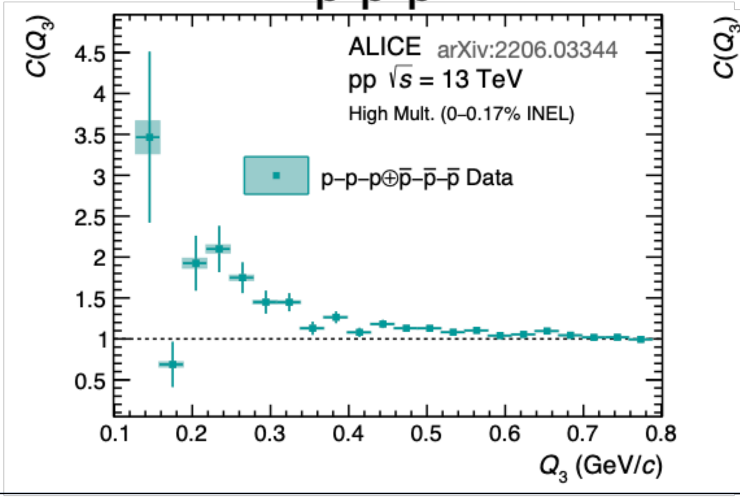
Three-particle femtoscopy



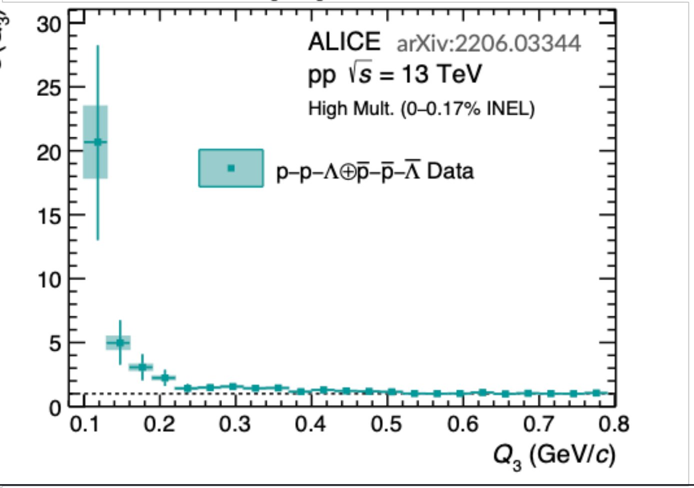
$$C(\vec{p}_a, \vec{p}_b, \vec{p}_c) = \int S(\vec{x}_a, \vec{x}_b, \vec{x}_c) \left| \psi_{\vec{p}_a, \vec{p}_b, \vec{p}_c}(\vec{x}_a, \vec{x}_b, \vec{x}_c) \right|^2 d^3\vec{x}_a \, d^3\vec{x}_b \, d^3\vec{x}_c$$

Three-body scattering wave function

p-p-p ALICE Coll., EPJ A 59, 145 (2023)



p-p-Λ



Interpretation of the measurements

- Step 0: Cumulant expansion method, to answer the question:
  "Is there any genuine three-body effect in the measured correlation function?"

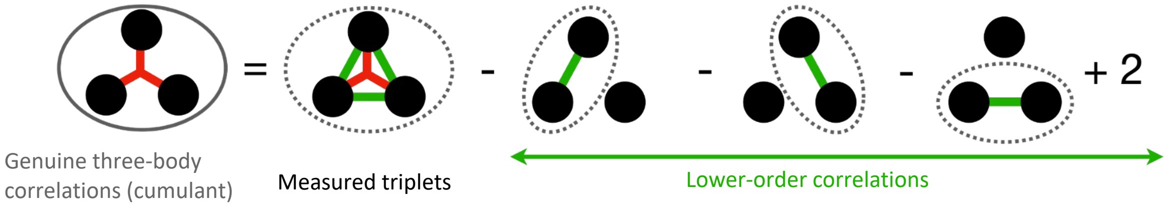
- Step 1: Calculate the three-particle scattering wave function and compute the correlation function

Step 0: Cumulants

Cumulants in femtoscopy

Genuine three-body effects can be isolated using the Kubo’s cumulant expansion method

J. Phys. Soc. Jpn. 17, pp. 1100-1120 (1962)



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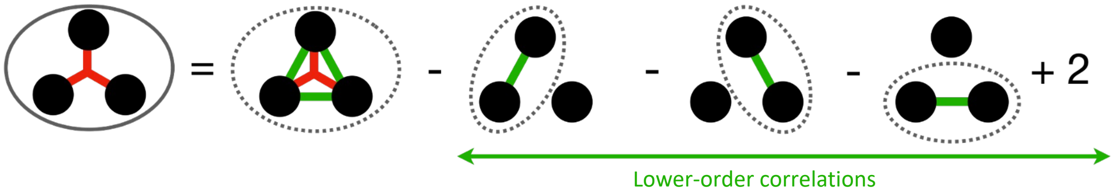
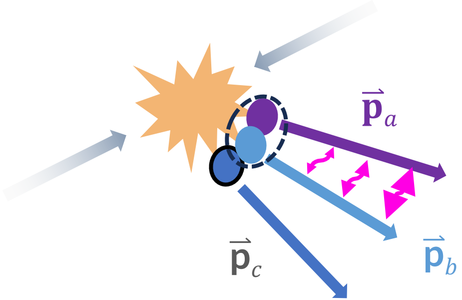


Data-driven method:

One particle from another collision



Projector method:

Each term of the lower-order correlations can be calculated using two-particle correlation functions *R. Del Grande, L. Serksnyte et al, EPJC 82 (2022)*

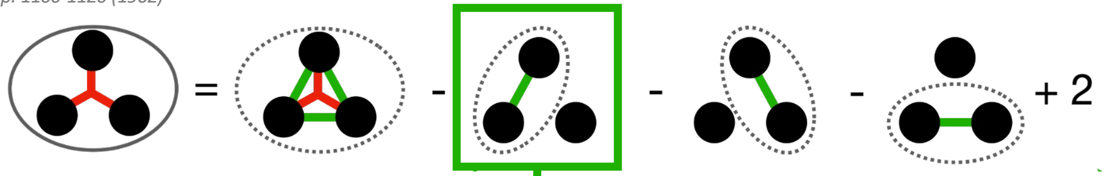
$$\psi_{\vec{p}_a,\vec{p}_b,\vec{p}_c}(\vec{x}_a,\vec{x}_b,\vec{x}_c) = \psi_{\vec{p}_a,\vec{p}_b}(\vec{x}_a,\vec{x}_b)\, e^{i\,\vec{p}_c\cdot\vec{x}_c}$$

Correlated pair — Uncorrelated particle

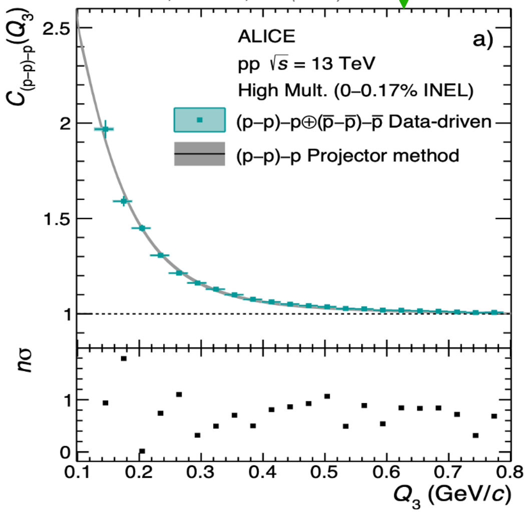
Cumulants in femtoscopy

Genuine three-body effects can be isolated using the Kubo's cumulant expansion method

J. Phys. Soc. Jpn. 17, pp. 1100-1120 (1962)



ALICE Coll., EPJ A 59, 145 (2023)



Cumulants in femtoscopy

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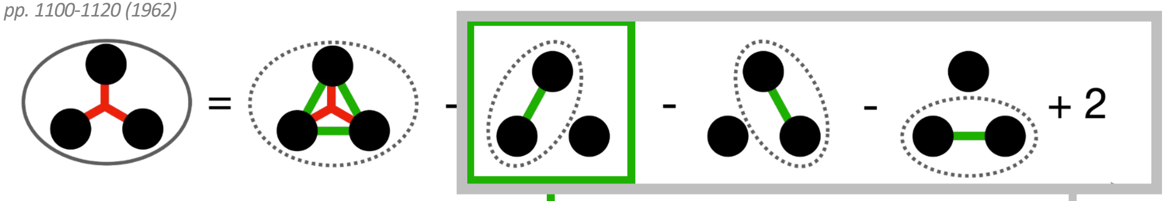

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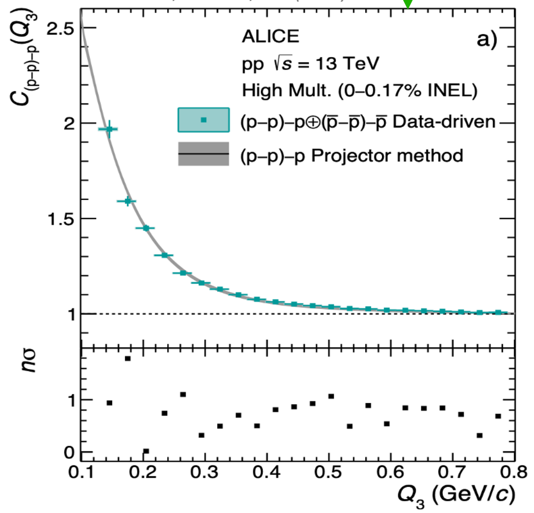
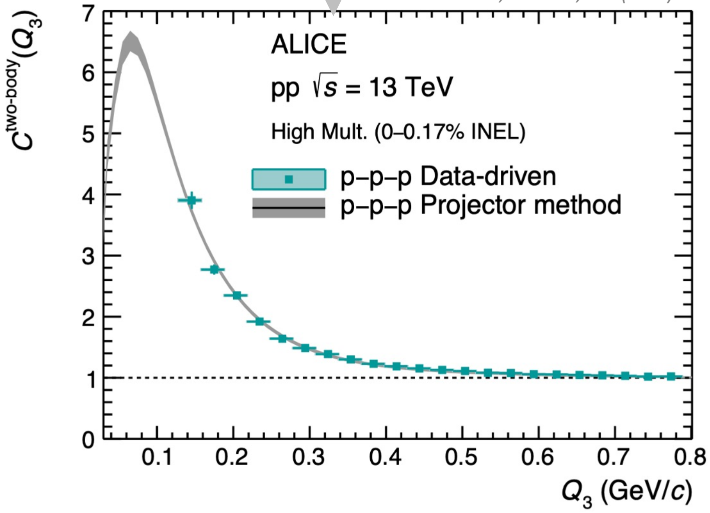
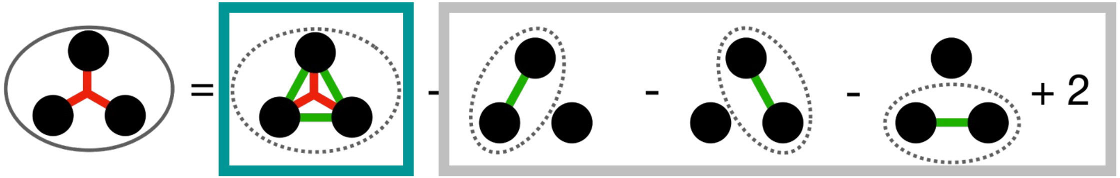
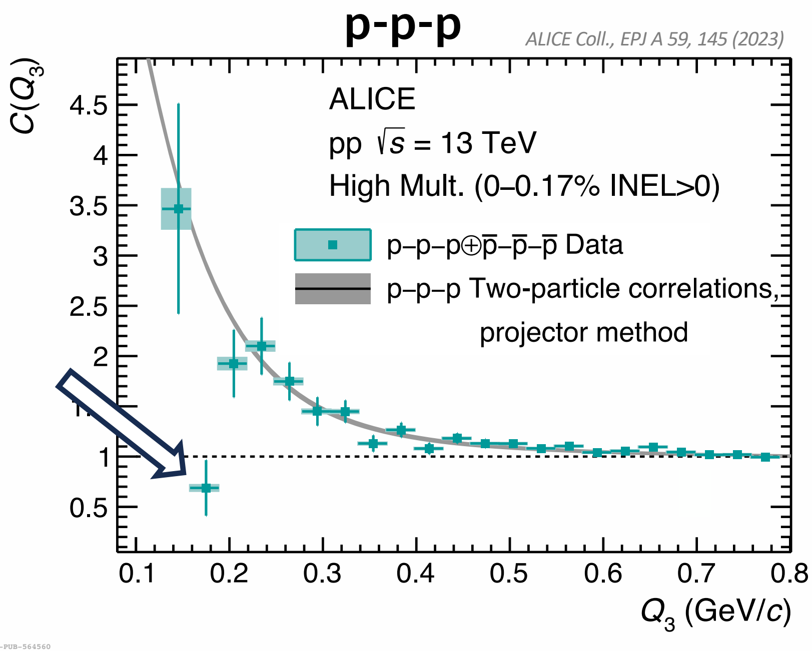


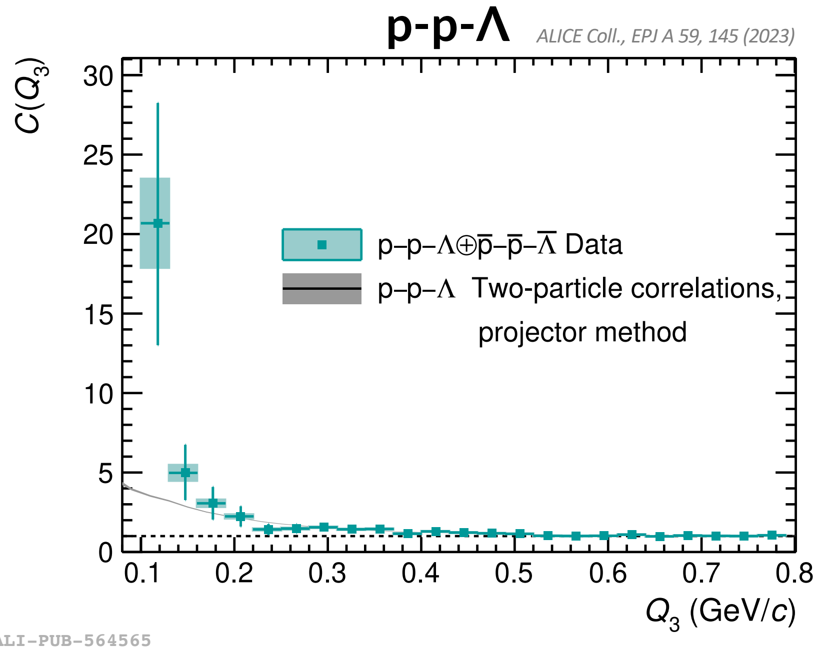
ALICE Coll., EPJ A 59, 145 (2023)



p-p-p and p-p-Λ correlations







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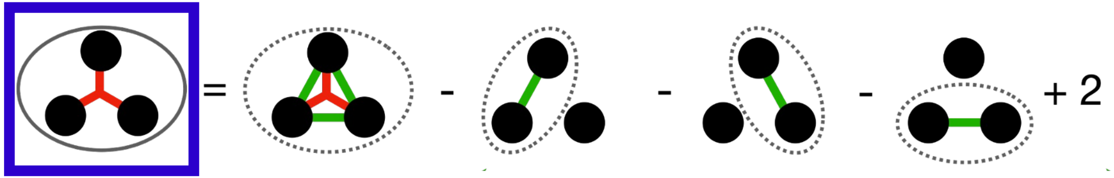
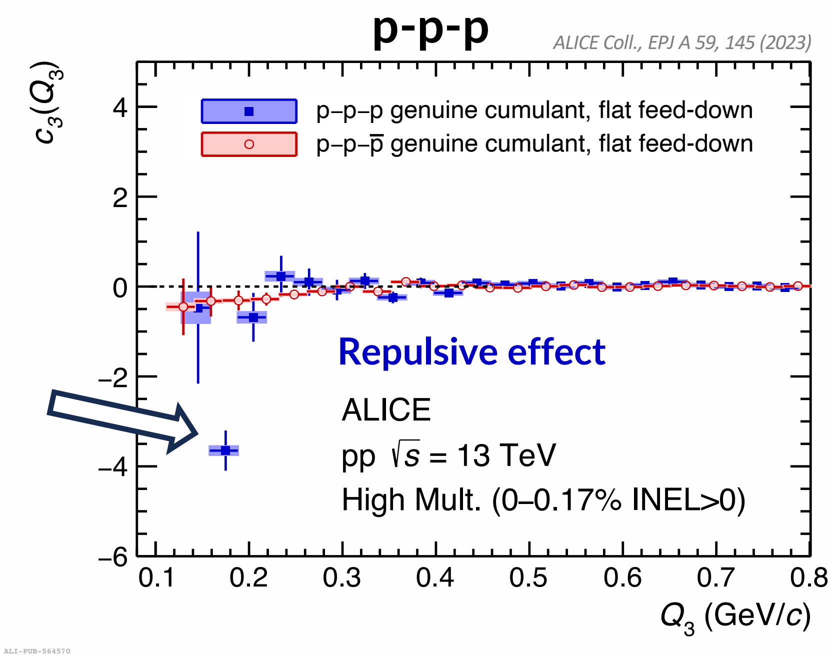
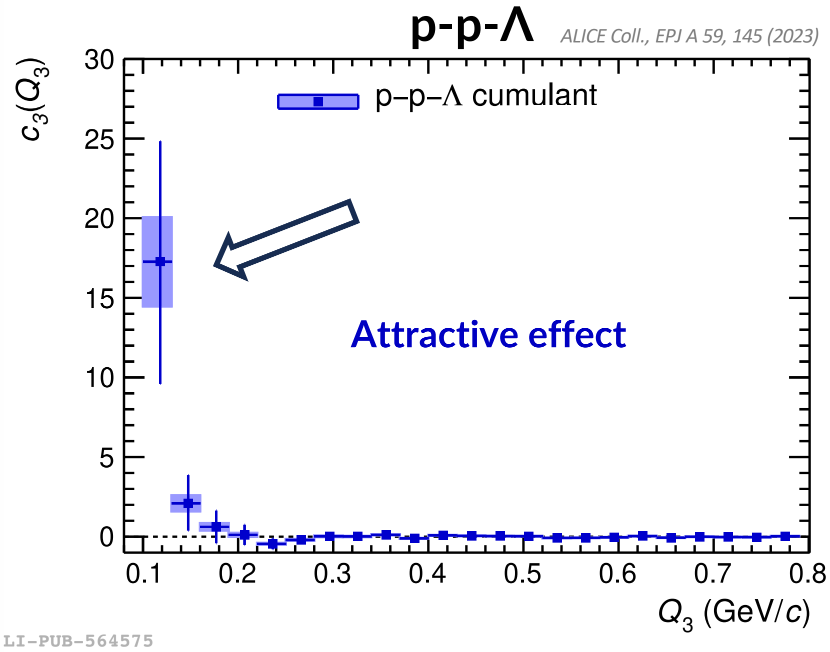
p-p-p and p-p-Λ cumulants







Step 1: Calculation of the three-body wave function

Hyperspherical coordinates

- Decomposition of the wave function in hyperspherical harmonics:

$$\psi = \sum_{[K]} \psi_{[K]} = \sum_{[K]} \rho^{-5/2} u_{[K]}(\rho) Y_{[K]}(\Omega)$$

- Schrödinger equation with the interaction:

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2 u_{[K]}(\rho)}{\partial \rho^2} - \frac{(K+3/2)(K+5/2)}{\rho^2} u_{[K]}(\rho)\right) + \sum_{[K']} U_{[K][K']}(\rho) u_{[K']}(\rho) = E u_{[K]}(\rho)$$

- The hypercentral potential is obtained as

$$U_{[K][K']}(\rho) = \int d\Omega\, Y^*_{[K]}(\Omega)[V_{12} + V_{23} + V_{31} + V_{123}] Y_{[K']}(\Omega)$$

p-p-p correlations

- Three-body correlation function:

$$C(Q_3) = \int S(\rho)|\psi(Q_3,\rho)|^2 \rho^5 \, d\rho$$

in collaboration with A. Kievsky, M. Viviani, L. Marcucci and E. Garrido.

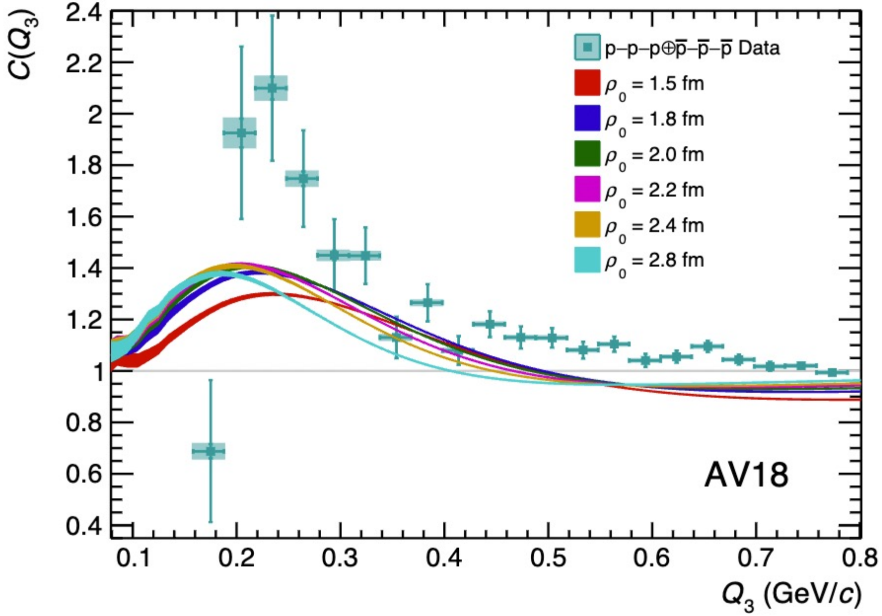
A. Kievsky et al., arXiv:2310.10428(2023), accepted by PRC

- Hyper-Source radius: $\rho_0 = 2\,R_{\mathrm{M}}$

- Wave function:
  AV18 + Coulomb + Antisymmetrisation

- Interaction at K <= 2, free solution for K > 2

A. Kievsky et al., arXiv:2310.10428(2023), accepted by PRC



p-p-Λ correlations

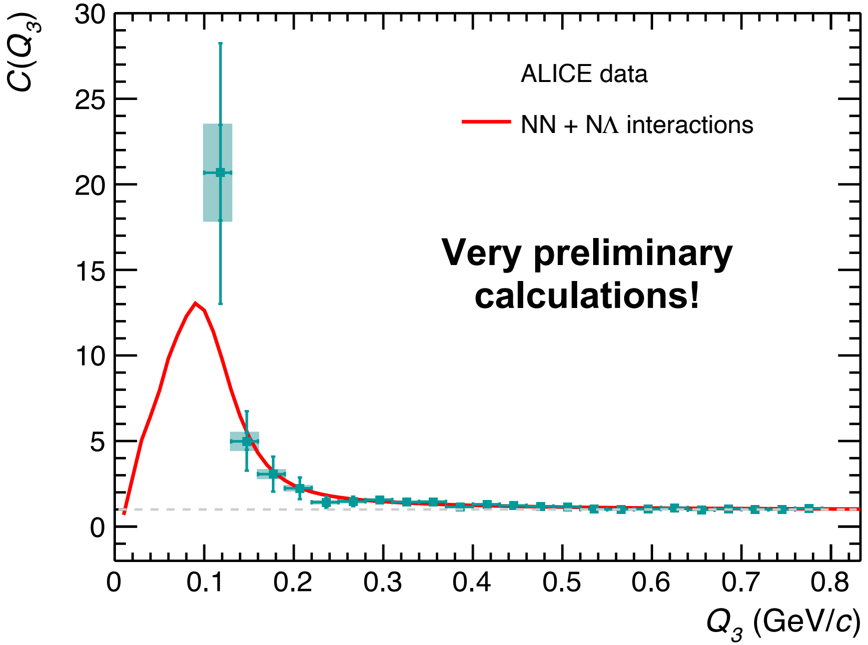
- Three-body correlation function:

$$C(Q_3) = \int S(\rho)|\psi(Q_3,\rho)|^2 \rho^5 \, d\rho$$

- Two-body interactions + Coulomb + Antisymmetrisation

- Hyper-source radius determined from the two-body p-p and p-Λ sources

- Calculations include the interaction for K <= 10



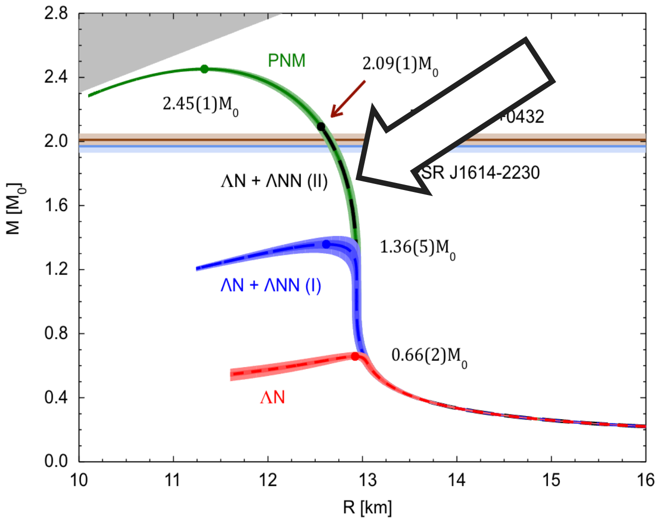
p-p-Λ correlations

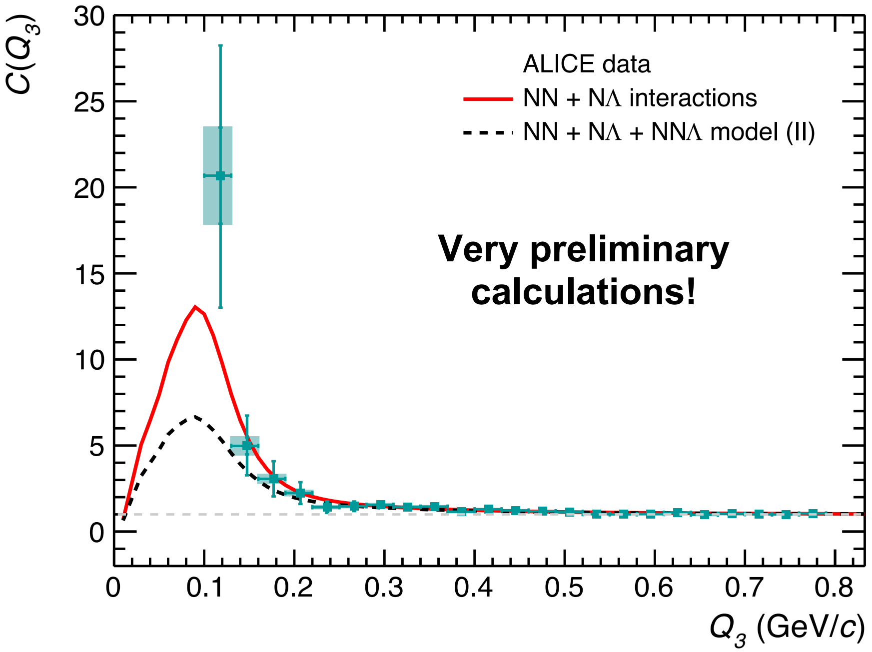
- Three-body correlation function:

$$C(Q_3) = \int S(\rho)|\psi(Q_3,\rho)|^2 \rho^5 \, d\rho$$

- Inclusion of the ΛNN (model II)

D. Lonardoni et al., PRL 114 (2019)





Larger statistics is needed to test three-body interaction models

Conclusions

Femtoscopy in pp collisions at the LHC

→ small particle source size (about 1 fm)

→ alternative method to study many-body systems

Results achieved by ALICE:

- p-d correlation function shows sensitivity to the full three-nucleon dynamics
- Free scattering of three hadrons measured for the fitst time:

  → p-p-p correlation function: evidence of antisymmetrisation effects

  → p-p-Λ correlation function: no strong evidence of a three-body repulsion (modelling has to be improved)

Backup

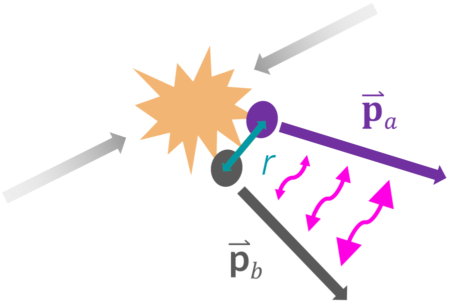
Source determination

Femtoscopy used in a "traditional" way: known interaction → determine the source size

p-p interaction:
Coulomb + Argonne v18

p-Λ interaction:
χEFT NLO and LO

$$C(k^*) = \int S(\vec{r}\,)\left|\psi(\vec{k}^*,\vec{r}\,)\right|^2 d^3\vec{r}$$



- Fit of the data as function of m_T
- Gaussian source

$$S(r) = G[r, r_{core}(m_T)] = \frac{1}{(4\pi r_{core}^2)^{3/2}} \exp\left(-\frac{r^2}{4r_{core}^2}\right)$$

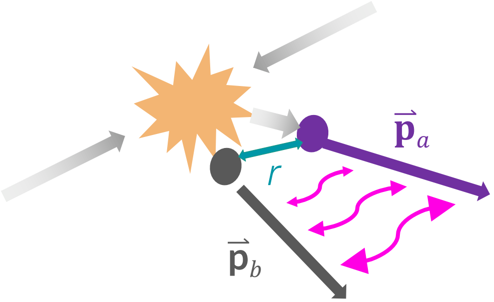
Source determination

Femtoscopy used in a “traditional” way: known interaction → determine the source size

p-p interaction:
Coulomb + Argonne v18

p-Λ interaction:
χEFT NLO and LO

$$C(k^*) = \int S(\vec{r}\,) \left|\psi(\vec{k}^*, \vec{r}\,)\right|^2 d^3\vec{r}$$



- Fit of the data as function of m_T
- Gaussian source

$$S(r) = G[r, r_{core}(m_T)] = \frac{1}{(4\pi r_{core}^2)^{3/2}} \exp\left(-\frac{r^2}{4 r_{core}^2}\right)$$

+ Resonances with cτ ~ r_{core} ~ 1fm (Δ^{++}, N^*, Σ^*)

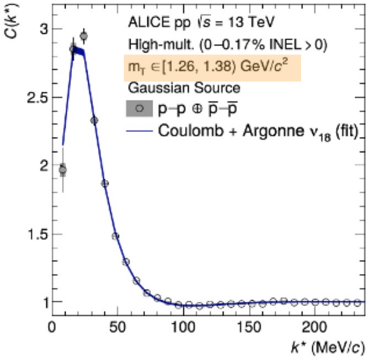
Particle	Primordial fraction	Resonances <cτ>
Proton	33 %	1.6 fm
Lambda	34 %	4.7 fm

U. Wiedemann and U. Heinz PRC 56 (1997)

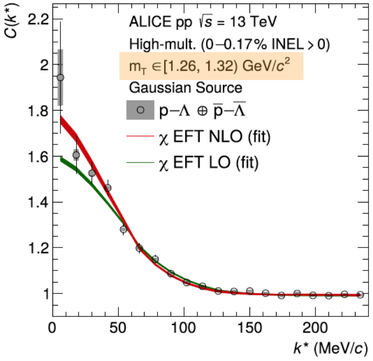
Source determination

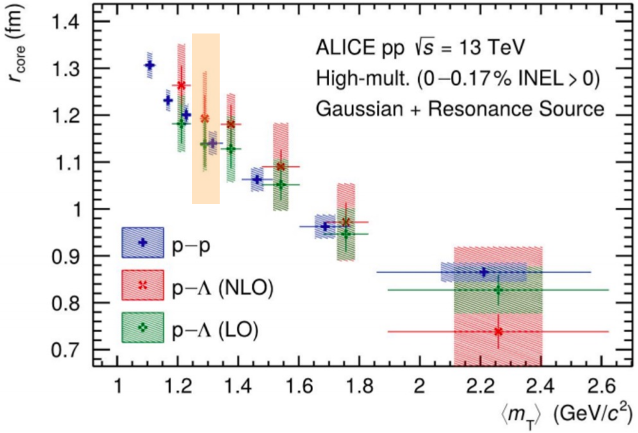
Femtoscopy used in a "traditional" way: known interaction → determine the source size

p-p interaction:
Coulomb + Argonne v18



p-Λ interaction:
χEFT NLO and LO





- Gaussian source

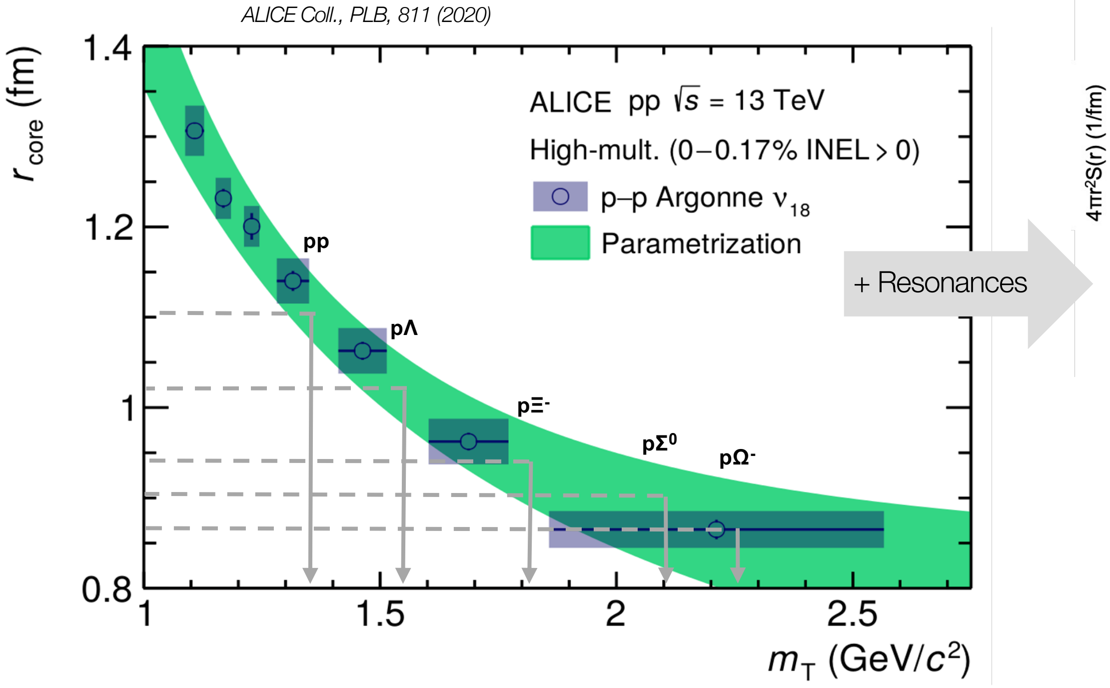
$$S(r) = G[r, r_{core}(m_T)] = \frac{1}{(4\pi r_{core}^2)^{3/2}} \exp\left(-\frac{r^2}{4r_{core}^2}\right)$$

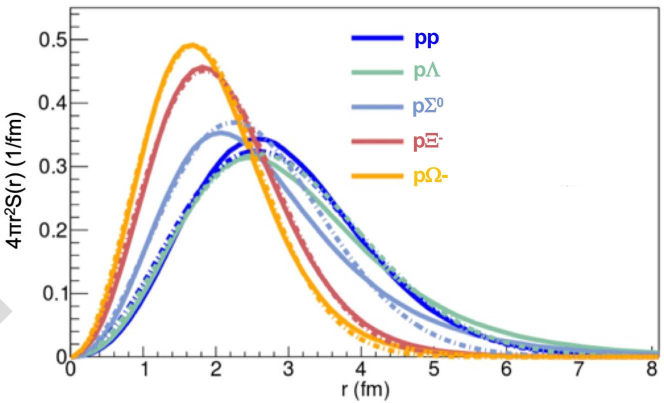
+ Resonances with cτ ~ r_{core} ~ 1fm (Δ^{++}, N^*, Σ^*)

One universal source for all hadrons

Similar results in K^+p, ππ

Source determination





Pair	r_{Core} [fm]	r_{Eff} [fm]
p-p	1.1	1.2
p-Λ	1.0	1.3
p-Σ^0	0.87	1.02
p-Ξ^-	0.93	1.02
p-Ω^-	0.86	0.95

Cumulants

- X_i denotes the general i-th stochastic variable
- The most general decomposition of 2-particle correlation is:

$$\langle X_1 X_2 \rangle = \langle X_1 \rangle \langle X_2 \rangle + \langle X_1 X_2 \rangle_c$$

- By definition, the 2nd term on the right is the 2-particle cumulant
- Cumulants cannot be measured directly, however:

$$\langle X_1 X_2 \rangle_c = \langle X_1 X_2 \rangle - \langle X_1 \rangle \langle X_2 \rangle$$

Cumulants

- The most general decomposition of 3-particle correlation is:

$$\begin{aligned}\langle X_1 X_2 X_3\rangle &= \langle X_1\rangle\langle X_2\rangle\langle X_3\rangle \\ &+ \langle X_1 X_2\rangle_c\langle X_3\rangle + \langle X_1 X_3\rangle_c\langle X_2\rangle + \langle X_2 X_3\rangle_c\langle X_1\rangle \\ &+ \langle X_1 X_2 X_3\rangle_c\end{aligned}$$

- Using the 2-particle cumulant: $\langle X_1 X_2\rangle_c = \langle X_1 X_2\rangle - \langle X_1\rangle\langle X_2\rangle$

- Working recursively from higher to lower orders, we have 3-particle cumulant expressed in terms of the measured 3-, 2-, and 1-particle averages:

$$\begin{aligned}\langle X_1 X_2 X_3\rangle_c &= \langle X_1 X_2 X_3\rangle \\ &- \langle X_1 X_2\rangle\langle X_3\rangle - \langle X_1 X_3\rangle\langle X_2\rangle - \langle X_2 X_3\rangle\langle X_1\rangle \\ &+ 2\langle X_1\rangle\langle X_2\rangle\langle X_3\rangle\end{aligned}$$

Cumulants

In terms of the probability distribution functions we have:

$$
\begin{aligned}
\langle X_1 X_2 \rangle_c &= \langle X_1 X_2 \rangle - \langle X_1 \rangle \langle X_2 \rangle = \\
&= \int X_1 X_2 f\left(X_1 X_2\right) dX_1 dX_2 - \int X_1 f_1\left(X_1\right) dX_1 \int X_2 f_1\left(X_2\right) dX_2 \\
&= \int X_1 X_2 \left[f\left(X_1 X_2\right) - f_1\left(X_1\right) f_1\left(X_2\right) \right] dX_1 dX_2 \\
&= \int X_1 X_2 f_c\left(X_1, X_2\right) dX_1 dX_2
\end{aligned}
$$

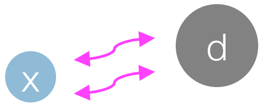
with

$$f_c\left(X_1, X_2\right) = f\left(X_1 X_2\right) - f_1\left(X_1\right) f_1\left(X_2\right)$$

For three particles:

$$f_c\left(X_1, X_2, X_3\right) = f\left(X_1, X_2, X_3\right) - f\left(X_1, X_2\right) f\left(X_3\right) - f\left(X_1, X_3\right) f\left(X_2\right) - f\left(X_3, X_2\right) f\left(X_1\right) + 2 f\left(X_1\right) f\left(X_2\right) f\left(X_3\right)$$

Hadron-deuteron correlations



- Coulomb + strong interaction using the Lednický model

$$\psi(\vec{k}^*,\vec{r}^*) = e^{i\delta_c}\sqrt{A_c(\eta)}\left[e^{-i\vec{k}^*\cdot\vec{r}^*}F(-i\eta,1,i\xi)+f_C(k^*)\frac{\tilde{G}(\rho,\eta)}{r*}\right]$$

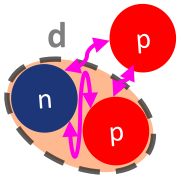
$$f_C(k^*) = \left(\frac{1}{f_0}+\frac{d_0\cdot k^{*2}}{2}-\frac{2h(k^*)}{a_c}-ik^*A_C(k^*)\right)^{-1}$$

- Point-like particle models anchored to scattering experiments

System	Spin averaged f_0(fm)	Spin averaged d_0(fm)	$S=1/2$ f_0(fm)	$S=1/2$ d_0(fm)	$S=3/2$ f_0(fm)	$S=3/2$ d_0(fm)
p–d			$-1.30^{+0.20}_{-0.20}$	—	$-11.40^{+1.20}_{-1.80}$	$2.05^{+0.25}_{-0.25}$
			$-2.73^{+0.10}_{-0.10}$	$2.27^{+0.12}_{-0.12}$	$-11.88^{+0.10}_{-0.40}$	$2.63^{+0.01}_{-0.02}$
			-4.0	—	-11.1	—
			-0.024	—	-13.7	—
			$0.13^{+0.04}_{-0.04}$	—	$-14.70^{+2.30}_{-2.30}$	—
K^+–d	-0.470	1.75				
	-0.540	0.0				

- Only s-wave interaction
- Source radius evaluated using the hadron-hadron universal m_T scaling

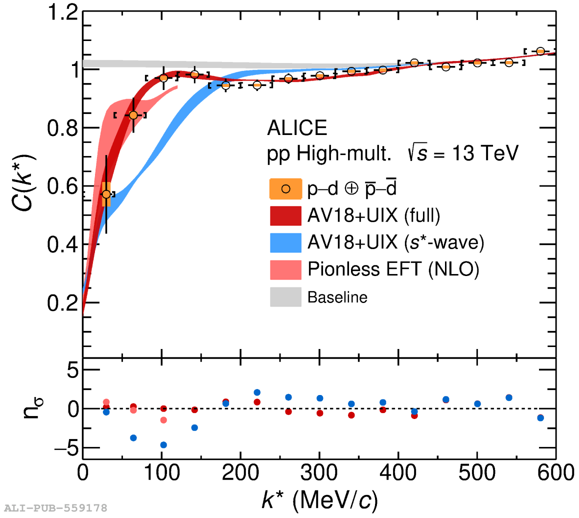
Proton-deuteron correlation

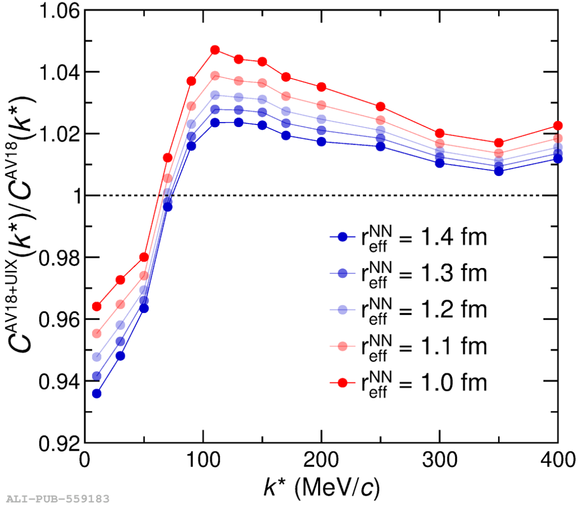


The measured p-d correlation function reflects the full three-nucleon dynamics:
Coulomb + strong interaction (NN and NNN) + Quantum Statistics

M. Viviani et al., arXiv:2306.02478 (2023)

ALICE Coll. arXiv:2308.16120 (2023)





- Sensitivity to the short inter-particle distances
- Hadron-nuclei correlations at the LHC can be used to study many-body dynamics

p-p-Λ correlations