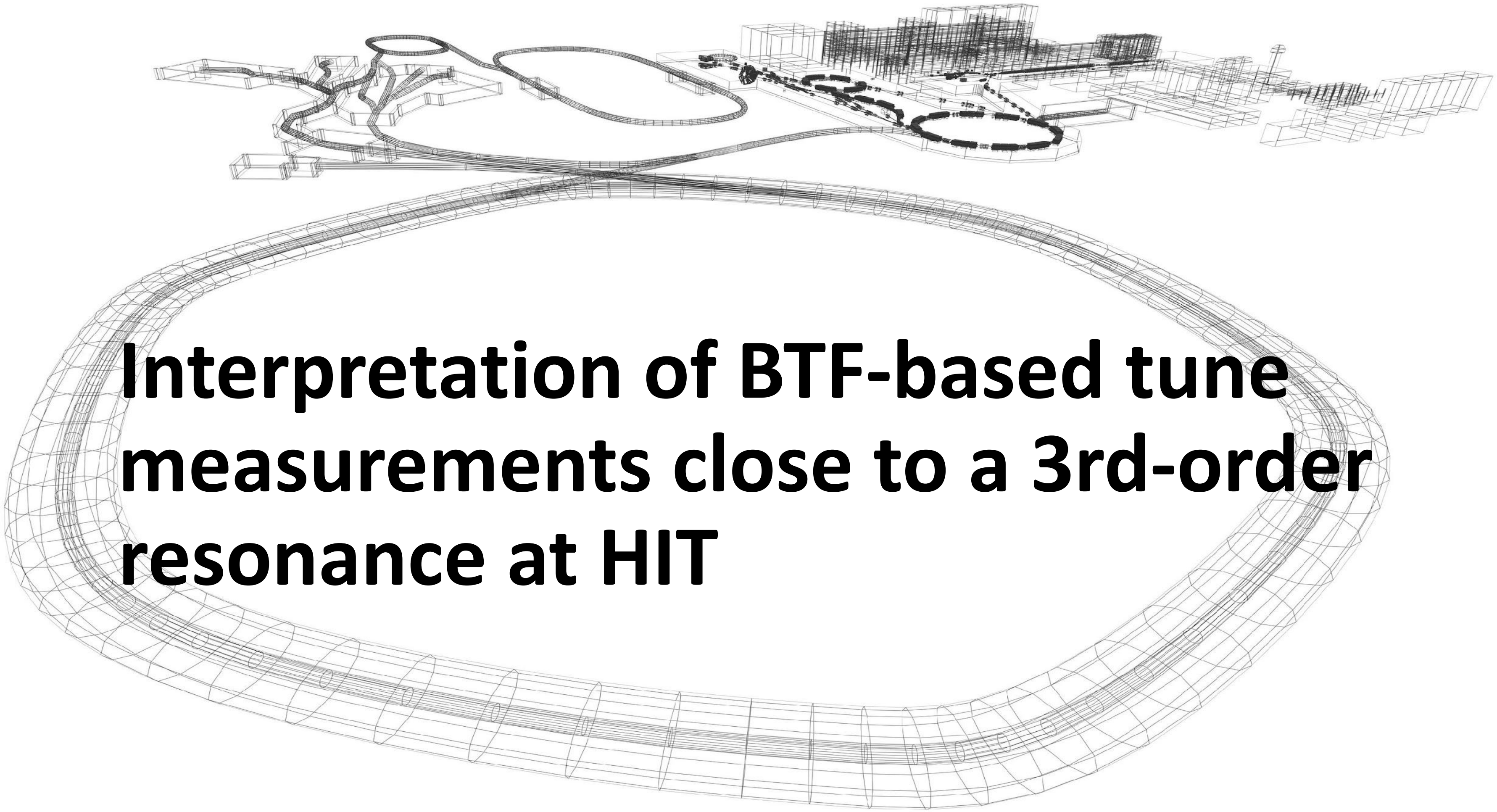


Interpretation of BTF-based tune measurements close to a 3rd-order resonance at HIT

E.C. Cortés García, **E. Benedetto, E. Feldmeier, T. Haberer, M. Hun, P. Niedermayer, R. Singh, R. Taylor**

Contents

Motivation and introduction

- Understand the dynamics near the third order resonance to excite the particles the most efficient way possible
- Application to resonant extraction

Beam Transfer Function measurement

- Observe beam reaction to different excitation frequencies and deduce the dynamics
- Established theoretical framework
- Experimentally available

Contents

Theory

Kobayashi Hamiltonian

- Tune near a third integer resonance

$$Q_x = n + \frac{1}{3} + \Delta q_x, n \in \mathbb{N}_0$$

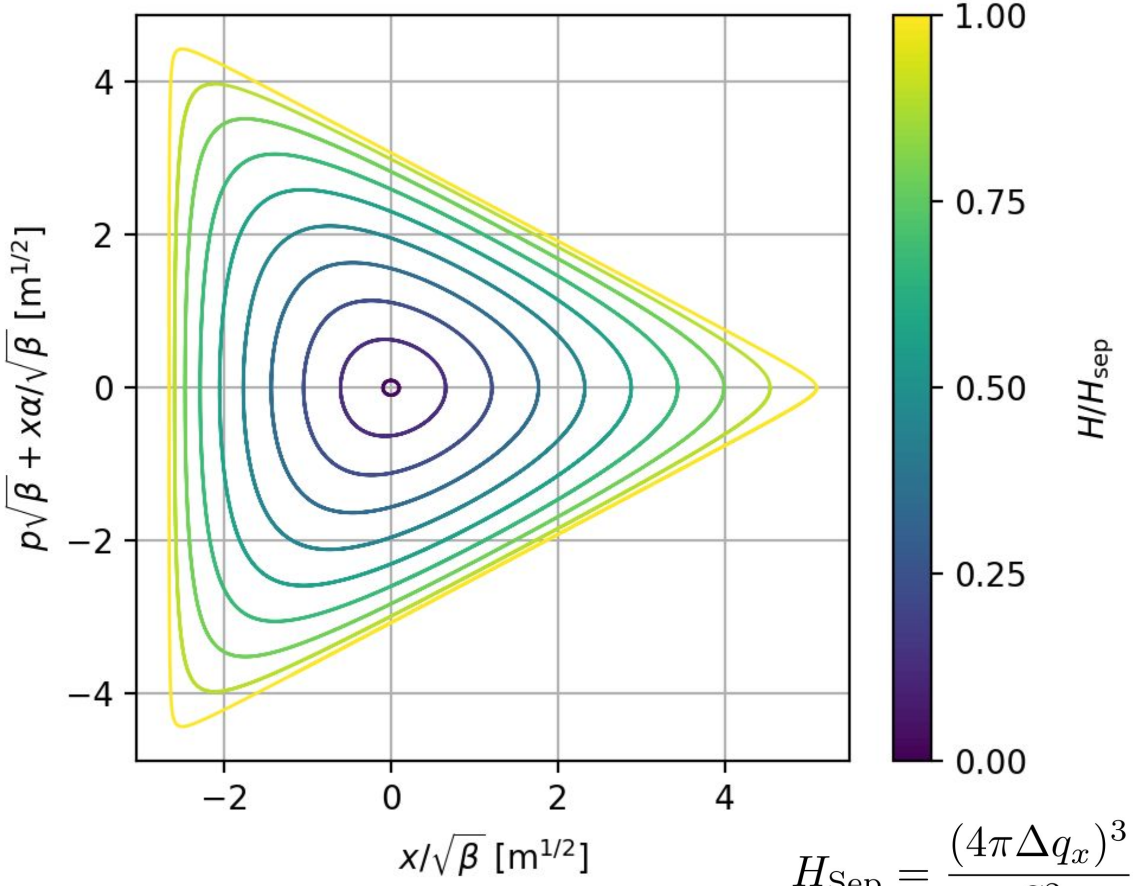
- Resonance driven by a sextupole component S
- The dynamics can be effectively described by [1]

Linear theory / **Non-linear term**

$$H = 3\pi\Delta q_x(X^2 + P^2) + \frac{S}{4}(3XP^2 - X^3)$$

$$X = x/\sqrt{\beta_x}, \quad P = p_x\sqrt{\beta_x} + \alpha_x X$$

Equipotential lines in normalized phase-space described by the Kobayashi Hamiltonian



$$H_{\text{Sep}} = \frac{(4\pi\Delta q_x)^3}{S^2}$$

[1] Y. Kobayashi and H. Takahashi, Improvement of the emittance in the resonant ejection, in *Proc. VIth Int. Conf. High Energy Accelerators* (Massachusetts, 1967) pp. 347–351.

Theory

Non-linear detuning

Linear term — Non-linear term

$$H = 3\pi\Delta q_x(X^2 + P^2) + \frac{S}{4}(3XP^2 - X^3)$$

- From normalized coordinates to action-angle variables

$$H = 6\pi\Delta q_x J + \frac{S}{\sqrt{2}} J^{3/2} \sin 3\phi$$

- Particle's tune (One-turn phase-advance)

Linear theory contribution — Non-linear detuning

$$\frac{1}{6\pi}\frac{\partial H}{\partial J} + \frac{n}{3} = q_{x,0} + q_{x,1}$$

$$q_{x,1} = \frac{3S}{\sqrt{2^5}\pi} J^{1/2} \sin 3\phi$$

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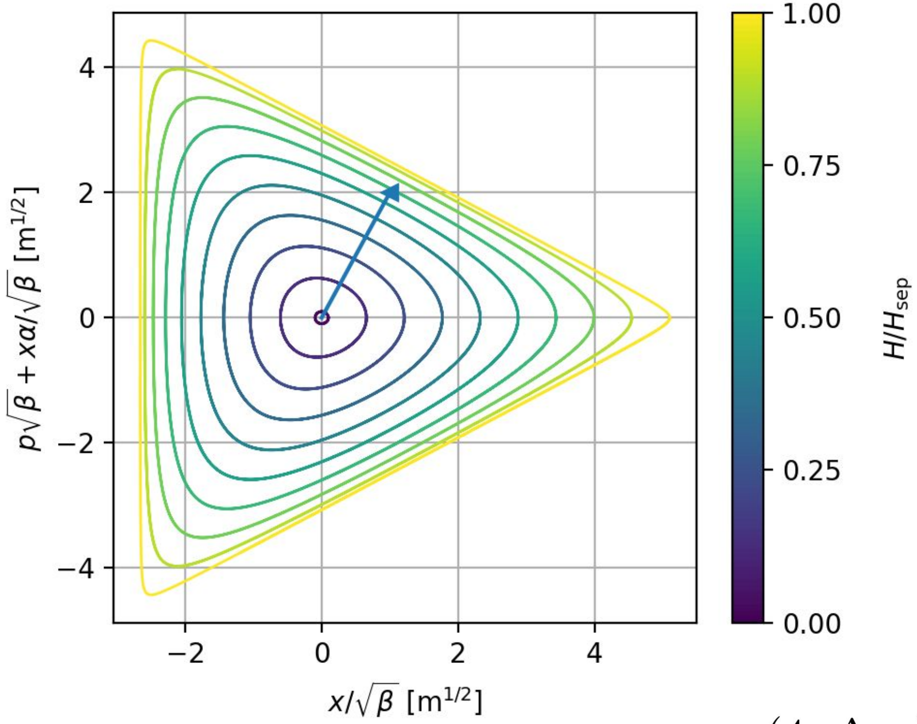

$$H_{\mathrm{Sep}} = \frac{(4\pi\Delta q_x)^3}{S^2}$$

Theory

Non-linear detuning

- Particle's one-turn phase-advance

Linear theory contribution / Non-linear detuning

$$\frac{1}{6\pi}\frac{\partial H}{\partial J} + \frac{n}{3} = q_{x,0} + q_{x,1}$$

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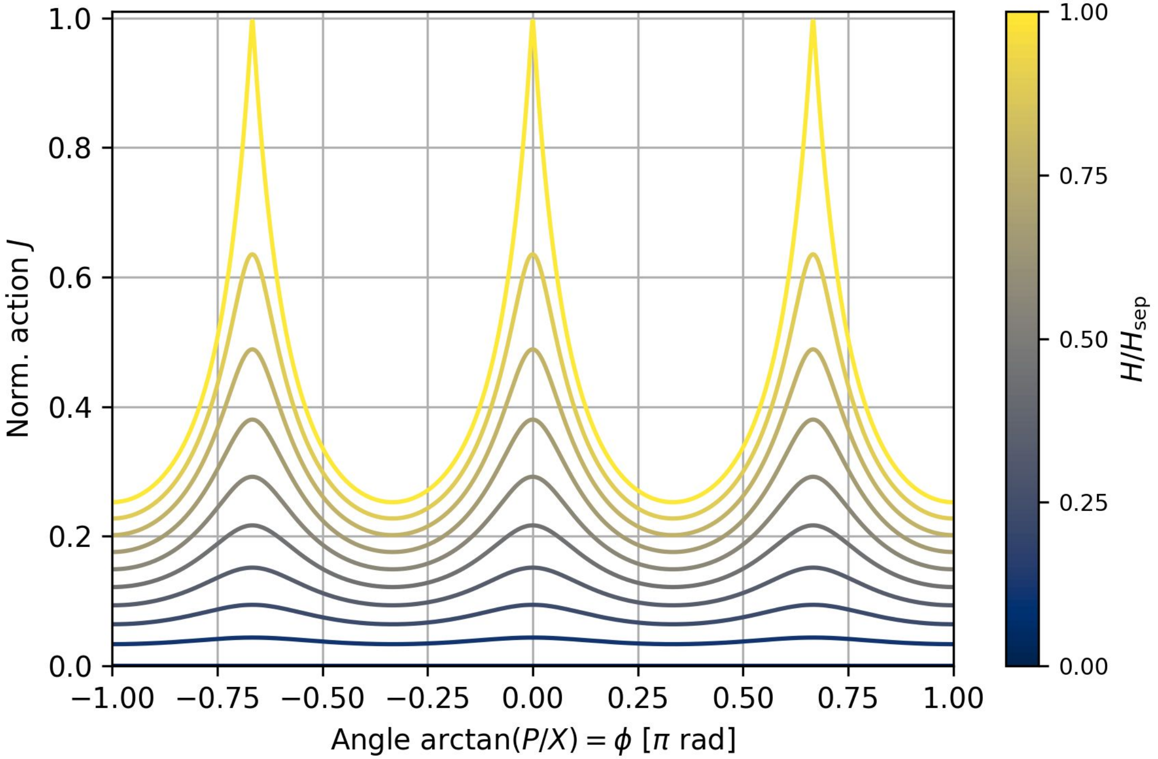
- Near the resonance there is a **phase-amplitude** modulation
- The average detuning over many turns gives a non-vanishing contribution
- The average detuning deviates in the direction of the nearest resonance

- Kobayashi Hamiltonian in action-angle variables

$$H = 6\pi\Delta q_x J + \frac{S}{\sqrt{2}} J^{3/2} \sin 3\phi$$



Theory

Non-linear detuning

- Particle's one-turn phase-advance

Linear theory contribution — Non-linear detuning

$$\frac{1}{6\pi}\frac{\partial H}{\partial J} + \frac{n}{3} = \boxed{q_{x,0}} + \boxed{q_{x,1}}$$

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- Near the resonance there is a **phase-amplitude** modulation
- The average detuning over many turns gives a non-vanishing contribution
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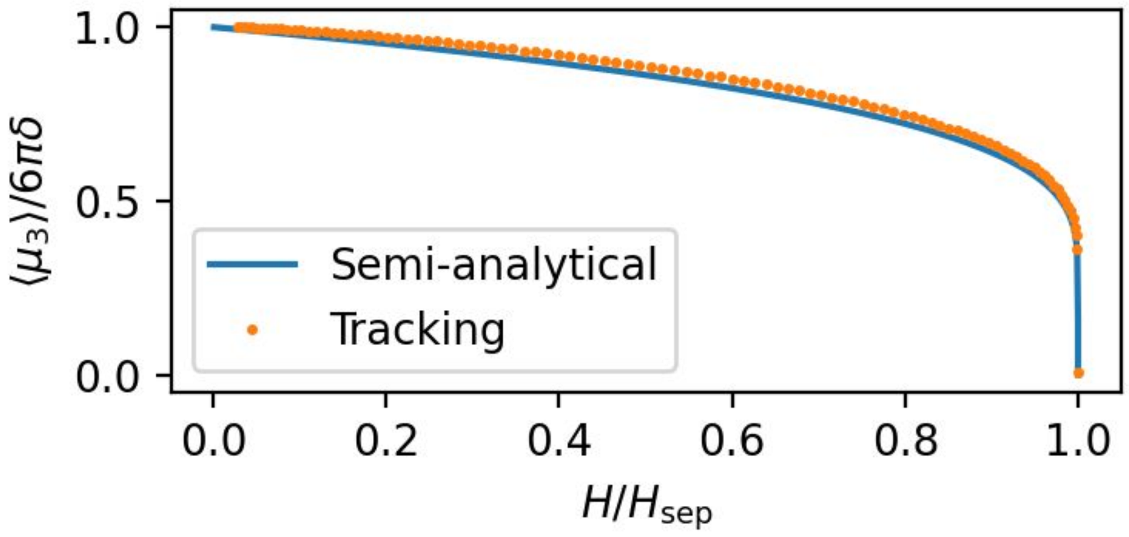
Average detuning over many turns

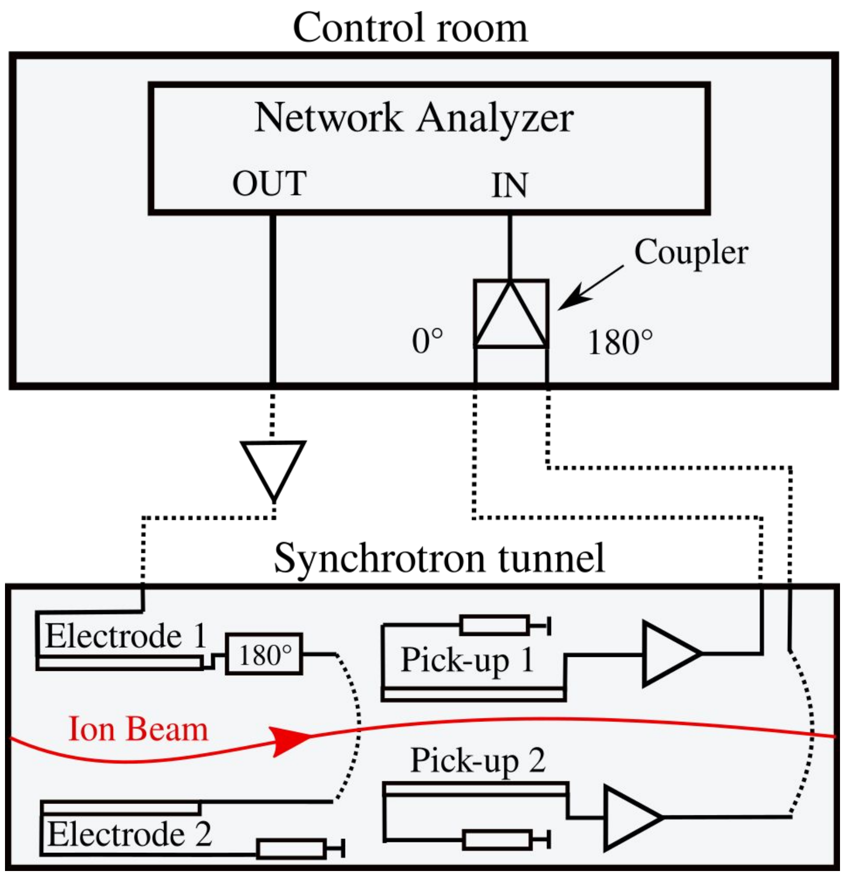
$$\frac{\langle \mu_3 \rangle}{6\pi\delta} = \left(\frac{1}{2\pi} \int_0^{2\pi} \frac{d\phi_x}{1 - \sqrt{2\tilde{J}} \cos 3\phi_x} \right)^{-1}$$



Contents

Measurements

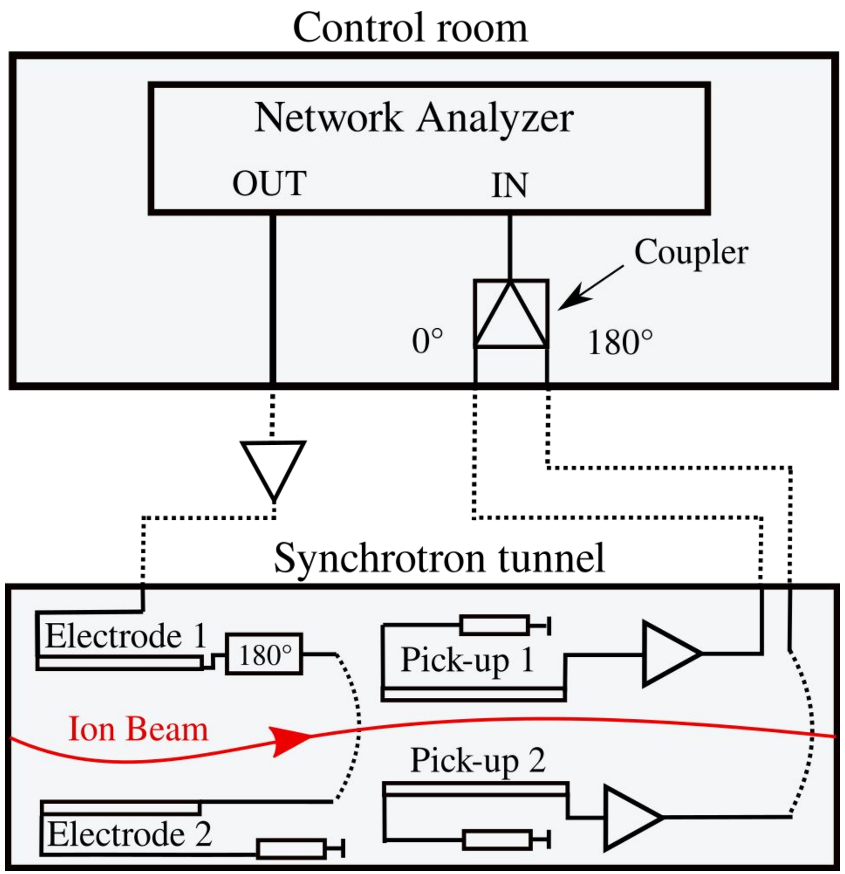
Beam Transfer Function measurement



1. Excite the beam with a single frequency (sinusoidal) wave
   - Generates a beam centroid oscillation with an amplitude of < 500 microns
   - Beam pipe radius is 8 cm
2. Observe centroid response signal
3. Extract the frequency component of the excitation signal
4. Go to next frequency and start from Step 1.

Measurements

Beam Transfer Function measurement

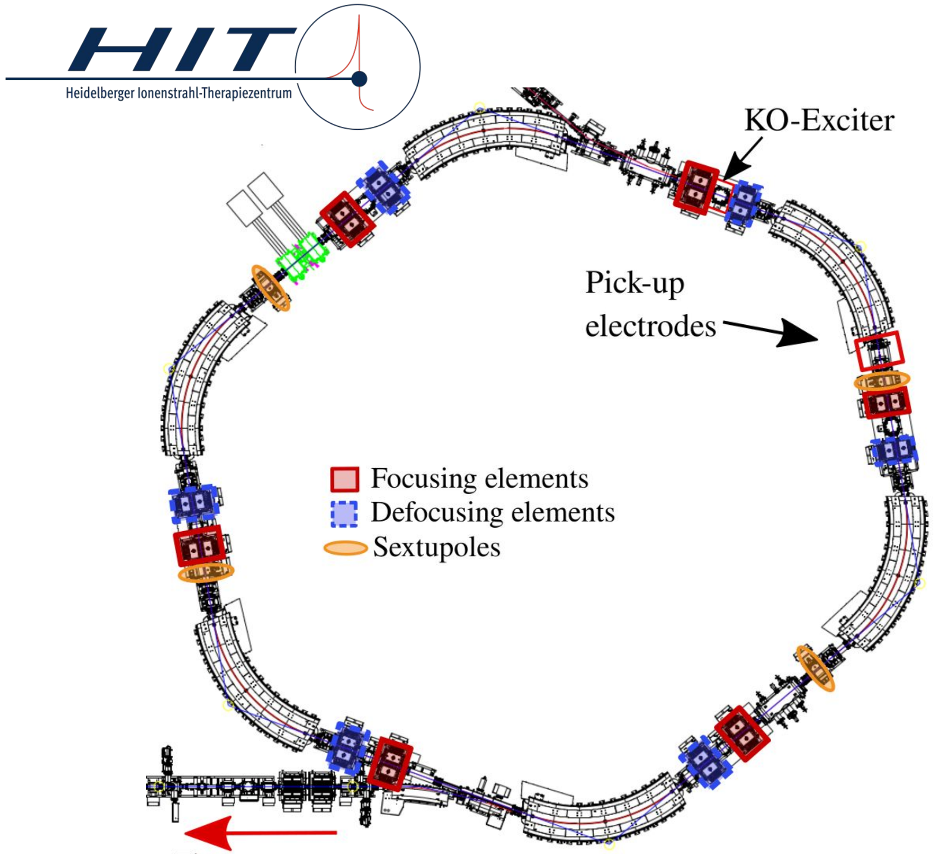


1. Excite the beam with a single frequency (sinusoidal) signal
2. Observe centroid response signal
3. Extract the frequency component of the excitation signal
4. Go to next frequency and start from Step 1.

- Investigation of **coasting beams**
- Low intensity (10^8 - 10^9 particles)
- Momentum spread ~ 10^{-3}
- Measurement campaigns at Heidelberg with Carbon-ions
- Measurement campaigns at GSI with Argon- and Uranium-ions

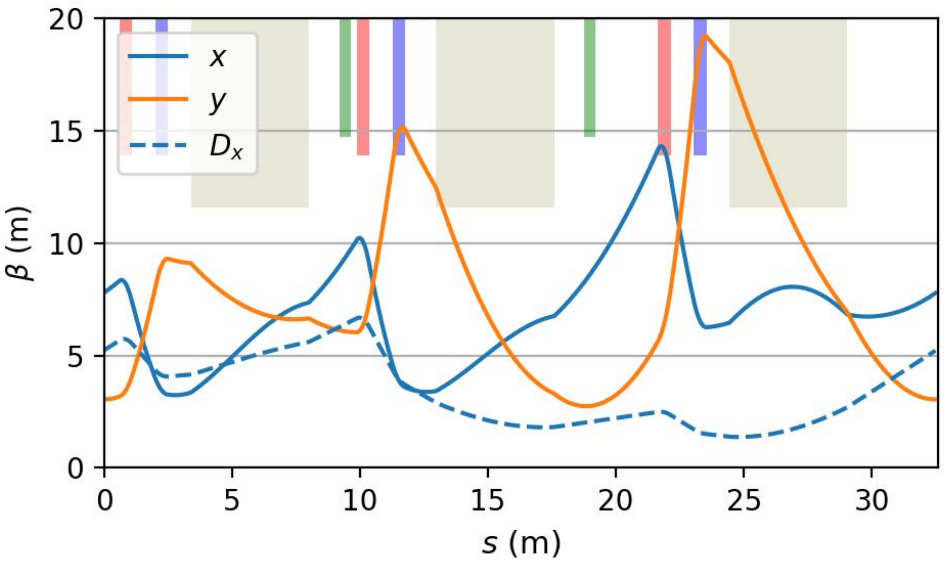
Measurements

Heidelberg Ion-Beam Therapy Center synchrotron



- Compact synchrotron designed for therapy

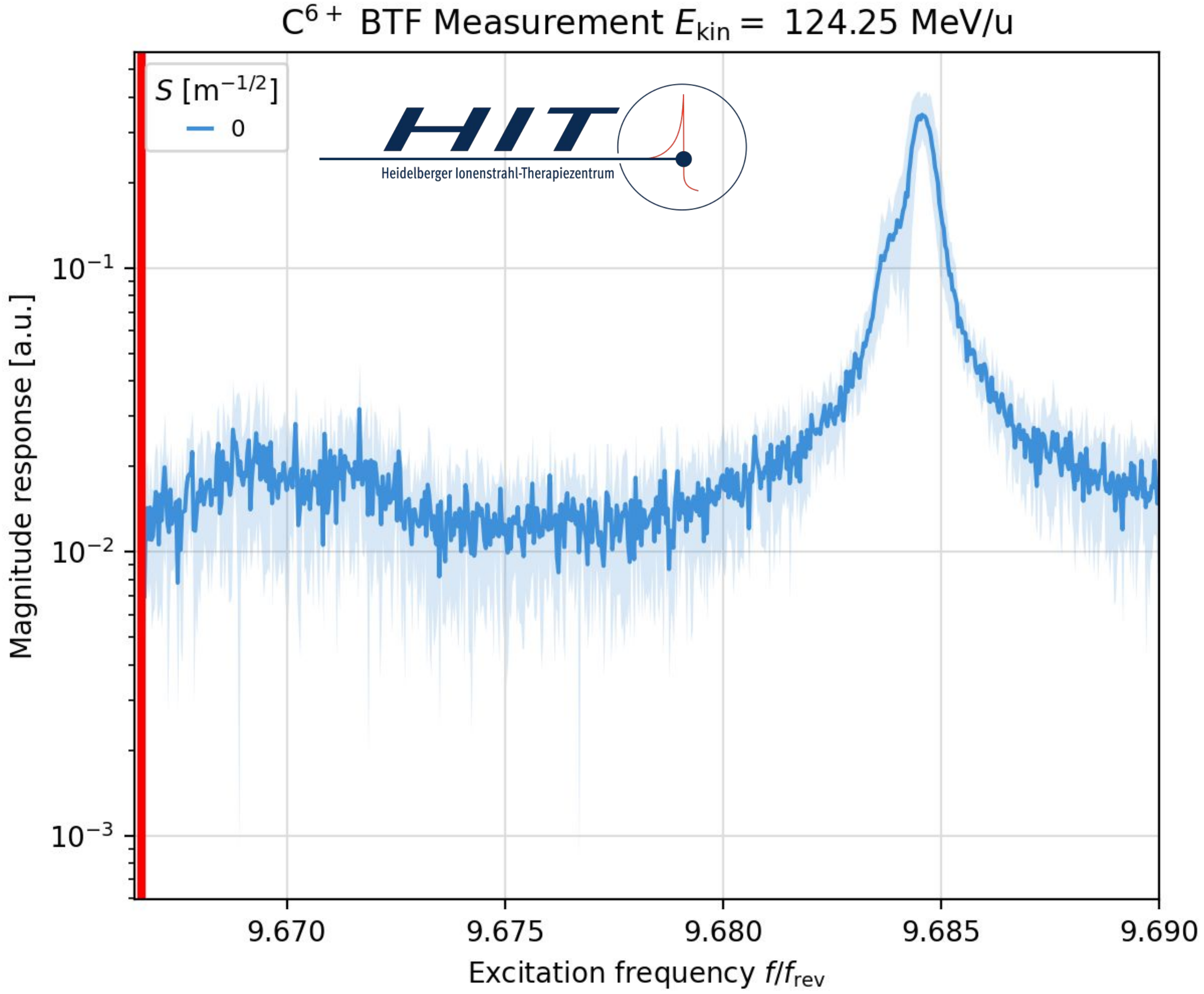
Parameter	HIT
Circumference	64.986 m
Tunes (Q_x, Q_y)	(1.67,1.74)
Chromaticity (ξ_x, ξ_y)	(-1.7, -1.6)
Harmonic n	2
Ion types	p^+, He^{2+}, C^{6+}, O^{8+}



Measurements

Scans over sextupole strength

- Excitation strength set to -30dBm (~50 nrad kick)
- 701 points
- 25 shots
- 10 s measurement time per shot
- Investigation of higher 9th betatron band
- Single peak (linear case)



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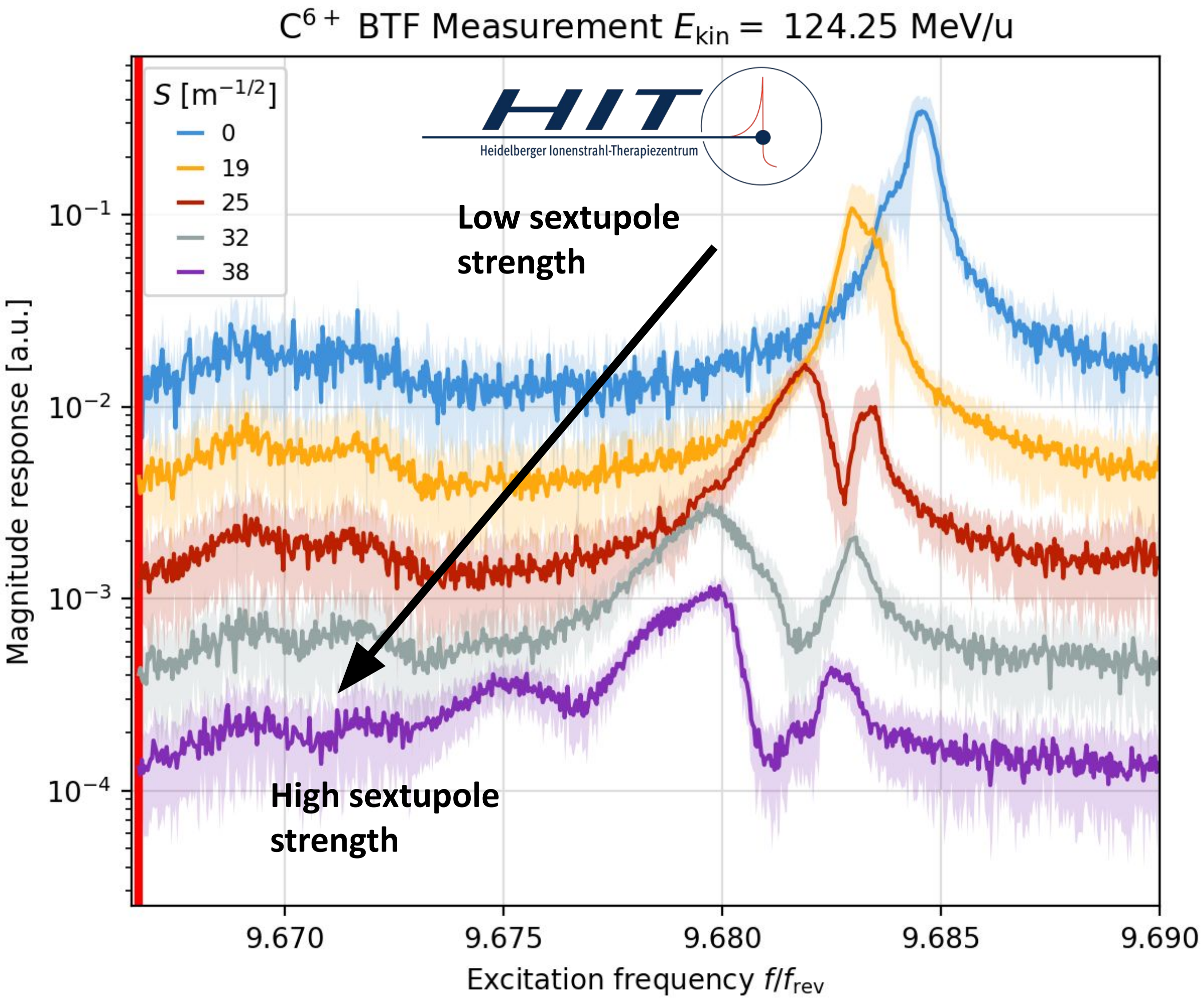
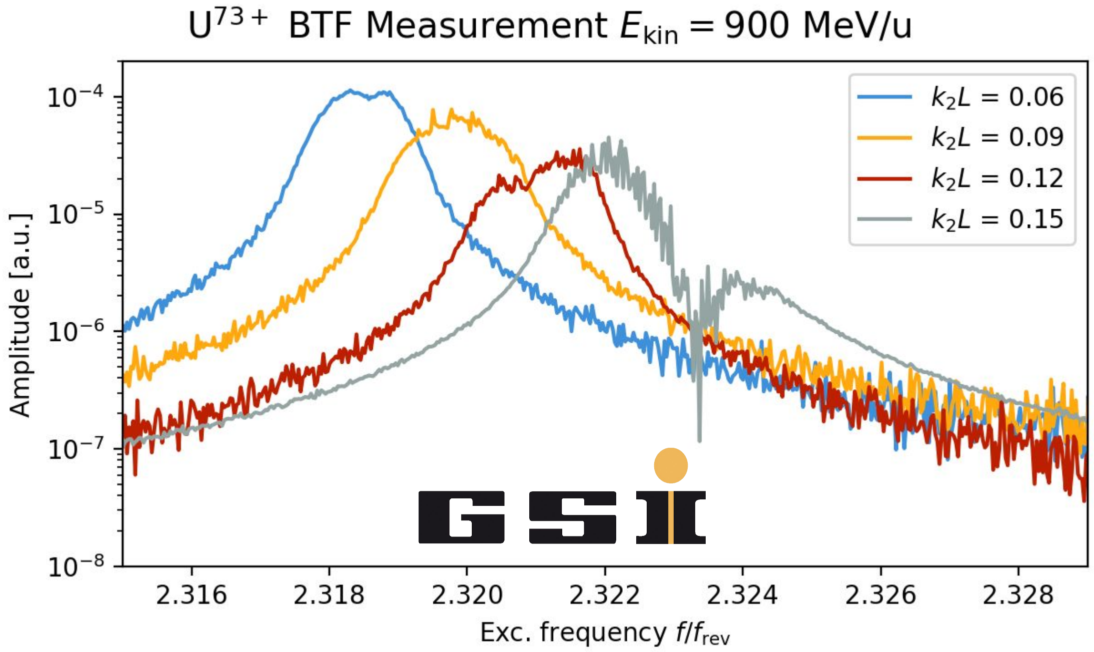
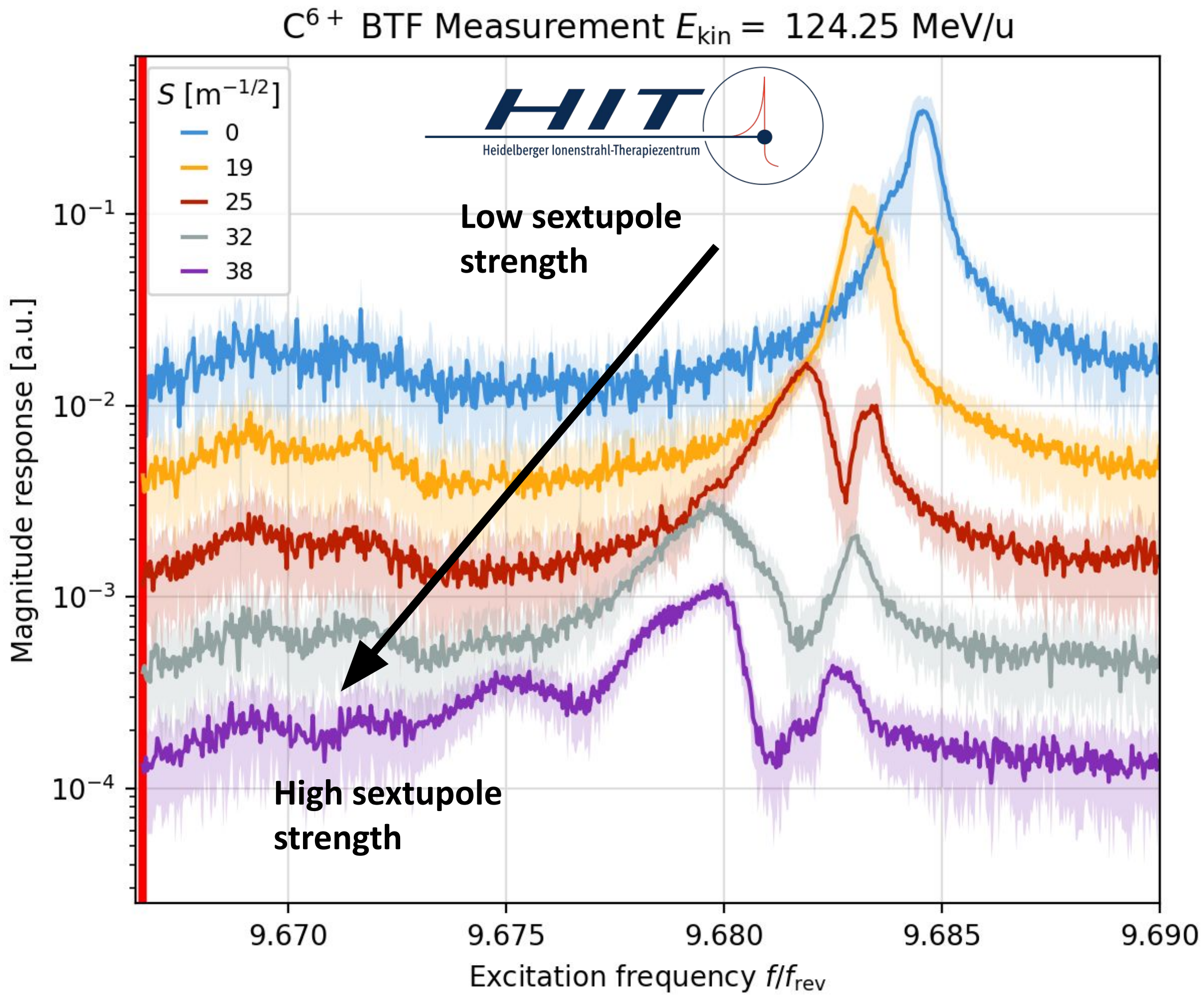


Measurements

Scans over sextupole strength

- Sextupole component distorts the signal
- Splitting is observed
- Qualitative behaviour confirmed with GSI measurements
- Initial conditions play a decisive role





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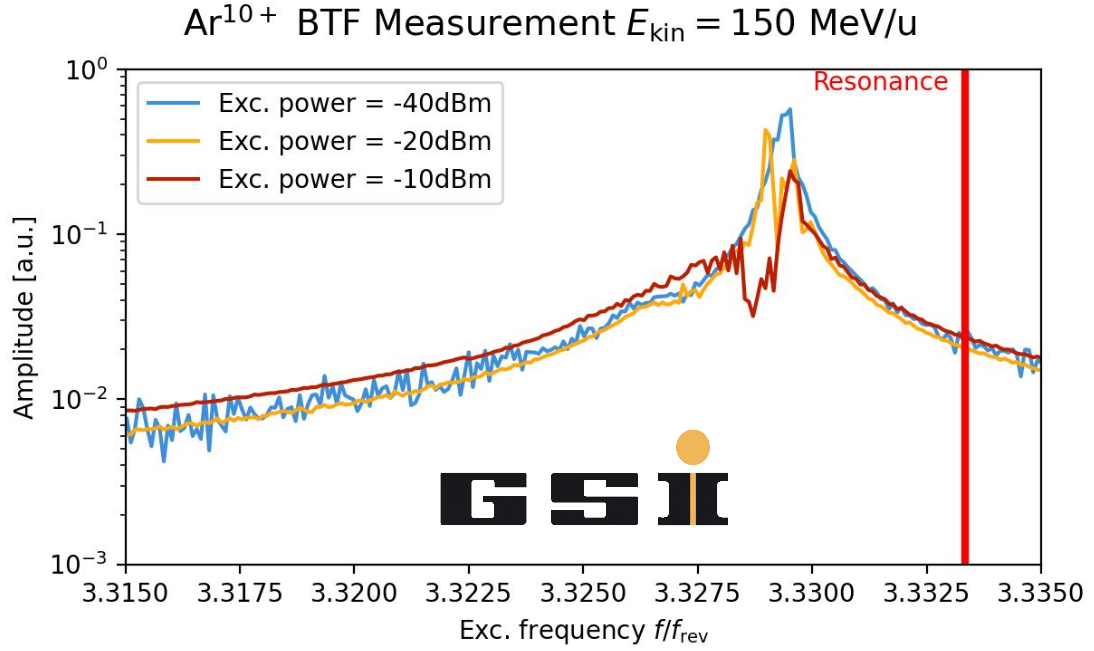
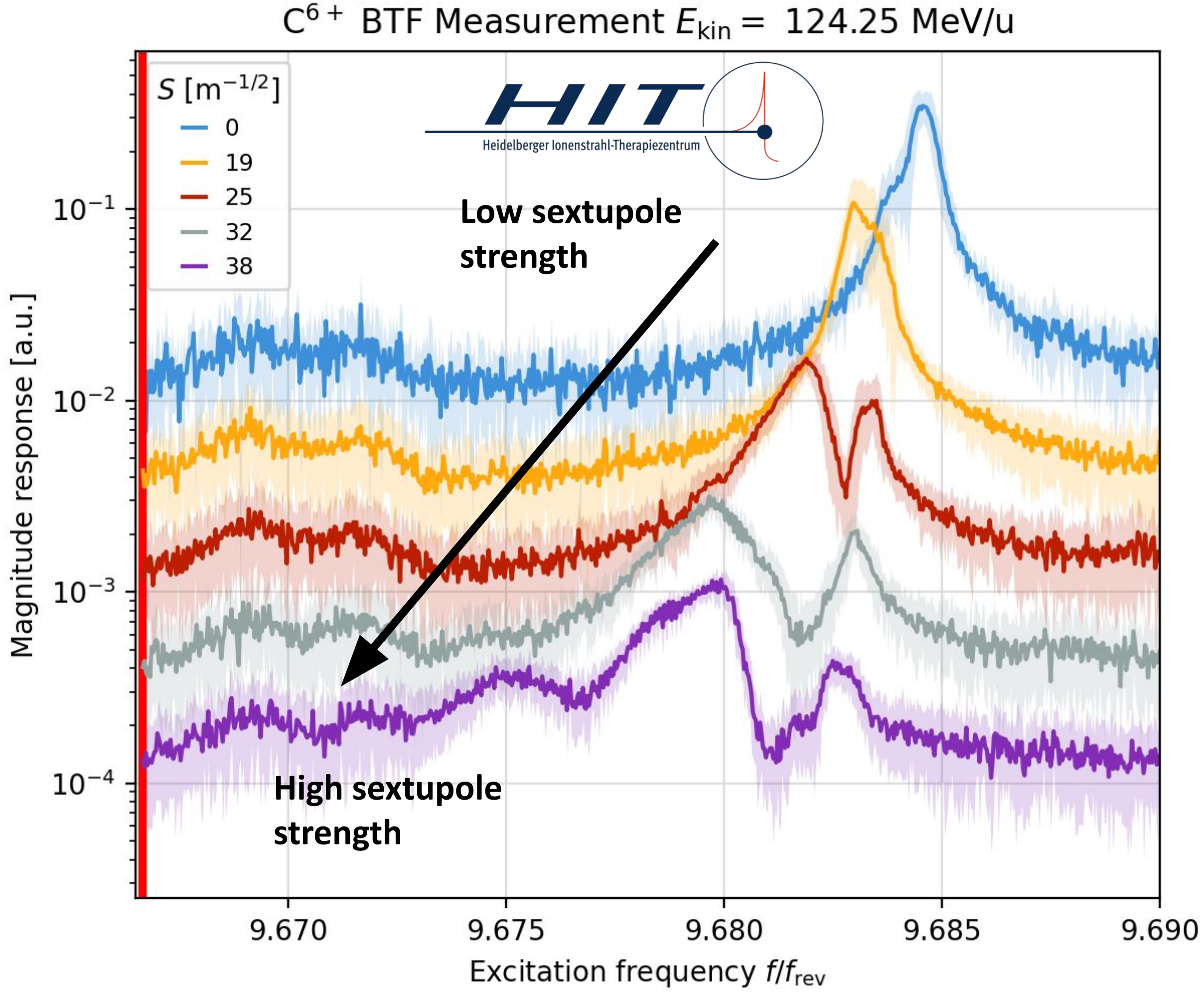




Contents

Simulation

Simulation results for the Heidelberg machine

- MAD-X
- Maptrack (performed by R. Taylor)
- **X-Suite**

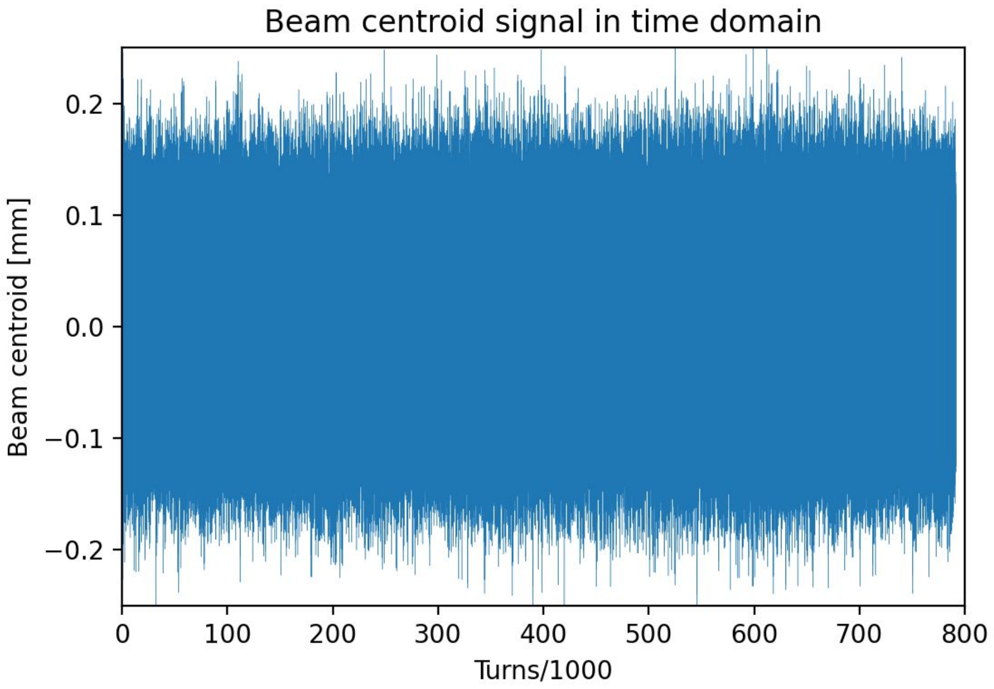
Typical parameters for simulation HPC (XSuite)

Parameter	Simulation	Experiment
N parts.	10^5	10^8
Turns per exc. freq.	30000	31 000
Excitation steps	701	701
Time	≤ 6 h	10 s
Samples per turn	≤ 20	continuous

Simulation

Simulation results for the Heidelberg machine

Typical signal from simulation



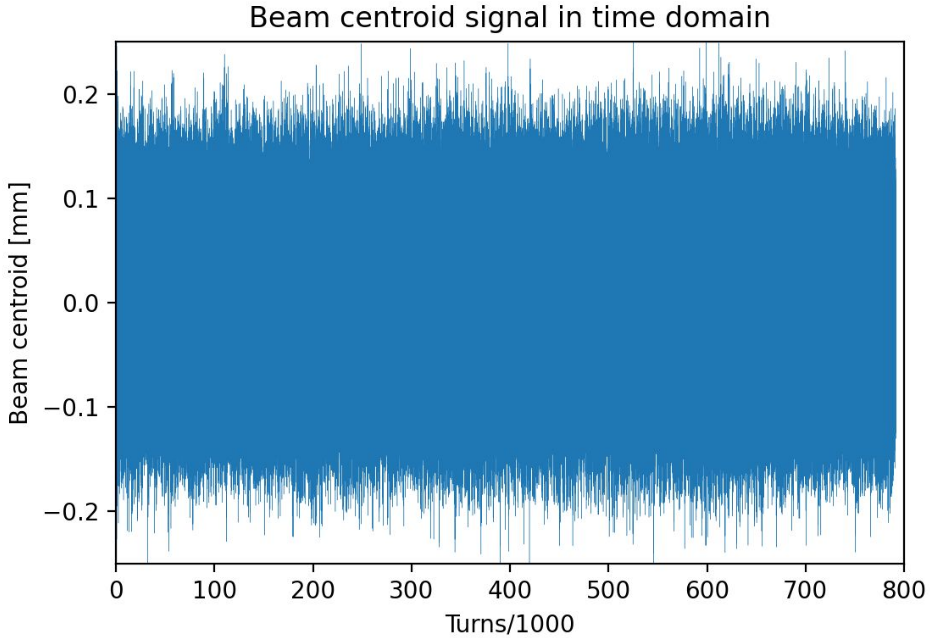
Data generation:

- Element by element tracking with XSuite
- Implementation of centroid monitor and beam size (https://github.com/xsuite/xtrack/pull/378)
- (Coasting) Beam sliced in longitudinal n-bins (user-defined)
- Data handling becomes less trivial -> 0.5GB generated for the beam centroid signal

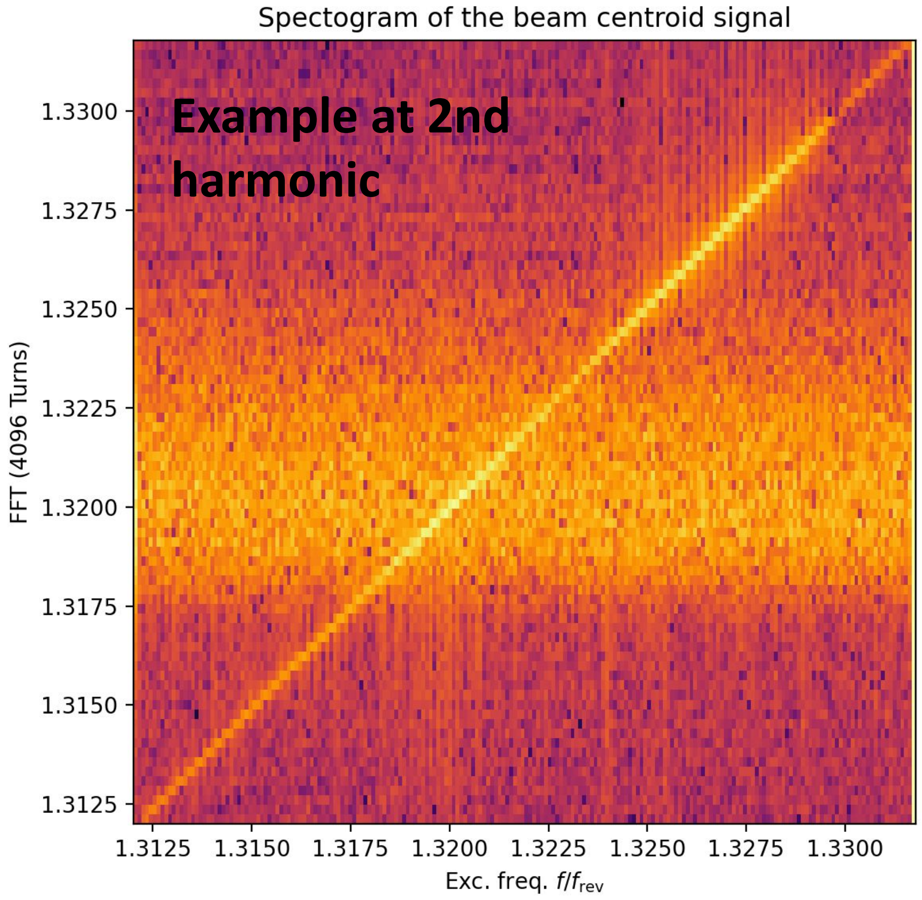
Simulation

Simulation results for the Heidelberg machine

Typical signal from simulation



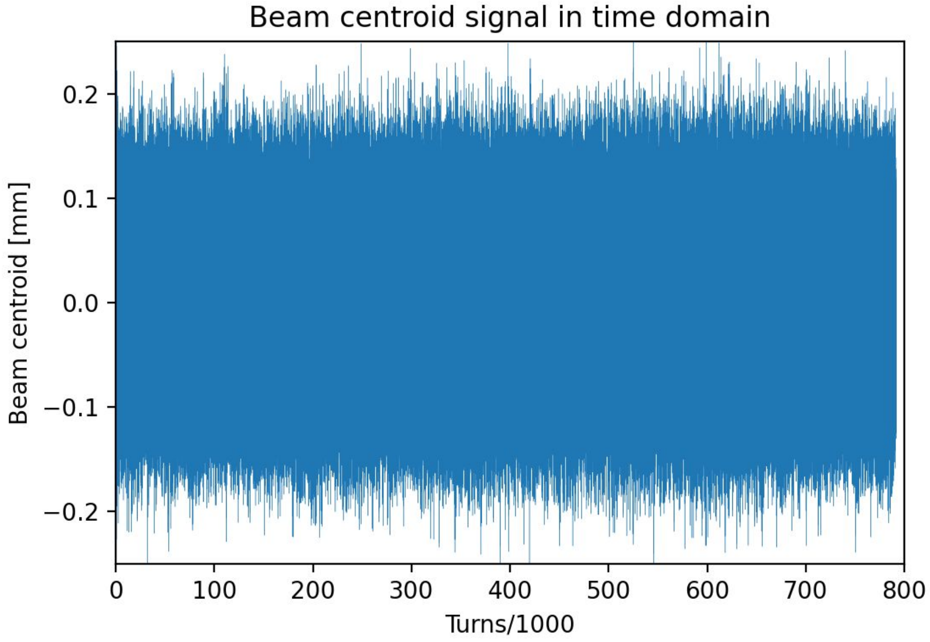
FFT



Simulation

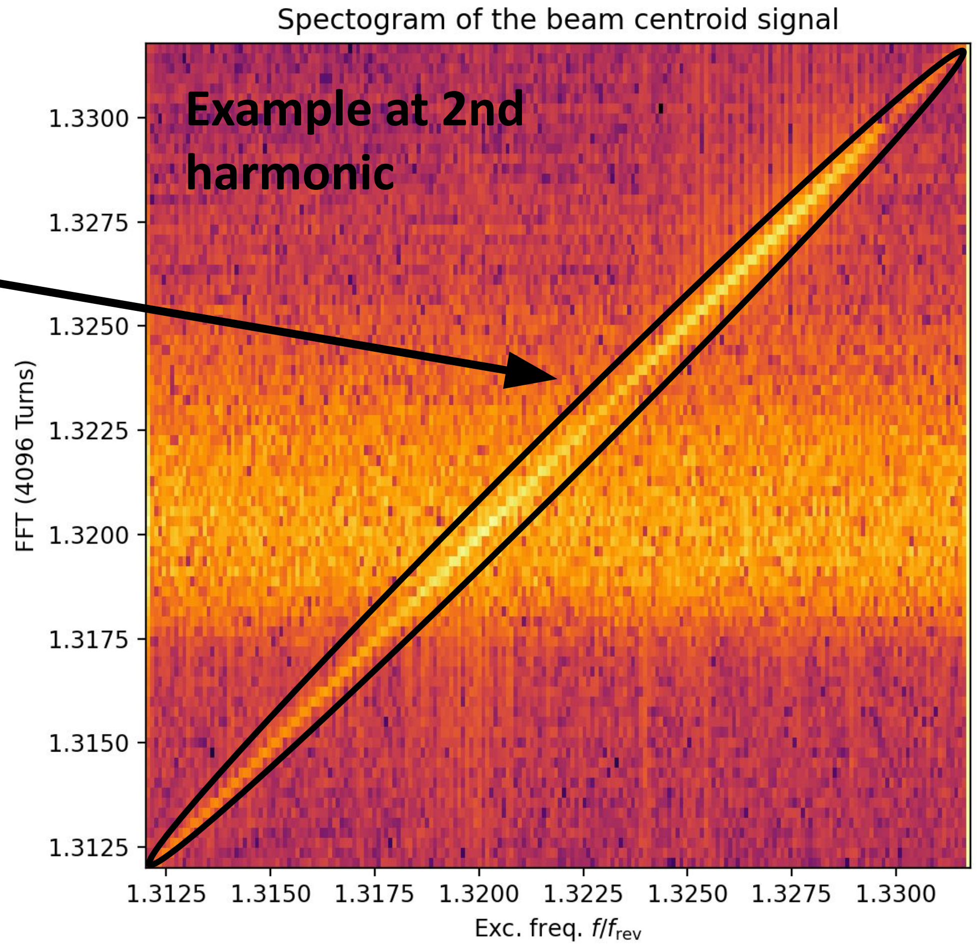
Simulation results for the Heidelberg machine

Typical signal from simulation



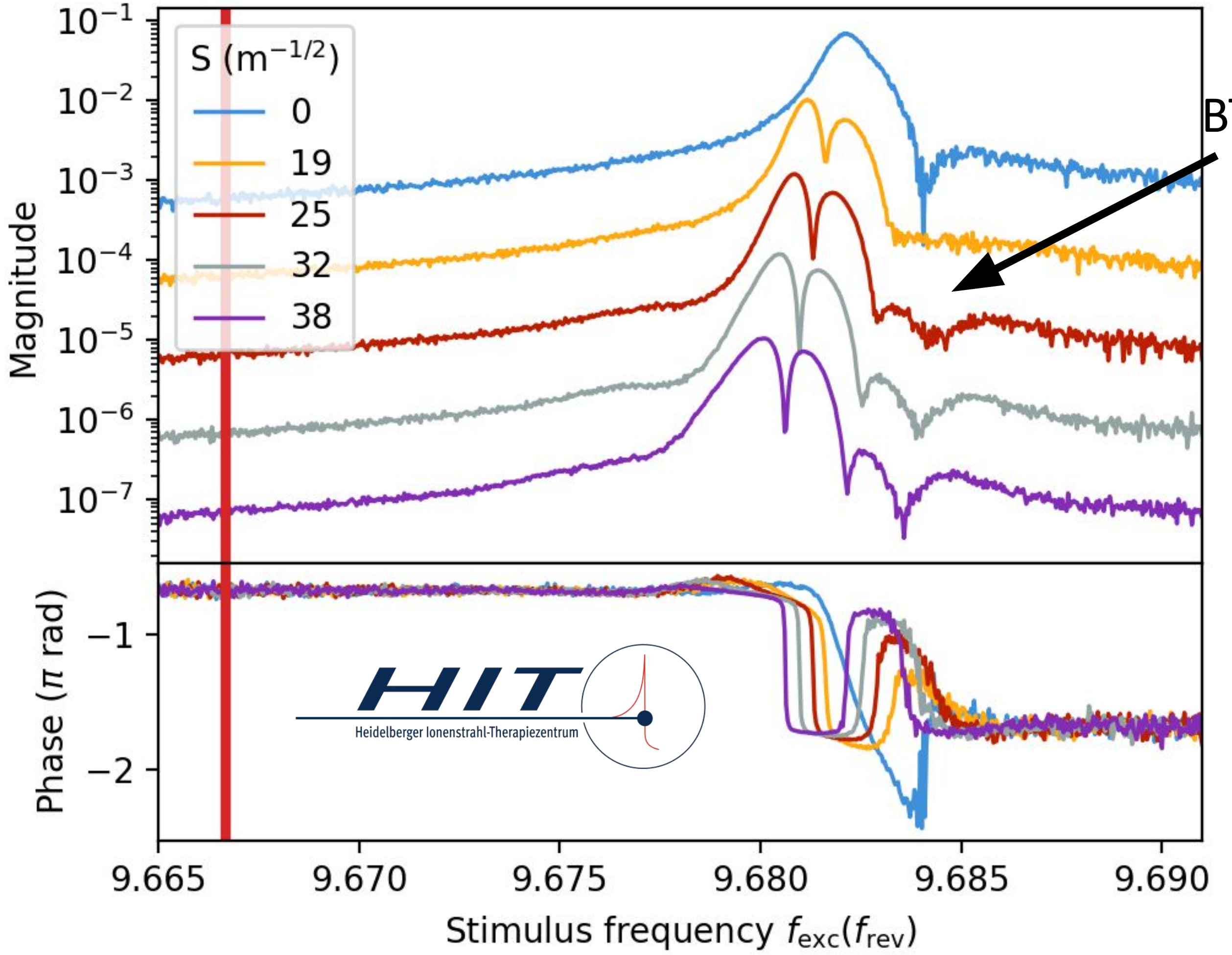
FFT

BTF signal

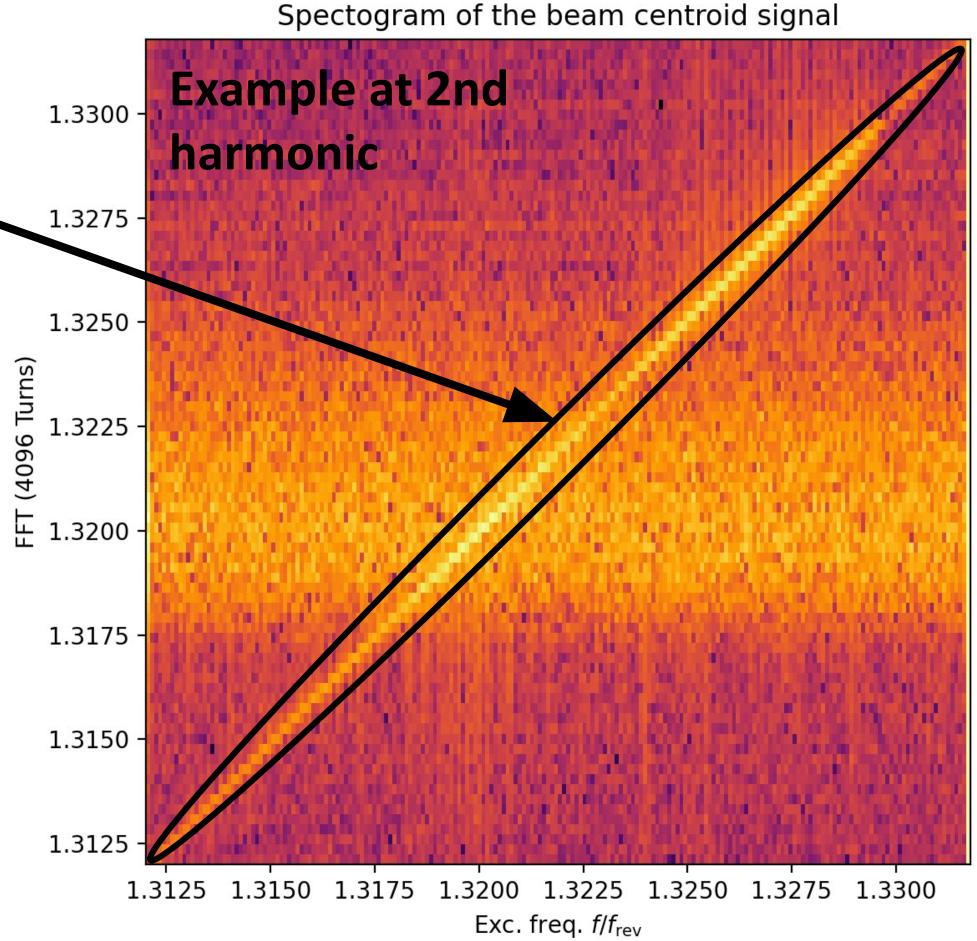


Simulation

Simulation results for the Heidelberg machine

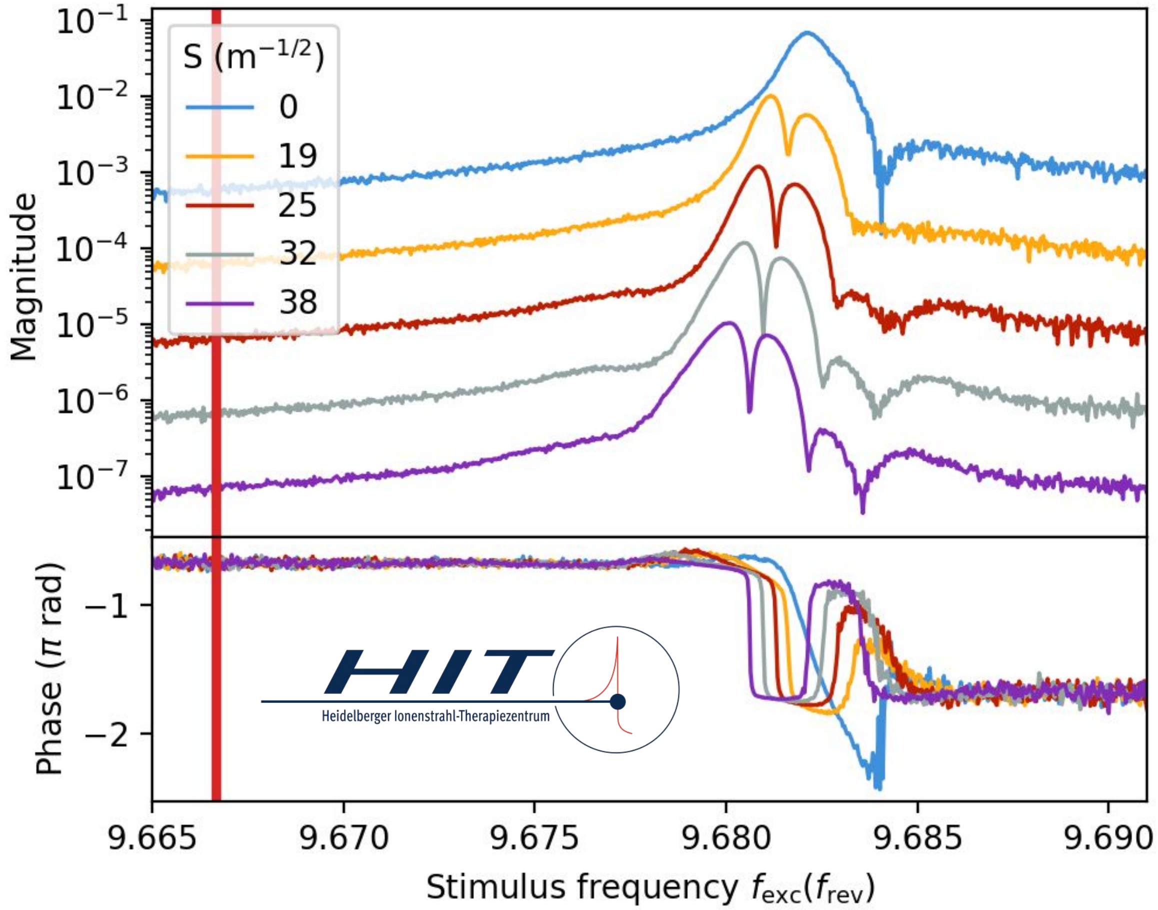


BTF signal



Simulation

Simulation results for the Heidelberg

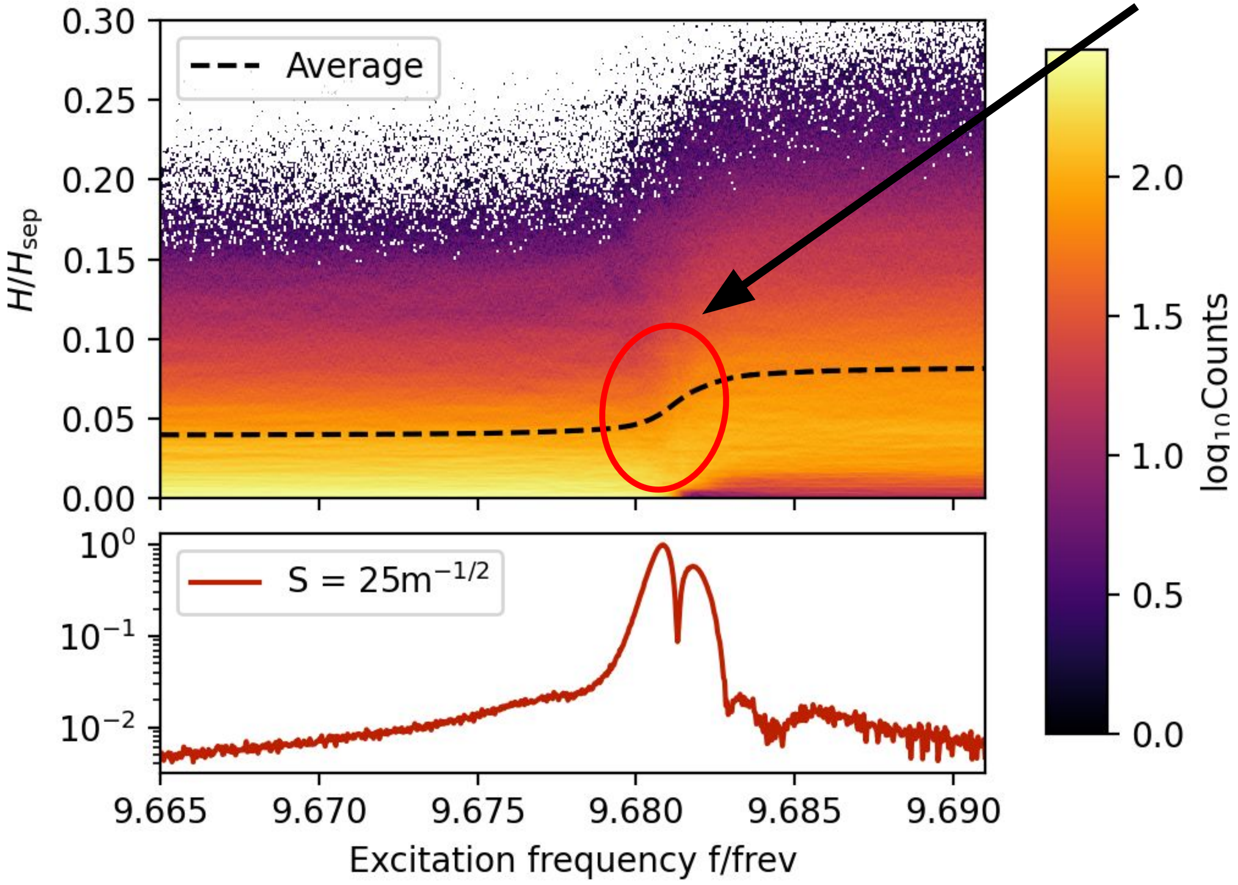


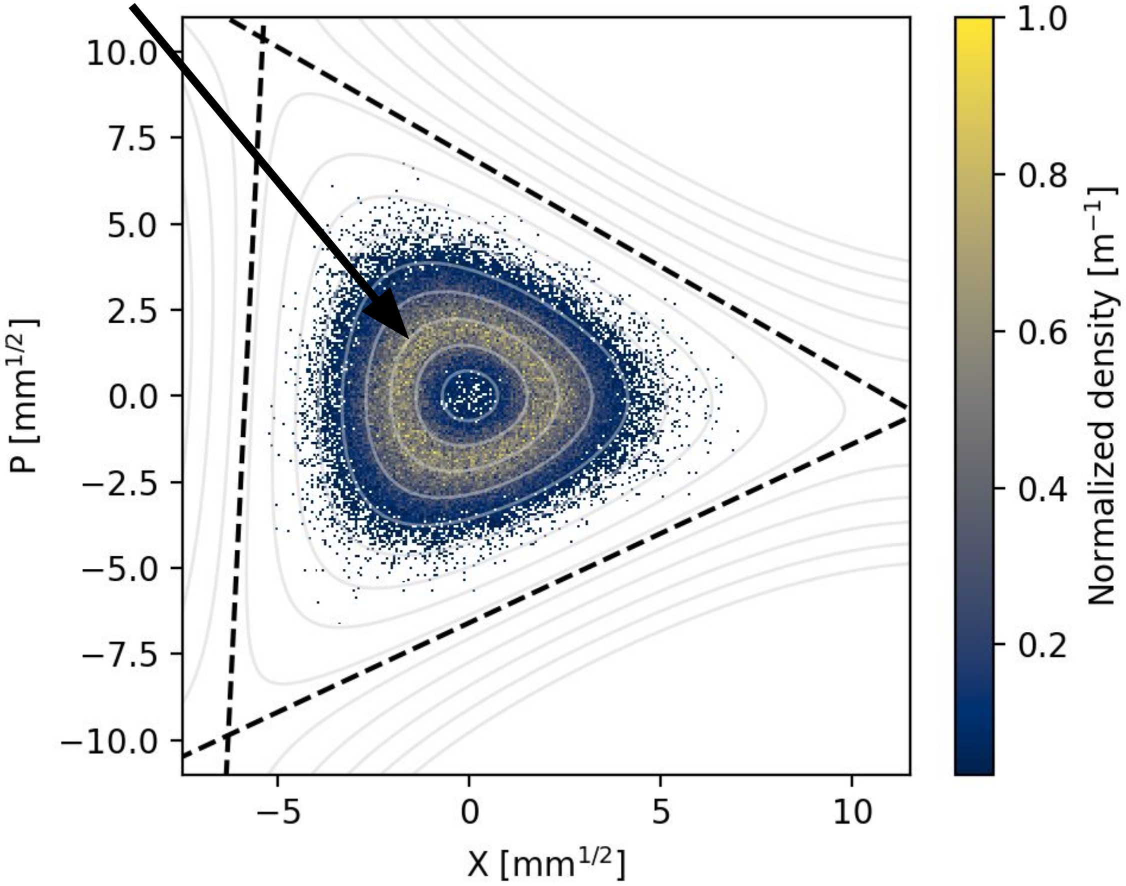
- Emittance is a free parameter but constrained
- Momentum spread is constrained but decapture influence is unknown (bunched -> coasting beam)
- Qualitative results agree with the simulation
- Excitation history plays an important role

Multi-particle dynamics

Effective Hamiltonian distribution during excitation process

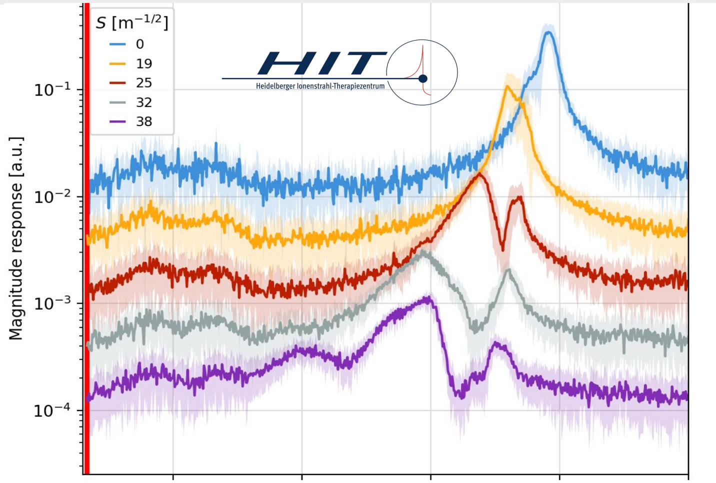
Shift and broadening of the distribution

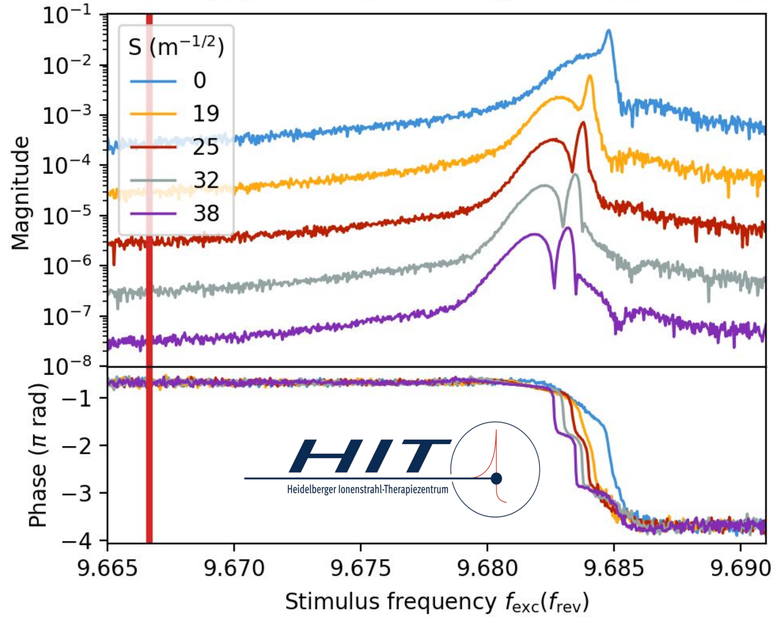




Summary

- There is a phase-amplitude detuning contribution
- The measured BTF signal splits asymmetrically towards the resonance into two peaks
- The simulation shows that there is an overall energy gain, which leads to a generation of a hollow beam
- Initial conditions are key to understanding the underlying non-linear dynamics





Many thanks to
P. Forck, the HIT and GSI machine operation teams

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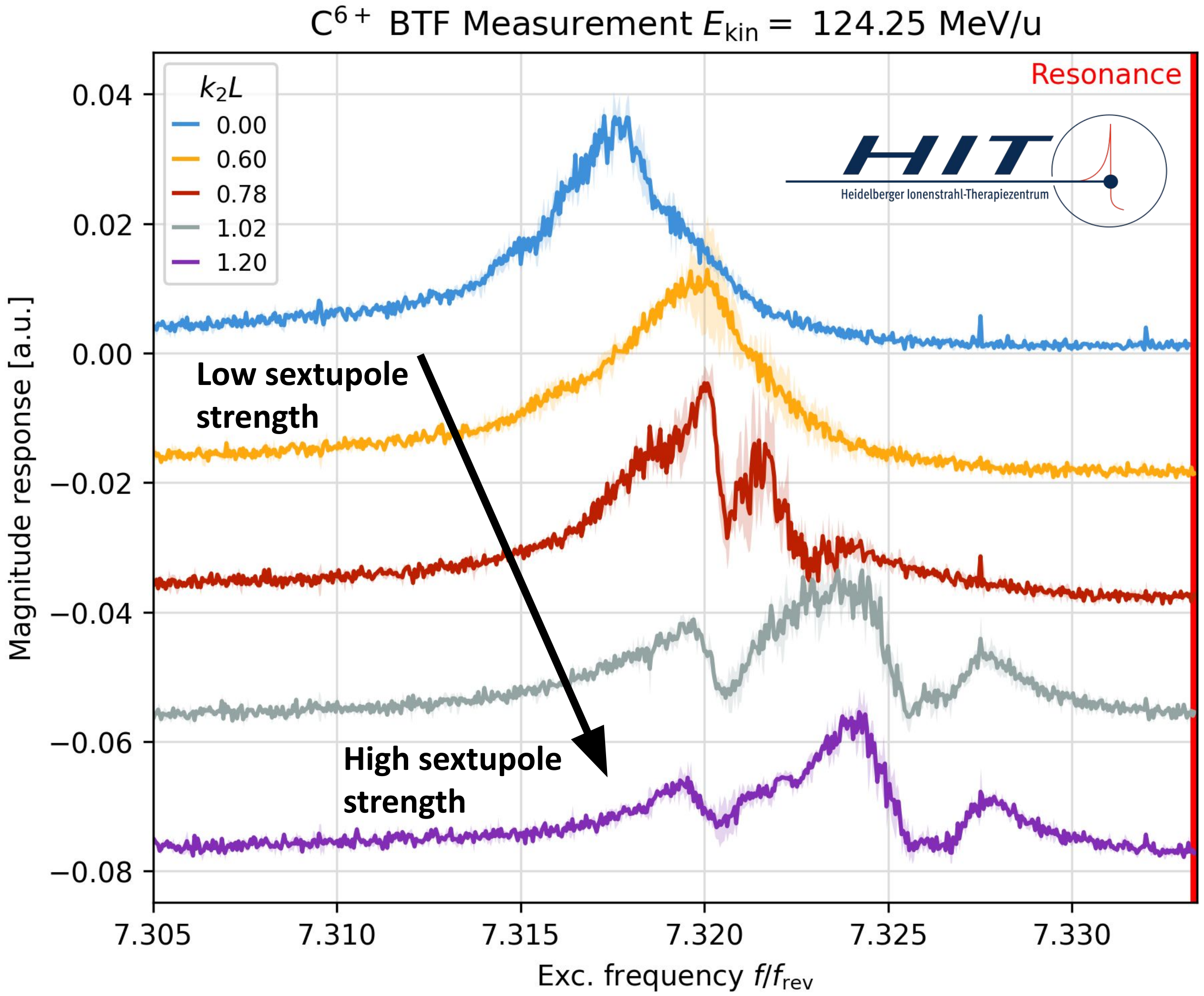
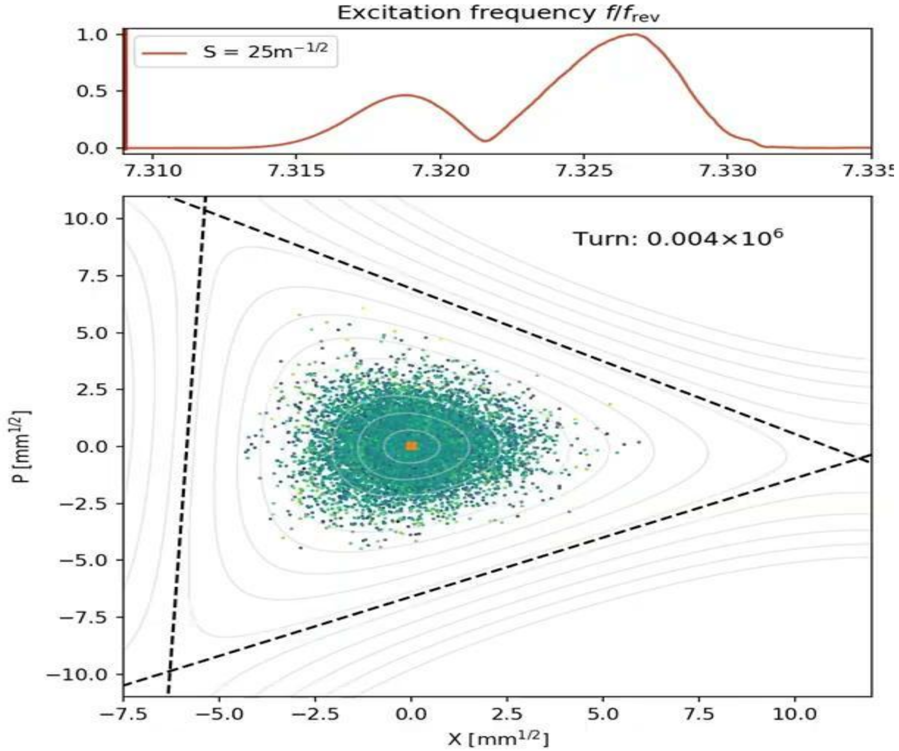
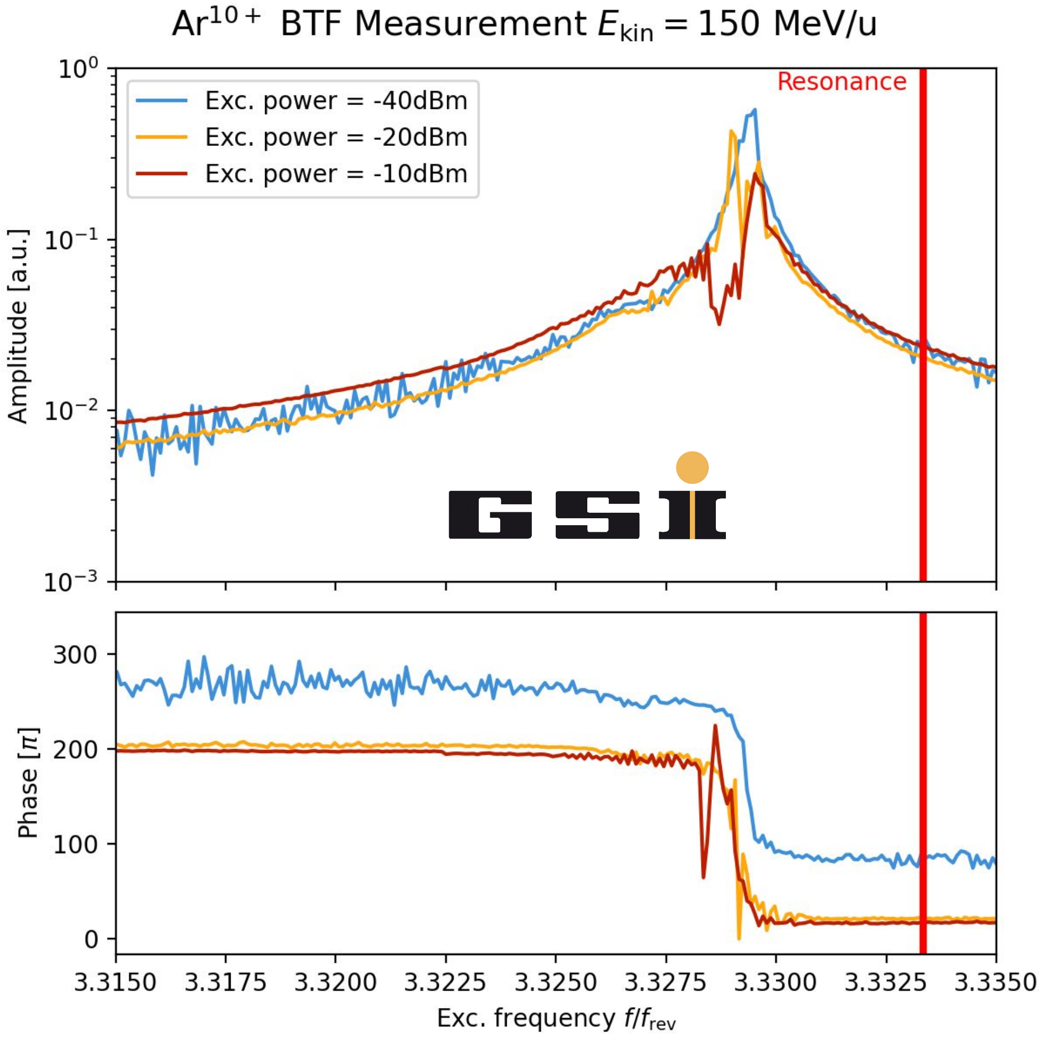
DESY, Notkestraße 85
D-22607 Hamburg

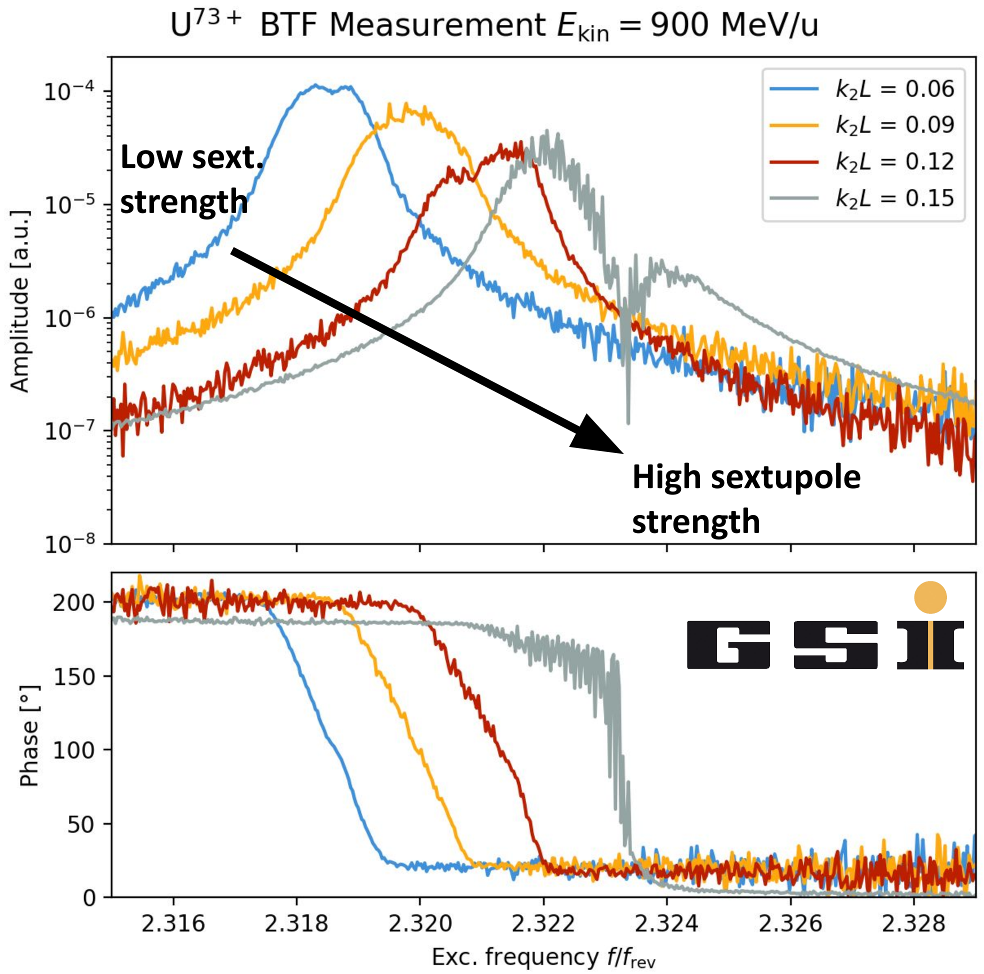
Simulation and measurement comparison





GSI measurements





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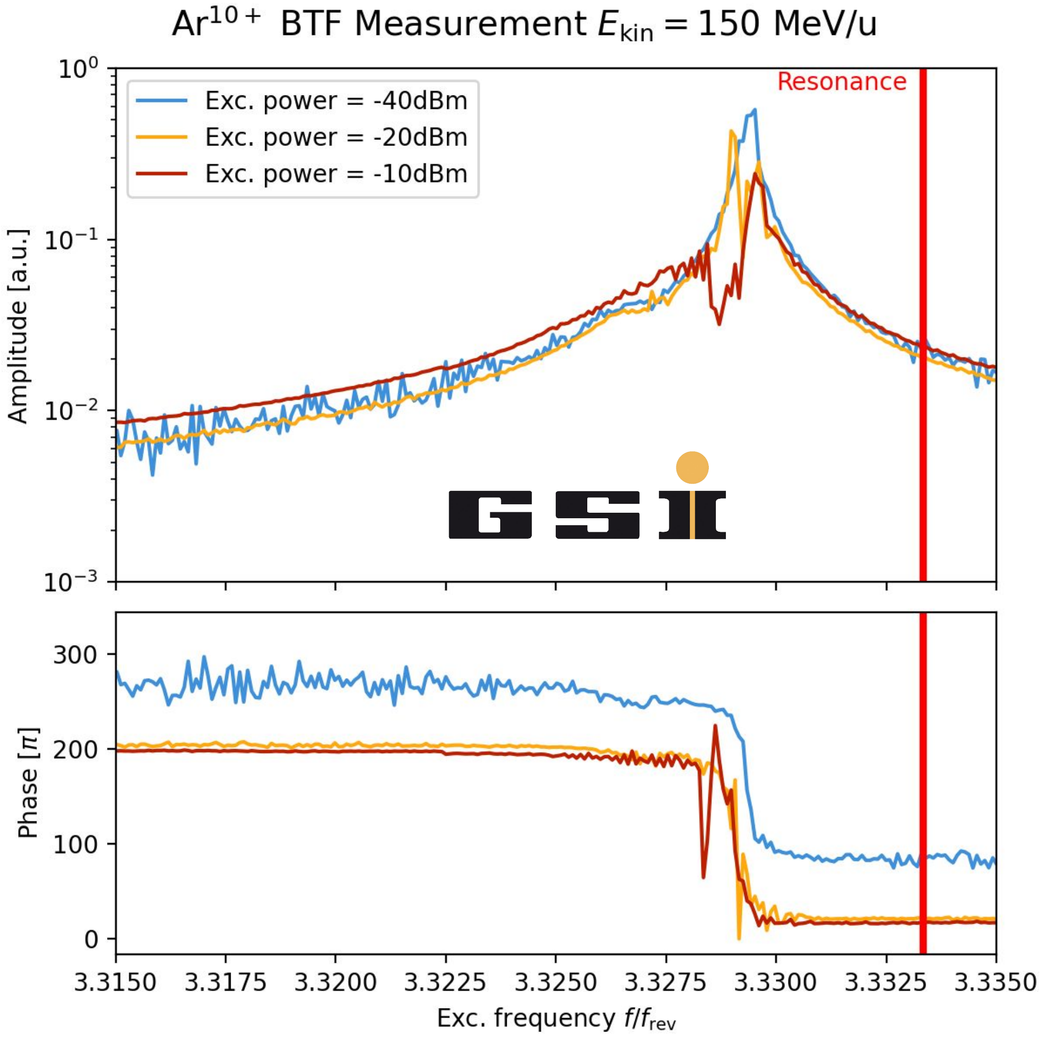
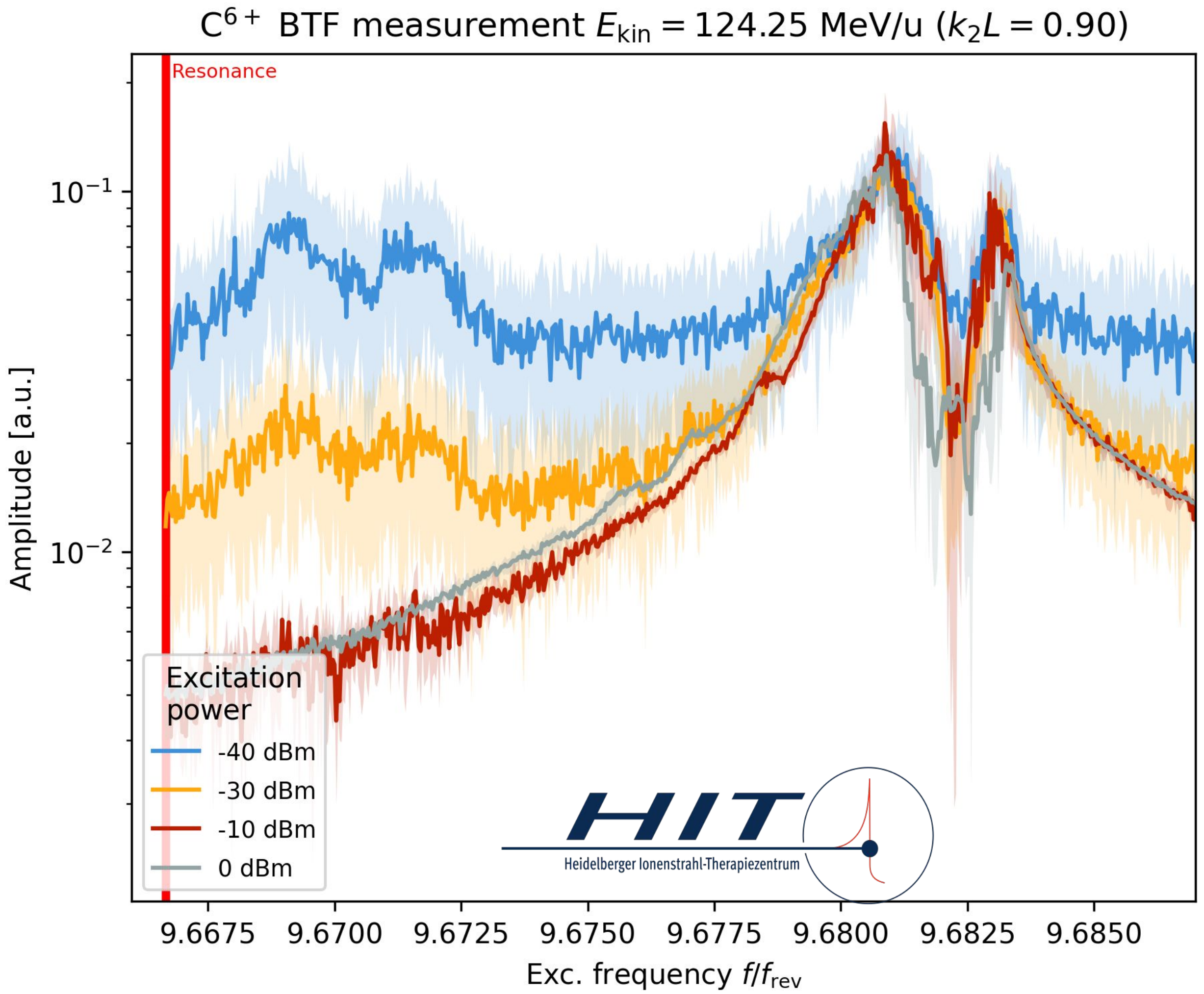
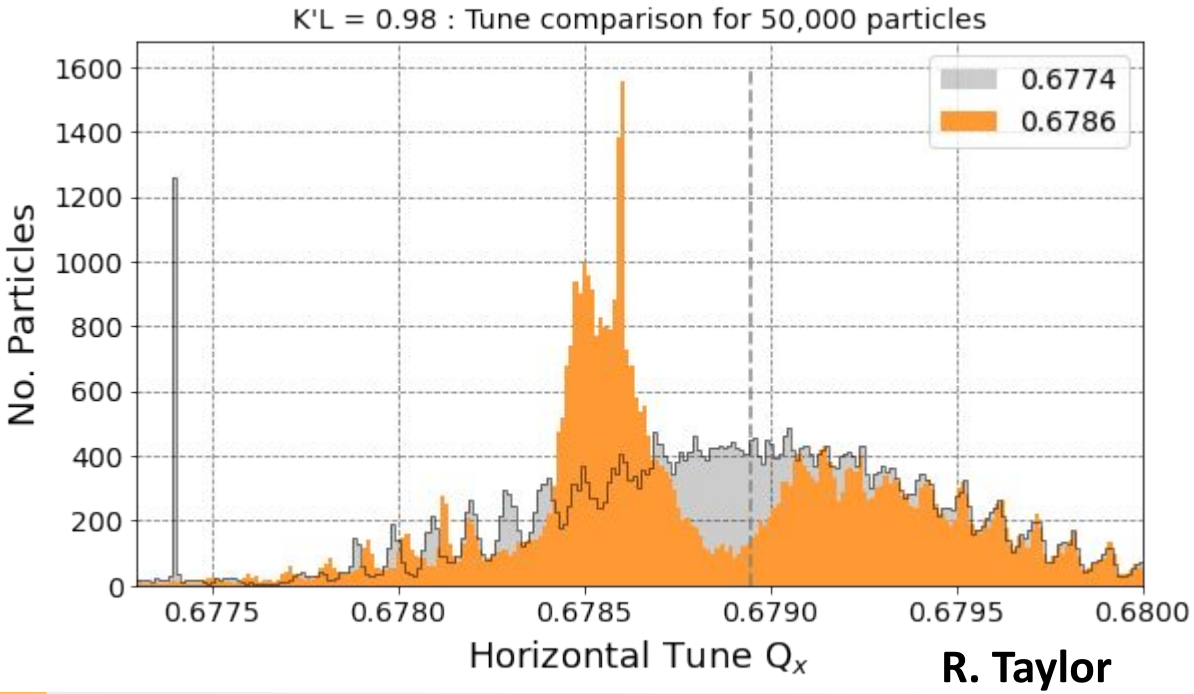


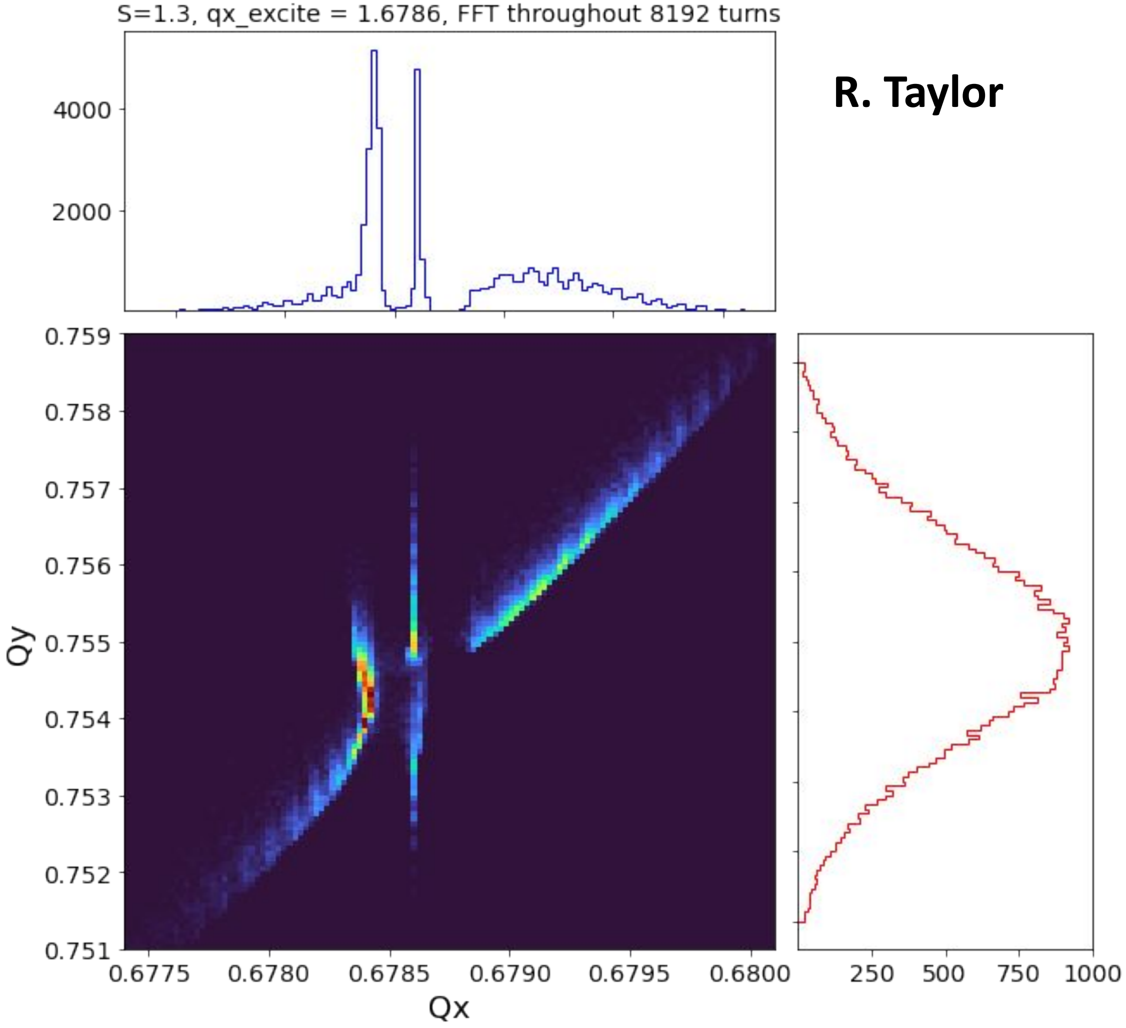


Maptrack simulation

- Single frequency excitation for 5 000 turns
- 10,000 particles
- Horizontal tune is the **fundamental frequency**
- Vertical tune distribution is not influenced
- Excitation frequency dominates the motion
- Kick strength 2 x 10^{-6} rad



R. Taylor



R. Taylor

Simulation and measurement comparison