Hyperon-nucleon interaction and light hypernuclei in chiral effective field theory

Johann Haidenbauer

IAS, Forschungszentrum Jülich, Germany

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(Hoai Le, Ulf-G. Meißner, Andreas Nogga)

Outline

1. Introduction
2. YN interaction in chiral effective field theory
3. Femtoscopy
4. Light Λ hypernuclei
5. Summary

Hyperon physics - recent developments

- Role of hyperons in neutron stars ("hyperon puzzle")
  Neutron stars with masses $\geq 2M_\odot \Rightarrow$ stiff equation of state (EoS)
  With increasing density $n \to \Lambda \Rightarrow$ softening of the EoS
  $\Rightarrow$ Conventional explanations of observed mass-radius relation fail
- New measurements of Λp cross sections by the CLAS Collaboration at JLab
  New extended measurements of ΣN observables in the E40 experiment at J-PARC
  differential cross sections for $\Sigma^+ p$, $\Sigma^- p$
- Measurements of two-particle momentum correlation functions by the STAR, HADES, and ALICE Collaborations
  (Λp, $\Lambda\Lambda$, $\Xi^- p$, ...)
- HAL QCD: Lattice QCD simulations for YN interactions for quark masses close to the physical point ($M_\pi \approx 145$ MeV)
- Progress in *ab initio* methods like no-core shell model (NCSM)
  microscopic calculations of hypernuclei up to $A \geq 10$
- New and precise value for the Λ decay parameter α

BB interaction in chiral effective field theory

Baryon-baryon interaction in SU(3) χEFT à la Weinberg (1990)

Advantages:

- Power counting
  systematic improvement by going to higher order
- Possibility to derive two- and three-baryon forces and external current operators in a consistent way

- degrees of freedom: octet baryons (*N*, Λ, Σ, Ξ), pseudoscalar mesons (π, *K*, η)

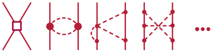
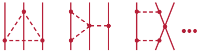
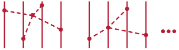
- pseudoscalar-meson exchanges
- contact terms – represent unresolved short-distance dynamics
  involve low-energy constants (LECs) that need to be fixed
  by a fit to data

Λ*N*-Σ*N* interaction
LO: H. Polinder, J.H., U.-G. Meißner, NPA 779 (2006) 244
NLO13: J.H., S. Petschauer, N. Kaiser, U.-G. Meißner, A. Nogga, W. Weise, NPA 915 (2013) 24
NLO19: J.H., U.-G. Meißner, A. Nogga, EPJA 56 (2020) 91
SMS NLO, NNLO: J.H., U.-G. Meißner, A. Nogga, H.Le, EPJA 59 (2023) 63

(*BB* systems with strangeness $S = -1$ to -6)

Extension of chiral EFT interaction up to NNLO

(Nucleon-nucleon forces in chiral EFT (E. Epelbaum))

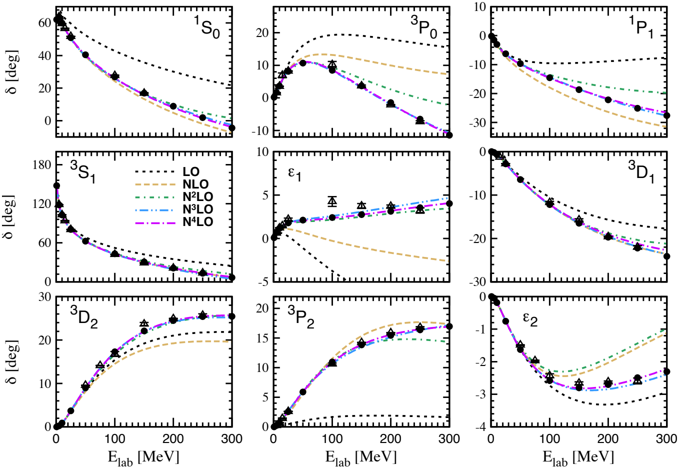
	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO (Q^0)		—	—
NLO (Q^2)		—	—
N^{2}LO (Q^3)			—
N^{3}LO (Q^4)			

N^{2}LO: no new (additional) LECs in the two-body sector

leading-order three-body forces (3BFs)

NN interaction in chiral EFT

Semilocal momentum-space (SMS) regularized chiral *NN* potential

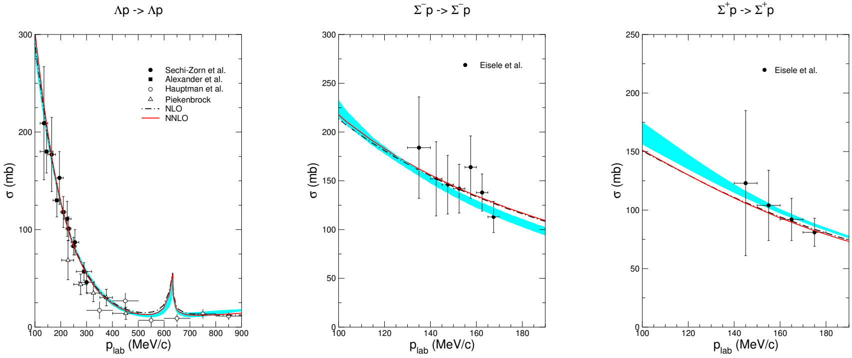


(Reinert, Krebs, Epelbaum, EPJA 54 (2018) 86) [up to N^{4}LO (N^{4}LO$^+$) !!]

LO to NLO: drastic change in all partial waves

NLO to N^{2}LO: changes mostly in *P*-waves and higher partial waves

Results for SMS chiral YN interactions



SMS YN potentials up to NLO, NNLO (with $\Lambda = 550$ MeV)

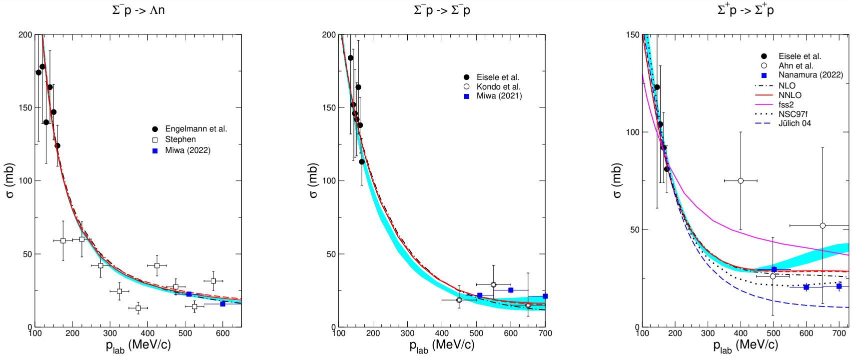
(J.H., U.-G. Meißner, A. Nogga, H. Le, EPJ A 59 (2023) 63)

NLO19: J.H., U.-G. Meißner, A. Nogga, EPJA 56 (2020) 91

quality of the fit – total χ^2 (36 data points):
NLO19(600): 16.0 SMS NLO: 15.2 SMS NNLO: 15.6

cross sections dominated by S-waves (are already well described at NLO)
$\rightarrow$ (as expected) practically no change when going to NNLO

Results for SMS YN interactions



integrated cross sections at higher energies not included in the fitting process!

$\Sigma^+ p \to \Sigma^+ p$ and $\Sigma^- p \to \Sigma^- p$ cross sections:

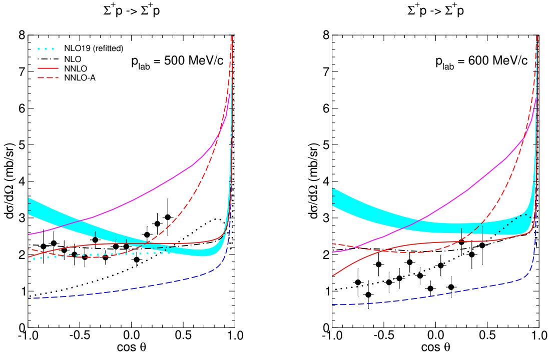
$$\sigma = \frac{2}{\cos\theta_{\rm max} - \cos\theta_{\rm min}} \int_{\cos\theta_{\rm min}}^{\cos\theta_{\rm max}} \frac{d\sigma(\theta)}{d\cos\theta}\, d\cos\theta$$

$\cos\theta_{\rm min} = -0.5; \;\cos\theta_{\rm max} = 0.5$

fss2 ... Fujiwara et al. (constitutent quark model)   Jülich 04, Nijmegen NSC97f ... meson-exchange potentials

Results for SMS YN interactions

Σ^+p (T. Nanamura et al., PTEP 2022 (2022) 093D01)



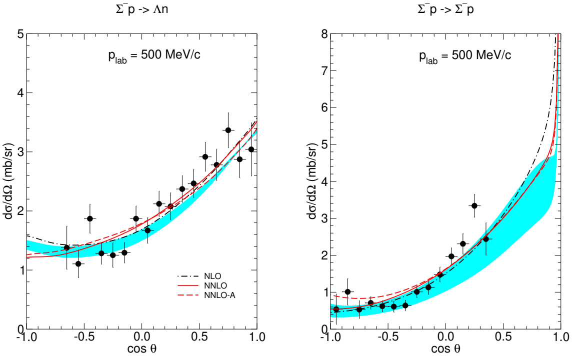
LECs in the $^1S_0$, $^3S_1$-$^3D_1$ fixed from low-energy cross sections

SMS NLO: LECs in ^3P-waves taken over from NN fit (RKE)
(strict SU(3) symmetry: $V_{NN} \equiv V_{\Sigma^+p}$ in the $^1S_0$, $^3P_{0,1,2}$ partial waves!)

SMS NNLO: LECs in P-waves fitted to the E40 data (two examples)!

data for $(550 \leq p \leq 650)$ MeV/c are overestimated (influence of $\Lambda p\pi^+$ threshold?)

Results for SMS YN interactions



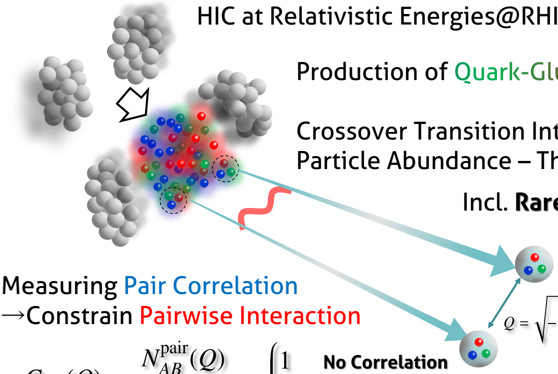
$\Sigma^- p \to \Lambda n$: quite well reproduced by NLO19 (NLO13) and SMS YN potentials
$\Sigma^- p \to \Sigma^- p$: behavior at forward angles remains unclear

$\Sigma^- p$ and $\Sigma^- p \to \Lambda n$ data for $(550 \leq p \leq 650)$ MeV/c are reproduced with comparable quality

- no unique determination of all P-wave LECs possible
- one needs data from additional channels (Λp, $\Sigma^- p \to \Sigma^0 n$, ...)
- one needs additional differential observables (polarizations, ...)

Two-particle correlation function (Kenji Morita)

How HIC Can Tell Us Interaction?



HIC at Relativistic Energies@RHIC, LHC

Production of Quark-Gluon Plasma

Crossover Transition Into Hadron
Particle Abundance – Thermal Eq.

Incl. **Rare Particles**

$$Q = \sqrt{-\left(\frac{p_1 - p_2}{2} - \frac{(p_1 - p_2)\cdot P}{P^2}P\right)^2}$$

Measuring Pair Correlation
→Constrain Pairwise Interaction

$$C_{AB}(Q) = \frac{N_{AB}^{\text{pair}}(Q)}{N_A N_B(Q)} = \begin{cases} 1 & \textbf{No Correlation} \\ \text{others} & \textbf{Interaction} \\ & \textbf{Interference etc} \end{cases}$$

Two-particle correlation function

Koonin-Pratt formalism

Correlation function for identical particles ($\Lambda\Lambda$, $\Sigma^+\Sigma^+$, ...)

$$C(k) \simeq 1 - \frac{1}{2}\exp(-4k^2R^2) + \frac{1}{2}\int_0^\infty 4\pi r^2\, dr\, S_{12}(\mathbf{r}) \left[|\psi(k,r)|^2 - |j_0(kr)|^2\right]$$

Correlation function for non-identical particles (Λp, $\Xi^- p$, $K^- p$, ...)

$$C(k) \simeq 1 + \int_0^\infty 4\pi r^2\, dr\, S_{12}(\mathbf{r}) \left[|\psi(k,r)|^2 - |j_0(kr)|^2\right]$$

Extension to multi-channel problem

$$|\psi(k,r)|^2 \to \sum_\beta \omega_\beta |\psi_{\beta\alpha}(k_\alpha, r)|^2$$

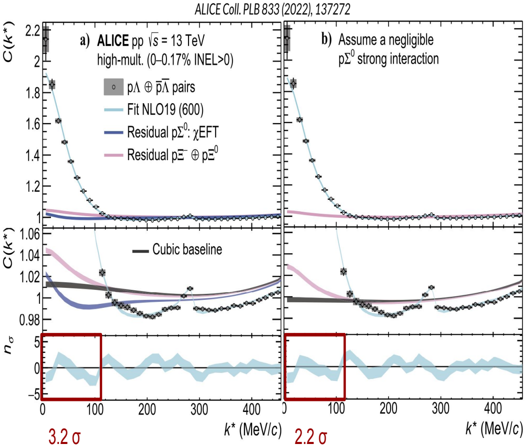
$$C_\alpha(k_\alpha) \simeq 1 + \sum_\beta \omega_\beta \int_0^\infty 4\pi r^2\, dr\, S_\beta(\mathbf{r}) \left[\left|\psi_{\beta\alpha}(k_\alpha, r)\right|^2 - \delta_{\beta\alpha}\, |j_0(k_\alpha r)|^2\right]$$

$\sum_\beta$ ... over all two-body intermediate states that couple to α

ω_β ... weights of the various components (often put to 1)

assume a static and spherical Gaussian source with radius R:
$S_{12}(\mathbf{r}) = \exp(-r^2/4R^2)/(2\sqrt{\pi}R)^3$

Λp momentum correlation function at $\sqrt{s} = 13$ TeV



ALICE Collaboration: *pp* collisions at 13 TeV (S. Acharya et al., PLB 833 (2022) 137272)

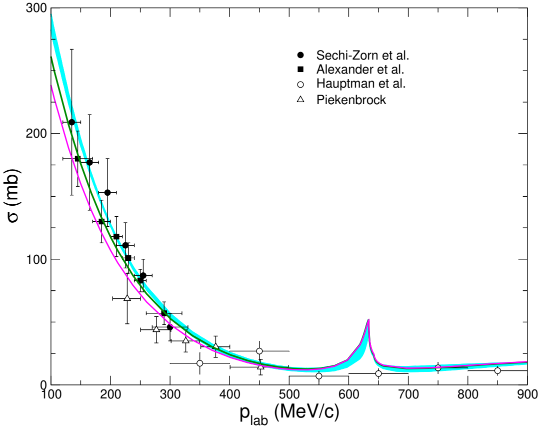
⇒ prediction of NLO19 is fairly well in line with data

sensitive to the assumption about the contribution of the $\Sigma^0 p$ feed-down

Λp: Slightly weaker energy dependence? Reduced overall strength?

Mihaylov & Gonzalez (arXiv:2305.08441): $a_t = -1.15 \pm 0.07$ fm

Reduced strength of the ΛN interaction in the $^3S_1$ state



NLO19(600) is used as starting point

$$a_t = -1.41 \text{ fm} \quad \Rightarrow \quad a_t = -1.30 \text{ fm} \quad [-1.15 \text{ fm}]$$

$$\chi^2 = 2.09 \quad \Rightarrow \quad \chi^2 = 3.45 \quad [7.14] \text{ (Sechi – Zorn)}$$

$$\chi^2 = 1.29 \quad \Rightarrow \quad \chi^2 = 1.15 \quad [6.00] \text{ (Alexander)}$$

$$n_\sigma = 3.2 \quad \Rightarrow \quad n_\sigma = 2.2 \text{ (with residual } \Sigma^0 p \text{ interaction included)}$$

(reduction in the $^1S_0$ state is limited since we want/need the $^3_\Lambda$H to be bound!)

Hypernuclei within the NCSM

ab initio no-core shell model (NCSM)

Basic idea: use harmonic oscillator states and soft interactions

- m-scheme uses single particle states (center-of-mass motion not separated)
- antisymmetrization for nucleons easily performed (Slater determinant)
- larger dimensions (applications to *p*-shell hypernuclei by Wirth & Roth)

Jacobi-NCSM

- uses relative (Jacobi) coordinates (Hoai Le et al., EPJA 56 (2020) 301)
- explicit separation of center-of-mass motion possible
- antisymmetrization for nucleons difficult but feasible for $A \leq 9$
- small dimensions

Soft interactions: Similarity renormalization group (SRG) (unitary transformation)

$$\frac{dH(s)}{ds} = [[T, H(s)], H(s)] \qquad H(s) = T + V(s) \qquad V(s) : V^{NN}(s), V^{YN}(s)$$

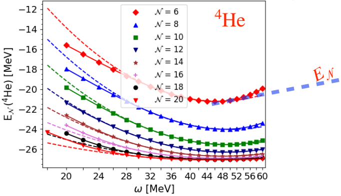
- Flow equations are solved in momentum space
- parameter (cutoff) $\lambda = \left(4\mu_{BN}^2/s\right)^{1/4}$ is a measure of the width of the interaction in momentum space
- $V(s)$ is phase equivalent to original interaction
- transformation leads to induced 3BFs, 4BFs, ...

  (induced 3BFs included in the work of Wirth & Roth and in our recent studies)
  (induced 4BFs are most likely very small)

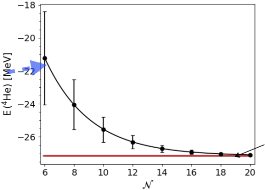
Procedure

slide from Hoai Le:

- **extrapolation of energies:**
  - $E_b(\omega, \mathcal{N}) = E_{\mathcal{N}} + \kappa(\log(\omega) - \log(\omega_{opt}))^2$



  - $E_{\mathcal{N}} = E_\infty + Ae^{-b\mathcal{N}}$



NN: SMS $\mathrm{N^4LO^+}(450)$

$\lambda = 7\ \mathrm{fm}^{-1}$

$E_{FY} = -27.15 \pm 0.02$ MeV

$E_\infty = -27.146 \pm 0.062$ MeV

$\boxed{\delta E = E_\infty - E_{\mathcal{N}_{max}}}$

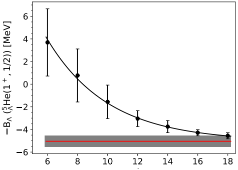
- lowest $E_{\mathcal{N},\omega_{opt}}$ are used for $\mathcal{N}$-space extrapolation ✓
- estimated uncertainties are rather conservative

- **extrapolation of Λ separation energies:** $B_\Lambda = E_{nucl} - E_{hyp}$
  - strong correlations between $E_{nucl}(\mathcal{N})$, $E_{hypnucl}(\mathcal{N})$

$\rightarrow$ $B_{\Lambda,\mathcal{N}} = E_{nucl}(\mathcal{N}) - E_{hypnucl}(\mathcal{N})$

$$\boxed{B_{\Lambda,\mathcal{N}} = B_{\Lambda,\infty} + A_1 e^{-b_1\mathcal{N}}}$$

HL, J. Haidenbauer, U.-G. Meißner, A. Nogga EPJA 56 (2020)



YN: SMS $\mathrm{N^2LO}(550)$

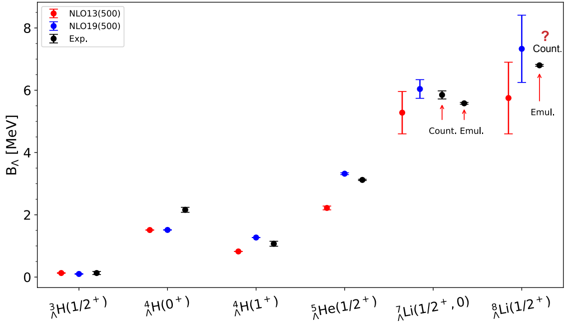
$\lambda_{YN} = 7\ \mathrm{fm}^{-1}$

Results for $B_\Lambda (A \le 8)$

Hoai Le et al., PRC 107 (2023) 024002

- **NLO13** and **NLO19** are almost **phase equivalent** in the 2-body sector
- **NLO13** characterised by a stronger $\Lambda N - \Sigma N$ transition potential (especially in $^3S_1$)

→ **manifest in higher-body observables** **(J. Haidenbauer et al. NPA 915 (2019))**



NN:SMS N^{4}LO+(450)

+3N: N^{2}LO(450)

+YN: NLO13,19(CSB)

+SRG-induced YNN

Experiment:

M. Agnello et al. PLB 681

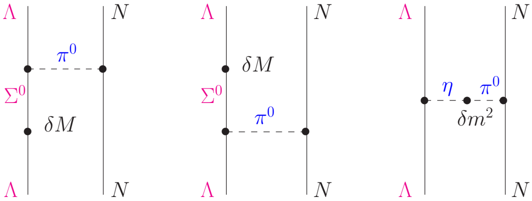
M. Juric NPB 52(1973)

→ $^{4}_{\Lambda}\mathrm{H}(1^{+})$, $^{5}_{\Lambda}\mathrm{He}$, $^{7}_{\Lambda}\mathrm{Li}$, $^{8}_{\Lambda}\mathrm{Li}$ are fairly well described by **NLO19**;

NLO13 underestimates separation energies

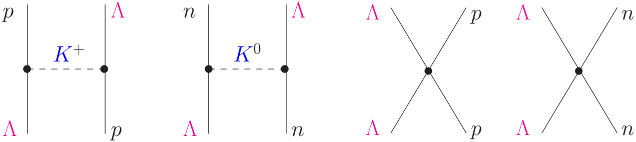
clear signal for (missing) chiral *Y NN* forces

Charge symmetry breaking in the ΛN interaction



CSB due to $\Lambda - \Sigma^0$ mixing: long-ranged contribution to the ΛN interaction

(R.H. Dalitz & F. von Hippel, PL 10 (1964) 153)



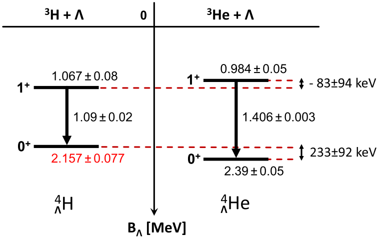
CSB in chiral EFT: additional short-range contributions ⇒ contact terms

(NN: Epelbaum, Glöckle, Meißner, NPA 747 (2005) 362; etc.)

J.H., U.-G. Meißner, A. Nogga, FBS 62 (2021) 105

Charge symmetry breaking in ${}^4_\Lambda$H-${}^4_\Lambda$He

- $\Delta E(0^+) = E^{0^+}_\Lambda({}^4_\Lambda\mathrm{He}) - E^{0^+}_\Lambda({}^4_\Lambda\mathrm{H})$
  $= 233 \pm 92$ keV
- $\Delta E(1^+) = E^{1^+}_\Lambda({}^4_\Lambda\mathrm{He}) - E^{1^+}_\Lambda({}^4_\Lambda\mathrm{H})$
  $= -83 \pm 94$ keV



adjust CSB contact terms to ΔE's

(Schulz et al., 2016; Yamamoto et al., 2015)

(fm // keV)	$a_s^{\Lambda p}$	$a_s^{\Lambda n}$	$a_t^{\Lambda p}$	$a_t^{\Lambda n}$	$\Delta E(0^+)$	$\Delta E(1^+)$
NLO19(500)	-2.649	-3.202	-1.580	-1.467	249	-75
NLO19(550)	-2.640	-3.205	-1.524	-1.407	252	-72
NLO19(600)	-2.632	-3.227	-1.473	-1.362	243	-67
NLO19(650)	-2.620	-3.225	-1.464	-1.365	250	-69

CSB in singlet (${}^1S_0$) much larger than in triplet (${}^3S_1$)
practically independent of cutoff; same results for NLO13
without CSB: $a_s^{\Lambda p} \approx a_s^{\Lambda n} \approx -2.9$ fm

- CSB in $A = 7, 8$ Λ-hypernuclei (Hoai Le et al., PRC 107 (2023) 024002)

Summary

Hyperon-nucleon interaction within chiral EFT

- ΛN-ΣN interaction within semilocal momentum-space regularized chiral EFT
  confirm our previous YN results (up to NLO) based on a nonlocal regulator
  successful extension to NNLO
  new $\Sigma^\pm p$ differential cross sections around $p_{lab} \approx 500$ MeV/c can be described
  unique determination of the P-waves is not yet possible

Hypernuclei

- three-body forces: should be small for (${}^3_\Lambda\mathrm{H}$) or moderate (${}^4_\Lambda\mathrm{H}$, ${}^4_\Lambda\mathrm{He}$, ${}^5_\Lambda\mathrm{He}$)
  needs to be quantified/confirmed by explicit inclusion of 3BFs
- charge-symmetry breaking in ${}^4_\Lambda\mathrm{H} - {}^4_\Lambda\mathrm{He}$
  can be reproduced when taking into account the full leading CSB potential within chiral EFT
- charge-symmetry breaking in $A = 7 - 8$ Λ-hypernuclei
  predicted CSB splitting for ${}^7_\Lambda$Be, ${}^7_\Lambda$Li*, ${}^7_\Lambda$He is in line with experiments
  CSB splitting for ${}^8_\Lambda$Be, ${}^8_\Lambda$Li is overestimated

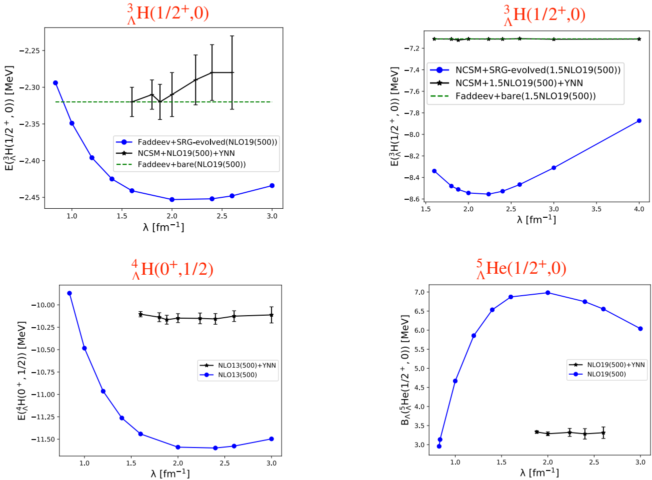
Λp momentum correlation functions

- ALICE measurement: indications that the Λp is possibly somewhat weaker than what the cross section data from the 1960ies suggest

backup slides

A=3-5 Λ hypernuclei with SRG-induced YNN force

Hoai Le, EPJ Web Conf. 271 (2022) 01004 (HYP2022)



NN:SMS N^{4}LO+(450)

3N: N^{2}LO(450)

⇒ contributions of SRG-induced *YNNN* forces are negligible

(R. Wirth, R. Roth, PRL 117 (2016); PRC 100 (2019))

Separation energies for A=3-8 Λ hypernuclei (MeV)

- NLO13 and NLO19 are practically phase equivalent ($\chi^2 \approx 16$ for 36 YN data points)
- NLO13 characterized by a stronger ΛN-ΣN coupling potential (${}^3S_1$-${}^3D_1$)

	${}^3_\Lambda\mathrm{H}$ [Faddeev]	${}^4_\Lambda\mathrm{H}(0^+)$	${}^4_\Lambda\mathrm{H}(1^+)$	${}^5_\Lambda\mathrm{He}$	${}^7_\Lambda\mathrm{Li}$	${}^8_\Lambda\mathrm{Li}$
NLO13	0.135	1.55 ± 0.01	0.82 ± 0.01	2.22 ± 0.06	5.28 ± 0.68	5.75 ± 1.08
NLO19	0.100	1.51 ± 0.01	1.27 ± 0.01	3.32 ± 0.03	6.04 ± 0.30	7.33 ± 1.15
Exp.	0.13 ± 0.05 0.41 ± 0.12 [S] 0.072 ± 0.063 [A]	2.16 ± 0.08	1.07 ± 0.08	3.12 ± 0.02	5.85 ± 0.13 5.58 ± 0.03	6.80 ± 0.03

NN: SMS N^{4}LO+(450) + 3NF: N^{2}LO(450) + SRG-induced YNN force
[S] ... STAR Collaboration, [A] ... ALICE Collaboration

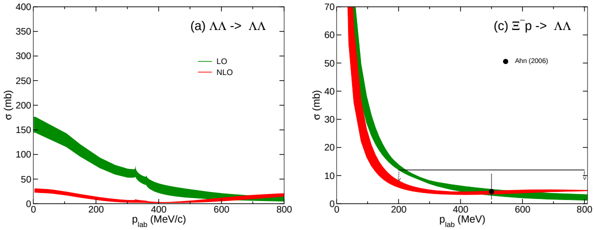
NLO19 (500): ${}^4_\Lambda\mathrm{H}(1^+)$, ${}^5_\Lambda\mathrm{He}$, ${}^7_\Lambda\mathrm{Li}$ fairly well described
NLO13 (500) underestimates the separation energies

clear signal for (missing) chiral YNN forces:

in (standard) chiral EFT 3BFs appear at N^{2}LO
with decuplet saturation at NLO (LECs: 1 ΛNN + 1 ΣNN)

→ could be fixed from separation energies of, e.g.,
${}^4_\Lambda$H (0^+, 1^+) or ${}^4_\Lambda$H (0^+, 1^+), ${}^5_\Lambda$He

Selected results for $S = -2$



ΛΛ effective range parameters

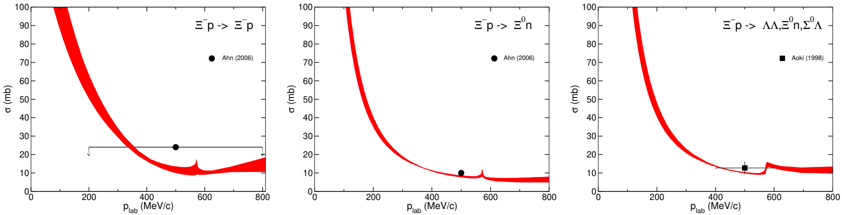
	NLO				LO			
Λ	500	550	600	650	550	600	650	700
$a_{^1S0}$	−0.62	−0.61	−0.66	−0.70	−1.52	−1.52	−1.54	−1.67
$r_{^1S0}$	7.00	6.06	5.05	4.56	0.82	0.59	0.31	0.34

empirical: $a_{\Lambda\Lambda}$ = -1.2 ± 0.6 fm (Gasparyan et al.) $-1.92 < a_{\Lambda\Lambda} < -0.50$ fm (A. Ohnishi et al.)

J.H., U.-G. Meißner, S. Petschauer, NPA 954 (2016) 273

Selected results for the ΞN system

(J.K. Ahn et al., PLB 633 (2006) 214; S. Aoki et al., NPA 644 (1998) 365)



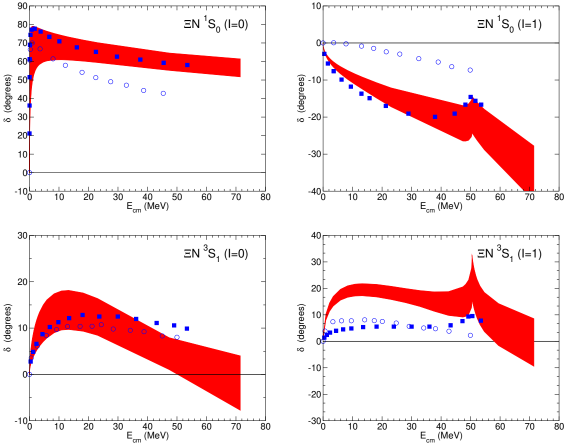
ΞN scattering lengths [in fm]:

	$I=0,\ ^1S_0$	$I=1,\ ^1S_0$		$I=0,\ ^3S_1$		$I=1,\ ^3S_1$	
potential	a_s	a_s	r_s	a_t	r_t	a_t	r_t
NLO (500)	-7.71-i2.03	0.37	-4.80	-0.33	-6.86	-1.17	3.44
NLO (550)	-7.24-i20.79	0.39	-4.95	-0.39	-1.77	-1.15	3.80
NLO (600)	-10.89-i14.91	0.34	-7.20	-0.62	1.00	-1.13	3.95
NLO (650)	-8.14-i2.43	0.31	-9.16	-0.85	1.42	-0.90	4.27

- scattering lengths $|a| \lesssim 1$ fm, except for $I=0,\ ^1S_0$
- ΞN interaction is fairly weak

J.H., U.-G. Meißner, EPJA 58 (2019) 23

ΞN: Comparison with HAL QCD results



HAL QCD Collaboration (almost at physical point, $m_\pi \approx 145$ MeV):
open circles from E. Hiyama et al., PRL 124 (2020) 092501 (no $\Lambda\Sigma$, $\Sigma\Sigma$)
filled squares from M. Kohno & K. Miyagawa, PTEP 2021 (2021) 103D04

Nuclear matter properties

$U_\Xi(p_\Xi = 0)$ [in MeV] at saturation density, $k_F = 1.35\ \mathrm{fm}^{-1}$ $(\rho_0 = 0.166\ \mathrm{fm}^{-3})$

potential	I	$^1S_0$	$^3S_1$	S-waves	P-waves	total
NLO (500)	0	-2.6	-3.3			
	1	12.7	-11.8	-5.0	-0.4	-5.5
NLO (550)	0	-2.9	-3.1			
	1	12.4	-9.5	-3.1	-0.7	-3.8
NLO (550)*	0	-3.15	-3.24			
	1	9.64	-11.0	-7.7	-1.1	-8.8
HAL QCD	0	-3.15	-5.36			
	1	7.12	-2.41	-4.11	-	-4.11
Ehime	0	-0.80	0.47			
(1.82)	1	-1.5	-8.6	-10.43	-11.4	-21.8

"traditional" value for the depth of the Ξ single-particle potential: ≈ -15 MeV

E. Friedman & A. Gal (optical potential, PLB 820 (2021) 136555): $U_\Xi \leq -20$ MeV

Y. Tanimura et al. (relativistic mean field, PRC 105 (2022) 044324): $U_\Xi \approx -12$ MeV

(from analyzing $^{15}_{\Xi}$C and $^{12}_{\Xi}$Be events)

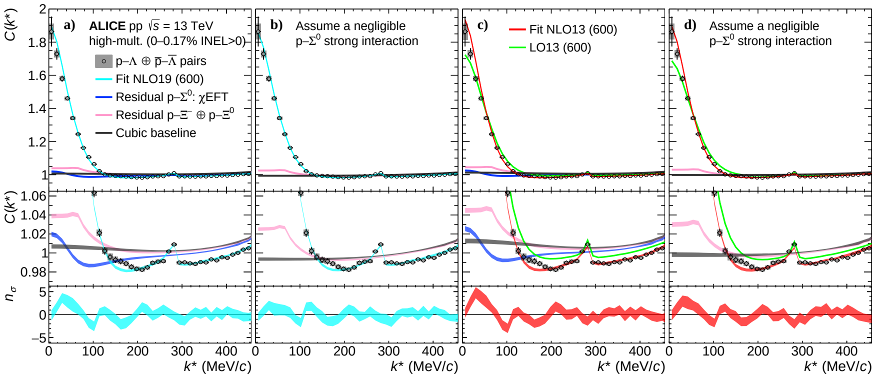
NLO (550)* : M. Kohno, PRC 100 (2019) 024313 (continuous prescription)

HAL QCD: T. Inoue, AIP Conf. Proc. 2130 (2019) 020002

Ehime: M. Yamaguchi et al., PTP 105 (2001) 627

Femtoscopic studies by ALICE at LHC/CERN

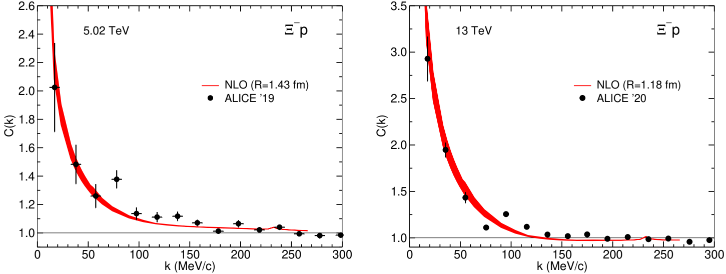
Λp momentum correlation function measured in *pp* collisions at $\sqrt{s} = 13$ TeV



ALICE Collaboration (Shreyasi Acharya et al.), arXiv:2104.04427

⇒ prediction of NLO19 is fairly well in line with data
sensitive to the assumption about the contribution of the $\Sigma^0 p$ feed-down
"true" Λp amplitude could have slightly weaker energy dependence
(a_t could be about $10 - 15$ % smaller; $\simeq -1.3$ fm instead of $\simeq -1.5$ fm)

ΞN: two-particle momentum correlation functions



$C_{\rm th}(k) = \frac{1}{4} C_{1_{S_0}}(k) + \frac{3}{4} C_{3_{S_1}}(k);$ $\quad C_\alpha(k) \simeq 1 + \int_0^\infty 4\pi r^2\, dr\, S_{12}(\mathbf{r}) \left[|\psi(k,r)|^2 - |j_0(kr)|^2 \right]$

$C(k) = (a + b\,k)\,(1 + \lambda\,(C_{\rm th}(k) - 1));$ $\quad S_{12}(\mathbf{r}) = \exp(-r^2/4R^2)/(2\sqrt{\pi}R)^3$

a, b, λ, R ... additional parameters that need to be determined (→ talk of Yuki Kamiya)

ALICE Collaboration: p-Pb at 5.02 TeV (PRL 123 (2019) 112002) $\quad$ pp at 13 TeV (Nature 588 (2020) 232)

$R = 1.427$ fm; $\lambda = 0.513$ $\quad$ $R = 1.02$ fm; $\lambda = 1$

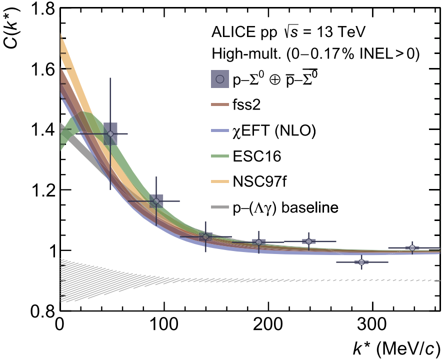
we adopt $R = 1.427$ fm & 1.18 fm, respectively
(same source radii as found in corresponding fits to *pp* correlation functions)

(J.H., U.-G. Meißner, arXiv:2201.08238)

Y. Kamiya et al., PRC 105 (2022) 014915, using HAL QCD potential: $R = 1.27$ fm & 1.05 fm
Z.-W. Liu et al., arXiv:2201.04997, cov. χEFT mimicking the HAL QCD potential: $R = 1.427$ fm & 1.182 fm

Femtoscopic studies by ALICE at LHC/CERN

$\Sigma^0 p$ momentum correlation function measured in *pp* collisions at $\sqrt{s} = 13$ TeV

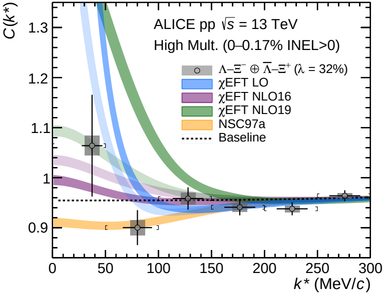


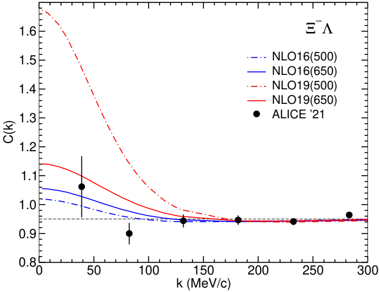
ALICE Collaboration (Shreyasi Acharya et al.), PLB 805 (2020) 135419

open channels ($\Sigma^+ n$, Λp) make theoretical analysis more complicated,

cf. J.H., NPA 981 (2019) 1

Results for $S = -3$: $\Lambda\Xi^-$





ALICE Collaboration, arXiv:2204.10258: *pp* at 13 TeV

$R = 1.03$ fm; $\lambda = 0.36$

LO potential (J.H., U.-G. Meißner, PLB 684 (2010) 275):
produces a bound state $\longrightarrow$ not supported by measurement

LO rel. χEFT potential (Z.-W. Liu et al., PRC 103 (2021) 025201): likewise too attractive

NLO19:

$a_s = -0.99 \cdots - 0.89$ fm, $r_s = 4.63 \cdots 5.77$ fm; $a_t = -0.42 \cdots -1.66$ fm, $r_t = 6.33 \cdots 1.49$ fm

NLO16:

$a_s = -0.99 \cdots - 0.89$ fm, $r_s = 4.63 \cdots 5.77$ fm; $a_t = 0.026 \cdots -0.12$ fm, $r_t = 32.0 \cdots 702$ fm

(J.H., U.-G. Meißner, arXiv:2201.08238)

$SU(3)$ structure of contact terms for BB

$SU(3)$ structure for scattering of two octet baryons $\rightarrow$
$8 \otimes 8 = 1 \oplus 8_a \oplus 8_s \oplus 10^* \oplus 10 \oplus 27$

BB interaction can be given in terms of LECs corresponding to the $SU(3)_f$ irreducible representations: C^1, C^{8a}, C^{8s}, C^{10^*}, C^{10}, C^{27}

	Channel	I	V_α	V_β	$V_{\beta\to\alpha}$
$S = \ \ 0$	$NN \rightarrow NN$	0	–	$C^{10^*}_\beta$	–
	$NN \rightarrow NN$	1	C^{27}_α	–	–
$S = -1$	$\Lambda N \rightarrow \Lambda N$	$\frac{1}{2}$	$\frac{1}{10}\left(9C^{27}_\alpha + C^{8_s}_\alpha\right)$	$\frac{1}{2}\left(C^{8_a}_\beta + C^{10^*}_\beta\right)$	$-C^{8_{sa}}$
	$\Lambda N \rightarrow \Sigma N$	$\frac{1}{2}$	$\frac{3}{10}\left(-C^{27}_\alpha + C^{8_s}_\alpha\right)$	$\frac{1}{2}\left(-C^{8_a}_\beta + C^{10^*}_\beta\right)$	$-3C^{8_{sa}}$
					$C^{8_{sa}}$
	$\Sigma N \rightarrow \Sigma N$	$\frac{1}{2}$	$\frac{1}{10}\left(C^{27}_\alpha + 9C^{8_s}_\alpha\right)$	$\frac{1}{2}\left(C^{8_a}_\beta + C^{10^*}_\beta\right)$	$3C^{8_{sa}}$
	$\Sigma N \rightarrow \Sigma N$	$\frac{3}{2}$	C^{27}_α	C^{10}_β	–

$\alpha = {}^1S_0, {}^3P_0, {}^3P_1, {}^3P_2$, $\beta = {}^3S_1, {}^3S_1 - {}^3D_1, {}^1P_1$

No. of contact terms: LO: 2 (NN) + 3 (YN) + 1 (YY)
NLO: 7 (NN) + 11 (YN) + 4 (YY)

(No. of spin-isospin channels in NN+YN: 10 $S = -2, -3, -4$: 27)

Contact terms for YN – partial-wave projected

spin-momentum structure up to NLO

$$\begin{aligned}
V(^1S_0) &= \tilde{C}_{^1S_0} + C_{^1S_0}(p^2 + p'^2) \\
V(^3S_1) &= \tilde{C}_{^3S_1} + C_{^3S_1}(p^2 + p'^2) \\
V(\alpha) &= C_\alpha \, p \, p' \qquad \alpha \hat{=} \, ^1P_1, \; ^3P_0, \; ^3P_1, \; ^3P_2 \\
V(^3D_1 - \, ^3S_1) &= C_{^3S_1 - \, ^3D_1} \, p'^2
\end{aligned}$$

$$\begin{aligned}
V(^1P_1 - \, ^3P_1) &= C_{^1P_1 - \, ^3P_1} \, p \, p' \\
V(^3P_1 - \, ^1P_1) &= C_{^3P_1 - \, ^1P_1} \, p \, p'
\end{aligned}$$

(antisymmetric spin-orbit force: $(\vec{\sigma}_1 - \vec{\sigma}_2) \cdot (\vec{q} \times \vec{k})$)

- $\tilde{C}_\alpha$, C_α ... low-energy constants (LECs)
- need to be fixed by a fit to (NN, YN, ...) data

chiral YN potential up to NNLO

Reinert, Krebs, Epelbaum, EPJA 54 (2018) 86

- Λ: 350 – 550 MeV ... 450 MeV give best results

YN interaction: approximate SU(3) flavor symmetry
$m_\pi = 138$ MeV, $m_K = 495$ MeV, $m_\eta = 547$ MeV

want to keep effects from SU(3) symmetry breaking generated by the single-meson exchange contributions
⇒ Λ: 500 – 600 MeV

two-meson exchange contributions: πK, $\pi\eta$, ... are represented by contact terms

⇒ some SU(3) symmetry breaking in the YN LECs

(S. Petschauer, N. Kaiser, NPA 916 (2013) 1)

$$V^{cont} = \tilde{C}^\alpha + C^\alpha(p^2 + p'^2) + C^\chi(m_K^2 - m_\pi^2)$$

$\tilde{C}^\alpha$, C^α, $\alpha = \{27\}, \{10^*\}, \{10\}, \{8_s\}, \{8_a\}, \{1\}$, ... "regular" contact terms in SU(3) chiral EFT
C_i^χ: SU(3) symmetry breaking contact terms

(in NLO13 and NLO19 ΛN-ΣN potentials we assumed that $C_i^\chi = 0$)

chiral YN potential up to NNLO

adopt the framework of Reinert, Krebs, Epelbaum, EPJA 54 (2018) 86:

"Semilocal momentum-space regularized (SMS) chiral NN potentials"

- employ a regulator that minimizes artifacts from cutoff Λ

nonlocal cutoff $(\vec{q} = \vec{p}\,' - \vec{p})$

$$V_{1\pi}^{\mathrm{reg}} \propto \frac{e^{-\frac{p'^4+p^4}{\Lambda^4}}}{\vec{q}^{\,2}+m_\pi^2} \to \frac{1}{\vec{q}^{\,2}+m_\pi^2}\left[1-\frac{p'^4+p^4}{\Lambda^4}+\mathcal{O}(\Lambda^{-8})\right]$$

local cutoff:

$$V_{1\pi}^{\mathrm{reg}} \propto \frac{e^{-\frac{\vec{q}^{\,2}+m_\pi^2}{\Lambda^2}}}{\vec{q}^{\,2}+m_\pi^2} \to \frac{1}{\vec{q}^{\,2}+m_\pi^2} - \frac{1}{\Lambda^2} + \frac{\vec{q}^{\,2}+m_\pi^2}{\Lambda^4} + \ldots$$

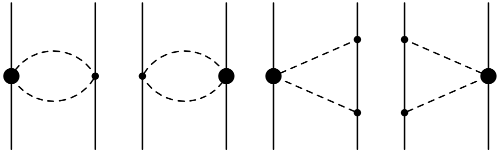
does not affect long-range physics at any order in the $1/\Lambda^2$ expansion

applicable to 2π exchange too:

$$V_{2\pi} = \frac{2}{\pi}\int_{2m_\pi}^{\infty} \mu d\mu \frac{\rho(\mu)}{\vec{q}^{\,2}+\mu^2} \to V_{2\pi}^{\mathrm{reg}} = e^{-\frac{\vec{q}^{\,2}}{2\Lambda^2}} \frac{2}{\pi}\int_{2m_\pi}^{\infty} \mu d\mu \frac{\rho(\mu)}{\vec{q}^{\,2}+\mu^2} e^{-\frac{\mu^2}{2\Lambda^2}} \quad + \ldots$$

chiral YN interaction up to NNLO

- no new BB contact terms (no additional LECs) enter
- sub-leading meson-baryon vertices enter at NNLO



πN: fixed from calculating pion-nucleon scattering in chiral perturbation theory
sub-leading (up to Q^2) πN LECs: $c_1 = -0.74;\ c_3 = -3.61;\ c_4 = 2.44$

(cf. RKE 2018)

$\pi\Lambda$, $\pi\Sigma$, $\pi\Lambda \leftrightarrow \pi\Sigma$:
involve additional LECs: d_1, d_2, d_3, b_D, b_F, b_0, b_1, b_2, b_3, b_4
fixed from resonance saturation via decuplet baryons ($\Sigma^*(1385)$)

(cf. Petschauer et al., NPA 957 (2017) 347)

Coupled channels Lippmann-Schwinger Equation

$$\begin{aligned} T^{\nu'\nu,J}_{\rho'\rho}(p',p) &= V^{\nu'\nu,J}_{\rho'\rho}(p',p) \\ &+ \sum_{\rho'',\nu''}\int_0^\infty \frac{dp''p''^2}{(2\pi)^3}\, V^{\nu'\nu'',J}_{\rho'\rho''}(p',p'')\frac{2\mu_{\rho''}}{p^2-p''^2+i\eta}\, T^{\nu''\nu,J}_{\rho''\rho}(p'',p) \end{aligned}$$

ρ', ρ = ΛN, ΣN ($\Lambda\Lambda$, ΞN, $\Lambda\Sigma$, $\Sigma\Sigma$)

LS equation is solved for particle channels (in momentum space)

Coulomb interaction is included via the Vincent-Phatak method

SMS: A nonlocal regulator is applied to the contact terms

$$V^{\nu'\nu,J}_{\rho'\rho}(p',p) \to f^\Lambda(p')\,V^{\nu'\nu,J}_{\rho'\rho}(p',p)f^\Lambda(p); \quad f^\Lambda(p) = e^{-(p/\Lambda)^2}$$

consider values $\Lambda = 500$ - 600 MeV [guided by *NN*, achieved χ^2]

NLO19 (NLO13): A a nonlocal regulator is applied to the whole potential

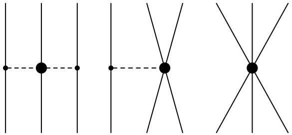
$$V^{\nu'\nu,J}_{\rho'\rho}(p',p) \to f^\Lambda(p')\,V^{\nu'\nu,J}_{\rho'\rho}(p',p)f^\Lambda(p); \quad f^\Lambda(p) = e^{-(p/\Lambda)^4}$$

with values $\Lambda = 500$ - 650 MeV

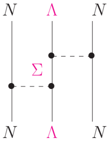
Three-body forces

- SU(3) χEFT 3BFs at NNLO (S. Petschauer et al., PRC 93 (2016) 014001)
- however, 5 LECs for ΛNN 3BF alone! (only 2 LECs for NNN)

SU(3) χEFT :

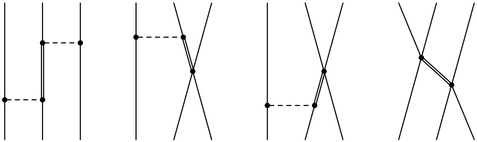


(at LO!) :



solve coupled channel (ΛN-ΣN) Faddeev-Yakubovsky equations:
$\Rightarrow$ ΛNN “3BF” from Σ coupling is automatically included

- 3BFs with inclusion of decuplet baryons (S. Petschauer et al., NPA 957 (2017) 347)



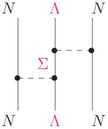
estimate ΛNN 3BF based on the $\Sigma^*(1385)$ excitation (appear at NLO!)

- only 1 LEC for ΛNN (2 LECs for YNN in general)

Estimation of 3BFs based on NLO results

- ${}^{3}_{\Lambda}\mathrm{H}$
  (a) cutoff variation: ΔE_Λ (3BF) $\leq$ 50 keV
  (b) “3BF” from ΛN-ΣN coupling:

  switch off ΛN-ΣN coupling
  in Faddeev-Yakubovsky equations:
  ΔE_Λ (3BF) $\approx$ 10 keV
  expect similar/smaller ΔE_Λ from $\Sigma^*(1385)$ excitation



  (c) ${}^{3}\mathrm{H}$: 3NF $\sim Q^3 \, |\langle V_{NN}\rangle|_{{}^{3}\mathrm{H}} \sim$ 650 keV
  ($|\langle V_{NN}\rangle|_{{}^{3}\mathrm{H}} \sim$ 50 MeV; $Q \sim m_\pi/\Lambda_b$; $\Lambda_b \simeq 600$ MeV)
  ${}^{3}_{\Lambda}\mathrm{H}$: $|\langle V_{\Lambda N}\rangle|_{{}^{3}_{\Lambda}\mathrm{H}} \sim$ 3 MeV $\rightarrow \Delta E_\Lambda$ (3BF) $\approx Q^3 \, |\langle V_{\Lambda N}\rangle|_{{}^{3}_{\Lambda}\mathrm{H}} \simeq$ 40 keV

- ${}^{4}_{\Lambda}\mathrm{H}$, ${}^{4}_{\Lambda}\mathrm{He}$
  (a) cutoff variation: ΔE_Λ (3BF) $\approx$ 200 keV (0^+) and $\approx$ 300 keV (1^+)
  (b) “3BF” from ΛN-ΣN coupling:
  ΔE_Λ (3BF) $\approx$ 230 – 340 keV (0^+), $\approx$ 150 – 180 keV (1^+)

${}^{3}_{\Lambda}\mathrm{H}$ and ${}^{4}_{\Lambda}\mathrm{H}$(He) calculations with explicit inclusion of 3BFs utilizing the decuplet saturation are planned for the future