

# Effective Field Theory for Halo Nuclei

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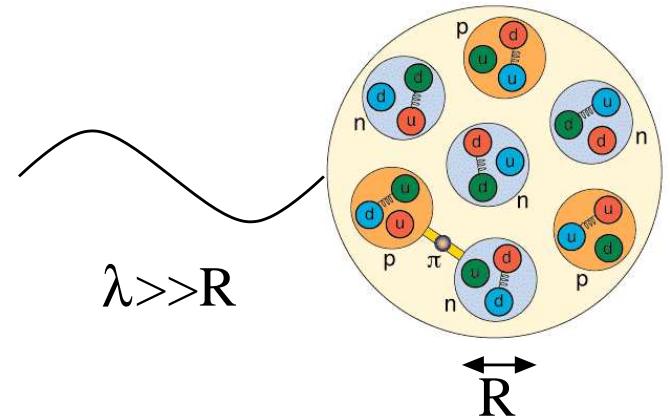
“The Extreme Matter Physics of Nuclei: From Universal Properties to Neutron-Rich Extremes”,  
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# Outline

- Introduction
- Halo Effective Field Theory
- Applications
  - Electric properties of  $^{11}\text{Be}$  (with D.R. Phillips)
  - Occupation numbers in EFT (with R.J. Furnstahl)
- Summary and Outlook

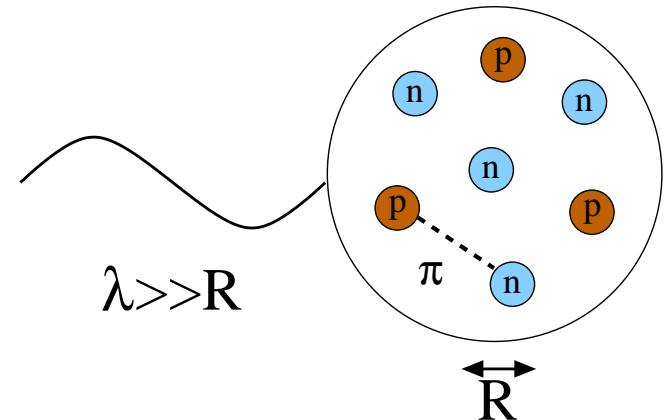
# Effective Theory

- Separation of scales:  
 $1/k = \lambda \gg R$
- Limited resolution at low energy:  
→ expand in powers of  $kR$

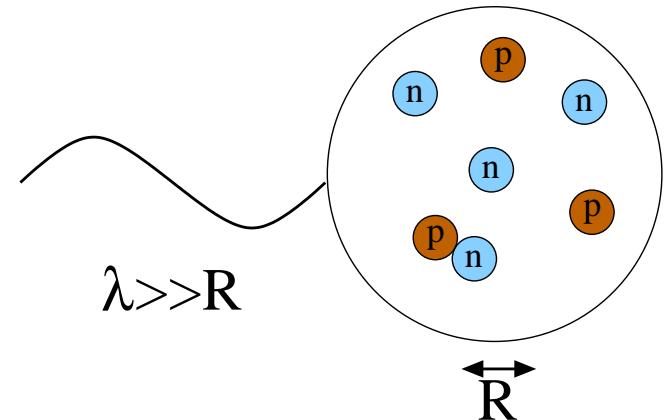


# Effective Theory

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 $1/k = \lambda \gg R$
- Limited resolution at low energy:  
→ expand in powers of  $kR$
- Short-distance physics not resolved  
→ capture in low-energy constants using renormalization  
→ include long-range physics explicitly



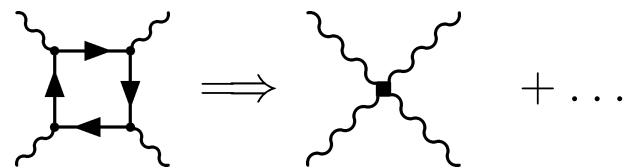
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- Short-distance physics not resolved
  - capture in low-energy constants using renormalization
  - include long-range physics explicitly
- Systematic, model independent → universal properties
- Classic example: light-light-scattering (Euler, Heisenberg, 1936)

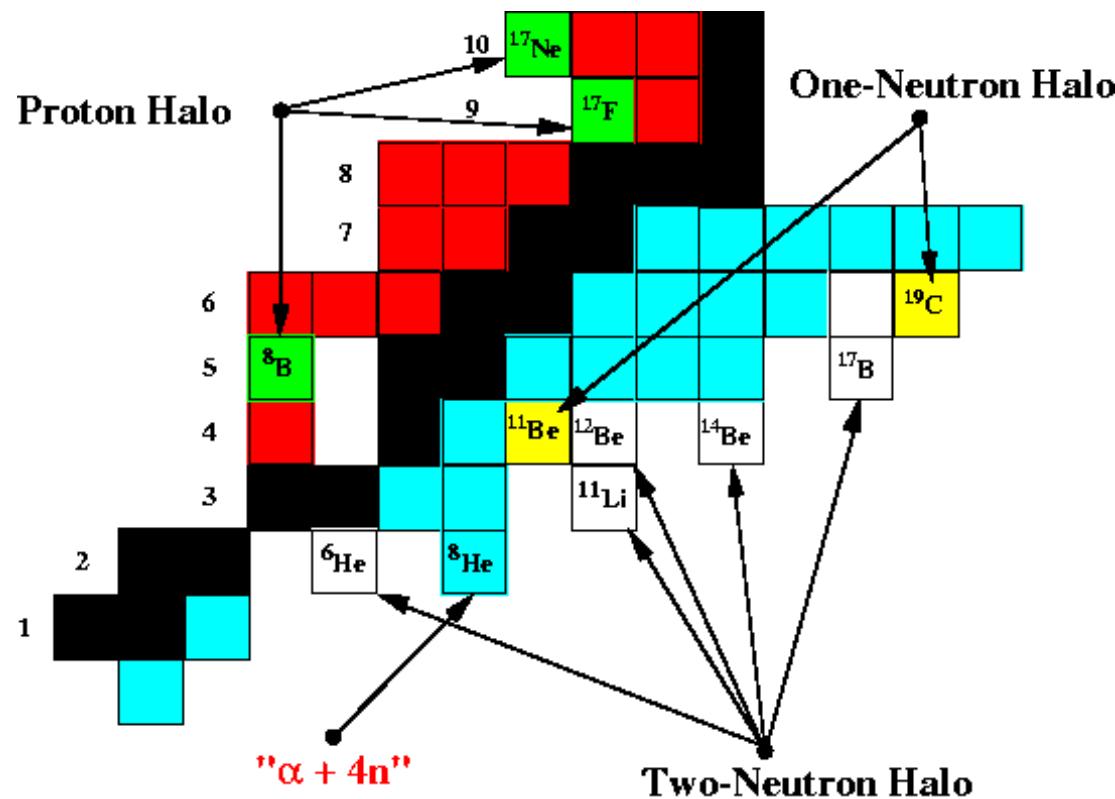
Contact interactions for  $\omega \ll m_e$ :

$$\mathcal{L}_{QED}[\psi, \bar{\psi}, A_\mu] \rightarrow \mathcal{L}_{eff}[A_\mu]$$



# Halo Nuclei

- Low separation energy of valence nucleons:  $B_{valence} \ll B_{core}, E_{ex}$   
 → close to “nucleon drip line” → scale separation → halo EFT

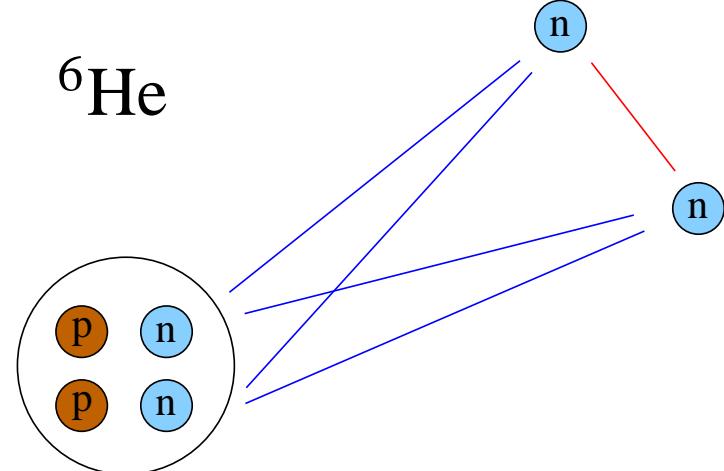


<http://www.nupecc.org>

- EFT for halo nuclei (Bedaque, Bertulani, HWH, van Kolck , 2002)

# Antisymmetrization Issues

- Antisymmetrization with respect to neutrons in core?
- Core neutrons not active dof in halo EFT
- Physics: exchange of core nucleon and halo nucleon only contributes to observables if there is spatial overlap between wave functions of core and halo nucleon
  - ⇒ small for  $R_{core} \ll R_{halo}$
- Effects subsumed in low-energy constants, included perturbatively in expansion in  $R_{core}/R_{halo}$



# Electromagnetic Structure of $^{11}\text{Be}$

- Properties of  $^{11}\text{Be}$ 
    - Ground state:  $J^P = 1/2^+$ , neutron separation energy: 504 keV
    - Excited state:  $J^P = 1/2^-$ , neutron separation energy: 184 keV
  - Properties of  $^{10}\text{Be}$ 
    - Ground state:  $J^P = 0^+$
    - First excitation: 3.4 MeV above g.s.
  - Separation of scales:  $E_{lo}/E_{hi} \approx \frac{0.5}{3.5} = \frac{1}{7} \Rightarrow R_{core}/R_{halo} \approx 0.4$
- ⇒ one neutron halo picture for  $^{11}\text{Be}$  appropriate
- Effective range theory (Typel, Baur, 2004, 2005, 2008)
  - EFT  $\implies$  straightforward coupling to external currents
  - Study EM properties in halo EFT picture (HWH, Phillips, NPA 865 (2011) 17)

# Effective Field Theory for $^{11}\text{Be}$

- Introduce fields for neutron/core with S- and P-wave interactions
- Effective Lagrangian at NLO
  - (cf. Bertulani, HWH, van Kolck, 2002; Bedaque, HWH, van Kolck, 2003)

$$\begin{aligned}\mathcal{L} = & c^\dagger \left( i\partial_t + \frac{\nabla^2}{2M} \right) c + n^\dagger \left( i\partial_t + \frac{\nabla^2}{2m} \right) n \\ & + \sigma^\dagger \left[ \eta_0 \left( i\partial_t + \frac{\nabla^2}{2M_{nc}} \right) + \Delta_0 \right] \sigma + \pi_j^\dagger \left[ \eta_1 \left( i\partial_t + \frac{\nabla^2}{2M_{nc}} \right) + \Delta_1 \right] \pi_j \\ & - g_0 \left[ \sigma n^\dagger c^\dagger + \sigma^\dagger n c \right] - \frac{g_1}{2} \left[ \pi_j^\dagger (n \overset{\leftrightarrow}{i\nabla}_j c) + (c^\dagger \overset{\leftrightarrow}{i\nabla}_j n^\dagger) \pi_j \right] + \dots\end{aligned}$$

- Parameters:
  - Leading order:  $g_0, \Delta_1, g_1 \Leftarrow B_0, B_1, a_1$  or  $B(E1)(1/2^+ \rightarrow 1/2^-)$
  - Next-to-leading order:  $\Delta_0 \Leftarrow B(E1)(1/2^+ \rightarrow 1/2^-)$  or  $dB/dE$

# Effective Field Theory for $^{11}\text{Be}$

- EFT generates S- and P-wave states from core-neutron contact interactions
- Reproduces correct asymptotics of wave functions for S- and P-wave states

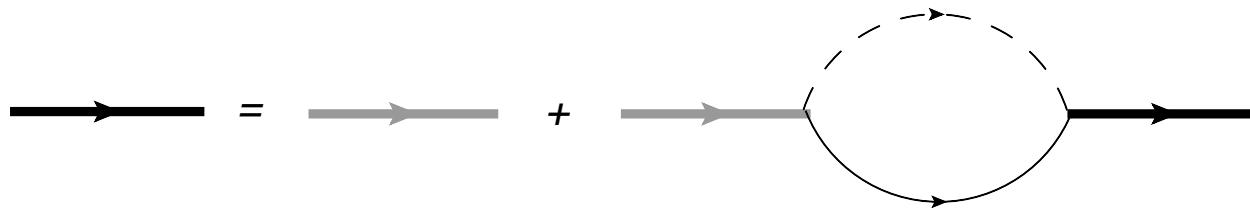
$$u_0(r) = A_0 \exp(-\gamma_0 r)$$

$$u_1(r) = A_1 \exp(-\gamma_1 r) \left( 1 + \frac{1}{\gamma_1 r} \right)$$

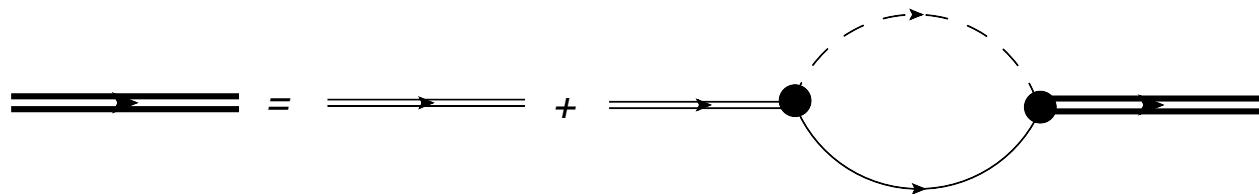
- Focus on observables: no discussion of  $n$ -core interaction at short distances, spectroscopic factors, ...
- Halo EFT: expansion in  $R_{core}/R_{halo}$
- Generating the S- and P-wave states in Halo EFT:  
 $\implies$  sum the  $nc$  bubbles

# Generating S- and P-Wave States

- **S-wave state:**  $g_0^2/\Delta_0 \sim R_{halo}$ ,  $nc$  loop  $\sim 1/R_{halo} \Rightarrow$  sum bubbles  
 (van Kolck, 1997, 1999; Kaplan, Savage, Wise, 1998)



- **P-wave state:**



- Determine parameters from bound state pole and/or scattering parameters

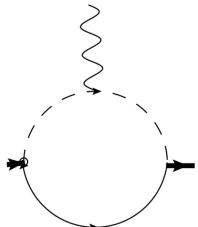
$$D_\pi(p) \propto \frac{1}{r_1 + 3\gamma_1} \frac{1}{p_0 - \mathbf{p}^2/(2M_{nc}) + B_1} + \text{regular}$$

where  $\gamma_1 = \sqrt{2m_R B_1}$

# Including Photons

- Minimal substitution:  $\partial_\mu \rightarrow D_\mu = \partial_\mu + ie\hat{Q}A_\mu$

- S-wave form factor (LO):



$$G_c(|\mathbf{q}|) = \frac{2\gamma_0}{f|\mathbf{q}|} \arctan \left( \frac{f|\mathbf{q}|}{2\gamma_0} \right) \quad \text{where} \quad f = m_R/M = 1/11$$

- Charge radius of  $^{11}\text{Be}$  relative to  $^{10}\text{Be}$ :

$$\langle r_c^2 \rangle_{^{11}\text{Be}} = \langle r_c^2 \rangle_{^{10}\text{Be}} + \frac{f^2}{2\gamma_0^2} \frac{1}{1 - \gamma_0 r_0}$$

- Results:

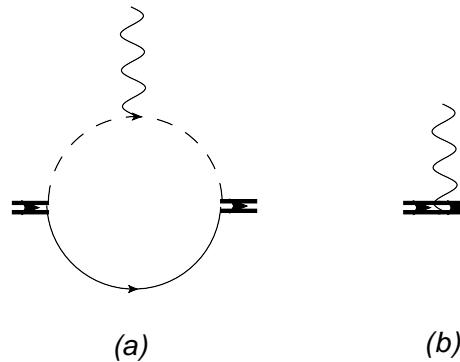
- At LO:  $\langle r_c^2 \rangle_{^{11}\text{Be}} - \langle r_c^2 \rangle_{^{10}\text{Be}} = 0.19 \text{ fm}^2$

- At NLO:  $\langle r_c^2 \rangle_{^{11}\text{Be}} - \langle r_c^2 \rangle_{^{10}\text{Be}} = 0.27 \dots 0.32 \text{ fm}^2$

- Comparison to experimental values  
Nörtshäuser et al., Phys. Rev. Lett. **102** (2009) 062503
- Using the experimental value:  $\sqrt{\langle r_c^2 \rangle_{^{10}\text{Be}}} = 2.357(18) \text{ fm}$ 
  - At LO:  $\sqrt{\langle r_c^2 \rangle_{^{11}\text{Be}}} = 2.40 \text{ fm}$
  - At NLO:  $\sqrt{\langle r_c^2 \rangle_{^{11}\text{Be}}} = 2.42 \text{ fm}$
- Experimental value:  $\sqrt{\langle r_c^2 \rangle_{^{11}\text{Be}}} = 2.463(16) \text{ fm}$

# P-wave Form Factors

- P-wave form factor (LO):

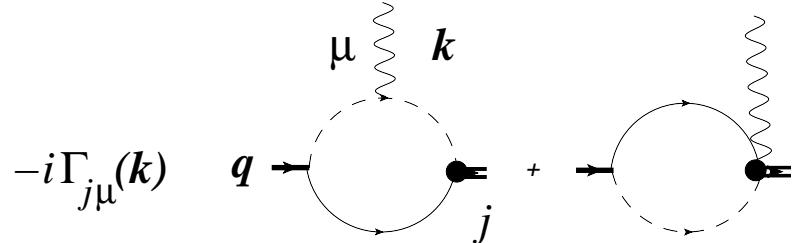


- ## Charge form factor:

$$G_c(|\mathbf{q}|) = \frac{1}{r_1 + 3\gamma_1} \left[ r_1 + \frac{1}{qf} \left( 2qf\gamma_1 + (q^2f^2 + 2\gamma_1^2) \arctan \left( \frac{f|\mathbf{q}|}{2\gamma_1} \right) \right) \right]$$

- Charge radius of  $^{11}\text{Be}^*$ :  $\langle r_c^2 \rangle_{^{11}\text{Be}^*} = \langle r_c^2 \rangle_{^{10}\text{Be}} - \frac{5f^2}{2\gamma_1 r_1}$
  - Using the experimental value (Nörtshäuser et al., PRL **102** (2009) 062503)
    - At LO:  $\sqrt{\langle r_c^2 \rangle_{^{11}\text{Be}^*}} = (2.43 \pm 0.1) \text{ fm}$
    - At NLO: unknown counterterm
  - Quadrupole form factor also predicted (not measurable in  $J = 1/2$  state)

# S-to-P Transition



- Irreducible transition vertex

$$\Gamma_{ji} = \delta_{ji}\Gamma_E + (k_j q_i + \cancel{q_j k_i})\Gamma_M \quad \text{for} \quad \mathbf{k} \cdot \mathbf{q} = 0, \quad \mathbf{k} \cdot \boldsymbol{\epsilon} = 0$$

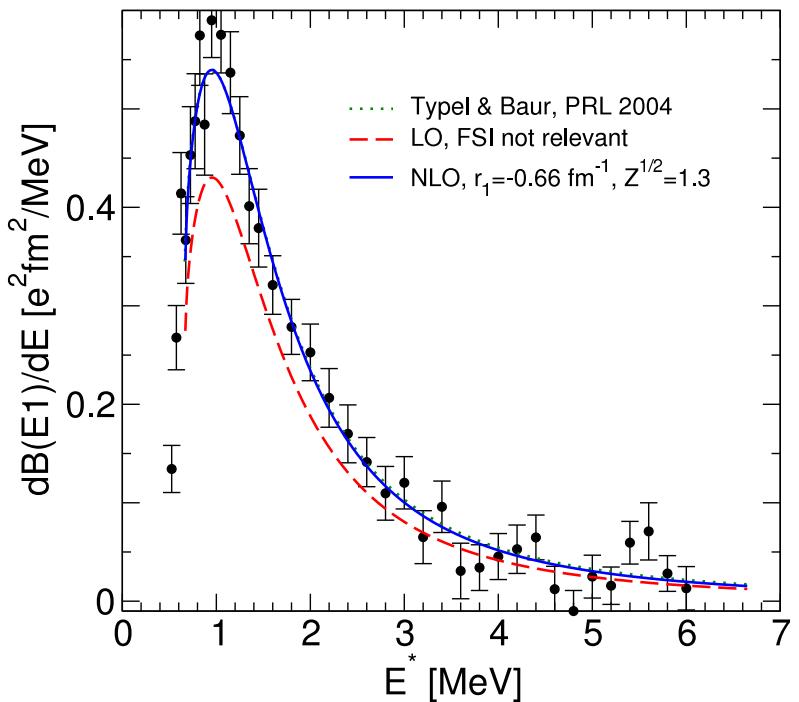
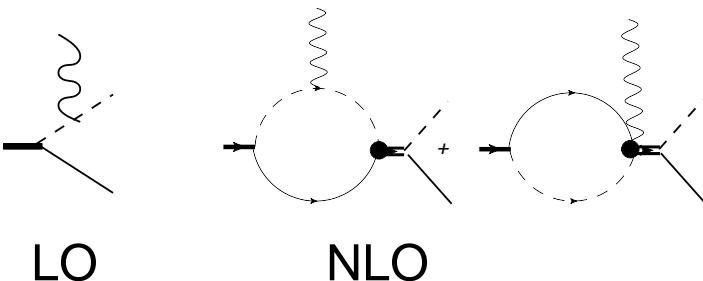
- Current conservation:  $k_\mu \Gamma_{j\mu} = 0 \implies \omega \Gamma_{j0} = k_j \Gamma_E$
- $B(E1)$  transition strength:

$$B(E1) = \frac{1}{4\pi} \left( \frac{\Gamma_E}{\omega} \right)^2 = \frac{Z_{eff}^2 e^2}{3\pi} \frac{\gamma_0}{-r_1} \left[ \frac{2\gamma_1 + \gamma_0}{(\gamma_0 + \gamma_1)^2} \right]^2 + \dots$$

- No cutoff required: divergences cancel!
- Experiment:  $B(E1) = 0.105 \dots 0.116 \text{ } e^2 \text{ fm}^2$   
 (Summers et al., PLB **650** (2007) 124; Millener et al., PRC **28** (1983) 497)
- Strategy: determine  $r_1 = -0.66 \text{ fm}^{-1}$  at LO

# Coulomb Dissociation

- Transition to the continuum:



- Reasonable convergence
- At LO: no FSI
- At NLO:
  - $r_1 = -0.66 \text{ fm}^{-1}$  [ $B(E1)$ ]
  - $\sqrt{Z_\sigma} = 1.3 \Rightarrow r_0 = 2.7 \text{ fm}$
- Detector resolution folded in

Data: Palit et al., PRC **68** (2003) 034318

# Counter Terms

- Counter term contributions (not generated by minimal substitution)

$$\begin{aligned}
 \mathcal{L}_{EM} = & -L_{C0}^{(\sigma)} \sigma_l^\dagger \underbrace{(\nabla^2 A_0 - \partial_0(\nabla \cdot \mathbf{A}))}_{\nabla \cdot \mathbf{E}} \sigma_l \\
 & - L_{E1}^{(1/2)} \sum_{ll'j} \sigma_l \pi_{l'}^\dagger \left( \frac{1}{2} l \frac{1}{2} l' \Big| 1j \right) \underbrace{(\nabla_j A_0 - \partial_0 A_j)}_{\mathbf{E}_j} \\
 & - L_{C0}^{(\pi)} \pi_l^\dagger \underbrace{(\nabla^2 A_0 - \partial_0(\nabla \cdot \mathbf{A}))}_{\nabla \cdot \mathbf{E}} \pi_l + \dots
 \end{aligned}$$

- Where do they come in?

- $L_{C0}^{(\sigma)}$ :  $\langle r_c^2 \rangle^{(\sigma)}$  at N3LO
- $L_{C0}^{(\pi)}$ :  $\langle r_c^2 \rangle^{(\pi)}$  at NLO  $\implies$  accuracy of models
- $L_{E1}^{(1/2)}$ :  $B(E1)$  at NLO

# Universal Correlations

- EFT gives correlations between different observables
- Example:  $B(E1)$  and radius of P-wave state

$$B(E1) = \frac{2e^2 Q_c^2}{15\pi} \left( \langle r_c^2 \rangle_{^{11}\text{Be}^*} - \langle r_c^2 \rangle_{^{10}\text{Be}} \right) x \left[ \frac{1+2x}{(1+x)^2} \right]^2 + \dots,$$

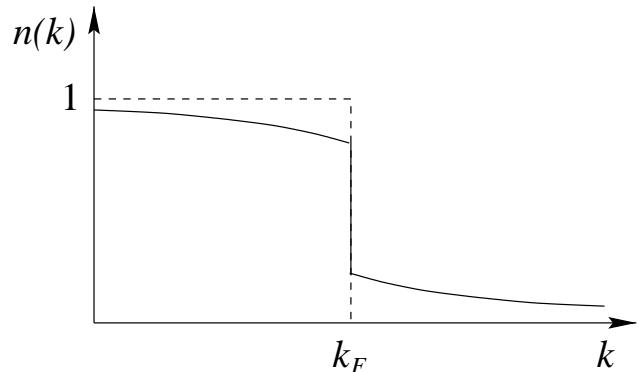
where  $x = \sqrt{B_1/B_0}$

- Adapt strategy to experimental situation
- P-wave radius relative to  $^{10}\text{Be}$  core from  $B(E1)$

$$\langle r_c^2 \rangle_{^{11}\text{Be}^*} - \langle r_c^2 \rangle_{^{10}\text{Be}} = 0.35...0.39 \text{ fm}^2$$

**Universality:** can be applied to any one-neutron halo nucleus with shallow S- and/or P-Wave State

# Occupation Numbers



- Spectroscopic factor in infinite system  $\Rightarrow$  occupation number
- Unique definition?
- Answer seems obvious: yes !  $\implies$  expectation value of  $a_{\mathbf{k}}^\dagger a_{\mathbf{k}}$
- True only for definite conventions/form of Hamiltonian
- Compare to parton distributions in DIS:
  - $\sigma = \int \text{parton distributions} \otimes \text{coefficient functions}$
  - factorization is scheme dependent
  - Study using the framework of EFT and field redefinitions
  - Observables are invariant under field redefinitions

on-shell Green's functions  $\iff$  S-matrix elements

# A Simple Example

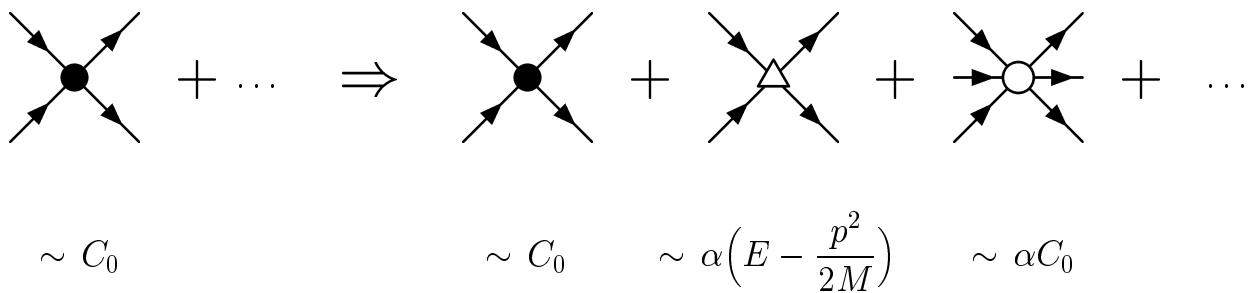
- Consider infinite, homogeneous fermion system with short-range interactions (HWH, Furnstahl, PLB **531** (2002) 203)

$$\mathcal{L} = \psi^\dagger [i\partial_t + \frac{\nabla^2}{2M}] \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2 + \dots$$

- $C_0$  given by *s*-wave scattering length:  $C_0 = \frac{4\pi a}{M}$
- Perform local field redefinition:

$$\psi \rightarrow \psi + \frac{4\pi\alpha}{\Lambda^3} (\psi^\dagger \psi) \psi, \quad \psi^\dagger \rightarrow \psi^\dagger + \frac{4\pi\alpha}{\Lambda^3} \psi^\dagger (\psi^\dagger \psi)$$

- Generates infinite class of physically equivalent  $\mathcal{L}_\alpha$



- Thermodynamic observables (energy density,...) independent of arbitrary parameter  $\alpha$  (cf. Furnstahl, HWH, Tiffessa, Nucl. Phys. **A689** ('01) 846)

# Total Particle Number

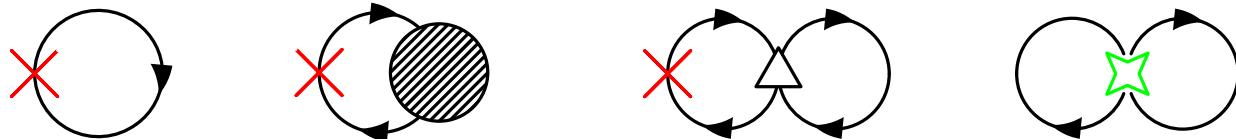
- Naively:  $\hat{N} = \psi^\dagger \psi$ , but correct for  $\alpha \neq 0$ ?
- Particle number conservation due to phase symmetry:

$$\psi(x) \rightarrow e^{-i\phi}\psi(x), \quad \psi^\dagger(x) \rightarrow e^{i\phi}\psi^\dagger(x)$$

- Noether theorem  $\Rightarrow$  number density operator

$$\hat{N}^\alpha \equiv \frac{\delta}{\delta(\partial_t \phi)} \tilde{\mathcal{L}}_\alpha[\psi, \psi^\dagger; \phi(x)] = \psi^\dagger \psi + \frac{4\pi\alpha}{\Lambda^3} 2 (\psi^\dagger \psi)^2$$

- $\alpha$ -dependent contributions cancel order-by-order



- Total number density unchanged:  $n = g k_F^3 / (6\pi^2)$

# Momentum Occupation Number

- Naively:  $\hat{n}_{\mathbf{k}} = a_{\mathbf{k}}^\dagger a_{\mathbf{k}}$   $\Rightarrow$  correct result in noninteracting limit
- Using field operators:  $\hat{\psi}(\mathbf{x}) = \int \frac{d^3 k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} a_{\mathbf{k}}, \dots$

$$a_{\mathbf{k}}^\dagger a_{\mathbf{k}} = \int d^3 x \underbrace{\int d^3 y e^{i\mathbf{k}\cdot\mathbf{y}} \hat{\psi}^\dagger(\mathbf{x} + \mathbf{y}) \hat{\psi}(\mathbf{x})}_{\hat{n}_{\mathbf{k}}(\mathbf{x})}$$

- In terms of Green's function  $G(\omega, \mathbf{k}) = [\omega - \mathbf{k}^2/(2m) - \Sigma^*(\omega, \mathbf{k})]^{-1}$   
the occupation number is

$$n(k) = \langle \hat{n}_{\mathbf{k}}(\mathbf{x}) \rangle = \lim_{\eta \rightarrow 0^+} (-i)g \int \frac{d\omega}{2\pi} e^{i\omega\eta} G(\omega, \mathbf{k})$$

(Migdal, Sov. Phys. JETP **5** (1957) 333)

- Definition used for dilute Fermi gas

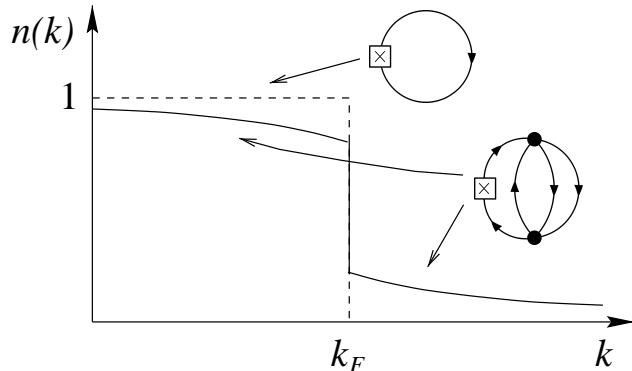
Belyakov, Sov. Phys. JETP **13** (1961) 850, Sartor, Mahaux, Phys. Rev. C **21** (1980) 1546

# Momentum Occupation Number

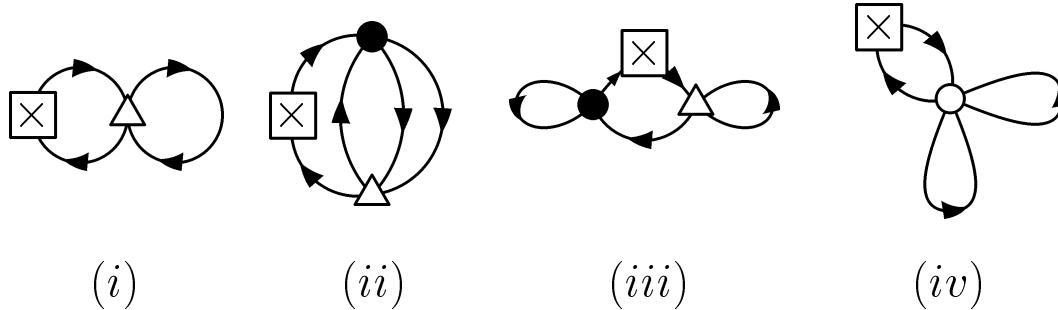
- Equivalent method: operator insertion for  $\hat{n}_{\mathbf{k}}(\mathbf{x})$

$$(-i)(2\pi)^3 \lim_{\eta \rightarrow 0^+} e^{i\omega\eta} \delta^3(\mathbf{p} - \mathbf{k}) \delta_{\alpha\beta}$$

- Momentum occupation number for  $\alpha = 0$



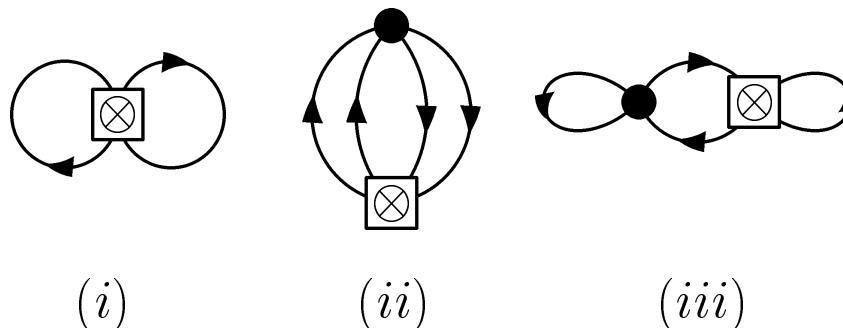
- Additional diagrams for  $\alpha \neq 0$



- $n(k)$  depends on  $\alpha$ !

# Momentum Occupation Number

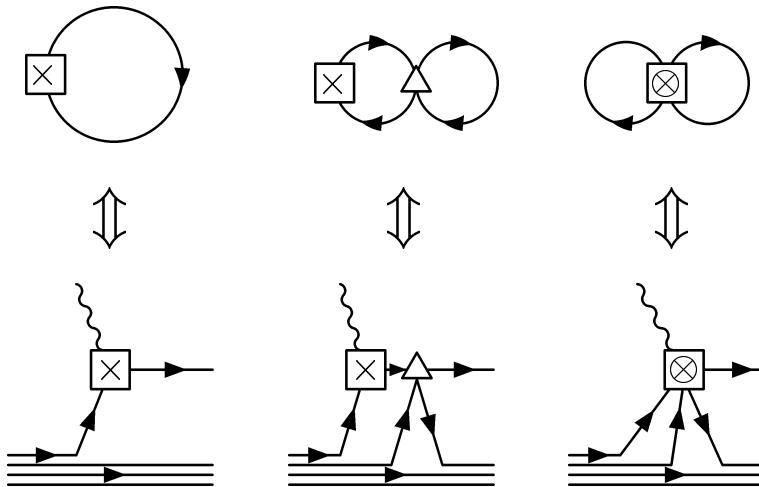
- Transform  $\hat{n}_{\mathbf{k}}(\mathbf{x})$  at the same time as  $\mathcal{L}$ 
  - generates four-fermion vertex insertion
  - $n(k)$  independent of  $\alpha$  by construction



- But:  $\hat{n}_{\mathbf{k}}(\mathbf{x})$  not a generator of global symmetry
  - definition of  $\hat{n}_{\mathbf{k}}^{\alpha}(\mathbf{x})$  is ambiguous
- $n(k)$  is not an observable
- Relevance to  $(e, e' p)$  experiments?
  - consider external source coupled to fermion number

# Connection to $(e, e' p)$ experiments

- Consider external source coupled to fermion number



- FSI, contact terms (MEC), and noninteracting contribution cannot be unambiguously separated
- Relative contributions are changed under field redefinitions
- Ambiguity is of natural size:  $\alpha \sim 1$  (cf.  $\psi \rightarrow \psi + \frac{4\pi\alpha}{\Lambda^3}(\psi^\dagger\psi)\psi$ )
- Explanation for difference in occupation numbers between relativistic and nonrelativistic Brueckner calculations?

- Estimate ambiguity from toy model ( $\alpha \sim 1$ )

$$\Delta n \sim \frac{4\pi\rho}{\Lambda^3}$$

- Nuclear matter:  $k_F = 280 \text{ MeV}$ ,  $\Lambda \sim 500 \dots 1000 \text{ MeV}$

$$\longrightarrow \Delta n \sim 2 \dots 15\%$$

- Depletion of Fermi sea  $\kappa = 1 - n(\langle p \rangle)$

relativistic:  $5 \dots 13\%$

nonrelativistic:  $\sim 20\%$

“empirical”:  $15 \pm 5\%$

(F. de Jong, H. Lenske Phys. Rev. C **54** (1996) 1488)

# Conclusions

- Occupation numbers: no unique definition using low-energy dof
  - not observable in the usual sense
  - depend on conventions for Hamiltonian etc.
- Usual assumption: “correct answer” can be extracted from experiment using “correct procedure”
- EFT: cannot be extracted in principle using low-energy degrees of freedom
- Must be able to convert between conventions
- Similar ambiguities in other areas of physics:
  - quark and gluon distributions in QCD
  - condensate fraction in BEC's
  - three-body forces in few-body physics
  - ...