What information can be extracted from transfer reactions?

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Stripping Reactions and the Structure of Light and Intermediate Nuclei*

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I. INTRODUCTION

A REDUCED width for the emission of a single nucleon in a transition between two specific nuclear states can be regarded as a product of two factors. Of these factors, the first is a measure of the probability that, in the initial nuclear state, all but one of the nucleons will find themselves in an arrangement corresponding to the final state; the second factor measures the probability that, when this happens, the two components will actually separate. The factorization is formally expressed by

$$\theta^2 = \mathbb{S}\theta_0^2. \tag{I.1}$$

52 years!

Goal

Phenomenological SFs



Independent particle shell model (IPSM)

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How much did we learn: S > 0.5 0.6-1.0 single-particle state
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S < 0.5 0.1 - 0.5

Different aspects to discuss. Theoretical calculations

Phenomenological SFs (from experiment)

1 Theory: fundamental problem SE is not obser

1. Theory: fundamental problem- SF is not observable.

Exact many-body approach:

Phys. 65, 94 (2010). R. J. Furnstahl and A. Schwenk, J. Phys. G 37, 064005 (2010).

R. J. Furnstahl and A. Schwenk, J. Phys. G 37, 064005 (2010). H. Feldmeier *et al.*, *Nucl. Phys. A 632*, *61 (1998)*.

UCOM – unitary correlation operator method

$$UCOM$$
 – unitary correlation operator method $U=e^{iG}$

$$\Psi = U\widetilde{\Psi}$$

$$< \Psi \mid H \mid \Psi > = < \widetilde{\Psi} \mid \widetilde{H} \mid \widetilde{\Psi} >$$

At distances large than correlation range
$$\Psi = \widetilde{\Psi}$$

The scattering amplitude – the amplitude of the outgoing scattered wave is invariant; unitary related potentials are phase-equivalent

A.M. And A.S. Kadyrov, PRC 82, 051601(R) (2010)

The most general model-independent definition of the SF for $\mathbf{P} = (\mathbf{v}, \mathbf{A})$ is the square of the norm of the overlap function:

$$B = (nA)$$
 is the square of the norm of the overlap function:

$$S = \langle I_A^B | I_A^B \rangle$$

$$I_A^B = (A+1)^{1/2} \langle \varphi_A | \varphi_B \rangle \quad \text{- overlap function}$$

 $U = U_{nA}U_A$ - cluster property (Feldmeier et al 1998) SF is contributed by the overlap function at small

distances where the effect of unitary transformations, which take into account short-range nucleon correlations, can be significant

$$\begin{split} I_A^B &= <\varphi_A \,|\, \varphi_B> = <\widetilde{\varphi}_A \,|\, U_A^{-1} U_A U_{nA} \,|\, \widetilde{\varphi}_B> \\ &= <\widetilde{\varphi}_A \,|\, U_{nA} \,|\, \widetilde{\varphi}_B> = \sum_i <\widetilde{\varphi}_A \,|\, U_{nA} \,|\, \widetilde{\varphi}_{A(i)}> \widetilde{I}_{A(i)}^B \\ r_{nA} &> R_{nA} \quad U_{nA} \to 1 \quad \text{ and } \quad I_A^B \to \widetilde{I}_A^B \end{split}$$

At distances larger then correlation region the overlap function is intact, while it is distorted at small distances.

Invariance of the sum rule.

$$\begin{split} &<\varphi_{B}\mid\varphi_{B}>=\sum_{i}<[<\varphi_{B}\mid\varphi_{A(i)}><\varphi_{A(i)}\mid\varphi_{B}>]>=\sum_{i}< I_{A(i)}^{B}\mid I_{A(i)}^{B}> \\ &=<\widetilde{\varphi}_{B}\mid U_{nA}^{-1}U_{nA}\mid\widetilde{\varphi}_{B}>=<\widetilde{\varphi}_{B}\mid\widetilde{\varphi}_{B}> \\ &=\sum_{i}<[<\widetilde{\varphi}_{B}\mid\widetilde{\varphi}_{A(i)}><\widetilde{\varphi}_{A(i)}\mid\widetilde{\varphi}_{B}>]>=\sum_{i}<\widetilde{I}_{A(i)}^{B}\mid\widetilde{I}_{A(i)}^{B}> \\ &=\sum_{i}\widetilde{S}_{nA(i)}=1 \end{split}$$

Another important theorem

Reaction amplitudes are invariant under short-range unitary transformations

Let us consider the transfer reaction A(a,b)B

$$\Psi_i^{(+)} = \Psi_i^{(0)} - \sum_{\alpha} \frac{\mu_{\alpha}}{2\pi} M_{\alpha} u^{(+)}(r_{\alpha}) \Phi_{\alpha} \quad \text{asymptotic behavior}$$

Rearrangement channel: $\alpha = b + B$, $\Phi_{\alpha} = \varphi_b \varphi_B$

Projection of asymptotic term of $\Psi_i^{(+)}$ on α :

$$<\varphi_{B}\varphi_{b} \mid \Psi_{i}^{(+)}> = <\widetilde{\varphi}_{B}\widetilde{\varphi}_{b} \mid U_{b} \mid_{B}\widetilde{\Psi}_{i}^{(+)}> \xrightarrow{r_{bB}\to\infty} <\widetilde{\varphi}_{B}\widetilde{\varphi}_{b} \mid \widetilde{\Psi}_{i}^{(+)}>$$

$$= -\frac{\mu_{\alpha}}{2\pi}M_{\alpha}u^{(+)}(r_{\alpha}) = -\frac{\mu_{\alpha}}{2\pi}\widetilde{M}_{\alpha}u^{(+)}(r_{\alpha})$$

$$M_{\alpha} = \widetilde{M}_{\alpha}$$
 reaction amplitude is invariant

B(e, e'p)A -the theorem holds

$$M = <\Psi_{f}^{(-)} | V_{eB} - U_{eB} | \varphi_{B} \chi_{eB}^{(+)} >$$

$$= <\Psi_{f}^{(-)} | V_{eB} + V_{B} - U_{eB} - V_{B} | \varphi_{B} \chi_{eB}^{(+)} >$$

$$= <\Psi_{f}^{(-)} | V_{eB} + V_{B} - U_{eB} - V_{B} | \varphi_{B} \chi_{eB}^{(+)} >$$

$$= <\Psi_{f}^{(-)} | E - \overline{T} - (E - \overline{T}) | \varphi_{B} \chi_{eB}^{(+)} >$$

$$= - <\Psi_{f}^{(-)} | \overline{T} - \overline{T} | \varphi_{B} \chi_{eB}^{(+)} >$$

$$= - <\Psi_{f}^{(-)} | \overline{T}_{eB} + \overline{T}_{pA} - \overline{T}_{eB} - \overline{T}_{pA} | \varphi_{B} \chi_{eB}^{(+)} >$$

$$= - <\Psi_{f}^{(-)} | \overline{T}_{eB} - \overline{T}_{eB} | \varphi_{B} \chi_{eB}^{(+)} >$$

$$= <\widetilde{\Psi}_{f}^{(-)} | \overline{T}_{eB} - \overline{T}_{eB} | U_{B}^{-1} U_{B} \widetilde{\varphi}_{B} \chi_{eB}^{(+)} >$$

$$= <\widetilde{\Psi}_{f}^{(-)} | \overline{T}_{eB} - \overline{T}_{eB} | \widetilde{\varphi}_{B} \chi_{eB}^{(+)} >$$
invariant under short-range unitary transformations

Green's theorem:

$$\begin{split} &\int_{r\leqslant R} \mathrm{d}\mathbf{r} f(\mathbf{r}) [\overleftarrow{T} - \overrightarrow{T}] g(\mathbf{r}) \\ &= -\frac{1}{2\mu} \oint_{r=R} \mathrm{d}\mathbf{S} [g(\mathbf{r}) \nabla_{\mathbf{r}} f(\mathbf{r}) - f(\mathbf{r}) \nabla_{\mathbf{r}} g(\mathbf{r})] \\ &= -\frac{1}{2\mu} R^2 \int \mathrm{d}\Omega_{\mathbf{r}} \left[g(\mathbf{r}) \frac{\partial f(\mathbf{r})}{\partial r} - f(\mathbf{r}) \frac{\partial g(\mathbf{r})}{\partial r} \right]_{r=R}. \end{split}$$

$$\begin{split} M^{DW} &= <\chi_{eA}^{(-)}\chi_{pA}^{(-)}\varphi_{A} \,|\, V_{eB} - U_{eB} \,|\, \varphi_{B}\chi_{eB}^{(+)} > \\ &= <\chi_{eA}^{(-)}\chi_{pA}^{(-)}\varphi_{A} \,|\, V_{eB} + V_{B} - [U_{eB} + V_{B}] \,|\, \varphi_{B}\chi_{eB}^{(+)} > \\ &= <\chi_{eA}^{(-)}\chi_{pA}^{(-)}\varphi_{A} \,|\, [U_{eA} + U_{pA} + V_{A}] + (V_{eB} - U_{eA}) + (V_{B} - U_{+pA} - V_{A}) \\ &- [V_{B} + U_{eB}] \,|\, \varphi_{B}\chi_{eB}^{(+)} > = <\chi_{eA}^{(-)}\chi_{pA}^{(-)}\varphi_{A} \,|\, E - \overline{T} + (V_{eB} - U_{eA}) \\ &+ (V_{B} - U_{pA} - V_{A}) - E + \overline{T} \,|\, \varphi_{B}\chi_{eB}^{(+)} > \\ &= - <\chi_{eA}^{(-)}\chi_{pA}^{(-)}\varphi_{A} \,|\, \overline{T} - \overline{T} \,|\, \varphi_{B}\chi_{eB}^{(+)} > + <\chi_{eA}^{(-)}\chi_{pA}^{(-)}\varphi_{A} \,|\, V_{eB} - U_{eB} \,|\, \varphi_{B}\chi_{eB}^{(+)} > \\ &<\chi_{eA}^{(-)}\chi_{pA}^{(-)}\varphi_{A} \,|\, (V_{B} - U_{pA} - V_{A}) \,|\, \varphi_{B}\chi_{eB}^{(+)} > \\ &\approx <\chi_{eA}^{(-)}\chi_{pA}^{(-)}\varphi_{A} \,|\, (V_{eB} - U_{eA}) \,|\, \varphi_{B}\chi_{eB}^{(+)} > \\ &\approx <\chi_{eA}^{(-)}\chi_{pA}^{(-)}\varphi_{A} \,|\, V_{ep} \,|\, \varphi_{B}\chi_{eB}^{(+)} > \\ &\approx <\chi_{eA}^{(-)}\chi_{pA}^{(-)}\varphi_{A} \,|\, V_{ep} \,|\, \varphi_{B}\chi_{eB}^{(+)} > \\ &\approx <\chi_{eA}^{(-)}\chi_{pA}^{(-)}\varphi_{A} \,|\, V_{ep} \,|\, \varphi_{B}\chi_{eB}^{(+)} > \\ \end{aligned}$$

 $=<\chi_{eA}^{(-)}\chi_{nA}^{(-)}\varphi_{A}|U_{nA}V_{en}|\varphi_{R}\chi_{eB}^{(+)}>$

not invariant under short-range unitary transformations

- W. H. Dickhoff, J. Phys. G 37, 064007 (2010).
- O. Jensen, G. Hagen, M. Hjorth-Jensen, B. Alex Brown, and A. Gade, PRL. 107, 032501 (2011)
- Microscopic coupled-cluster calculations of the spectroscopic factors for proton removal from the closed-shell oxygen isotopes

 NN interaction at next-to-next-to-leading order.
- 1. Significant quenching of the SFs due to the coupling-to-continuum.
- 2. Role of correlations beyond of mean field: SF for p1/2 proton removal from 24 O: SRG with $\lambda = 3.2, 3.4, 3.6 \text{ fm}^{-1}$
- Equation of motion coupled clusters reduction of 20 25% over the range of λ considered. Confirms importance of correlations beyond the mean-field.
 - Not clear about the size of reduction if the range of λ is increased, but it is clear that the short-range correlations play important role causing ambiguity of the microscopically calculated SFs.
- O. Jensen, G. Hagen, T. Papenbrock, D. J. Dean, and J. S. Vaagen, PRC 82, 014310 (2010).
- The spectroscopic factor is not an observable, as it depends on the employed Hamiltonian or model. In nuclear physics, the high-momentum parts of the interaction are unconstrained and modeled in different ways. Thus, the short-ranged part of the wave function is model dependent, and so is an overlap between wave functions. Therefore, the spectroscopic factor is merely a theoretical quantity and cannot be measured.
- R. J. Furnstahl and H. W. Hammer, Phys. Lett. B **531**, **203** (2002).
- R. J. Furnstahl and A. Schwenk, arXiv:1001.0328v1

Does the spectroscopic factor "provides a useful basis for the comparison of experiment and current nuclear models"? Macfarlane and French

2. Phenomenology- extraction from experiment.

Drastic approximations.

In the exact coupled channels approach no SF appears.

Faddeev equations – solution gives reaction amplitudes, not SFs.

One of the main approximations:

$$I_A^B \approx S_{nA}^{1/2} \varphi_n$$

Many-body object

Single-particle object

Exact only in the external region, where n-A nuclear interaction can be neglected

$$(-\varepsilon_{nA} - T_{nA})I_A^B = \langle \varphi_A | V_{nA} | \varphi_B \rangle \qquad ^{W}_{N}$$

W. T. Pinkston and G. R. Satchler, Nucl. Phys. 72, 641 (1965)

$$\varphi_{B} = \sum_{i} I_{A(i)}^{B} \varphi_{A(i)}$$

$$(-\varepsilon_{nA} - T_{nA}) \varphi_{nA} = \overline{V}_{nA} \varphi_{nA}$$

$$(-\varepsilon_{nA} - T_{nA}) I_{A}^{B} = 0,$$

$$(-\varepsilon_{nA} - T_{nA}) \varphi_{nA} = 0$$

$$r_{nA} > R_{nA}$$

$$(-\varepsilon_{nA} - T_{nA}) \varphi_{nA} = 0$$

$$I_A^B(r_{nA}) pprox C_{nA}^B rac{e^{-\kappa_{nA}r_{nA}}}{r_{nA}}$$

$$\varphi_{nA}(r_{nA}) pprox b_{nA} rac{e^{-\kappa_{nA}r_{nA}}}{r}$$

$$C_{nA}^B = S_{nA}^{1/2}b_{nA}$$

$$C_{nA}^B = S_{nA}^{1/2} b_{nA}$$

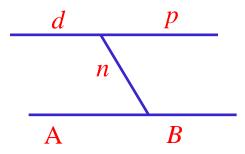
This equation is correct only in the external region and S_{nA} is nothing but normalization coeffcient between the ANC C_{nA}^{B} and single-particle ANC b_{nA}

Only extension of this equation to the whole region makes S_{nA} SF:

$$< I_A^B \mid I_A^B > = S_{nA} < \varphi_{nA} \mid \varphi_{nA} > = S_{nA}$$

How robust is parameterization of the nuclear transfer reaction in terms of SFs?

1. Nuclear reaction cross section is quite complicated



$$\begin{split} M^{post} &= <\chi_{pB}^{(-)} \varphi_{B} \mid \Delta V_{pB} \mid \Psi_{i}^{(+)}> = <\Psi_{f}^{(-)} \mid \Delta V_{dA} \mid \varphi_{d} \varphi_{A} \chi_{dA}^{(+)}> = M^{prior} \\ \Delta V_{pB} &= V_{pB} - U_{pB} = V_{pA} + V_{pn} - U_{pB} \\ \Delta V_{dA} &= V_{dA} - U_{dA} = V_{pA} + V_{nA} - U_{dA} \end{split}$$

1-st approximation: transition from exact to distorted waves. The accuracy is unclear. The contribution from coupled channels is lost. Effectively is taken into account through OP. Both post and prior form are equal but reflect different physics.

$$\Psi_{i}^{(+)} \rightarrow \varphi_{d} \varphi_{A} \chi_{dA}^{(+)}$$

$$\Psi_{f}^{(-)} \rightarrow \varphi_{B} \chi_{pB}^{(-)}$$

$$M^{DW(post)} = \langle \chi_{pB}^{(-)} \varphi_{B} | \Delta V_{pB} | \varphi_{d} \varphi_{A} \chi_{dA}^{(+)} \rangle$$

$$= \langle \chi_{pB}^{(-)} \varphi_{B} | \Delta V_{dA} | \varphi_{d} \varphi_{A} \chi_{dA}^{(+)} \rangle = M^{DW(prior)}$$

We still don't have overlap function to introduce the SF 2-nd approximation: introduce overlap

3. Approximation of the overlap function I_A^B by φ_{nA} is questionable. As many-body object I_A^B contains not only mean-field effects but also residual interactions important on the surface (in the shell-model language). Short-range correlations in microscopic approach.

Approximation by the single-particle wave function φ_{nA} in the mean-field (Hartree-Fock) may not be adequate.

4. $C_{nA}^{B} = S_{nA}^{1/2} b_{nA}^{B}$ must be fulfilled whenever $I_{A}^{B} \approx S_{nA}^{1/2} \varphi_{n}^{B}$ is being used. Puts strong limitations on SF and provides check of consistency of the single-particle approximation, because ANC can be measured or calculated.

A.M. and F. M. Nunes, PRC 72, 017602 (2005) A.M., F. M. Nunes, and P. Mohr, PRC 77, 051601(R) (2008) (talk by P. Capel)

$$\begin{split} M^{DW(post)} &= <\chi_{pB}^{(-)}I_{A}^{B} \mid \Delta \overline{V}_{pB} \mid \varphi_{d}\chi_{dA}^{(+)}> \\ &= <\chi_{pB}^{(-)}I_{A}^{B} \mid \Delta \overline{V}_{dA} \mid \varphi_{d}\chi_{dA}^{(+)}> = M^{DW(prior)} \\ \Delta \overline{V}_{pB} &= <\varphi_{A} \mid V_{pA} \mid \varphi_{A}> + V_{pn} - U_{pB} \approx U_{pA} + V_{pn} - U_{pB} \\ \Delta \overline{V}_{dA} &= <\varphi_{A} \mid V_{pA} \mid \varphi_{A}> + <\varphi_{A} \mid V_{nA} \mid \varphi_{A}> - U_{dA} \approx U_{pA} + U_{nA} - U_{dA} \end{split}$$

$$I_A^B(r_{nA}) = S_{nA}^{1/2} \varphi_{nA}(r_{nA})$$

$$M^{DW(post)} = S_{nA}^{1/2} < \chi_{pB}^{(-)} \varphi_{nA} | \Delta \overline{V}_{pB} | \varphi_d \chi_{dA}^{(+)} >$$

For peripheral reaction

$$I_A^B(r_{nA}) \approx C_{nA} \frac{e^{-\kappa_{nA}r_{nA}}}{r_{nA}}, \quad \varphi_{nA}(r_{nA}) \approx b_{nA} \frac{e^{-\kappa_{nA}r_{nA}}}{r_{nA}}, \quad r_{nA} > R_{nA}$$

$$M^{DW(post)} = C_{nA} < \chi_{pB}^{(-)} \frac{e^{-\kappa_{nA}r_{nA}}}{r_{nA}} |\Delta \overline{V}_{pB}| \varphi_d \chi_{dA}^{(+)} >$$
 doesn't depend on b_{nA}

$$M^{DW(post)} = S_{nA}^{1/2} b_{nA} < \chi_{pB}^{(-)} \frac{e^{-\kappa_{nA} r_{nA}}}{r_{nA}} |\Delta \overline{V}_{pB}| \varphi_d \chi_{dA}^{(+)} >$$

$$\frac{M^{DW(post)}}{b_{nA}} = S_{nA}^{1/2} < \chi_{pB}^{(-)} \frac{e^{-\kappa_{nA}r_{nA}}}{r_{nA}} |\Delta \overline{V}_{pB}| \varphi_d \chi_{dA}^{(+)} >$$

$$^{48}\text{Ca}(d,p)^{49}\text{Ca}$$
 $(C_{mA}^B)^2 = 32.0 \pm 3.2 \text{ fm}^{-1}$ from sub-Coulomb stripping $I = 1$ $I = 3/2$ $\varepsilon = 5.14 \text{ MeV}$

$$l = 1, J = 3/2, \varepsilon = 5.14 \text{ MeV}$$

$$\frac{d\sigma^{\text{exp}}}{d\Omega} = S_{nA} \frac{d\sigma^{DW}}{d\Omega} = \frac{C_{nA(\text{exp})}^2}{b_{nA}^2} \frac{d\sigma^{DW}(b_{nA})}{d\Omega} \rightarrow C_{\text{exp}}^2 = \frac{\frac{d\sigma^{DW}}{d\Omega}}{\frac{d\sigma^{DW}}{d\sigma(b_{nA})}}$$

$$C_{\text{exp}}^{2} = \frac{\frac{d\sigma^{\text{exp}}}{d\Omega}}{\frac{d\sigma(b_{nA})}{d\Omega}} b_{nA}^{2}$$

Standard approach with Hartree-Fock mean field.

Jenny Lee et al., PRC 73, 044608 (2006)

$$S = 0.74 \pm 0.08$$
 exp

$$r_0 = 1.245$$
 fm,

$$(C_{mA}^B)^2 = 24.4 \text{ fm}^{-1}$$

If information about ANC is used

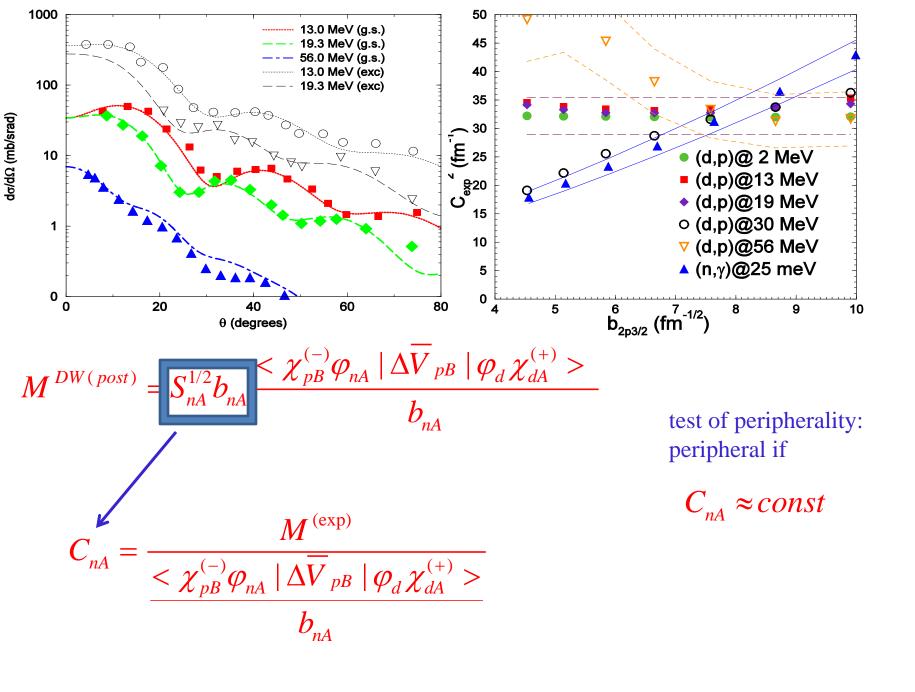
$$b_0 = 7.8 \pm 1.2 \text{ fm}^{-1} \quad (r_0 = 1.45 \text{ fm}),$$

 $S = 0.55 \pm 0.25$

We don't check the theory, the uncertainty is large. Optical potential uncertainty and DWBA accuracy. CDCC is better but not resolve the problem.

The only quantity, which can be extracted is the ANC.

I challenge any result of (d,p) analysis and require to provide the ANC along with SF. The peripheral part gives the dominant contribution at energies < 50 MeV/A, more than 80%. Underestimation of the ANC just by 10% will increase the SF by 50%.



3. New insight into deuteron stripping populati g bound states and resonances. From surface integral formalism to the generalized R-matrix.

Here I will demonstrate what we really measure in deuteron stripping at low energies

In collaboration with I. Thompson, J. Escher, LLNL

Start with DWBA and end up with CDCC. Both transfer to bound states and especially to resonances are considered.

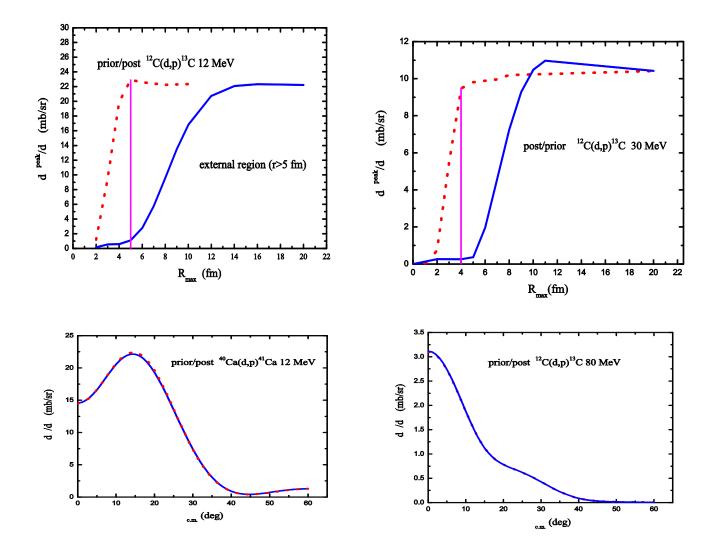
A(d,p)B
$$\frac{d}{n} \qquad r_{nA}, r_{pB} \text{ -Jacobian variables}$$

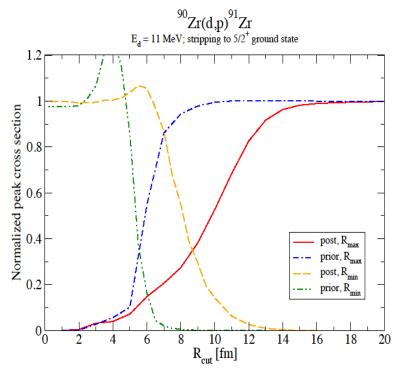
$$\frac{d}{n} \qquad R_{nA}, r_{pB} \text{ -Jacobian variables}$$

$$M^{DW(post)}(\mathbf{k}_{pB}, \mathbf{k}_{dA}) = M^{DW(post)}_{int}(\mathbf{k}_{pB}, \mathbf{k}_{dA})$$

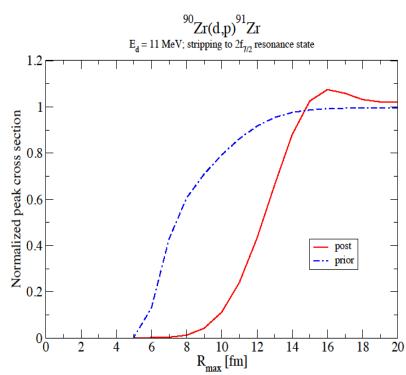
$$+ M^{DW(prior)}_{ext}(\mathbf{k}_{pB}, \mathbf{k}_{dA}) + M^{DW}_{S_{R-A}}(\mathbf{k}_{pB}, \mathbf{k}_{dA})$$

The amplitude for deuteron stripping to bound states and resonances is parameterized in terms of the reduced widths and boundary conditions- generalized R-matrix for stripping

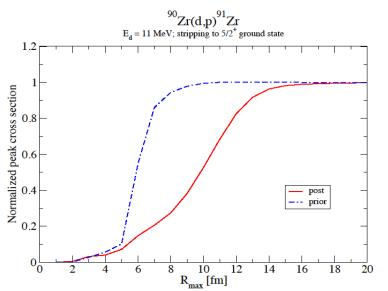




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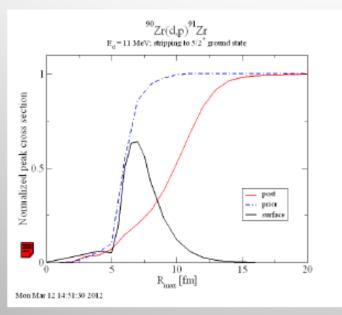


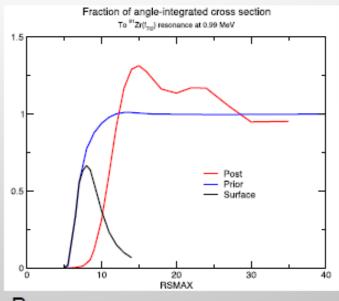
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Size of the Surface Term





Bound state

Resonance

Current work:

Generalized Faddeev equations in the AGS form taking into account target excitations – ultimate goal to couple different channels with realistic potentials (in collaboration with C. Elster and F. Nunes)

$$T_{\beta n,\alpha m}^{\sigma \rho} = V_{\beta n,\alpha m}^{\sigma \rho} + \sum_{\gamma i \tau \nu} V_{\beta n,\gamma i}^{\sigma \tau} G_{\gamma i}^{\tau \nu} T_{\gamma i,\alpha m}^{\nu \rho}$$