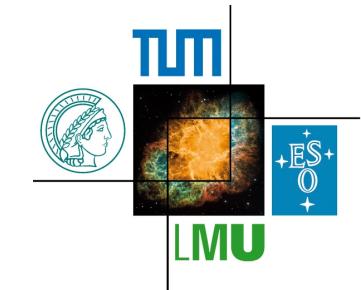


Quasiparticle interaction in nuclear and neutron matter

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with
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Outline

- **Fermi liquid theory: concepts and applications**

- ✚ Quasiparticle interaction (QPI) and Fermi liquid parameters (FLPs)
 - ✚ Constraints from symmetric nuclear matter
 - ✚ Physical applications

- **Symmetric nuclear matter: benchmarking MBPT**

- ✚ Perturbative calculation with chiral two- and three-nucleon interactions
 - ✚ Comparison to empirical data

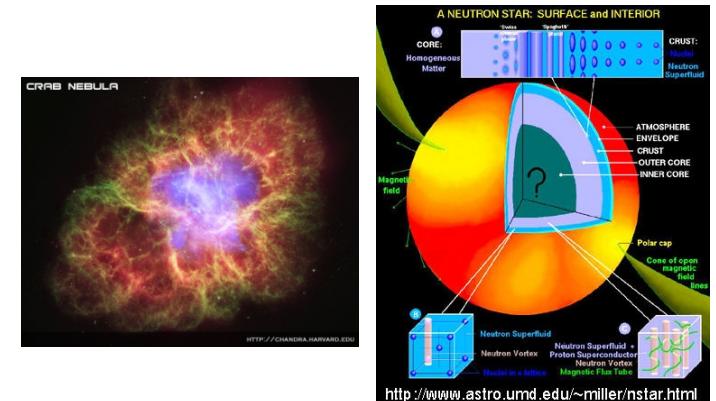
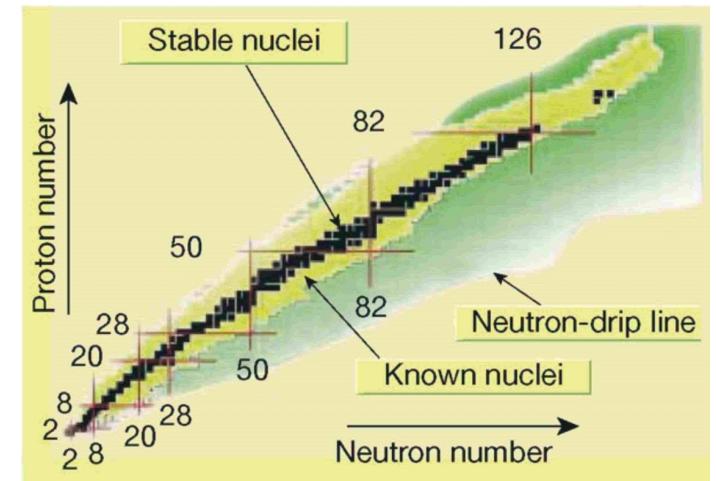
- **Pure neutron matter: path to neutron star structure/evolution**

- ✚ Central + noncentral components of quasiparticle interaction
 - ✚ Contributions from three-nucleon interactions

- **Summary/Outlook**

Introduction: Neutron-rich matter

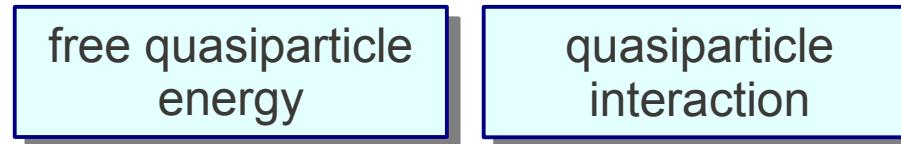
- Nuclei far from stability
 - ✚ R-process nucleosynthesis
 - ✚ Shell evolution, ...
- Neutrino processes (scattering, absorption and production)
 - ✚ Dynamics of supernova explosion
 - ✚ Neutron star evolution, cooling
- Role of the density-dependent isospin asymmetry energy
 - ✚ Neutron star cooling
 - ✚ Neutron star composition, radius, ...
- Response of neutron matter to strong magnetic fields



Landau's theory of normal Fermi liquids

- **Quasiparticle description** of small excitations about the ground state of a strongly-interacting normal Fermi system [L.D. Landau, (1950's)]
 - Quasiparticles well-defined only near the Fermi surface: $\tau \sim (\epsilon - \epsilon_F)^{-2}$
- Applicable to **normal Fermi systems** at low temperature

$$\delta\mathcal{E} = \sum_{\vec{p}_1} \epsilon_{\vec{p}_1}^{(0)} \delta n(\vec{p}_1) + \frac{1}{2} \sum_{\vec{p}_1, \vec{p}_2} \mathcal{F}(\vec{p}_1, \vec{p}_2) \delta n(\vec{p}_1) \delta n(\vec{p}_2)$$



$$\begin{aligned}\mathcal{F}(\vec{p}_1, \vec{p}_2) = & f(\vec{p}_1, \vec{p}_2) + f'(\vec{p}_1, \vec{p}_2) \vec{\tau}_1 \cdot \vec{\tau}_2 + [g(\vec{p}_1, \vec{p}_2) + g'(\vec{p}_1, \vec{p}_2) \vec{\tau}_1 \cdot \vec{\tau}_2] \vec{\sigma}_1 \cdot \vec{\sigma}_2 \\ & + \text{noncentral components}\end{aligned}$$

Non-central quasiparticle interactions

- Exchange-tensor interaction: $S_{12}(\hat{q}) = 3\vec{\sigma}_1 \cdot \hat{q}\vec{\sigma}_2 \cdot \hat{q} - \vec{\sigma}_1 \cdot \vec{\sigma}_2$ $\vec{q} = \vec{p}_1 - \vec{p}_2$
[Haensel & Dabrowski (1975)]
- Presence of Fermi sea allows additional contributions to effective interaction
(absent in free-particle scattering) [Friman & Schwenk (2004)]
 - Center-of-mass tensor: $S_{12}(\hat{P}) = 3\vec{\sigma}_1 \cdot \hat{P}\vec{\sigma}_2 \cdot \hat{P} - \vec{\sigma}_1 \cdot \vec{\sigma}_2$ $\vec{P} = \vec{p}_1 + \vec{p}_2$
 - Spin non-conserving interactions -- arising from polarization (particle-hole) contributions to effective interaction:

★ $A_{12}(\vec{q}, \vec{P}) = (\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot (\vec{q} \times \vec{P})$ Cross vector

$$\left(\begin{array}{ccc|c} 0 & 0 & 0 & \langle 11 | A | 00 \rangle \\ 0 & 0 & 0 & \langle 10 | A | 00 \rangle \\ 0 & 0 & 0 & \langle 1-1 | A | 00 \rangle \\ \hline \langle 00 | A | 11 \rangle & \langle 00 | A | 10 \rangle & \langle 00 | A | 1-1 \rangle & 0 \end{array} \right)$$

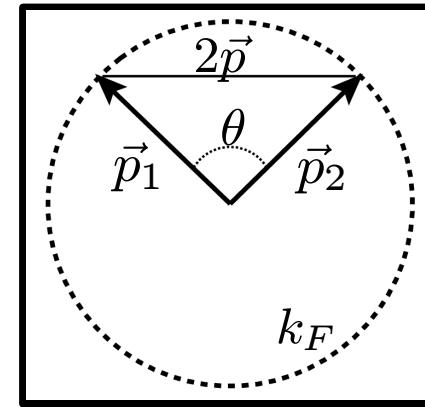
SM_S

Constraints from nuclear matter observables

- Legendre polynomial expansion:

$$f(\vec{p}_1, \vec{p}_2) = \sum_L f_L P_L(\cos \theta)$$

$$g(\vec{p}_1, \vec{p}_2) = \sum_L g_L P_L(\cos \theta)$$
$$\vdots$$
$$\vdots$$



- Small number of parameters characterizing physics about the Fermi surface
- Properties of the bulk medium and quasiparticles (valid at any density)

$$\frac{M^*}{M_N} = 1 + \frac{F_1}{3} \quad \mathcal{K} = \frac{3\hbar^2 k_F^2}{M^*} (1 + F_0) \quad \beta = \frac{\hbar^2 k_F^2}{6M^*} (1 + F'_0) \quad \delta g_l = \frac{F'_1 - F_1}{3(1 + F_1/3)}$$

Effective mass

[0.7 – 1.0]

Compression modulus

[200 – 300 MeV]

Symmetry energy

[30 – 36 MeV]

Anomalous orbital g-factor

[0.20 – 0.26]

$$F(\vec{p}_1, \vec{p}_2) = \frac{2M^* k_F}{\pi^2} f(\vec{p}_1, \vec{p}_2), \dots \quad (\text{dimensionless parameters})$$

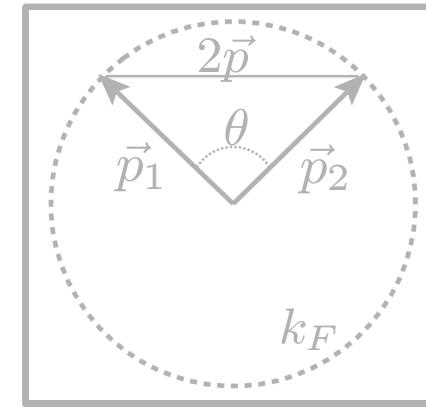
Constraints from nuclear matter observables

- Legendre polynomial expansion:

$$f(\vec{p}_1, \vec{p}_2) = \sum_L f_L P_L(\cos \theta)$$

$$g(\vec{p}_1, \vec{p}_2) = \sum_L g_L P_L(\cos \theta)$$

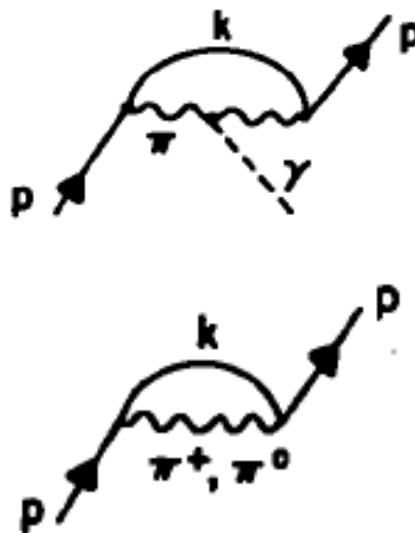
⋮
⋮



Natural inclusion of meson-exchange current contributions

- Properties of the bulk medium an

$\frac{M^*}{M_N} = 1 + \frac{F_1}{3}$	$\mathcal{K} = \frac{3\hbar^2 k_F^2}{M^*} ($
Effective mass	Com m
[0.7 – 1.0]	[200 –



at any density)

$$+ F'_0)$$

Anomalous orbital g-factor

$$\delta g_l = \frac{F'_1 - F_1}{3(1 + F_1/3)}$$

[0.20 – 0.26]

V]

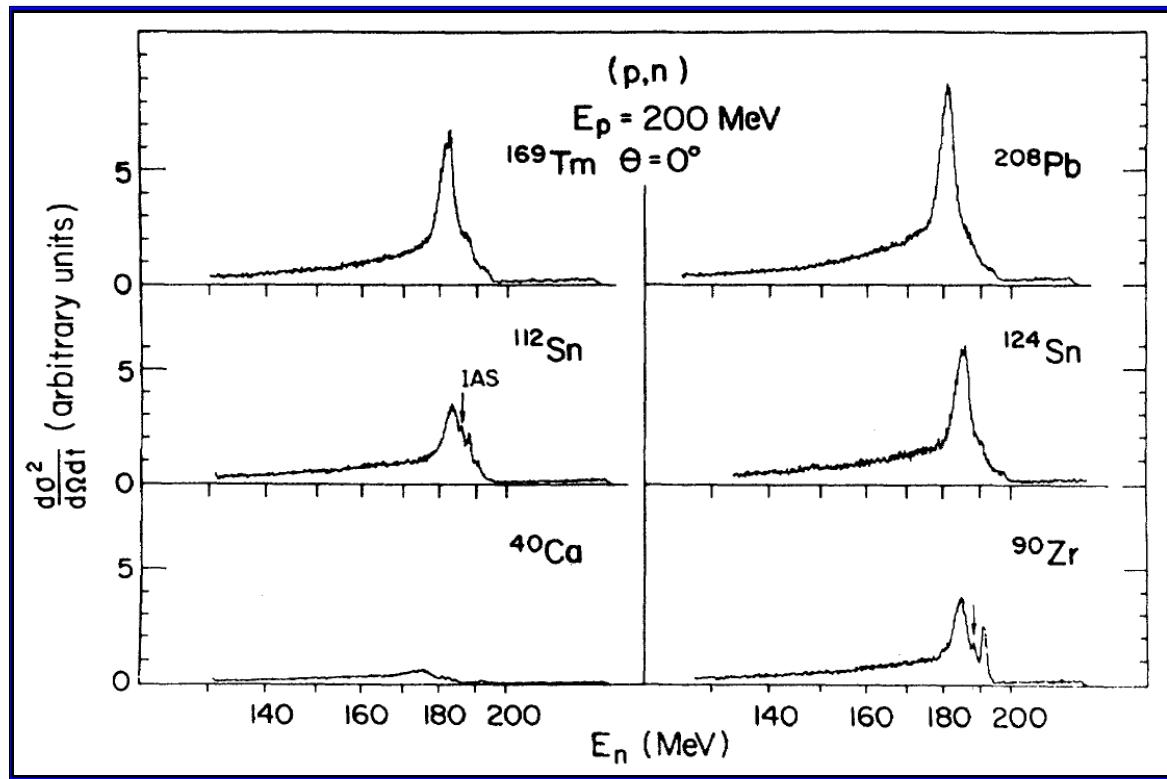
parameters)

$$F(\vec{p}_1, \vec{p}_2) = \frac{2M^*k_F}{\pi^2} f(\vec{p}_1, \vec{p}_2)$$

[Brown & Rho (1979)]

Constraints from collective excitations

Giant Gamow-Teller resonances



C. Gaarde et al., NPA (1981)

- Collective **spin-isospin excitations** in nuclei
- Probed in intermediate-energy charge-exchange reactions
- Controlled by spin-isospin Fermi liquid parameter g'_0
- Pion condensation, quenching of axial coupling constant, ...

$$V_{\sigma\tau}(\vec{r}_1, \vec{r}_2) = \frac{g_{\pi N}^2}{4M_N^2} g'_{NN} \delta^3(\vec{r}_1 - \vec{r}_2) \vec{\sigma}_1 \cdot \vec{\sigma}_2 \vec{\tau}_1 \cdot \vec{\tau}_2 + \text{zero-range } \Delta N \text{ and } \Delta\Delta \text{ interactions}$$

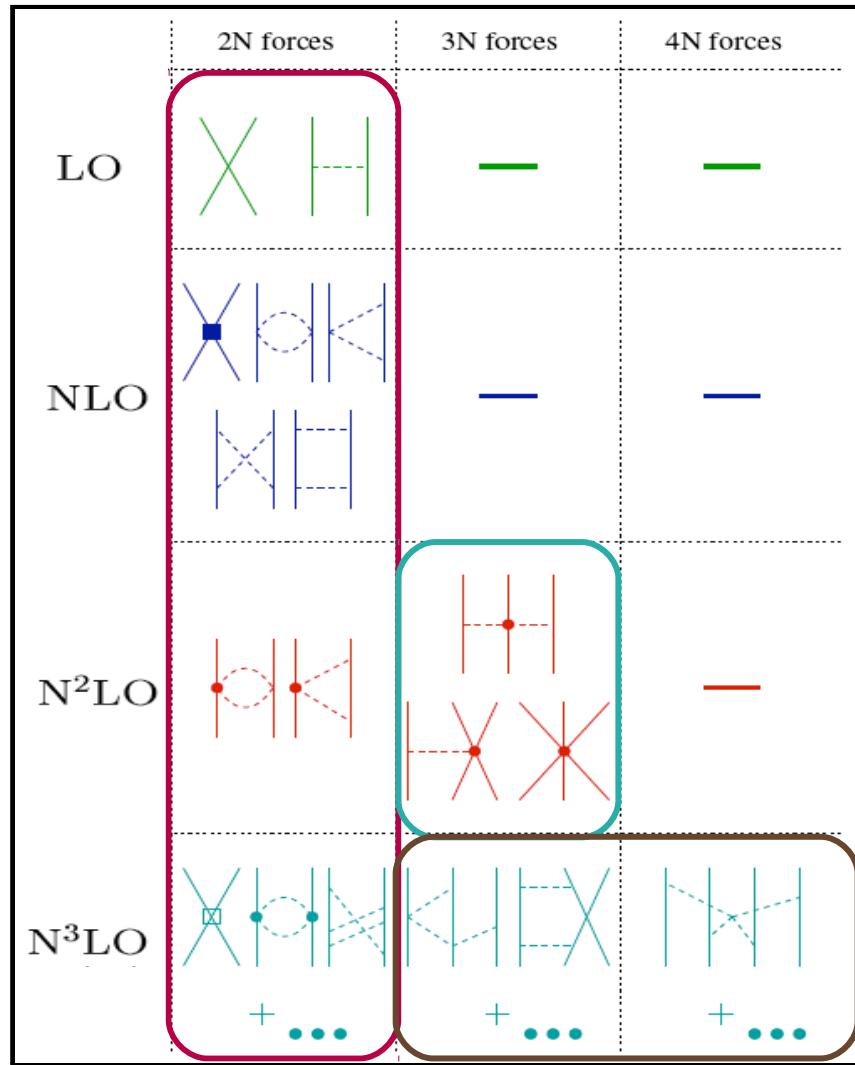
- Agreement with observed resonance energies for $g'_{NN} \simeq 0.6 - 0.7$

SYMMETRIC NUCLEAR MATTER

Chiral nuclear interactions

- **SYSTEMATIC EXPANSION** in powers of Q/Λ_χ : $\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{NN}^{(0)} + \dots$

$$Q = p, m_\pi$$

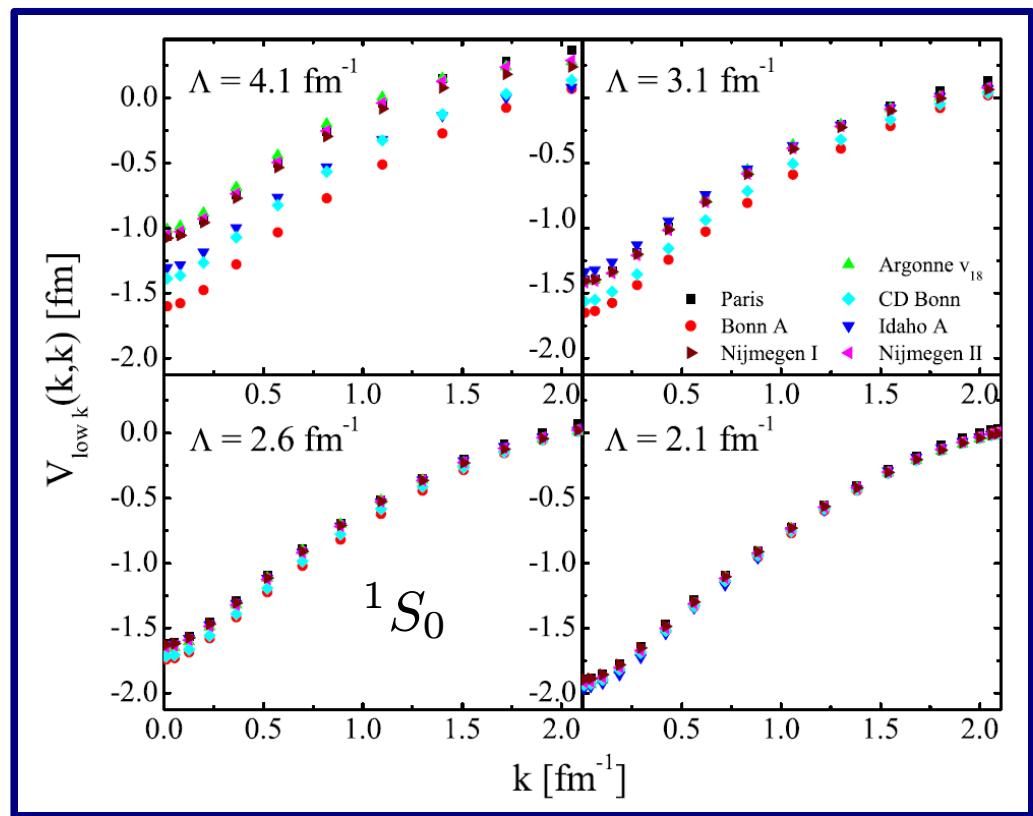


Low-momentum NN interactions

- Renormalization-group evolution yields “soft” NN potentials

$$T_{\text{low-}k}(p', p) = V_{\text{low-}k}(p', p) + \frac{2}{\pi} \mathcal{P} \int_0^\Lambda \frac{V_{\text{low-}k}(p', q) T_{\text{low-}k}(q, p)}{p^2 - q^2} q^2 dq$$

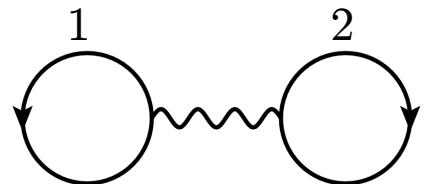
$$T_{\text{low-}k}(p', p) = T(p', p), \quad \text{for } p', p < \Lambda$$



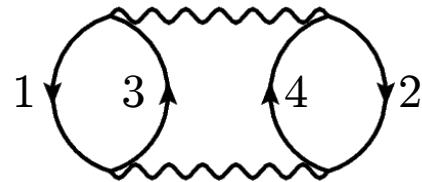
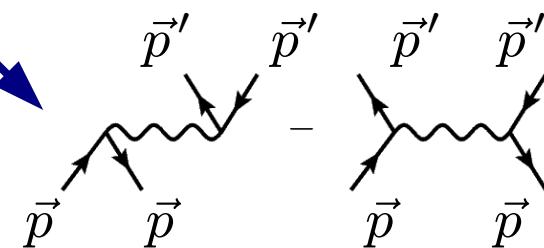
Bogner, Kuo, Schwenk (2003)

- Nearly universal two-nucleon interaction at $\Lambda \simeq 2 \text{ fm}^{-1}$
- Enhanced convergence in many-body perturbation theory
- Tool for assessing theoretical errors
- Requires consistent evolution of nuclear many-body forces

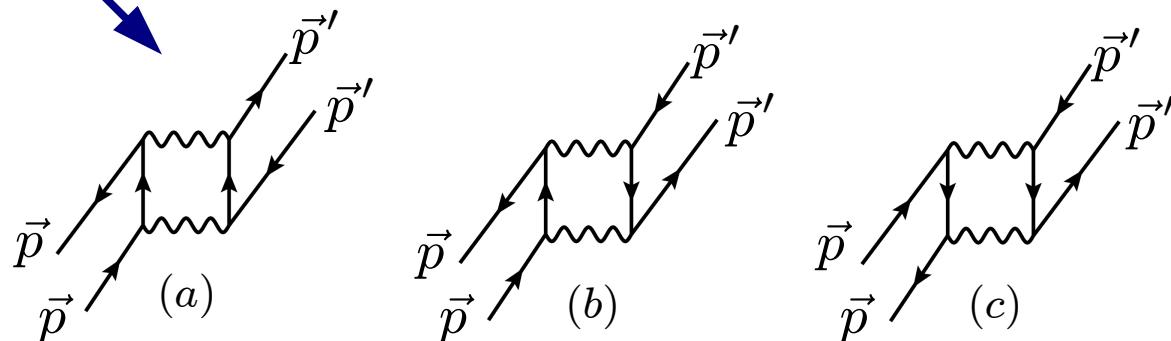
Perturbative calculation of quasiparticle interaction



$$\frac{E_{NN}^{(1)}}{V} = \frac{1}{2} \text{Tr}_{\sigma_1, \tau_1} \text{Tr}_{\sigma_2, \tau_2} \int \frac{d^3 k_1}{(2\pi)^3} \int \frac{d^3 k_2}{(2\pi)^3} n_{\vec{k}_1} n_{\vec{k}_2} \langle 12 | V(1 - P_{12}) | 12 \rangle$$



$$\begin{aligned} \frac{E_{NN}^{(2)}}{V} = & -\frac{1}{4} \prod_{i=1}^4 \left(\text{Tr}_{\sigma_i, \tau_i} \int \frac{d^3 k_i}{(2\pi)^3} \right) n_{\vec{k}_1} n_{\vec{k}_2} (1 - n_{\vec{k}_3})(1 - n_{\vec{k}_4}) \\ & \times \frac{|\langle 12 | V(1 - P_{34}) | 34 \rangle|^2}{\epsilon_{k_3} + \epsilon_{k_4} - \epsilon_{k_1} - \epsilon_{k_2}} (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2 - \vec{k}_3 - \vec{k}_4) \end{aligned}$$

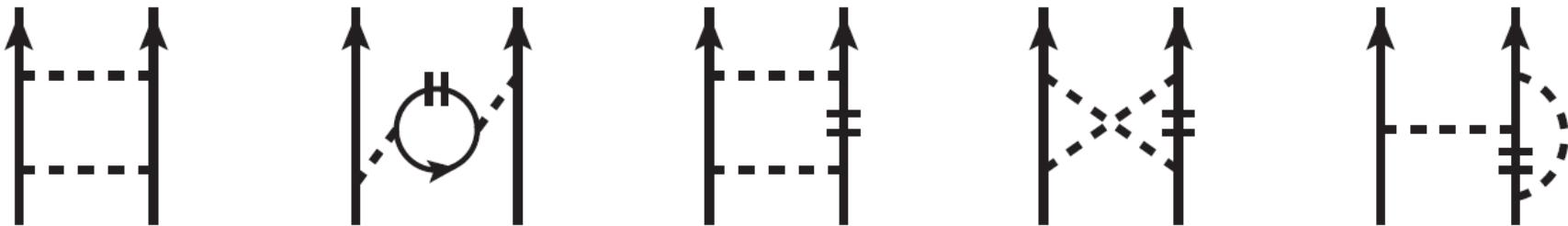


Analytically solvable models

[N. Kaiser, NPA (2006)]

- Alternative decomposition of second-order terms (number of “medium insertions”)

$$\begin{aligned} G(p_0, \vec{p}) &= i \left(\frac{\theta(|\vec{p}| - k_F)}{p_0 - \vec{p}^2/2M_N + i\epsilon} + \frac{\theta(k_F - |\vec{p}|)}{p_0 - \vec{p}^2/2M_N - i\epsilon} \right) \\ &= \frac{i}{p_0 - \vec{p}^2/2M_N + i\epsilon} - 2\pi\delta(p_0 - \vec{p}^2/2M_N)\theta(k_F - |\vec{p}|) \end{aligned}$$



- Average over both quasiparticle angles:

$$\begin{aligned} \mathcal{F}_L(k_F) &= \frac{2L+1}{(4\pi)^2} \int d\Omega_1 d\Omega_2 \langle \vec{p}_1 \vec{p}_2 | V_{\text{eff}} | \vec{p}_1 \vec{p}_2 \rangle P_L(\hat{p}_1 \cdot \hat{p}_2) \\ &= f_L(k_F) + g_L(k_F) \vec{\sigma}_1 \cdot \vec{\sigma}_2 + f'_L(k_F) \vec{\tau}_1 \cdot \vec{\tau}_2 + g'_L(k_F) \vec{\sigma}_1 \cdot \vec{\sigma}_2 \vec{\tau}_1 \cdot \vec{\tau}_2 \end{aligned}$$

Comparison of analytical and numerical results

[JWH, N. Kaiser and W. Weise, NPA (2011)]

$$V_C(q) = -\frac{g_s^2}{m_s^2 + q^2}$$

$$\{m_s = 500 \text{ MeV}, g_s = 2.5\}$$

$$V_T(\vec{q}) = -g_\pi^2 \frac{(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q})}{(m_\pi^2 + q^2)^2} \vec{\tau}_1 \cdot \vec{\tau}_2$$

$$\{m_\pi = 400 \text{ MeV}, g_\pi = 2.5\}$$

Scalar-isoscalar boson exchange ($k_F = 1.33 \text{ fm}^{-1}$)								
	f_0 [fm 2]	g_0 [fm 2]	f'_0 [fm 2]	g'_0 [fm 2]	f_1 [fm 2]	g_1 [fm 2]	f'_1 [fm 2]	g'_1 [fm 2]
1st	-0.809	0.164	0.164	0.164	0.060	0.060	0.060	0.060
2nd(pp)	-0.186	0.056	0.056	0.056	0.038	-0.006	-0.006	-0.006
2nd(hh)	-0.033	0.010	0.010	0.010	0.042	-0.013	-0.013	-0.013
2nd(ph)	0.198	0.061	0.061	0.061	0.100	0.085	0.085	0.085
Total	-0.830	0.291	0.291	0.291	0.240	0.127	0.127	0.127
Exact	-0.830	0.292	0.292	0.292	0.242	0.127	0.127	0.127

Modified pion exchange ($k_F = 1.33 \text{ fm}^{-1}$)								
	f_0 [fm 2]	g_0 [fm 2]	f'_0 [fm 2]	g'_0 [fm 2]	f_1 [fm 2]	g_1 [fm 2]	f'_1 [fm 2]	g'_1 [fm 2]
1st	0.244	-0.081	-0.081	0.027	-0.079	0.026	0.026	-0.009
2nd(pp)	-0.357	-0.062	0.269	0.104	0.018	-0.005	0.027	0.009
2nd(hh)	-0.017	-0.002	0.009	0.003	0.029	0.003	-0.014	-0.005
2nd(ph)	0.146	-0.023	0.027	0.008	0.008	0.010	0.036	-0.003
Total	0.017	-0.169	0.224	0.142	-0.024	0.035	0.075	-0.009
Exact	0.017	-0.169	0.224	0.142	-0.023	0.035	0.074	-0.009

First-order NN contributions

V_{NN}^{N3LO} ($k_F = 1.33 \text{ fm}^{-1}$)

l	$f_l [\text{fm}^2]$	$g_l [\text{fm}^2]$	$f'_l [\text{fm}^2]$	$g'_l [\text{fm}^2]$
0	-1.274	0.298	0.200	0.955
1	-1.018	0.529	0.230	0.090
2	-0.333	0.244	0.147	0.051

$V_{\text{low-}k}^{2.1}$ ($k_F = 1.33 \text{ fm}^{-1}$)

l	$f_l [\text{fm}^2]$	$g_l [\text{fm}^2]$	$f'_l [\text{fm}^2]$	$g'_l [\text{fm}^2]$
0	-1.919	0.327	0.497	1.099
1	-1.034	0.475	0.409	0.178
2	-0.378	0.243	0.168	0.056

$$\frac{M^*}{M_N} = 1 + \frac{F_1}{3} = 0.70$$

$$\mathcal{K} = \frac{3\hbar^2 k_F^2}{M^*} (1 + F_0) = -44 \text{ MeV}$$

$$\beta = \frac{\hbar^2 k_F^2}{6M^*} (1 + F'_0) = 21 \text{ MeV}$$

$$\delta g_l = \frac{F'_1 - F_1}{3(1 + F_1/3)} = 0.27$$

$$\frac{M^*}{M_N} = 1 + \frac{F_1}{3} = 0.69$$

$$\mathcal{K} = \frac{3\hbar^2 k_F^2}{M^*} (1 + F_0) = -220 \text{ MeV}$$

$$\beta = \frac{\hbar^2 k_F^2}{6m^*} (1 + F'_0) = 25 \text{ MeV}$$

$$\delta g_l = \frac{F'_1 - F_1}{3(1 + F_1/3)} = 0.31$$

- Nuclear matter unstable to scalar-isoscalar density fluctuations
 - Nuclear matter does not saturate with 'low-momentum' NN interactions alone
 - Three-nucleon forces & 2nd-order NN contributions

First- and second-order contributions (two-body forces)

[JWH, N. Kaiser and W. Weise, NPA (2011)]

Idaho N ³ LO potential for $k_F = 1.33 \text{ fm}^{-1}$								
	$f_0 [\text{fm}^2]$	$g_0 [\text{fm}^2]$	$f'_0 [\text{fm}^2]$	$g'_0 [\text{fm}^2]$	$f_1 [\text{fm}^2]$	$g_1 [\text{fm}^2]$	$f'_1 [\text{fm}^2]$	$g'_1 [\text{fm}^2]$
1st	-1.274	0.298	0.200	0.955	-1.018	0.529	0.230	0.090
2nd(pp)	-1.461	0.023	0.686	0.255	0.041	-0.059	0.334	0.254
2nd(hh)	-0.271	0.018	0.120	0.041	0.276	0.041	-0.144	-0.009
2nd(ph)	1.642	-0.057	0.429	0.162	0.889	-0.143	0.130	0.142
Total	-1.364	0.281	1.436	1.413	0.188	0.367	0.550	0.477

	F_0	G_0	F'_0	G'_0	F_1	G_1	F'_1	G'_1	M^*/M_N	$\mathcal{K} [\text{MeV}]$	$\beta [\text{MeV}]$	δg_l	g'_{NN}
V_{N3LO}	-1.64	0.35	1.39	1.59	-0.13	0.50	0.58	0.47	0.96	-148	30.5	0.12	0.67
$V_{\text{low-k}}^{2.1}$	-1.98	0.58	1.94	2.14	0.38	0.83	0.87	0.80	1.13	-191	31.8	0.07	0.77

- Large **second-order** contributions to
 - ✚ Isospin asymmetry energy (F'_0)
 - ✚ Quasiparticle effective mass (F_1)
 - ✚ Spin-isospin response (G'_0)

- ★ Compression modulus remains **negative**
- ★ Large effective mass has dramatic effect on **effective orbital magnetic moment**
- ★ Considerable scale dependence → *relevant physics is missing!*

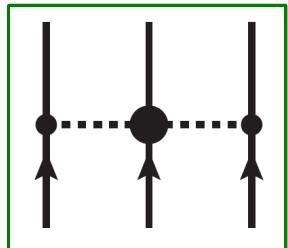
Scale dependence (two-body forces)

Idaho N ³ LO potential for $k_F = 1.33 \text{ fm}^{-1}$								
	f_0 [fm ²]	g_0 [fm ²]	f'_0 [fm ²]	g'_0 [fm ²]	f_1 [fm ²]	g_1 [fm ²]	f'_1 [fm ²]	g'_1 [fm ²]
1st	-1.274	0.298	0.200	0.955	-1.018	0.529	0.230	0.090
2nd(pp)	-1.461	0.023	0.686	0.255	0.041	-0.059	0.334	0.254
2nd(hh)	-0.271	0.018	0.120	0.041	0.276	0.041	-0.144	-0.009
2nd(ph)	1.642	-0.057	0.429	0.162	0.889	-0.143	0.130	0.142
Total	-1.364	0.281	1.436	1.413	0.188	0.367	0.550	0.477

$V_{\text{low}-\mathbf{k}}(\Lambda = 2.1 \text{ fm}^{-1})$ for $k_F = 1.33 \text{ fm}^{-1}$								
	f_0 [fm ²]	g_0 [fm ²]	f'_0 [fm ²]	g'_0 [fm ²]	f_1 [fm ²]	g_1 [fm ²]	f'_1 [fm ²]	g'_1 [fm ²]
1st	-1.919	0.327	0.497	1.099	-1.034	0.475	0.409	0.178
2nd(pp)	-0.864	-0.079	0.507	0.164	-0.130	0.011	0.236	0.174
2nd(hh)	-0.386	0.022	0.195	0.085	0.355	0.034	-0.195	-0.049
2nd(ph)	2.033	0.164	0.493	0.292	1.620	0.098	0.234	0.412
Total	-1.135	0.434	1.692	1.640	0.812	0.617	0.684	0.715

[JWH, N. Kaiser and W. Weise, NPA (2011)]

Contributions from (N^2LO) chiral three-nucleon force

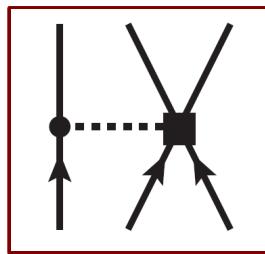
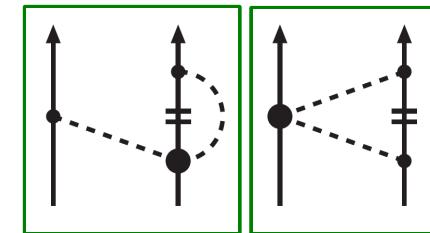
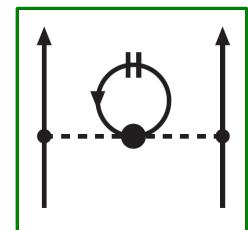


$$V_{3N}^{(2\pi)} = \sum_{i \neq j \neq k} \frac{g_A^2}{8f_\pi^4} \frac{\vec{\sigma}_i \cdot \vec{q}_i \vec{\sigma}_j \cdot \vec{q}_j}{(\vec{q}_i^2 + m_\pi^2)(\vec{q}_j^2 + m_\pi^2)} F_{ijk}^{\alpha\beta} \tau_i^\alpha \tau_j^\beta$$

$$F_{ijk}^{\alpha\beta} = \delta^{\alpha\beta} (-4c_1 m_\pi^2 + 2c_3 \vec{q}_i \cdot \vec{q}_j) + c_4 \epsilon^{\alpha\beta\gamma} \tau_k^\gamma \vec{\sigma}_k \cdot (\vec{q}_i \times \vec{q}_j)$$

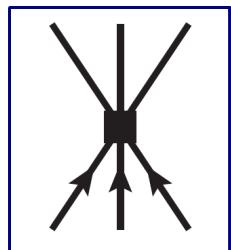
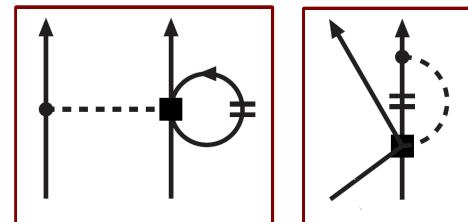
$$c_1 = -0.76, \quad c_3 = -4.78, \quad c_4 = 3.96 \quad [\text{GeV}^{-1}]$$

(Rentmeester et al., PRC (2003))



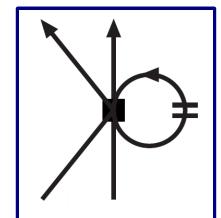
$$V_{3N}^{(1\pi)} = - \sum_{i \neq j \neq k} \frac{g_A c_D}{8f_\pi^4 \Lambda_\chi} \frac{\vec{\sigma}_j \cdot \vec{q}_j}{\vec{q}_j^2 + m_\pi^2} \vec{\sigma}_i \cdot \vec{q}_j \vec{\tau}_i \cdot \vec{\tau}_j$$

$$c_D(2.1 \text{ fm}^{-1}) = -2.06 \quad (\text{Nogga et al., PRC (2004)})$$

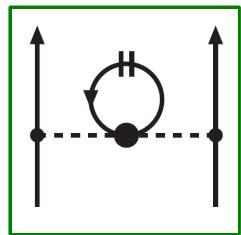


$$V_{3N}^{(ct)} = \sum_{i \neq j \neq k} \frac{c_E}{2f_\pi^4 \Lambda_\chi} \vec{\tau}_i \cdot \vec{\tau}_j$$

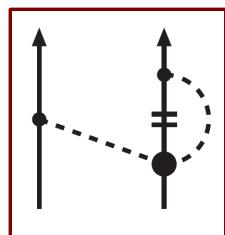
$$c_E(2.1 \text{ fm}^{-1}) = -0.63 \quad (\text{Nogga et al., PRC (2004)})$$



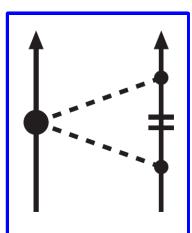
$L = 0$ quasiparticle interaction [JWH, N. Kaiser and W. Weise, NPA (2012)]



$$\left\{ \begin{aligned} \mathcal{F}_0(k_f)_{\text{ex}}^{(\text{med},1)} &= (3 - \sigma_1 \cdot \sigma_2)(3 - \tau_1 \cdot \tau_2) \frac{g_A^2 m_\pi^3}{(6\pi)^2 f_\pi^4} \left\{ \frac{(2c_1 - c_3)u^3}{1 + 4u^2} \right. \\ &\quad \left. - c_3 u^3 + (c_3 - c_1) \frac{u}{2} \ln(1 + 4u^2) \right\} \end{aligned} \right.$$

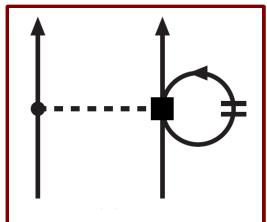


$$\left\{ \begin{aligned} \mathcal{F}_0(k_f)_{\text{ex}}^{(\text{med},2)} &= (3 - \sigma_1 \cdot \sigma_2)(3 - \tau_1 \cdot \tau_2) \frac{g_A^2 m_\pi^3}{(24\pi)^2 f_\pi^4} \\ &\quad \times \left\{ \frac{3c_1}{8u^5} \left[4u^2 - \ln(1 + 4u^2) \right] \left[8u^4 + 4u^2 - (1 + 4u^2) \ln(1 + 4u^2) \right] \right. \\ &\quad + c_3 \left[\frac{2}{u^2} \left(4u^2 - \ln(1 + 4u^2) \right) \arctan 2u + \frac{48u^4 + 16u^2 + 3}{64u^7} \ln^2(1 + 4u^2) \right. \\ &\quad + \frac{12u^4 - 16u^6 - 30u^2 - 9}{24u^5} \ln(1 + 4u^2) + \frac{20u^3}{3} - 11u + \frac{1}{u} + \frac{3}{4u^3} \left. \right] \\ &\quad + c_4 \left[\frac{4}{u^2} \left(\ln(1 + 4u^2) - 4u^2 \right) \arctan 2u + \frac{3 + 16u^2 - 48u^4}{64u^7} \ln^2(1 + 4u^2) \right. \\ &\quad \left. \left. + \frac{80u^6 + 12u^4 - 30u^2 - 9}{24u^5} \ln(1 + 4u^2) - \frac{28u^3}{3} + 13u + \frac{1}{u} + \frac{3}{4u^3} \right] \right\} \end{aligned} \right.$$

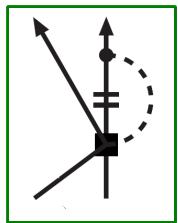


$$\left\{ \begin{aligned} \mathcal{F}_0(k_f)_{\text{dir-ex}}^{(\text{med},3)} &= \frac{g_A^2 m_\pi^3}{(4\pi)^2 f_\pi^4} \left\{ 24(c_3 - c_1)u - 8c_3 u^3 + (3c_3 - 4c_1) \frac{3}{u} \ln(1 + 4u^2) \right. \\ &\quad + 6(6c_1 - 5c_3) \arctan 2u + (3 - \sigma_1 \cdot \sigma_2)(3 - \tau_1 \cdot \tau_2) \frac{c_4}{9} \int_0^u dx (Y^2 - X^2) \\ &\quad \left. + (1 + \sigma_1 \cdot \sigma_2)(1 + \tau_1 \cdot \tau_2) \int_0^u dx \left[3c_1 Z^2 + \frac{c_3}{2}(X^2 + 2Y^2) \right] \right\} \end{aligned} \right.$$

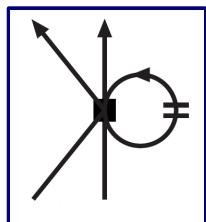
$L = 0$ quasiparticle interaction [JWH, N. Kaiser and W. Weise, NPA (2012)]



$$\mathcal{F}_0(k_f)_{\text{ex}}^{(\text{med},4)} = (3 - \sigma_1 \cdot \sigma_2)(3 - \tau_1 \cdot \tau_2) \frac{g_A c_D m_\pi^3}{(24\pi)^2 f_\pi^4 \Lambda_\chi} \left[4u^3 - u \ln(1 + 4u^2) \right]$$



$$\begin{aligned} \mathcal{F}_0(k_f)_{\text{dir-ex}}^{(\text{med},5)} = & (3 - \sigma_1 \cdot \sigma_2 - \tau_1 \cdot \tau_2 - \sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2) \frac{g_A c_D m_\pi^3}{16\pi^2 f_\pi^4 \Lambda_\chi} \left\{ \frac{2u^3}{3} \right. \\ & \left. - u + \arctan 2u - \frac{1}{4u} \ln(1 + 4u^2) \right\} \end{aligned}$$



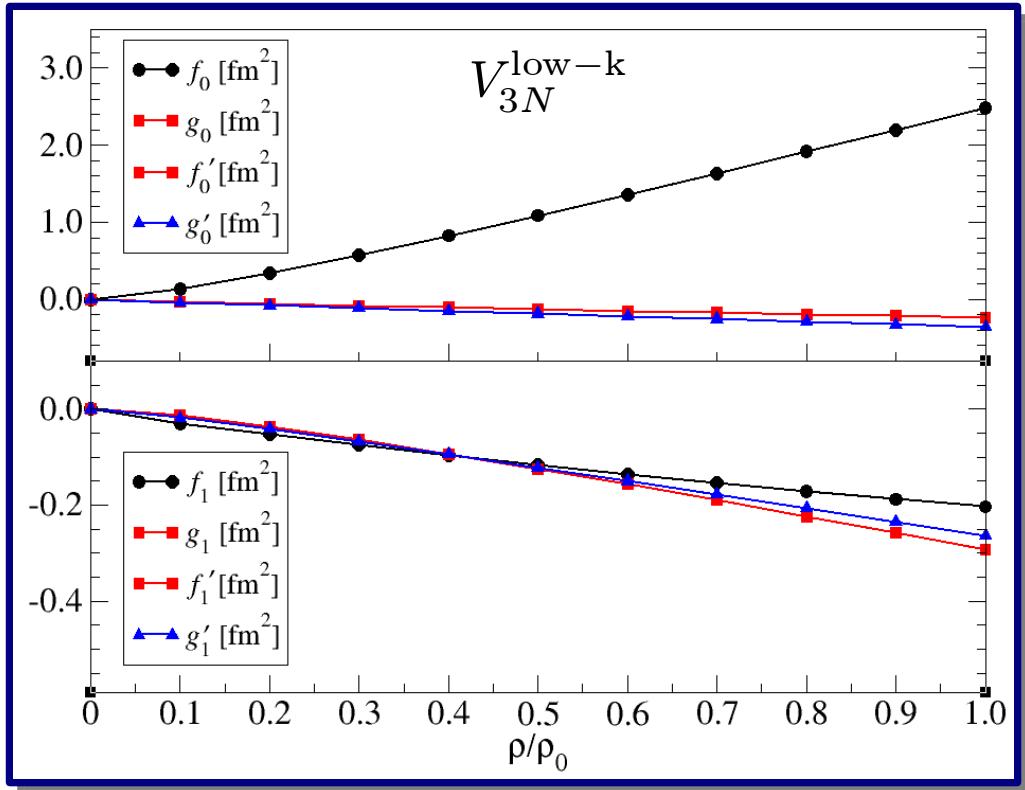
$$\mathcal{F}_0(k_f)_{\text{dir-ex}}^{(\text{med},6)} = (\sigma_1 \cdot \sigma_2 + \tau_1 \cdot \tau_2 + \sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2 - 3) \frac{c_E k_f^3}{4\pi^2 f_\pi^4 \Lambda_\chi}$$

- $u = \frac{k_F}{m_\pi}$ and X, Y, Z result from Fermi sphere integrals over two pion propagators
- Similar expressions for $L = 1$ Landau parameters:

$$\bar{\mathcal{F}}_1(k_F) = \frac{3}{(4\pi)^2} \int d\Omega_1 d\Omega_2 \langle \vec{p}_1 \vec{p}_2 | V_{\text{eff}}(1 - P_{12}) | \vec{p}_1 \vec{p}_2 \rangle P_1(\hat{p}_1 \cdot \hat{p}_2)$$

Three-nucleon forces at first order

[JWH, N. Kaiser and W. Weise, NPA (2012)]



V_{3N}^{N3LO} ($k_F = 1.33 \text{ fm}^{-1}$)				
l	$f_l [\text{fm}^2]$	$g_l [\text{fm}^2]$	$f'_l [\text{fm}^2]$	$g'_l [\text{fm}^2]$
0	1.218	0.009	0.009	-0.295
1	-0.073	-0.232	-0.232	-0.179

$V_{3N}^{\text{low-}k}$ ($k_F = 1.33 \text{ fm}^{-1}$)				
l	$f_l [\text{fm}^2]$	$g_l [\text{fm}^2]$	$f'_l [\text{fm}^2]$	$g'_l [\text{fm}^2]$
0	2.488	-0.237	-0.237	-0.359
1	-0.203	-0.293	-0.293	-0.263



$$f_0 [\text{fm}^2] = 1.41 - 1.17 + 2.15 - 0.188 - 0.369 + 0.655$$

$$g_0, f'_0 [\text{fm}^2] = -0.471 + 0.390 - 0.124 + 0.063 + 0.123 - 0.218$$

$$g'_0 [\text{fm}^2] = 0.157 - 0.130 - 0.270 - 0.021 + 0.123 - 0.218$$

- Large repulsion in spin- and isospin-independent interaction

Nuclear matter with leading-order 3NF

$V_{\text{N}3\text{LO}}^{(1+2)} \quad (k_F = 1.33 \text{ fm}^{-1})$				
l	F_l	G_l	F'_l	G'_l
0	-1.64	0.35	1.39	1.59
1	-0.13	0.50	0.58	0.47

$V_{\text{low-}k}^{(1+2)} \quad (k_F = 1.33 \text{ fm}^{-1})$				
l	F_l	G_l	F'_l	G'_l
0	-1.98	0.58	1.94	2.14
1	0.38	0.83	0.87	0.80

$V_{\text{N}3\text{LO}}^{(1+2)} + V_{3N}^{(1)} \quad (k_F = 1.33 \text{ fm}^{-1})$				
l	F_l	G_l	F'_l	G'_l
0	-0.15	0.35	1.36	1.20
1	-0.22	0.21	0.28	0.24

Reduced scale dependence



$V_{\text{low-}k}^{(1+2)} + V_{3N}^{(1)} \quad (k_F = 1.33 \text{ fm}^{-1})$				
l	F_l	G_l	F'_l	G'_l
0	1.48	0.22	1.45	1.48
1	0.08	0.37	0.41	0.39

$$\frac{M^*}{M_N} = 1 + \frac{F_1}{3} = 0.93$$

$$\mathcal{K} = \frac{3\hbar^2 k_F^2}{M^*} (1 + F_0) = 200 \text{ MeV}$$

$$\beta = \frac{\hbar^2 k_F^2}{6M^*} (1 + F'_0) = 31 \text{ MeV}$$

$$\delta g_l = \frac{F'_1 - F_1}{3(1 + F_1/3)} = 0.09$$

$$g'_{NN} = 0.52$$

$$[0.7 - 1.0]$$

$$[200 - 300 \text{ MeV}]$$

$$[30 - 36 \text{ MeV}]$$

$$[0.20 - 0.26]$$

$$[0.6 - 0.7]$$

$$\frac{M^*}{M_N} = 1 + \frac{F_1}{3} = 1.03$$

$$\mathcal{K} = \frac{3\hbar^2 k_F^2}{M^*} (1 + F_0) = 530 \text{ MeV}$$

$$\beta = \frac{\hbar^2 k_F^2}{6M^*} (1 + F'_0) = 29 \text{ MeV}$$

$$\delta g_l = \frac{F'_1 - F_1}{3(1 + F_1/3)} = 0.05$$

$$g'_{NN} = 0.58$$

NEUTRON MATTER

Response of neutron matter to neutrino probes

- Dynamics of stellar core collapse and subsequent evolution of residual compact star sensitive to neutrino processes (scattering, absorption and production) → input for *numerical simulations*
- Neutrino momenta small compared to the neutron Fermi momentum
 - Linear response theory (for vector and axial vector probes) applicable

- Neutrino mean free path

$$\frac{1}{\lambda(\vec{k}_i, T)} = \frac{G_F^2}{32\pi^3} \int d^3 k_f [(1 + \cos \theta) S^{(0)}(\omega, \vec{q}, T) + g_A^2 (3 - \cos \theta) S^{(1)}(\omega, \vec{q}, T)]$$

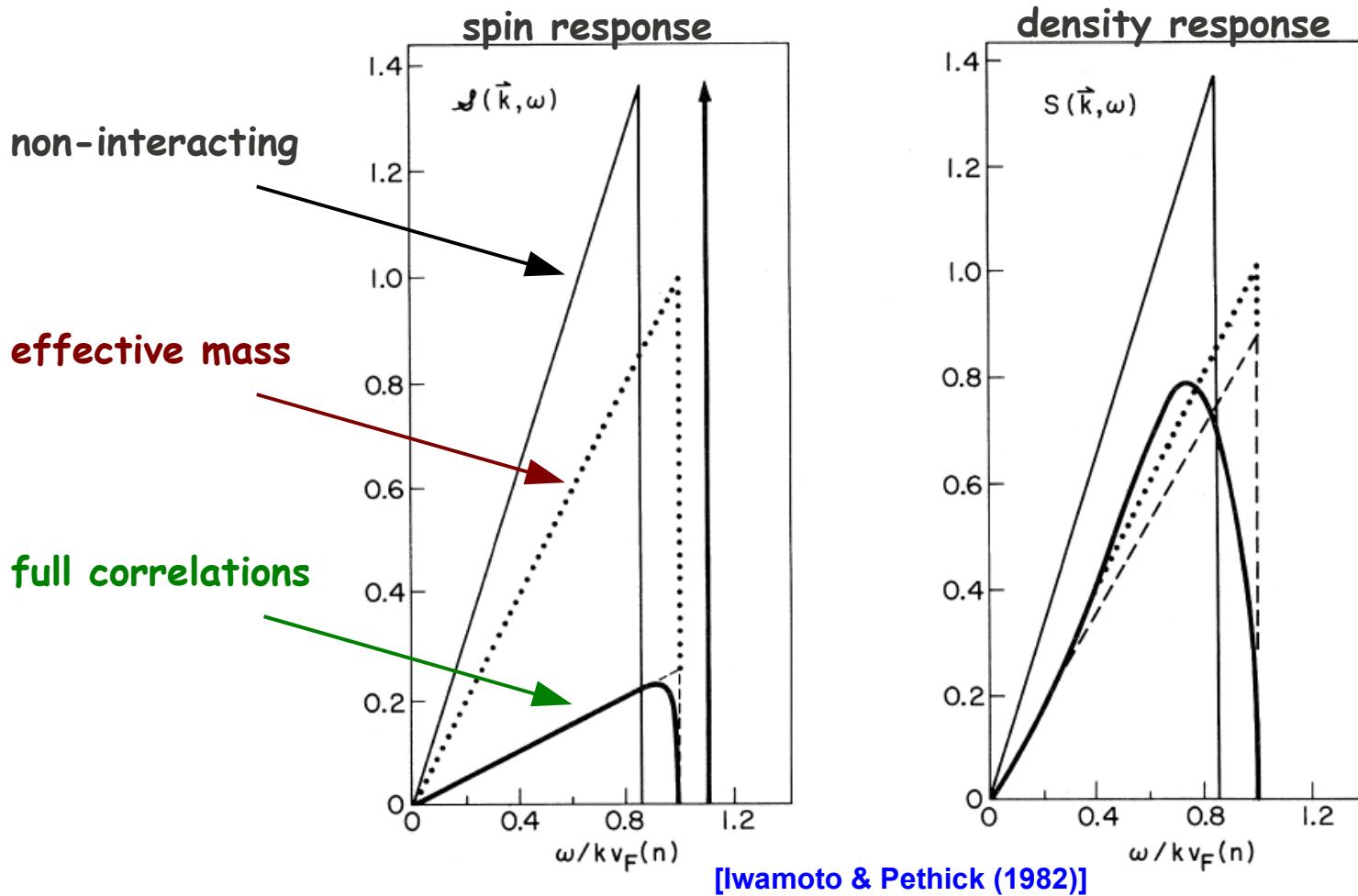
- **Structure factors** $S^{(S)}(\omega, \vec{q}, T)$ given by the imaginary part of response function
 - Particle-hole excitations and collective modes

- **Spin response function**

$$\chi_\sigma(\vec{q}, \omega) = \frac{N_0}{V} \frac{g(\lambda)}{1 + [G_0 + \lambda^2 G_1 / (1 + G_1/3)] g(\lambda)}$$

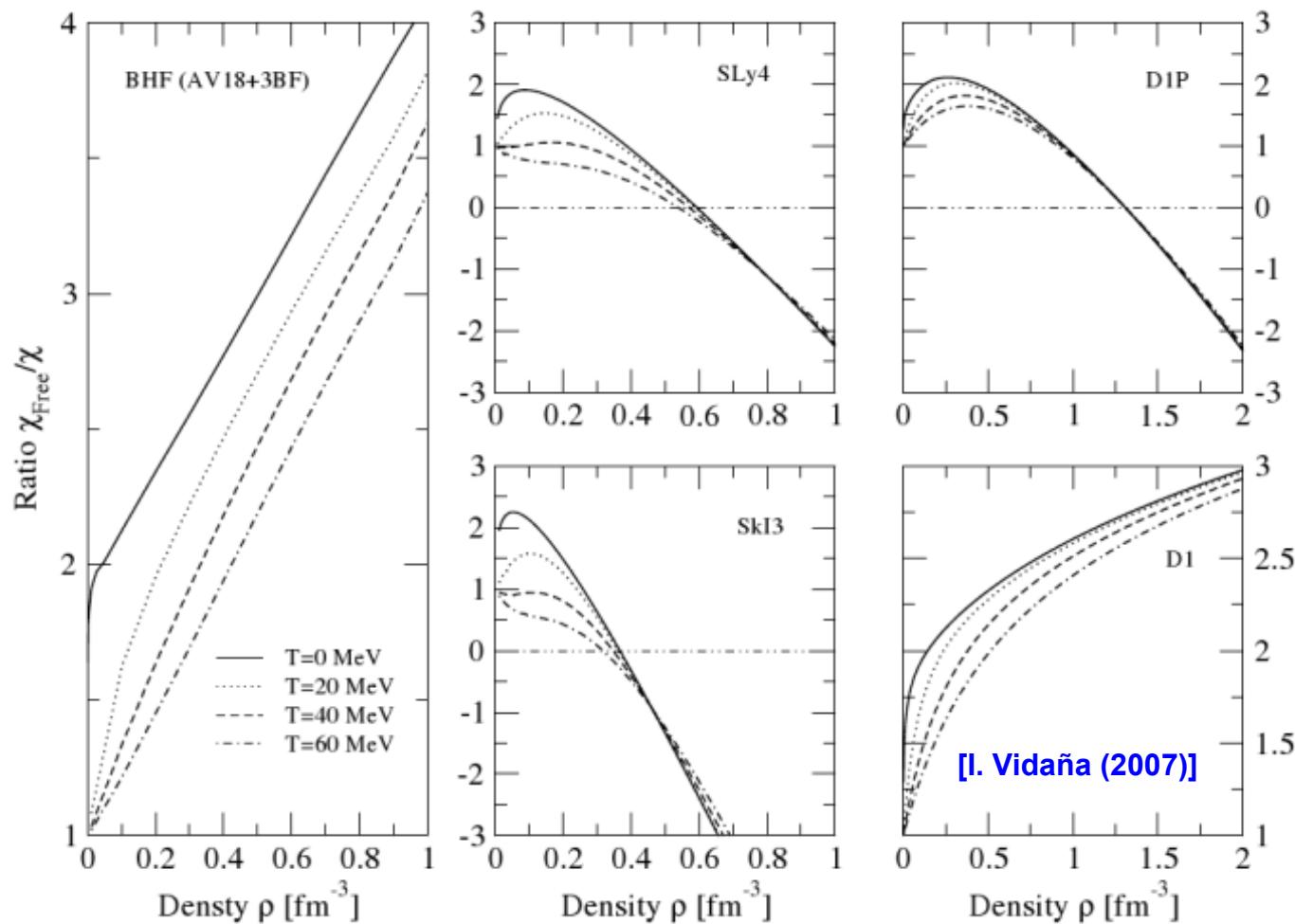
$$g(\lambda) = 1 - \frac{\lambda}{2} \ln \left| \frac{\lambda + 1}{\lambda - 1} \right| + \frac{i\pi}{2} \lambda \theta(1 - |\lambda|) \quad \lambda = \frac{\omega}{qv_F}$$

Nuclear correlation effects on structure functions



- Effect of nuclear interactions is to reduce the structure functions and **increase the neutrino mean free path**
- Collective spin excitations present

Neutron matter magnetic susceptibility



- Qualitative difference between **microscopic** and **phenomenological** forces
- Spin susceptibility (central quasiparticle interaction): $\chi = \frac{\mu_0^2 N(0)}{1 + G_0}$

Extraction of operator structures [JWH, N. Kaiser and W. Weise, in prep.]

$$\begin{aligned}\mathcal{F}(\vec{p}_1, \vec{p}_2) &= f(\vec{p}_1, \vec{p}_2) + g(\vec{p}_1, \vec{p}_2) \vec{\sigma}_1 \cdot \vec{\sigma}_2 + h(\vec{p}_1, \vec{p}_2) S_{12}(\hat{q}) \\ &\quad + k(\vec{p}_1, \vec{p}_2) S_{12}(\hat{P}) + l(\vec{p}_1, \vec{p}_2) (\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot (\hat{q} \times \hat{P})\end{aligned}$$

$$\vec{q} = \vec{p}_1 - \vec{p}_2 \propto \hat{z}$$

$$\vec{P} = \vec{p}_1 + \vec{p}_2 \propto \hat{x}$$

Central interactions

$$\left(\begin{array}{ccc|c} g & 0 & 0 & 0 \\ 0 & g & 0 & 0 \\ 0 & 0 & g & 0 \\ \hline 0 & 0 & 0 & -3g \end{array} \right) \quad \left(\begin{array}{ccc|c} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & f & 0 \\ \hline 0 & 0 & 0 & f \end{array} \right) \quad \begin{aligned} f &= (2\mathcal{F}_{11}^t + \mathcal{F}_{00}^t + \mathcal{F}_{00}^s)/4 \\ g &= (2\mathcal{F}_{11}^t + \mathcal{F}_{00}^t - 3\mathcal{F}_{00}^s)/12 \end{aligned}$$

Tensor interactions

$$\left(\begin{array}{ccc|c} 2h & 0 & 0 & 0 \\ 0 & -4h & 0 & 0 \\ 0 & 0 & 2h & 0 \\ \hline 0 & 0 & 0 & 0 \end{array} \right) \quad \left(\begin{array}{ccc|c} -k & 0 & 3k & 0 \\ 0 & 2k & 0 & 0 \\ 3k & 0 & -k & 0 \\ \hline 0 & 0 & 0 & 0 \end{array} \right) \quad \begin{aligned} h &= (\mathcal{F}_{11}^t + \mathcal{F}_{1-1}^t - \mathcal{F}_{00}^t)/6 \\ k &= (\mathcal{F}_{1-1}^t)/3 \end{aligned}$$

Cross vector interaction

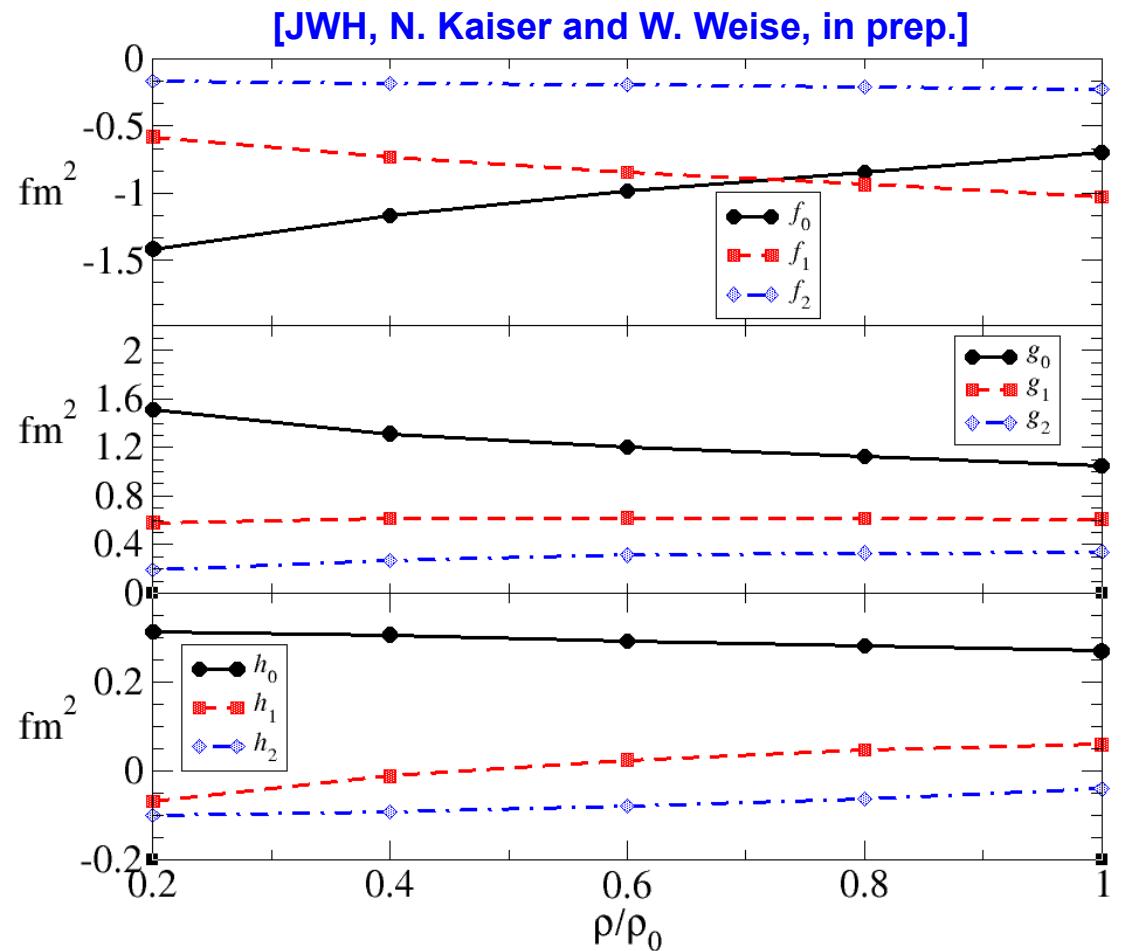
$$\left(\begin{array}{ccc|c} 0 & 0 & 0 & \sqrt{2}l \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{2}l \\ \hline \sqrt{2}l & 0 & \sqrt{2}l & 0 \end{array} \right) \quad l = (\mathcal{F}_{10}^{ts})/\sqrt{2}$$

Neutron matter FLPs: 1st order NN interactions

$$\mathcal{F}_L(k_F; Sm_s m'_s) = 8\pi(2L+1) \sum_{ll'J} i^{l-l'} (1 - (-1)^{l+S+J}) \sqrt{(2l+1)(2l'+1)} \\ \times \langle l0 Sm_s | JM \rangle \langle l'0 Sm'_s | JM \rangle \int_0^{k_F} dp \frac{p}{k_F^2} \langle pl SJM | V | pl' SJM \rangle P_L(1 - 2p^2/k_F^2)$$

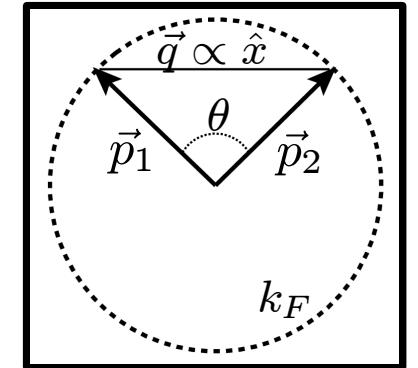
$V_{\text{N3LO}}^{(1)}$ ($k_F = 1.7 \text{ fm}^{-1}$)				
.	0	1	2	3
$f_L [\text{fm}^2]$	-0.70	-1.03	-0.23	-0.11
$g_L [\text{fm}^2]$	1.05	0.61	0.34	0.20
$h_L [\text{fm}^2]$	0.27	0.06	-0.04	-0.01

$V_{\text{low-k}}^{(1)}$ ($k_F = 1.7 \text{ fm}^{-1}$)				
.	0	1	2	3
$f_L [\text{fm}^2]$	-1.19	-0.68	-0.30	-0.11
$g_L [\text{fm}^2]$	1.21	0.65	0.35	0.20
$h_L [\text{fm}^2]$	0.24	0.17	-0.05	-0.08



Second-order (particle-particle & hole-hole)

$$\begin{aligned} \mathcal{F}_{pp}(\vec{p}_1 \vec{p}_2; Sm_s S' m'_s) &= -\frac{1}{2} \sum_{\bar{S} \bar{m}_s} \int \frac{d^3 k_3}{(2\pi)^3} \frac{d^3 k_4}{(2\pi)^3} (1 - n_{\vec{k}_3})(1 - n_{\vec{k}_4}) \\ &\times \frac{\langle \vec{p}_1 \vec{p}_2 Sm_s | \tilde{V} | \vec{k}_3 \vec{k}_4 \bar{S} \bar{m}_s \rangle \langle \vec{k}_3 \vec{k}_4 \bar{S} \bar{m}_s | \tilde{V} | \vec{p}_1 \vec{p}_2 S' m'_s \rangle}{\epsilon_{k_3} + \epsilon_{k_4} - \epsilon_{p_1} - \epsilon_{p_2}} (2\pi)^3 \delta(\vec{p}_1 + \vec{p}_2 - \vec{k}_3 - \vec{k}_4) \end{aligned}$$



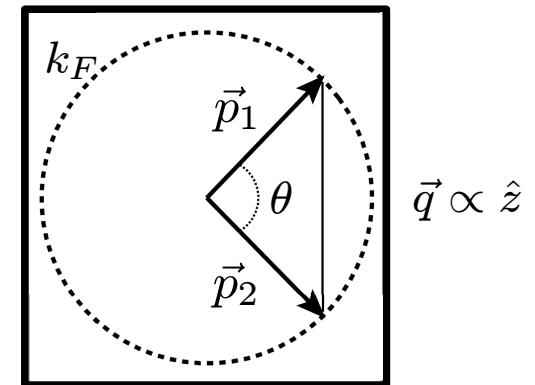
- $S = \bar{S} = S'$ from parity conservation
- $|m_s - m'_s| = 0, 2$
 - Integration over $\vec{k} = (\vec{k}_3 - \vec{k}_4)/2$ azimuthal angle yields $m_{l_2} = m_{l_3}$
 - $m_s + m_{l_1} = m'_s + m_{l_4}$
 - $m_s - m'_s = m_{l_4} - m_{l_1} = 2n \quad (P_{l_1}^{m_1}(\cos \theta_q) P_{l_4}^{m_1 \pm (2n+1)}(\cos \theta_q) \propto \cos \theta_q \rightarrow 0)$

$$\left(\begin{array}{cccc} f + g - h + 2k & 0 & 3h & 0 \\ 0 & f + g + 2h - 4k & 0 & 0 \\ 3h & 0 & f + g - h + 2k & 0 \\ \hline 0 & 0 & 0 & f - 3g \end{array} \right)$$

- Only non-central interactions are **exchange tensor** and **center-of-mass tensor**

Second-order (particle-hole)

$$\begin{aligned} \mathcal{F}_{ph}^{(a)}(\vec{p}_1 \vec{p}_2; s_1^a s_2^a s_2^b s_1^b) &= \sum_{s_3 s_4} \int \frac{d^3 k_3}{(2\pi)^3} \frac{d^3 k_4}{(2\pi)^3} n_{\vec{k}_3} (1 - n_{\vec{k}_4}) \\ &\times \frac{\langle \vec{p}_1 \vec{k}_3 s_1^a s_3 | \tilde{V} | \vec{p}_2 \vec{k}_4 s_2^a s_4 \rangle \langle \vec{p}_2 \vec{k}_4 s_2^b s_4 | \tilde{V} | \vec{p}_1 \vec{k}_3 s_1^b s_3 \rangle}{\epsilon_{p_2} + \epsilon_{k_4} - \epsilon_{p_1} - \epsilon_{k_3}} (2\pi)^3 \delta(\vec{p}_1 + \vec{k}_3 - \vec{p}_2 - \vec{k}_4) \end{aligned}$$



- Evaluate for all possible spin projections
- Form of quasiparticle interaction much less constrained
 - Spin-nonconserving interactions expected
- Expected form of interaction

$$\left(\begin{array}{cccc} f+g+2h-k & 0 & 3k & \sqrt{2}l \\ 0 & f+g-4h+2k & 0 & 0 \\ 3k & 0 & f+g+2h-k & \sqrt{2}l \\ \hline \sqrt{2}l & 0 & \sqrt{2}l & f-3g \end{array} \right)$$

Model interactions (noncentral components)

[JWH, N. Kaiser and W. Weise, in prep.]

$$V_\pi(\vec{q}) = -g_\pi^2 \frac{(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q})}{(m_\pi^2 + q^2)^2}$$

$$\{m_\pi = 300 \text{ MeV}, g_\pi = 2.5\}$$

Modified pion exchange ($k_F = 1.7 \text{ fm}^{-1}$)						
	h_0 [fm 2]	k_0 [fm 2]	l_0 [fm 2]	h_1 [fm 2]	k_1 [fm 2]	l_1 [fm 2]
2nd(pp)	0.0084	-0.0089	0.0000	-0.0104	0.0033	0.0000
2nd(hh)	0.0017	-0.0038	0.0000	-0.0036	0.0046	0.0000
2nd(ph)	0.0871	-0.0001	0.0000	0.0570	-0.0001	0.0000
Total	0.0972	-0.0128	0.0000	0.0430	0.0079	0.0000
Analytical	0.0969	-0.0130	0.0000	0.0449	0.0078	0.0000

- Agreement between numerical and (partial) analytical results within 2%

$$V_{LS} = -2g_s^2 \frac{i(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot (\vec{q} \times \vec{p})}{(m_s^2 + q^2)^2}$$

$$\{m_s = 800 \text{ MeV}, g_s = 6\}$$

particle-hole

$$\begin{pmatrix} & & & & \\ & 0.184 & 0.000 & -0.079 & 0.000 \\ & 0.000 & 0.368 & 0.000 & 0.000 \\ & -0.079 & 0.000 & 0.184 & 0.000 \\ \hline & 0.000 & 0.000 & 0.000 & 0.000 \end{pmatrix}$$

- Spin non-conserving interactions arise not through tensor or spin-orbit interactions **alone**
- Spin-orbit-tensor and spin-orbit-spin-spin mixing do produce nonzero l terms

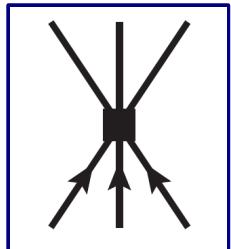
Realistic chiral NN interactions

[JWH, N. Kaiser and W. Weise, in prep.]

Chiral N ³ LO NN potential ($k_F = 1.7 \text{ fm}^{-1}$)										
	$f_0 \text{ [fm}^2]$	$g_0 \text{ [fm}^2]$	$h_0 \text{ [fm}^2]$	$k_0 \text{ [fm}^2]$	$l_0 \text{ [fm}^2]$	$f_1 \text{ [fm}^2]$	$g_1 \text{ [fm}^2]$	$h_1 \text{ [fm}^2]$	$k_1 \text{ [fm}^2]$	$l_1 \text{ [fm}^2]$
1st	-0.70	1.05	0.27	0.00	0.00	-1.03	0.61	0.06	0.00	0.00
2nd(pp)	-0.73	0.59	-0.10	-0.08	0.00	0.50	0.11	0.11	0.06	0.00
2nd(hh)	-0.15	0.01	-0.05	-0.04	0.00	0.17	0.09	0.08	0.05	0.00
2nd(ph)	0.99	0.06	-0.06	-0.06	0.14	0.48	-0.02	-0.09	-0.02	-0.03
Total	-0.59	1.71	0.06	-0.18	0.14	0.12	0.79	0.16	0.09	-0.03

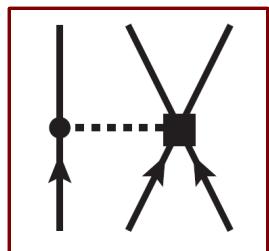
- Large renormalization of f_1 (effective mass) and g_0 (spin susceptibility)
- Reduction of isotropic **exchange tensor** interaction at second order h_0
- Coherent second-order contributions to the isotropic **center-of-mass tensor** interaction k_0
- Qualitatively similar statements hold for low-momentum interactions

Three-nucleon interactions in neutron matter



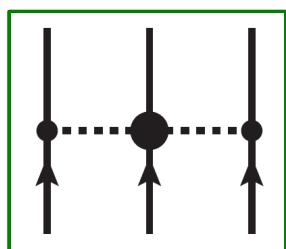
$$V_{3N}^{(ct)} = \sum_{i \neq j \neq k} \frac{c_E}{2f_\pi^4 \Lambda_\chi} \vec{\tau}_i \cdot \vec{\tau}_j$$

(vanishes)



$$V_{3N}^{(1\pi)} = - \sum_{i \neq j \neq k} \frac{g_A c_D}{8f_\pi^4 \Lambda_\chi} \frac{\vec{\sigma}_j \cdot \vec{q}_j}{\vec{q}_j^2 + m_\pi^2} \vec{\sigma}_i \cdot \vec{q}_j \vec{\tau}_i \cdot \vec{\tau}_j$$

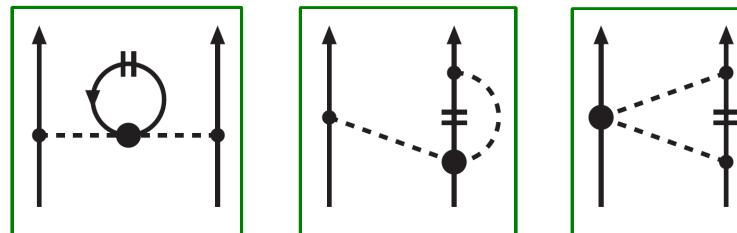
(vanishes)



$$V_{3N}^{(2\pi)} = \sum_{i \neq j \neq k} \frac{g_A^2}{8f_\pi^4} \frac{\vec{\sigma}_i \cdot \vec{q}_i \vec{\sigma}_j \cdot \vec{q}_j}{(\vec{q}_i^2 + m_\pi^2)(\vec{q}_j^2 + m_\pi^2)} F_{ijk}^{\alpha\beta} \tau_i^\alpha \tau_j^\beta$$

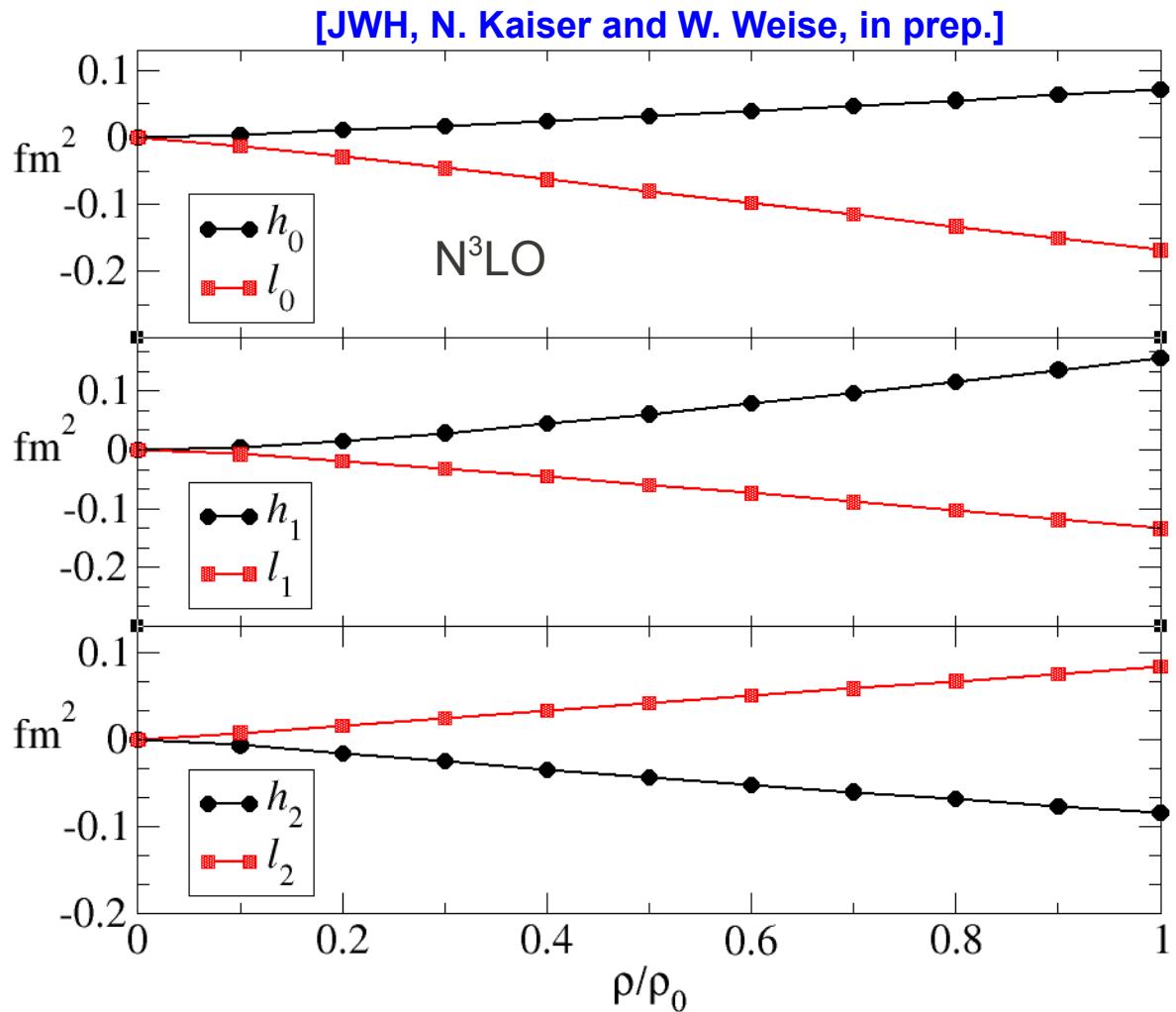
$$F_{ijk}^{\alpha\beta} = \delta^{\alpha\beta} (-4c_1 m_\pi^2 + 2c_3 \vec{q}_i \cdot \vec{q}_j) + c_4 \epsilon^{\alpha\beta\gamma} \tau_k^\gamma \vec{\sigma}_k \cdot (\vec{q}_i \times \vec{q}_j)$$

Only c_1 and c_3
terms survive



Density dependence of non-central interactions

$$\begin{aligned}\mathcal{F}(\vec{p}_1, \vec{p}_2) = & f(\vec{p}_1, \vec{p}_2) + g(\vec{p}_1, \vec{p}_2) \vec{\sigma}_1 \cdot \vec{\sigma}_2 + h(\vec{p}_1, \vec{p}_2) S_{12}(\hat{q}) \\ & + k(\vec{p}_1, \vec{p}_2) S_{12}(\hat{P}) + l(\vec{p}_1, \vec{p}_2) (\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot (\vec{q} \times \vec{P})\end{aligned}$$



- Low-energy constants
 - $c_1 = -0.81 \text{ GeV}^{-1}$
 - $c_3 = -3.2 \text{ GeV}^{-1}$
- Contributions to exchange-tensor
 -
 -
- Contribution to cross vector
 -

Final results (preliminary)

Chiral 2N+3N potential ($k_F = 1.7 \text{ fm}^{-1}$)										
	f_0 [fm 2]	g_0 [fm 2]	h_0 [fm 2]	k_0 [fm 2]	l_0 [fm 2]	f_1 [fm 2]	g_1 [fm 2]	h_1 [fm 2]	k_1 [fm 2]	l_1 [fm 2]
1st (2N)	-0.70	1.05	0.27	0.00	0.00	-1.03	0.61	0.06	0.00	0.00
1st (3N)	1.32	-0.28	0.07	0.00	-0.17	-0.04	-0.36	0.16	0.00	-0.13
2nd (2N)	0.08	0.46	-0.14	-0.13	0.10	0.81	0.13	0.07	0.06	-0.02
Total	0.70	1.23	0.20	-0.13	-0.07	-0.26	0.38	0.29	0.06	-0.15

$V_{\text{low}-\mathbf{k}}^{2.1} + 3\text{N}$ potential ($k_F = 1.7 \text{ fm}^{-1}$)										
	f_0 [fm 2]	g_0 [fm 2]	h_0 [fm 2]	k_0 [fm 2]	l_0 [fm 2]	f_1 [fm 2]	g_1 [fm 2]	h_1 [fm 2]	k_1 [fm 2]	l_1 [fm 2]
1st (2N)	-1.19	1.21	0.24	0.00	0.00	-0.68	0.65	0.17	0.00	0.00
1st (3N)	1.92	-0.42	0.11	0.00	-0.24	-0.07	-0.52	0.23	0.00	-0.20
2nd (2N)	0.29	0.05	-0.13	-0.08	-0.06	0.66	0.13	-0.01	0.12	-0.10
Total	1.02	0.84	0.22	-0.08	-0.30	-0.09	0.26	0.39	0.12	-0.30

Summary/Outlook

- Quasiparticle interaction from **chiral two- and three-nucleon interactions** provides accurate description of bulk nuclear matter properties
- Accurate numerical calculations of central + noncentral components of QPI at second-order in perturbation theory (chiral and low-momentum NN potentials)
- First calculation of **three-nucleon force contributions** to the quasiparticle interaction in pure neutron matter (including also noncentral interactions)

- Noncentral quasiparticle interactions in *symmetric nuclear matter*
- Applications to neutron star structure and evolution
- Spin observables in finite nuclei