Making sense of scale- and scheme-dependent observables in low-energy nuclear physics

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April, 2012

An overview of questions about spectroscopic factors (and the like) from a renormalization group perspective.

Partial list of 'non-observables' references

- *Equivalent Hamiltonians in scattering theory*, H. Ekstein, (1960)
- *Measurability of the deuteron D state probability*, J.L. Friar, (1979)
- *Problems in determining nuclear bound state wave functions*, R.D. Amado, (1979)
- *Nucleon nucleon bremsstrahlung: An example of the impossibility of measuring off-shell amplitudes*, H.W. Fearing, (1998)
- *Are occupation numbers observable?*, rjf and H.-W. Hammer, (2002)
- *Unitary correlation in nuclear reaction theory: Separation of nuclear reactions and spectroscopic factors*, A.M. Mukhamedzhanov and A.S. Kadyrov, (2010)
- *Non-observability of spectroscopic factors*, B.K. Jennings, (2011)
- *How should one formulate, extract, and interpret 'non-observables' for nuclei?*, rjf and A. Schwenk, (2010) [in J. Phys. G focus issue on Open Problems in Nuclear Structure Theory, edited by J. Dobaczewski]

‘Non-observables’ vs. Scheme-dependent observables

- Some quantities are *in principle* not observable
  - T.D. Lee: “The root of all symmetry principles lies in the assumption that it is impossible to observe certain basic quantities; these will be called ‘non-observables’.”
  - E.g., you can’t measure absolute position or time or a gauge

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- Directly measurable quantities are “clean” observables
  - E.g., cross sections and energies
  - Note: Association with a Hermitian operator is not enough!

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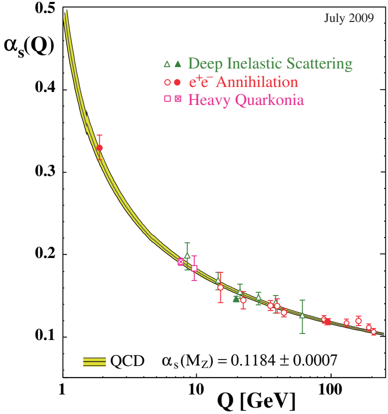
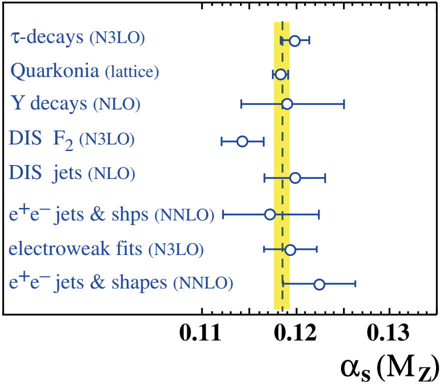
- Directly measurable quantities are “clean” observables
  - E.g., cross sections and energies
  - Note: Association with a Hermitian operator is not enough!
- Scale- and scheme-dependent derived quantities
  - Critical questions to address for each quantity:
    - What is the ambiguity or convention dependence?
    - Can one convert between different prescriptions?
    - Is there a consistent extraction from experiment such that they can be compared with other processes and theory?
  - Physical quantities can be *in-practice* clean observables if scheme dependence is negligible (e.g., $(e, 2e)$ from atoms)
  - How do we deal with dependence on the Hamiltonian?

Measuring the QCD Hamiltonian: Running $\alpha_s(Q^2)$



- Extractions from experiment can be compared (here at M_Z):

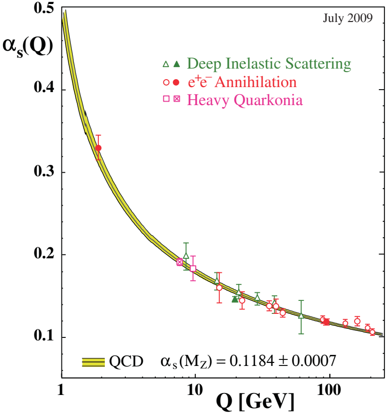


- The QCD coupling is *scale* dependent (“running”): $\alpha_s(Q^2) \approx [\beta_0 \ln(Q^2/\Lambda^2_{\mathrm{QCD}})]^{-1}$
- The QCD coupling strength α_s is *scheme* dependent (e.g., “V” scheme used on lattice, or $\overline{\mathrm{MS}}$)
- cf. QED, where $\alpha_{em}(Q^2)$ is effectively constant for soft Q^2:

  $\alpha_{em}(Q^2 = 0) \approx 1/137$

  $\therefore$ fixed H for quantum chemistry

Running QCD $\alpha_s(Q^2)$ vs. running nuclear V_λ



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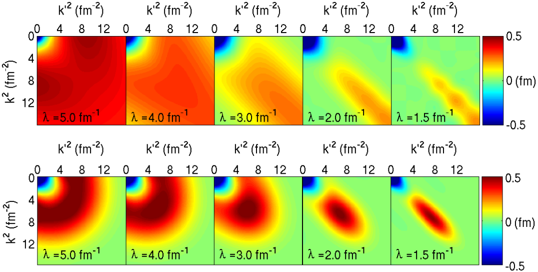
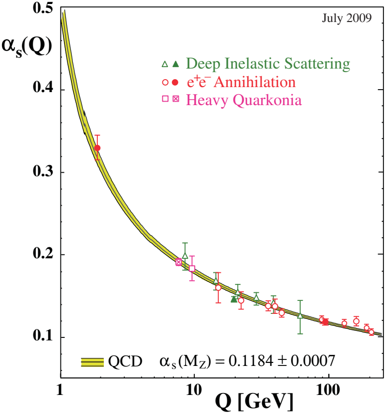
- Vary scale ("resolution") with RG
- Scale dependence: SRG (or $V_{\mathrm{low}\,k}$) running of initial potential with λ (decoupling or separation scale)



- Scheme dependence: AV18 vs. N^{3}LO (plus associated 3NFs)
- But all are (NN) phase equivalent!
- Shift contributions between interaction and sums over intermediate states

Running QCD $\alpha_s(Q^2)$ vs. running nuclear V_λ



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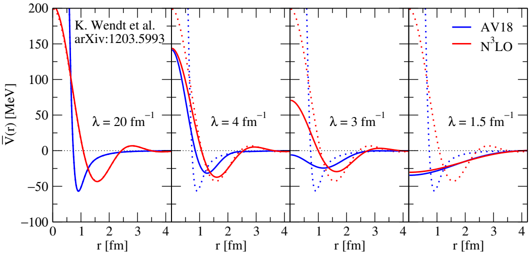
- Vary scale ("resolution") with RG
- Scale dependence: SRG (or $V_{\mathrm{low}\,k}$) running of initial potential with λ (decoupling or separation scale)
- Project non-local NN potential to visualize: $\overline{V}_\lambda(r) = \int d^3r'\, V_\lambda(r, r')$



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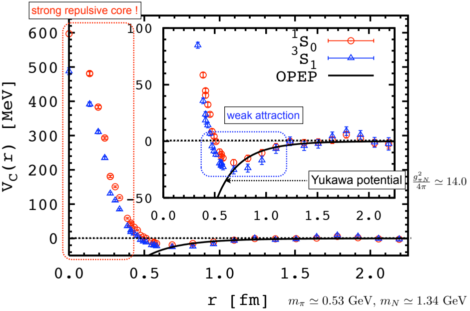
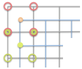
Determining the nuclear potential from lattice QCD

[S. Aoki, *Hadron interactions in lattice QCD*, arXiv:1107.1284]

NN (effective) central potentials $m_\pi \simeq 0.53$ GeV $t - t_s = 6$

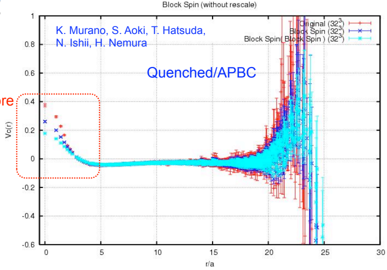


Smearing and potentials Preliminary



Wave function is smeared. ➡ "smeared" potential

Repulsive core becomes weaker.

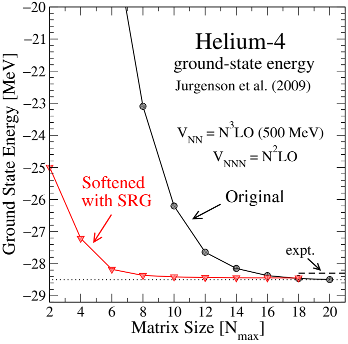


- "…the potential depends on the choice of nucleon operator…" which "…is considered to be a 'scheme' to define the potential."
- "Is such a scheme-dependent quantity useful? The answer to this question is probably 'yes', since the potential is useful to understand or describe the phenomena."
- Claim: useful to choose a scheme that yields good convergence of the velocity expansion (which means close to local)

Scale for structure: Nuclei with *soft* interactions

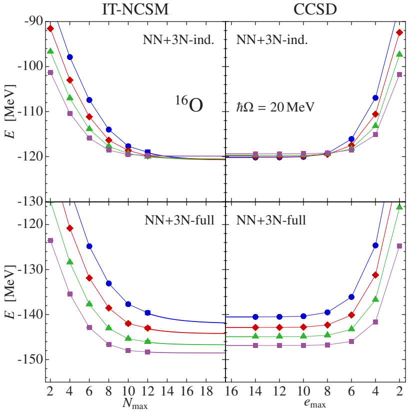
Softened potentials (SRG, $V_{\mathrm{low}\,k}$, UCOM, ...) enhance convergence

- Convergence for no-core shell model (NCSM):



- (Already) soft chiral EFT potential and evolved (softened) SRG potentials, including NNN

- Softening allows importance truncation (IT) and converged coupled cluster (CCSD)



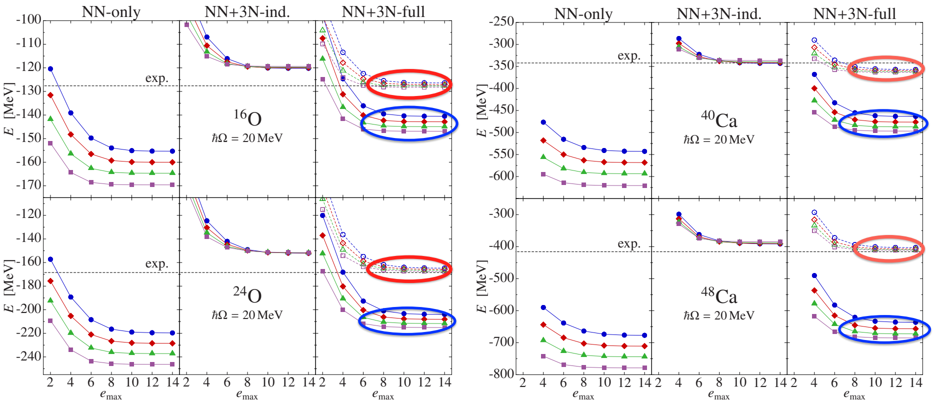
[Roth et al., arXiv:1112.0287]

- Also enables ab initio nuclear reactions with NCSM/RGM [Navratil et al.]

Scale for structure: Nuclei with *soft* interactions

R. Roth et al. SRG-evolved N^{3}LO with NNN [arXiv:1112.0287]

- Coupled cluster with interactions $H(\lambda)$: λ is a decoupling scale
  - NN-only: doesn't include induced NNN $\Longrightarrow$ λ dependent
  - NN+3N-induced: λ independent energies *but different NNN for each* λ
  - NN+3N-full: includes (two) initial NNN fit to $A = 3, 4$ properties



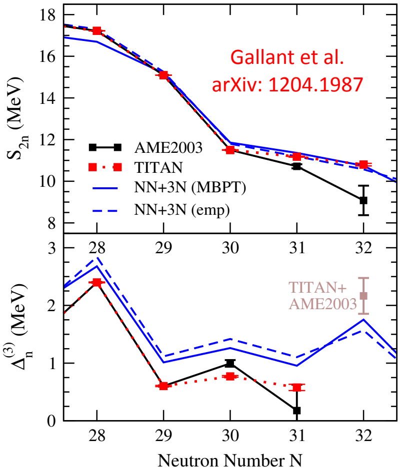
- Same predictions for λ's! (but still issues about NNN to resolve)

Scale for structure: Nuclei with *soft* interactions

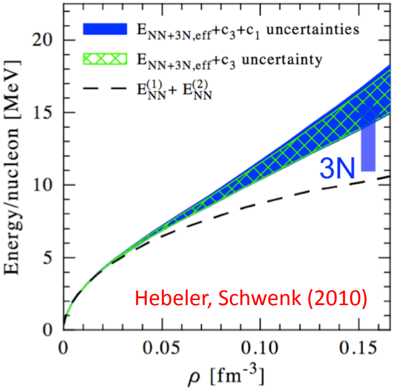
Lowered scale enables many-body perturbation theory (MBPT)

- Evolve NN to low momentum, fit NNN to $A = 3, 4$ at each scale

- Quantitative prediction for Ca isotope S_{2n} trends (verified!)



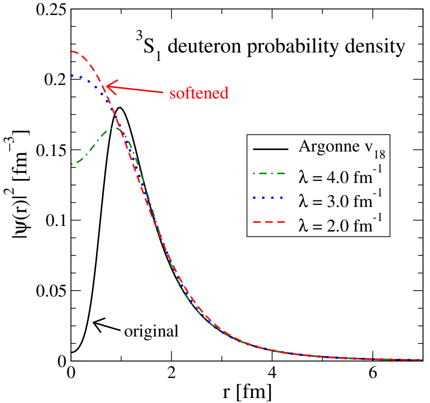
- Neutron/nuclear matter

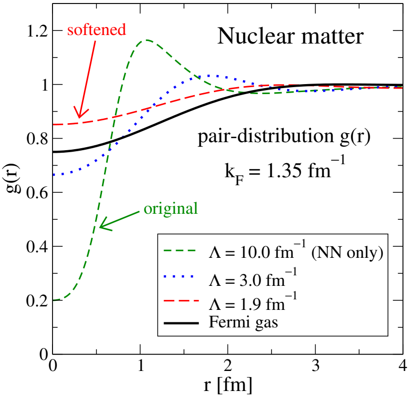


- Constrain neutron stars: $R = 10\text{–}14\,\text{km}$ for $1.4\,M_{\rm sun}$ [Hebeler et al. (2010)]

Scale for structure: Nuclei with *soft* interactions

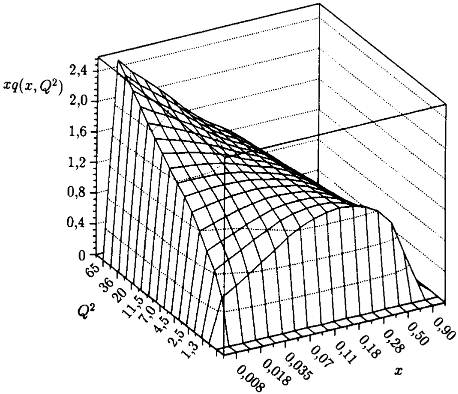
But soft potentials don't lead to short-range correlations (SRC)!





- Continuously transformed potential $\Longrightarrow$ variable SRC's in wf!
- Therefore, it seems that SRC's are *very scale/scheme dependent*
- Is there an analog in high energy QCD?

Parton vs. nuclear momentum distributions



- The quark distribution $q(x, Q^2)$ is scheme *and* scale dependent
- $x\ q(x, Q^2)$ measures the share of momentum carried by the quarks in a particular x-interval
- $q(x, Q^2)$ and $q(x, Q_0^2)$ are related by RG evolution equations

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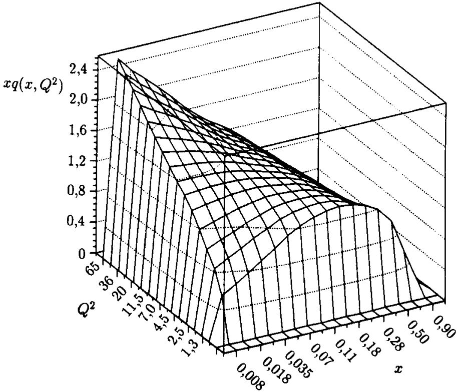

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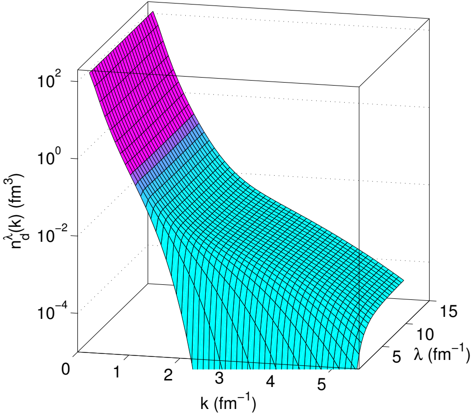
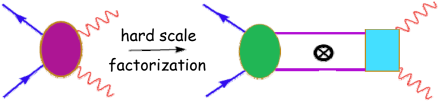


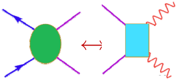
- Deuteron momentum distribution is scheme *and* scale dependent
- Initial AV18 potential evolved with SRG from $\lambda = \infty$ to $\lambda = 1.5\,\mathrm{fm}^{-1}$
- High momentum tail shrinks as λ decreases (lower resolution)

Factorization: high-E QCD vs. low-E nuclear



$$F_2(x, Q^2) \sim \sum_a f_a(x, \mu_f) \otimes \widehat{F}_2^a(x, Q/\mu_f)$$

long-distance parton density



short-distance Wilson coefficient

- Separation between long- and short-distance physics is not unique $\Longrightarrow$ introduce μ_f
- Choice of μ_f defines border between long/short distance
- Form factor F_2 is independent of μ_f, but pieces are not
- Choice of scheme: re-shuffles between parton distributions and Wilson coefficients

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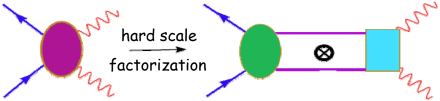

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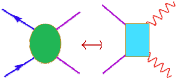

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- Also has factorization assumptions (e.g., from D. Bazin ECT* talk, 5/2011)

Observable: cross section

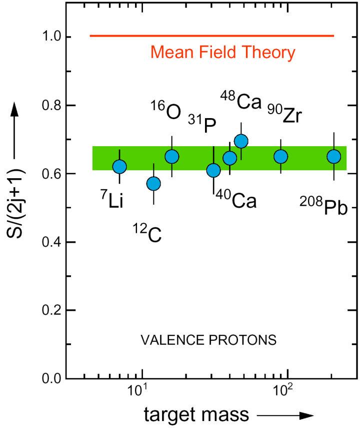
Structure model: spectroscopic factor

Reaction model: single-particle cross section

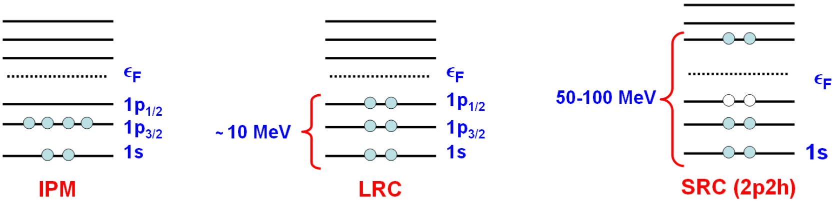
$$\sigma^{if} = \sum_{|J_f - J_i| \le j \le J_f + J_i} S_j^{if} \sigma_{sp}$$

- Is the factorization general/robust? (Process dependence?)
- What does it mean to be *consistent* between structure and reaction models? Can they be treated separately?
- How does scale/scheme dependence come in?
- What are the trade-offs? (Does simpler structure part always mean more complicated reaction part?)

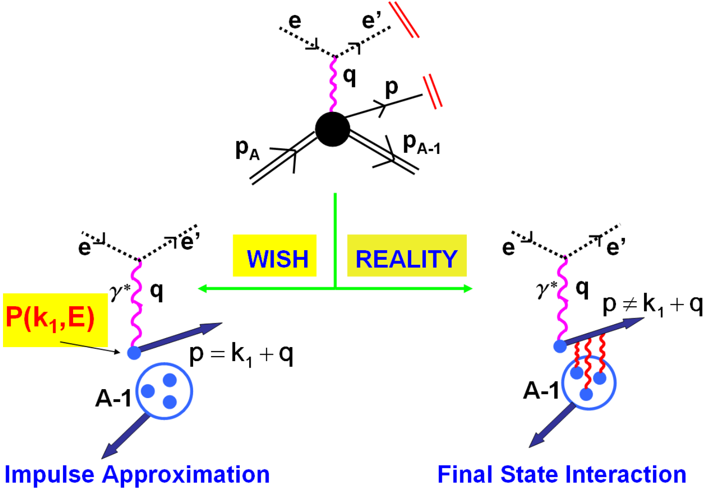
Scale/scheme dependence: spectroscopic factors



- Spectroscopic factors for valence protons have been extracted from $(e, e'p)$ experimental cross sections (e.g., NIKHEF 1990's at left)
- Used as canonical evidence for "correlations", particularly short-range correlations (SRC's)
- But if SFs are scale/scheme dependent, how do we explain the cross section?

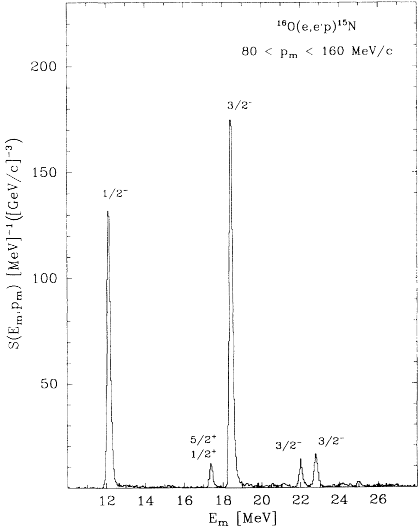


Standard story for $(e, e'p)$ [from C. Ciofi degli Atti]

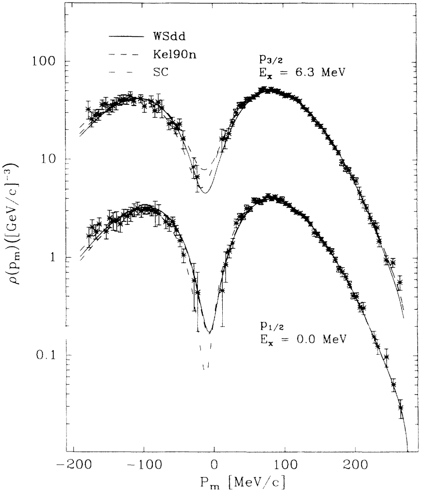


- In IA: "missing" momentum $p_m = k_1$ and energy $E_m = E$
- Choose E_m to select a discrete final state for range of p_m
- FSI treated as managable *add-on* theoretical correction to IA

(Assumed) factorization of $(e, e'p)$ cross section



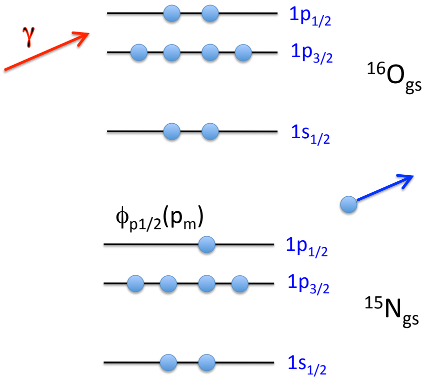
Missing energy spectrum for
$^{16}O(e, e'p)^{15}N$ [Leuschner (1994)]

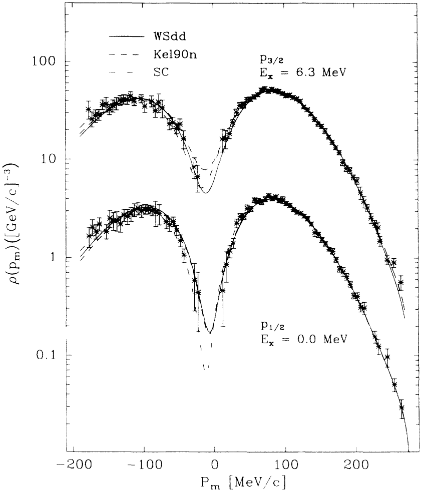


$$\frac{d\sigma}{d\mathbf{p}'_e d\mathbf{p}'_N} = K\sigma_{ep} \times \rho(\mathbf{p}_m) \;\; \propto |\phi_\alpha(\mathbf{p}_m)|^2$$

$\Longrightarrow p_{1/2}$ spectroscopic factor ≈ 0.63

(Assumed) factorization of $(e, e'p)$ cross section





- Knock out $p_{1/2}$ proton from ^{16}O to ^{15}N ground state in IPM
- Adjust s.p. well depth and radius to identify $\phi_\alpha(\mathbf{p}_m)$
- Final state interactions (FSI) added using optical potential(s)

$$\frac{d\sigma}{d\mathbf{p}'_e d\mathbf{p}'_N} = K\sigma_{ep} \times \rho(\mathbf{p}_m) \ \propto |\phi_\alpha(\mathbf{p}_m)|^2$$

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Source of scale-dependence for low-E structure

- Measured cross section as convolution: reaction $\otimes$ structure
  - but separate parts are not unique, *only* the combination
- Short-range unitary transformation U leaves m.e.'s invariant:

  $$O_{mn} \equiv \langle \Psi_m | O | \Psi_n \rangle = \left(\langle \Psi_m | U^\dagger \right) UOU^\dagger \left(U | \Psi_n \rangle \right) = \langle \widetilde{\Psi}_m | \widetilde{O} | \widetilde{\Psi}_n \rangle \equiv \widetilde{O}_{\widetilde{m}\widetilde{n}}$$

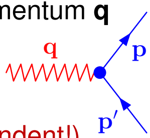
  But the matrix elements of operator O itself between the transformed states are in general modified:

  $$O_{\widetilde{m}\widetilde{n}} \equiv \langle \widetilde{\Psi}_m | O | \widetilde{\Psi}_n \rangle \neq O_{mn} \quad \Longrightarrow \quad \text{e.g., } \langle \Psi_n^{A-1} | a_\alpha | \Psi_0^A \rangle \text{ changes}$$

- In a low-energy effective theory, transformations that modify *short-range* unresolved physics $\Longrightarrow$ equally valid states.
  So $\widetilde{O}_{mn} \neq O_{mn} \Longrightarrow$ scale/scheme dependent observables.
- [Field theory version: the equivalence principle says that only on-shell quantities can be measured. Field redefinitions change off-shell dependence only.]
- Plan: Use (RG) unitary transformations to characterize and explore scale and scheme and process dependence!

Generic knockout reaction [à la Dickhoff/Van Neck text]

- Consider a scalar external probe that just transfers momentum $\mathbf{q}$



$$\rho(\mathbf{q}) = \rho_0 \sum_{j=1}^{A} e^{-i\mathbf{q}\cdot\mathbf{r}} \implies \widehat{\rho}(\mathbf{q}) = \rho_0 \sum_{\mathbf{p},\mathbf{p}'} \langle \mathbf{p}|e^{-i\mathbf{q}\cdot\mathbf{r}}|\mathbf{p}'\rangle a^\dagger_{\mathbf{p}} a_{\mathbf{p}'}$$

  - Typical assumption: one-body operator (scale dependent!)

- Then the cross section from Fermi's golden rule is

$$d\sigma \sim \sum \delta(\omega + E_i - E_f)|\langle \Psi_f|\widehat{\rho}(\mathbf{q})|\Psi_i\rangle|^2$$

- Complication: ejected final particle A interacts on way out (FSI)

$$H_A = \sum_{i=1}^{A} \frac{p_i^2}{2m} + \sum_{i<j=1}^{A} V(i,j) = H_{A-1} + \frac{p_A^2}{2m} + \sum_{i=1}^{A-1} V(i,A)$$

  - If we neglect this interaction $\implies$ PW (no FSI)

$$|\Psi_i\rangle = |\Psi_0^A\rangle\ , \qquad |\Psi_f\rangle = a^\dagger_{\mathbf{p}}|\Psi_n^{A-1}\rangle \implies \langle\Psi_f| = \langle\Psi_n^{A-1}|a_{\mathbf{p}}$$

$\implies$ factorized knockout cross section $\propto$ hole spectral fcn:

$$d\sigma \sim \rho_0^2 \sum_n \delta(E_m - E_0^A + E_n^{A-1})|\langle\Psi_n^{A-1}|a_{\mathbf{p}_m}|\Psi_0^A\rangle|^2 = \rho_0^2\, S_h(\mathbf{p}_m, E_m)$$

- Does it still factorize when corrected for (scale dependent!) FSI?

Now repeat with a unitary transformation $\widehat{U}$

- The cross section is *guaranteed* to be the same from $\widehat{U}^\dagger \widehat{U} = 1$

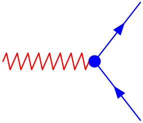
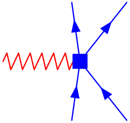
$$
\begin{aligned}
d\sigma &\sim \sum \delta(\omega + E_i - E_f) |\langle \Psi_f | \widehat{\rho}(\mathbf{q}) | \Psi_i \rangle|^2 \\
&= \sum \delta(\omega + E_i - E_f) |\langle \Psi_f | (\widehat{U}^\dagger \widehat{U}) \widehat{\rho}(\mathbf{q}) (\widehat{U}^\dagger \widehat{U}) | \Psi_i \rangle|^2 \\
&= \sum \delta(\omega + E_i - E_f) |(\langle \Psi_f | \widehat{U}^\dagger)(\widehat{U} \widehat{\rho}(\mathbf{q}) \widehat{U}^\dagger)(\widehat{U} | \Psi_i \rangle)|^2
\end{aligned}
$$

  but the pieces are different now.

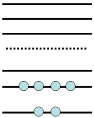
- Schematically, the SRG has $\widehat{U} = 1 + \frac{1}{2}(U - 1) a^\dagger a^\dagger a a + \cdots$
  - U is found by solving for the unitary transformation in the $A = 2$ system (this is the easy part!)
  - The $\cdots$'s represent higher-body operators
  - One-body operators ($\propto a^\dagger a$) gain many-body pieces (EFT: there are always many-body pieces at some level!)
  - Both initial and final states are modified (and therefore FSI)

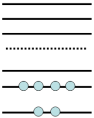
New pieces after the unitary transformation

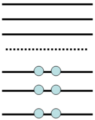
- The current is no longer just one-body (cf. EFT current):

$\widehat{U}\widehat{\rho}(\mathbf{q})\widehat{U}^\dagger =$  $+\,\alpha$  $+\cdots$

- New correlations have appeared (or disappeared):

$\widehat{U}|\Psi_0^A\rangle = \widehat{U}$ 

$+\cdots \Longrightarrow$ 

$+\,\alpha$ 

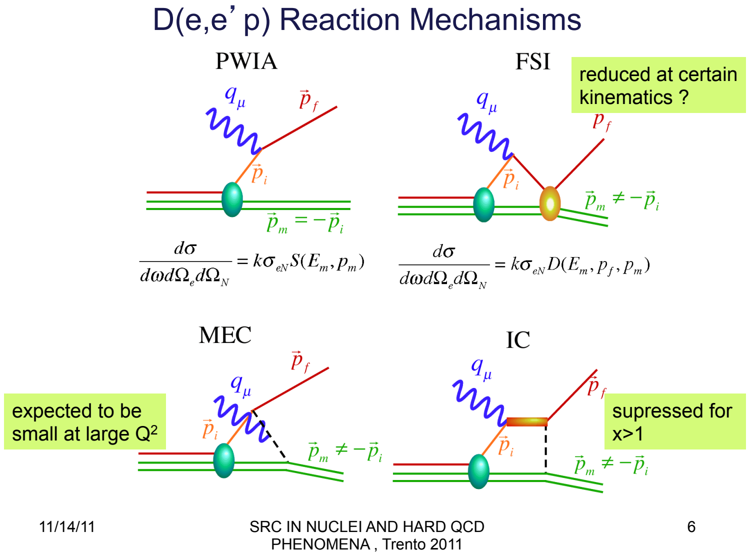
$+\cdots$

  - Similarly with $|\Psi_f\rangle = a_{\mathbf{p}}^\dagger|\Psi_n^{A-1}\rangle$
  - So the spectroscopic factors are modified

- Final state interactions are also modified by $\widehat{U}$

- Bottom line: the cross section is unchanged *only* if all pieces are included, with the same U: $H(\lambda)$, current operator, FSI, ...

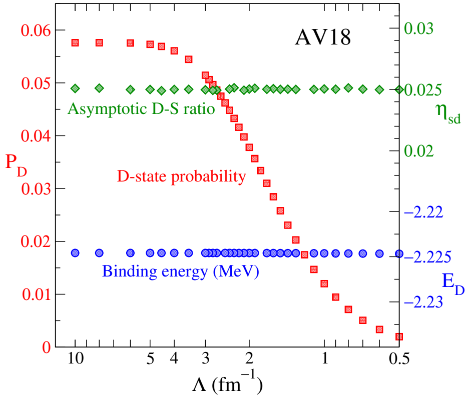
Can we treat corrections independently? [Boeglin ECT*]

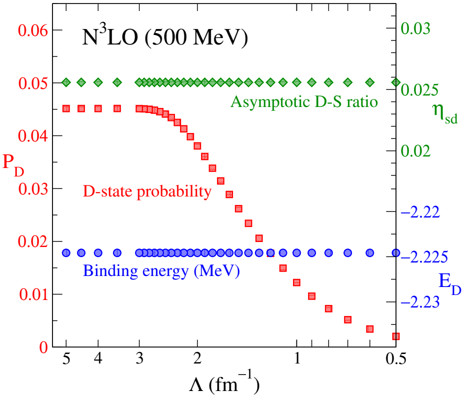


Answer: Mixtures are scale/scheme dependent (cf. 3NF)

But how much are the pieces changed as λ varies? (in progress!)

Deuteron scale-(in)dependent observables

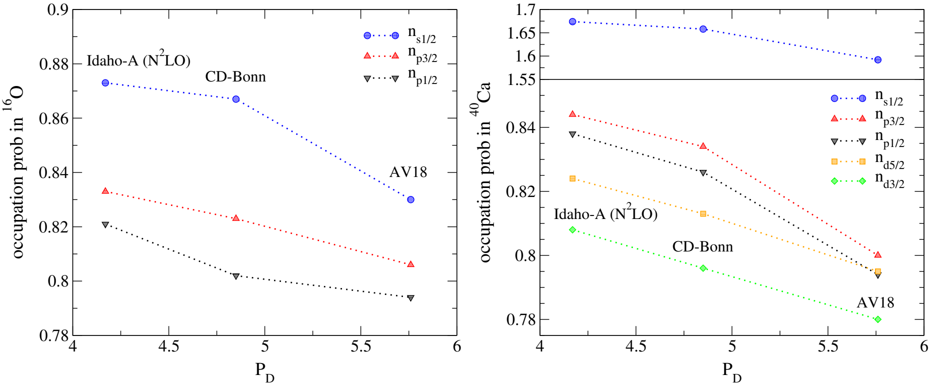




- $V_{\text{low }k}$ RG transformations labeled by Λ (different V_Λ's)
  $\Longrightarrow$ soften interactions by lowering resolution (scale)
  $\Longrightarrow$ reduced short-range and tensor correlations
- Energy and asymptotic D-S ratio are unchanged (cf. ANC's)
- But D-state probability changes (cf. spectroscopic factors)
- Plan: Make analogous calculations for $A > 2$ quantities (like SFs)

Correlation of P_D with spectroscopic factors?

Calculations from Gad and Muether, Phys. Rev. C **66**, 044361 (2002)



- Increased occupation probability with increased non-locality and correlated reduction in short-range tensor strength
- Are these calculations sufficiently complete/consistent?
- If so, is the correlation quantitatively predictable?

Scale dependence in coupled cluster calculations

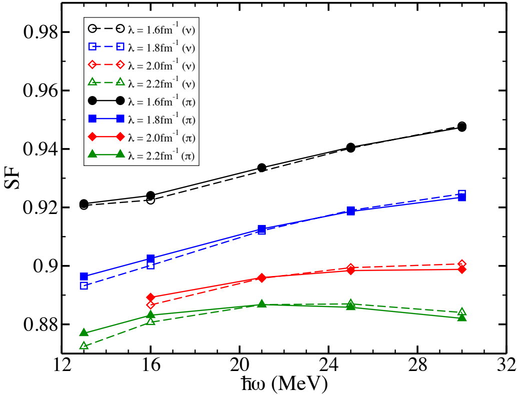


FIG. 4: (Color online) Spectroscopic factor SF$(1/2^-)$ for neutron and proton removal as a function of the oscillator spacing $\hbar\omega$ for nucleon-nucleon interactions with different cutoffs in a model space with $N = 6$.

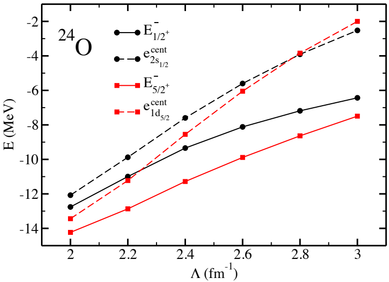
^{16}O spectroscopic factors (SFs) [From Ø. Jensen et al., PRC **82**, 014310 (2010)]

- SF increases as SRG resolution λ decreases from 2.2 to 1.6 fm^{-1}
- But significant $\hbar\omega$ dependence and no NNN
- Need to check that direct measurables are invariant

Wave functions become less correlated as Λ/λ decreases; how does the nature of reaction operators change?

See T. Duguet and G. Hagen, arXiv:1110.2468 for first steps

Resolution scale dependence [Slide courtesy of T. Duguet]



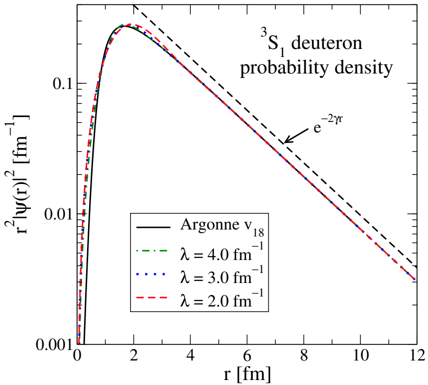
One-neutron removal in ^{24}O

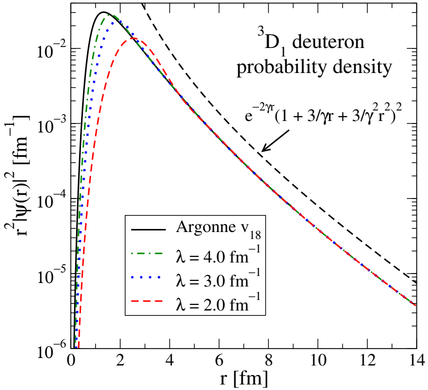
- $\rightsquigarrow$ E^-_ν and e^{cent}_p versus Λ_{RG}
- $\rightsquigarrow$ $\Lambda_{RG} \in [2.0; 3.0]$ fm^{-1}

Non-observability of ESPEs

1. Scale dependence of E^-_ν from omitted induced forces and clusters
2. Intrinsic scale dependence of $e^{cent}_p \approx 6$ MeV for $\Lambda_{RG} \in [2.0, 3.0]$ fm^{-1}
3. Extracting *the* shell structure from $(E^\pm_k, \sigma^\pm_k)$ is an illusory objective
   - $\rightsquigarrow$ One shell structure per (preferably low) resolution scale Λ_{RG}
   - $\rightsquigarrow$ Using *consistent* structure and reaction models is mandatory
   - $\rightsquigarrow$ Requires consistent many-body techniques and same $H(\Lambda_{RG})$

Why are ANC's different? Coordinate space





- ANC's, like phase shifts, are asymptotic properties
  $\Longrightarrow$ short-range unitary transformations do not alter them
  [e.g., see Mukhamedzhanov/Kadyrov, PRC **82** (2010)]
- In contrast, SF's rely on *interior* wave function overlap
- (Note difference in S-wave and D-wave ambiguities)

Why are ANC's different? Momentum space

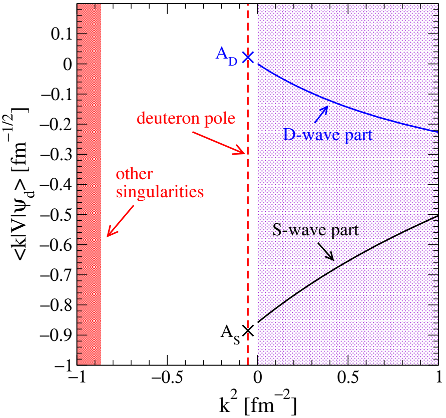
[based on R.D. Amado, PRC **19** (1979)]

1. $$\frac{k^2}{2\mu}\langle \mathbf{k}|\psi_n\rangle + \langle \mathbf{k}|V|\psi_n\rangle = -\frac{\gamma_n^2}{2\mu}\langle \mathbf{k}|\psi_n\rangle$$
$$\Longrightarrow \langle \mathbf{k}|\psi_n\rangle = -\frac{2\mu\langle \mathbf{k}|V|\psi_n\rangle}{k^2+\gamma_n^2}$$

2. $$\langle \mathbf{r}|\psi_n\rangle = \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{r}}\langle \mathbf{k}|\psi_n\rangle \xrightarrow{|\mathbf{r}|\to\infty} A_n e^{-\gamma_n r}/r$$

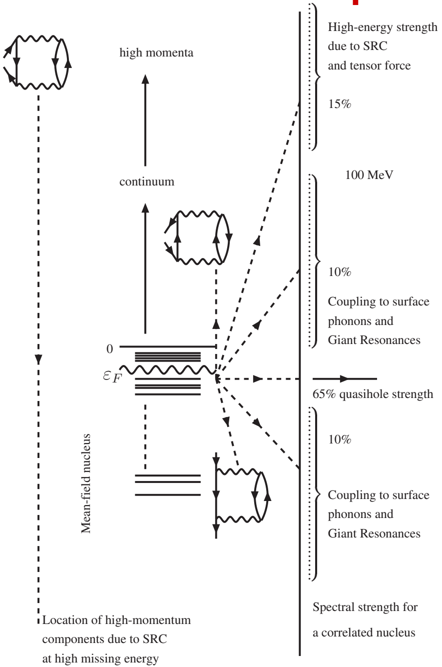
3. integral dominated by pole from 1.

4. extrapolate $\langle \mathbf{k}|V|\psi_n\rangle$ to $k^2 = -\gamma_n^2$



- Or, residue from extrapolating on-shell T-matrix to deuteron pole $\Longrightarrow$ invariant under unitary transformations
- Next vertex singularity at $-(\gamma + m_\pi)^2 \Longrightarrow$ same for FSI
- How far can we get solely with quantities that are invariant under (short-range) unitary transformations?

What is the scale dependence of s.p. strength?



Schematic illustration of the distribution of single-particle strength in stable closed shell nuclei
[figure from W. Dickhoff essay, J. Phys. G (2010)]

- What about the strength near the Fermi surface from long-range correlations?
- How does the interpretation of experiments to find high-energy strength vary with resolution (choice of separation scale)?

What about long-range correlations?

- SF calculations with FRPA
- Chiral N^{3}LO Hamiltonian
  - Soft $\Longrightarrow$ small SRC
  - SRC contribution to SF changes dramatically with lower resolution
- Compare short-range correlations (SRC) to long-range correlations from particle-vibration coupling
- LRC $\gg$ SRC!!

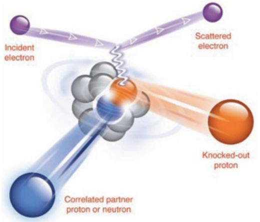
- How scale/scheme dependent are long-range correlations?
- Additional microscopic calculations are needed!

C. Barbieri, PRL 103 (2009)

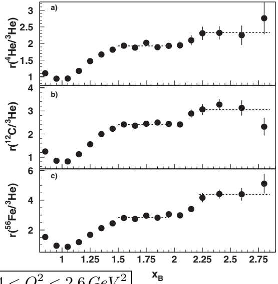
TABLE I. Spectroscopic factors (given as a fraction of the IPM) for valence orbits around ^{56}Ni. For the SC FRPA calculation in the large harmonic oscillator space, the values shown are obtained by including only SRC, SRC and LRC from particle-vibration couplings (full FRPA), and by SRC, particle-vibration couplings and extra correlations due to configuration mixing (FRPA + ΔZ_α). The last three columns give the results of SC FRPA and SM in the restricted $1p0f$ model space. The ΔZ_αs are the differences between the last two results and are taken as corrections for the SM correlations that are not already included in the FRPA formalism.

	10 osc. shells			Exp. [29]	$1p0f$ space		
	FRPA (SRC)	Full FRPA	FRPA $+\Delta Z_\alpha$		FRPA	SM	ΔZ_α
^{57}Ni:							
$\nu 1p_{1/2}$	0.96	0.63	0.61		0.79	0.77	-0.02
$\nu 0f_{5/2}$	0.95	0.59	0.55		0.79	0.75	-0.04
$\nu 1p_{3/2}$	0.95	0.65	0.62	0.58(11)	0.82	0.79	-0.03
^{55}Ni:							
$\nu 0f_{7/2}$	0.95	0.72	0.69		0.89	0.86	-0.03
^{57}Cu:							
$\pi 1p_{1/2}$	0.96	0.66	0.62		0.80	0.76	-0.04
$\pi 0f_{5/2}$	0.96	0.60	0.58		0.80	0.78	-0.02
$\pi 1p_{3/2}$	0.96	0.67	0.65		0.81	0.79	-0.02
^{55}Co:							
$\pi 0f_{7/2}$	0.95	0.73	0.71		0.89	0.87	-0.02

Looking for missing strength at large Q^2



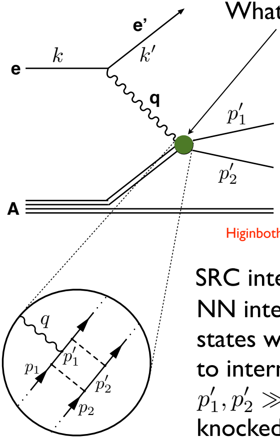
Subedi et al., Science 320, 1476 (2008)



$1.4 < Q^2 < 2.6\, GeV^2$

Egiyan et al. PRL 96, 1082501 (2006)

What is this vertex?



$$q = k - k'$$

$$\nu = E_k - E_{k'}$$

$$Q^2 = -q^2$$

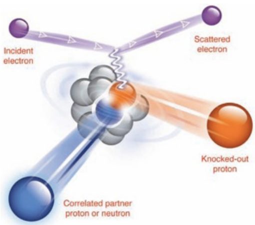
$$x_B = \frac{Q^2}{2m_N \nu}$$

Higinbotham, arXiv:1010.4433

SRC interpretation:
NN interaction can scatter states with $p_1, p_2 \lesssim k_F$ to intermediate states with $p'_1, p'_2 \gg k_F$ which are knocked out by the photon

- SRC explanation relies on high-momentum nucleons in structure!

Looking for missing strength at large Q^2



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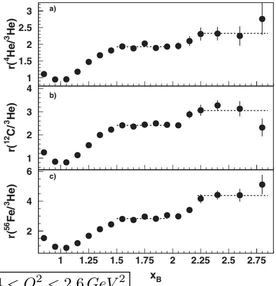

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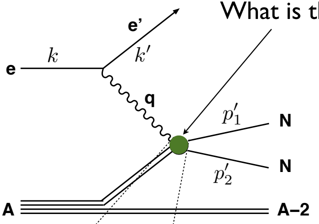

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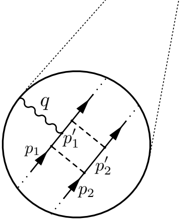

SRC interpretation:
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How to explain cross sections in terms of low-momentum interactions?

Vertex depends on the resolution!

- SRC explanation relies on high-momentum nucleons in structure!

Questions about short-range correlations (SRCs)

For this afternoon and all week!

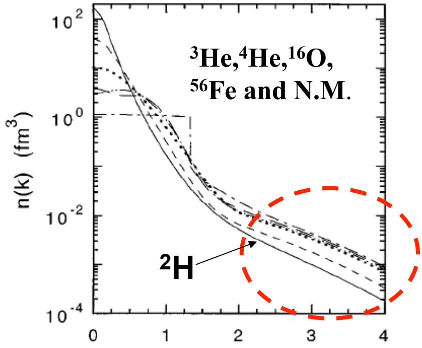
- How should we interpret the universal features of SRCs for different nuclei?
- Can SRCs inform us about high density matter (e.g., the EOS or physics of neutron stars)?
- Are SRCs important for understanding low-energy nuclear structure?
- How can we understand the observed correlation between the A-dependence of the EMC slope and scaling factors from $x > 1$?
- How does one explain cross sections from (e, e'), $(e, e'p)$ and $(e, e'pN)$ experiments with soft interactions that have minimal SRCs?
- How should one interpret the high-momentum tails of momentum distributions in nuclei, which vary significantly with different Hamiltonians?
- Under what conditions are asymptotic (high Q^2) assumptions for potentials valid?

Here: some brief comments about the low-resolution perspective

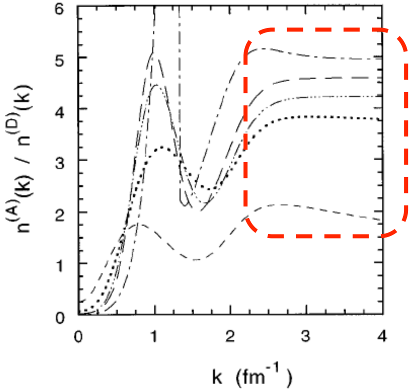
Deuteron-like scaling at high momenta

C. Ciofi and S. Simula, *Phys.Rev* **C53**, 1689(1996)

Momentum Distributions n(k)



Ratio to the Deuteron



n(k) at high Momentum regions are similar to it of the Deuteron

Almost Flat!

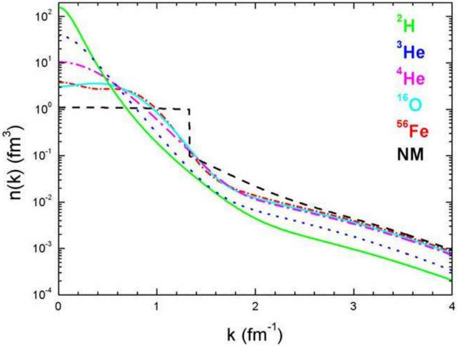
High resolution: Dominance of V_{NN} and SRCs (Frankfurt et al.)

How do we understand this scaling with low-resolution interactions?

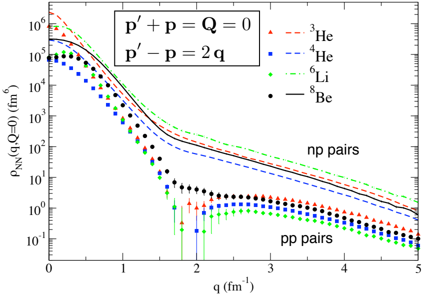
$\Longrightarrow$ Lower resolution means lower separation scale

Changing the scale separation with RG evolution

- Conventional analysis has (implied) high momentum scale
  - Based on potentials like AV18 and one-body current operator



[From C. Ciofi degli Atti and S. Simula]



Schiavilla et al. PRL 98, 132501 (2007)

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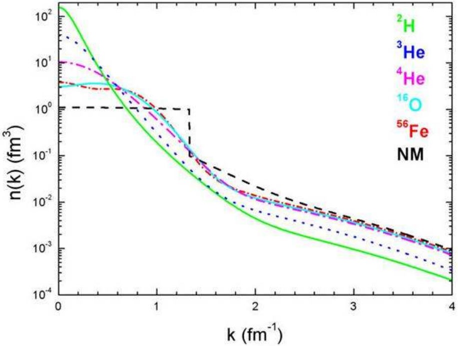

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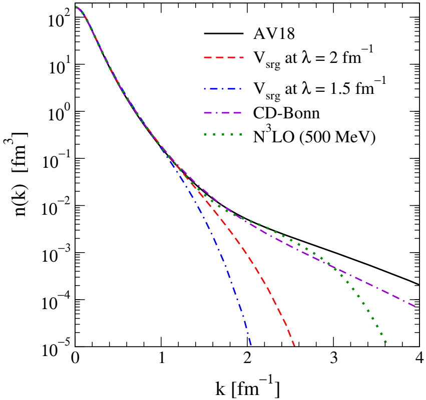


- With RG evolution, probability of high momentum decreases, but

$$n(k) \equiv \langle A|a^\dagger_{\mathbf{k}} a_{\mathbf{k}}|A\rangle = \left(\langle A|\widehat{U}^\dagger\right)\, \widehat{U} a^\dagger_{\mathbf{k}} a_{\mathbf{k}} \widehat{U}^\dagger \,\left(\widehat{U}|\Psi_n\rangle\right) = \langle \widetilde{A}|\widehat{U} a^\dagger_{\mathbf{k}} a_{\mathbf{k}} \widehat{U}^\dagger|\widetilde{A}\rangle$$

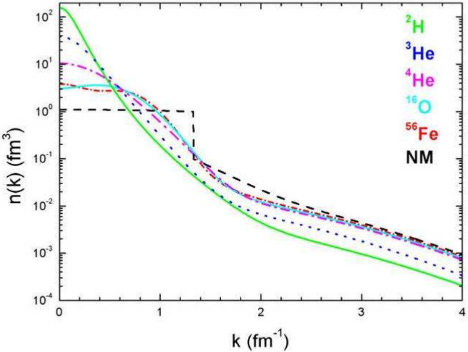
is unchanged! $|\widetilde{A}\rangle$ is easier to calculate, but is operator harder?

Nuclear scaling from factorization (schematic!)

- Factorization: when $k < \lambda$ and $q \gg \lambda$, $U_\lambda(k,q) \to K_\lambda(k) Q_\lambda(q)$

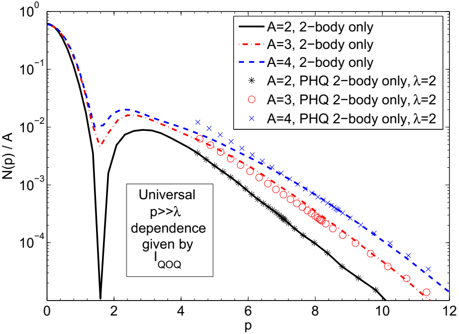
$$\frac{n_A(q)}{n_d(q)} = \frac{\langle\widetilde{A}|\widehat{U} a^\dagger_{\mathbf{q}} a_{\mathbf{q}} \widehat{U}^\dagger|\widetilde{A}\rangle}{\langle\widetilde{d}|\widehat{U} a^\dagger_{\mathbf{q}} a_{\mathbf{q}} \widehat{U}^\dagger|\widetilde{d}\rangle} = \frac{\langle\widetilde{A}|\int U_\lambda(k',q')\delta_{q'q} U^\dagger_\lambda(q,k)|\widetilde{A}\rangle}{\langle\widetilde{d}|\int U_\lambda(k',q')\delta_{q'q} U^\dagger_\lambda(q,k)|\widetilde{d}\rangle}$$

$\Longrightarrow n_A(q) \approx C_A n_D(q)$ at large q



[From C. Ciofi degli Atti and S. Simula]

Test case: A bosons in toy 1D model



[Anderson et al., arXiv:1008.1569]

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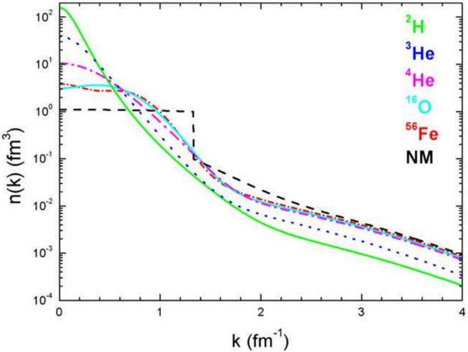

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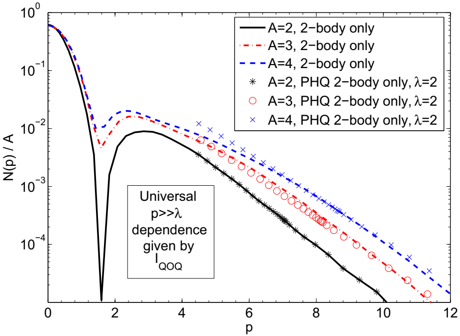

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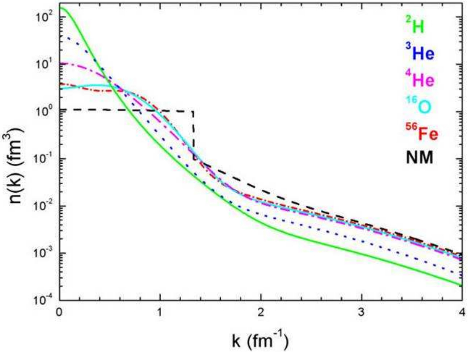
$\Longrightarrow n_A(q) \approx C_A n_D(q)$ at large q



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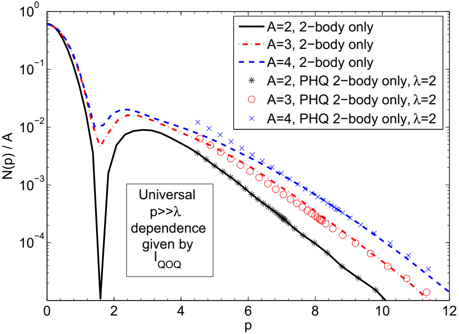

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Factorization with SRG [Anderson et al., arXiv:1008.1569]

- Factorization: $U_\lambda(k,q) \to K_\lambda(k)Q_\lambda(q)$ when $k < \lambda$ and $q \gg \lambda$
- Operator product expansion for nonrelativistic wf's (see Lepage)

$$\Psi^\infty_\alpha(q) \approx \gamma^\lambda(q)\int_0^\lambda p^2dp\, Z(\lambda)\Psi^\lambda_\alpha(p) + \eta^\lambda(q)\int_0^\lambda p^2dp\, p^2\, Z(\lambda)\,\Psi^\lambda_\alpha(p) + \cdots$$

- Construct unitary transformation to get $U_\lambda(k,q) \approx K_\lambda(k)Q_\lambda(q)$

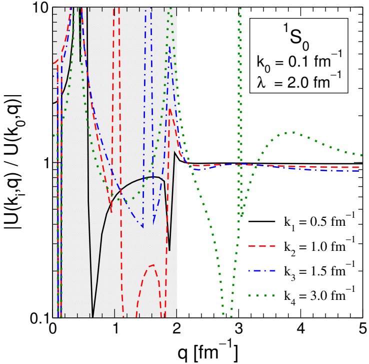
$$U_\lambda(k,q) = \sum_\alpha \langle k|\psi^\lambda_\alpha\rangle\langle\psi^\infty_\alpha|q\rangle \to \Big[\sum_\alpha^{\alpha_{low}}\langle k|\psi^\lambda_\alpha\rangle \int_0^\lambda p^2dp\, Z(\lambda)\Psi^\lambda_\alpha(p)\Big]\gamma^\lambda(q) + \cdots$$

- Test of factorization of U:

$$\frac{U_\lambda(k_i,q)}{U_\lambda(k_0,q)} \to \frac{K_\lambda(k_i)Q_\lambda(q)}{K_\lambda(k_0)Q_\lambda(q)},$$

so for $q \gg \lambda \Rightarrow \dfrac{K_\lambda(k_i)}{K_\lambda(k_0)} \xrightarrow{\text{LO}} 1$

- Look for plateaus: $k_i \lesssim 2\,\text{fm}^{-1} \lesssim q$ $\Longrightarrow$ it works!
- Leading order $\Longrightarrow$ contact term!



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$$\Psi^\infty_\alpha(q) \approx \gamma^\lambda(q) \int_0^\lambda p^2 dp\, Z(\lambda) \Psi^\lambda_\alpha(p) + \eta^\lambda(q) \int_0^\lambda p^2 dp\, p^2\, Z(\lambda)\, \Psi^\lambda_\alpha(p) + \cdots$$

- Construct unitary transformation to get $U_\lambda(k,q) \approx K_\lambda(k) Q_\lambda(q)$

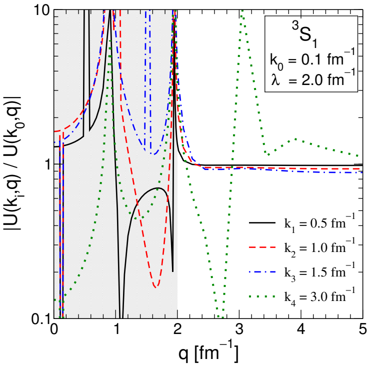
$$U_\lambda(k,q) = \sum_\alpha \langle k|\psi^\lambda_\alpha\rangle \langle \psi^\infty_\alpha|q\rangle \to \Big[\sum_\alpha^{\alpha_{low}} \langle k|\psi^\lambda_\alpha\rangle \int_0^\lambda p^2 dp\, Z(\lambda) \Psi^\lambda_\alpha(p)\Big] \gamma^\lambda(q) + \cdots$$

- Test of factorization of U:

$$\frac{U_\lambda(k_i,q)}{U_\lambda(k_0,q)} \to \frac{K_\lambda(k_i) Q_\lambda(q)}{K_\lambda(k_0) Q_\lambda(q)},$$

so for $q \gg \lambda \Rightarrow \dfrac{K_\lambda(k_i)}{K_\lambda(k_0)} \xrightarrow{\text{LO}} 1$

- Look for plateaus: $k_i \lesssim 2\,\text{fm}^{-1} \lesssim q$ $\Longrightarrow$ it works!
- Leading order $\Longrightarrow$ contact term!



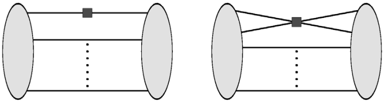
Universality of the EMC effect

- EFT treatment by Chen and Detmold [Phys. Lett. B 625, 165 (2005)]

$$F_2^A(x) = \sum_i Q_i^2\, x q_i^A(x) \qquad \Longrightarrow \qquad R_A(x) = F_2^A(x)/AF_2^N(x)$$

“The x dependence of $R_A(x)$ is governed by short-distance physics, while the overall magnitude (the A dependence) of the EMC effect is governed by long distance matrix elements calculable using traditional nuclear physics.”

- Match matrix elements: leading-order nucleon operators to isoscalar twist-two quark operators



$$\Longrightarrow \langle x^2\rangle_q v^{\mu_0}\cdots v^{\mu_n} N^\dagger N[1+\alpha_n N^\dagger N] + \cdots$$

$$R_A(x) = \frac{F_2^A(x)}{AF_2^N(x)} = 1 + g_{F_2}(x)\mathcal{G}(A) \quad \text{where} \quad \mathcal{G}(A) = \langle A|(N^\dagger N)^2|A\rangle/A\Lambda_0$$

$\Longrightarrow$ the slope $\frac{dR_A}{dx}$ scales with $\mathcal{G}(A)$ [Why is this not cited?]

Final comments and questions

- Summary (and follow-up) points
  - While scale and scheme-dependent observables can be (to good approximation) unambiguous for *some* systems, they are often (generally?) not for nuclei!
  - Scale/scheme includes *consistent* Hamiltonian and operators. How dangerous is it to treat experimental analysis in pieces?
  - Unitary transformations reveal *natural* scheme dependence
  - Parton distribution functions as a paradigm
    $\Longrightarrow$ Can we have controlled factorization at low energies?
- Questions for which RG/EFT perspective + tools can help
  - How should one choose a scale/scheme?
  - Can we (should we) use a reference Hamiltonian?
  - What *is* the scheme-dependence of SF's and other quantities?
  - What is the role of short-range/long-range correlations?
  - How do we match Hamiltonians and operators?
  - When is the assumption of one-body operators viable?
  - What can EFT or RG say about N-nucleus optical potentials?
  - . . . and many more!

Extra Slides

Questions and some possible answers

How should one choose a scale/scheme?

- To make calculations easier or more convergent
  - QCD running coupling and scale: improved perturbation theory; choosing a gauge: e.g., Coulomb or Lorentz
  - (Near-) local potential: quantum Monte Carlo methods work
  - Low-*k* potential: many-body perturbation theory works, or to make microscopic connection to shell model
- Better interpretation or intuition $\Longrightarrow$ predictability
- Use range of scales to test calculations and physics
  - Use renormalization group to consistently relate scales and quantitatively probe ambiguity

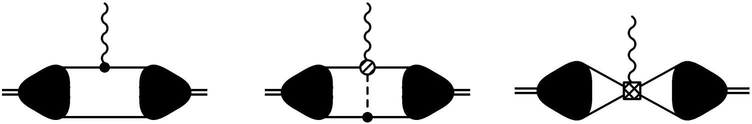
Can we (should we) use a reference Hamiltonian?

- That is, to allow us to make comparisons
- If so, which one? (Cleanest extraction from experiment?)
  - Can one "optimize" validity of impulse approximation?

More questions and some possible answers

How do we consistently match Hamiltonians and operators?

- Use EFT perspective
  - E.g., electromagnetic currents [D.R. Phillips, nucl-th/0503044]



$O(e)$ $O(eP^3)$ $O(eP^5)$

  - Model independent because complete (up to some order)
  - Can identify consistent operator and interaction
  - Tells you when new info is required
- Use RG as tool to evolve consistent operators

Can EFT or RG help to construct optical potentials?

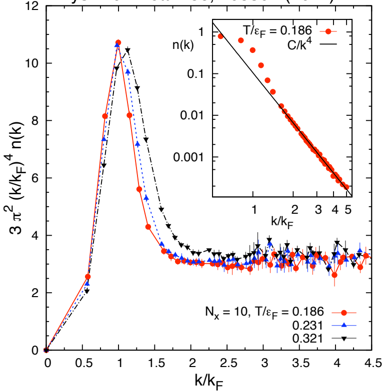
Unitary cold atoms: Is $n(k)$ observable?

- Tail of momentum distribution + contact [Tan; Braaten/Platter]

$$n(k) \xrightarrow{k\to\infty} \frac{C}{k^4}$$

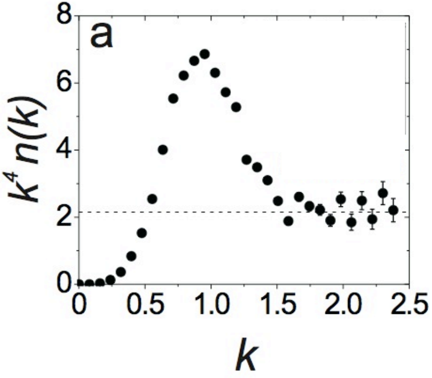
Theory (lattice)

J. E. Drut, T. A. Lähde, T. Ten
Phys. Rev. Lett. **106**, 205302 (2011)



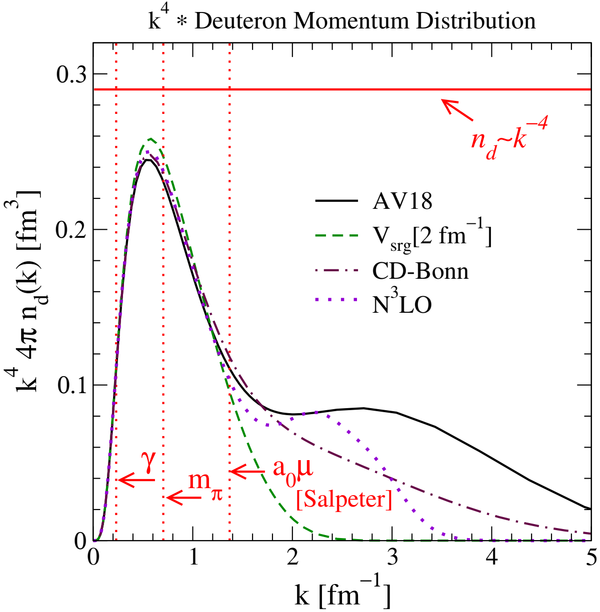
Experiment

J. T. Stewart et al
PRL **104**, 235301 (2010)



- When $R/a_s \ll 1$ and $kR \ll 1 \Longrightarrow$ tiny scheme dependence

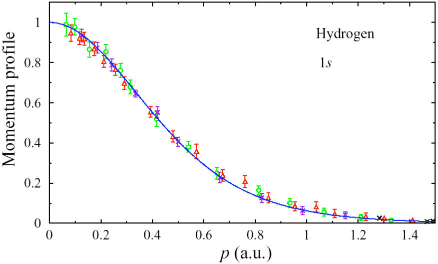
Is the tail of $n(k)$ for nuclei measurable? (cf. SRC's)



- E.g., extract from electron scattering?
- No region where $1/a_s \ll k \ll 1/R$
- Scheme dependent high-momentum tail!
- $n(k)$ from V_{SRG} has *no* high-momentum components!
- But $n(k)$ from $U a_k^\dagger a_k U^\dagger$ is unchanged $\Longrightarrow$ two-body operator!

When are wave functions measurable? [W. Dickhoff]

Atoms studied with the (e,2e) reaction

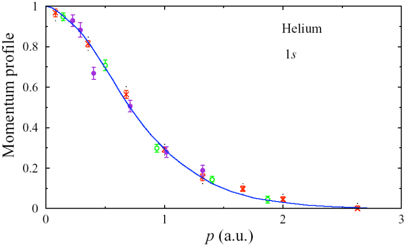


$$\varphi_{1s}(p) = 2^{3/2}\pi \frac{1}{(1+p^2)^2}$$

Hydrogen 1s wave function
"seen" experimentally
Phys. Lett. 86A, 139 (1981)

And so on for other atoms …

Helium
in Phys. Rev. A8, 2494 (1973)



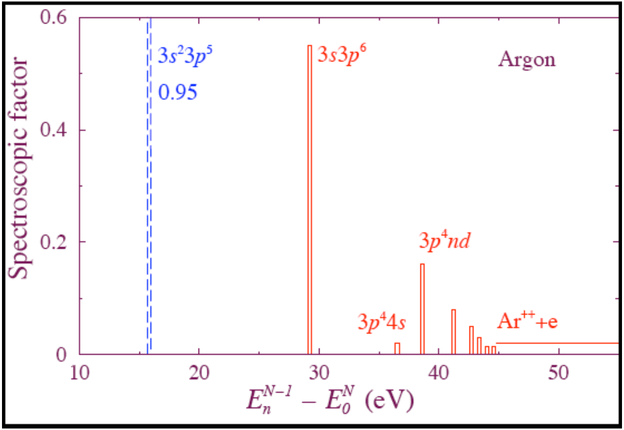
- But compare approximations for $(e,2e)$ on atoms to those for $(e,e'p)$ on nuclei! (Impulse approx., FSI, vertex, …)

Spectroscopic factors in atoms

For a bound final *N*-1 state the spectroscopic factor is given by $S = \int d\vec{p} \left| \left\langle \Psi_n^{N-1} \middle| a_{\vec{p}} \middle| \Psi_0^N \right\rangle \right|^2$

For H and He the 1*s* electron spectroscopic factor is 1

For Ne the valence 2*p* electron has *S*=0.92 with two additional fragments, each carrying 0.04, at higher energy.



Argon
3*p* and 3*s* strength

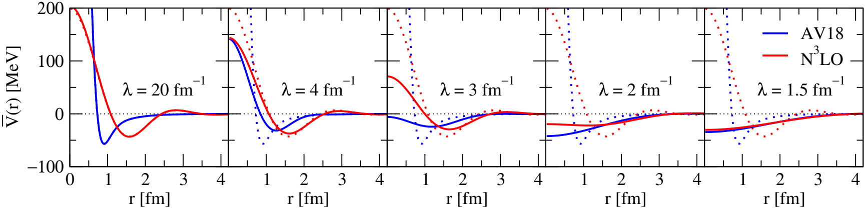
Closed-shell atoms
$n(\alpha) = 0$ or 1

One-body scattering, small scheme dependence $\Longrightarrow$ robust SF

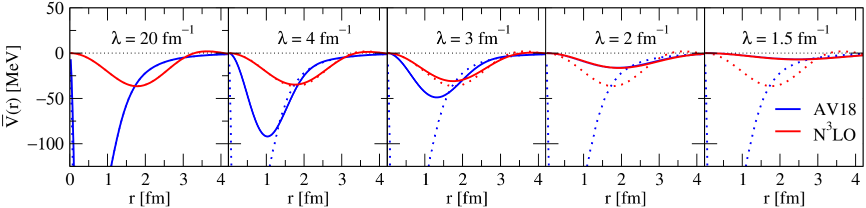
Scale for structure: Nuclei with *soft* interactions

Changing the scale/scheme: (short-range) NN potential

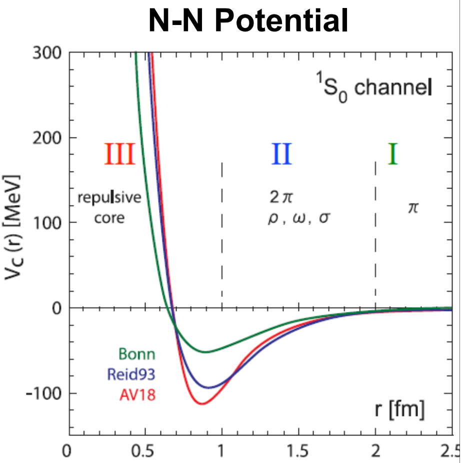
- Project non-local NN potential: $\overline{V}_\lambda(r) = \int d^3r'\, V_\lambda(r, r')$
    - Roughly gives action of potential on long-wavelength nucleons
- Central part (S-wave) [Note: The V_λ's are all phase equivalent!]



- Tensor part (S-D mixing) [graphs from K. Wendt]



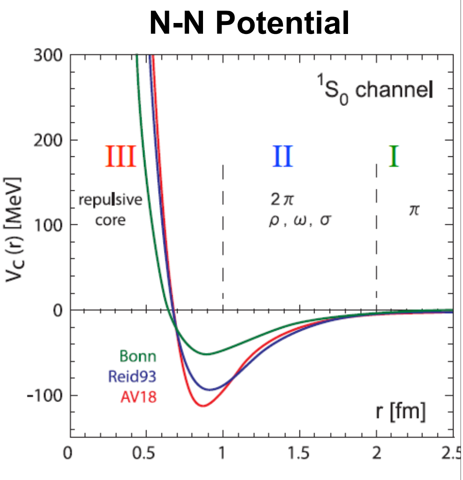
But isn't there a correct Hamiltonian? (no!)



Bonn: Machleidt, Phys.Rev. C63('01)024001
Reid93: Stoks et al., Phys. Rev. C49('94)2950.
AV18: Wiringa et al., Phys.Rev. C51('95) 38.

- For low-energy effective theories, short-range part can be modified dramatically (cf. interparticle spacing in nuclei $>$ 1 fm)
- What about inverse scattering theorems? Unique potential only if fixed form (e.g., local) and phase shifts known to infinite energy (but still arbitrarily bound-state properties)
- Cf. running couplings in QED or QCD (or field redefinitions)

But isn't there a correct Hamiltonian? (no!)



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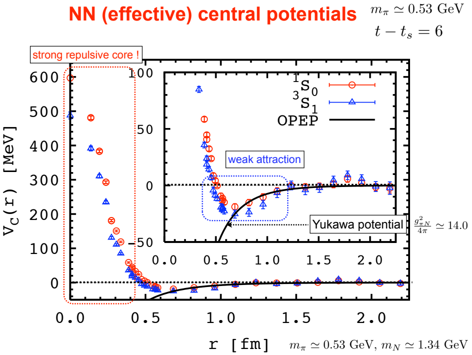
- W. Dickhoff, J. Phys. G (2010): "Recent even softer chiral interactions generate correspondingly less depletion. The latter result appears to be inconsistent with the experimental confirmation of a global depletion of the nuclear Fermi sea for protons in ^{208}Pb of about 15%. In addition, recent lattice calculations of the nucleon–nucleon interaction suggest that a strongly repulsive core will arise once the pion mass is taken to realistic values."

Determining the nuclear potential from lattice QCD

[S. Aoki, *Hadron interactions in lattice QCD*, arXiv:1107.1284]



Bethe-Salpeter amplitude

$$\boxed{\varphi_E(\vec{r}) = \langle 0|N(\vec{x},0)N(\vec{y},0)|2N, E\rangle}$$

Nucleon fields

2N state with energy E

- define non-local $U(\mathbf{x}, \mathbf{y})$

$$[E - H_0]\varphi_E(\mathbf{x}) = \int d^3y\, U(\mathbf{x}, \mathbf{y})\varphi_E(\mathbf{y})$$

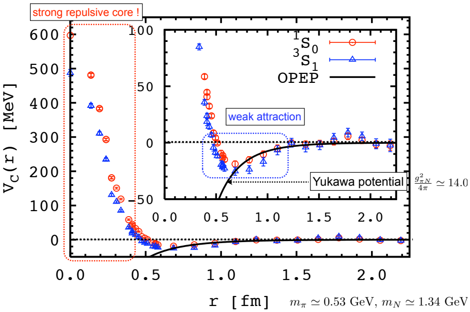
- Expand $U(\mathbf{x}, \mathbf{y}) = V(\mathbf{x}, \nabla)\delta(\mathbf{x} - \mathbf{y})$ to get AV18 form of local V

- Why not just calculate energy as function of separation $\Longrightarrow V(r)$?
  - Only works in heavy mass limit (e.g., works for B-mesons)
- But is this unique? No!
  - choice of nucleon interpolating field $\Longrightarrow$ different $V(\mathbf{x})$
  - choice of “wave function” smearing (changes overlap)

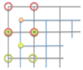
Determining the nuclear potential from lattice QCD

[S. Aoki, *Hadron interactions in lattice QCD*, arXiv:1107.1284]

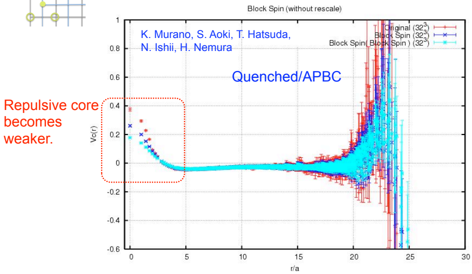
NN (effective) central potentials $m_\pi \simeq 0.53$ GeV $t - t_s = 6$



Smearing and potentials Preliminary



Wave function is smeared. → "smeared" potential



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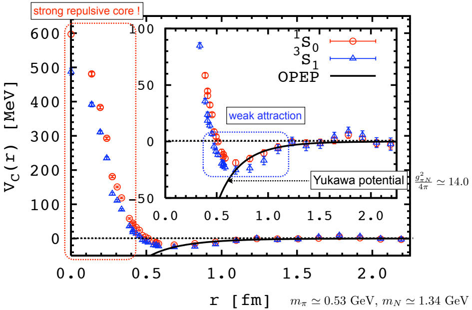
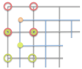


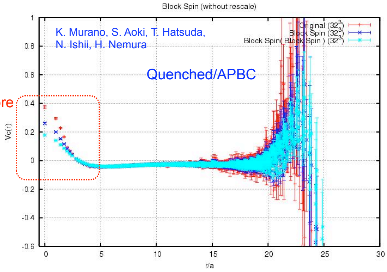
Smearing and potentials

Preliminary



Wave function is smeared. ➡ “smeared” potential

Repulsive core becomes weaker.



- “. . . the potential depends on the choice of nucleon operator. . . ” which “. . . is considered to be a ‘scheme’ to define the potential.”
- “Is such a scheme-dependent quantity useful? The answer to this question is probably ‘yes’, since the potential is useful to understand or describe the phenomena.”
- Claim: useful to choose a scheme that yields good convergence of the velocity expansion (close to local)

When can you measure a potential?

- Think about quantum mechanical convolution for energy

$$E = \int d\mathbf{x}\, \Psi^*(\mathbf{x})(T+V)\Psi(\mathbf{x})$$

  - (Schematic: e.g., here $\mathbf{x} = \{\mathbf{x}_1, \mathbf{x}_2\}$)
  - When can we isolate $H = T + V$ from $|\Psi(\mathbf{x})|^2$?
- Need very heavy particles or long-distances so that wave functions can be approximated as delta functions
- Examples
  - classical limit (e.g., gravitational potential)
  - heavy quark potential on a lattice
  - Coulomb potential in atoms/molecules
- In nuclear case, can change both $\Psi(\mathbf{x})$ and $V(\mathbf{x})$ at short distance and leave E unchanged $\Longrightarrow$ not measurable
- In field theory formulation, freedom to shift between interaction vertex and propagator for exchanged particle

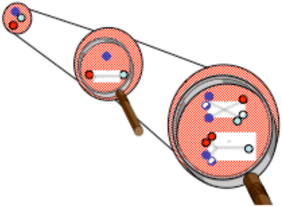
Impulse approximation

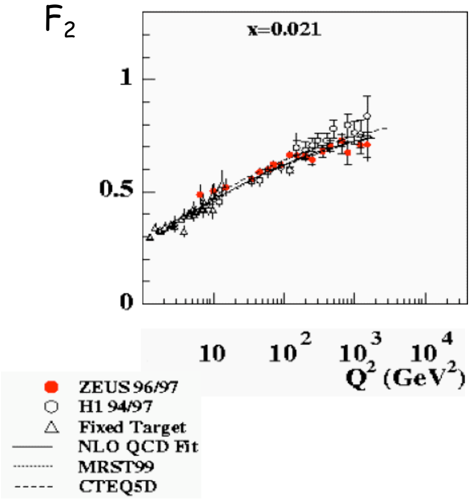
- The discussion always starts with: “If we assume . . . ”
  - Usually that the impulse approximation is good (one-body current and one active nucleon), and increasingly good with larger momentum transfer
  - Final state interactions neglected (and then assumed to be accounted for in a model-independent way)
- This brings to mind some quotes:
  - “If my grandmother had wheels, she’d be a bicycle.”
  - “Hope is not a plan!” (or a reliable guide to experiment)
- How well the impulse approximation works depends on the system and probe (process dependent)
  - Works well: electron scattering from atoms, neutron scattering from liquid helium (??? maybe not in detail)
  - Large corrections: nuclear reactions!
- Should we choose a scheme in which the impulse approximation is best satisfied?

Parton distributions as paradigm: Factorization

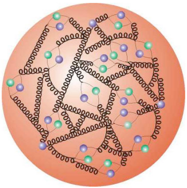
- PDF analysis: part of convolution for cross section can be calculated reliably for given experimental conditions so that the remaining part can be extracted as a universal quantity, to be related to other processes and kinematic conditions
- For hard-scattering processes with large momentum transfer scale Q, *factorization* allows separation of momentum and distance scales in reaction
  - The time scale for binding interactions in the rest frame is time dilated in the center-of-mass frame, so the interaction of an electron with a hadron in deep-inelastic scattering is with single non-interacting partons
  - Short-distance part calculated systematically in low-order perturbative QCD; long-distance part identified in PDF's (momentum distribution for partons in hadrons)
- PDF's relate deep inelastic scattering of leptons, Drell-Yan, jet production, and more
  - Measure in limited set of reactions and then perturbative calculations of hard scattering and PDF evolution enable first principles predictions of cross sections for other processes

Parton distributions as paradigm [C. Keppel]



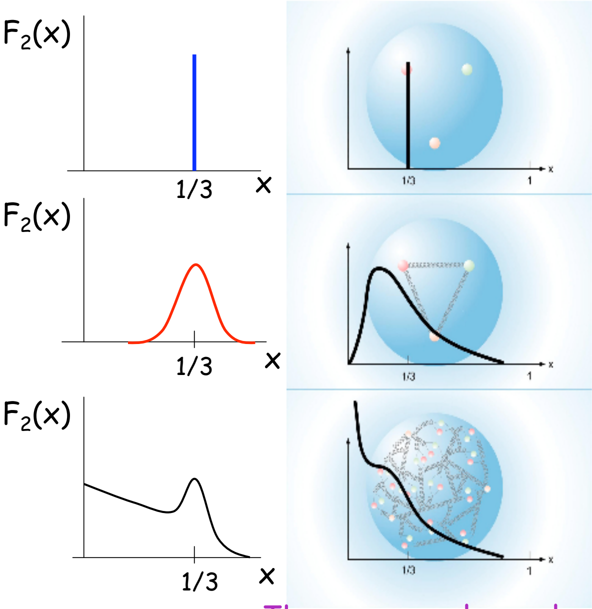


Higher the resolution (i.e. higher the Q^2) more low x partons we "see".



So what do we expect F_2 as a function of x at a fixed Q^2 to look like?

Parton distributions as paradigm [C. Keppel]



Three quarks with 1/3 of total proton momentum each.

Three quarks with some momentum smearing.

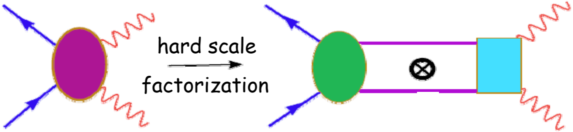
The three quarks radiate partons at low x.

....The answer depends on the Q^2!

Parton distributions as paradigm [Marco Stratman]

Factorization schemes

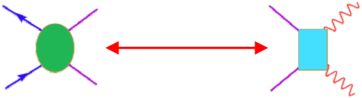
pictorial representation of factorization:



e.g. $F_2(x,Q^2) \sim \sum_a f_a(x,\mu_f) \otimes \hat{F}_2^a(x,\frac{Q}{\mu_f})$

the separation between long- and short-distance physics is not unique

long-distance parton density



short-distance Wilson coefficient

1. **choice of** μ_f: defines borderline between long-/short-distance
2. **choice of scheme**: re-shuffling finite pieces

Parton distributions as paradigm [Marco Stratman]

Deep-inelastic scattering (DIS) according to pQCD

the physical structure fct. is independent of μ_f
(this will lead to the concept of renormalization group eqs.)

both, pdf's and the short-dist. coefficient depend on μ_f
(choice of μ_f: shifting terms between long- and short-distance parts)

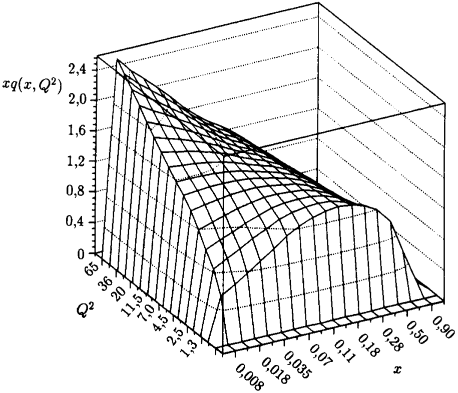
$$F_2(x,Q^2) = x \sum_{a=q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} f_a(\xi,\mu_f^2) \left[\delta(1-\frac{x}{\xi}) + \frac{\alpha_s(\mu_r)}{2\pi} \underbrace{\left[P_{qq}\left(\frac{x}{\xi}\right) \ln\frac{Q^2}{\mu_f^2} + (C_2^q - z_{qq})\left(\frac{x}{\xi}\right)\right]}_{\text{short-distance "Wilson coefficient"}}\right]$$

yet another scale: μ_r
due to the renormalization
of ultraviolet divergencies

short-distance "Wilson coefficient"

choice of the factorization scheme

Parton distributions as paradigm: Evolution

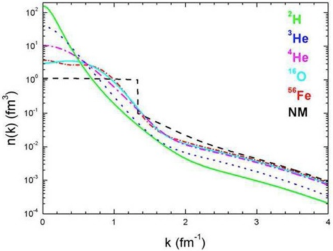


- The quark distribution $q(x, Q^2)$ is both scheme and scale dependent
- $x\ q(x, Q^2)$ measures the share of momentum carried by the quarks in a particular x-interval
- $q(x, Q^2)$ and $q(x, Q_0^2)$ can be related by well-controlled evolution equations

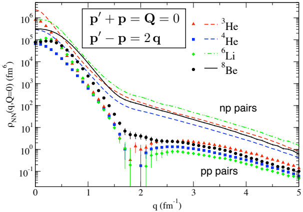
Parton distributions as paradigm: Lessons

- The momentum distribution for a given hadron is not unique
  - With parton distributions one would not talk about the results at a particular Q^2 as being “the” quark or gluon momentum distribution as opposed to distributions for lower or higher Q^2.
  - Dependence on Q^2, which serves as the resolution scale and can be changed by renormalization group (RG) evolution, and the PDF analysis at NLO must be performed in a specific renormalization and factorization scheme (e.g., MS or DIS)
  - Controlled factorization allows PDF’s from one process to be used in other processes (and at other scales)!
  - For consistency, hard-scattering cross section calculations used for the input processes or that use the extracted PDFs have to be implemented with the same scheme
- Can we formulate our stucture/reaction theory to have the same control as with PDFs using factorization?

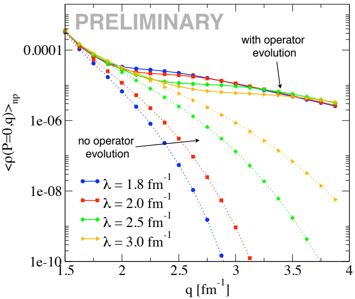
Simpler calculations of pair densities [Anderson, Hebeler]

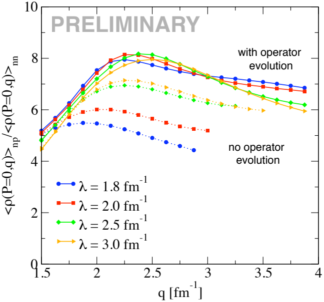


taken from Ciofi degli Atti, Simula PRC 53, 1689 (1996)



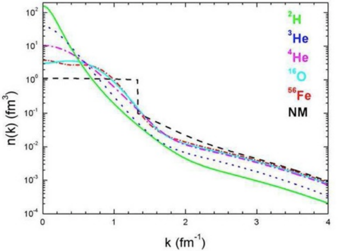
Schiavilla et al. PRL 98, 132501 (2007)





Many-body perturbation theory may be sufficient at low resolution!

Simpler calculations of pair densities [Anderson, Hebeler]



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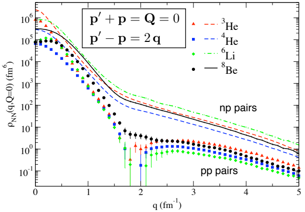
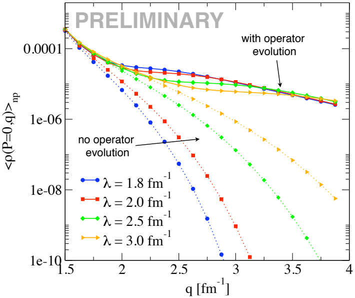
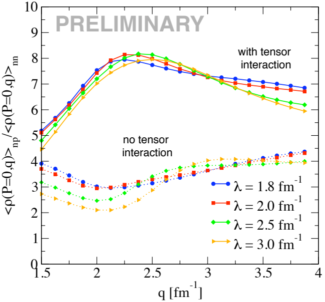


Schiavilla et al. PRL 98, 132501 (2007)





Many-body perturbation theory may be sufficient at low resolution!

Quantities that vary with convention or scheme

- deuteron D-state probability [e.g., Friar, PRC **20** (1979)]
- off-shell effects (e.g., from NN bremsstrahlung) [Fearing/Scherer, PRC **62** (2000)]
- occupation numbers [Hammer/rjf, PLB **531** (2002)]
- spectroscopic factors [Mukhamedzhanov/Kadyrov, PRC **82** (2010)]
- proton radius (cf. charge radius) [Polyzou, PRC **58** (1998)]
- short-range part of wave functions (SRC's)
- wound integrals
- short-range potentials; e.g., contribution of short-range 3-body forces
- and so on . . .

[L. Cardman, FRIB Superusers 2011]

The Impact of Correlations on Nuclear Spectral Functions

Electron-induced proton knock-out has been studied systematically since high duty-factor electron beams became available, first at Saclay (70's), then at NIKHEF (80's) with ~100 keV energy resolution.

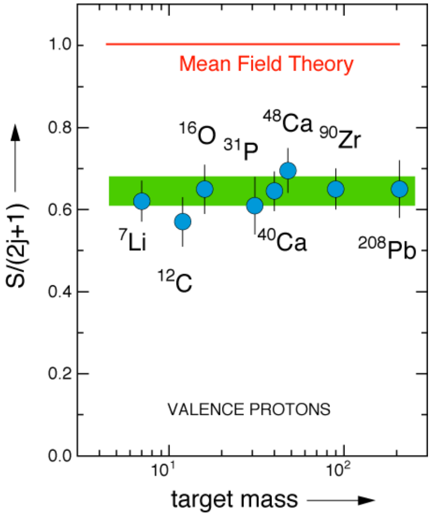
For complex (A>4) nuclei, the spectroscopic strength S for valence protons was found to be 60-65% of the IPSM value

$$\boxed{S_\alpha = 4\pi \int S(E_m, p_m) p_m^2 dp_m \delta(E_m - E_\alpha)}$$

Long-range correlations account for about 10%, but the rest was ascribed to short-range N-N correlations, by which strength was pushed to energies well above the Fermi edge.

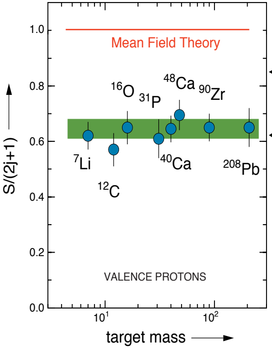
These kinematics were not accessible at the accelerators of that era, but they are at CEBAF.

Hall C E97-006: direct search for this "missing" strength at large E_m and p_m



[H.P. Blok, “Probing nuclei with $(e, e'p)$ . . .”

Spectroscopic Factors of Valence Transitions



RPA (LRC), G-matrix

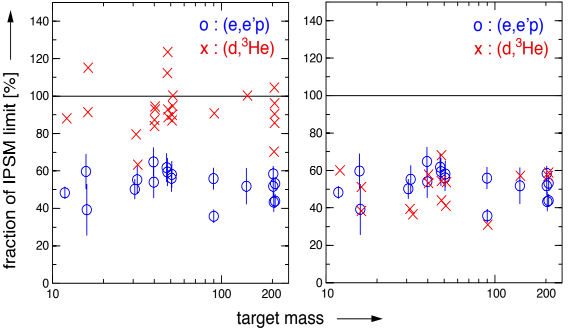
NM (SRC) +RPA (LRC)

35 % reduction w.r.t MFT

- 10-15 % due to LRC for finite nuclei in RPA
- 10-15 % effect due to SRC calculated for infinite NM

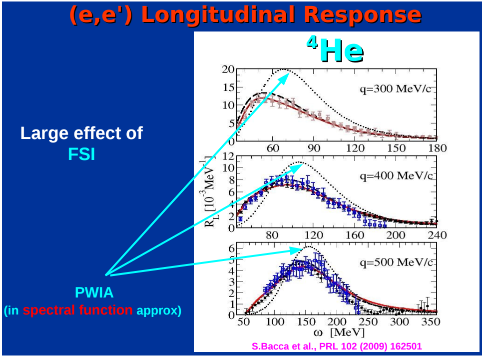
Original data
(d,^{3}He) Local/Zero-range
arbitrary BSWF

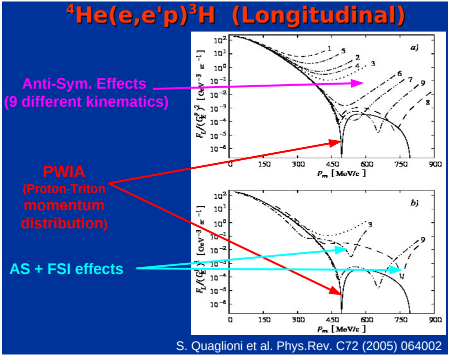
NIKHEF Reanalysis
(d,^{3}He) Non-Local/Finite-range
BSWF from (e,e'p)



Seeming discrepancy between (d,^{3}He) and (e,e’p) data solved

Ab initio electron scattering with LIT [from G. Orlandini]





- Ab initio calculations of longitudinal (e, e') response functions show importance of FSI for quasi-elastic regime
  - PWIA fails for quasi-elastic peak and at low ω
  - FSI effects decrease with q in peak but not at low ω
- Direct proton knockout and neglect of FSI tested for $(e, e'p)$
  - Both antisymmetrization effects and FSI play important roles
  - Approximate estimates of FSI effects can be poor