Density and Spin Response of a Fermi Gas in the Attractive and Quasi-Repulsive Regime

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density-density $\chi_n(\mathbf{r}\tau, \mathbf{r}'\tau') = -\langle\, T_\tau \left[\rho(\mathbf{r}\tau)\rho(\mathbf{r}'\tau')\right]\rangle$

spin-spin $\chi_s(\mathbf{r}\tau, \mathbf{r}'\tau') = -\langle\, T_\tau \left[S_z(\mathbf{r}\tau) S_z(\mathbf{r}'\tau')\right]\rangle$

where T_τ = imaginary-time ordering operator and

$$\psi^\dagger_\alpha(\mathbf{r}\tau^+)\,\psi_\beta(\mathbf{r}\tau) = e^{(H-\mu N)\tau}\psi^\dagger_\alpha(\mathbf{r})\,\psi_\beta(\mathbf{r})e^{-(H-\mu N))\tau}$$

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- For a homogeneous system in $\mathbf{r}$ and τ:

$$\begin{aligned}\chi_{n/s}(\mathbf{q},\Omega_\nu) &= \int_0^{1/(k_B T)} d(\tau-\tau')\,e^{i\Omega_\nu(\tau-\tau')} \\ &\times \int d(\mathbf{r}-\mathbf{r}')\,e^{-i\mathbf{q}\cdot(\mathbf{r}-\mathbf{r}')}\,\chi_{n/s}(\mathbf{r}\tau,\mathbf{r}'\tau')\end{aligned}$$

$\mathbf{q}$ = wave vector and $\Omega_\nu = 2\pi\nu T$ (ν integer)
at temperature T.

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- For non-interacting fermions:

  $\chi_n^{(0)} = 2N_0$ and $\chi_s^{(0)} = 2N_0\mu_B^2$

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- The above relations for χ_n and χ_s are Ward identities that connect single- (n and M) and two-particle (= response functions) properties.

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Regularize V_{eff} in terms of the scattering length a_F of the 2-body problem:

$$\frac{m}{4\pi a_F} = \frac{1}{v_0} + \int^{k_0} \frac{d\mathbf{k}}{(2\pi)^3} \frac{m}{\mathbf{k}^2}$$

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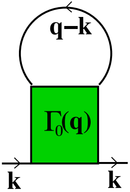
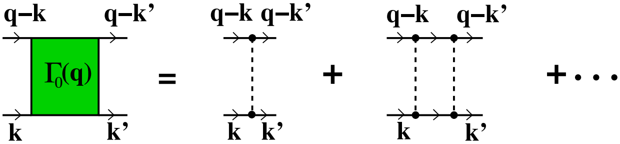
- Above T_c, a reasonable description is obtained in terms of the t-matrix (Galitskii) with self-energy:

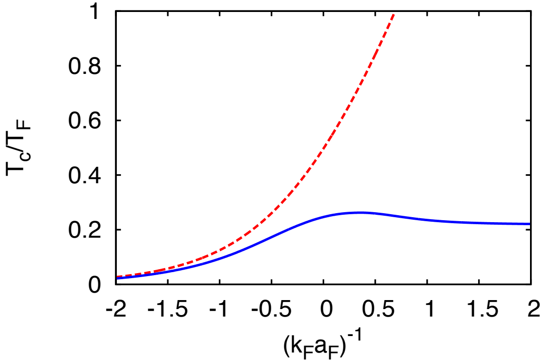
The t-matrix self-energy:





In principle , all single-particle lines should be self-consistent for the theory to be "conserving" (Baym). In practice , this is most often avoided.

The critical temperature T_c according to the t-matrix:



The Thouless criterion $\Gamma_0^{-1}(\mathbf{q}=0, \Omega_\nu=0; \mu)=0$ plus the density equation to fix $\mu(T \rightarrow T_c^+)$ determine T_c throughout the BCS-BEC crossover in terms of the coupling parameter $(k_F a_F)^{-1}$.

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  μ_B = bosonic chemical potential

  $T_c \longrightarrow T_{BEC} = \frac{3.31}{2m} \left(\frac{n}{2}\right)^{2/3}$

  and $\Gamma_0(\mathbf{q}, \Omega_\nu) \simeq -\left(\frac{8\pi}{m^2 a_F}\right) \frac{1}{i\Omega_\nu - \frac{\mathbf{q}^2}{4m} + \mu_B}$

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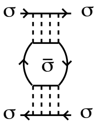
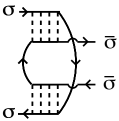
- What about the two-particle response about equilibrium?

Response kernels out of the t-matrix:

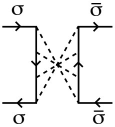
The kernel of the Bethe-Salpeter equation is:

(a) 

⇒ Aslamazov-Larkin (AL1) diagram

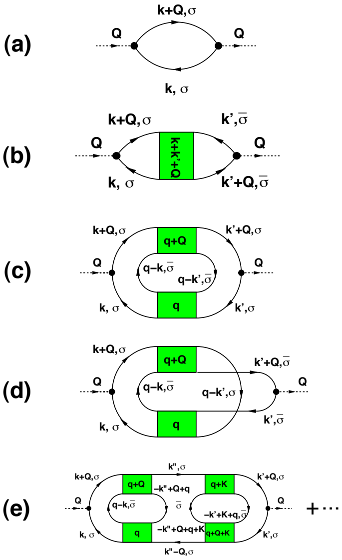
(b) 

⇒ Aslamazov-Larkin (AL2) diagram

(c) 

⇒ Maki-Thomson (MT) diagram

The static limit χ_n (compressibility):



If all the $G \to G_0$, only the (a) DOS, (b) MT, (c) AL1, and (d) AL2 diagrams contribute to the compressibility $\frac{dn}{d\mu}$.

In the BEC limit, only AL1 and AL2 survive!

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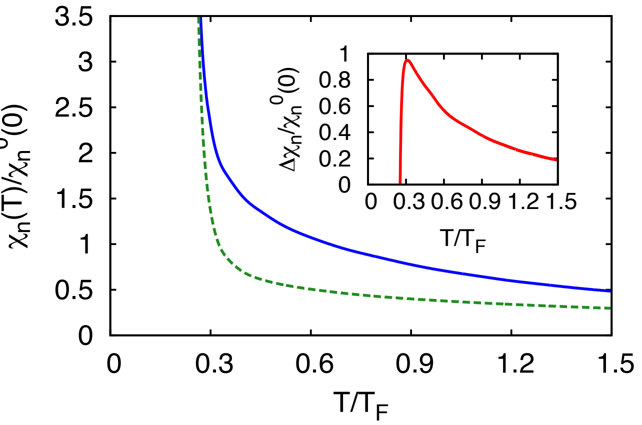
- In the calculation of $\frac{dn}{d\mu}$ with the t-matrix only the $\Omega_\nu = 0$ mode gives rise to this divergence!

The divergence of $\frac{dn}{d\mu}$ when $T \to T_c^+$:



——— (blue)	$=$	full calculation (t-matrix)
- - - - (green)	$=$	$\Omega_\nu = 0$ only
——— (red)	$=$	difference between the two

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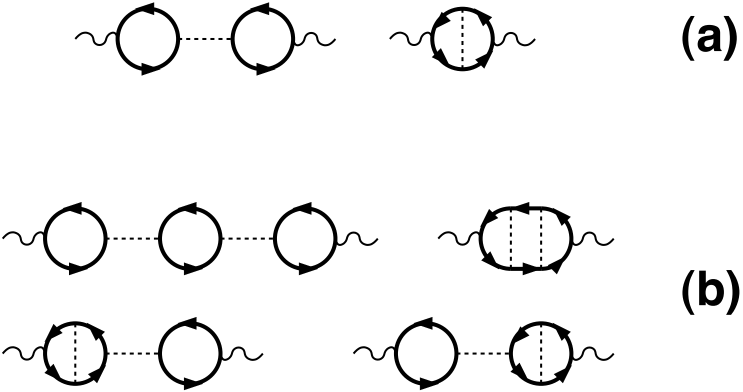

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$$(T \to T_c^+) \quad -\frac{1}{2\,g_B} = -\left(\frac{\partial n_B}{\partial \mu_B}\right)_{T,V}$$

where $g_B = \frac{4\pi a_B}{m_B}$ and $\mu_B = 2\,g_B\,n_B$ at the Hartree-Fock (Popov) level.

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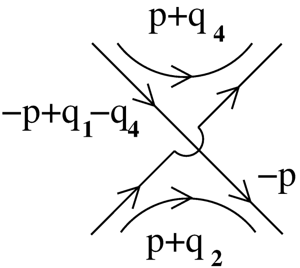
and the residual interaction between composite bosons is identified as follows:



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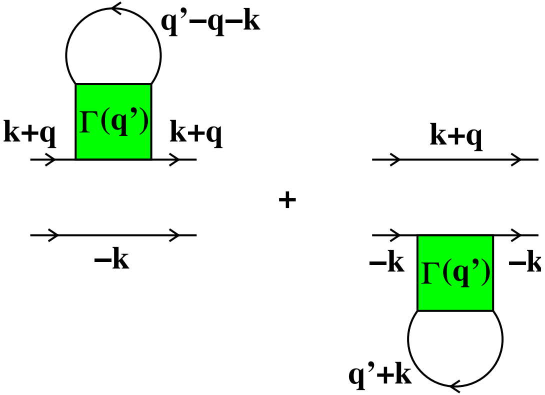

in the normal phase above T_c ($\Longleftrightarrow$ a little bit of self-consistency in the single-particle G !).

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- From the density profile $n(r)$ (with $r = |\mathbf{r}|$):

$$\frac{dn(r)}{dr} = \frac{dn(r)}{d\mu(r)} \frac{d\mu(r)}{dr} = - m \, \omega_0 \, r \, \frac{dn(r)}{d\mu(r)}$$

where $\frac{dn(r)}{d\mu(r)} = -\chi_n^{\text{(homo)}}(n(r), T)$ within LDA.

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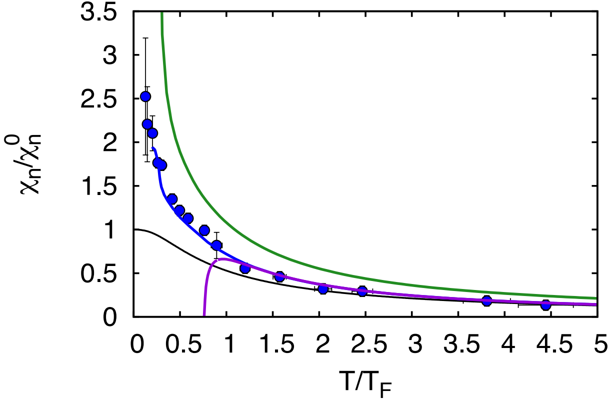
where $\frac{dn(r)}{d\mu(r)} = -\chi_n^{(\text{homo})}(n(r), T)$ within LDA.

- At the unitary limit $(k_F a_F)^{-1} = 0$, all regions of the trap share the same coupling !

The MIT experiment (Zwierlein, 2011)



● = experimental data

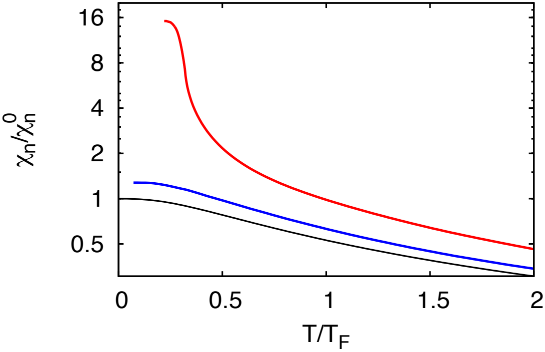
—— = full calculation (Popov)

—— = t-matrix

—— = high-T (virial) expansion

—— = non-interacting Fermi gas

- Note how the residual interaction between fluctuating Cooper pairs above T_c accounts the virial expansion when $T \gtrsim T_F$.
- Away from the unitary limit:



$(k_F a_F)^{-1} = +1.0$

$(k_F a_F)^{-1} = -1.0$

$\leftrightarrow$ non-interacting Fermi gas

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$G_{11}(k) \leftrightarrow$ normal single-particle propagator

$G_{12}(k) \leftrightarrow$ anomalous single-particle propagator

The static limit χ_s (spin susceptibility):

In the static limit (Yosida - 1958):

$$\lim_{q \to 0} \chi_{zz}^{(BCS)}(q) = -\frac{1}{2} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{\partial f_F(E_\mathbf{k})}{\partial E_\mathbf{k}} \quad \xrightarrow[T \to 0]{} \quad 0$$

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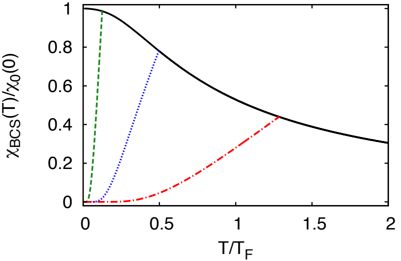

$(k_F a_F)^{-1} = -1.0$ (dashed)
$(k_F a_F)^{-1} = 0.0$ (dotted)
$(k_F a_F)^{-1} = +1.0$ (dash-dot)
$\longleftrightarrow$ non interacting (solid)

Spin susceptibility beyond BCS:

With the inclusion of pairing fluctuations (t-matrix) beyond mean field, the normal and anomalous BCS response diagrams are replaced by the DOS and MT contributions:

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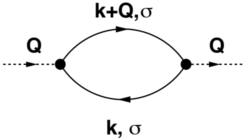
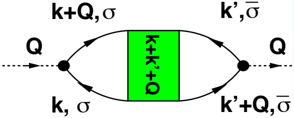
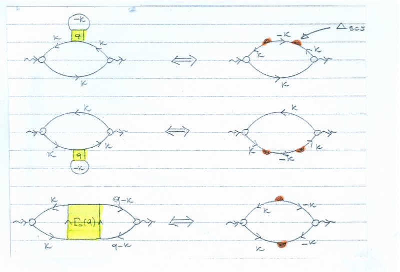


(b)





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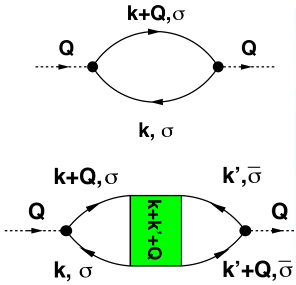

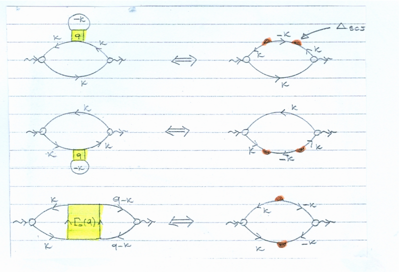

N.B. The two AL diagrams do not contribute because their contributions cancel each other!

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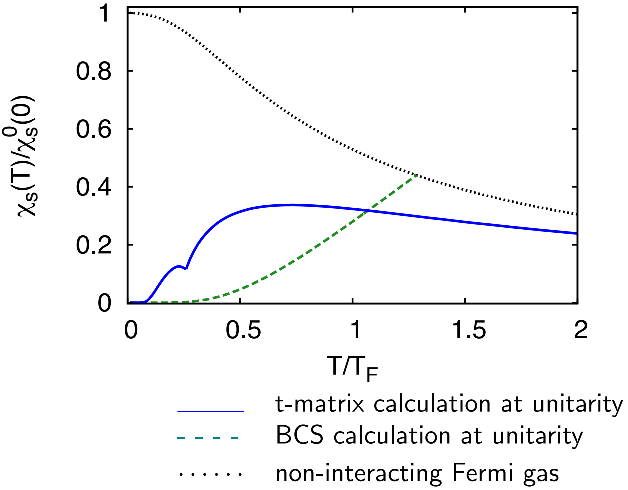

Comparison with MIT experiment (Ketterle) at equilibrium & trapped:

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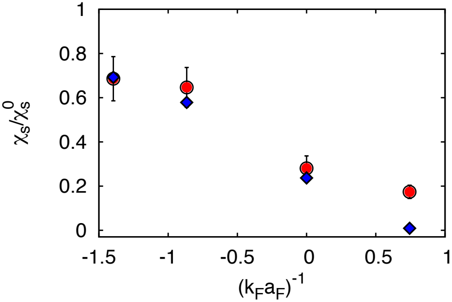
Extract χ_s for the whole trap by resolving the spin fluctuations [PRL **106**, 010402 (2011)]:

$$\langle (N_\uparrow - N_\downarrow)^2 \rangle = \frac{3}{2} N \left(\frac{T}{T_F} \right) \frac{\chi_s}{\chi_s^{(0)}}$$



● experimental data (with error bars)

◆ t-matrix calculation for the trap

left data: $T/T_F = 0.13$; right data $T/T_F = 0.19$

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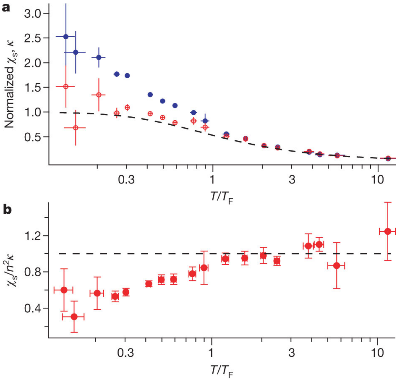

panel (a): ● compressibility χ_n at unitarity
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panel (b): ● ratio χ_s/χ_n
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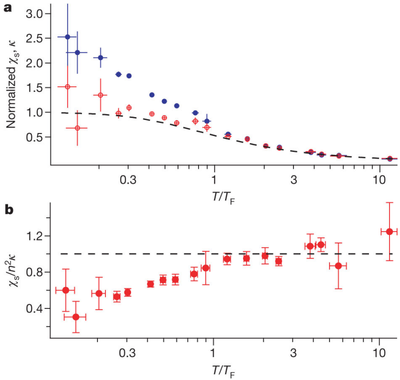

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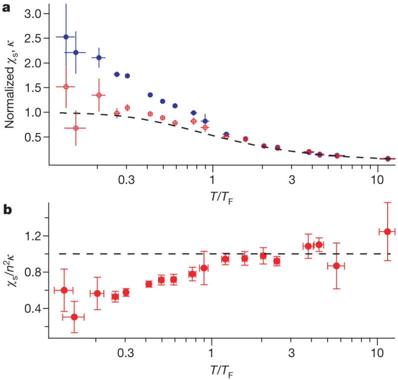

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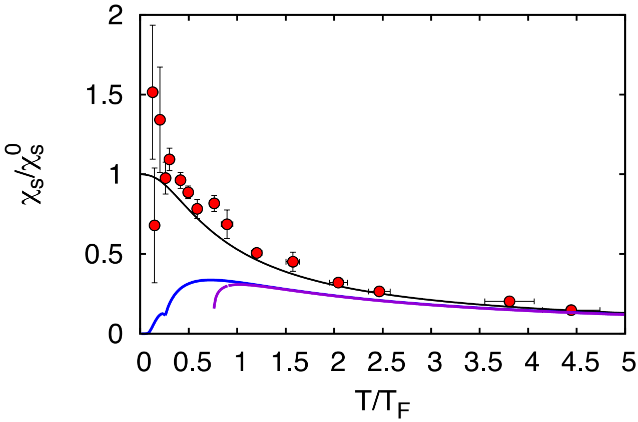
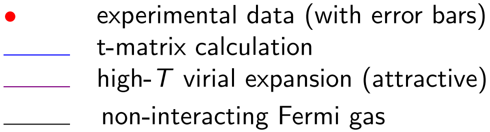
What's going on ? ! ?

First-hand comparison with theory for χ_s "at equilibrium":





In this experiment, χ_s is obtained for the homogeneous system via a complicated procedure by making two clouds (one with spin $\uparrow$and the other with spin $\downarrow$) to collide against each other:

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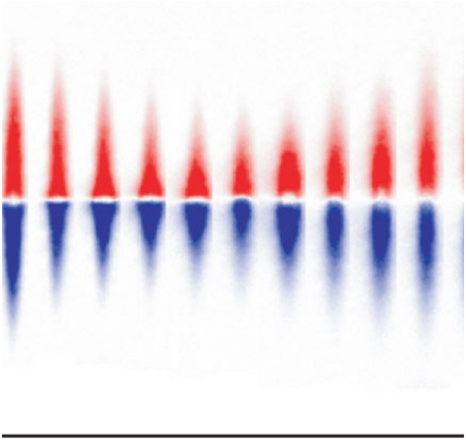

Difference in column densities of the two clouds taken versus time at intervals of 1 ms apart (spin $\uparrow$, spin $\downarrow$).

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- This would correspond to the famous *Arab phoenix*: *"That it exists, everybody agrees, where it is, nobody knows !"*
  (Pietro Metastasio, "Demetrio", 1731)

The “upper” branch of the Fermi gas:

An attractive interaction reflects itself in negative value of the scattering length a_F:

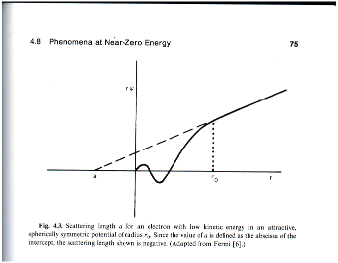
4.8 Phenomena at Near-Zero Energy 75

Fig. 4.3. Scattering length a for an electron with low kinetic energy in an attractive, spherically symmetric potential of radius r_0. Since the value of a is defined as the abscissa of the intercept, the scattering length shown is negative. (Adapted from Fermi [6].)

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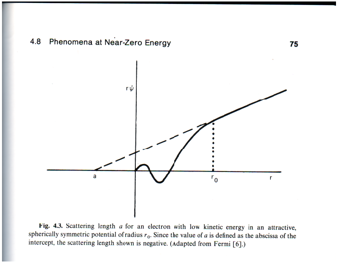

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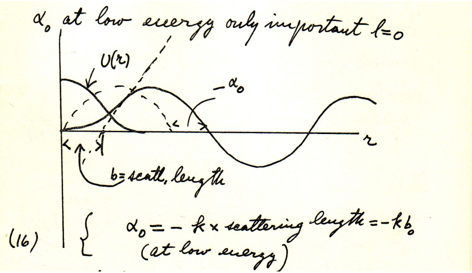
When the bound state sets in a_F turns positive, even though the inter-particle interaction is still attractive $\Longleftrightarrow$



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- At equilibrium, the system has had enough time for pairs to fell into the bound state (with the help of 3-body forces).

- But what about if pairs do not have this time and remain unbound at threshold with $a_F > 0$?

- We argue that this is precisely what's happening in the Zwierlein's experiment with two bouncing clouds of opposite spins.

Interpreting the Zwierlein's experiment:

Within the t-matrix approach, the bound state appears in the ladder propagator

$$\Gamma_0(\mathbf{q}, i\Omega_\nu) = -\int_{-\infty}^{+\infty} \frac{d\omega}{\pi} \frac{\mathrm{Im}\Gamma_0^R(\mathbf{q}, \omega)}{i\Omega_\nu - \omega}$$

where $\Gamma_0^R(\mathbf{q}, \omega) = \Gamma_0(\mathbf{q}, i\Omega_\nu \to \omega + i\eta)$ $(\eta = 0^+)$.

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On the BEC side of the crossover, $\mathrm{Im}\Gamma_0^R(\mathbf{q}, \omega)$ has a delta-like contribution that corresponds to the bound state plus a continuum that starts at $\omega_c(\mathbf{q}) = \mathbf{q}^2/(4m) - 2\mu$ for given $\mathbf{q}$.

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One would thus expect that, to eliminate the contribution of the bound state, it would be sufficient to begin the ω-integration from $\omega_c(\mathbf{q})$.

The "quasi-repulsive" regime with $a_F > 0$:

However, it turns out that this is not enough to reproduce the behavior of a weakly repulsive Fermi gas when $(k_F a_F)^{-1} \gg 1$.

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$$\Gamma_0^{\mathrm{rep}}(\mathbf{p}, \mathbf{q}, i\Omega_\nu) = -\int_{\omega_c(\mathbf{q})}^{+\infty} \frac{d\omega}{\pi} \, \frac{\mathrm{Im}\Gamma_0^R(\mathbf{q}, \omega)}{i\Omega_\nu - \omega} - \frac{8\pi/(m a_F)}{a_F^{-2} + \mathbf{p}^2}$$

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N.B. One is familiar with the presence of a similar constant term, for instance, in the energy dependence of the scattering amplitude for zero scattering angle [Landau-Lifshitz, *Quantum Mechanics*, §129]:

$$f(0, E) = -\int_0^{+\infty} \frac{dE'}{\pi} \frac{\mathrm{Im} f(0, E')}{E - E'} + \sum_n \frac{d_n}{E - E_n} + f_{\mathrm{Born}}$$

“Quasi-repulsive” gas: Phase diagram

We use the above expression for Γ_0^{rep} to calculate the thermodynamics and the spin response function of this out-of-equilibrium system.

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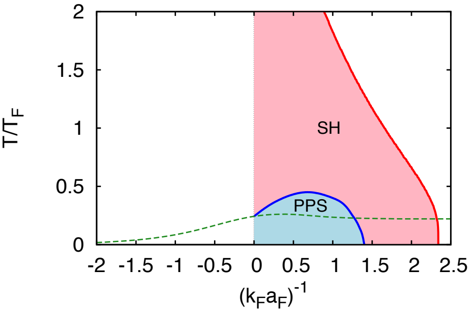
In the T-$(k_F a_F)^{-1}$ phase diagram we identify a "forbidden region" where it is not possible to solve the density equation using Γ_0^{rep}:



- - - - T_c (t-matrix)
PPS forbidden region
SH forbidden region

[SH $\leftrightarrow$ Shenoy & Ho, PRL **107**, 210401 (2011)]

"Quasi-repulsive" gas: Chemical potential

For $(k_F a_F)^{-1} \gg 1 \implies$ recover the weakly-repulsive Fermi gas (Galitskii):

$$\mu = E_F \left[1 + \frac{4}{3\pi} k_F a_F + \frac{4}{5\pi^2} (21 - 2\ln 2) (k_F a_F)^2 + \cdots \right]$$

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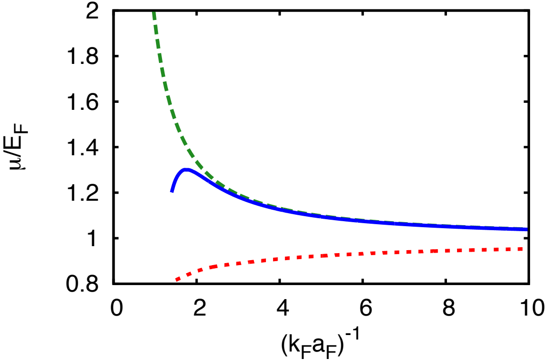

- - - - Galitskii repulsive

——— t-matrix repulsive with frequency-independent term

· · · · · · t-matrix repulsive without frequency-independent term (wrong sign of the linear term!) $[T = 0.1\, T_F]$

"Quasi-repulsive" gas: Spin susceptibility

Calculate χ_s using Γ_0^{rep} in the DOS and MT diagrams: $[T = 0.1\,T_F]$

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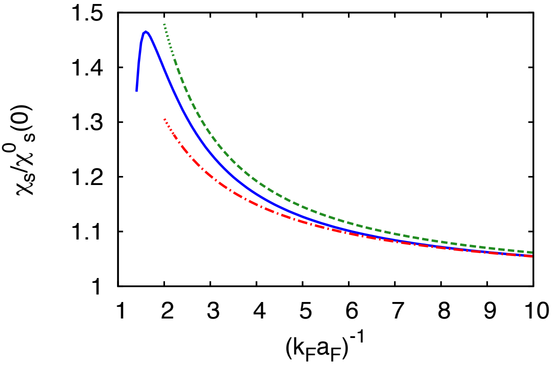

- - - - Galitskii repulsive (up to 2^{nd}-order)

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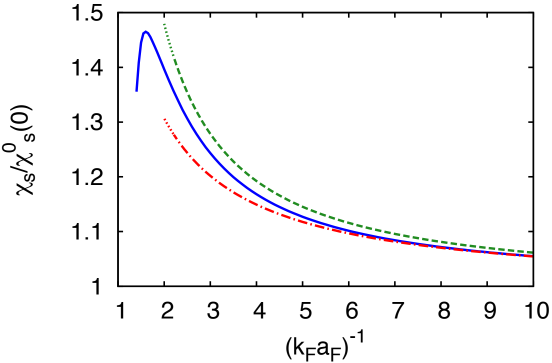

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——— t-matrix repulsive (with frequency-independent term)

$\Longrightarrow$ our t-matrix reproduces well the results of a truly repulsive Fermi gas (Galitskii) when $(k_F a_F)^{-1} \gtrsim 2$

"Quasi-repulsive" gas: MIT experiment

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Assumption: By extrapolating the shape of the spin susceptibility before it drops at $(k_F a_F)^{-1} \approx 2$, one should end up by reaching an “excited configuration” as if an avoided level crossing were present with a dynamics determined by Landau-Zeener processes:

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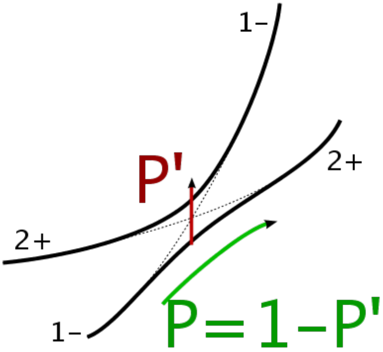
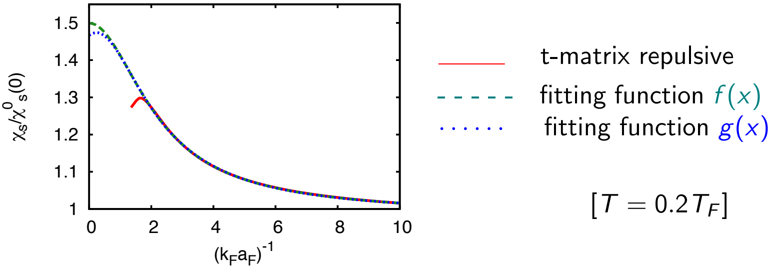
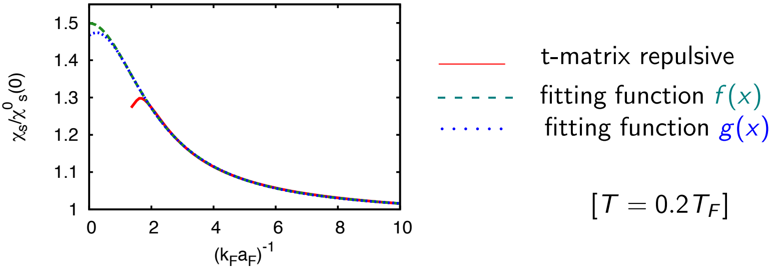


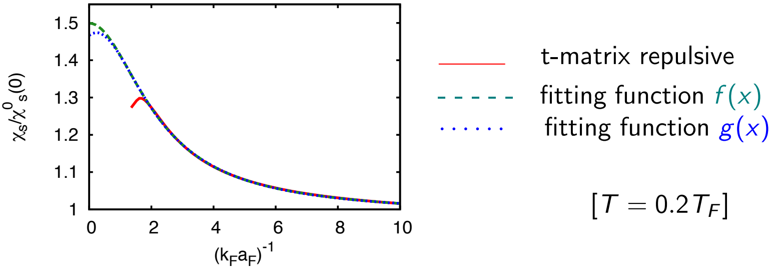


$\chi_s/\chi^0_s(0)$
1.5
1.4
1.3
1.2
1.1
1
0
2
4
6
8
10
$(k_F a_F)^{-1}$
t-matrix repulsive
fitting function $f(x)$
fitting function $g(x)$
$[T = 0.2 T_F]$



$$f(x) = \chi_s^0(T) + \frac{a}{\sqrt{x^2+b^2}} + \frac{c}{x^2+d^2}$$

$$g(x) = \chi_s^0(T)\left(1 + \frac{b\,x + c}{x^2 + d\,x + e}\right) \qquad [x = (k_F a_F)^{-1}]$$



$[T = 0.2\, T_F]$

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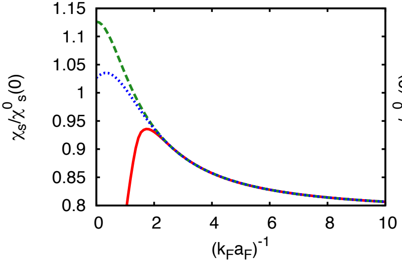
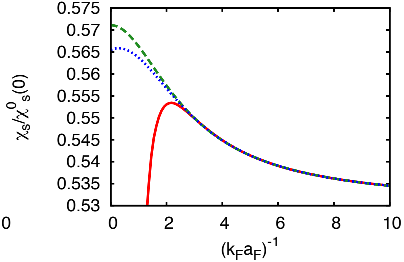
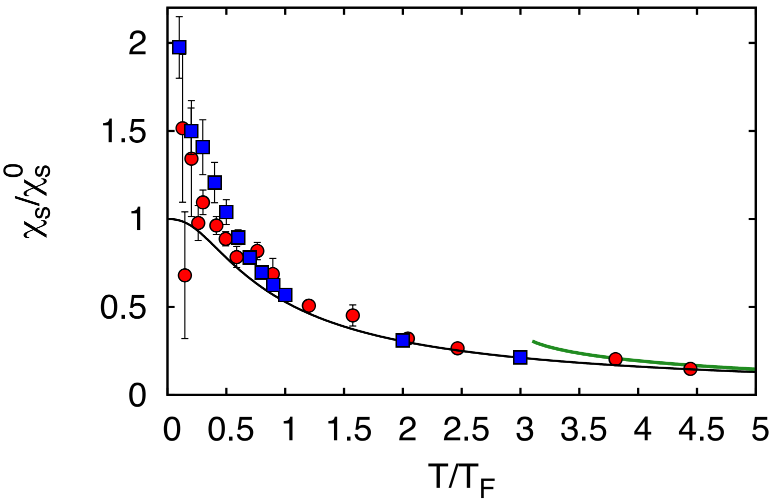


$[T = 0.5\, T_F]$



$[T = 1.0\, T_F]$

Comparison with MIT experiment:



● experimental data ■ our extrapolation

—— virial expansion —— non-interacting gas

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- Thank you for your attention and best wishes for your brand new EMMI enterprise !

Supplemental Material: Bragg spectroscopy with ultra-cold Fermi atoms

Dynamic spin response of a strongly interacting Fermi gas

S. Hoinka[1], M. Lingham[1], M. Delehaye[1,2], and C. J. Vale[1]
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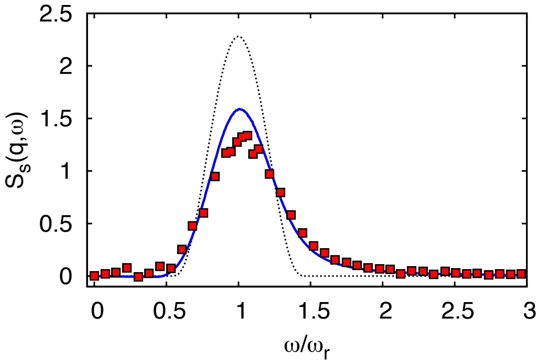
[2] *Departement de Physique, Ecole Normale Superieure, 24 rue Lhomond, 75005 Paris, France*
(Dated: March 22, 2012)

We present an experimental investigation of the dynamic spin response of a strongly interacting Fermi gas using Bragg spectroscopy. By varying the detuning of the Bragg lasers, we show that it is possible to measure the response in the spin and density channels separately. At low Bragg energies, the spin response is suppressed due to pairing, whereas the density response is enhanced. These experiments provide the first independent measurements of the spin-parallel and spin-antiparallel dynamic and static structure factors and open the way to a complete study of the structure factors at any momentum. At high momentum the spin-antiparallel dynamic structure factor displays a universal high frequency tail, proportional to $\omega^{-5/2}$, where $\hbar\omega$ is the probe energy.

PACS numbers: 03.75.Hh, 03.75.Ss, 05.30.Fk

arXiv:1203.4657v1 [cond-mat.quant-gas] 21 Mar 2012

Comparison with the experimental data for the spin dynamic structure factor (1):

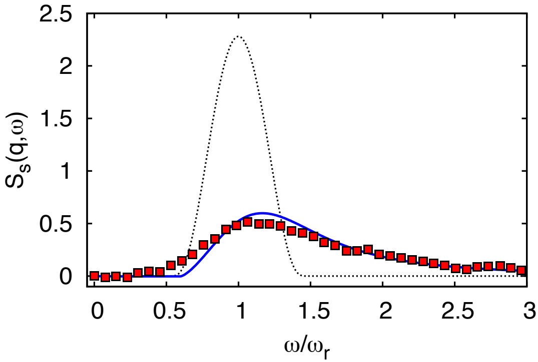


■ experimental data —— BCS theory (trap averaged)

······ non-interacting gas

$(k_F a_F)^{-1} = 0.0$ $\quad T \simeq 0.05\, T_F$ $\quad q = 4.5 k_F$

Comparison with the experimental data for the spin dynamic structure factor (2):



■ experimental data ——— BCS theory (trap averaged)

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$(k_F a_F)^{-1} = +1.0$ $\quad T \simeq 0.05\, T_F$ $\quad q = 4.5 k_F$