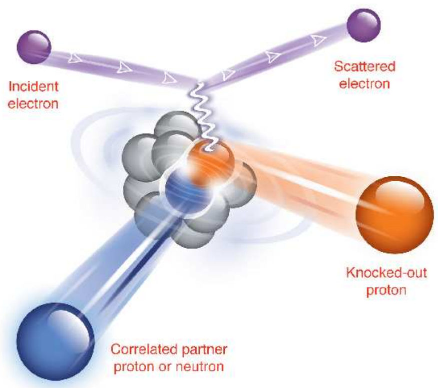
Universality of short-range nucleon-nucleon correlations



Hans Feldmeier

Thomas Neff

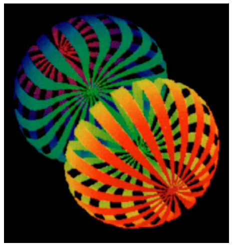
Wataru Horiuchi

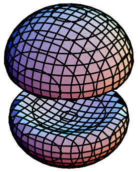
Yasuyuki Suzuki

First EMMI Program, 2012-04-16 to 2012-05-11

The Extreme Matter Physics of Nuclei: From Universal Properties to Neutron-Rich Extremes

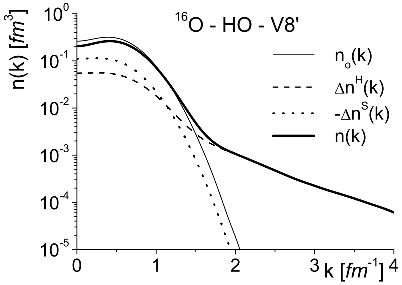
Motivation





Deuteron dumbbell
Argonne Potential
short-range repulsion, tensor
no good for SM or HF

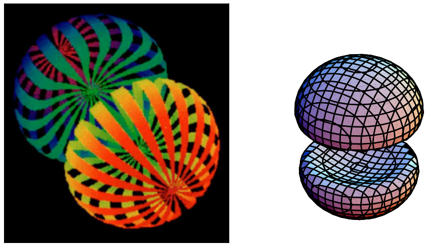
Argonne group, RevModPhys70(1998)743



High momentum tails (real?)
HF and SM can not have those

Alvioli et al. PRC72(2005)054310

Motivation



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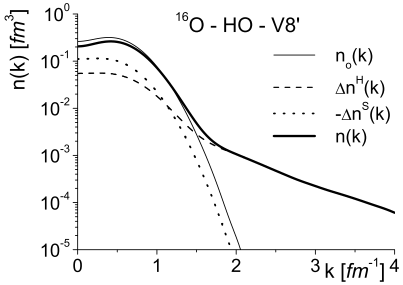

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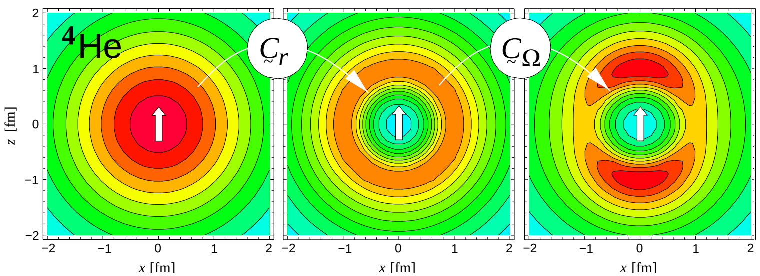
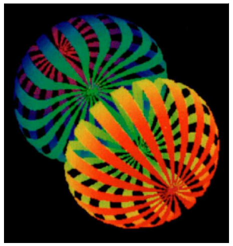
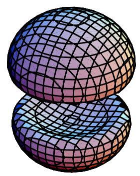


UCOM $C = C_\Omega C_r\ ,\ C^\dagger C = 1$
imprint short-range correlations
tame the NN-interaction
keep phase-shifts

Feldmeier, Neff, Roth, NPA632(1998)61, NPA713(2003)311

Motivation

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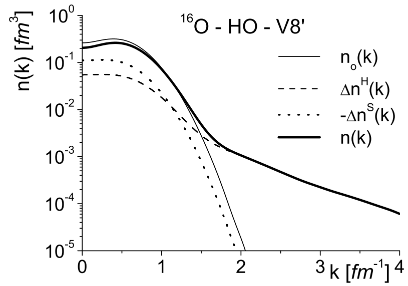

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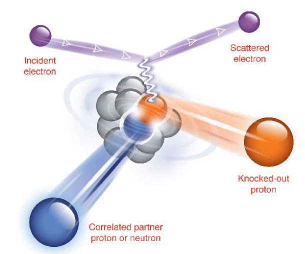
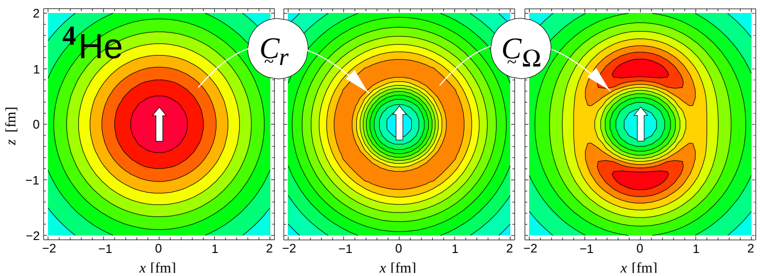


back-to-back high k
p-p pairs observed
Piasetzky et al.
PRL97(2006)162504



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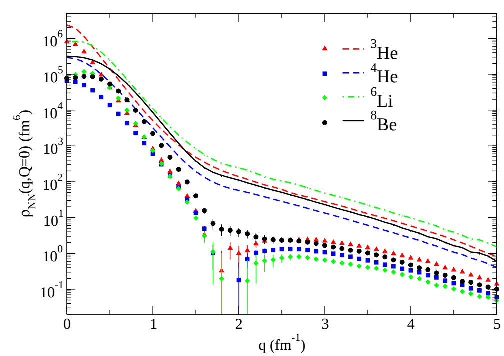
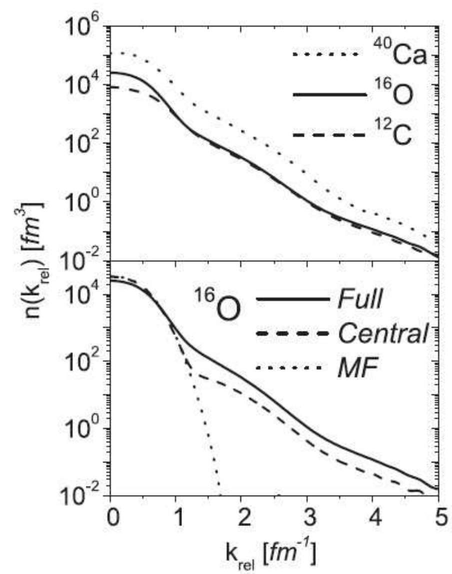


$\mathrm{k_{rel}}$-distribution
different for p-n and p-p
$\mathrm{n(k_{rel})}$ similar for all nuclei
universal?
Schiavilla et al. PRL98(2007)132501



Alvioli et al. PRL100(2008)162503

Outline

✔ Motivation

- Two-body densities of exact many-body states ^{2}H, ^{3}H, ^{3}He, ^{4}He, ^{4}He* as function of relative distance r and relative momentum k for different S,T channels
- At short distance two-body densities are one-to-one cast of potential (AV8')
- Perfect universality up to r $\lessapprox$ 1 fm and k $\gtrapprox$ 3 fm^{-1}
- Correlations induced by UCOM $\hat{C} = \hat{C}_{\Omega}\hat{C}_{r}$
- No-Core Shell Model results when softening the interaction

Results taken from: Feldmeier, Horiuchi, Neff, Suzuki, PRC 84(2011)054003

Many-Body States

- Given Hamiltonian $\hat{H} = \hat{T} + \hat{V}_{NN}$
  two-body interaction $\hat{V}_{NN}$= Argonne V8'

Solve many-body Schrödinger equation exactly (correlated Gaussian basis):

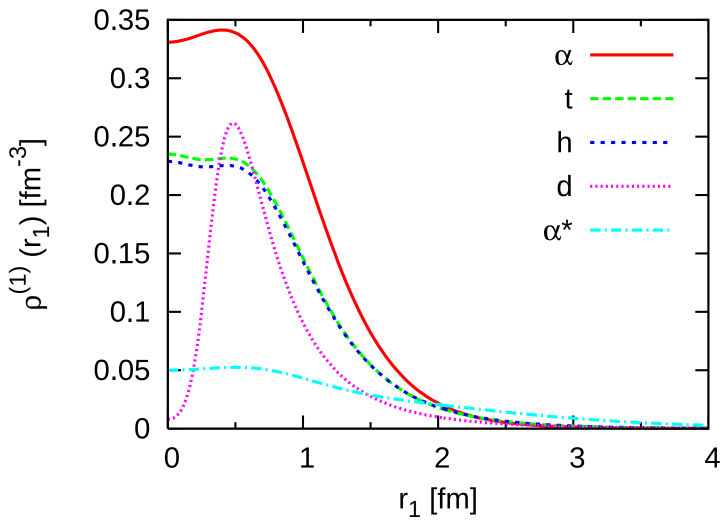
$$\hat{H}\left|\Psi; JM\right\rangle = E\left|\Psi; JM\right\rangle$$

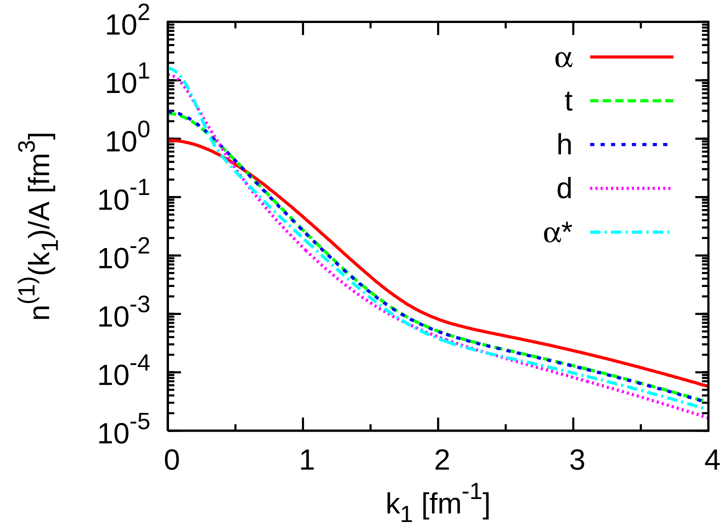
- exact many-body eigenstates $\left|\Psi; JM\right\rangle$ which contain all many-body correlations induced by V_{NN}
  (short-, middle- and long-ranged)

We investigate five many-body states $\left|\Psi; JM\right\rangle$

α = ^{4}He ground state
t = ^{3}H
h = ^{3}He
d = ^{2}H
α^*= excited 0^+ (20.25 MeV) state of ^{4}He.

one-body density





one-body momentum density

Two-Body Densities

- Exact many-body state $|\Psi; JM\rangle$ contains all many-body correlations induced by $\hat{V}_{NN}$

Two-body density as function of distance $\mathbf{r}$ between nucleon pair with spin S, M_S and isospin T, M_T:

- $$\rho^{\text{rel}}_{S M_S, T M_T}(\mathbf{r}) = \frac{1}{2J+1} \sum_M \langle \Psi; JM | \sum_{i<j}^{A} \hat{P}^{S M_S}_{ij} \hat{P}^{T M_T}_{ij} \delta^3(\hat{\mathbf{r}}_i - \hat{\mathbf{r}}_j - \mathbf{r}) | \Psi; JM \rangle$$

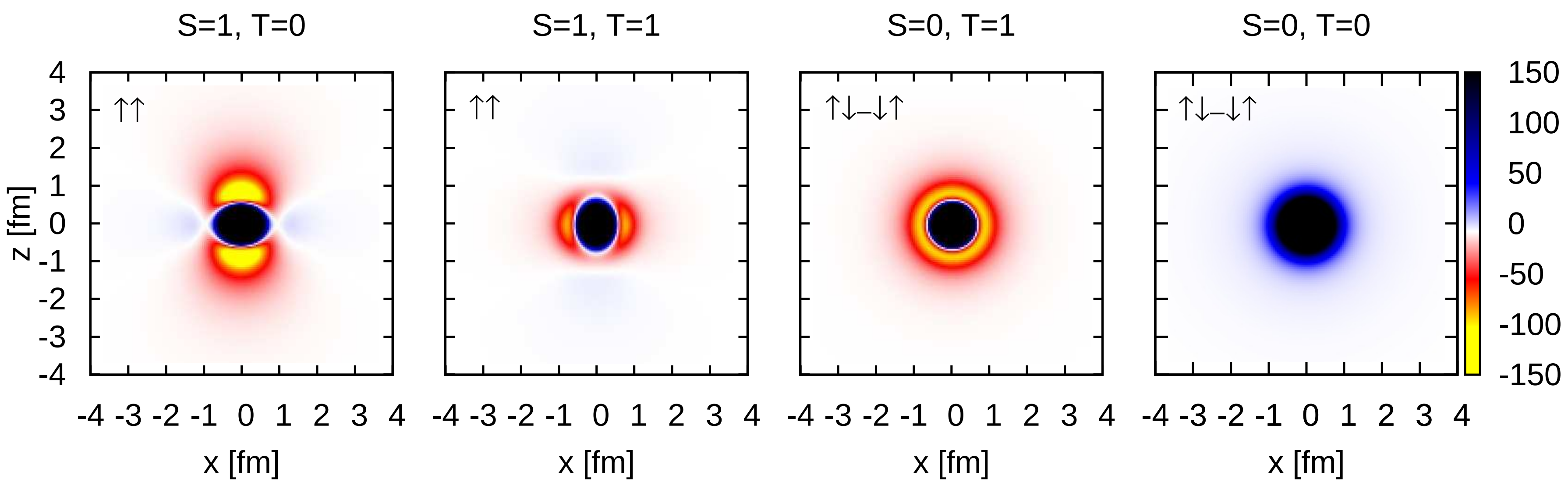
- $$\rho^{\text{rel}}_{S,T}(r) = \sum_{M_S, M_T} \rho^{\text{rel}}_{S M_S, T M_T}(\mathbf{r})$$

Two-body momentum density as function of relative momentum $\mathbf{k}$ between nucleon pair with spin S, M_S and isospin T, M_T:

$$n^{\text{rel}}_{S M_S, T M_T}(\mathbf{k}) = \frac{1}{2J+1} \sum_M \langle \Psi; JM | \sum_{i<j}^{A} \hat{P}^{S M_S}_{ij} \hat{P}^{T M_T}_{ij} \delta^3(\tfrac{1}{2}(\hat{\mathbf{k}}_i - \hat{\mathbf{k}}_j) - \mathbf{k}) | \Psi; JM \rangle$$

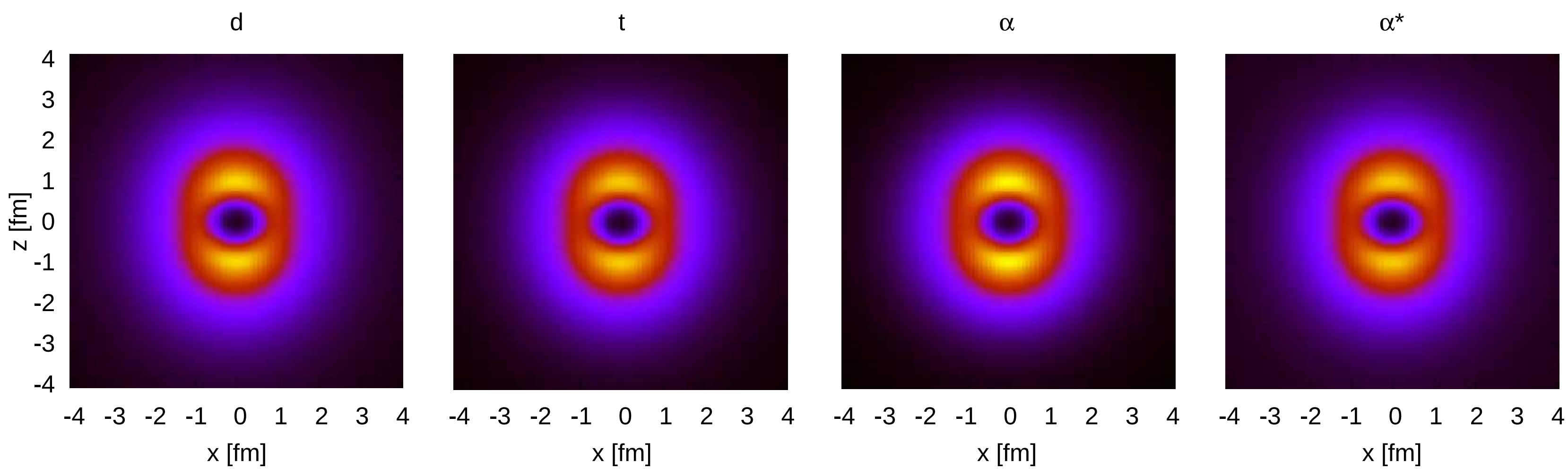
- $$n^{\text{rel}}_{S,T}(k) = \sum_{M_S, M_T} n^{\text{rel}}_{S M_S, T M_T}(\mathbf{k})$$

Argonne V8' Potential

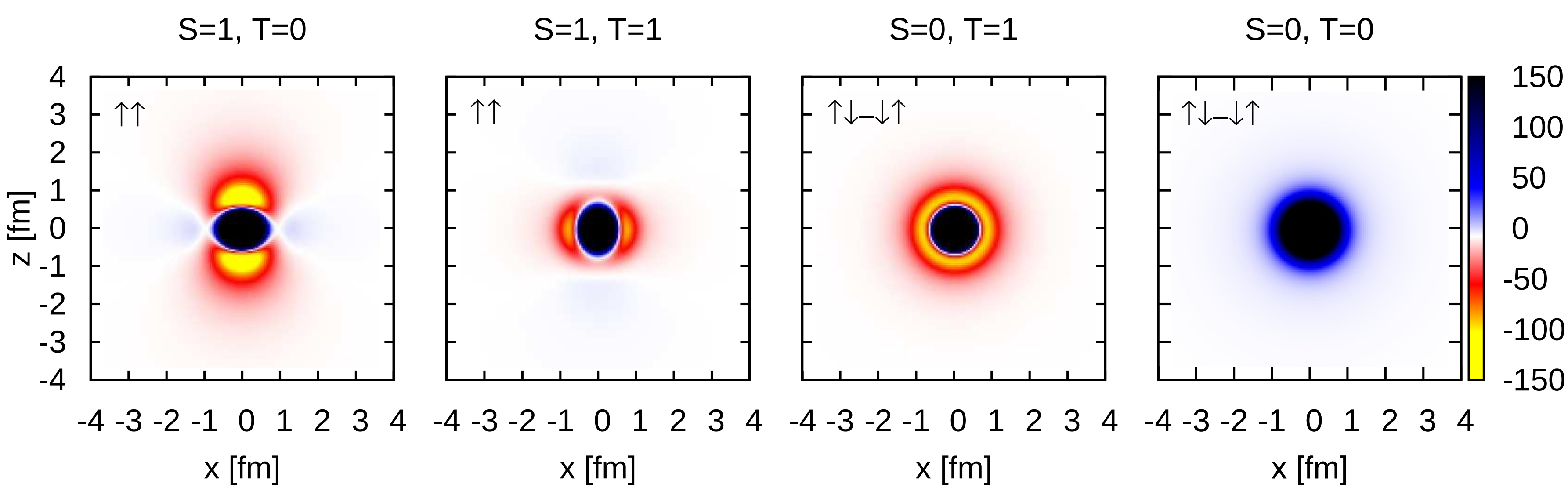


Argonne V8′ in different spin-isospin channels as a function of distance vector $\mathbf{r} = (x, y = 0, z)$.
In the $S = 1$ channels the total spin is aligned with the z axis. Units are in MeV.

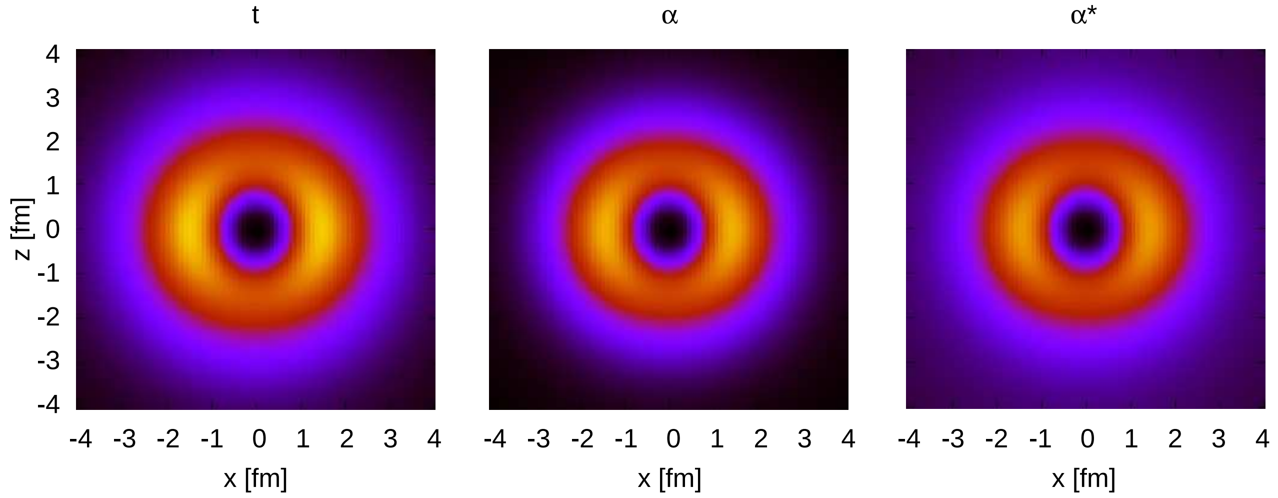
Two-body density for $S = 1, M_S = 1, T = 0$ pairs



Two-body densities in coordinate space for a pair of nucleons with $S = 1, M_S = 1$ and $T = 0$ in ground states of ^{2}H = d, ^{3}H = t and ^{4}He = α and the 20.21 MeV excited state of ^{4}He = α^*. Densities have rotational symmetry around the z axis.



Two-body density for $S = 1, M_S = 1, T = 1$ pairs



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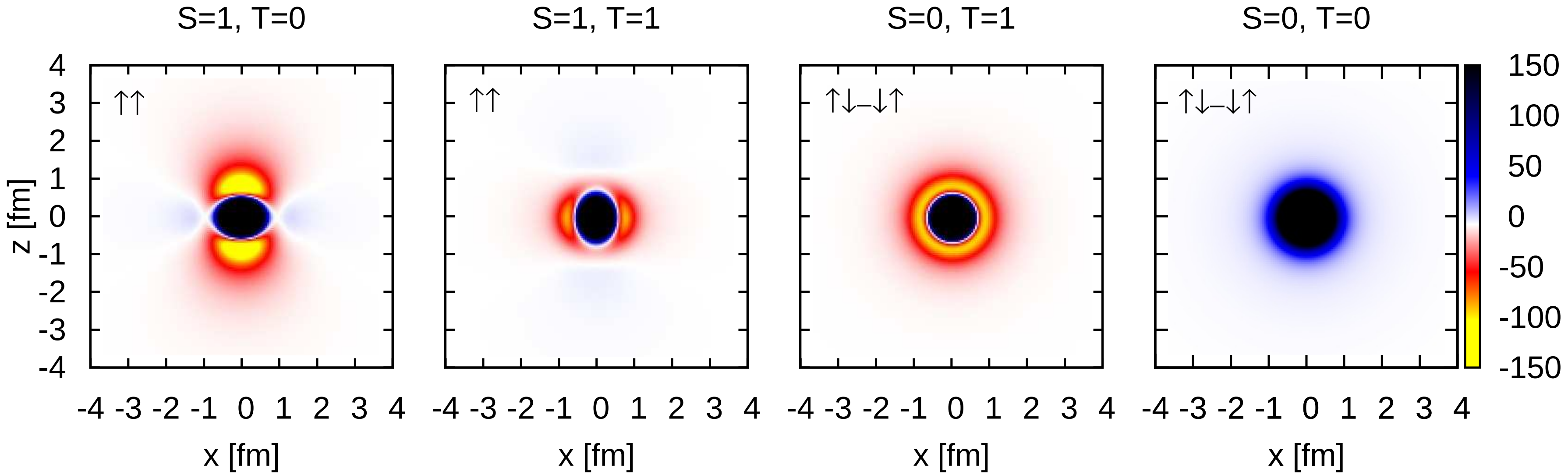
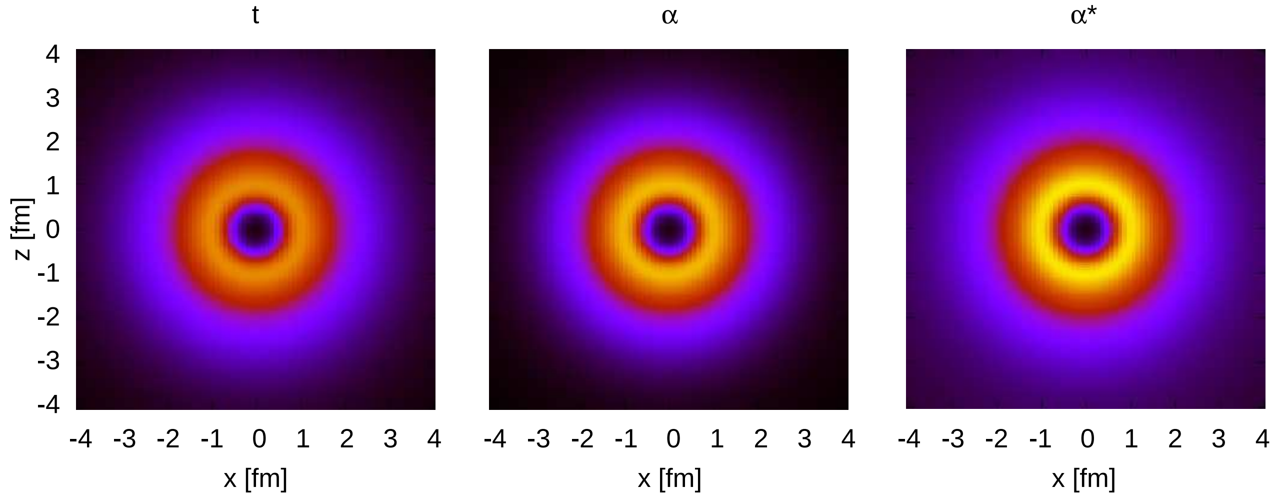


Two-body density for $S = 0, T = 1$ pairs



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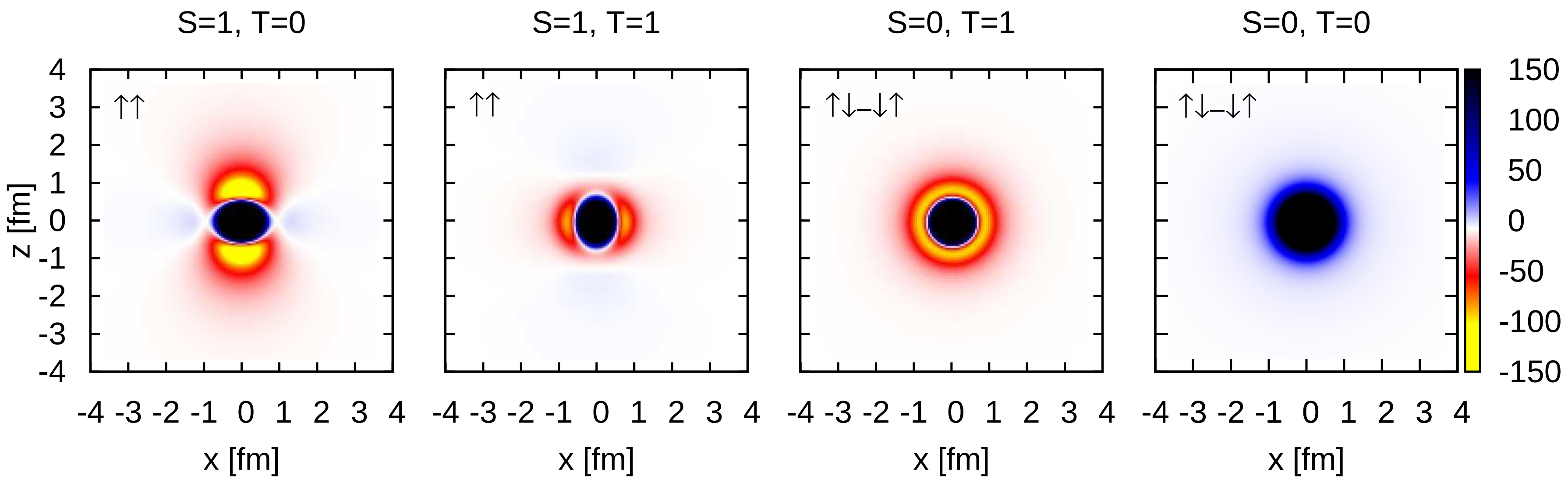
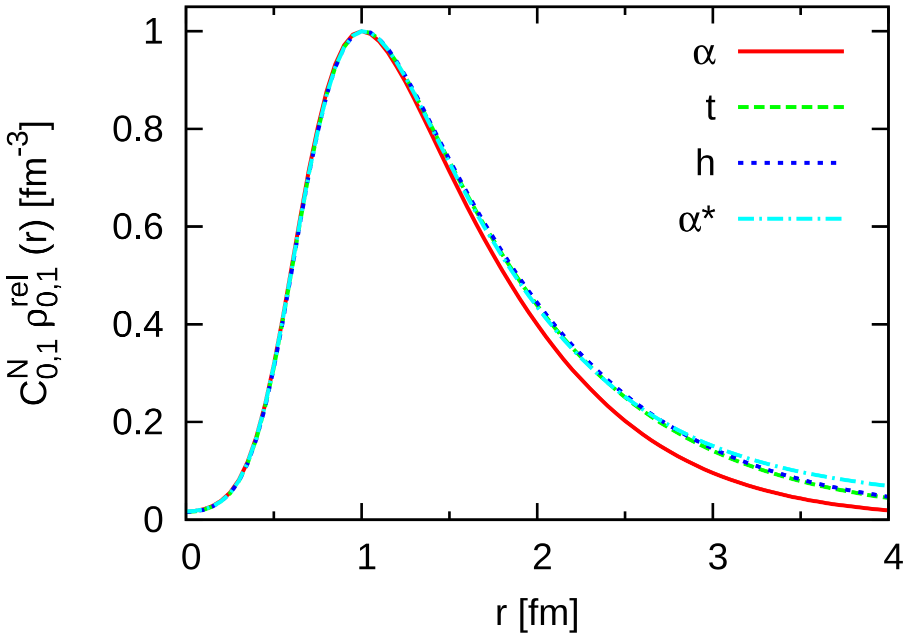


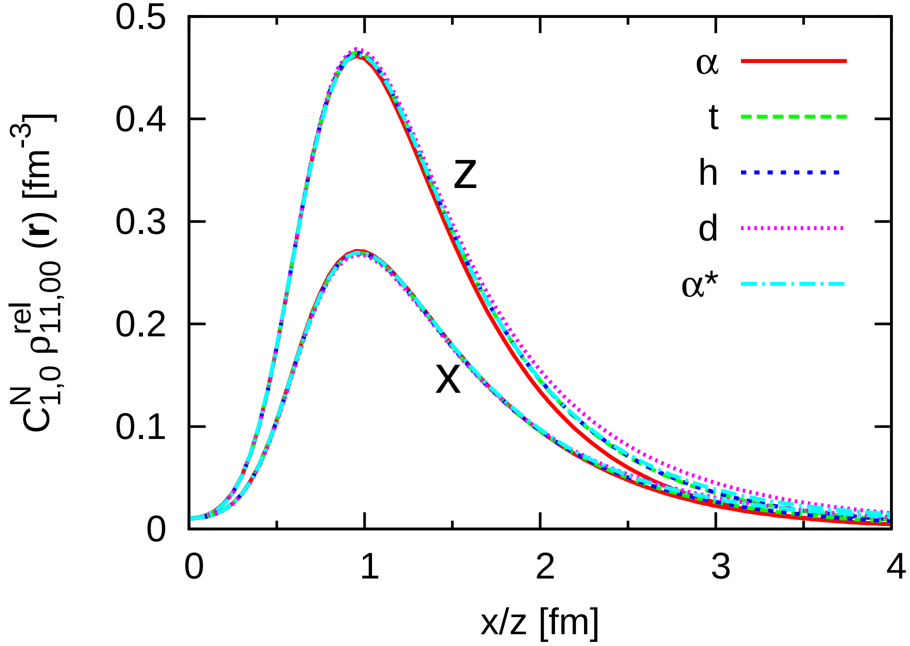
Universality in coordinate space

Two-body densities $\rho^{\mathrm{rel}}_{S,T}(r)$ normalized to 1 fm^{-3} at r=1 fm

S=0, T=1



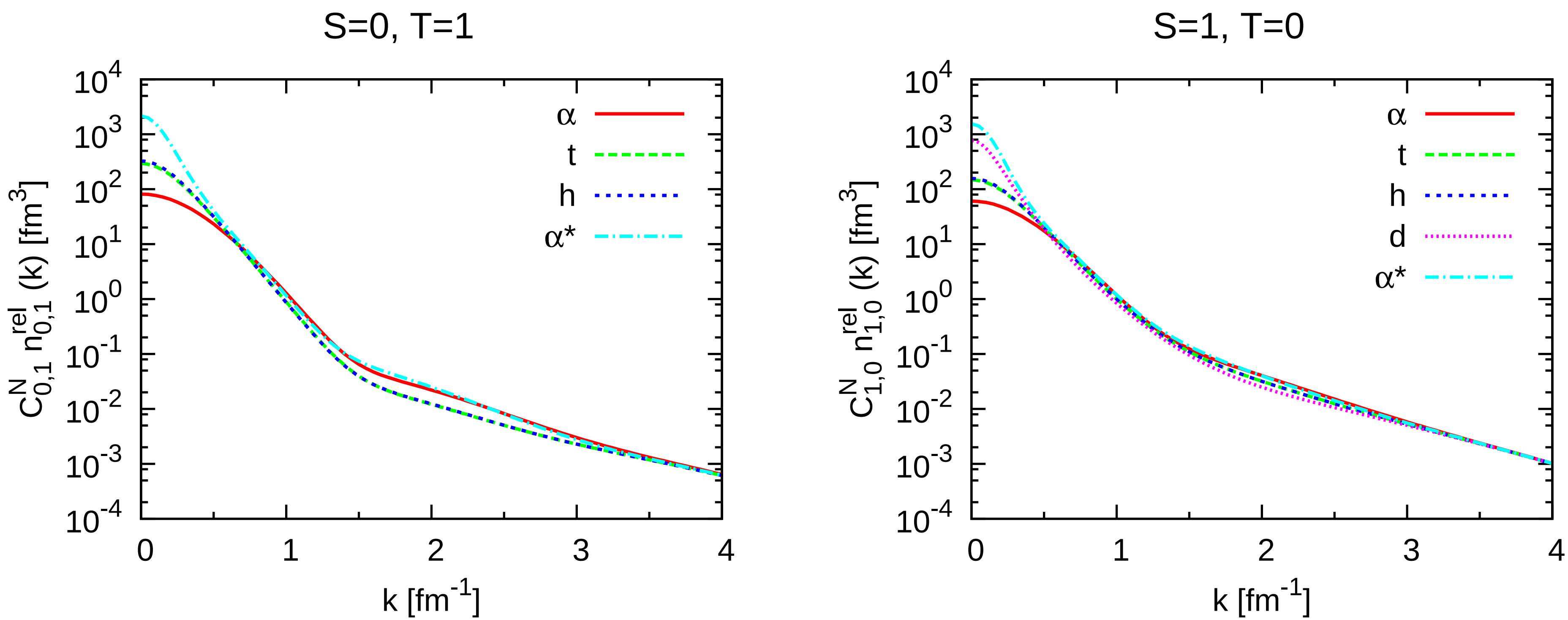
S=1, M$_S$=1, T=0, cuts along x- and z-axis



➨ Universal behaviour of central and tensor correlations up to r ≈ 1 fm for all many-body states

Universality in momentum space

Two-body densities $\rho^{\rm rel}_{S,T}(r)$ normalized to 1 fm^{-3} at r=1 fm in coordinate space !



- 0 < k < 0.7 fm^{-1} large differences
  0.7 fm^{-1} < k < 3 fm^{-1} universal for S=1, T=0
  less universal for S=0, T=1, 3-body correlations
  3 fm^{-1} < k very short-range correlations, perfect universality

Admixture of wrong parity

Number of pairs in ^{4}He ground states

(ST)	(10)	(01)	(11)	(00)
parity rel.motion	even	even	odd	odd
exact AV8′	2.992	2.572	0.428	0.008
shell model $(s_{1/2})^4$	3.000	3.000	0	0

- Why does (ST)=(01) even channel give away 0.428 pairs to (ST)=(11) odd ?
- Odd channel is less attractive
  V$_{NN}$ does not scatter from even to odd
  Unitary 2-body correlator also keeps parity

Admixture of wrong parity by 3-body correlations

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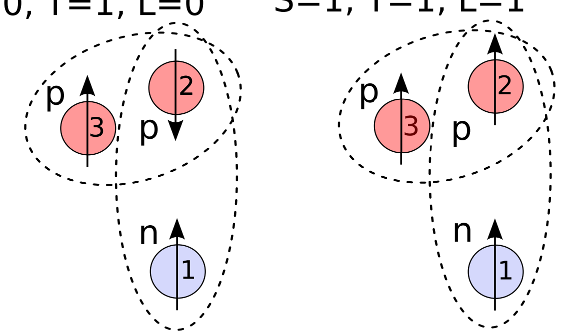
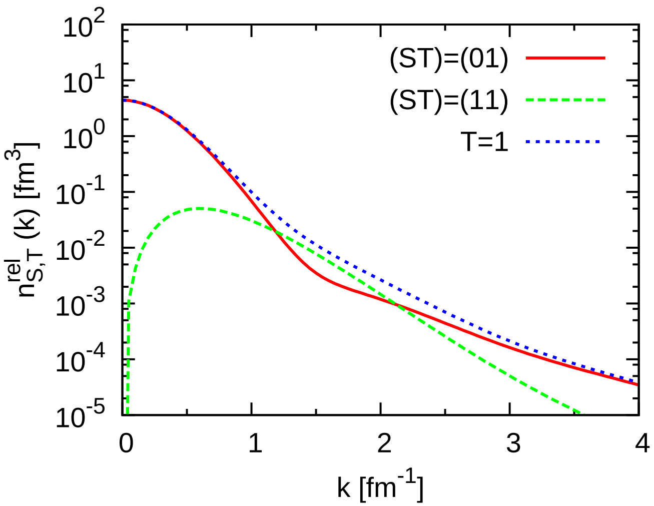
Explanation: 3-body correlations



Strong long-range tensor breaks S=0,T=1 pair (2,3) and aligns spins of S=1, T=0 pair (1,2)
$\Rightarrow$ more binding



^{4}He
rel. momentum distribution

(ST)=(01) and (ST)=(11) channels and sum

(ST)=(11) from 3-body corr. contributes most where tensor-corr. are in (ST)=(10)

UCOM and SRG

Unitary Correlation Operator Method

- UCOM imprints tensor and central correlations into SM-like many-body states $\left|\Phi\right\rangle$
$$\left|\Psi\right\rangle = \hat{C}\left|\Phi\right\rangle = \hat{C}_\Omega \hat{C}_r \left|\Phi\right\rangle, \quad \hat{C}^\dagger \hat{C} = 1$$

- Unitary trafo of $\hat{H}$ to soft $\hat{H}_{\text{eff}} = \hat{C}^\dagger \hat{H}\,\hat{C}$
$$\left\langle\Psi'\right|\hat{H}\left|\Psi\right\rangle = \left\langle\Phi'\right|\hat{C}^\dagger \hat{H}\,\hat{C}\left|\Phi\right\rangle = \left\langle\Phi'\right|\hat{H}_{\text{eff}}\left|\Phi\right\rangle$$
and unitary trafo of observables $\hat{O}$
$$\left\langle\Psi'\right|\hat{O}\left|\Psi\right\rangle = \left\langle\Phi'\right|\hat{C}^\dagger \hat{O}\,\hat{C}\left|\Phi\right\rangle = \left\langle\Phi'\right|\hat{O}_{\text{eff}}\left|\Phi\right\rangle$$

Hergert, Roth, Phys. Rev. C **75**, 051001(R) (2007)
Bogner *et. al.*, Phys. Rev. C **75**, 061001(R) (2007)

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- SRG transformation of $\hat{H}$ by flow equation

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- Get UCOM correlation fcts. which define $\hat{C}_\Omega$, $\hat{C}_r$ by mapping S- and P-wave two-body scattering solutions for $E = 0$: $\left|\Psi_{12}\right\rangle = \underset{\sim}{C}_\Omega \underset{\sim}{C}_r \left|\Phi_{12}(\alpha)\right\rangle$ gives mapping $\hat{U}(\alpha) \Rightarrow \underset{\sim}{C}_r \underset{\sim}{C}_\Omega$

Hergert, Roth, Phys. Rev. C **75**, 051001(R) (2007)
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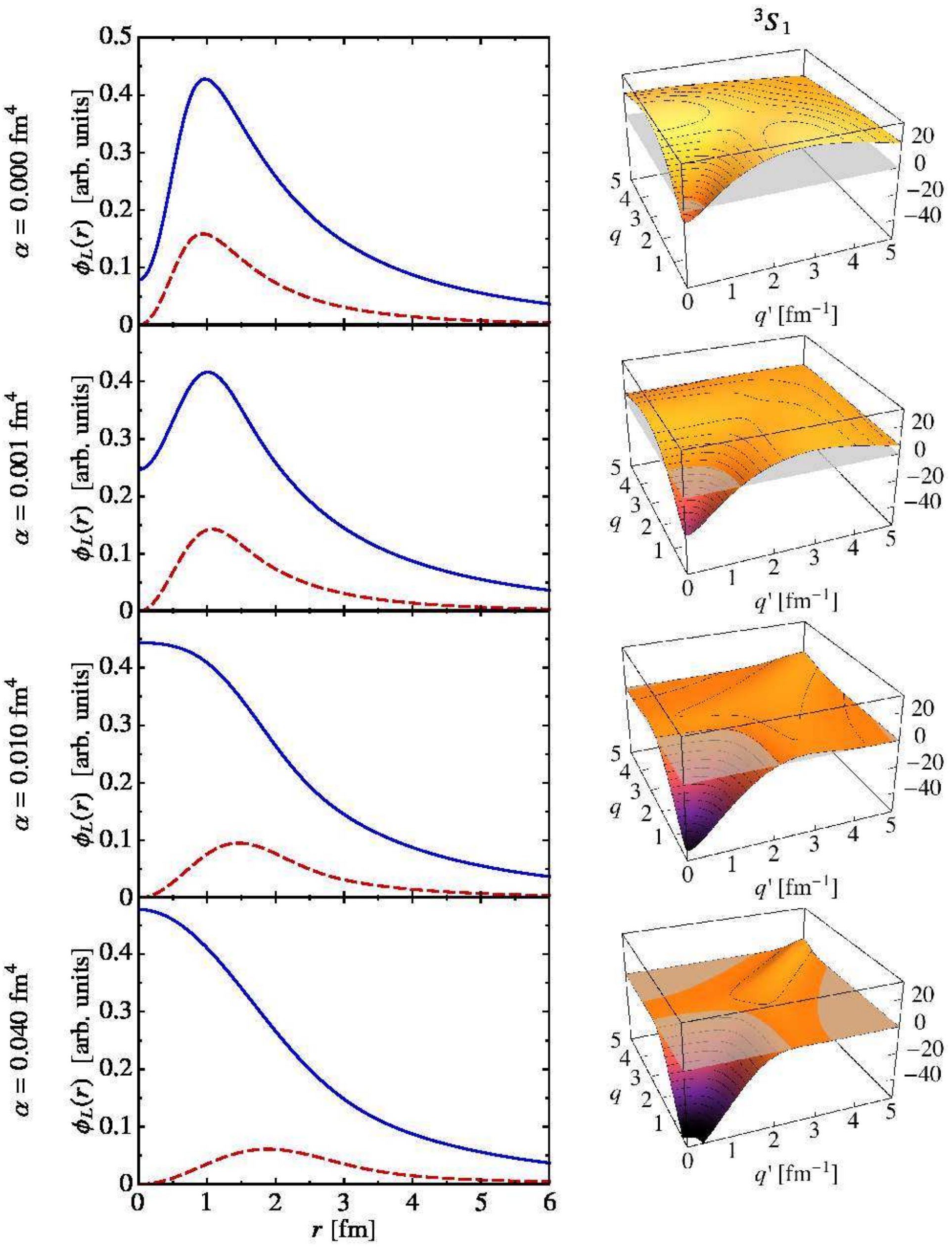
Hergert, Roth, Phys. Rev. C **75**, 051001(R) (2007)
Bogner *et. al.*, Phys. Rev. C **75**, 061001(R) (2007)

Approximation: In the following all trafos only in 2-body space, neglecting induced n-body contributions

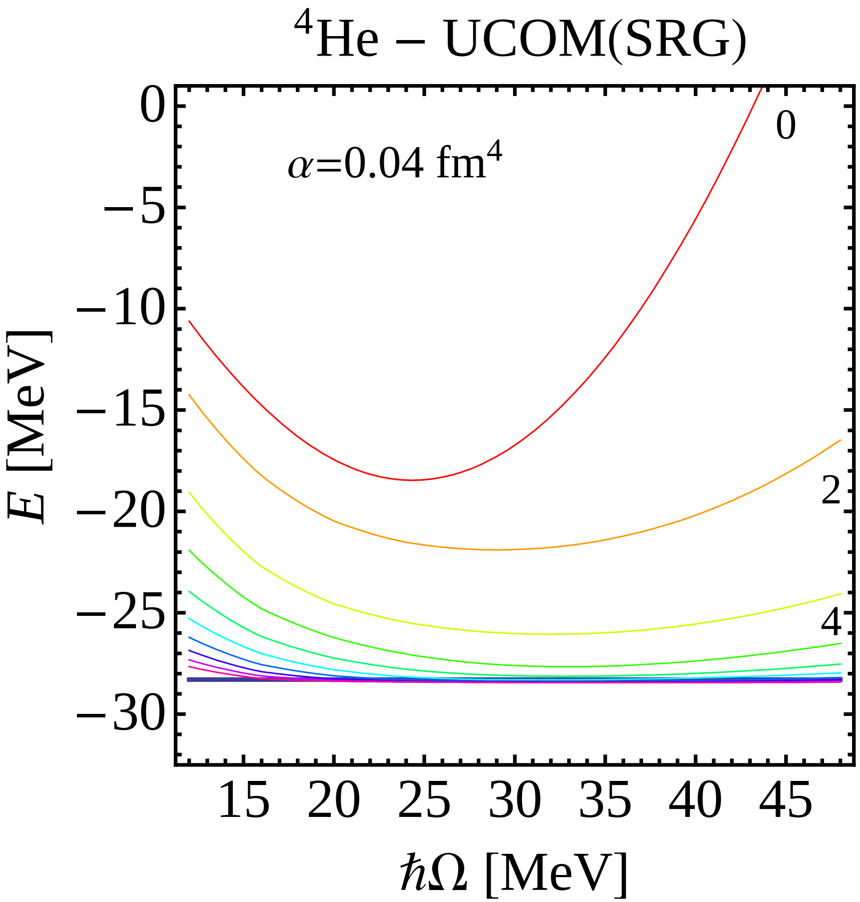
SRG and Deuteron wave functions

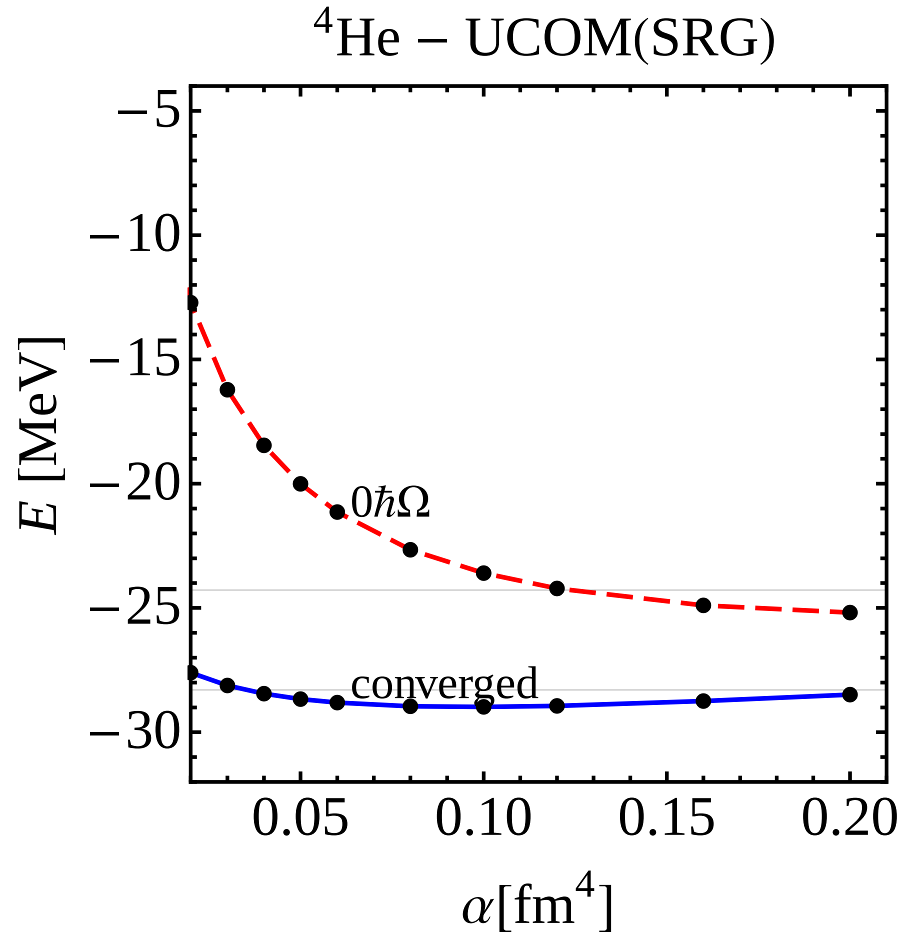


Increasing α or range of UCOM correlator $\hat{C}$

- SRG-evolved ^{2}H wave function $\left|\Phi_{12}(\alpha)\right\rangle$ short-range correlation hole is eliminated D-wave admixture gets reduced
- SRG-evolved interaction $\hat{V}(\alpha)$ decouples low- and high momentum states becomes softer

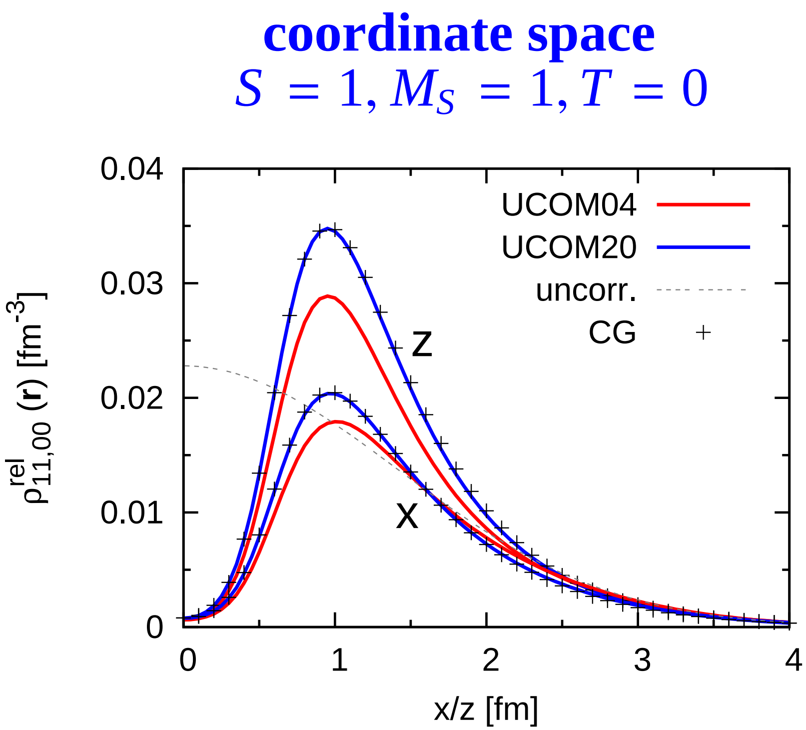
No-Core Shell Model Calculations



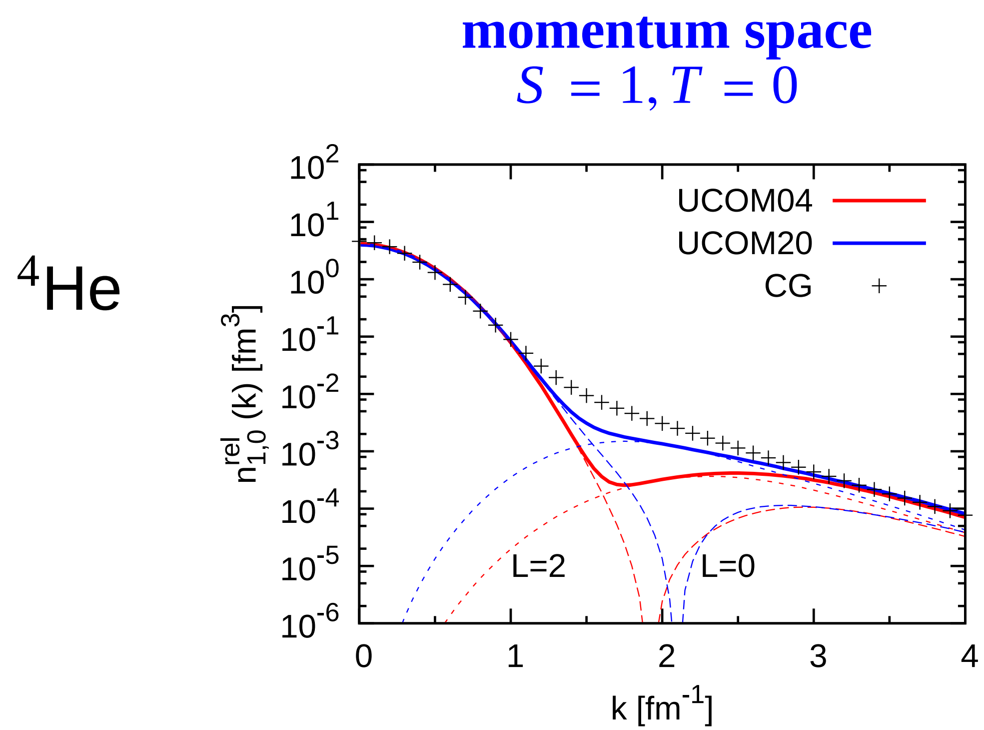


- convergence much improved compared to bare interaction
- in 2-body approximation effective interaction gives different energy then bare interaction induced 3-, 4-body missing, but also genuine 3-body missing

Two-body densities



$\langle (s_{1/2})^4 \,|\, \hat{C}^\dagger \hat{\rho}^{rel}_{11,00}(\mathbf{r})\, \hat{C} \,|\, (s_{1/2})^4 \rangle$

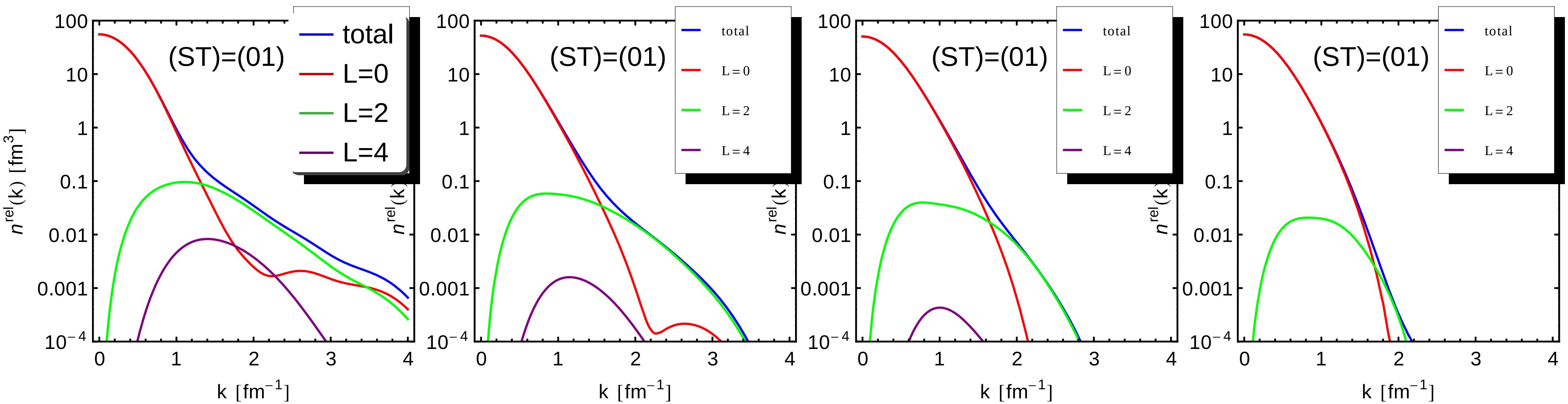


$\langle (s_{1/2})^4 \,|\, \hat{C}^\dagger \hat{n}^{rel}_{1,0}(k)\, \hat{C} \,|\, (s_{1/2})^4 \rangle$

- two-body densities calculated from $0\hbar\Omega$ ^{4}He uncorrelated state and correlated density operators
- UCOM20 derived from $\alpha = 0.04\ \mathrm{fm}^4$ or $\lambda \approx 2.2\ \mathrm{fm}^{-1}$ SRG trafo reproduces coordinate space 2-body density and high-momentum components well
- tensor correlations dominate
- long-range many-body correlations should fill up around Fermi momentum

(Uncorrelated) ^{4}He Relative Momentum Distributions

AV18	UCOM01	UCOM04	UCOM20
$\alpha = 0$	$\alpha = 0.01\ \text{fm}^4$	$\alpha = 0.04\ \text{fm}^4$	$\alpha = 0.20\ \text{fm}^4$
$\lambda = \infty$	$\lambda = 3.2\ \text{fm}^{-1}$	$\lambda = 2.2\ \text{fm}^{-1}$	$\lambda = 1.5\ \text{fm}^{-1}$



(ST)=(11) pairs: 0.41	0.29	0.18	0.07

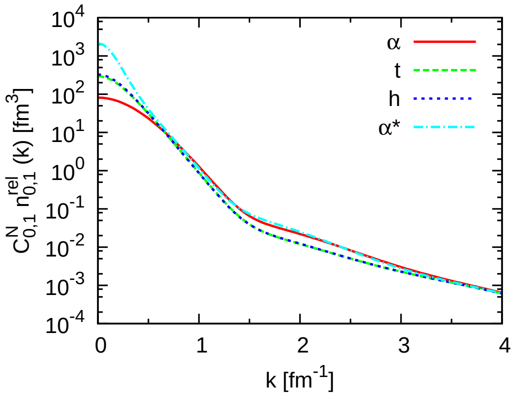
- two-body momentum density $n^{\text{rel}}_{1,0}(k) = \langle\, \Phi(\alpha) \,|\, \hat{n}^{\text{rel}}_{10}(k) \,|\, \Phi(\alpha)\,\rangle$ (with uncorrelated operators)
  $|\,\Phi(\alpha)\,\rangle$ NCSM eigenstate of softened $\hat{H}(\alpha)$
- with increasing α or range of correlator $\hat{C}$ Hamiltonian gets softer and correlations are reduced
- correlations dominated by tensor, low-momentum components remain unchanged

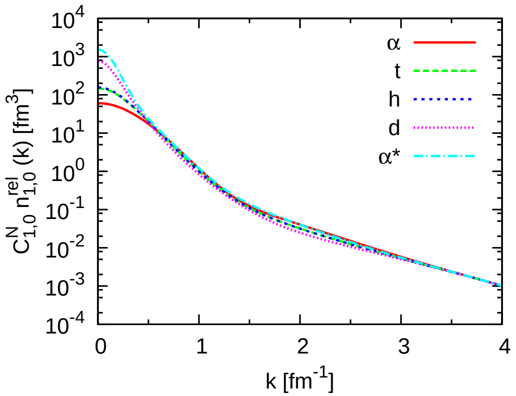
T. Neff, unpublished

Conclusions and Outlook

- Correlations induced by AV8' perfectly universal $r < 1$ fm and $k > 3$ fm^{-1}
- 2-body tensor induces 3-body correlations
- UCOM(SRG) demonstrates softening of potential and loss of short-range correlations
- Proposal:

$0 < k \lesssim 1$ fm^{-1}	long-range correlations
	vibrations, deformation, mean-field ...
$1 \lesssim k \lesssim 3$ fm^{-1}	mid-range correlations
	tensor, induced 3-body, ...
3 fm$^{-1} \lesssim k$	short-range
	not accessible with nuclear d.o.f.





Outlook

- Abandon 2-body approximation, include 3-body everywhere
- Develop new SRG-generator in 3-body space such that contributions from induced 4- and higher body operators remain very small (→ R. Roth, A. Calci)