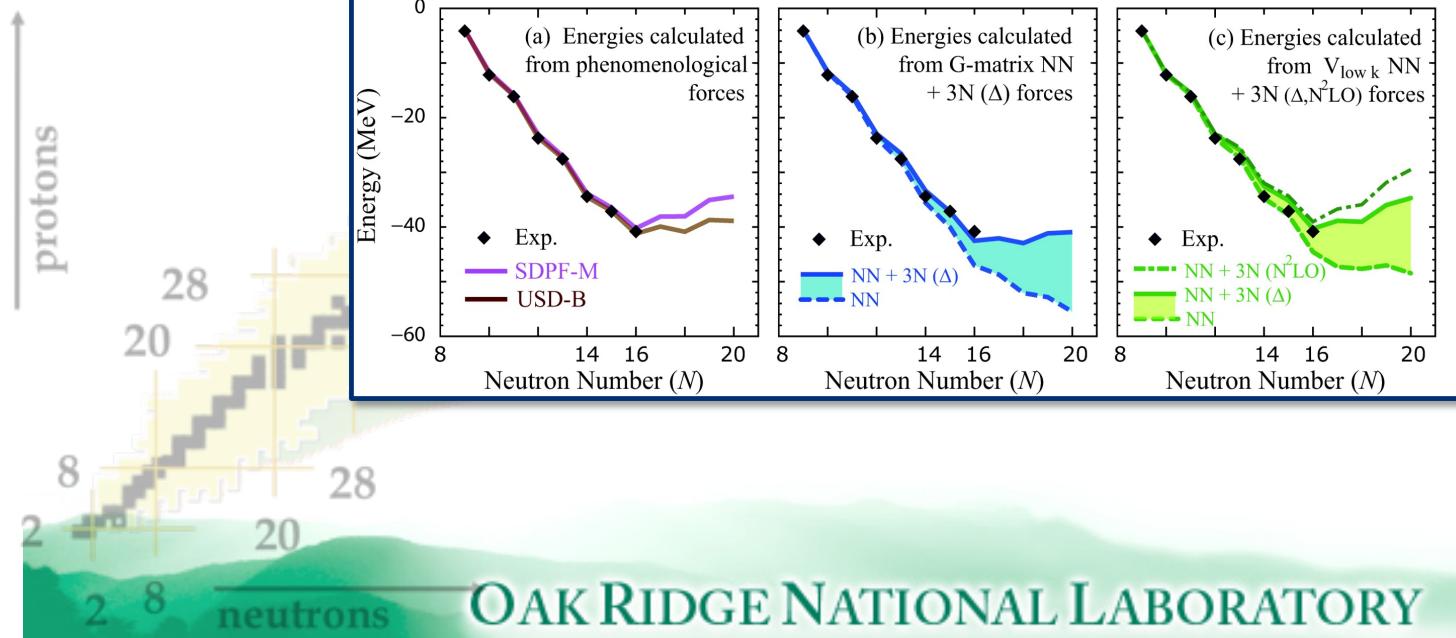


Three-Nucleon Forces and Masses of Neutron-Rich Nuclei

Jason D. Holt

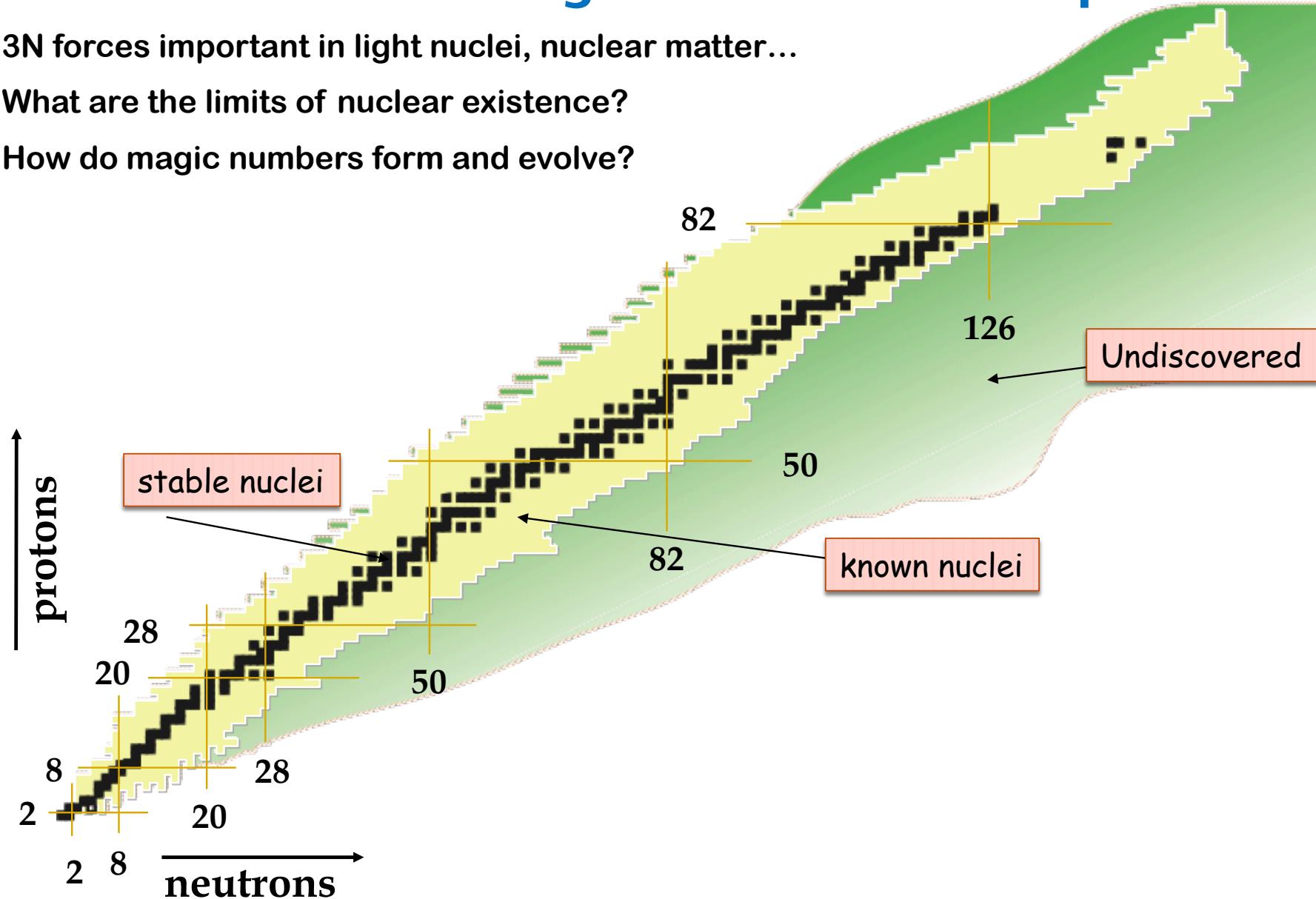


Drip Lines and Magic Numbers: The Evolving Nuclear Landscape

3N forces important in light nuclei, nuclear matter...

What are the limits of nuclear existence?

How do magic numbers form and evolve?



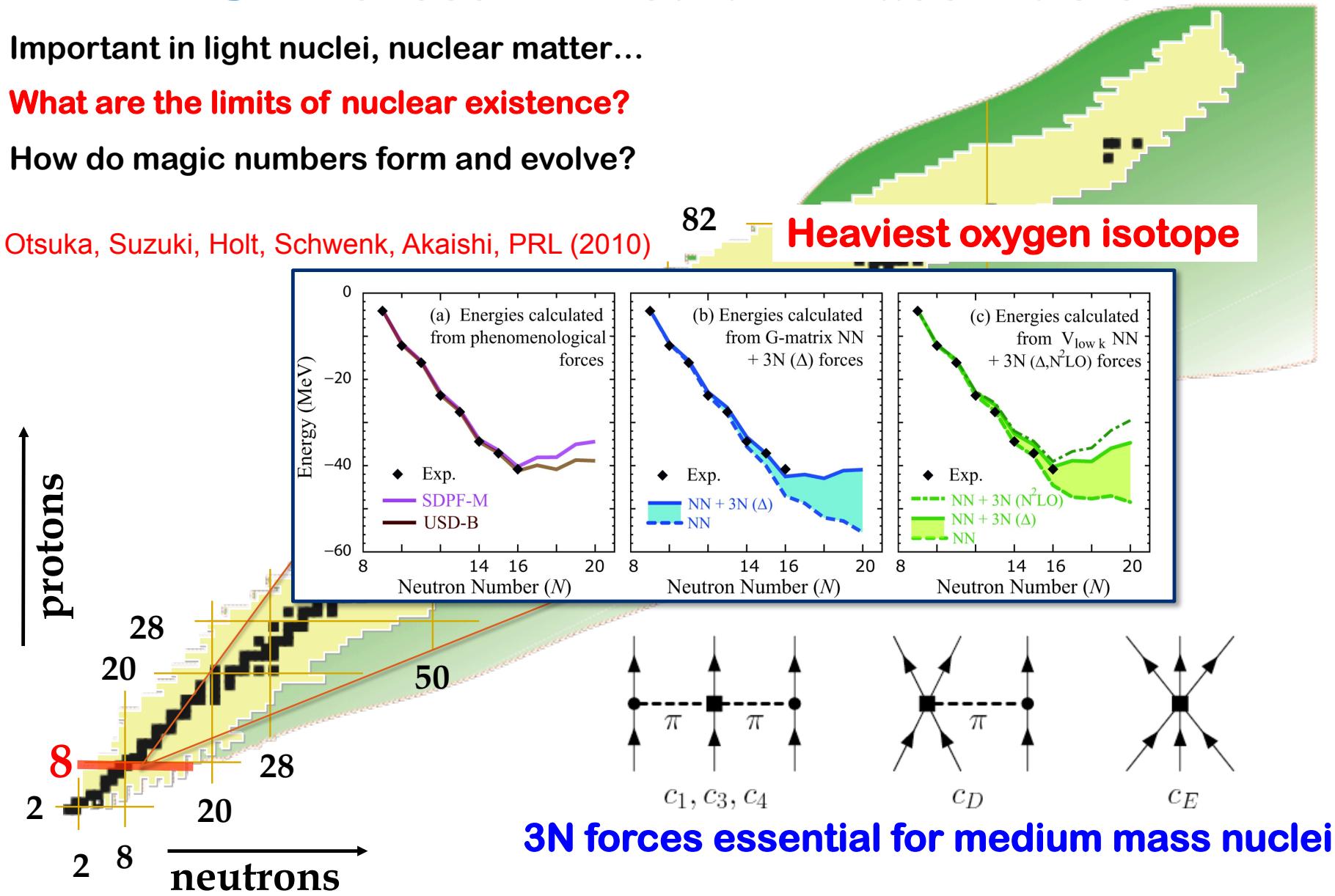
Drip Lines and Magic Numbers: 3N Forces in Medium-Mass Nuclei

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Otsuka, Suzuki, Holt, Schwenk, Akaishi, PRL (2010)



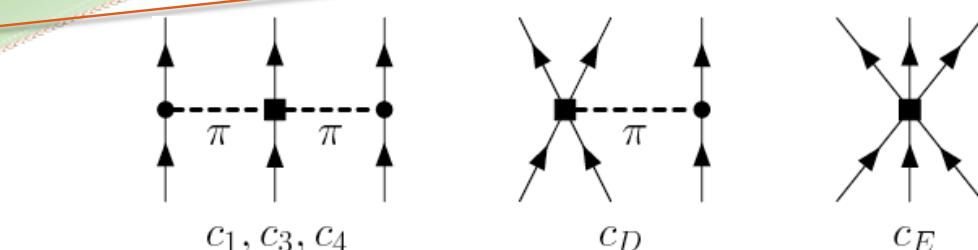
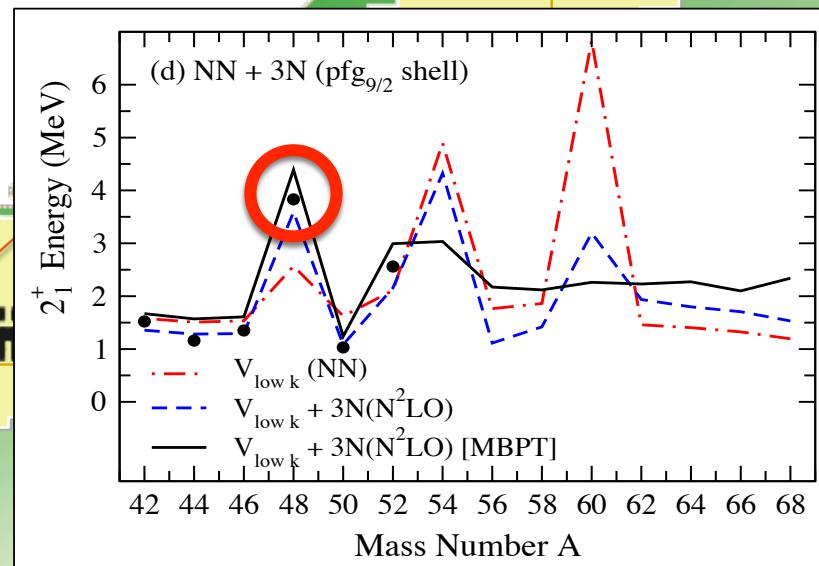
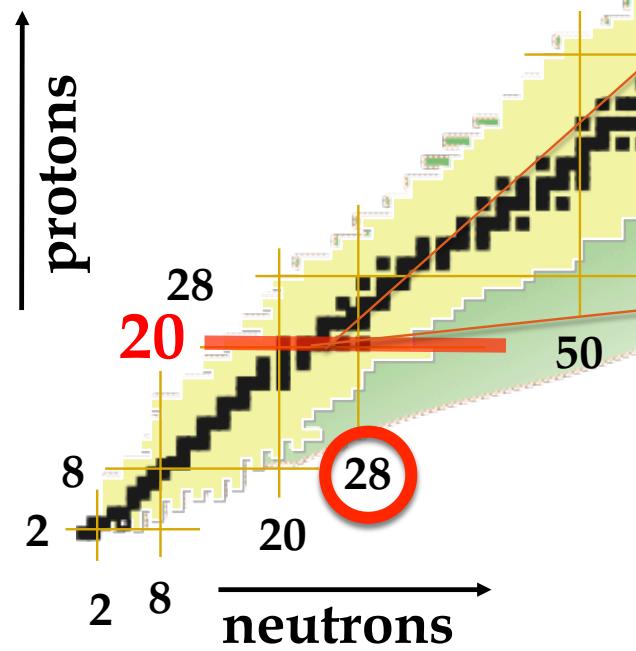
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Holt, Otsuka, Schwek,
Suzuki, arXiv:1009.5984



3N forces essential for medium mass nuclei



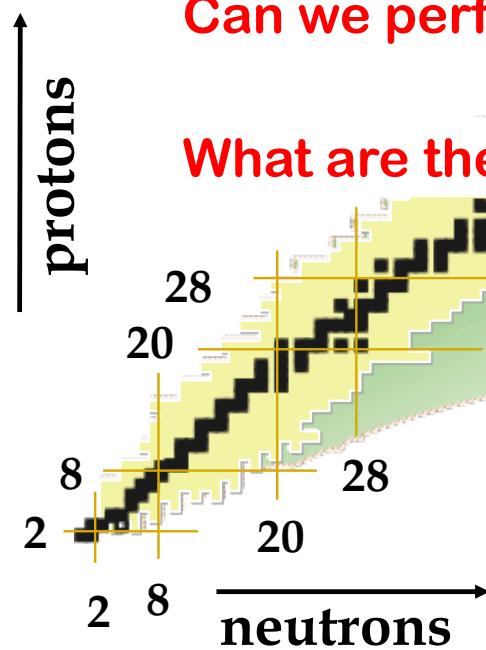
N=28 magic number in calcium

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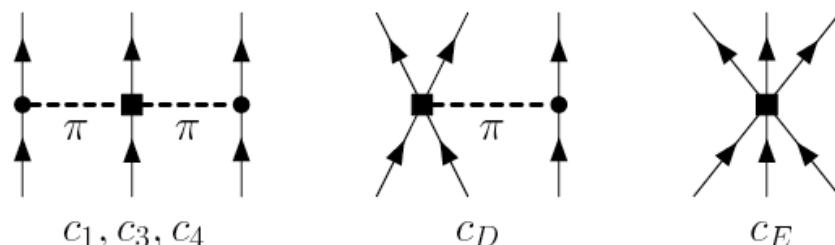
How do magic numbers form and evolve?



Advances in microscopic theory:
Calculate all theoretical inputs consistently with NN+3N

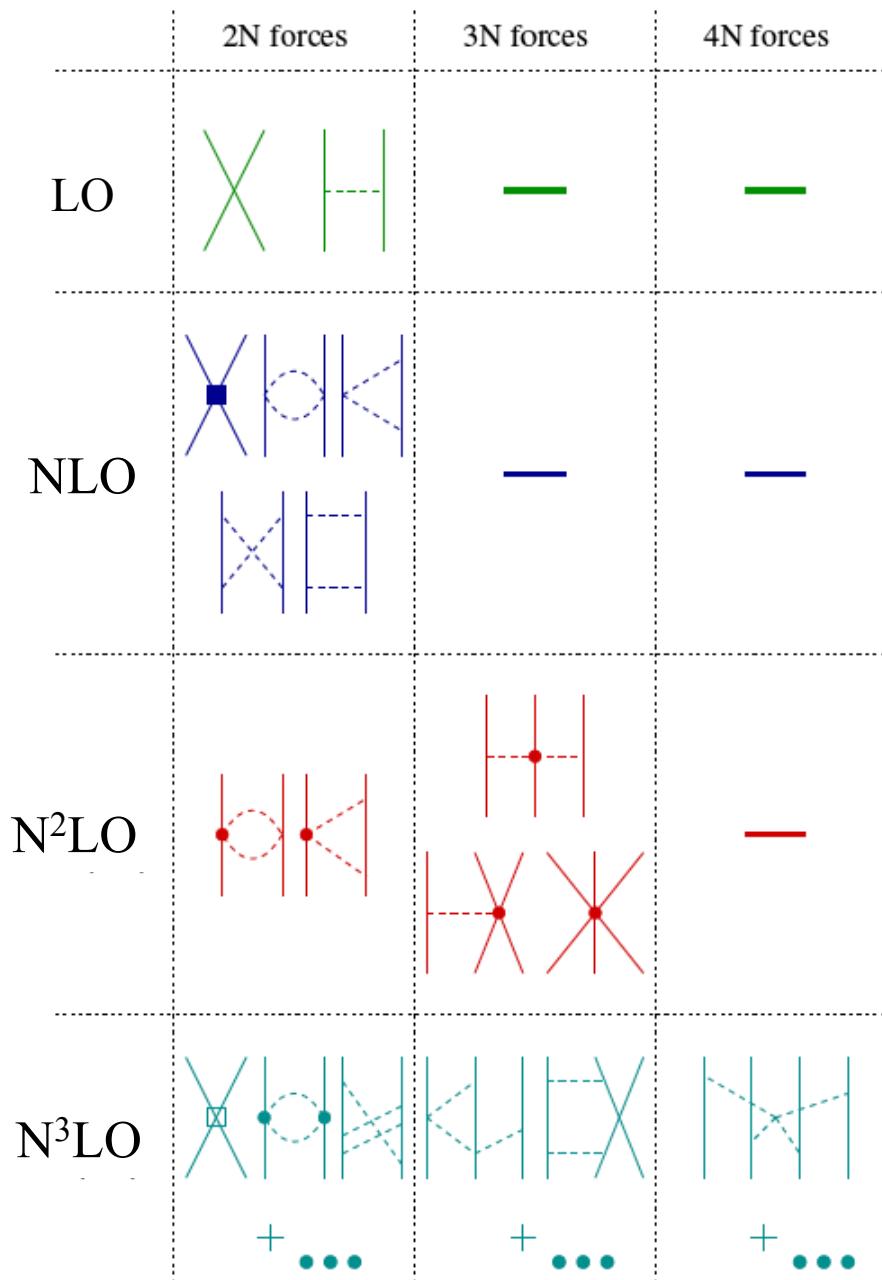
Can we perform adequate spectroscopic calculations?

What are the effects of 3N to higher-order in MBPT?



3N forces essential for medium mass nuclei

Chiral Effective Field Theory: Nuclear Forces



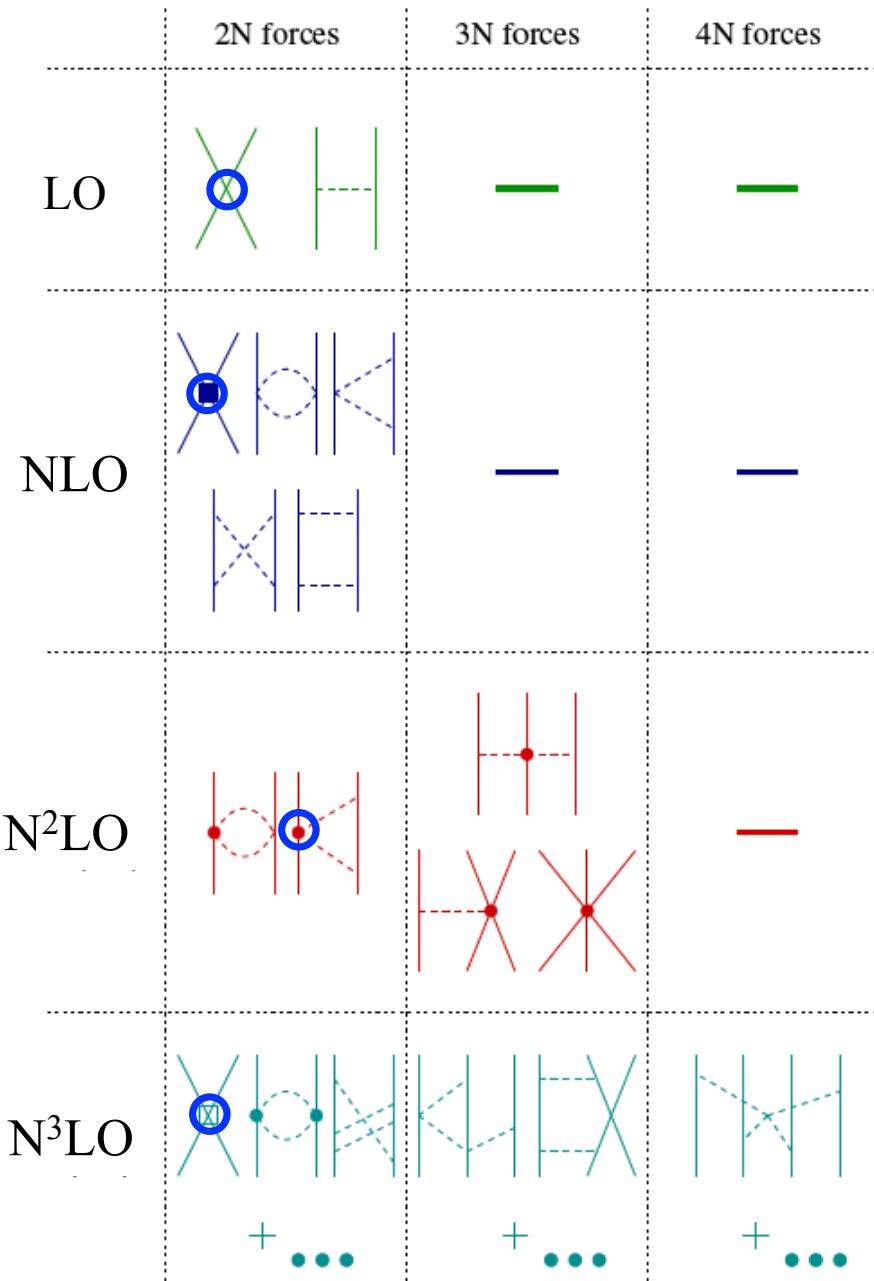
Nucleons interact via pion exchanges and contact interactions

Hierarchy: $V_{NN} > V_{3N} > \dots$

Consistent treatment of NN, 3N, ... electroweak operators

Weinberg, van Kolck, Kaplan, Savage, Wise,
Epelbaum, Kaiser, Meissner,...

Chiral Effective Field Theory: Nuclear Forces



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Consistent treatment of NN, 3N, ... electroweak operators

Couplings fit to experiment once

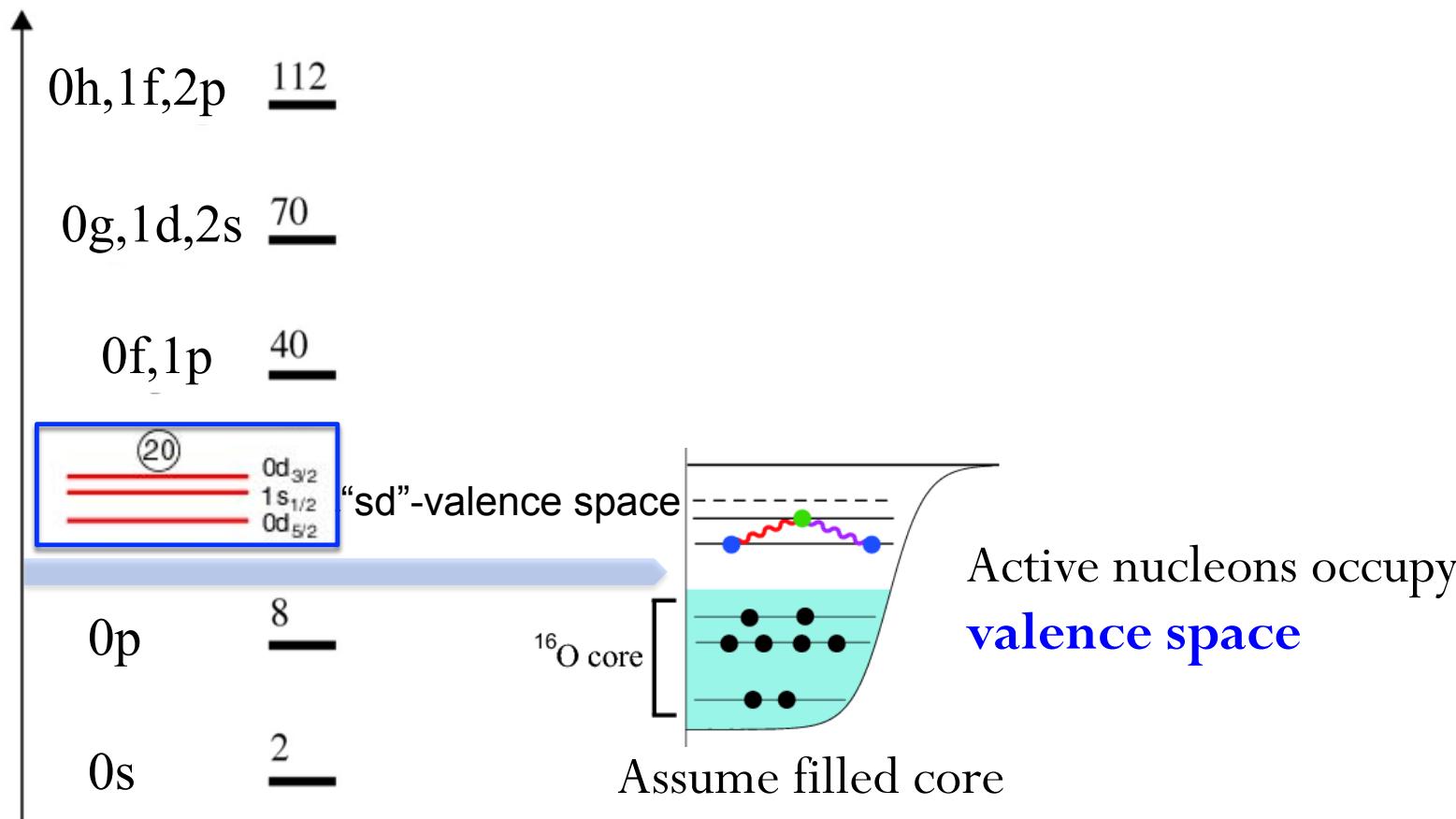
Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Meissner,...

Solving the Nuclear Many-Body Problem

Nuclei understood as many-body system starting from closed shell, add nucleons

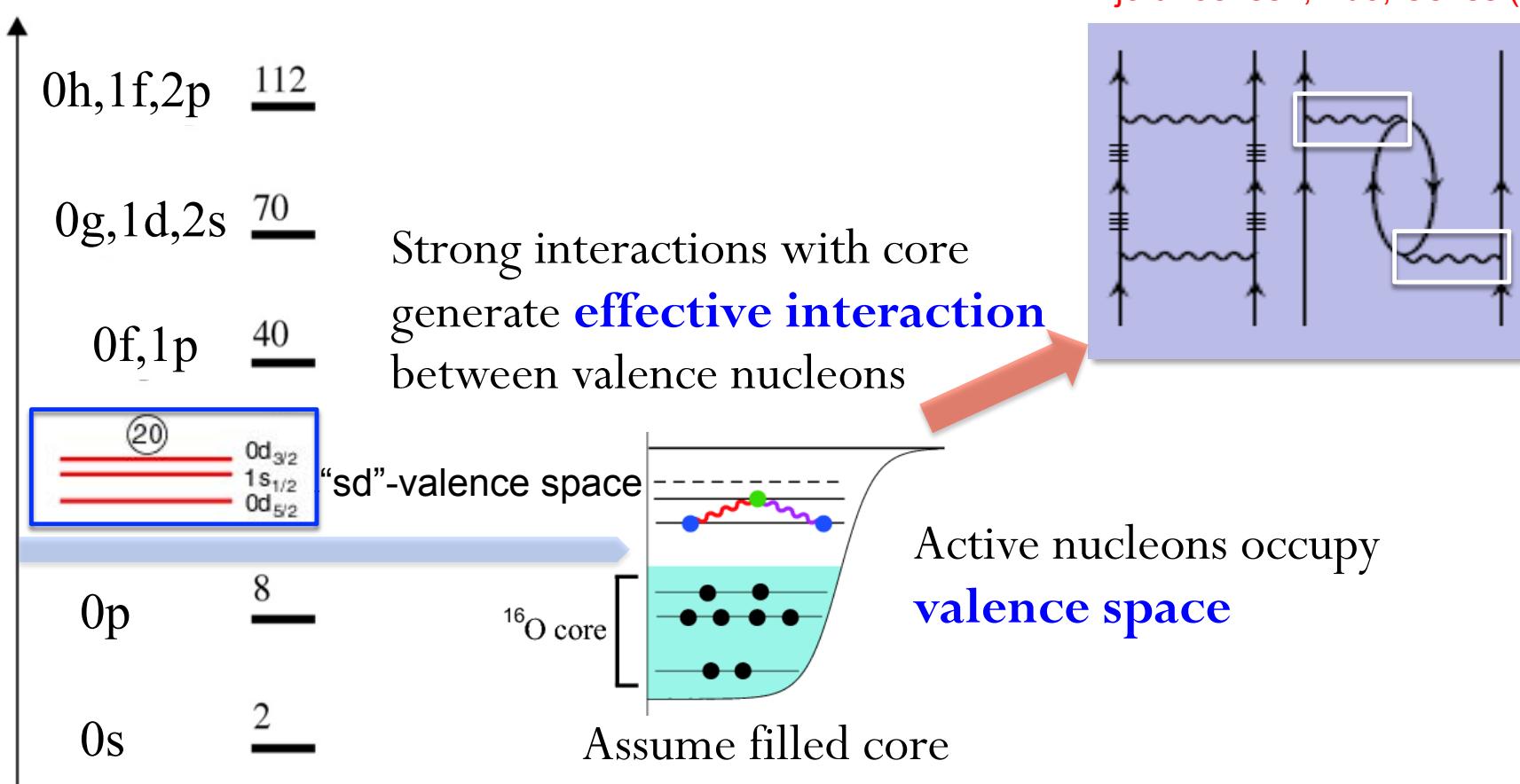
Interaction and energies of valence space orbitals from original $V_{\text{low } k}$

This alone does not reproduce experimental data



Solving the Nuclear Many-Body Problem

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This alone does not reproduce experimental data – allow explicit breaking of core



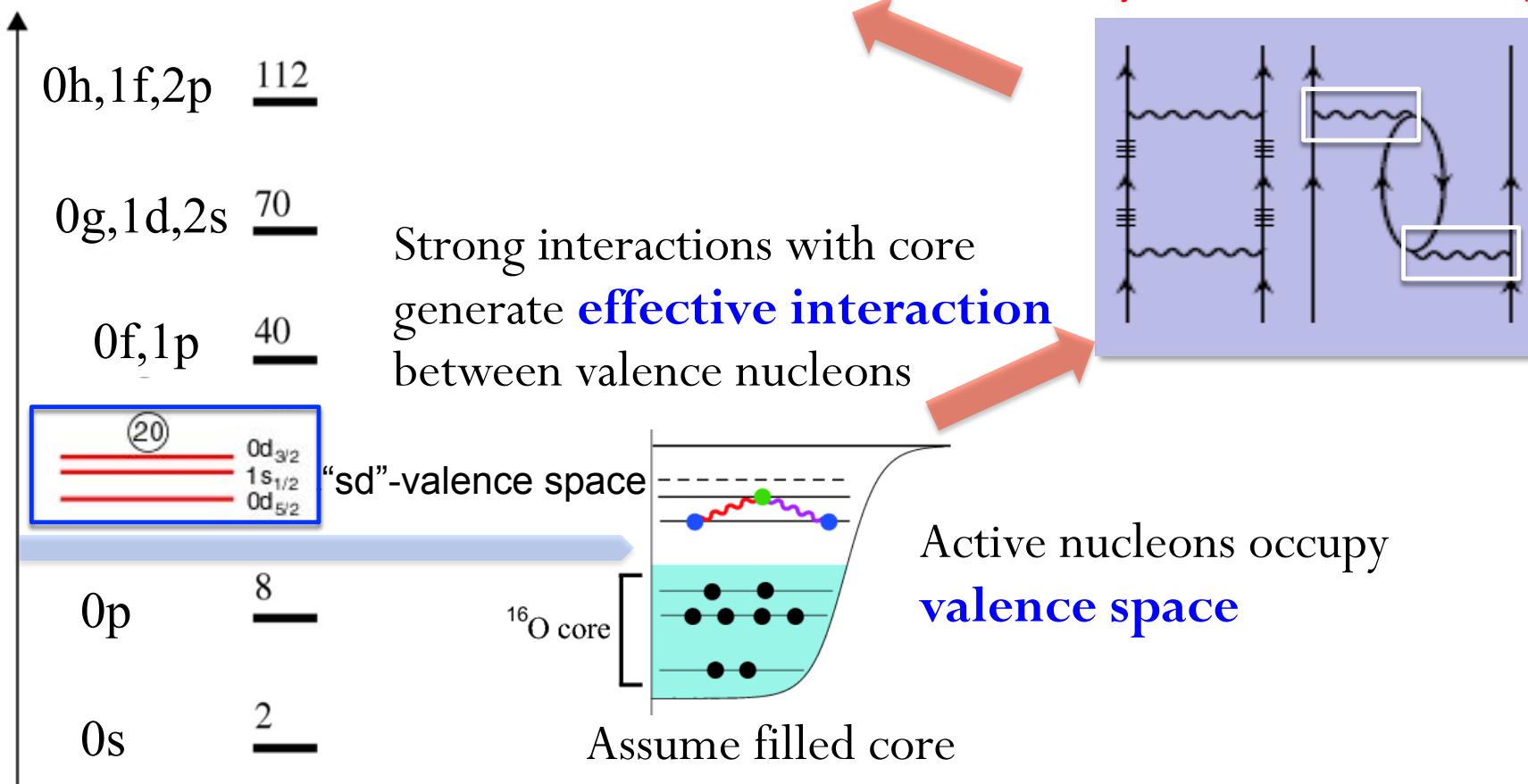
Solving the Nuclear Many-Body Problem

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Effective two-body matrix elements Single-particle energies (SPEs)

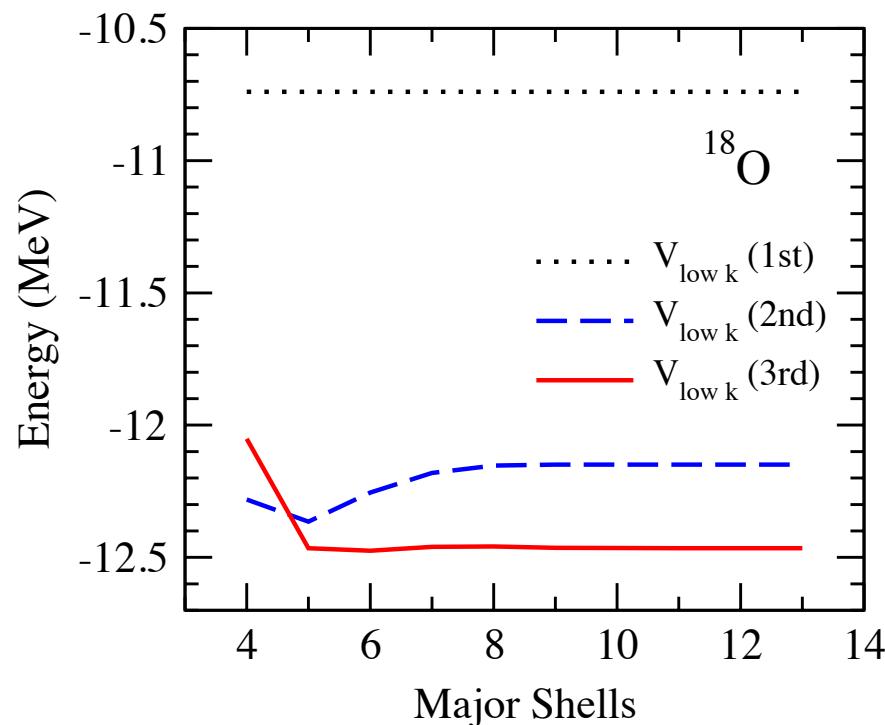


Calculation Details

Convergence in terms of Harmonic Oscillator basis size

NN matrix elements derived from:

- Chiral N³LO (Machleidt, 500 MeV) using smooth-regulator $V_{\text{low } k}$
- 3rd-order in perturbation theory
- 13 major HO shells for intermediate state configurations (converged)

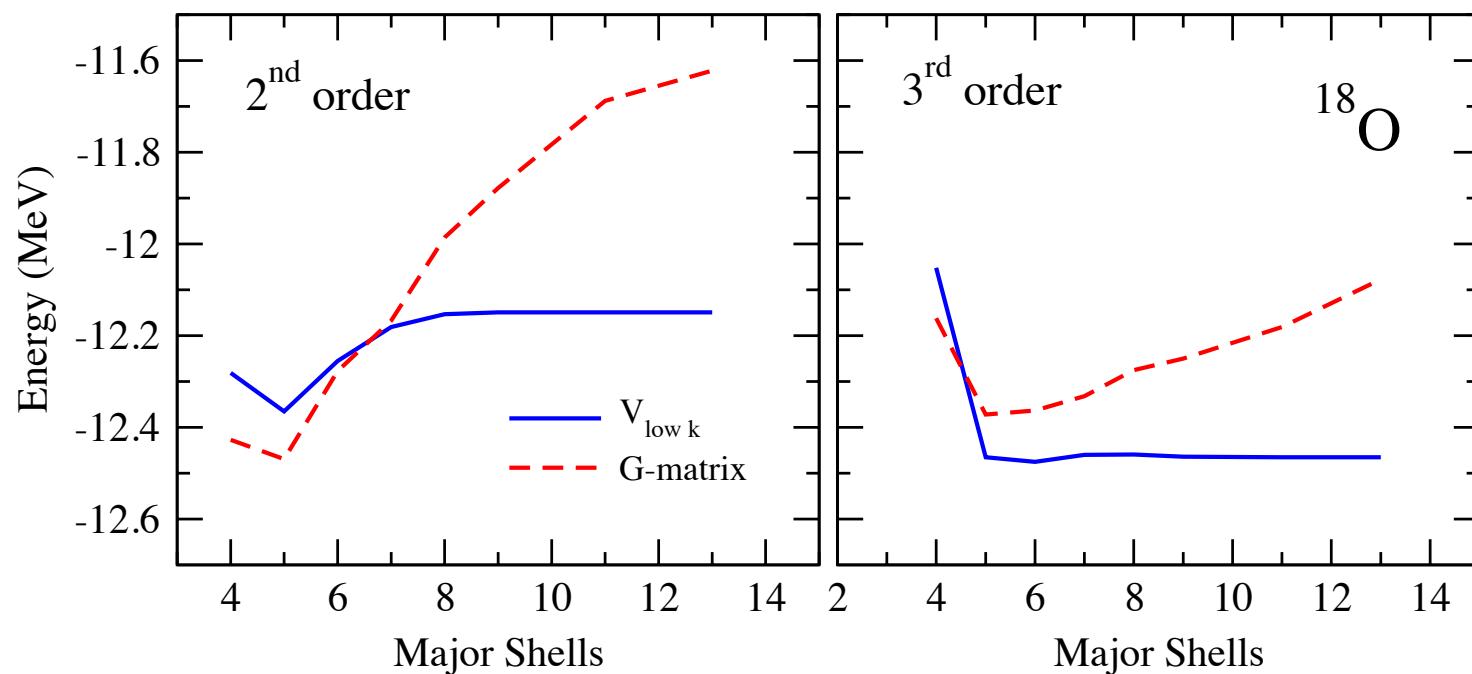


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Monopole	5 ms	7 ms	9 ms	11 ms	13 ms
p _{1/2} p _{1/2}	-0.45922	-0.50144	-0.41379	-0.40983	-0.40956
p _{3/2} p _{1/2}	-0.69051	-0.96226	-0.91887	-0.90077	-0.90012
p _{3/2} p _{3/2}	-0.80796	-1.15054	-1.10253	-1.08504	-1.08432
f _{5/2} p _{1/2}	-0.00596	0.14333	0.15494	0.15505	0.15505
f _{5/2} p _{3/2}	-0.14579	0.05654	0.06977	0.06879	0.06875
f _{5/2} f _{5/2}	-0.06151	-0.05002	-0.03476	-0.03412	-0.03412
f _{7/2} p _{1/2}	-0.33051	-0.20719	-0.18425	-0.18095	-0.18076
f _{7/2} p _{3/2}	-0.24654	-0.12230	-0.09767	-0.09494	-0.09479
f _{7/2} f _{5/2}	-0.44166	-0.34340	-0.33354	-0.33360	-0.33368
f _{7/2} f _{7/2}	-0.26185	-0.19436	-0.18413	-0.18397	-0.18399

Limits of Nuclear Existence: Oxygen Anomaly

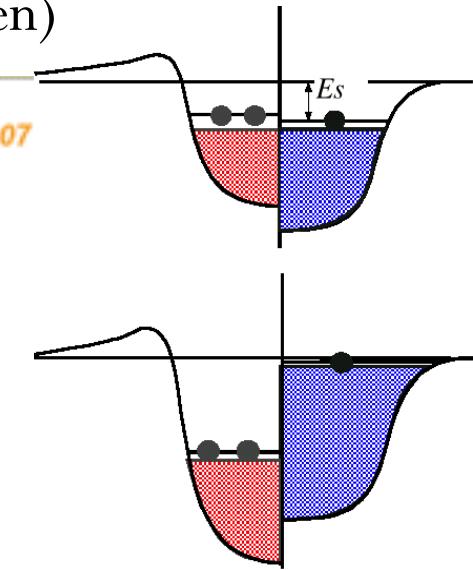
Where is the nuclear dripline?

Limits defined as last isotope with positive neutron separation energy

- Nucleons “drip” out of nucleus

Neutron dripline experimentally established to Z=8 (Oxygen)

^{28}Si	^{29}Si	^{30}Si	^{31}Si	^{32}Si	^{33}Si	^{34}Si	^{35}Si	^{36}Si	^{37}Si	^{38}Si	^{39}Si	^{40}Si	^{41}Si	^{42}Si	^{43}Si	^{44}Si
^{27}Al	^{28}Al	^{29}Al	^{30}Al	^{31}Al	^{32}Al	^{33}Al	^{34}Al	^{35}Al	^{36}Al	^{37}Al	^{38}Al	^{39}Al	^{40}Al	^{41}Al	^{42}Al	^{43}Al
^{26}Mg	^{27}Mg	^{28}Mg	^{29}Mg	^{30}Mg	^{31}Mg	^{32}Mg	^{33}Mg	^{34}Mg	^{35}Mg	^{36}Mg	^{37}Mg	^{38}Mg	^{40}Mg			
^{57}Na	^{26}Na	^{27}Na	^{28}Na	^{29}Na	^{30}Na	^{31}Na	^{32}Na	^{33}Na	^{34}Na	^{35}Na		^{37}Na				
^{24}Ne	^{25}Ne	^{26}Ne	^{27}Ne	^{28}Ne	^{29}Ne	^{30}Ne	^{31}Ne	^{32}Ne			^{34}Ne					
^{23}F	^{24}F	^{25}F	^{26}F	^{27}F			^{29}F		^{31}F							
^{22}O	^{23}O	^{24}O														
^{21}N	^{22}N	^{23}N														
^{20}C			^{22}C													



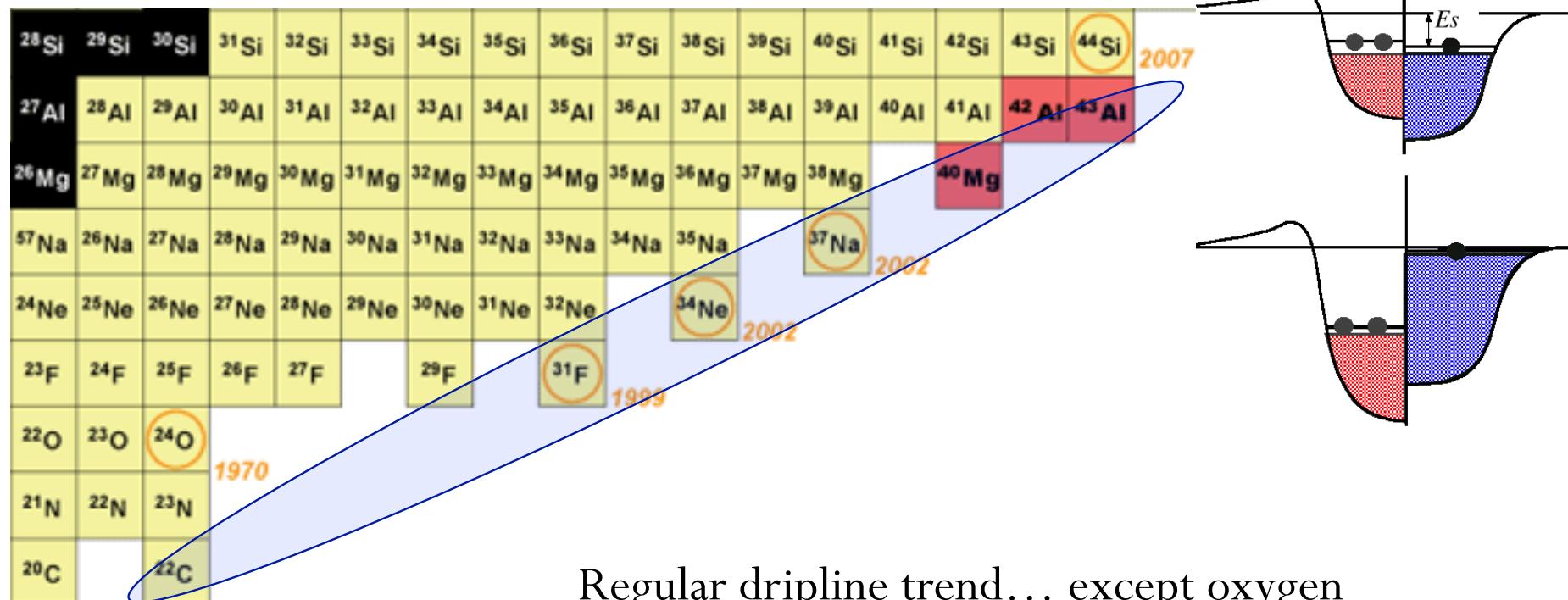
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Regular dripline trend... except oxygen
Adding one proton binds 6 additional neutrons

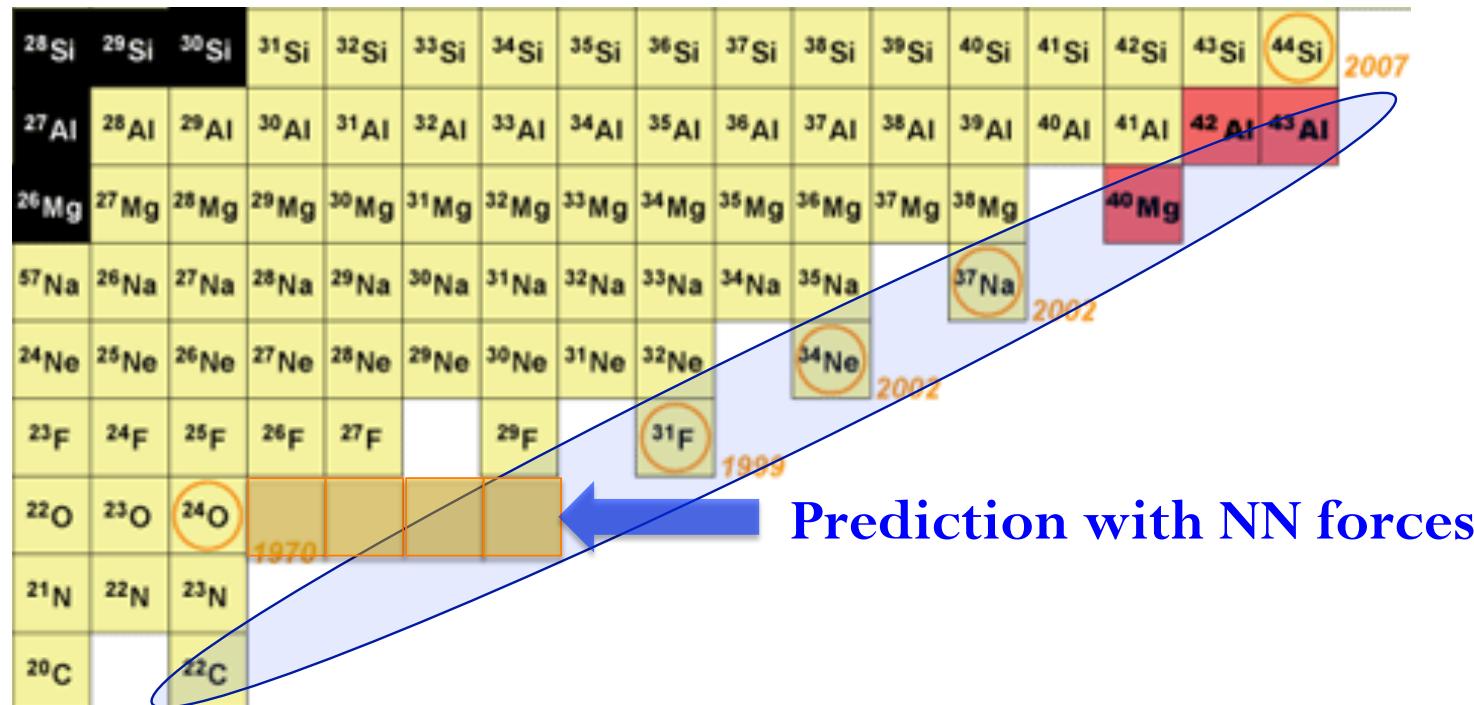
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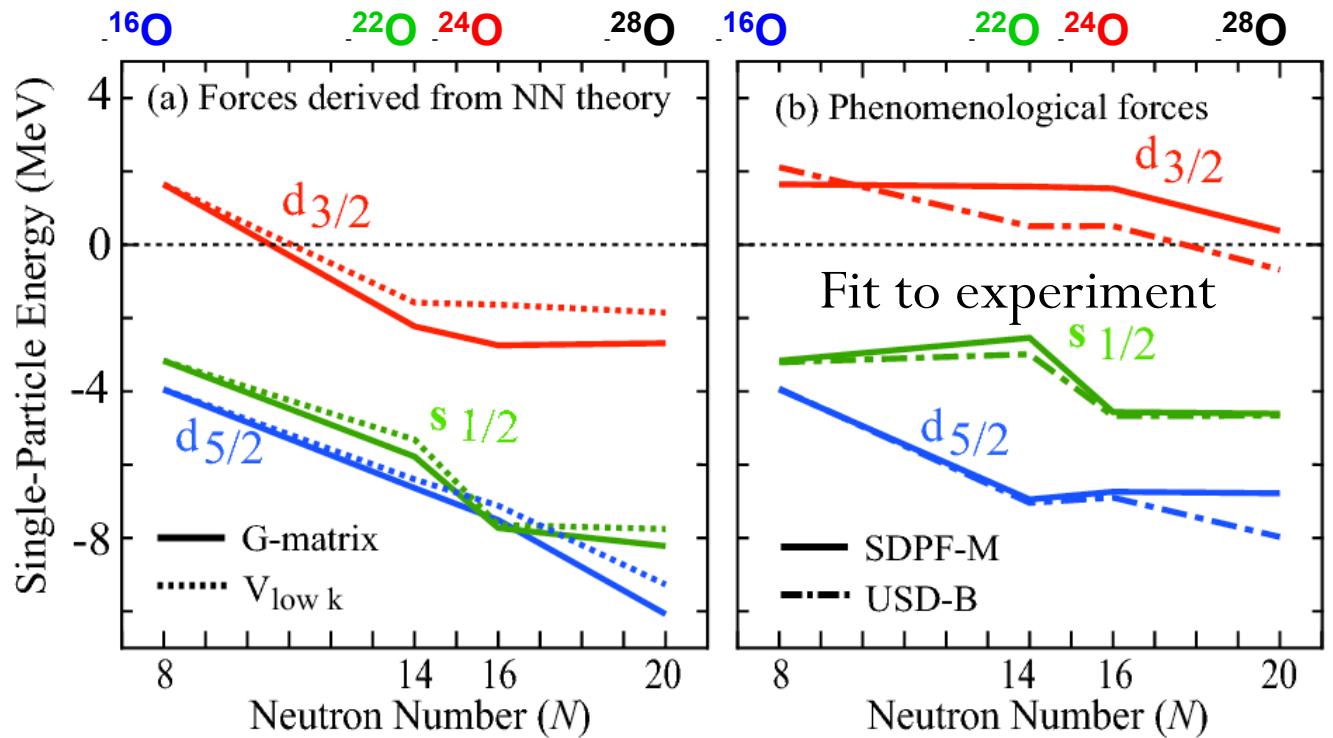
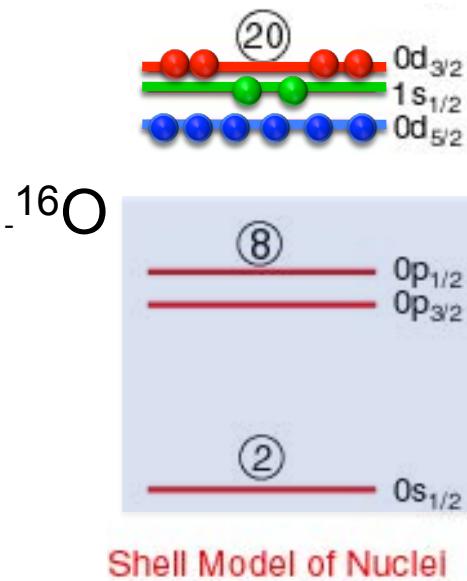


Microscopic picture: **NN-forces too attractive**

Incorrect prediction of dripline

Physics in Oxygen Isotopes

Calculate evolution of *sd*-orbital energies from interactions



Microscopic NN Theories

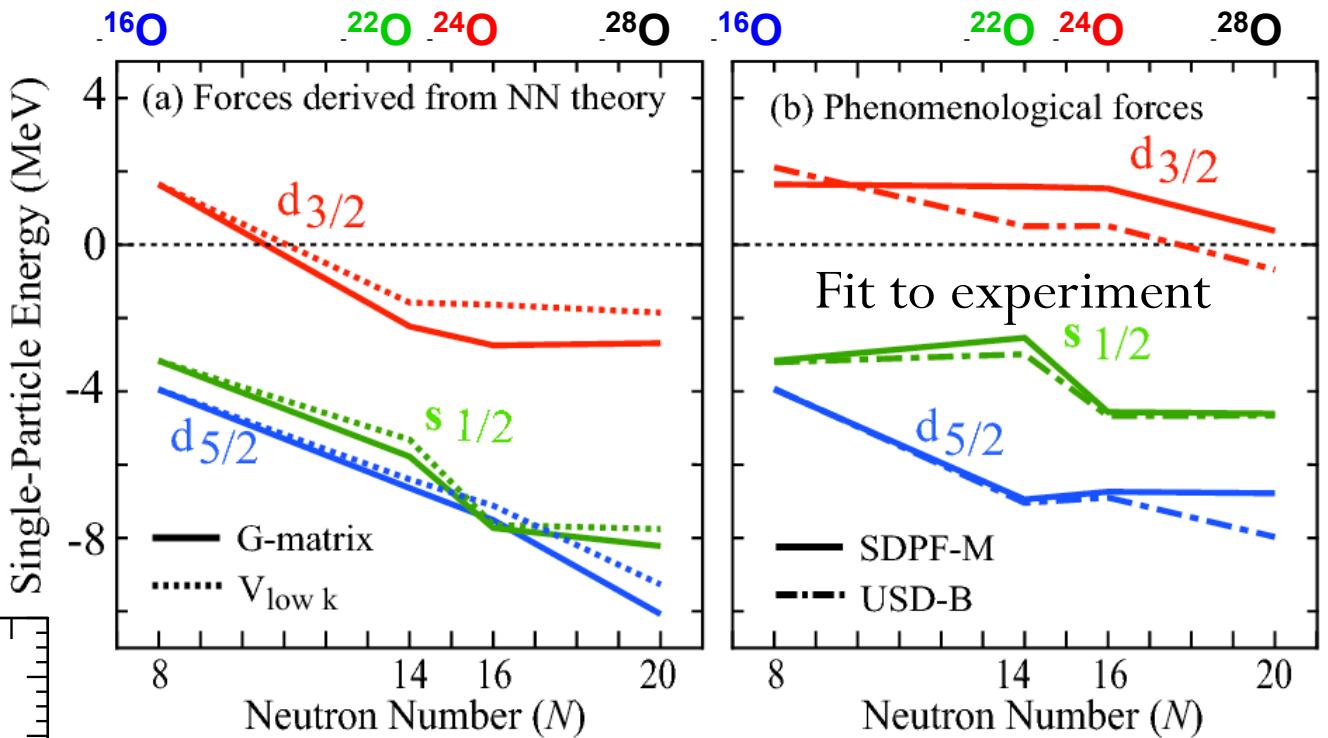
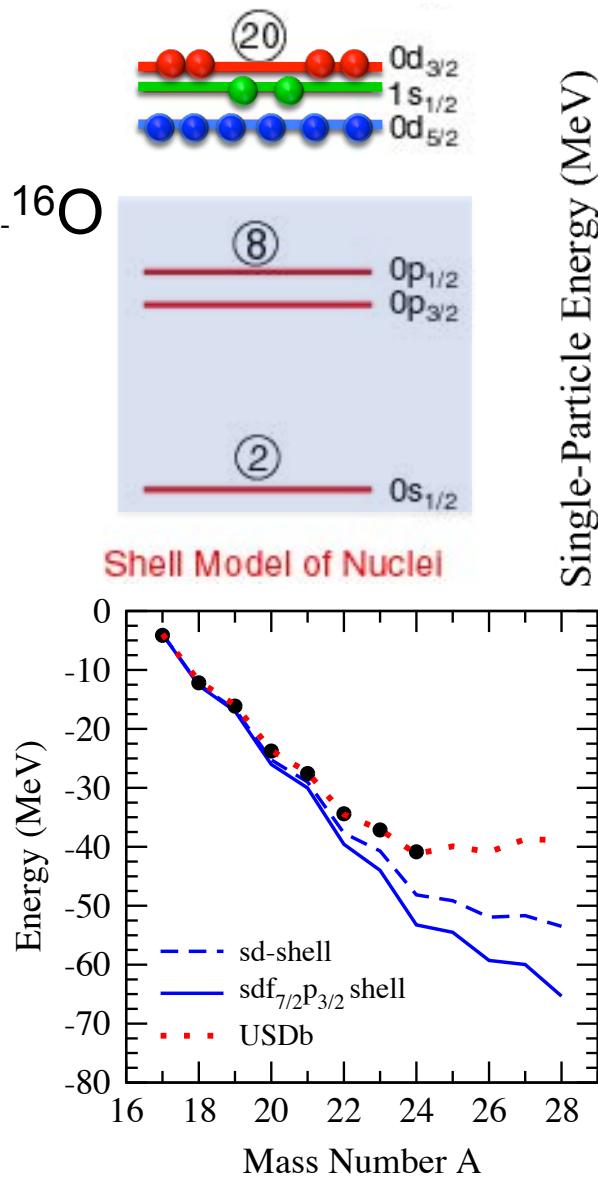
$d_{3/2}$ orbit bound to ^{28}O

Phenomenological Models

$d_{3/2}$ orbit unbound

Physics in Oxygen Isotopes

Calculate evolution of *sd*-orbital energies from interactions



Microscopic NN Theories

$d_{3/2}$ orbit bound to ^{28}O
Dripline at ^{28}O

Phenomenological Models

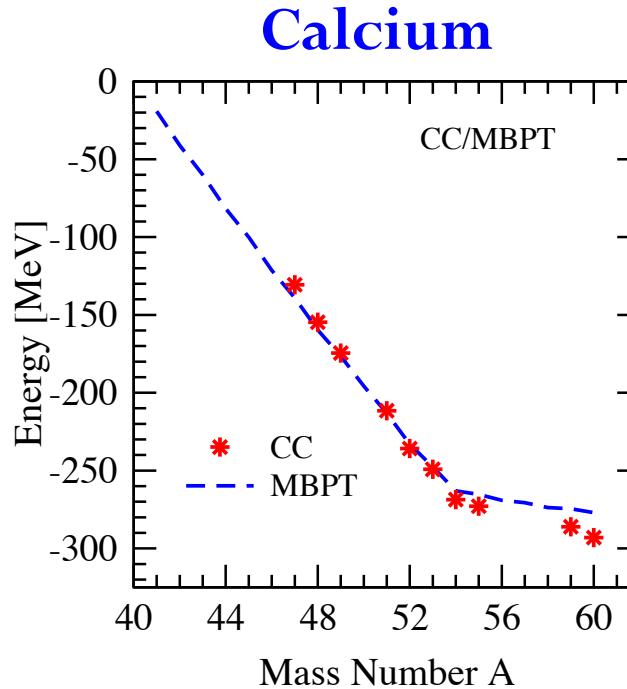
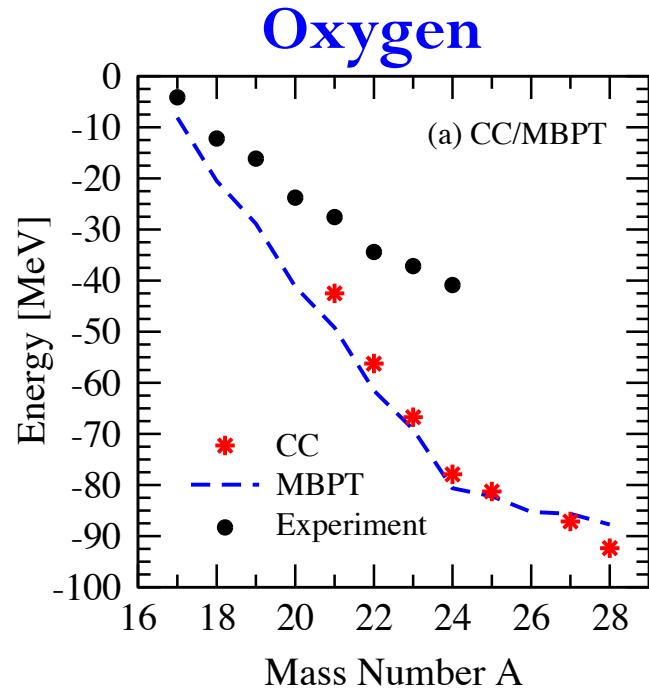
$d_{3/2}$ orbit unbound
Dripline at ^{24}O

Oxygen anomaly unexplained with NN forces

Comparison to Coupled Cluster

Many-body method insufficient?

Benchmark against *ab-initio* Coupled Cluster at NN-only level

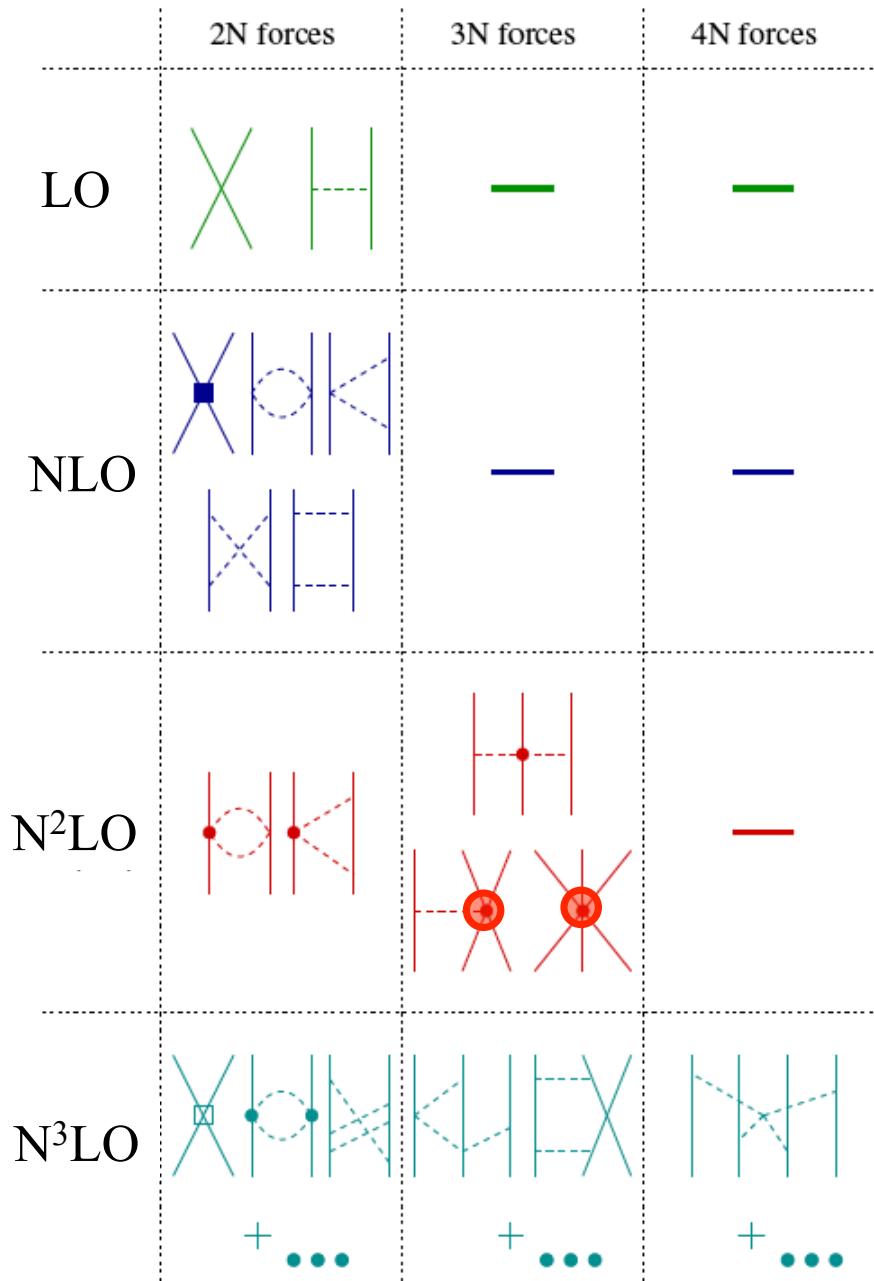


SPEs: one-particle attached CC energies in ^{17}O and ^{41}Ca

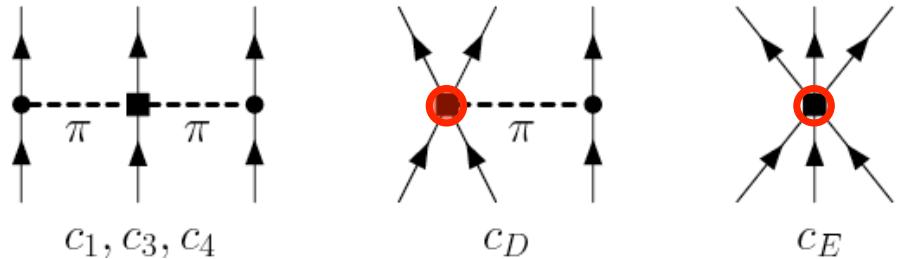
Small difference in many-body methods

Include **3N forces**

Chiral Effective Field Theory: 3N Forces



Only two new couplings at N^2LO



c terms given from NN fits:
constrained by NN, πN data

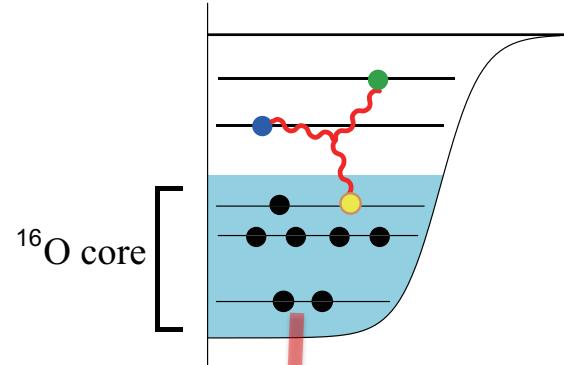
c_D c_E fit to properties of light nuclei:
Triton binding energy, ${}^4\text{He}$ radius

No new 3N, 4N couplings at N^3LO

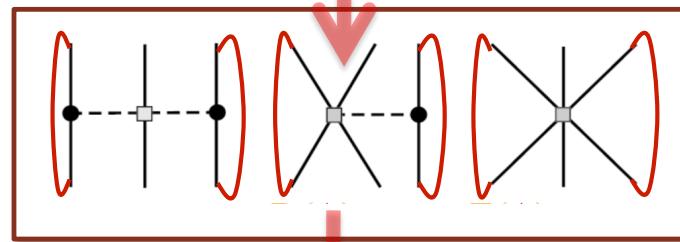
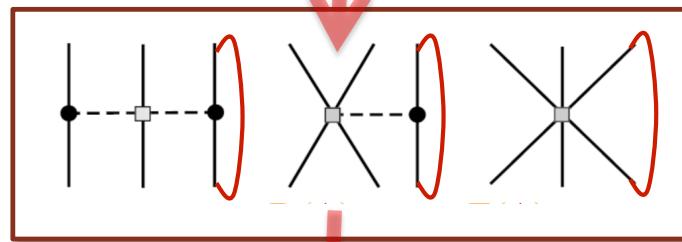
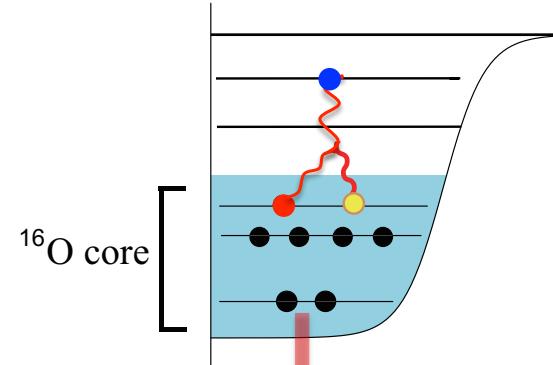
3N Forces for Valence-Shell Theories

Normal-ordered 3N: contribution to valence neutron interactions

Effective two-body



Effective one-body



$$\langle ab | V_{3N,\text{eff}} | a' b' \rangle = \sum_{\alpha=\text{core}} \langle \alpha ab | V_{3N} | \alpha a' b' \rangle$$

$$\langle a | V_{3N,\text{eff}} | a' \rangle = \frac{1}{2} \sum_{\alpha\beta=\text{core}} \langle \alpha\beta a | V_{3N} | \alpha\beta a' \rangle$$

Combine with microscopic NN: eliminate empirical adjustments

Calculation Details

Convergence in terms of Harmonic Oscillator basis size

NN matrix elements derived from:

- Chiral N³LO (Machleidt, 500 MeV) using smooth-regulator $V_{\text{low } k}$
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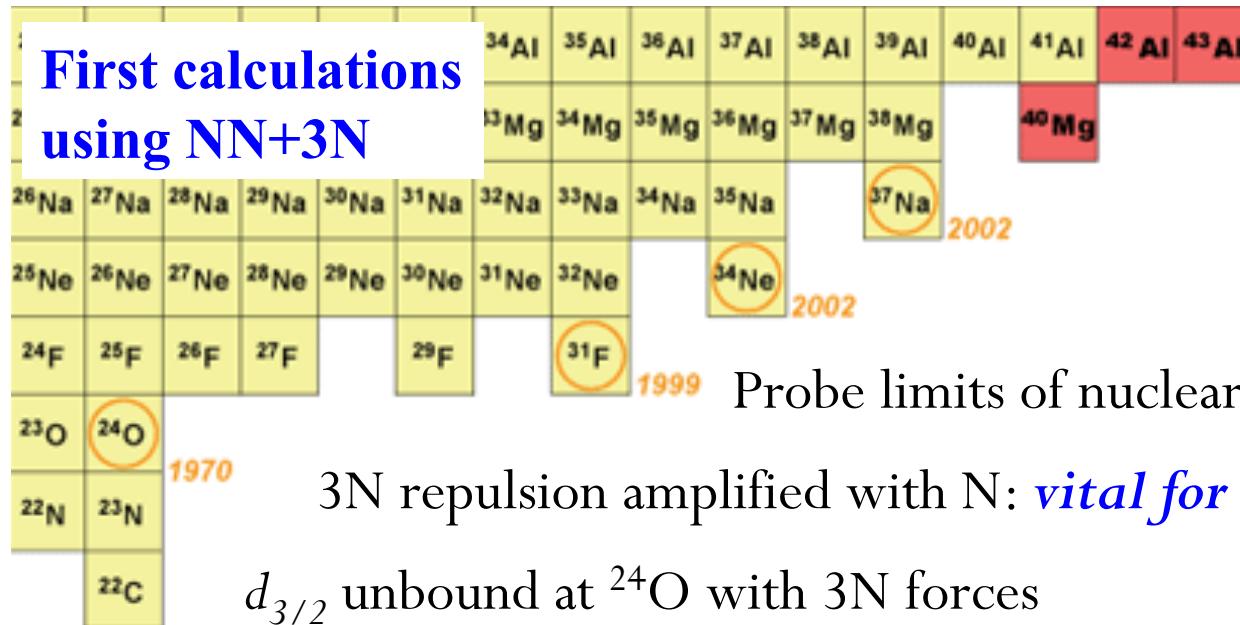
3N force contribution to TBMEs and SPEs

- Chiral N²LO: c_D , c_E fit to properties of light nuclei
with above NN $V_{\text{low } k}$ ($\Lambda = 2.0 \text{ fm}^{-1}$)
- **Normal-ordered two-body** part included to **Third Order** in MBPT
- Calculated within **5 major HO shells**

Are 3N forces fit in light nuclei appropriate for medium-mass systems?

Oxygen Anomaly

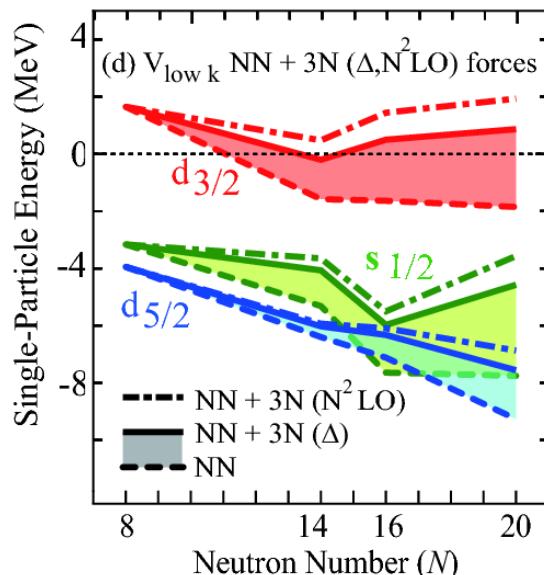
First calculations
using NN+3N



Probe limits of nuclear existence with 3N forces

3N repulsion amplified with N: *vital for neutron-rich nuclei*

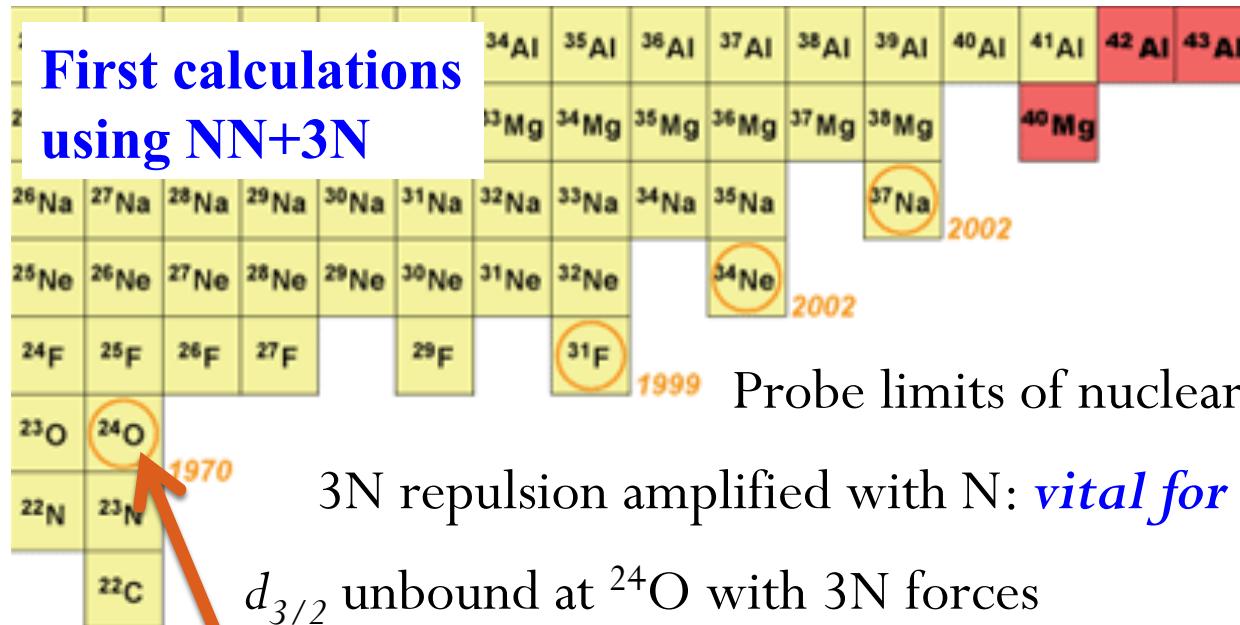
$d_{3/2}$ unbound at ^{24}O with 3N forces



Otsuka, Suzuki, Holt, Schwenk, Akaishi, PRL (2010)

Oxygen Anomaly

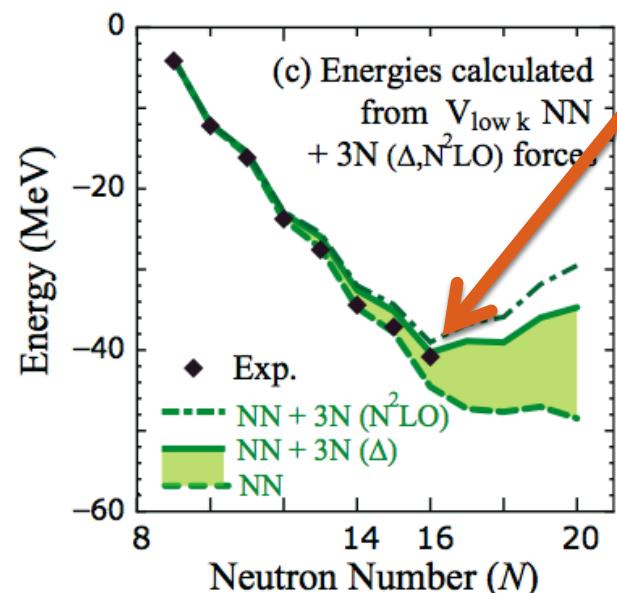
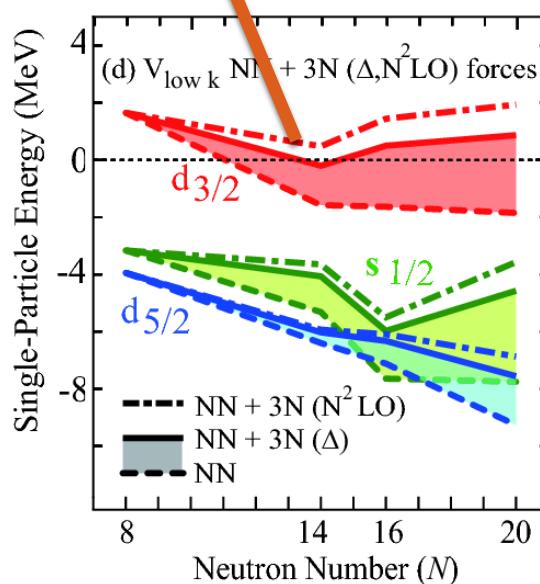
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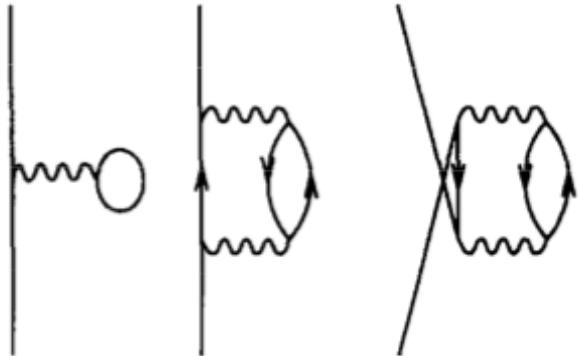


Isotopes unbound beyond ^{24}O

First microscopic explanation of oxygen anomaly

One-Body 3N: Single Particle Energies

NN-only microscopic SPEs yield poor results – rely on empirical adjustments

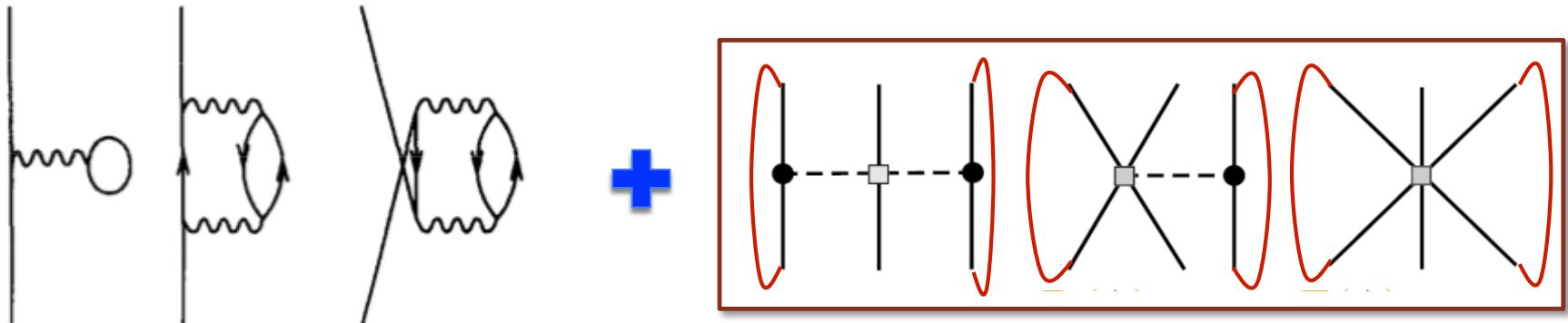


sd-shell: SPEs much too bound, unreasonable splitting

Orbit	“Exp”	USD b	$T + V_{NN}$
$d_{5/2}$	-4.14	-3.93	-5.43
$s_{1/2}$	-3.27	-3.21	-5.32
$d_{3/2}$	0.944	2.11	-0.97

One-Body 3N: Single Particle Energies

NN-only microscopic SPEs yield poor results – rely on empirical adjustments



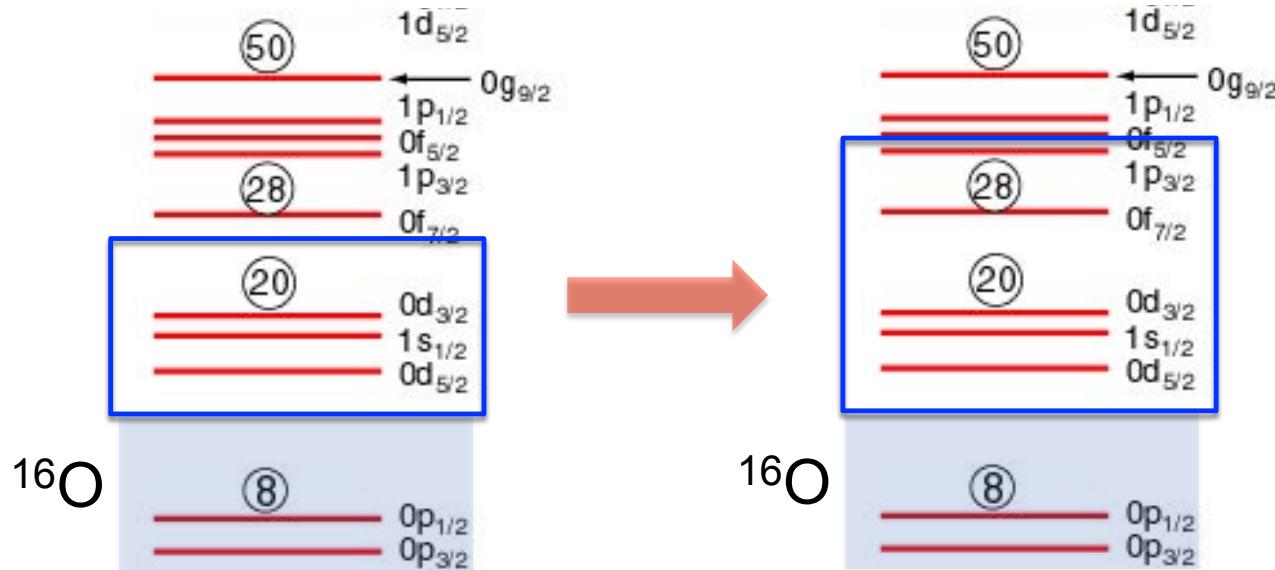
sd-shell: SPEs much too bound, unreasonable splitting

3N forces: additional repulsion – reasonable values!

Orbit	USD b	$T + V_{NN} + V_{3N}$
$d_{5/2}$	-3.93	-3.78
$s_{1/2}$	-3.21	-2.42
$d_{3/2}$	2.11	1.45

One-Body 3N: Single Particle Energies

Effects of correlations beyond one major oscillator shell:



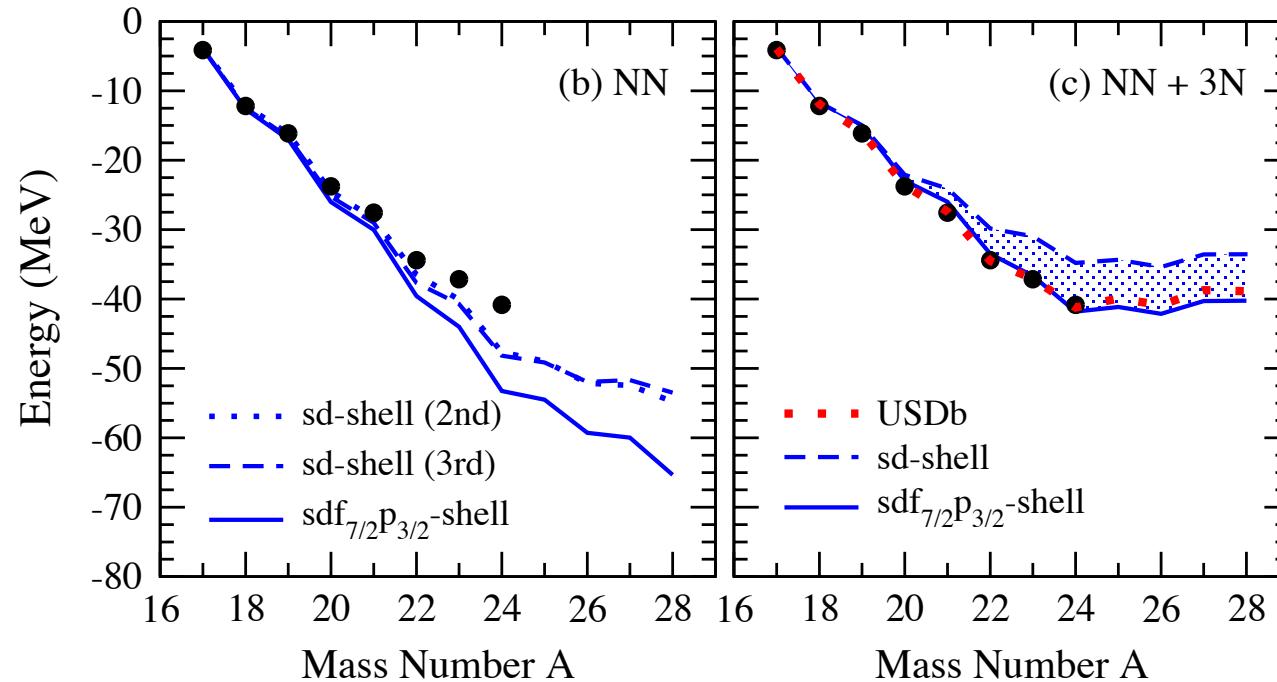
Orbit	USD b	$T + V_{NN} + V_{3N}$	SDPF-M	$T + V_{NN} + V_{3N}$
$d_{5/2}$	-3.93	-3.78	-3.95	-3.46
$s_{1/2}$	-3.21	-2.42	-3.16	-2.20
$d_{3/2}$	2.11	1.45	1.65	1.92
$f_{7/2}$			3.10	3.71
$p_{3/2}$			3.10	7.72

Fully microscopic framework and extended valence space

Binding Energies of Oxygen Isotopes

Interaction and self-consistent SPEs from NN+3N (**3rd-order**)

Empirical SPEs for NN-only



NN+3N: more modest trend past ^{24}O

Holt, Menendez, Schwenk, arXiv:1108.2680

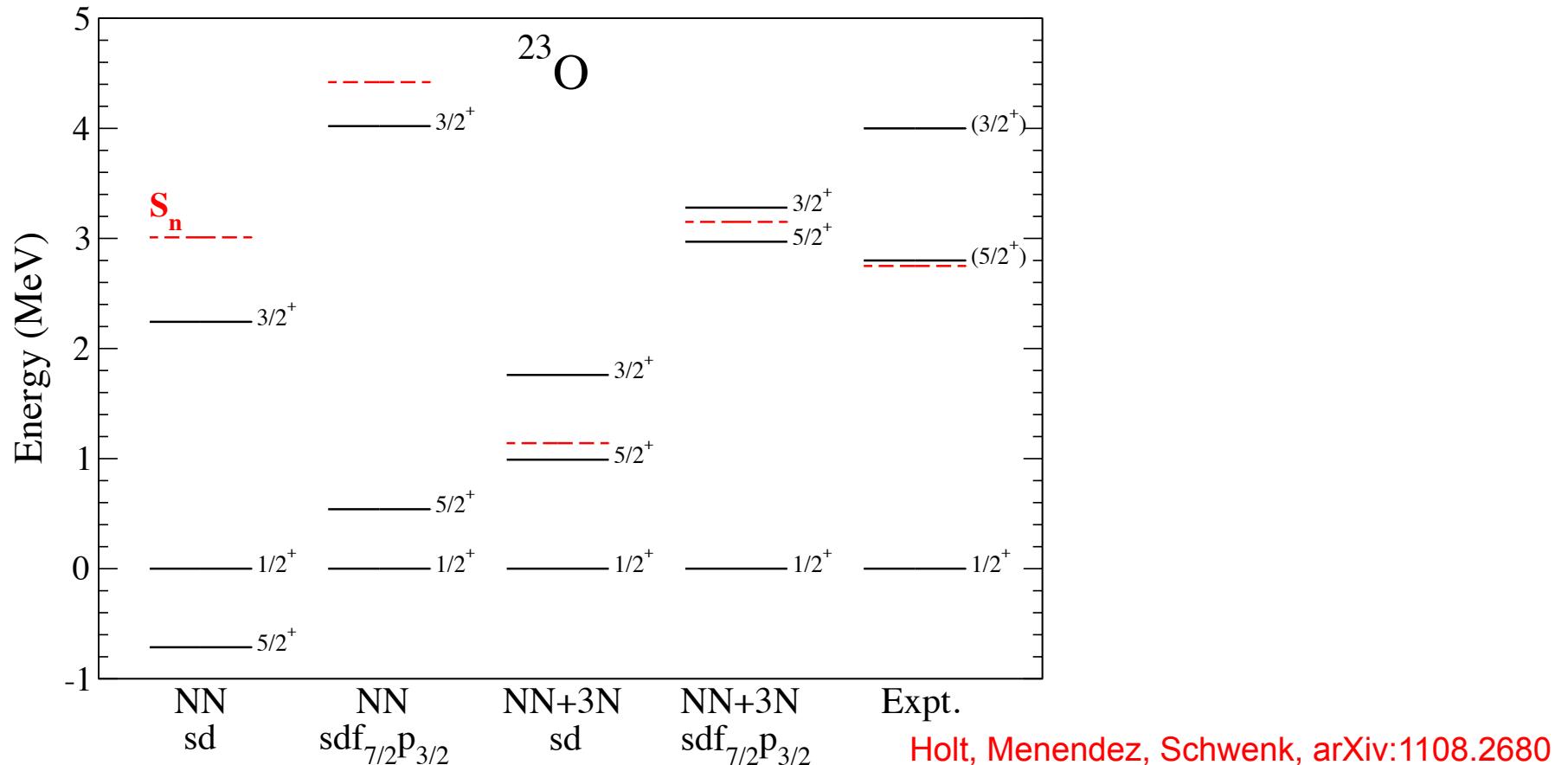
sd-shell results underbound; improved in $sdf_{7/2} p_{3/2}$

Continuum: ~300keV more binding beyond ^{24}O (from CC)

Impact on Spectra: ^{23}O

Neutron-rich oxygen spectra with NN+3N (3rd order)

$5/2^+$, $3/2^+$ indicate position of $d_{5/2}$ and $d_{3/2}$ orbits

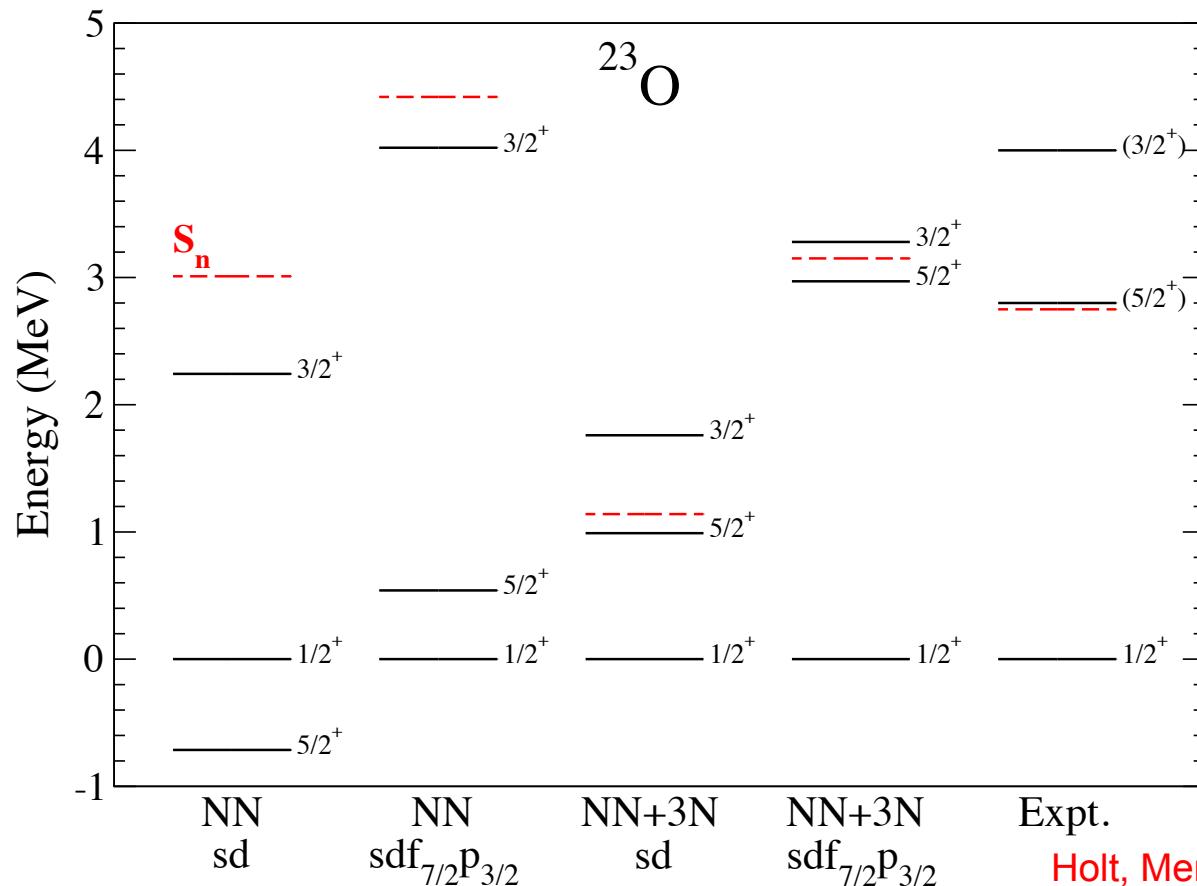


Holt, Menendez, Schwenk, arXiv:1108.2680

Impact on Spectra: ^{23}O

Neutron-rich oxygen spectra with NN+3N (3rd order)

$5/2^+$, $3/2^+$ indicate position of $d_{5/2}$ and $d_{3/2}$ orbits



sd-shell NN-only

Wrong ground state!

$5/2^+$ too low

$3/2^+$ bound

Microscopic NN+3N

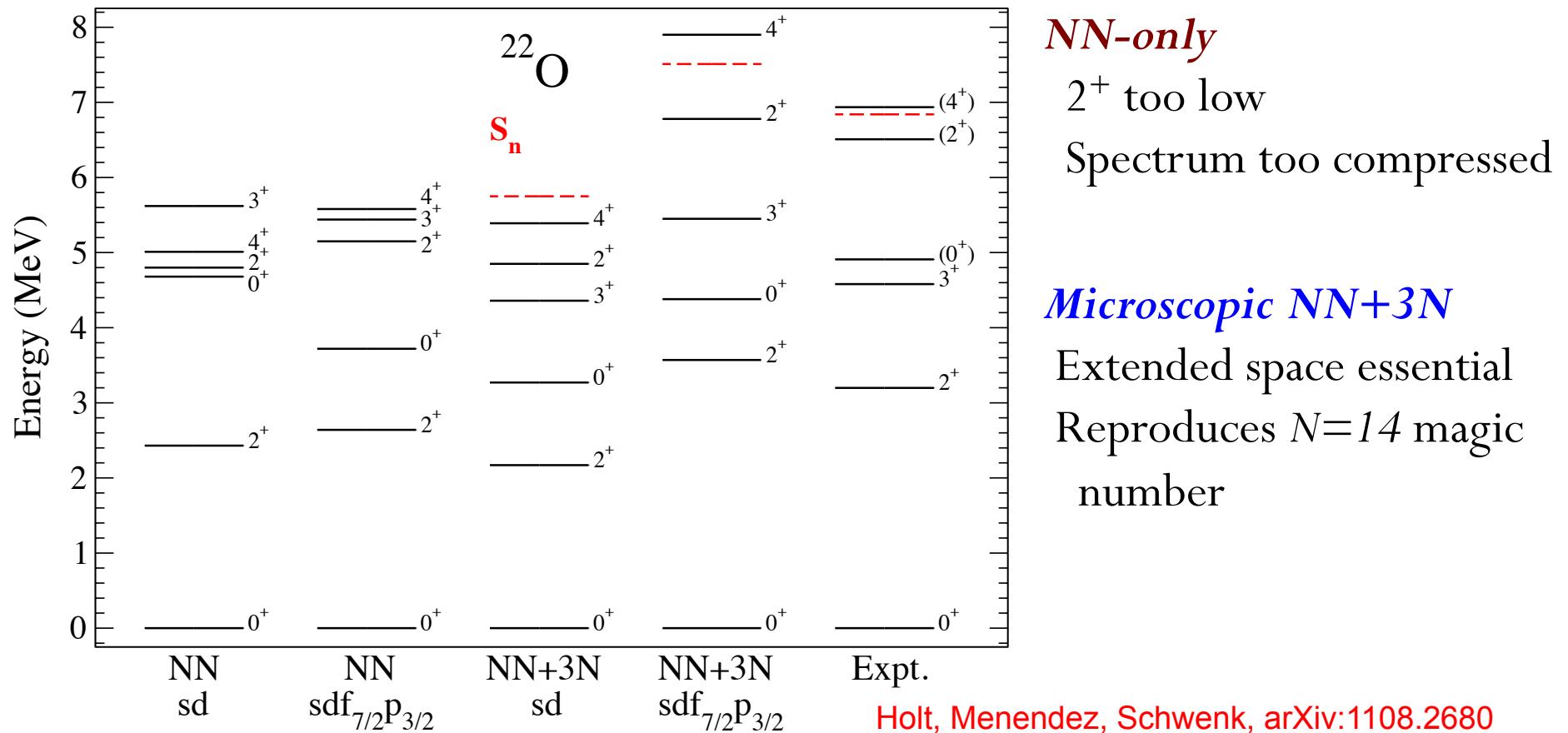
Improvements in
extended valence space

Holt, Menendez, Schwenk, arXiv:1108.2680

Impact on Spectra: ^{22}O

Neutron-rich oxygen spectra with NN+3N

^{22}O : $N=14$ new magic number – not reproduced with NN



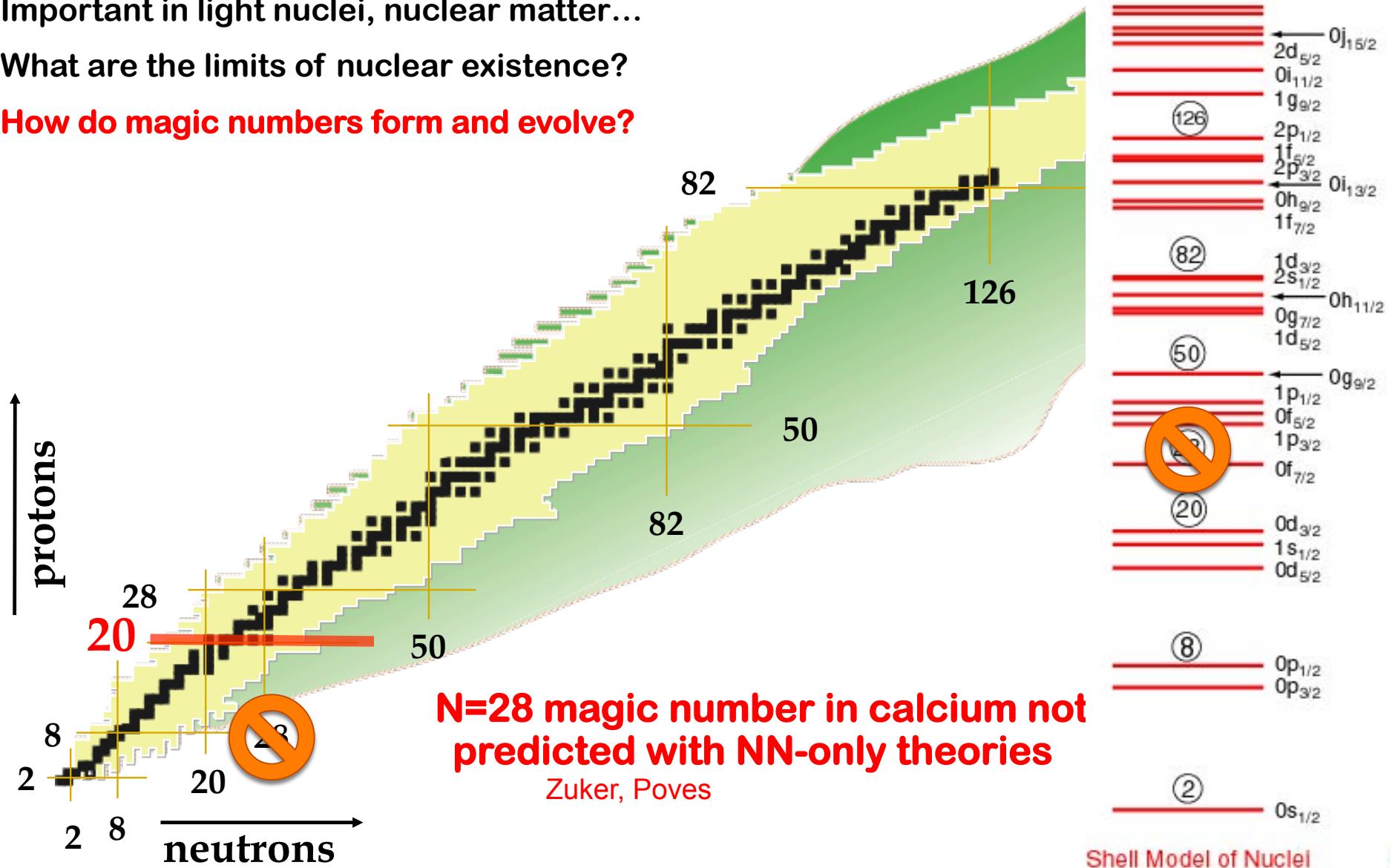
Contributions from 3N and extended valence orbitals important

Shell Formation/Evolution in Calcium Isotopes

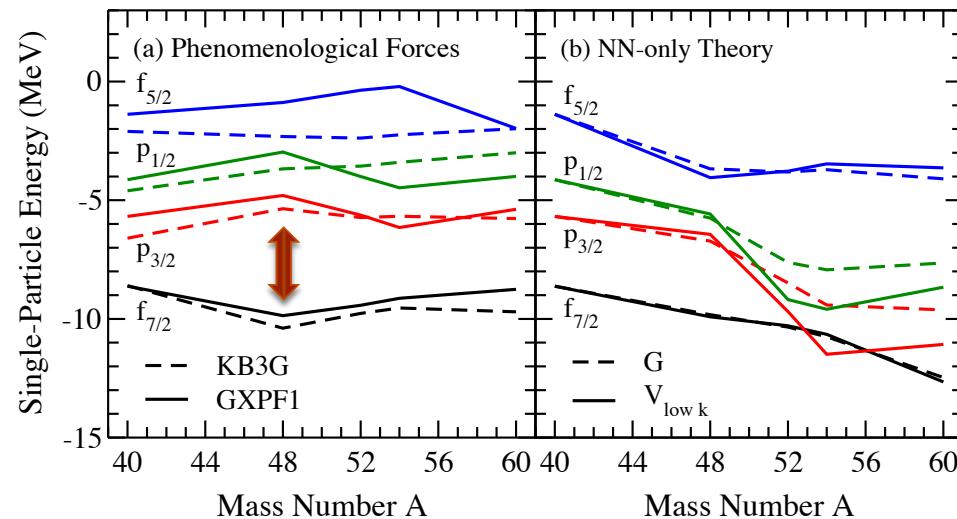
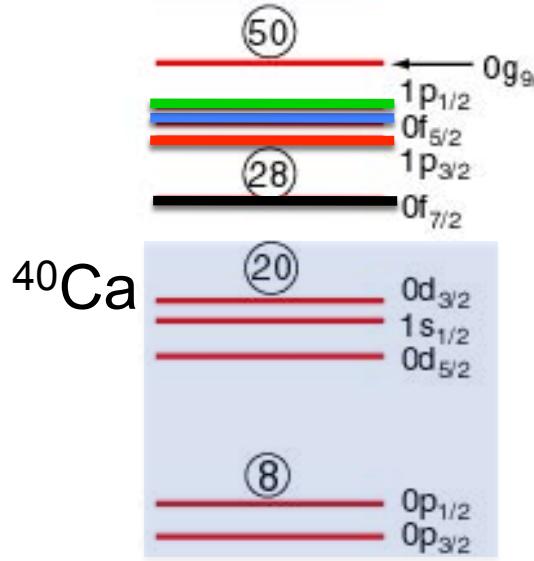
Important in light nuclei, nuclear matter...

What are the limits of nuclear existence?

How do magic numbers form and evolve?

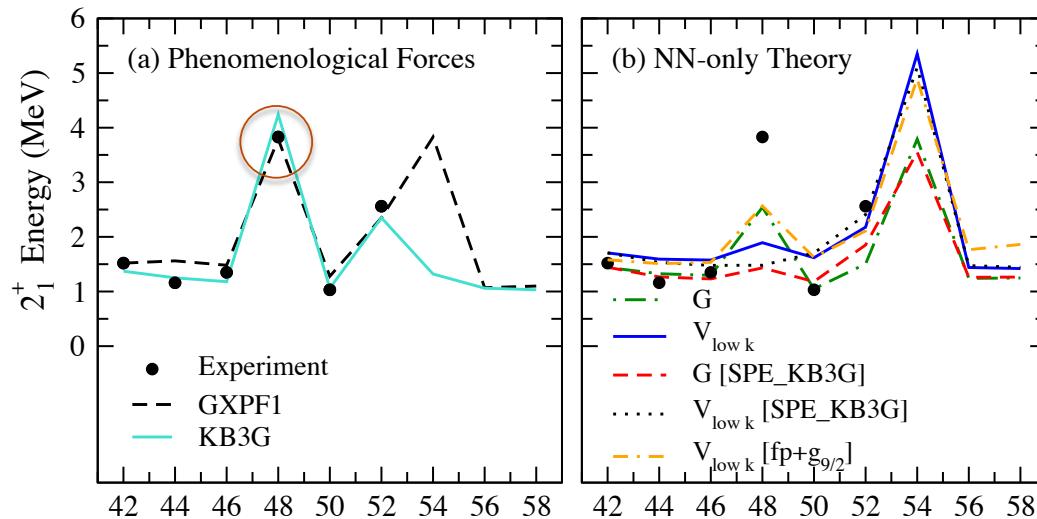


Calcium Isotope Physics: Magic Numbers



GXPF1: Honma, Otsuka, Brown, Mizusaki (2004)

KB3G: Poves, Sanchez-Solano, Caurier, Nowacki (2001)



Phenomenological Forces

Large gap at ^{48}Ca

Discrepancy at $N=34$

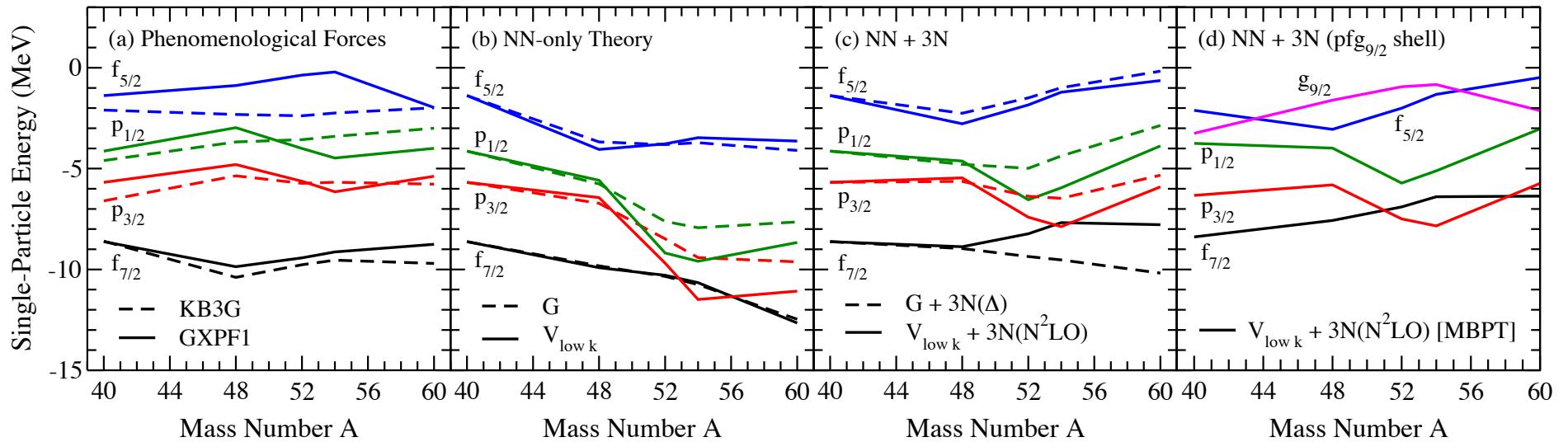
Microscopic NN Theory

Small gap at ^{48}Ca

N=28: first standard magic number not reproduced in microscopic NN theories

Evolution of Shell Structure

SPE evolution with 3N forces in pf and $pfg_{9/2}$ spaces:



NN+3N pf -shell:

Holt, Otsuka, Schwenk, Suzuki arXiv:1009.5984

Trend across: improved binding energies

Increased gap at ^{48}Ca : enhanced closed-shell features

Include $g_{9/2}$ orbit, calculated SPEs

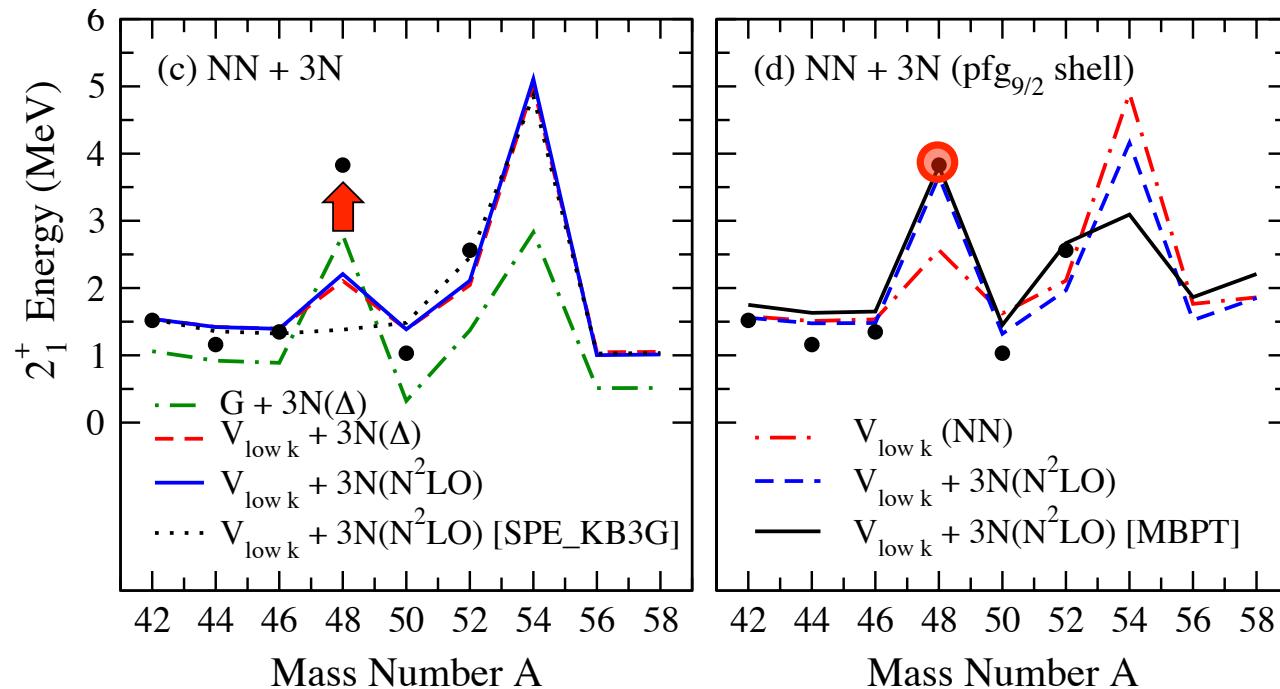
Different behavior of ESPEs (not observable, model dependent)

Small gap can give large 2^+ energy: due to many-body correlations (physics beyond monopole part of Hamiltonian)

Duguet, Hagen, arXiv:1110.2468

N=28 Magic Number in Calcium

First excited 2^+ energies in calcium isotopes with NN+3N



Holt, Otsuka, Schwenk, Suzuki arXiv:1009.5984

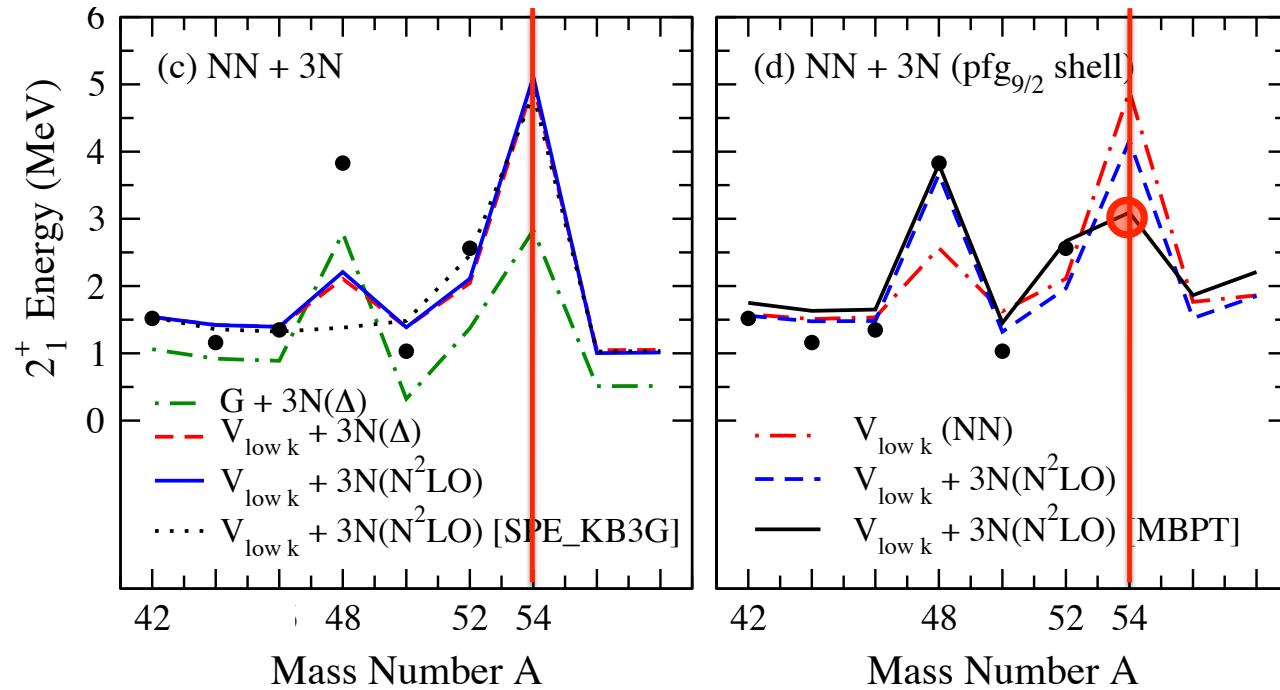
pf-shell: robust but modest improvement in 2^+ energies, below experiment

pfg_{9/2}-shell: reproduce experimental 2^+ in ^{48}Ca

Both 3N and extended space essential

Evolution of Magic Numbers: N=34

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Holt, Otsuka, Schwenk, Suzuki arXiv:1009.5984

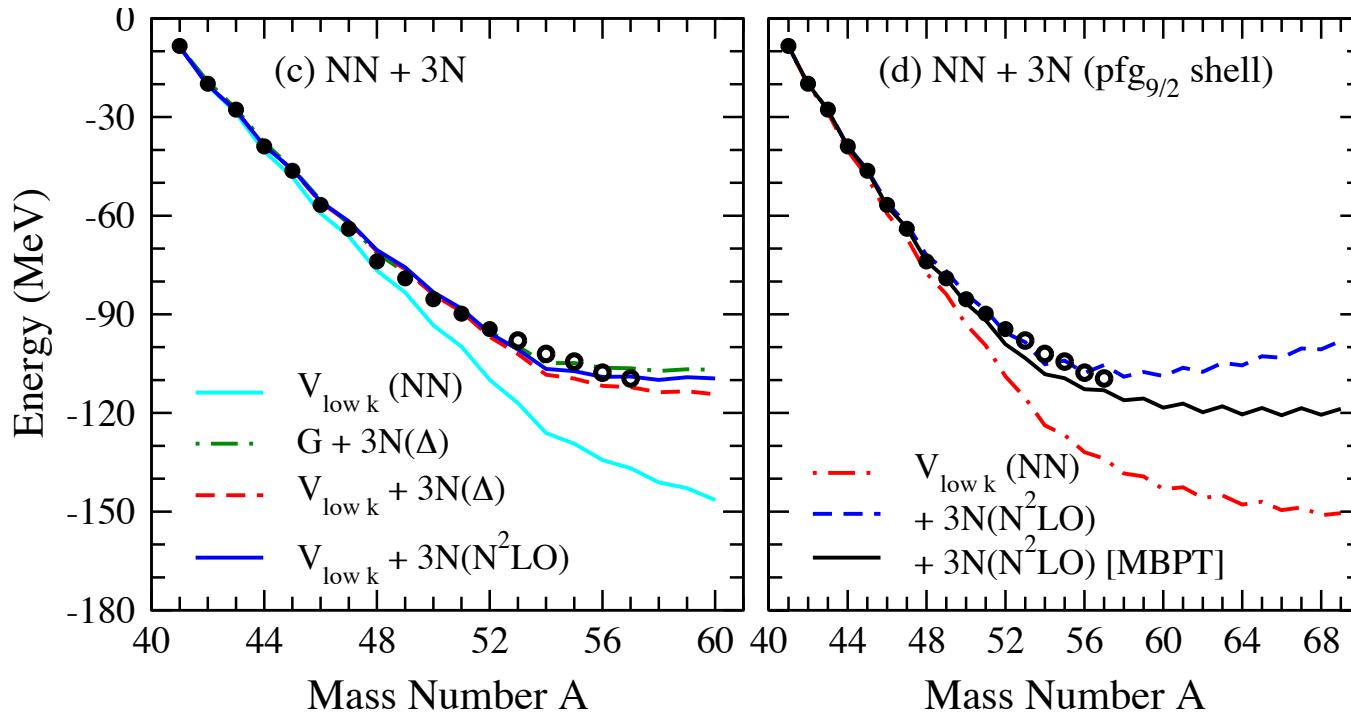
pf -shell: Very pronounced closed-shell properties

$pfg_{9/2}$ -shell: More modest, similar to ^{52}Ca

Calcium Ground State Energies and Dripline

Ground state energies using NN+3N

NN-only: overbinds beyond $\sim {}^{46}\text{Ca}$



Holt, Otsuka, Schwenk, Suzuki arXiv:1009.5984

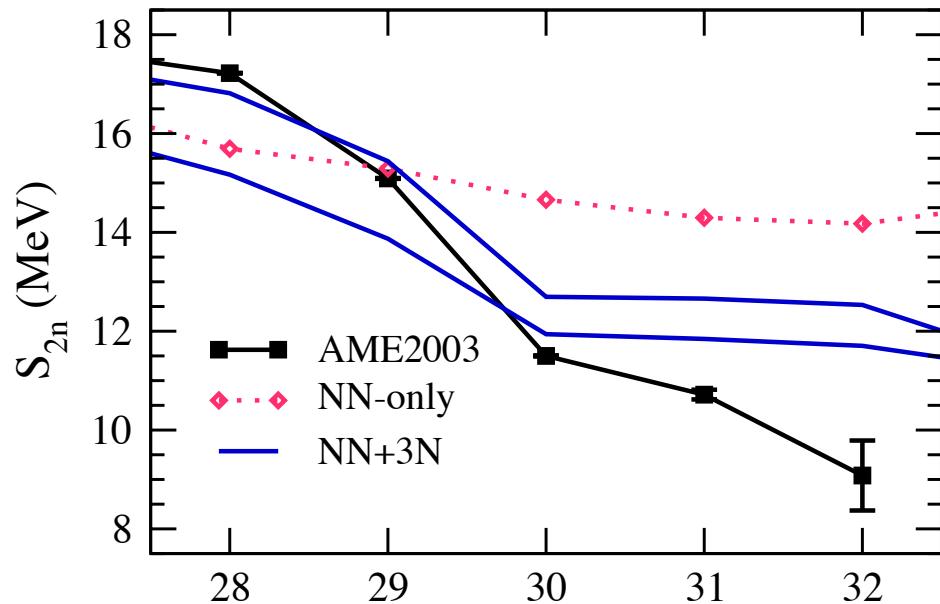
***p*f-shell**: 3N forces correct binding energies; good experimental agreement

***pfg_{9/2}*-shell**: calculate to ${}^{70}\text{Ca}$; modest overbinding beyond ${}^{52}\text{Ca}$

Predict heaviest calcium isotope $\sim {}^{58-60}\text{Ca}$; ${}^{70}\text{Ca}$ not unreasonable

Experimental Connection: Mass of ^{52}Ca

S_{2n} energies for exotic calcium isotopes:



NN-only

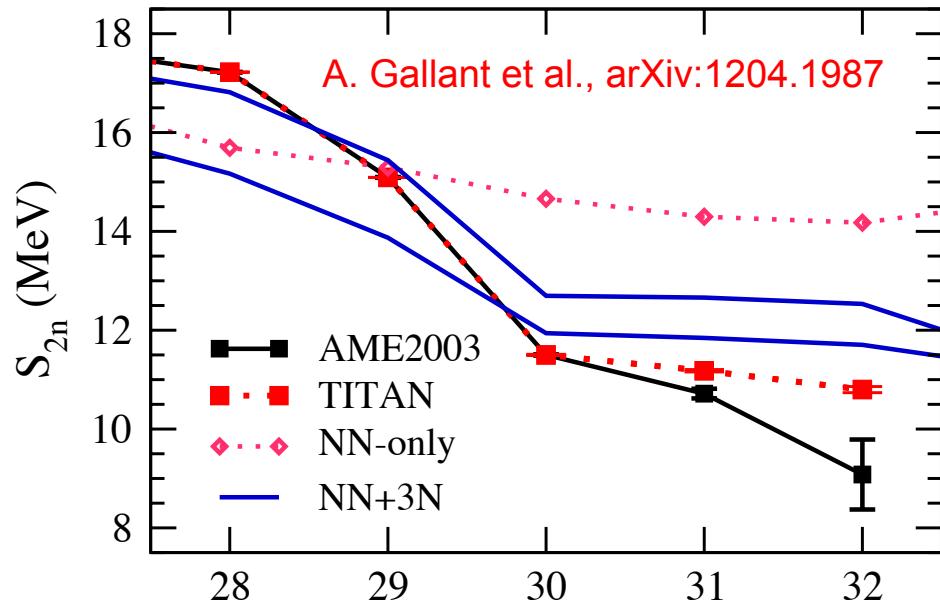
poor experimental agreement

NN+3N

Improvement for lighter calcium,
wrong behavior past ^{50}Ca

Experimental Connection: Mass of ^{52}Ca

New mass measurements of $^{51,52}\text{Ca}$ at **TITAN**: Penning trap experiment



NN-only

poor experimental agreement

NN+3N

Improved agreement with new experimental trend

TITAN Measurement

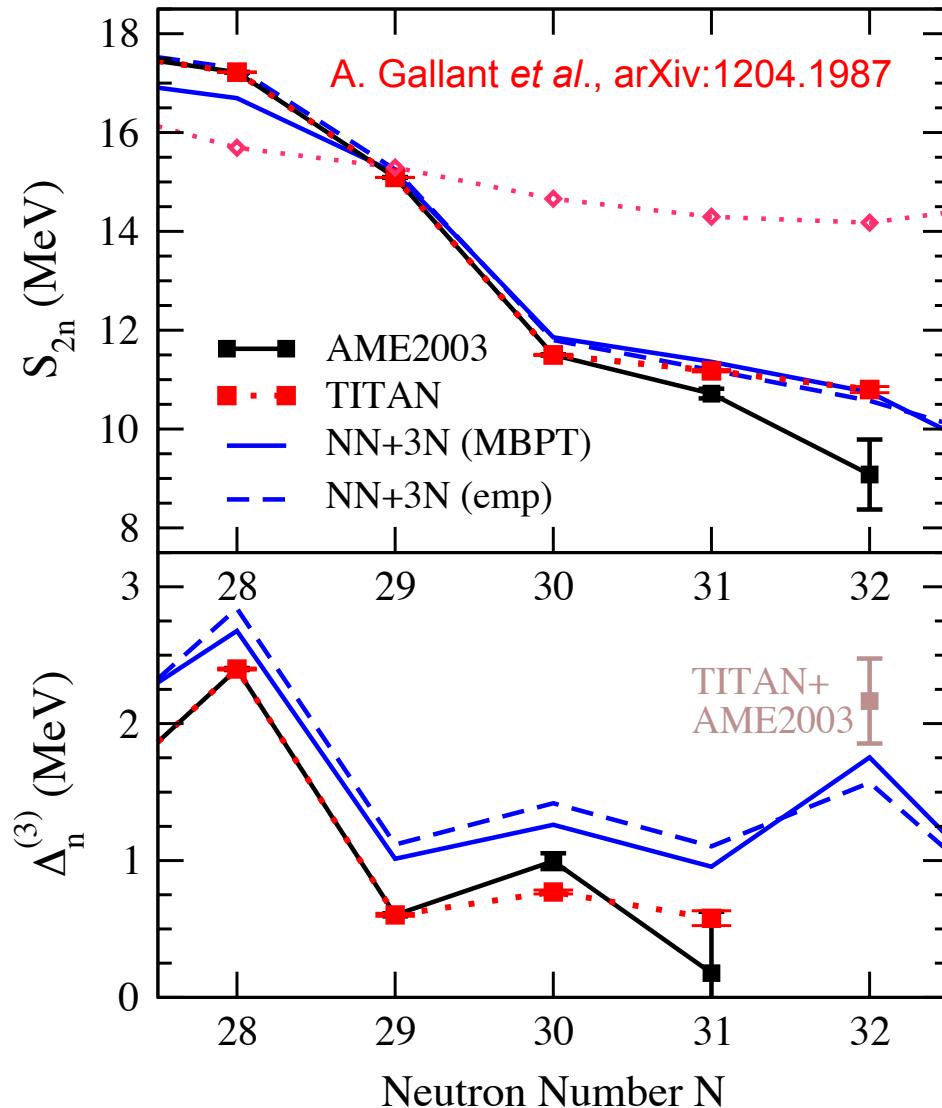
^{52}Ca mass 1.75MeV

more bound than

AME2003 value!

Experimental Connection: Mass of ^{52}Ca

New mass measurements of $^{51,52}\text{Ca}$ at **TITAN**: Penning trap experiment



NN-only

poor experimental agreement

NN+3N

Improved agreement with new experimental trend

TITAN Measurement

^{52}Ca mass 1.75 MeV

more bound than

AME2003 value!

NN+3N(3rd order)

Remarkable experimental agreement

Reduced uncertainty from SPEs

Good reproduction of pairing gaps

Cont. in talk of J. Menendez!

Conclusion

- Exciting era for nuclear structure – **experimentally and theoretically**
- Exploring frontiers of nuclear structure theory of medium-mass nuclei with 3N forces
- Discovered robust and general repulsive 3N mechanism for $T=1$ neutron-rich nuclei
- 3N forces now to third order in MBPT

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- **Oxygen isotopes**: first fully-microscopic results with **NN+3N, extended spaces**
 - Cures NN-only failings: dripline, shell evolution, spectra in oxygen isotopes
- **Calcium isotopes** in pf - and $pfg_{9/2}$ -shells:
 - First microscopic prediction of $N=28$ magic number in ^{48}Ca
 - Shell evolution towards the dripline: modest $N=34$ closure, quenching of $N=40$
 - Dripline near ^{60}Ca

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 - Dripline near ^{60}Ca
- Clearly improvable upgrade path

Much more in next talk!

Acknowledgments

Collaborators



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