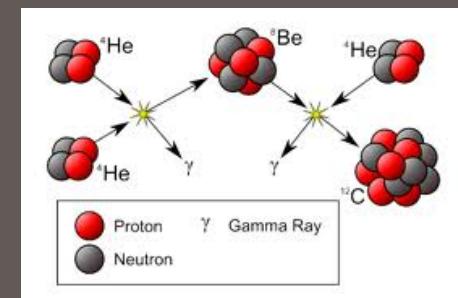
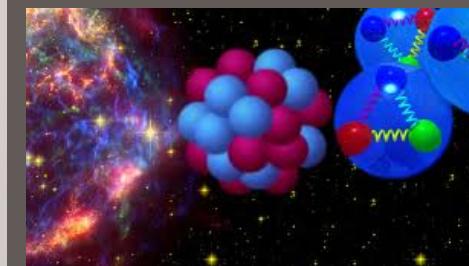
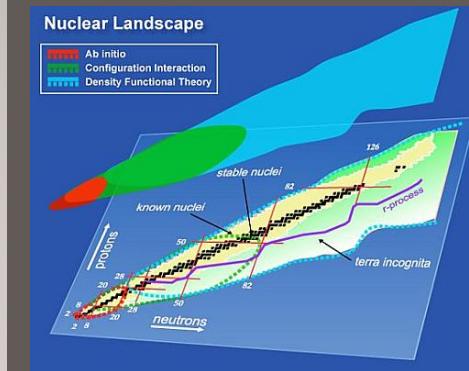


Local chiral 3N interaction in *ab initio* calculations

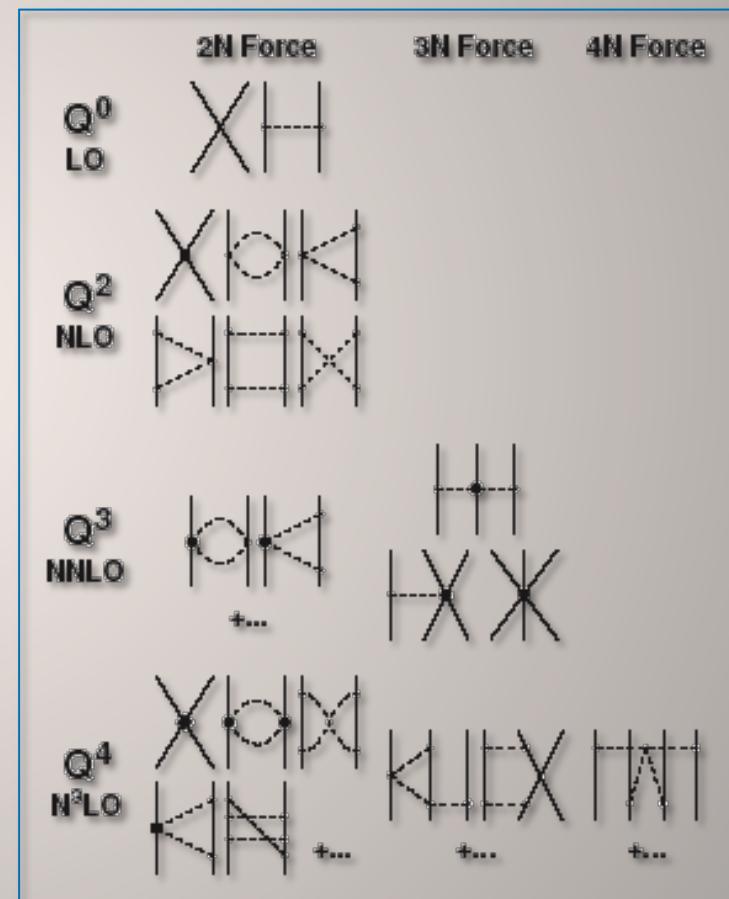
The Extreme Matter Physics of Nuclei:
 From Universal Properties to Neutron-Rich Extremes
 ExtreMe Matter Institute EMMI, GSI
 April 30, 2012

Petr Navratil | TRIUMF



Light nuclei from the first principles

- First principles for Nuclear Physics:
QCD
 - Non-perturbative at low energies
 - Lattice QCD in the future
- **For now a good place to start:**
- Inter-nucleon forces from chiral effective field theory
 - Based on the symmetries of QCD
 - Degrees of freedom: nucleons + pions
 - Systematic low-momentum expansion to a given order
 - Hierarchy
 - Consistency
 - Low energy constants (LEC)
 - Fitted to data
 - Can be calculated by lattice QCD

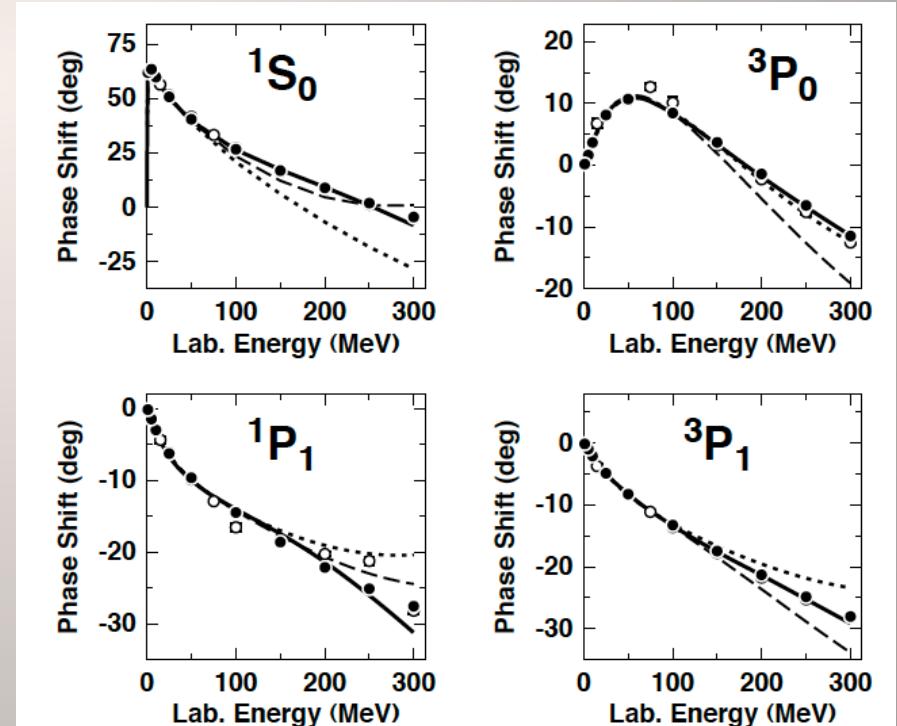
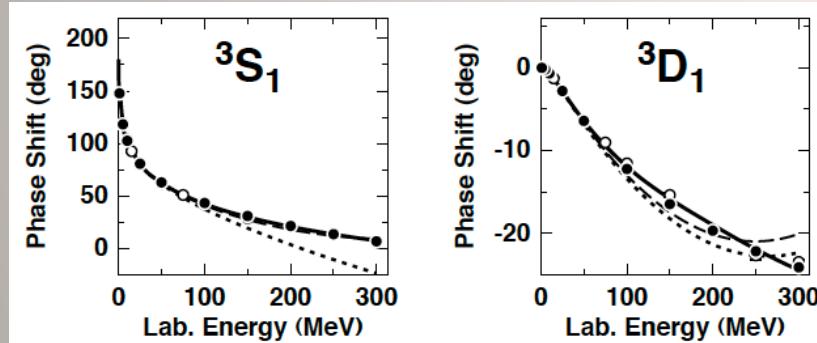


The NN interaction from chiral EFT

PHYSICAL REVIEW C **68**, 041001(R) (2003)

Accurate charge-dependent nucleon-nucleon potential at fourth order of chiral perturbation theory

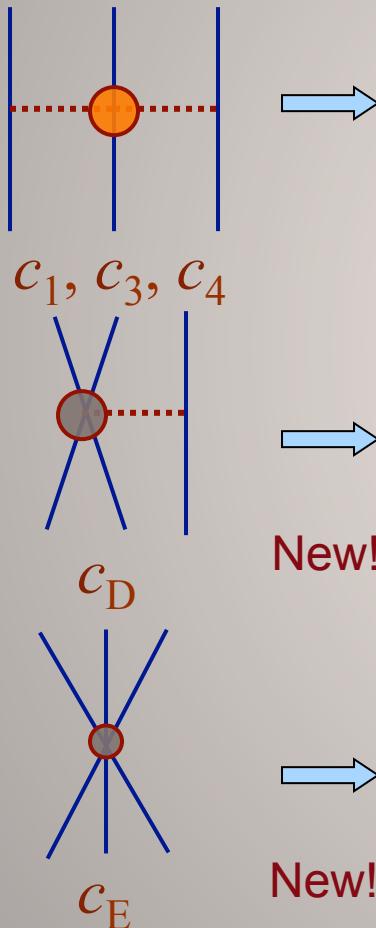
D. R. Entem^{1,2,*} and R. Machleidt^{1,†}



- 24 LECs fitted to the np scattering data and the deuteron properties
 - Including c_i LECs ($i=1-4$) from pion-nucleon Lagrangian

The NNN interaction from chiral EFT

N²LO



Two-pion exchange

c_1, c_3, c_4 LECs appear in the chiral NN interaction

- Determined in the $A=2$ system

One-pion-exchange-contact

New c_D LEC

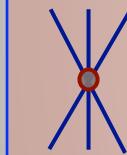
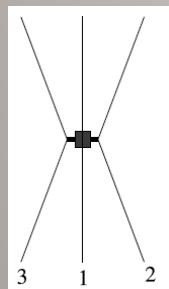
Contact

New c_E LEC

Must be determined
in $A \geq 3$ system

Local chiral N²LO NNN interaction

- Regulator depending on momentum transfer \Rightarrow local NNN interaction in coordinate space
 - Simpler to use, more like TM', UIX, IL
 - Different space-tensor structure (compared to regulation with nucleon momenta)
 - Appears to perform better in mid- p -shell nuclei
 - Example:** Even the simplest, the contact term, gets involved...



$$W_1^{\text{cont}} = E\vec{r}_2 \cdot \vec{r}_3 \delta(\vec{r}_1 - \vec{r}_2) \delta(\vec{r}_3 - \vec{r}_1)$$

$$= E\vec{r}_2 \cdot \vec{r}_3 \frac{1}{(2\pi)^6} \frac{1}{(\sqrt{3})^3} \int d\vec{\pi}_1 d\vec{\pi}_2 d\vec{\pi}'_1 d\vec{\pi}'_2 |\vec{\pi}_1 \vec{\pi}_2\rangle \langle \vec{\pi}'_1 \vec{\pi}'_2|$$

$$W_1^{\text{cont},Q} = E\vec{r}_2 \cdot \vec{r}_3 \frac{1}{(2\pi)^6} \frac{1}{(\sqrt{3})^3} \int d\vec{\pi}_1 d\vec{\pi}_2 d\vec{\pi}'_1 d\vec{\pi}'_2 |\vec{\pi}_1 \vec{\pi}_2\rangle F(\vec{Q}^2; \Lambda) F(\vec{Q}'^2; \Lambda) \langle \vec{\pi}'_1 \vec{\pi}'_2|$$

$$= E\vec{r}_2 \cdot \vec{r}_3 \int d\vec{\xi}_1 d\vec{\xi}_2 |\vec{\xi}_1 \vec{\xi}_2\rangle Z_0(\sqrt{2}\xi_1; \Lambda) Z_0(|\frac{1}{\sqrt{2}}\vec{\xi}_1 + \sqrt{\frac{3}{2}}\vec{\xi}_2|; \Lambda) \langle \vec{\xi}_1 \vec{\xi}_2|$$

Few Body Syst (2007) 41: 117–140
 DOI 10.1007/s00601-007-0193-3
 Printed in The Netherlands

Local three-nucleon interaction from chiral effective field theory

P. Navrátil*

Lawrence Livermore National Laboratory, Livermore, CA, USA

Few
Body
Systems

$$\vec{\xi}_1 = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2)$$

$$\vec{\xi}_2 = \sqrt{\frac{2}{3}}\left(\frac{1}{2}(\vec{r}_1 + \vec{r}_2) - \vec{r}_3\right)$$

$$\vec{\pi}_1 = \frac{1}{\sqrt{2}}(\vec{p}_1 - \vec{p}_2)$$

$$\vec{\pi}_2 = \sqrt{\frac{2}{3}}\left(\frac{1}{2}(\vec{p}_1 + \vec{p}_2) - \vec{p}_3\right)$$

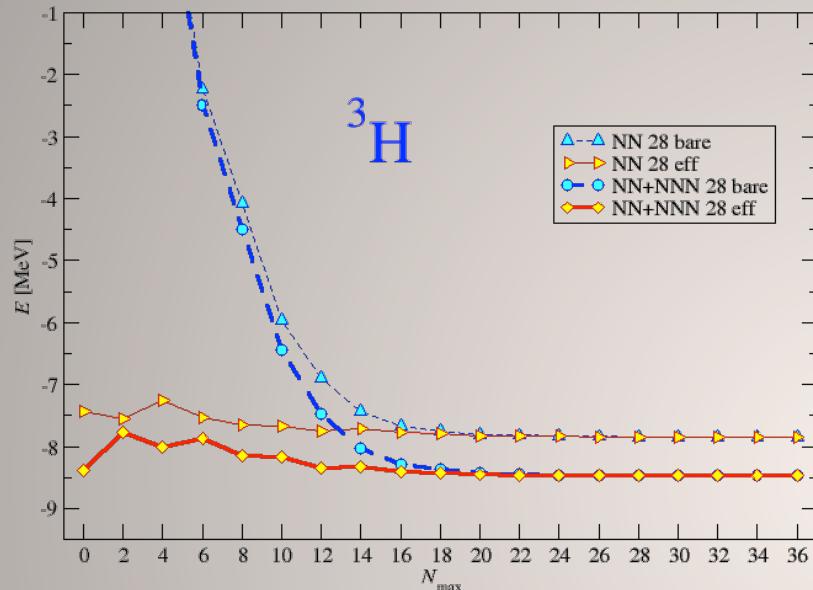
$$\vec{Q} = \vec{p}'_2 - \vec{p}'_1 = -\frac{1}{\sqrt{2}}(\vec{\pi}'_1 - \vec{\pi}_1) + \frac{1}{\sqrt{6}}(\vec{\pi}'_2 - \vec{\pi}_2)$$

$$\vec{Q}' = \vec{p}'_3 - \vec{p}'_2 = \sqrt{\frac{2}{3}}(\vec{\pi}_2 - \vec{\pi}'_2)$$

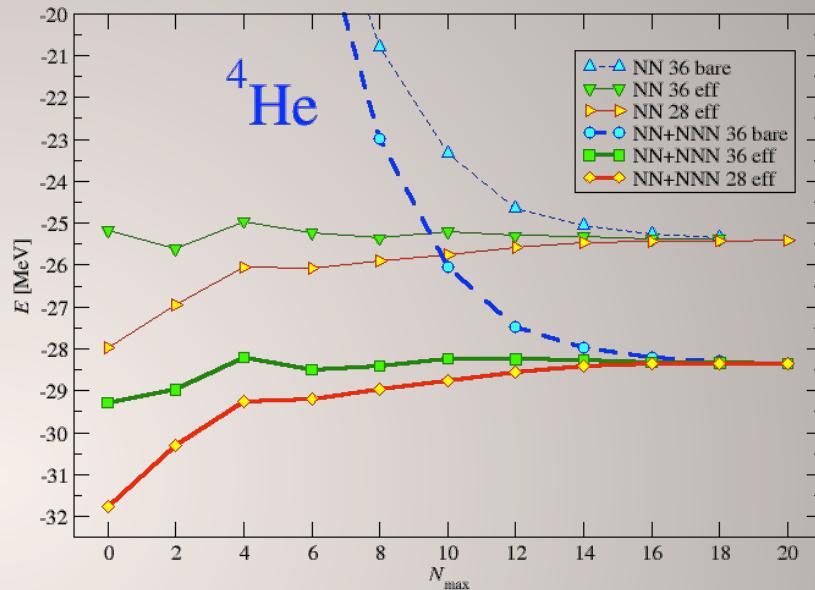
$$Z_0(r; \Lambda) = \frac{1}{2\pi^2} \int dq q^2 j_0(qr) F(q^2; \Lambda)$$

$$F(q^2; \Lambda) = \exp[-q^4/\Lambda^4]$$

^3H and ^4He with chiral EFT interactions



^3H



^4He

		$E_{\text{g.s.}}$	$\langle r_p^2 \rangle^{1/2}$
NN	NCSM [71]	-7.852(5)	1.650(5)
NN	HH [17]	-7.854	1.655
NN+NNN	NCSM [71]	-8.473(5)	1.608(5)
NN+NNN	HH [17]	-8.474	1.611
Expt.		-8.482	1.60

A=3 binding energy constraint
 $c_D=+1$, $c_E=-0.029$, $\Lambda=500$ MeV

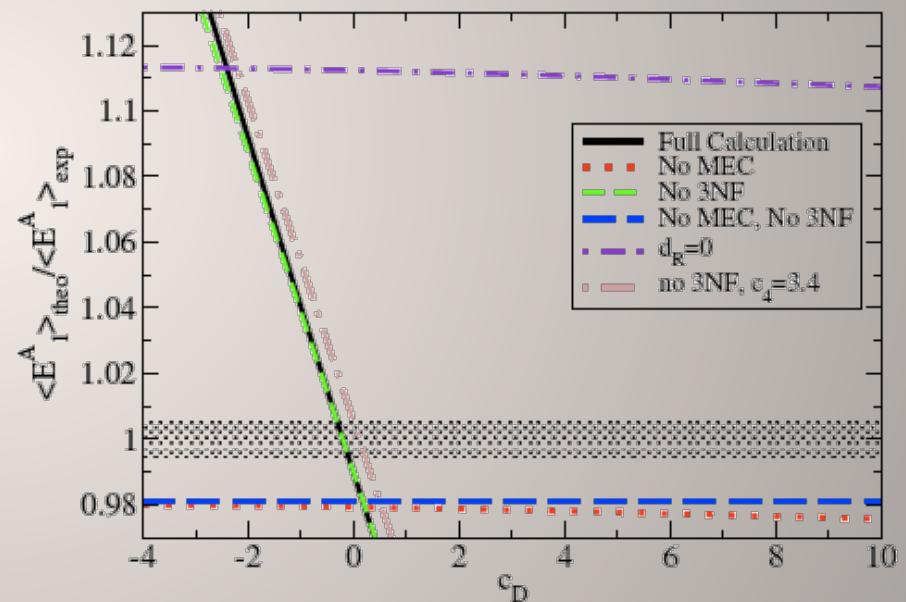
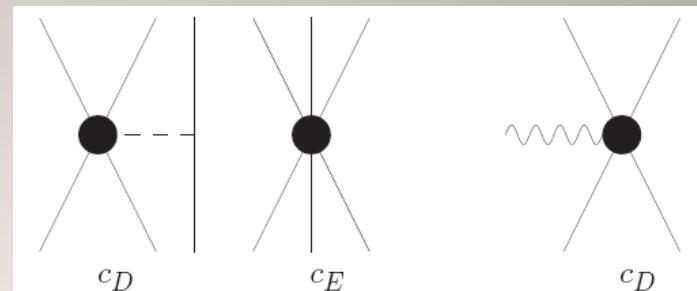
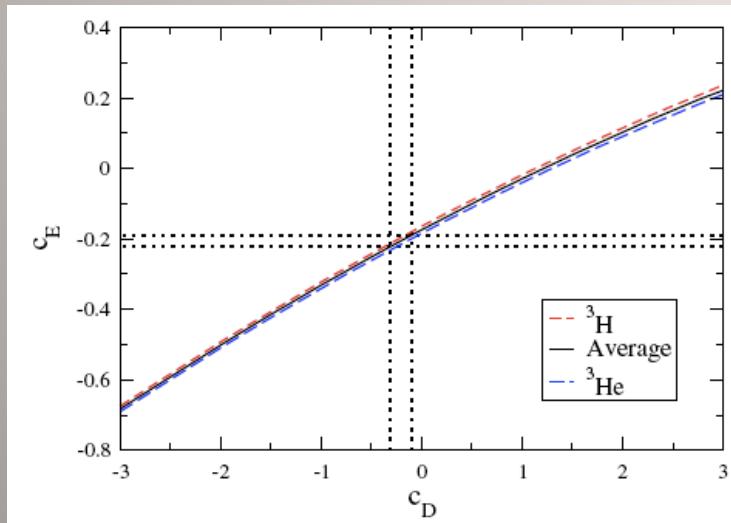
		$E_{\text{g.s.}}$	$\langle r_p^2 \rangle^{1/2}$
NN	NCSM [71]	-25.39(1)	1.515(2)
NN	HH [17]	-25.38	1.518
NN+NNN	NCSM [71]	-28.34(2)	1.475(2)
NN+NNN	HH [17]	-28.36	1.476
Expt.		-28.296	1.467(13)

[71] FBS **41**, 117 (2007)

[17] JPG **35**, 063101 (2008)

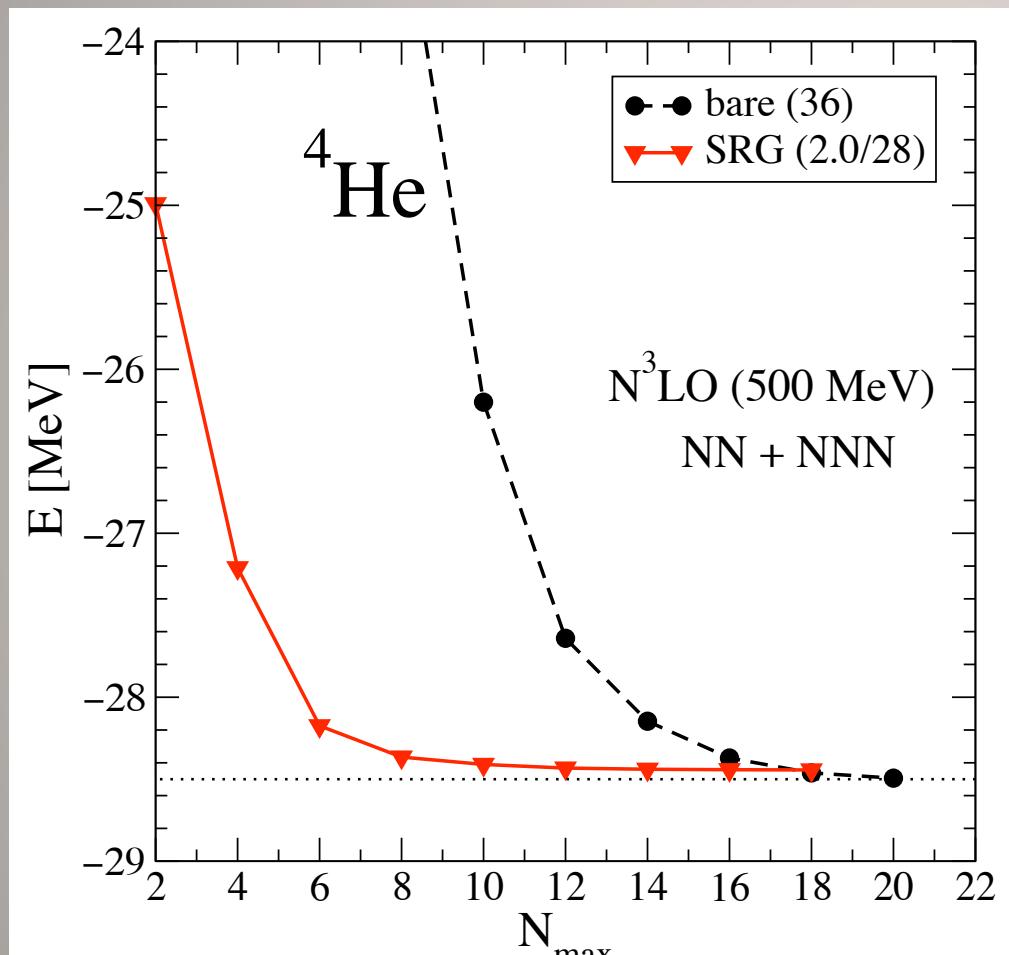
Determination of NNN constants c_D and c_E from the triton binding energy and the half life

- Chiral EFT: c_D also in the two-nucleon contact vertex with an external probe
- Calculate $\langle E_1^A \rangle = |\langle {}^3\text{He} || E_1^A || {}^3\text{H} \rangle|$
 - Leading order GT
 - N²LO: one-pion exchange plus contact
- A=3 binding energy constraint:
 $c_D = -0.2 \pm 0.1$ $c_E = -0.205 \pm 0.015$



PRL 103, 102502 (2009)

^4He from chiral EFT interactions: g.s. energy convergence



PRL 103, 082501 (2009)

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week ending
21 AUGUST 2009

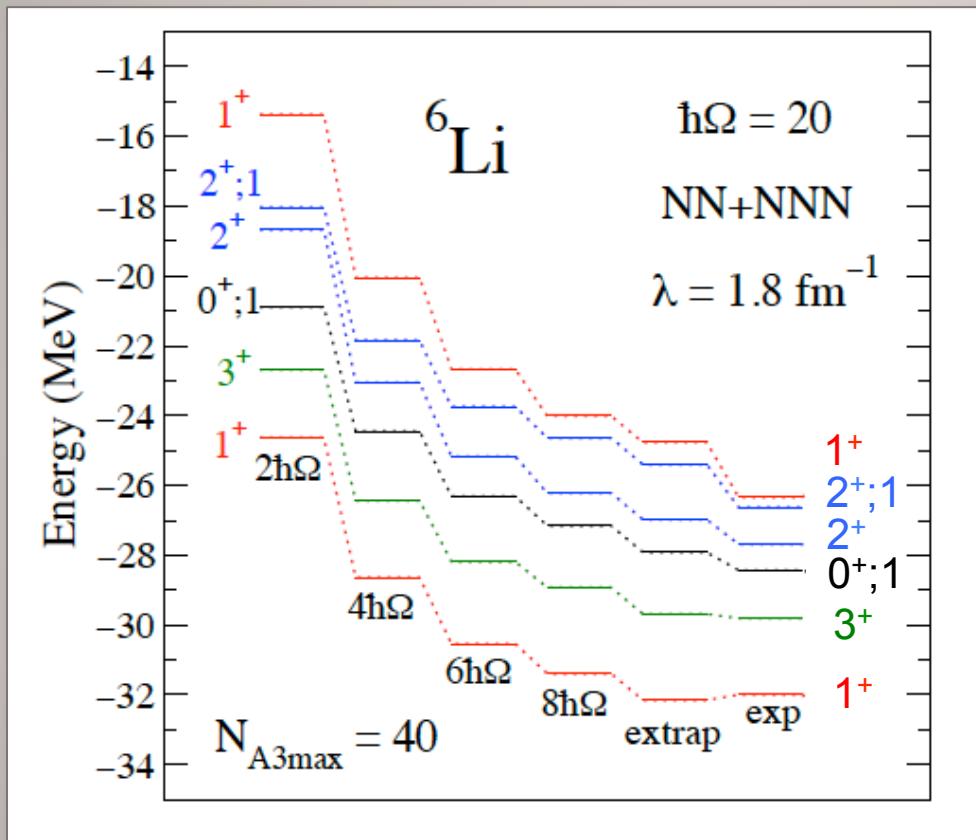
Evolution of Nuclear Many-Body Forces with the Similarity Renormalization Group

E. D. Jurgenson,¹ P. Navrátil,² and R. J. Furnstahl¹

Chiral $N^3\text{LO}$ NN plus $N^2\text{LO}$ NNN potential

- Bare interaction (black line)
 - Variational calculation
 - Strong short-range correlations
 - Large basis needed
- SRG evolved effective interaction (red line)
 - Unitary transformation
 - Two- plus *three*-body components, *four*-body omitted
 - Softens the interaction
 - Variational calculation
 - Smaller basis sufficient

^6Li from chiral EFT interactions: Ground-state & excitation energies



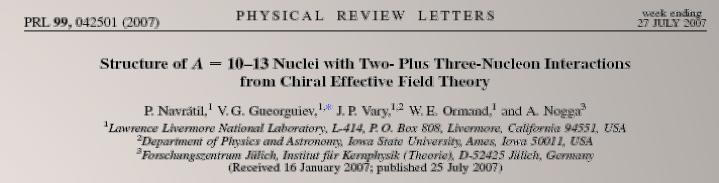
$A=3$ binding energy & half life constraint
 $c_D = -0.2$, $c_E = -0.205$, $\Lambda = 500$ MeV

SRG with 2- plus 3-body: Good convergence, extrapolation to infinite basis space possible

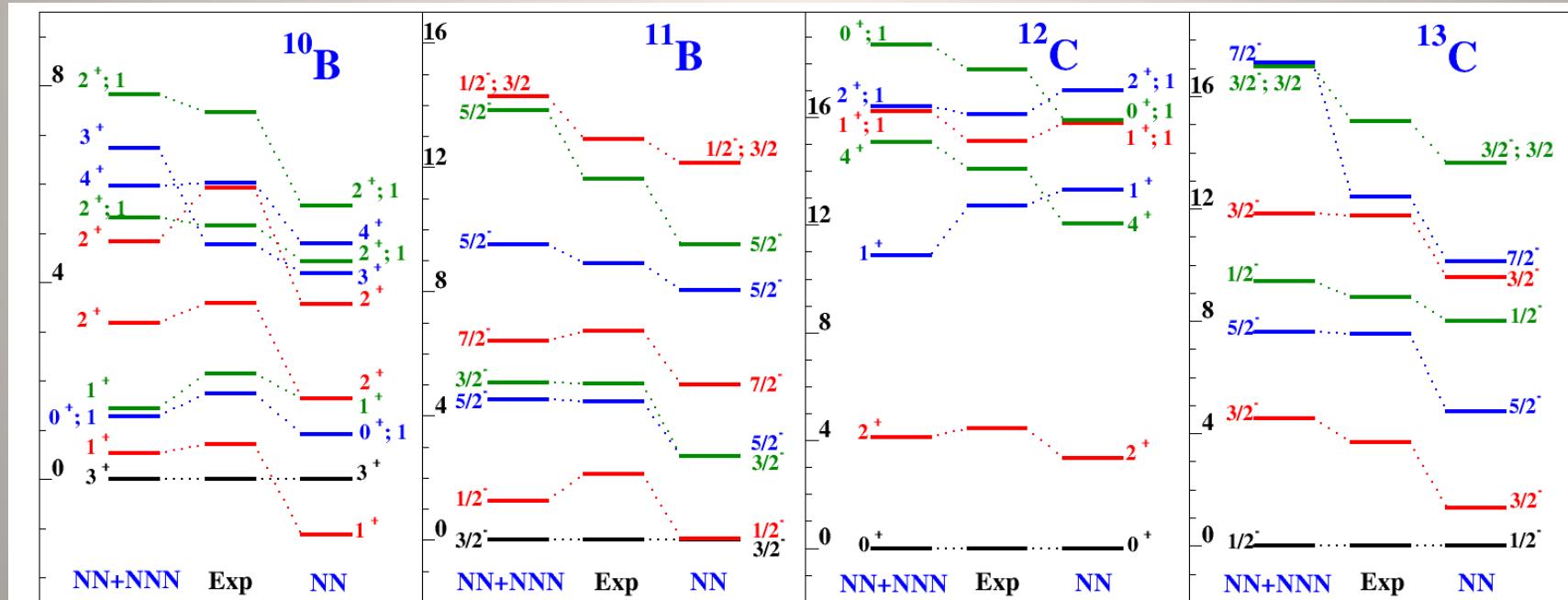
Structure of p -shell nuclei with chiral NN+NNN interactions

- NCSM is the only method to apply the chiral EFT NN+NNN interactions in the p -shell

- Technically challenging, large-scale computational problem
 - ~4000 processors used in $^{12,13}\text{C}$ calculations



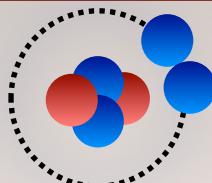
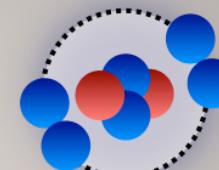
- NNN interaction essential to describe structure of light nuclei



$A=3$ binding energy constraint, $c_D=-1$, $c_E=-0.331$, $\Lambda=500$ MeV

NNN interaction effects in neutron rich nuclei: He isotopes

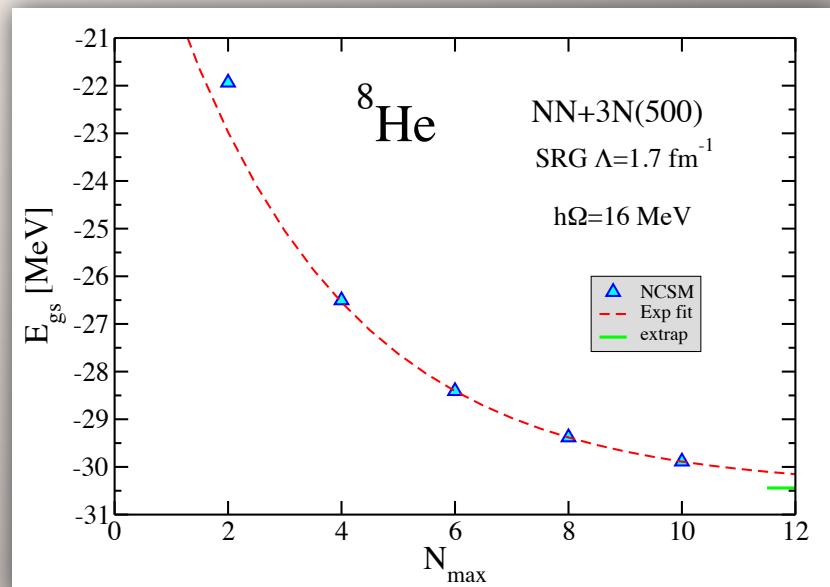
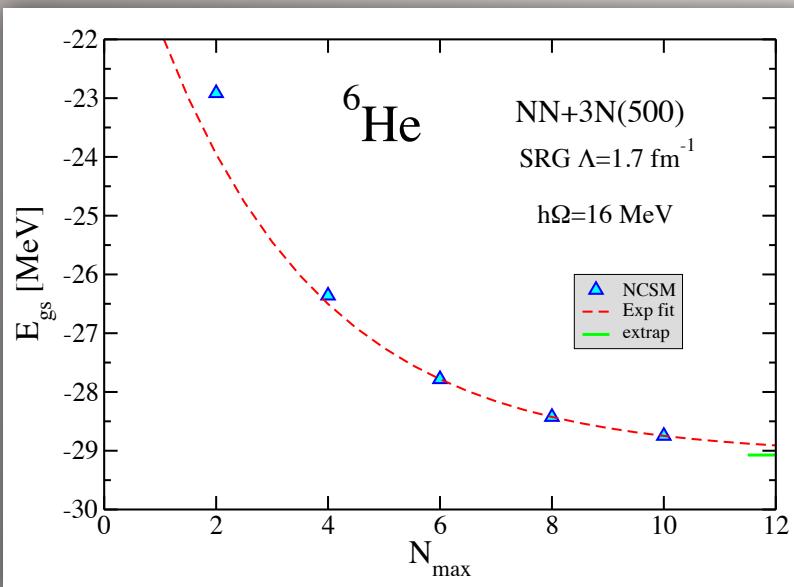
 ^4He

 ^6He

 ^8He


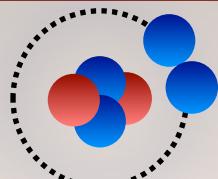
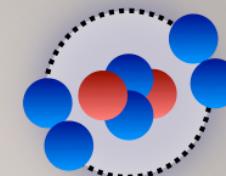
^6He and ^8He with SRG-evolved chiral N³LO NN + N²LO NNN

- 3N matrix elements in coupled- J single-particle basis:
 - Introduced and implemented by Robert Roth *et al.*
 - Now also in my codes: Jacobi-Slater-Determinant transformation & NCSD code
 - Example: ^6He , ^8He NCSM calculations up to $N_{\max}=10$ done with moderate resources

$A=3$ binding energy & half life constraint
 $c_D=-0.2$, $c_E=-0.205$, $\Lambda=500$ MeV



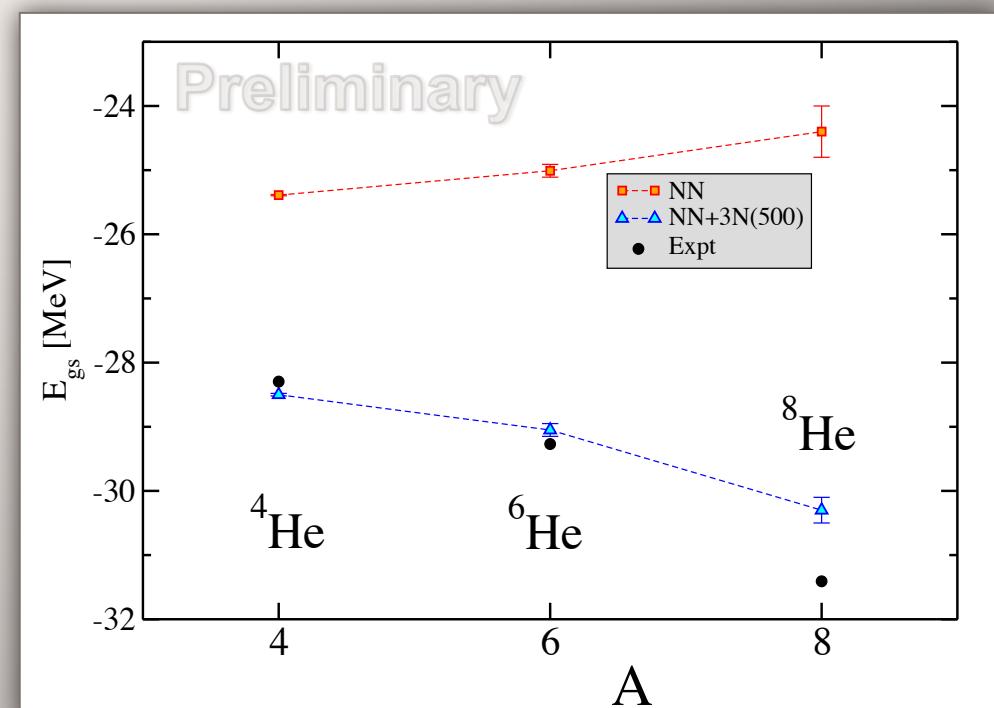
3N interaction effects in neutron rich nuclei: He isotopes

 ^4He  ^6He  ^8He 

- ^6He and ^8He with SRG-evolved chiral N³LO NN + N²LO 3N
 - chiral N³LO NN: ^4He underbound, ^6He and ^8He unbound
 - chiral N³LO NN + N²LO 3N(500): ^4He OK, both ^6He and ^8He bound

$A=3$ binding energy & half life constraint
 $c_D = -0.2$, $c_E = -0.205$, $\Lambda = 500$ MeV

NNN interaction important
to bind neutron rich nuclei



3N interaction effects in neutron rich nuclei: He isotopes

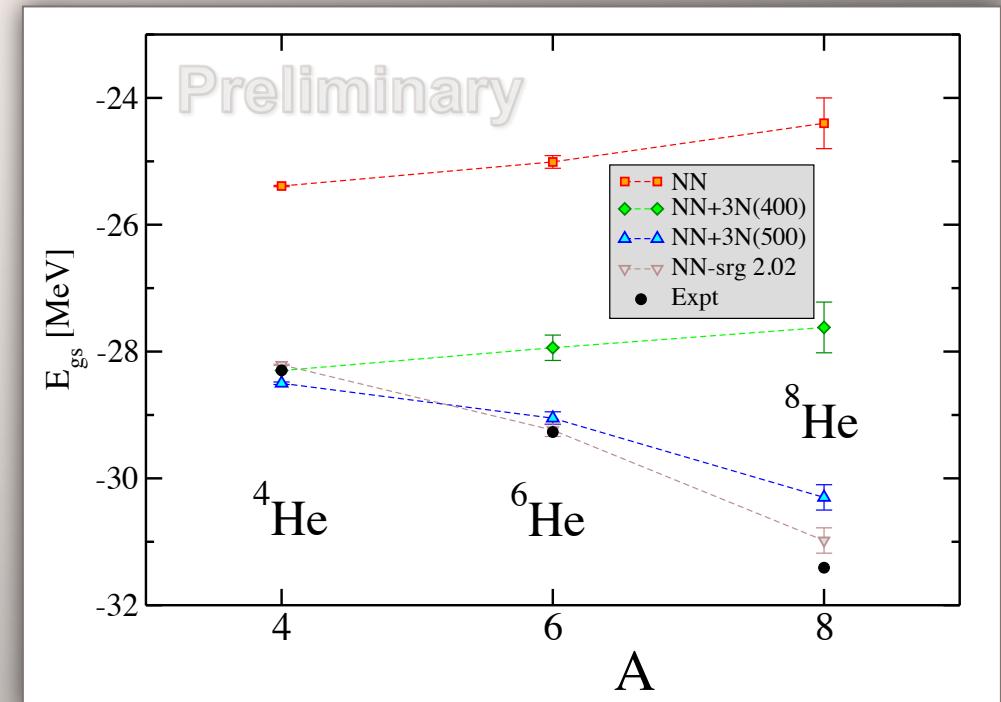
- ^6He and ^8He with SRG-evolved chiral N³LO NN + N²LO 3N
 - chiral N³LO NN: ^4He underbound, ^6He and ^8He unbound
 - chiral N³LO NN + N²LO 3N(400): ^4He fitted, ^6He barely unbound, ^8He unbound
 - describes quite well binding energies of ^{12}C , ^{16}O , ^{40}Ca , ^{48}Ca
 - chiral N³LO NN + N²LO 3N(500): ^4He OK, both ^6He and ^8He bound
 - does well up to $A=10$, overbinds ^{12}C , ^{16}O , Ca isotopes
 - SRG-N³LO NN $\Lambda=2.02 \text{ fm}^{-1}$: ^4He OK, both ^6He and ^8He bound
 - ^{16}O , Ca strongly overbound

^4He binding energy & ^3H half life constraint
 $c_D=-0.2$, $c_E=+0.098$, $\Lambda=400 \text{ MeV}$

$A=3$ binding energy & half life constraint
 $c_D=-0.2$, $c_E=-0.205$, $\Lambda=500 \text{ MeV}$

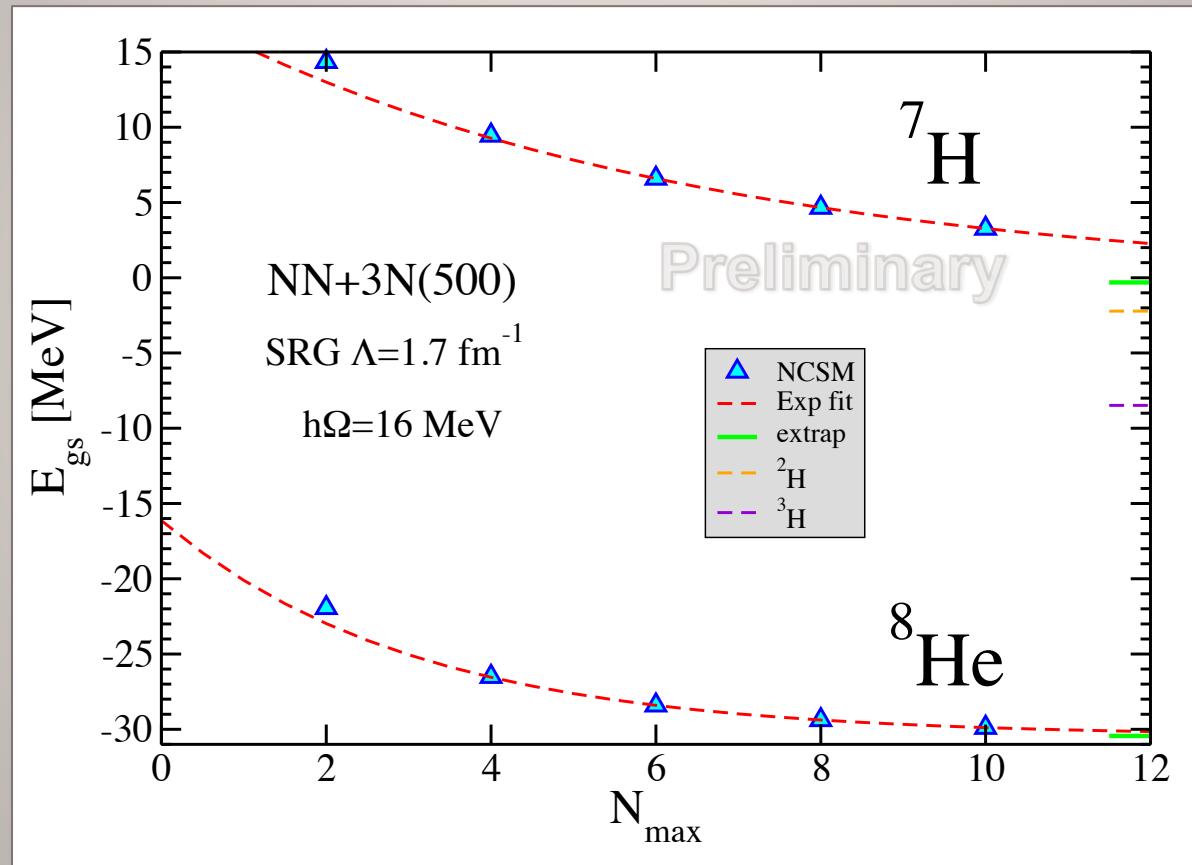
**NNN interaction important
to bind neutron rich nuclei**

**Our knowledge of the 3N interaction
is incomplete**



ExtreMe matter: ^7H

Proton knockout from ^8He : Is there a ^7H resonance?



NCSM calculation extrapolates to $E_{\text{gs}} < 0$. Not much can be concluded at this stage...

Superallowed Fermi β -decay

- **Test of the Standard Model**

$$ft_{1/2} = \frac{K}{G_V^2 |M_F|^2}$$

$$|M_F|^2 = 2(1 - \delta_c)$$

$$f = \int_1^{W_0} dW p_W (W_0 - W)$$

δ_c : Isospin-mixing correction

$$M_F = \langle f | \tau_+ | i \rangle = \sqrt{2} \text{ for } T = 1$$

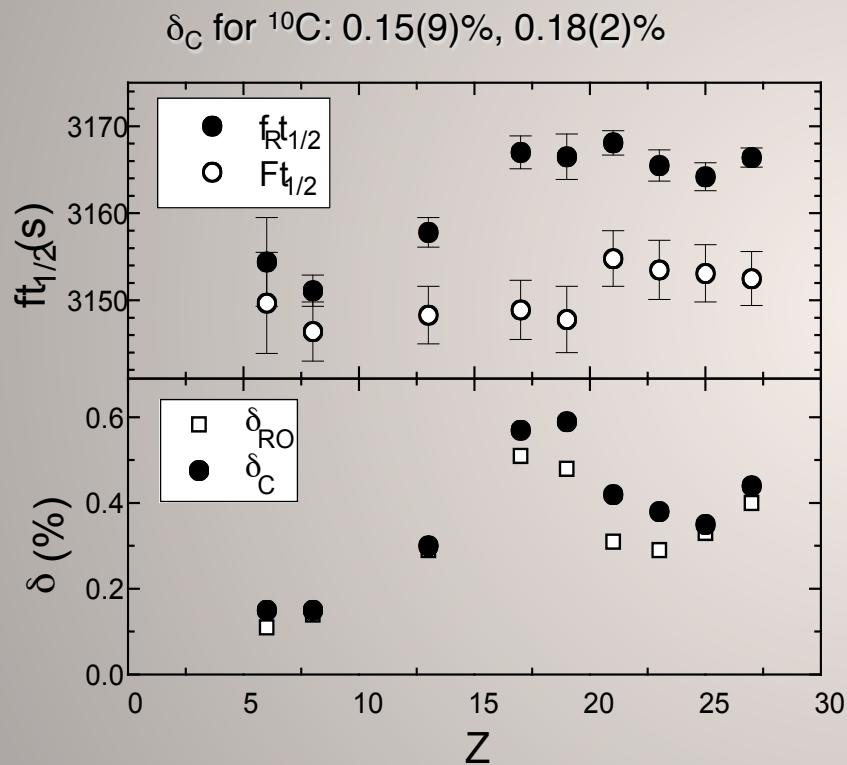
- **Cabibbo-Kobayashi-Maskawa matrix**

$$\begin{pmatrix} d_W \\ s_W \\ b_W \end{pmatrix} = \begin{pmatrix} v_{ud} & v_{us} & v_{ub} \\ v_{cd} & v_{cs} & v_{cb} \\ v_{td} & v_{ts} & v_{tb} \end{pmatrix} \begin{pmatrix} d_S \\ s_S \\ b_S \end{pmatrix}$$

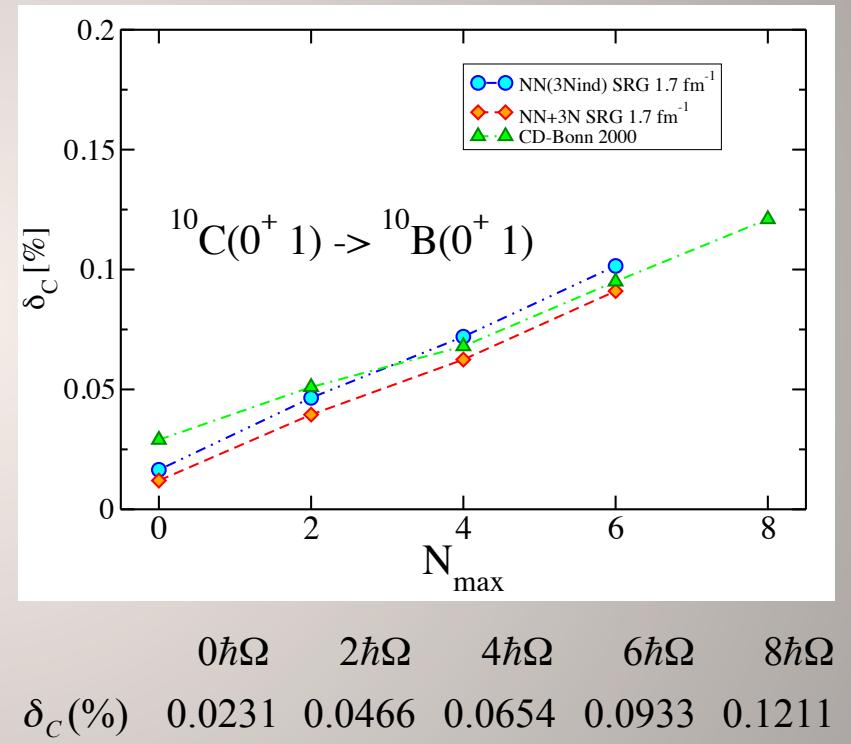
$$|v_{ud}|^2 + |v_{us}|^2 + |v_{ub}|^2 = 1$$

Superallowed Fermi β -decay

Phenomenological Shell Model calculations



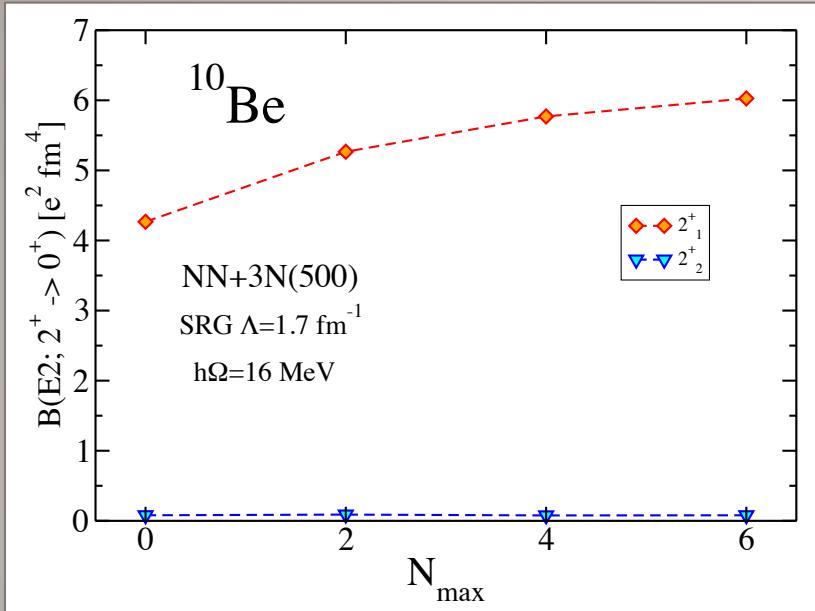
Ab initio calculation



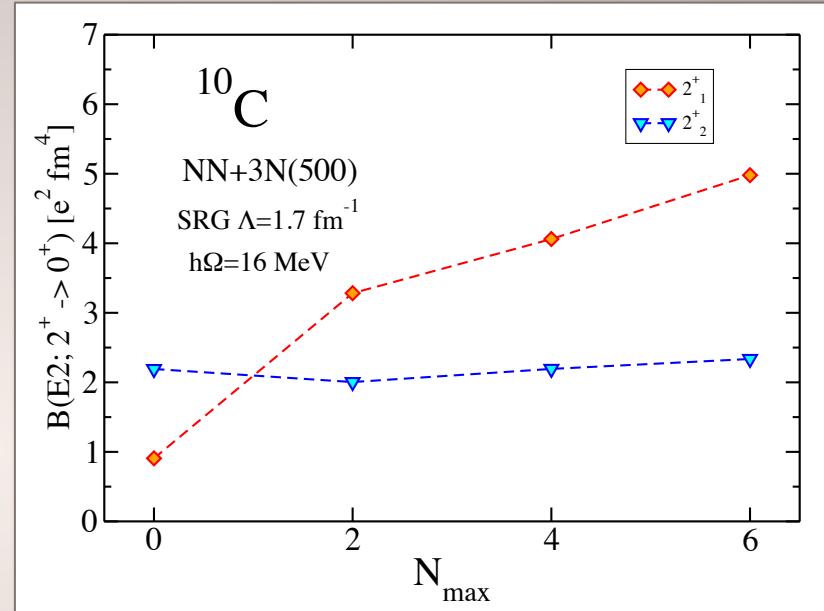
Unitarity condition:
 $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.99995 \pm 0.00061$

Useful to revisit:
 ^{10}C constraints the scalar current

E2 transitions in ^{10}Be and ^{10}C



$Q(2^+_1) < 0 ; Q(2^+_2) > 0$



$Q(2^+_1) < 0 ; Q(2^+_2) > 0$

Expt.: Phys. Rev. Lett. **103**, 192501 (2009)
 $2^+_1: 9.2(3) e^2 \text{ fm}^4 ; 2^+_2: 0.11(2) e^2 \text{ fm}^4$

GFMC: AV18+IL7
 $2^+_1: 8.8(4) e^2 \text{ fm}^4 ; 2^+_2: 1.8(1) e^2 \text{ fm}^4$

Expt.: arXiv: 1201.2960 [nucl-ex]
 $2^+_1: 8.8(3) e^2 \text{ fm}^4 ; 2^+_2: ?$

GFMC: AV18+IL7
 $2^+_1: 15.3(1.4) e^2 \text{ fm}^4 ; 2^+_2: 0.2(1) e^2 \text{ fm}^4$

Chiral NN+3N appears to do better than AV18+IL7 for E2 transitions in $A=10$



“Anomalous Long Lifetime of Carbon-14”

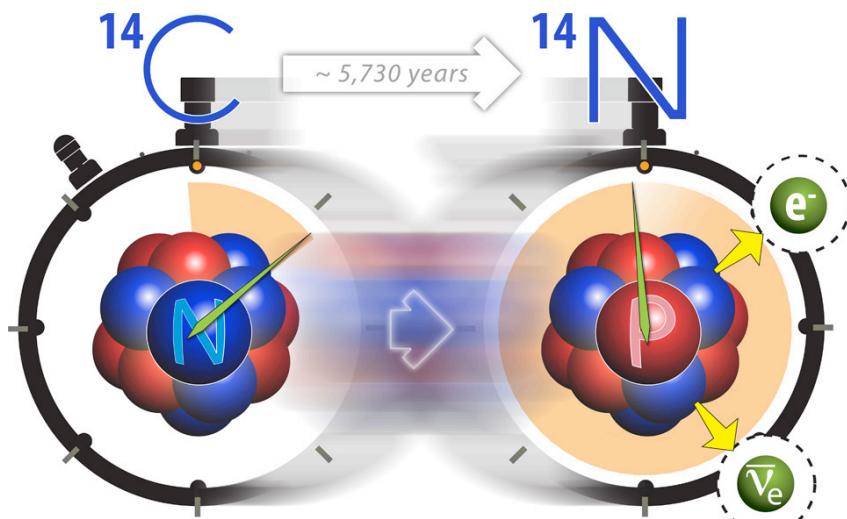


Objectives

- Solve the puzzle of the long but useful lifetime of ^{14}C
- Determine the microscopic origin of the suppressed β -decay rate

Impact

- Establishes a major role for strong 3-nucleon forces in nuclei
- Verifies accuracy of *ab initio* microscopic nuclear theory
- Provides foundation for guiding DOE-supported experiments



PRL 106, 202502 (2011)

PHYSICAL REVIEW LETTERS

week ending
20 MAY 2011

Origin of the Anomalous Long Lifetime of ^{14}C

P. Maris,¹ J. P. Vary,¹ P. Navrátil,^{2,3} W. E. Ormand,^{3,4} H. Nam,⁵ and D. J. Dean⁵

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TRIUMF

UNEDF SciDAC Collaboration
Universal Nuclear Energy Density Functional

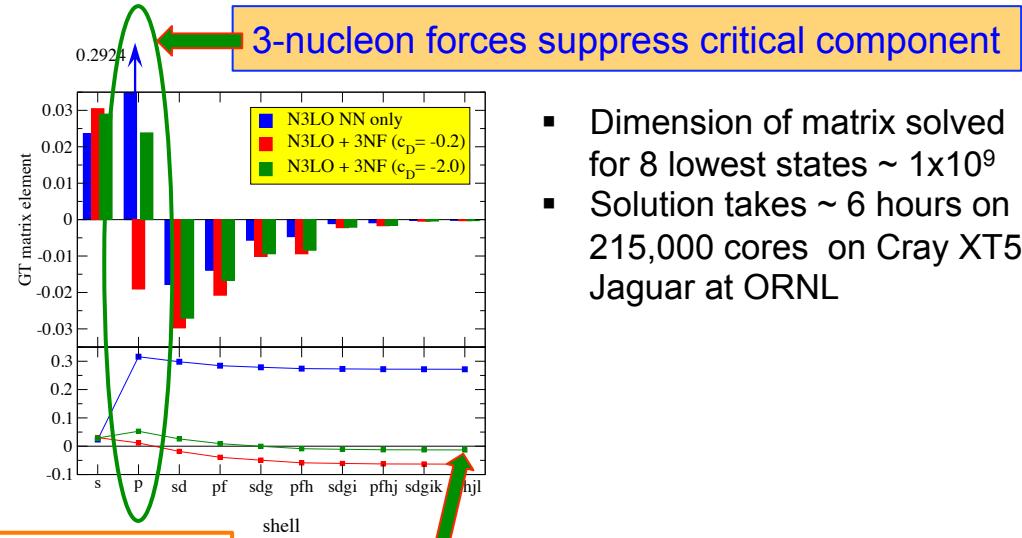
NNSA



LAWRENCE
LIVERMORE
NATIONAL SECURITY
LABORATORY



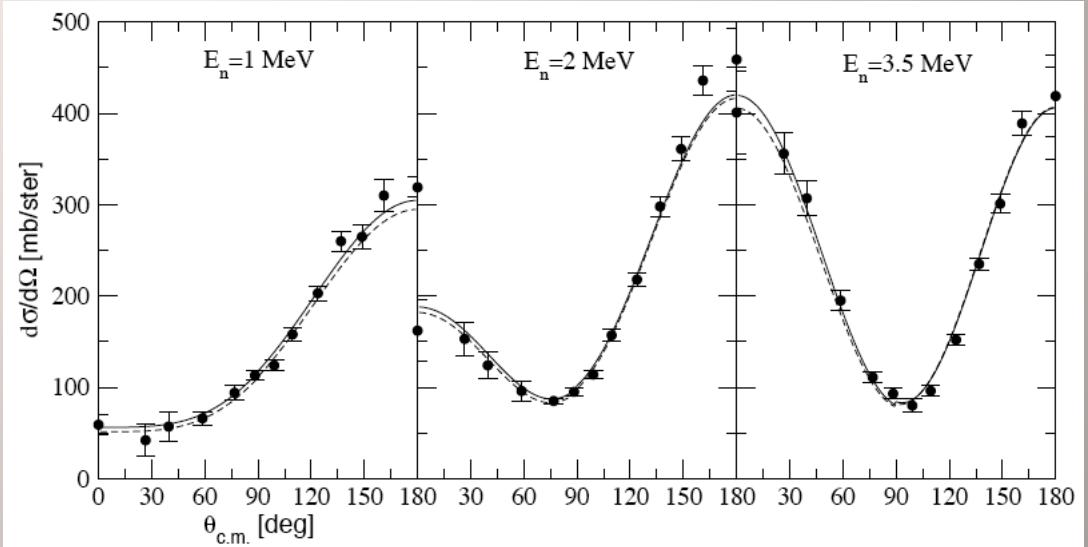
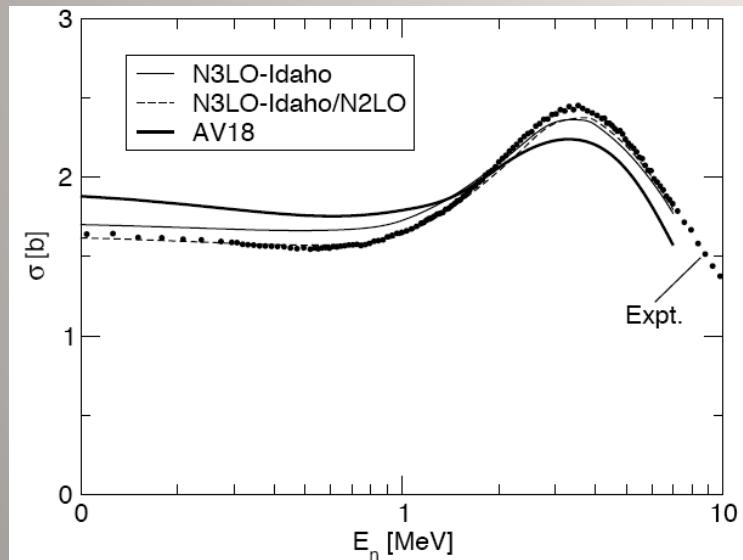
OAK RIDGE NATIONAL LABORATORY
Managed by UT-Battelle for the Department of Energy



Neutron-triton elastic scattering with χ EFT NN+NNN

- Hypherspherical-harmonics variational calculations
 - M. Viviani, A. Kievski, L. Girlanda, L. E. Marcucci, Few Body Syst. **45**, 119-121 (2009); arXiv:0812.3547
- Chiral N³LO NN plus local chiral N²LO NNN

A=3 binding energy constraint, $c_D=+1$, $c_E=-0.029$, $\Lambda=500$ MeV

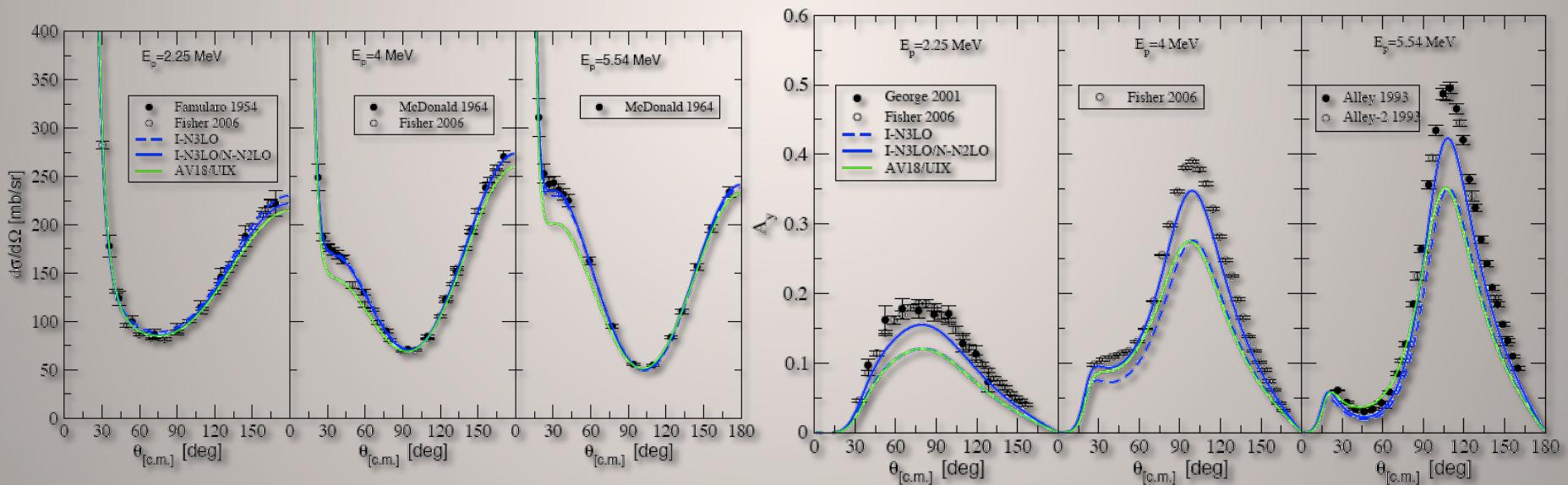


Cross sections obtained using chiral NN+NNN interaction are in the best agreement with experiment

Proton- ^3He elastic scattering with $\chi\text{EFT NN} + \text{NNN}$

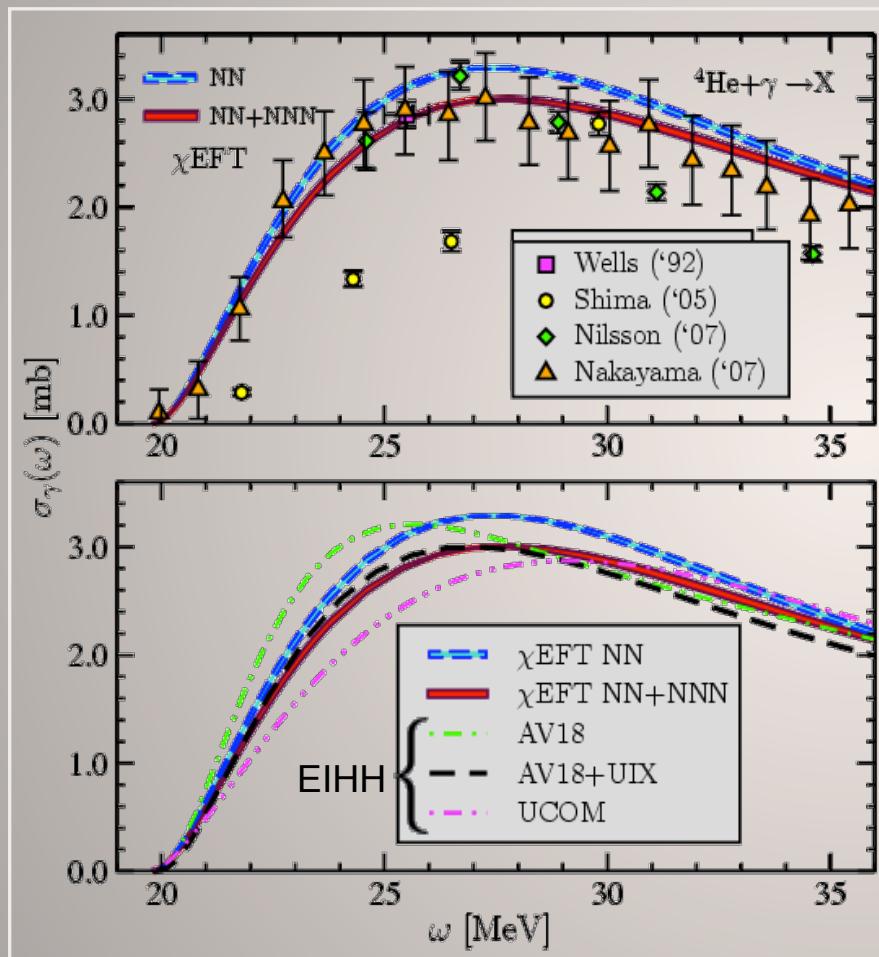
- Hypherspherical-harmonics variational calculations
 - M. Viviani, L. Girlanda, A. Kievski, L. E. Marcucci, and S. Rosati, arXiv:1004.1306
- A_y puzzle resolved with the chiral N³LO NN plus local chiral N²LO NNN
 - developed for and used within the NCSM

$A=3$ binding energy constraint,
 $c_D=+1$, $c_E=-0.029$, $\Lambda=500$ MeV



Chiral NN+NNN Hamiltonian provides the best agreement with the cross section and analyzing power data and with the new TUNL PSA analysis

^4He photo-absorption cross section with $\chi\text{EFT NN} + \text{NNN}$ forces



- Still large discrepancies between different experimental data
 - up to 100% disagreement on the peak-height

The NNN force induces a suppression of the peak

Overall better agreement with recent data by Nakayama *et al.*

In the peak region $\chi\text{EFT NN+NNN}$ and AV18 + UIX curves are relatively close:

- expect larger effects in p -shell nuclei

A=3 binding energy constraint,
 $c_D = -1$, $c_E = -0.331$, $\Lambda = 500$ MeV

S. Quaglioni and P. N.,
Phys. Lett. B 652 (2007) 37

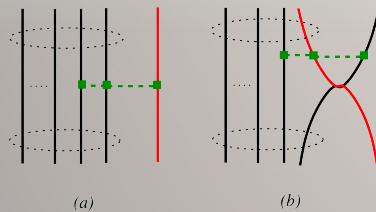
Sizable effect of NNN force. However, differences in the realistic calculations far below the experimental uncertainties: urgency for further experimental activity to clarify the situation.

Including 3N interaction in the NCSM/RGM Single-nucleon projectile:

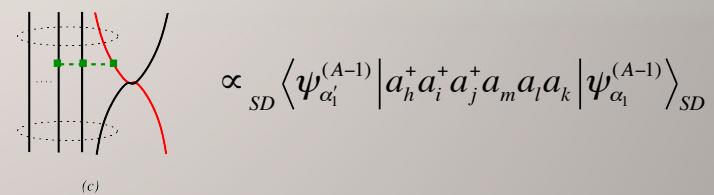
$$\left\langle \Phi_{\nu' r'}^{J^\pi T} \left| \hat{A}_{\nu'} V^{NNN} \hat{A}_\nu \right| \Phi_{\nu r}^{J^\pi T} \right\rangle = \left\langle \begin{array}{c} (A-1) \\ \text{red} \quad \text{blue} \\ \text{---} \quad \text{---} \\ r' \quad (a'=1) \end{array} \right| V^{NNN} \left(1 - \sum_{i=1}^{A-1} \hat{P}_{iA} \right) \left| \begin{array}{c} (A-1) \\ \text{red} \quad \text{blue} \\ \text{---} \quad \text{---} \\ (a=1) \quad r \end{array} \right\rangle$$

$$\mathcal{V}_{\nu' \nu}^{NNN}(r, r') = \sum R_{n'l'}(r') R_{nl}(r) \left[\frac{(A-1)(A-2)}{2} \left\langle \Phi_{\nu' n'}^{J^\pi T} \left| V_{A-2A-1A} (1 - 2P_{A-1A}) \right| \Phi_{\nu n}^{J^\pi T} \right\rangle \right. \\ \left. - \frac{(A-1)(A-2)(A-3)}{2} \left\langle \Phi_{\nu' n'}^{J^\pi T} \left| P_{A-1A} V_{A-3A-2A-1} \right| \Phi_{\nu n}^{J^\pi T} \right\rangle \right].$$

Direct potential: in the model space
(interaction is localized!)



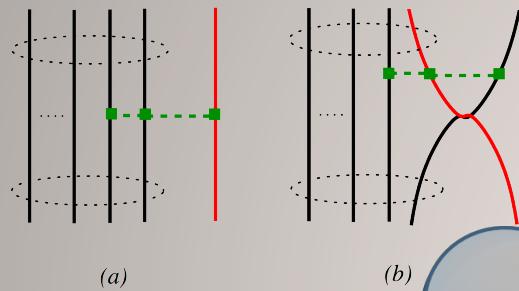
Exchange potential: in the model space
(interaction is localized!)



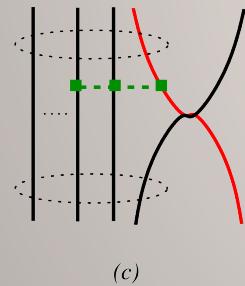
$$\propto_{SD} \left\langle \psi_{\alpha'_1}^{(A-1)} \left| a_i^+ a_j^+ a_l a_k \right| \psi_{\alpha_1}^{(A-1)} \right\rangle_{SD}$$

Including 3N interaction challenging: more than 2 body density required

Including 3N interaction in the NCSM/RGM: Direct and exchange terms



(b)



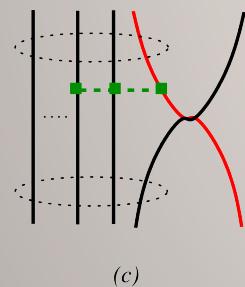
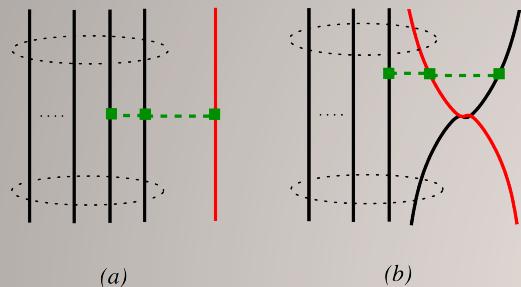
G. Hupin: Kernel derivations
with many-body densities.

Use of existing codes:
Applicable to $A=3,4$ targets

$$\begin{aligned}
 & \sum \hat{s}\hat{s}'\hat{j}\hat{j}'\hat{\tau}\hat{K} (-1)^{J'_1+j'+J}(-1)^{j_0+J_0-j+2j'}(-1)^{T_1+1/2+T}(-1)^{1/2+T_0+t_0+1} \\
 & \left\{ \begin{array}{ccc} J_1 & 1/2 & s \\ l & J & j \end{array} \right\} \left\{ \begin{array}{ccc} J'_1 & 1/2 & s' \\ l' & J & j' \end{array} \right\} \\
 & \left\{ \begin{array}{ccc} J_1 & K & J'_1 \\ j' & J & j \end{array} \right\} \left\{ \begin{array}{ccc} j'_0 & j_0 & K \\ j & j' & J_0 \end{array} \right\} \\
 & \left\{ \begin{array}{ccc} T_1 & \tau & T'_1 \\ 1/2 & T & 1/2 \end{array} \right\} \left\{ \begin{array}{ccc} t'_0 & t_0 & \tau \\ 1/2 & 1/2 & T_0 \end{array} \right\} \\
 & \left\langle \left[(n'_a l'_a j'_a : n'_b l'_b j'_b) j'_0 t'_0 : n' l' j' \right] J_0 T_0 \right| V_{A-2A-1A} (1 - 2P_{A-1A}) \left| \left[(n_a l_a j_a : n_b l_b j_b) j_0 t_0 : n l j \right] J_0 T_0 \right\rangle \\
 & \left\langle \alpha'_{A-1} J'_1 T'_1 \left| \left| \left| \left[\left[a_{n'_a l'_a j'_a}^\dagger a_{n'_b l'_b j'_b}^\dagger \right]^{j'_0 t'_0} \left[\tilde{a}_{n_a l_a j_a} \tilde{a}_{n_b l_b j_b} \right]^{j_0 t_0} \right]^{K\tau} \right| \right| \alpha_{A-1} J_1 T_1 \right\rangle
 \end{aligned}$$

$$\begin{aligned}
 & \sum \hat{s}\hat{s}'\hat{j}\hat{j}'\hat{\tau}\hat{K} \hat{J}_0 \hat{T}_0 \hat{g}' \hat{t}'_g \hat{j}'_0 \hat{t}'_0 \hat{k}' \hat{t}'_k (-1)^{J'_1+j'+J} (-1)^{j+j'_a+j'_b+j'_0+J_0+k'} (-1)^{T_1+1/2+T} (-1)^{1-T_0-\tau+t'_0+t'_k} \\
 & \left\{ \begin{array}{ccc} J_1 & 1/2 & s \\ l & J & j \end{array} \right\} \left\{ \begin{array}{ccc} J'_1 & 1/2 & s' \\ l' & J & j' \end{array} \right\} \\
 & \left\{ \begin{array}{ccc} J_1 & K & J'_1 \\ j' & J & j \end{array} \right\} \left\{ \begin{array}{ccc} g' & J_0 & K \\ j'_0 & k' & j'_b \end{array} \right\} \left\{ \begin{array}{ccc} k' & j'_0 & K \\ j' & j & j'_a \end{array} \right\} \\
 & \left\{ \begin{array}{ccc} T_1 & \tau & T'_1 \\ 1/2 & T & 1/2 \end{array} \right\} \left\{ \begin{array}{ccc} t'_g & T_0 & \tau \\ t'_0 & t'_k & 1/2 \end{array} \right\} \left\{ \begin{array}{ccc} t'_k & t'_0 & \tau \\ 1/2 & 1/2 & 1/2 \end{array} \right\} \\
 & \left\langle \left[(n' l' j' : n'_a l'_a j'_a) j'_0 t'_0 : n'_b l'_b j'_b \right] J_0 T_0 \right| V_{A-3A-2A-1} \left| \left[(n_\alpha l_\alpha j_\alpha : n_a l_a j_a) j_0 t_0 : n_b l_b j_b \right] J_0 T_0 \right\rangle \\
 & \left\langle \alpha'_{A-1} J'_1 T'_1 \left| \left| \left| \left[\left[a_{n'_a l'_a j'_a}^\dagger a_{n'_b l'_b j'_b}^\dagger \right]^{k' t'_k} a_{n'_b l'_b j'_b}^\dagger \right]^{g' t'_g} \left\{ \left[\tilde{a}_{n_\alpha l_\alpha j_\alpha} \tilde{a}_{n_a l_a j_a} \right]^{j_0 t_0} \tilde{a}_{n_b l_b j_b} \right\}^{J_0 T_0} \right]^{K\tau} \right| \right| \alpha_{A-1} J_1 T_1 \right\rangle
 \end{aligned}$$

Including 3N interaction in the NCSM/RGM: Direct and exchange terms

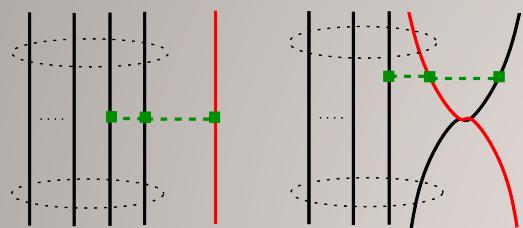


J. Langhammer:

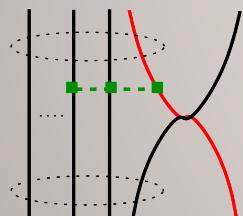
Kernel derivations without the angular momentum
re-coupling and the many-body density factorization.

Kernel calculations directly from the target eigenvectors:
Applicable to p -shell nuclei targets
The same strategy possible for multi-nucleon projectiles
and $A > 4$ targets

Including 3N interaction in the NCSM/RGM: Direct and exchange terms



(b)



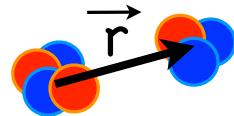
Computational challenge:
Large scale parallelization,
target eigenvectors for
multiple M values

$$\begin{aligned} & \langle \epsilon_{\nu'n'}^{J\pi T} | \hat{V}_{A-2 A-1 A} (1 - \hat{T}_{A-1,A} - \hat{T}_{A-2,A}) | \epsilon_{\nu n}^{J\pi T} \rangle \\ &= \sum_{M_1 m_j} \sum_{M_{T_1} m_t} \left(\begin{array}{cc} I_1 & j \\ M_1 & m_j \end{array} \middle| \begin{array}{c} J \\ M_J \end{array} \right) \left(\begin{array}{cc} T_1 & \frac{1}{2} \\ M_{T_1} & m_t \end{array} \middle| \begin{array}{c} T \\ M_T \end{array} \right) \\ & \quad \sum_{M'_1 m'_j} \sum_{M'_{T_1} m'_t} \left(\begin{array}{cc} I'_1 & j' \\ M'_1 & m'_j \end{array} \middle| \begin{array}{c} J \\ M'_J \end{array} \right) \left(\begin{array}{cc} T'_1 & \frac{1}{2} \\ M'_{T_1} & m'_t \end{array} \middle| \begin{array}{c} T \\ M'_T \end{array} \right) \\ & \quad \frac{1}{2(A-1)(A-2)} \sum_{\beta_{A-2}} \sum_{\beta_{A-1}} \sum_{\beta'_{A-2}} \sum_{\beta'_{A-1}} \\ & \quad \langle \Psi' I'_1 M'_1 T'_1 M'_{T_1} | \hat{a}_{\beta_{A-1}}^\dagger \hat{a}_{\beta_{A-2}}^\dagger \hat{a}_{\beta'_{A-2}} \hat{a}_{\beta'_{A-1}} | \Psi I_1 M_1 T_1 M_{T_1} \rangle \\ & \quad {}_a \langle \beta_{A-2} \beta_{A-1} n' l' j' m'_j m'_t | \hat{V} | \beta'_{A-2} \beta'_{A-1} n l j m_j m_t \rangle_a \end{aligned}$$

$$\begin{aligned} & \langle \epsilon_{\nu'n'}^{J\pi T} | \hat{V}_{A-3 A-2 A} \hat{T}_{A-1,A} | \epsilon_{\nu n}^{J\pi T} \rangle \\ &= \sum_{M_1 m_j} \sum_{M_{T_1} m_t} \left(\begin{array}{cc} I_1 & j \\ M_1 & m_j \end{array} \middle| \begin{array}{c} J \\ M_J \end{array} \right) \left(\begin{array}{cc} T_1 & \frac{1}{2} \\ M_{T_1} & m_t \end{array} \middle| \begin{array}{c} T \\ M_T \end{array} \right) \\ & \quad \sum_{M'_1 m'_j} \sum_{M'_{T_1} m'_t} \left(\begin{array}{cc} I'_1 & j' \\ M'_1 & m'_j \end{array} \middle| \begin{array}{c} J \\ M'_J \end{array} \right) \left(\begin{array}{cc} T'_1 & \frac{1}{2} \\ M'_{T_1} & m'_t \end{array} \middle| \begin{array}{c} T \\ M'_T \end{array} \right) \\ & \quad \frac{1}{6} \frac{1}{(A-1)(A-2)(A-3)} \sum_{\beta_{A-3}} \sum_{\beta_{A-2}} \sum_{\beta'_{A-3}} \sum_{\beta'_{A-2}} \sum_{\beta'_{A-1}} \\ & \quad \langle \Psi' I'_1 M'_1 T'_1 M'_{T_1} | \hat{a}_{nljm_j \frac{1}{2}m_t}^\dagger \hat{a}_{\beta_{A-2}}^\dagger \hat{a}_{\beta_{A-3}}^\dagger \hat{a}_{\beta'_{A-3}} \hat{a}_{\beta'_{A-2}} \hat{a}_{\beta'_{A-1}} | \Psi I_1 M_1 T_1 M_{T_1} \rangle \\ & \quad {}_a \langle \beta_{A-3} \beta_{A-2} n' l' j' m'_j \frac{1}{2} m'_t | \hat{V}_{A-3 A-2 A} | \beta'_{A-3} \beta'_{A-2} \beta'_{A-1} \rangle_a \end{aligned}$$

New developments: NCSM with continuum

NCSM/RGM



The idea behind the NCSMC

$$|\Psi_A^{J^\pi T}\rangle = \sum_\nu \int d\vec{r} \chi_\nu(\vec{r}) \hat{A} \Phi_{\nu\vec{r}}^{J^\pi T(A-a,a)}$$

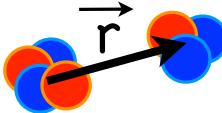
$$\mathcal{H}\chi = E\mathcal{N}\chi$$

$$\bar{\chi} = \mathcal{N}^{+\frac{1}{2}}\chi$$

$$(\mathcal{N}^{-\frac{1}{2}}\mathcal{H}\mathcal{N}^{-\frac{1}{2}})\bar{\chi} = E\bar{\chi}$$

New developments: NCSM with continuum

NCSM/RGM



The idea behind the NCSMC

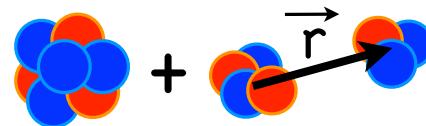
$$|\Psi_A^{J^\pi T}\rangle = \sum_\nu \int d\vec{r} \chi_\nu(\vec{r}) \hat{A} \Phi_{\nu\vec{r}}^{J^\pi T(A-a,a)}$$

$$\mathcal{H}\chi = E\mathcal{N}\chi$$

$$\bar{\chi} = \mathcal{N}^{+\frac{1}{2}}\chi$$

$$(\mathcal{N}^{-\frac{1}{2}}\mathcal{H}\mathcal{N}^{-\frac{1}{2}})\bar{\chi} = E\bar{\chi}$$

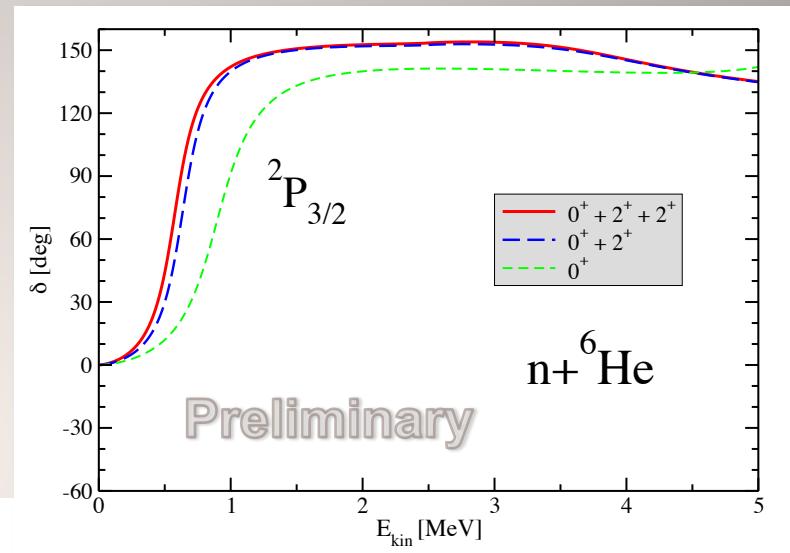
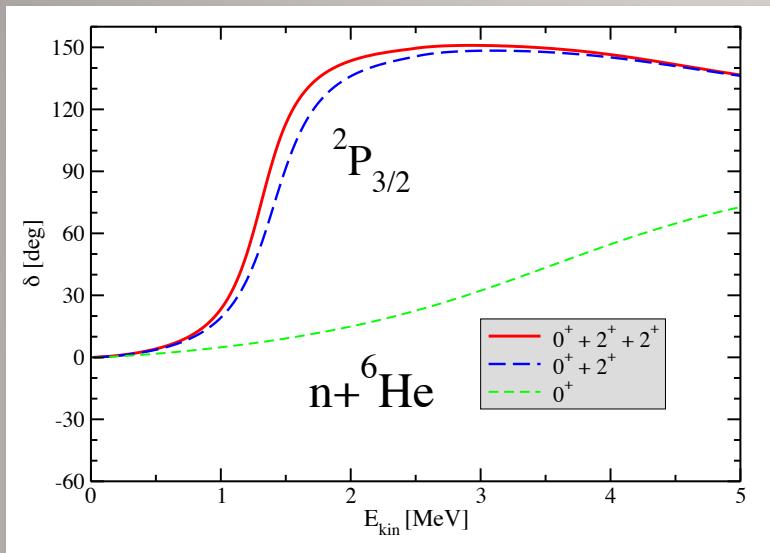
NCSMC



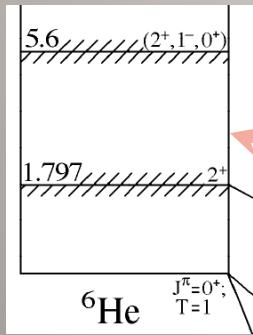
$$|\Psi_A^{J^\pi T}\rangle = \sum_\lambda c_\lambda |A\lambda J^\pi T\rangle + \sum_\nu \int d\vec{r} \left(\sum_{\nu'} \int d\vec{r}' \mathcal{N}_{\nu\nu'}^{-\frac{1}{2}}(\vec{r}, \vec{r}') \bar{\chi}_{\nu'}(\vec{r}') \right) \hat{A} \Phi_{\nu\vec{r}}^{J^\pi T(A-a,a)}$$

$$\begin{pmatrix} H_{NCSM} & \bar{h} \\ \bar{h} & \mathcal{N}^{-\frac{1}{2}}\mathcal{H}\mathcal{N}^{-\frac{1}{2}} \end{pmatrix} \begin{pmatrix} c \\ \bar{\chi} \end{pmatrix} = E \begin{pmatrix} 1 & \bar{g} \\ \bar{g} & 1 \end{pmatrix} \begin{pmatrix} c \\ \bar{\chi} \end{pmatrix}$$

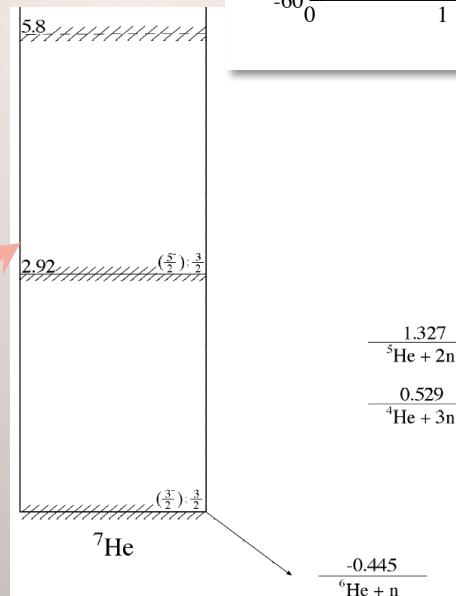
NCSM with continuum: ${}^7\text{He} \leftrightarrow {}^6\text{He} + n$



NCSM/RGM
with up to three ${}^6\text{He}$ states

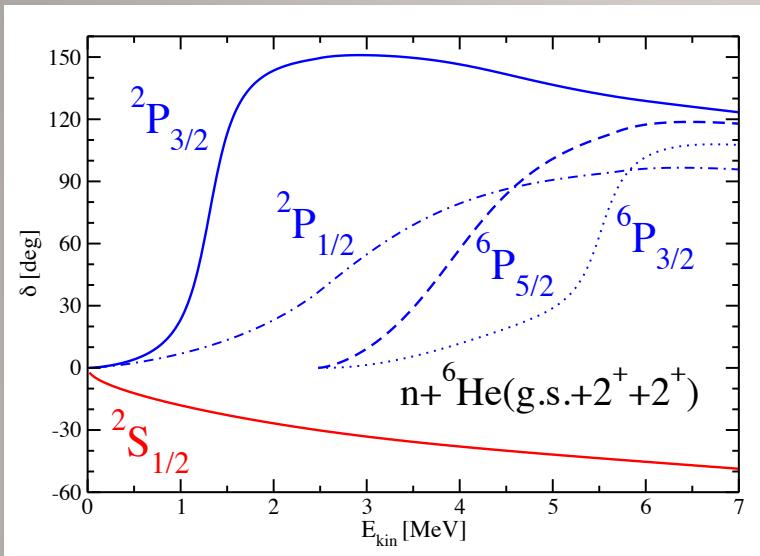


Expt.

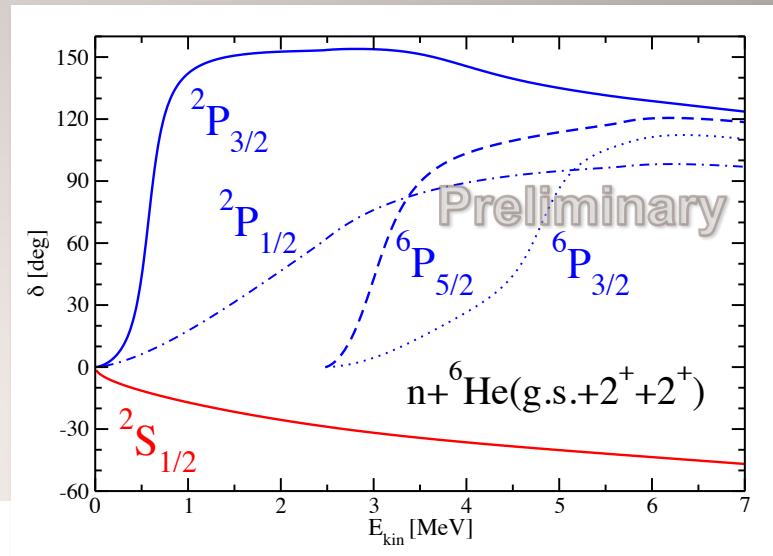
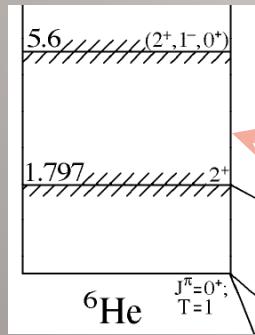


NCSMC
with up to three ${}^6\text{He}$ states
and three ${}^7\text{He}$ eigenstates
More **7-nucleon correlations**
Fewer target states needed

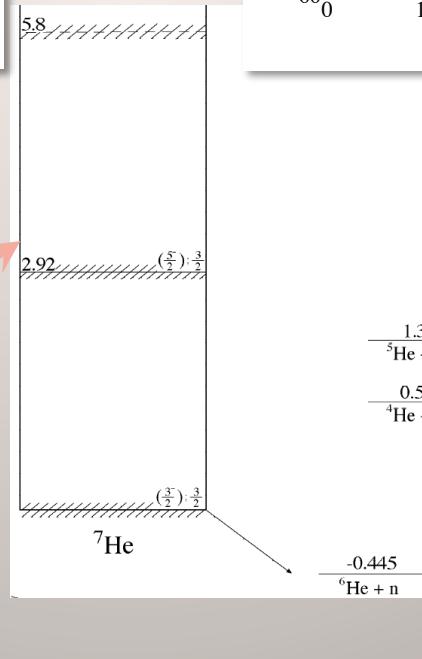
NCSM with continuum: ${}^7\text{He} \leftrightarrow {}^6\text{He} + n$



NCSM/RGM
with three ${}^6\text{He}$ states



NCSMC
with three ${}^6\text{He}$ states
and three ${}^7\text{He}$ eigenstates
More 7-nucleon correlations
Fewer target states needed



Experimental controversy:
Existence of low-lying $1/2^-$ state
... not seen in this calculations

Conclusions

- Chiral N³LO NN interaction plus local chiral N²LO NNN interaction with consistent LECs currently the best Hamiltonian for nuclei $A \leq 11$
 - Binding energies and radii of ^3H , ^3He , ^4He
 - Half-life of ^3H
 - proton- ^3He and neutron- ^3H elastic scattering
 - Photodisintegration of ^4He
 - Spectra and binding energies of p -shell nuclei
- However:
 - Overbinds for $A \geq 12$ nuclei
 - “overcorrects” spin-orbit sensitive observables ($1^+ 0$ state in ^{12}C too low...)
- Lowering the cutoff to 400 MeV and re-fitting c_E to ^4He binding energy:
 - Accurate binding energies for ^{12}C , ^{16}O , ^{24}O , ^{40}Ca , ^{48}Ca
 - overcorrection problem goes away
- Our knowledge of the 3N interaction is incomplete
 - Include N³LO NNN terms ...

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