SIS100 Dipole Magnet Design
Evaluation of the Field Quality

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Abstract

The SIS100 main dipole magnet has been thoroughly designed and optimised over the last years. The field quality of the magnet as described in the Technical Design Report was reinvestigated and its sources of field distortion were found. Based on these sources a insert for the magnet end was proposed and different methods already investigated which allow optimising the end profile shape so that a minimum total distortion is reached.

1 Introduction

SIS100: MAC History I/II

March 2009 status of the SIS100 magnets

- full size magnets, measured performance: field quality [1, 2], AC losses [3] → cycle repetition rate 0.3 Hz instead of 1 Hz
- vacuum chamber [4]
- curved single layer dipole: necessity [5], high current cable [6], model magnet, (2 turn quadrupole [7])

July 2009 SIS100 magnets: mechanical stability

- wire contact,
- cable: tube leak rate, measured stability [8], cabling machine [9]
- coil: support structure, measured stability [8],
- curved single layer dipole

SIS100: MAC History II/II

February 2010 SIS100 dipole magnet status, status report [10]

- magnet components
- mechanical stability
- magnetic steel selection [11]
- magnetic field design, measurement vs. calculation [1, 2, 11]
- cooling, AC losses, hydraulics
2 Theory

The horizontal plane (transverse to the beam) is given by the axes \( x \) and \( y \). Plane circular multipoles within this presentation follow the “European convention”, thus the complex magnetic field \( B(z) \) of the complex coordinate \( z = x + i(y) \) is given by

\[
B_y(x,y) + iB_x(x,y) = B(z) = \sum_{n=1}^{\infty} C_n \left( \frac{z}{R_{\text{Ref}}} \right)^{n-1},
\]

with \( z = x + iy \). The coefficients \( C_n = B_n + iA_n \) are presented here as normalised dimensionless coefficients \( c_n \). These are given by

\[
c_n = \frac{C_n}{C_m} \cdot 10^4
\]

with \( m \) the main multipole (\( m = 1 \ldots \text{dipole} \)). While the \( c_n \) are dimensionless, they are given in units with 1 unit = 100ppm.

2.1 Advanced Multipoles

2.1.1 Elliptic multipoles

Plane elliptic coordinates

\[
x = e \cosh \eta \cos \psi \\
y = e \sinh \eta \sin \psi.
\]

Confocal ellipses
\( \eta = \text{const.} \) (\( \cong \text{circles} \)).

Confocal hyperbolas
\( \psi = \text{const.} \) (\( \cong \text{radials} \)).

e = eccentricity foci:
\( \eta = 0, \ \psi = 0, \pi \)

\( C_{\text{ref}} : \text{ellipse} \ \eta = \eta_0, \)
\( a = e \cosh \eta_0, \ b = e \sinh \eta_0. \)
Elliptic Multipoles for Complex Magnetic Field

Complete system:

\[ \cosh[n(\eta + i\psi)], \quad n = \text{integer} \geq 0. \]

\[ \left| \frac{\cosh[n(\eta + i\psi)]}{\cosh(n\eta_0)} \right| \leq 1 \quad \text{for} \quad \eta \leq \eta_0. \]

Field expansion:

\[ w = \eta + i\psi \]

\[ B(w) = \frac{e_0}{2} + \sum_{n=1}^{\infty} e_n \frac{\cosh[n(\eta + i\psi)]}{\cosh(n\eta_0)} \]

Expansion coefficients:

\[ e_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} B(w = e \cosh(\eta_0 + i\psi)) \times \cos(n\psi) \, d\psi. \]

From Elliptic to Circular Multipoles

- Circular multipoles widespread (e.g. beam dynamics)
- calculation of the multipoles:
  - elliptic multipoles \( A_n, B_n \)
  - the inverse transformation matrix \( \hat{S} \) elliptic \( \rightarrow \) circular
- calculating \( \hat{S} \)
  - insert elliptic coordinates (7) and (8) into \((z/R_0)^m\) and \((z^*/R_0)^m\); \( m \geq 0 \).
  - expand resulting expression in harmonics of hyperbolic and trigonometric functions \( \rightarrow ce_n(\eta, \phi), se_n(\eta, \phi) \)
  - invert the matrix \( \hat{T} \rightarrow \hat{S} \)

Relations Elliptic \( \iff \) Circular Multipoles

\[ c_m(e/R_{ref})^m = \sum_{k=0}^{M} (e_k/\cosh(k\eta_0)) \, t_{km}, \]

\[ k = 0, 1, 2, \ldots, M, \quad m = 0, 1, 2, \ldots, M. \]

\( M \) = Number of multipoles used.
Closed analytic expressions for $t_{km}$, too complex to be shown here given in \[12\].

The conversion matrices are given for two sets of ellipses which were used for SIS100.

\[
a = 5.75, \quad b = 3.0, \quad R_{Ref} = 4.0
\]

\[
\begin{array}{cccccccccc}
\hline
m & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\hline
0 & 1.00 & -0.57 & 0.20 & -0.06 & 0.02 \\
1 & 0.70 & -0.84 & 0.45 & -0.20 & 0.08 \\
2 & 0.76 & -1.04 & 0.74 & -0.42 \\
3 & 0.74 & -1.20 & 1.06 & -0.71 \\
4 & 0.69 & -1.32 & 1.38 \\
5 & 0.64 & -1.41 & 1.70 \\
6 & 0.58 & -1.47 \\
7 & 0.53 & -1.51 \\
8 & 0.49 & \\
9 & 0.45 \\
\hline
\end{array}
\]

\[
a = 4.5, \quad b = 1.7, \quad R_{Ref} = 4.0
\]

\[
\begin{array}{cccccccccc}
\hline
m & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\hline
0 & 1.00 & -0.75 & 0.39 & -0.18 & 0.08 \\
1 & 0.89 & -1.60 & 1.29 & -0.83 & 0.48 \\
2 & 1.38 & -2.89 & 3.03 & -2.45 \\
3 & 1.97 & -4.76 & 6.11 & -5.93 \\
4 & 2.66 & -7.45 & 11.29 \\
5 & 3.51 & -11.26 & 19.68 \\
6 & 4.58 & -16.65 \\
7 & 5.93 & -24.19 \\
8 & 7.67 & \\
9 & 9.91 \\
\hline
\end{array}
\]
Measurement positions

not the whole ellipse is covered: measurement combination: weight calculated cubic polynomial

Combining the measurements

- measurement accuracy → relative better than absolute?
- match them at the interlapping position
- reconstruct the field on the ellipse weighting the measurements
- calculate elliptic and circular harmonics

full description and application [12, 13, 14]

Measured vs calculated

SIS100: straight model magnet
black...measured; yellow, green...calculated [1, 2]
2.1.2 Multipoles adapted to toroids

Local toroidal coordinates

\[ X + i Y = R_C \, h \, e^{i \phi}, \quad (3) \]
\[ Z = R_{Ref} \, \sin \theta, \quad (4) \]
\[ h = 1 + \epsilon \, \rho \, \cos \theta; \quad (5) \]
\[ \epsilon = \frac{R_{Ref}}{R_C}. \quad (6) \]

\( R_C \) major radius = radius of curvature;
\( R_{Ref} \) minor radius = reference radius;
\( \epsilon \) the inverse aspect ratio.

Local toroidal Multipoles

Toroidally uniform magnetic induction:

\[ \vec{B}(x, y) = \sum_{m=1}^{M} \left( \tilde{r}_m \, \vec{T}^{(n)}_m(x, y) + \tilde{s}_m \, \vec{T}^{(s)}_m(x, y) \right). \quad (7) \]

See COMPEL 2009 [15]

Toroidal Elliptic Multipoles

Potential Equation

\[ \Phi_{cn}(\bar{\eta}, \bar{\psi}) = (\bar{h})^{-1/2} \, c_n(\bar{\eta}, \bar{\psi}) + O(\bar{\epsilon}^2), \quad (8) \]
\[ = S(\bar{\eta}, \bar{\psi}) \, \cosh(n\bar{\eta}) \, \cos(n\bar{\psi}) + O(\bar{\epsilon}^2), \quad (9) \]
\[ \Phi_{sn}(\bar{\eta}, \bar{\psi}) = (\bar{h})^{-1/2} \, s_n(\bar{\eta}, \bar{\psi}) + O(\bar{\epsilon}^2), \quad (10) \]
\[ = S(\bar{\eta}, \bar{\psi}) \, \sinh(n\bar{\eta}) \, \sin(n\bar{\psi}) + O(\bar{\epsilon}^2). \quad (11) \]

with

\[ S(\bar{\eta}, \bar{\psi}) = (1 + \bar{\epsilon} \, \frac{1}{2} \, \cosh(n\bar{\eta}) \, \cos(n\bar{\psi})) \quad (12) \]

See IGTE 2010 [16]
2.2 Measuring toroidal multipoles

Excursus: measurement within a straight magnet  The output of a radial coil rotating with constant angular frequency $\omega$ in a magnetic field is Fourier decomposed as:

$$V(t) = -\omega \sum_{n=1}^{M} K_n \left( a_n \cos(n\omega t) + b_n \sin(n\omega t) \right),$$  \hspace{1cm} (13)

with $a_n$, $b_n$ the multipole components of the magnetic field. The upper limit of the sum, $M$ can be $\infty$. In practice a value between 10 and 20 is used. The sensitivity of a so called radial coil probe to the multipole $n$ is given by

$$K_n = \frac{NL}{n} \left[ \left( \frac{r_2}{R_{Ref}} \right)^n - \left( \frac{r_1}{R_{Ref}} \right)^n \right].$$  \hspace{1cm} (14)

with $L$ the length of the coil, $r_2$ and $r_1$ the other and inner radius of the coil \cite{17} and $R_{Ref}$ the reference radius.

Toroidal Circ. Multipoles: Measurement

$$a_n = -\sum_{m=1}^{M} \bar{s}_m C_{mn}$$

$$b_n = \sum_{m=1}^{M} \bar{r}_m D_{mn}$$  \hspace{1cm} (15)

rotating coil within curved magnet aperture

See IGTE 2010 \cite{16}, ASC10 \cite{18}
Matrices given above found by comparison and sorting of the coefficients. The relation between the two matrices is given by

\[(dC)_{m,1} = - \epsilon \left( \frac{d}{2R_{\text{Ref}}} \right)^m D = C + dC. \quad (16)\]

The matrix \(C\) consists of the conversion matrix \((C_{nm})\) can be written in the following form

\[C = I - \epsilon (U + D + L^{\text{co}} + L^{dr2} + L^{dr}), \quad (17)\]

with \(\epsilon = \frac{R_{\text{Ref}}}{R_C}, R_C, d\ldots\) horizontal offset of the coil.

Main terms: only \(U\) independent of \(d\). Toroidal “dipol” “feed up” to straight quadrupole...

\[U = \frac{m}{4(m-1)} \delta_{n,m+1} \quad (18)\]

<table>
<thead>
<tr>
<th>machine</th>
<th>(\epsilon)</th>
<th>(r_1 \rightarrow b_2)</th>
<th>(r_2 \rightarrow b_3)</th>
<th>(r_3 \rightarrow b_4)</th>
<th>(r_4 \rightarrow b_5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIS100</td>
<td>7.62</td>
<td>3.80</td>
<td>2.85</td>
<td>2.53</td>
<td>2.37</td>
</tr>
<tr>
<td>SIS18</td>
<td>65.</td>
<td>32.50</td>
<td>24.38</td>
<td>21.67</td>
<td>20.31</td>
</tr>
<tr>
<td>LHC</td>
<td>.06</td>
<td>.03</td>
<td>.02</td>
<td>.02</td>
<td>.02</td>
</tr>
</tbody>
</table>

SIS100: \(R_{\text{Ref}} = 40 \text{ mm}, R_C = 52.632 \text{ m}\)

SIS18: \(R_{\text{Ref}}^{\text{SIS18}} = 65 \text{ mm}, R_C = 10 \text{ m}\)

LHC: \(R_{\text{Ref}}^{\text{LHC}} = 17 \text{ mm}, R_C = 2804 \text{ m}\)

SIS18 /SIS100: only relevant feedup \(r_1 \rightarrow b_2\). Required for SIS18 reference data (section 3.1.1).

\(L^{\text{co}}\) describes the interaction of the coil length

\[L^{\text{co}} = \binom{n-2}{n-m-1} (n-1) \left( \frac{d}{R_{\text{Ref}}} \right)^{n-m-1} S^c \quad (19)\]
with $S^c$ given by

$$S^c = \left[ \frac{l^2}{24 R_{Ref}^2} - \frac{K_{m+2}}{4 (m+1) K_m} \right].$$  \hspace{1cm} (20)

$S^m$ describes the smear out of one toroidal multipole on the whole spectrum measured by the rotating coil depending on the curvature of the torus (described by $\epsilon$ and the coil geometry).

**Toroidal Circ. Multipoles: Measurement**

$$\epsilon L^{co} = \frac{l^2 \epsilon}{24 R_{Ref} d} (n - m) L^{dr}. \hspace{1cm} (21)$$

Offset: rotating coil in cylindric coodinates $\rightarrow L^{dr}$

rotating coil in cylindric torus $\rightarrow \approx \epsilon L^{co} + L^{dr}$

Coil size $\leftarrow$ given by expected axis offset

equalize feed down: cylindric and toroidal coordinates

$$l = \sqrt{\frac{24 d}{\epsilon R_C}} \approx \begin{cases} 0.8 m & \text{SIS100} \\ 0.6 m & \text{SIS18} \end{cases}$$  \hspace{1cm} (22)

Numerical values for $d = 1mm$

$R_C = 10 \text{ m for SIS18}, R_C \approx 52.5 \text{ m for SIS100};$ the numerical values are based on an expected offset of the coil from the coordinate centre (torus midline) by 1 mm.

These mathematics is needed later on to justify the comparison of the end field mappings of the SIS18 dipole to the calculated end fields of the SIS100 dipole.

### 2.3 Magnet field generation

Field generation per se is well understood since Maxwell’s theory was established and measurement equipment for material parameters developed.

A recall of the basis principles simplifies the motiviation and search steps in the magnetic field optimisation process.

**Magnetic field generation**

- air coil magnets: Biot-Savart law $\rightarrow \cos \theta$ magnets
- iron dominated magnets: refraction law of MQS

10
2D / 3D field generation

**Material boundary**

\[ \mu_a \rightarrow 1 \quad \mu_b \rightarrow \infty \]

\[ \lim_{\mu_b \rightarrow \infty} \frac{H_a}{\mu_b} = 0 \]

two solutions

- \( \tan(\alpha) \rightarrow 0: \mathcal{H} \) tangential
- \( \tan(\beta) \rightarrow \infty: \beta \rightarrow 90 \text{ Grad} \)
- field in medium \( a \) normal to the surface → iron dominated magnets

### 2.3.1 Iron dominated magnet

**Field: iron dominated dipol**

\[ H_g = \frac{NI}{\mu \frac{A_2}{A_1} l_1 + 2\mu \frac{A_0}{2} l_2 + g} \approx \frac{NI}{\mu \lim_{\mu \rightarrow \infty} g} \]

\[ B = t_f \cdot I \]

Example: SIS100 CSLD magnet:

\( g = 68 \text{ mm}, N = 8, I = 1 \text{kA} \rightarrow t_f \approx 147.84 \text{ mT / kA} \)

**Application: SIS100 CSLD**

\[ B = t_f \cdot I \]

Application: SIS100 “CSLD” magnet:

\( g = 68 \text{ mm}, N = 8, I = 1 \text{kA} \rightarrow t_f \approx 147.8 \text{ mT / kA} \)

SIS100 CSLD multipoles central part

<table>
<thead>
<tr>
<th>( I[kA] )</th>
<th>( B_1[T] )</th>
<th>( t_f[mT / kA] )</th>
<th>( B_1[T] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1319.1</td>
<td>-0.19502</td>
<td>-147.84</td>
<td>-0.1946</td>
</tr>
<tr>
<td>6595.5</td>
<td>-0.97508</td>
<td>-147.84</td>
<td>-0.9738</td>
</tr>
<tr>
<td>12531.4</td>
<td>-1.85264</td>
<td>-147.84</td>
<td>-1.8238</td>
</tr>
<tr>
<td>13191.0</td>
<td>-1.95015</td>
<td>-147.84</td>
<td>-1.9059</td>
</tr>
</tbody>
</table>

[19]
2.3.2 Air coil magnets

Pure Multipoles

End field

- coil → generates large sextupole
- compensated by iron in the near part
- → make negative sextupole in the iron → integrated reduced
- image currents ← $\mu_r \gg 1$

3 Field Quality

Field Quality Distortions

- Iron dominated
  - $\mu_r \gg 1$ pole shoe forms field homogeneity
  - iron: flux short cut
  - works only if iron not saturated
- current dominated (air coil)
  - coil head
  - its image current
3.1 SIS18: Reference

Comparison to SIS18

• Some specification
• not as detailed as e.g. CERN LHC 99-07 table
• define your own!

SIS18: Measurement setup

SIS18: $R_C = 10$ m, rectangle magnet, end angle $7.5^\circ$ length $\approx 2.62$ m, measurement length $\pm 0.5$ m, parallel to beam on exit

SIS18: Measurement procedure

• Magnetic field measured within a full box $l \pm 0.5m$, (mapper with Hall probe) only $B_y \rightarrow$ assumption: skew components negligible $\rightarrow$ larger area
• calculated on a straight elliptic cylinder $\rightarrow$ integral
• separate measurement (on ellipse) $\rightarrow$ field deviation in the central part $< 2$ units
• method to qualify the end field
• mapper $\rightarrow$ straight line, not beam path $\rightarrow$ standard correction $x \tan \psi \rightarrow$ quadrupole $\rightarrow 65mm \tan(7.5^\circ) \approx 85$ units
• multipoles calculated, quadrupole subtracted

\[ B_y \rightarrow \text{only normal components} \]

\[ B(z) = B_y(x + iy) + iB_x(x + iy) = \sum_{n=1}^{\infty} C_n \left( \frac{z}{R_{\text{Ref}}} \right)^{n-1} \]

\[ = \sum_{n=1}^{\infty} [B_n + iA_n] \left( \frac{r}{R_{\text{Ref}}} \right)^{n-1} (\cos \phi + i \sin \phi)^n \]

\[ = \sum_{m=0}^{\infty} r^m s \left[ B_n \cos(m\phi) - A_n \sin(m\phi) + iA_n \cos(m\phi) - iB_n \sin(m\phi) \right] \]

with \( m = n - 1 \) Therefore: Assumption skew components \( A_n \) negligible \[ (e^{i\phi})^m = e^{im\phi} = \cos(m\phi) + i \sin(m\phi) \]

Measurement results

![Graph 1](image1.png)

![Graph 2](image2.png)

with quadrupole

Central field \( \approx 1.0 \) T
Field integrated over 1 m: assumption
Field deviation – 8 units

without quadrupole
with quadrupole

Central field $\approx 1.75$ T

Field integrated over 1 m

Field deviation $\pm 20$ units

Difference $\leftrightarrow$ magnet saturated, $\frac{dB(x,y)}{dz}! = 0$ at the ends

3.1.1 SIS18: Reference data

Defining a reference radius for SIS18  The reference radius $R_{\text{Ref}}$ is the characteristic length (see also (I)). The eccentricity of an ellipse is given by

$$
\varepsilon = \sqrt{\frac{a^2 - b^2}{a^2}} = \sqrt{1 - \left(\frac{b}{a}\right)^2}
$$

(23)

and its focal length $f$ is given by

$$
f = \sqrt{a^2 - b^2} = \varepsilon a
$$

(24)

The coil aperture width of the CSLD for SIS100 is $\approx 71.42$ mm and the aperture height 34 mm. This gives a focal length $f$ of $\approx 62.8$ mm. These coil aperture dimensions and the defined good field region of $a = 57.5$ mm and $b = 30$ mm let to a reference radius $R_{\text{Ref}} = 40$ mm.

The coil aperture of the SIS18 magnet is $a \approx 71.42$ mm and $b = 45$ mm. This gives a focal length $f^{\text{SIS18}} \approx 107.4$ mm. Using the ratio $R_{\text{Ref}}/f$ one obtains

$$
R_{\text{Ref}}^{\text{SIS18}} = \frac{R_{\text{Ref}}}{f^{\text{SIS18}}} \approx 68.4 \approx 65$ mm
$$

(25)

Thus the multipoles for SIS18 are given for a reference radius of $R_{\text{Ref}}^{\text{SIS18}} = 65$ mm
Calculating the total harmonics  Beam dynamic calculations model magnets as lenses. Thus a straightforward approach is to compare the field quality of magnets for different machines is to calculate their total deviation they produce.

Using (1) and (2) a integral harmonic $C_n$ can be defined by

$$C_n = \int_{-\pi}^{\pi} \int_{-\infty}^{\infty} B(r, \phi, s) \, ds \, e^{-i n \phi} \, d\phi.$$  \hspace{1cm} (26)

with $s$ the longitudinal coordinate. $z = re^{i\phi}$. As $B$ fulfils Laplace equation one can interchange the integrals and define $C_n(s)$ by

$$C_n(s) = \int_{-\pi}^{\pi} B(r, \phi, s) \, ds \, e^{-i n \phi} \, d\phi.$$  \hspace{1cm} (27)

These $C_n(s)$ only fulfill the two dimensional Laplace equation, if the derivatives of the appropriate components is 0.

The magnet length of a dipole is defined by

$$B(0, 0, 0)l_m = \int_{-\pi}^{\pi} \int_{-\infty}^{\infty} B(r, \phi, s) \, ds \, e^{-i n \phi} \, d\phi.$$  \hspace{1cm} (28)

The normalised integral harmonics $c_n$ are then defined by

$$c_n = \frac{C_n}{Bl_m}.$$  \hspace{1cm} (29)

The field homogeneity of the SIS18 magnet within the centre region is ±2 units. The field of the central part was also mapped by the hall probe. Variations in the iron ($\mu_r$ curve, packing factor) give similar variations in $B_z$, thus the requirements for plane multipoles are not fulfilled. Therefore the integral (29) is split in three parts: a central part and two end parts. Here it is assumed that both end field are the same. Further it is assumed that all higher multipoles in the central part are zero.

Therefore the total normalised multipoles $c_n$ are calculated by

$$c_n = 2 \int_{-\pi}^{\pi} \int_{-\infty}^{\infty} B(r, \phi, s) \, ds \, e^{-i n \phi} \, d\phi.$$  \hspace{1cm} (30)

with $l_e$ the chosen limit of the end field. The measurement results show, that for SIS18 the central field will also contribute to the total field deviation.
SIS18: End field multipoles

<table>
<thead>
<tr>
<th>I [kA]</th>
<th>$B_1$ [T]</th>
<th>$b_3$</th>
<th>$b_4$</th>
<th>$b_5$</th>
<th>$b_6$</th>
<th>$b_7$</th>
<th>$b_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.0480</td>
<td>−5.72</td>
<td>0.01</td>
<td>−1.61</td>
<td>0.57</td>
<td>−1.25</td>
<td>−0.28</td>
</tr>
<tr>
<td>3.5</td>
<td>1.7236</td>
<td>−2.74</td>
<td>0.67</td>
<td>−1.72</td>
<td>1.13</td>
<td>−0.84</td>
<td>−0.42</td>
</tr>
</tbody>
</table>

$R_{SIS18}^{Ref} = 65$ mm

Measured Ellipse area: $a = 77.5$ mm, $b = 30.0$ mm

$B_1$ central dipole field

NB Measured straight, beam curved; appropriate description: Local Toroidal Elliptic Multipoles

It was shown above that the feed up of the toroidal multipoles will only spill into $b_2$, provided that the data need not to be corrected for an offset in the coordinate systems. So these values can be used for comparison, neglecting the appropriate description.

3.2 2D Field Quality

Field quality: 2D

- Large positive sextupole in the magnet ends
- 2D lamella reoptimised
- accepting a small negative sextupole in the middle

SIS100: 2D Lamella
The difference between the two designs is not too large. But one can see that a careful design allows reducing one harmonic in favour of another one.
3.3 3D Field Quality

All integral multipoles, presented below, are total harmonics with the same assumption as made for the SIS18 magnet: the central part does not contribute to the total harmonics.

3.3.1 End effects

FEM End model

- $2 \cdot \text{aperture width} \rightarrow dB_y/dz \approx 0$
- Only short end model
- considerably faster computation

Effect of the end coil loop height

- Used the coil as designed for the CSLD
- inserted several lengths in the straight part
  - original position... blue point
  - 10 mm up... green point
  - 20 mm up... red point
  - 40 mm up... cyan point
  - 100 mm up ... magenta point

Total effect $\rightarrow$ not too large
Effect of the end coil loop height

II/III

$b_3$
original position... blue points, 10 mm up... green diamond, 20 mm up... red diamond, 40 mm up... cyan diamond 100 mm up... magenta diamond
Small reduction in sextupole, small increase in dekapole, if too high, also bad

$b_5$

Effect of the end coil loop height

III/III

$b_7$
original position... blue points, 10 mm up... green diamond, 20 mm up... red diamond, 40 mm up... cyan diamond 100 mm up... magenta diamond Nearly no effect.
Effect of the end coil loop height

- coil loop height → small effect
- image currents?
- move it far away ...

The multipoles $c_n(s)$ were normalised by $C_n(s)/(B(0,0,0))$ (see also (27)).

End loop effect: cross check

$I/II$

$\begin{align*}
  b_3 & \text{ original (different field levels): solid line} \\
  \text{large end coil: dashed line} & \quad \text{multipoles → sextupole, dekapole, generated as by $\cos \theta$ magnets}
\end{align*}$

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End loop effect: cross check

3.3.2 Insert for field optimisation

End field optimisation

- magnet coil $\rightarrow$ troublemaker
- similar procedure as for $\cos \theta$ magnets
  - change end profile $\rightarrow$ generate multipoles of opposite shape
  - minimise the integral distortions
Insert: SIS18

block
taken out
Not directly applicable because of different flux flow (mixed H/window type ↔ window type)

Insert: SIS100 3D Sketch

end block & support
inserted
Insert: 2D Sketch

Flux flow change due to new air gap. The longitudinal length of the insert is rather large. The motivation for this length will be given below.

Insert: additional gaps

The insert shall be dismountable, these gaps are foreseen for a Kapton foil. That shall prevent the laminated blocks to get stuck.
without insert... blue point

a gap of 0.4 mm on the right ... green point

a gap of 0.2 mm at each side ... red point

without insert... blue point

a gap of 0.4 mm on the right ... green point
a gap of 0.2 mm at each side ... red point
The insert changes the total sextupole by less than 1 unit (or 10%); for the higher order multipoles it is even close to the accuracy of the calculation.

Combined function magnets provide a dipole with a quadrupole by adding a tilt angle between the pole shoes.

The end field is now dominated by a spurious sextupole. The idea is to add a sextupole component with a strength similar to the spurious one, but of opposite sign. Similar to the combined function this should be achievable adding a sextupole profile to the magnet lamella used in the ends.

**FEM End model**

- Rogowsky profile
- modified with an additional sextupole
- no lamella change
End profile: parabola

Curves: parabolae with different stepness

End profile: parabole

Curves: parabolae with different curvature The parabola here is only a modification of the Rogowsky profile in the end.

Parabola on Lamella

- parabola:
  - on top of the Rogowsky profile → limited size
• → change the 2D lamella in the ends

• total harmonic content change → compared afterwards

**Insert: adding a sextupole**

Change of the flat part of the lamella so that the sextupole is (hopefully) compensated. The solid line indicates the original lamella, while the profile change is indicated by a dashed line or dashed dotted line. The red ellipse is just an visual aid. It is desirable, that the area, available for the vacuum chamber is not reduced (thus the minimum gap height must be 68 mm). This profile needs to be continuous adjacent to the lamella found in the two 2D part of the magnet.
Insert: adding a sextupole: 3D profile

Change of the flat part of the lamella so that the sextupole is compensated at the magnet end. The shape of the lamella must have a continuous shape matching the shape of the dipole lamella. The adjustment for the Rogowsky profile is not given here.

The following plots show the $c_n(s)$

End profile: extended parabola

$I / II$

\begin{align*}
\begin{array}{c}
\text{End profile: extended parabola} \\
\includegraphics[width=\textwidth]{end_profile.png}
\end{array}
\end{align*}
End profile: extended parabola

Curves: parabola with different curvature

Parabola: Integrated harmonics

original data...blue circles, parabola on Rogovsky profile: V1...green diamonds, V2...red diamonds, V3...magenta diamonds, V4...cyan diamonds, 200 mm long parabola...black circles,
original data...blue circles, parabola on Rogovsky profile: V1...green diamonds, V2...red diamonds, V3...magenta diamonds, V4...cyan diamonds, 200 mm long parabola...black circles, These calculations show that paraoblæ of the tested shapes allow reducing the sextupole by up to a factor of 2 at low fields and by a third at high fields.

End field: widening the end aperture
End field: widening the end I / IV

$\mathbf{b}_3$  
original data...blue circles, end widening V1...magenta diamonds end widening V2...red diamonds

End field: widening the end II / IV

$\mathbf{b}_5$  

$\mathbf{b}_7$  
original data...blue circles, end widening V1...magenta diamonds end widening V2...red diamonds  
original data...blue circles. These changes allowed reducing the first allowed multipoles $\mathbf{b}_3, \mathbf{b}_5$. The higher order ones, however, are increased. This again shows that the harmonics to optimise have to be identified.
When the yoke aperture at the end of the magnet is widened, one can even invert the sign of the sextupole. But the higher order multipoles are much more disturbed then before. The green dot shows the first try, while the later ones were done with a much larger effect.

So this approach shows one further path of improvement, but one has to keep an
eye on the higher order multipoles.

End profile: possibilities

![Graph 1](image1)

![Graph 2](image2)

The improvements, to be tested on the insert, allow a reduction of the sextupole to a level similar to the harmonics found in SIS18.

End profile: possibilities

![Graph 3](image3)

![Graph 4](image4)

The original data... blue dots, parabolic insert... cyan diamond, deformed end pole V1... green diamond, deformed end pole V1... red diamond,
Optimising the end profile

1. Prerequisite: identify most severe harmonics

2. Different approaches:
   - superimpose a parabola on top of the Rogowsky profile
   - slightly change the shape of the lamella in the end part
   - widen the yoke aperture in the end

3. Steps to proceed
   - build and measure the magnet
   - insert only at one one → deterioration due to the insert
   - machine the insert and remeasure

4 Conclusion

Conclusion I/II

1. The end field of the SIS18 magnet was mapped and 2D planar circular multipoles were calculated. Theory shows that the curvature produces mainly a “feed up” to the quadrupole field.

2. Harmonics, scaled to the total magnet, were calculated for the SIS18 and SIS100 dipole. The reference radius for both machines was based on the focal length of their aperture ellipse (see section 3.1.1). These harmonics allow comparing the field quality based on their respective aperture site

3. The SIS18 end field produces a total sextupole $b_3$ of 3 – 5 units and a $b_5 \approx 1.5$ units. $R_{Ref} = 65 \, \text{mm}$

4. The SIS100 $b_3$ produces a total sextupole of 9 – 11 units, and a $b_5 \approx 4.5$ units.

Conclusion II/II

5. The effects of design variations (2D, 3D, yoke geometry, coil shape) on the field quality (multipoles) are clearly understood and can be adjusted to meet beam dynamic requirements.
6. The effect of the material and technological uncertainties will be tested and compensated on the CSLD model magnet modifying the insert.

7. Based on these results the final dipole can be optimised according to a clear specification to be defined by the beam dynamics requirements: for injection, on the field ramp and for the nominal extraction field.

8. The field quality could be further increased by widening the 2D aperture. But this is not preferable for the magnet operation: cable (current, temperature) margin, AC losses, cable mechanics. Therefore well justified arguments have to be provided by beam dynamics.

References


